Gravitational wave signatures from “magnetised" supermassive primordial black holes

Theodoros Papanikolaou

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Contents

Magnetic Fields in the Universe

- Magnetic fields (MFs) can play a key role in the process of **particle acceleration** through the intergalactic medium as well as on the **propagation of cosmic rays.**

- They can influence as well the **dynamical evolution of the primordial plasma** in the early Universe.

- Regarding the amplitudes of MFs, in **galactic scales** we observe a MF amplitude ~ 10^{-7}G [J. P. Vallée - 2004] while on **intergalactic scales**, there is strong evidence for a pre-galactic seed MF amplitude ~ 10^{-18}G [Dermer et al. - 2011].

- However, their **dynamical behavior, amplification** and above all, their **generation mechanism** are still not clear.

Primordial magnetic fields from PBH disks

[T. Papanikolaou, K. N. Gourgouliatos, Phys.Rev.D 107 (2023) 10, 103532, arXiv: 2301.10045 [astro-ph.CO]]

Primordial magnetic fields from PBH disks

- Primordial black holes form during the radiation-dominated era between BBN and recombination ⇒ $10^5 M_\odot < M < 10^{17} M_\odot$.

- A disk can easily form due to the vortex-like motion of the primordial plasma between BBN and recombination [Trivedi et al. - 2018].

- In such a physical setup, a seed primordial magnetic field (PMF) *à la Biermann* [Biermann - 1950] can naturally be generated reading as

$$\frac{\partial \vec{B}}{\partial t} = \nabla\left(\vec{u} \times \vec{B}\right) - \frac{ck_B}{e}\frac{\nabla\rho \times \nabla T}{\rho}. \quad (1)$$

- A seed MF is generated if the energy density and temperature gradients are not parallel to each other.

Locally Isothermal Disks

- Biermann battery induced seed MFs requires disk equation of states (EoS) where $\nabla\rho \times \nabla T \neq 0$. Thus, **isothermal or barotropic are ruled out.**

- Thus, a viable choice for the disk EoS without major ad hoc assumptions is the that of **locally isothermal disk** [G. D'Angelo and S. H. Lubow - 2010].

$$p(R,\phi,z) = \rho(R,\phi,z)c_s^2(R), \quad \frac{p}{\rho} = \epsilon^2\frac{GM}{R}, \quad (2)$$

$$\text{with} \quad \rho(R,z) = f(R)\exp\left(\frac{R-\sqrt{R^2+z^2}}{\epsilon^2\sqrt{R^2+z^2}}\right). \quad (3)$$

- Eq. (2) can describe quite well a **gas that radiates energy gained by socks** [S. H. Lubow et al. - 1999], here created by the **turbulent motion of the primordial plasma** between BBN and recombination era [P. Trivedi et al. - 2018].

The seed PMF

- Considering therefore an ideal gas EoS relating p and ρ one gets the temperature profile $T \sim 1/R$. At the end, Eq. (1) can be recast as

$$\frac{\partial \overrightarrow{B}}{\partial t} = -\frac{c\mu m_{\mathrm{e}} GM}{eR^2} \frac{zR}{(R^2+z^2)^{3/2}} \hat{\phi} \,. \quad (4)$$

  One then obtains a **toroidal seed MF** that is antisymmetric with respect to the equatorial plane.

- This **linear growth of** $\overrightarrow{B}$ is expected to saturate when ∇T and $\nabla \rho$ are smoothed out as it can be seen by Eq. (1). This saturation time t_{s} is defined as

$$t_{\mathrm{s}} = \min[t_{\mathrm{dis}}, t_{\mathrm{soun}}], \quad \text{where} \quad t_{\mathrm{dis}} \equiv (T/\nabla T)/u_{\mathrm{th,e}}, \quad t_{\mathrm{sound}} \equiv (\rho/\nabla \rho)/c_{\mathrm{s}} \,. \quad (5)$$

- Regarding now the dynamical time t_{dyn}, it is defined as the time needed to establish the vertical hydrostatic equilibrium, namely $t_{\mathrm{dyn}} \equiv H_{\mathrm{d}}/c_{\mathrm{s}}$. Thus, the duration of the linear growth of $\overrightarrow{B}$ will be

$$\Delta T = t_{\mathrm{s}} - t_{\mathrm{dyn}} \,. \quad (6)$$

The magnetic field amplitude

- Accounting for the mass distribution of PBHs, one gets for the Fourier transform of the $\overrightarrow{B}$ that

$$\mathbf{B}_k = \int_{M_{\min}}^{M_{\max}} \mathrm{d}M \frac{\mathrm{d}n}{\mathrm{d}M} \int \left(\int \mathbf{B}(\mathbf{x}-\mathbf{x}')\mathrm{d}^3\mathbf{x}' \right) e^{i\mathbf{k}\cdot\mathbf{x}} \mathrm{d}^3\mathbf{x} \,. \quad (7)$$

- To estimate the MF intensity, one should derive the MF power spectrum defined as $P_B \equiv \langle B_k B_k^* \rangle / V_k$, with $V_k = 4\pi(2\pi/k)^3/3$. At the end, one obtains that

$$\langle B \rangle_{\mathrm{s}} = \sqrt{\frac{k^3 P_B(k,t_{\mathrm{s}})}{2\pi^2}} \,. \quad (8)$$

- At this point, we need to stress that we introduce a coherent/correlation scale r_ξ which is roughly equal to the PBH mean separation scale, i.e

$$r_\xi \sim \bar{r}_{\mathrm{PBH}} = \left(\frac{M_{\mathrm{PBH}}}{\rho_{\mathrm{PBH}}}\right)^{1/3} = \left(\frac{4\gamma\pi\rho_{\mathrm{tot,f}} H_{\mathrm{f}}^{-3}/3}{\Omega_{\mathrm{PBH}}(t)\rho_{\mathrm{tot}}(t)}\right)^{1/3} \simeq 10\mathrm{kpc} \left(\frac{M}{10^{10}M_\odot}\right)^{1/2} \left(\frac{10^{-4}}{\Omega_{\mathrm{PBH,f}}}\right)^{1/3} \left(\frac{1\mathrm{meV}}{T}\right) \propto a \,.$$

- This correlation length can be viewed as a UV cutoff scale, below which the magnetic field will interfere,

$$k \leq k_{\mathrm{UV}} \sim 1/\bar{r}_{\mathrm{PBH}} = 10^{19} \Omega_{\mathrm{PBH,f}}^{1/3} \frac{M_\odot}{M} \left(\frac{a_{\mathrm{f}}}{a}\right) \mathrm{Mpc}^{-1} \,.$$

The magnetic field amplitude

- Assuming monochromatic PBH mass functions and accounting for the effect of cosmic expansion, i.e. $B \sim a^{-2}$, one gets

$$\langle |\mathbf{B}_{\mathbf{k}}| \rangle(z) \simeq 10^{-86} q \Omega_{\mathrm{PBH,f}} \ell_{\mathrm{R}}^2 \left(\frac{M}{M_\odot}\right)^2 \left(\frac{k}{1\mathrm{Mpc}^{-1}}\right)^3 (1+z)^2 \quad (\mathrm{G}), \quad (9)$$

$$\text{with} \quad q = \frac{H_{\mathrm{d}}}{R_{\mathrm{ISCO}}} \leq 1, \quad \text{and} \quad \ell_R = \frac{R_{\mathrm{d}}}{R_{\mathrm{ISCO}}}.$$

- For $z = 30, \quad k = 100\mathrm{Mpc}^{-1} \Rightarrow r = 10\mathrm{kpc}$ and accounting for the fact that $q \leq 1$ and $\Omega_{\mathrm{PBH,f}} < 10^{-9}\sqrt{M/M_\odot}$ (for $\Omega_{\mathrm{PBH,eq}} \leq 1$) one gets

$$B(k = 100\mathrm{Mpc}^{-1}, z = 30) \leq 10^{-30}\mathrm{G} \left(\frac{\ell_{\mathrm{R}}}{10^6}\right)^2 \left(\frac{M}{10^{14} M_\odot}\right)^{5/2}. \quad (10)$$

- For $\ell_R \leq 10^{11}$ [J. C. McKinney et al. - MNRAS (2012)] depending on the accretion rate and $M \geq 10^{10} M_\odot$ one gets a seed MF~$10^{-32} - 10^{-28}\mathrm{G}$ which is the minimum seed MF amplitude so as to generate a MF~$10^{-18}\mathrm{G}$ on intergalactic scales due to turbulent/galactic dynamo and instability processes [T. Vachaspati - 2021].

Magnetically induced gravitational waves (MIGWs) from supermassive PBHs

[T. Papanikolaou, K. N. Gourgouliatos, arXiv: 2306.05473 [astro-ph.CO]]

The magnetic anisotropic stress

- Regarding the stress-energy tensor associated to a magnetic field $\overrightarrow{B}$, this can be recast as:

$$T_{ij}^{(B)} \equiv \frac{1}{4\pi}\left[\frac{B^2 g_{ij}}{2} - B_i B_j\right]. \quad (11)$$

- From (10) one can define an associated anisotropic stress reading as

$$\Pi_{ij} \equiv \left(P_i^l P_j^m - \frac{P_{ij}P^{lm}}{2}\right) T_{lm}, \quad (12)$$

where P_{ij} is a projection operator defined as $P_{ij} \equiv \delta_{ij} - \hat{\mathbf{k}}_i \hat{\mathbf{k}}_j$ and $\hat{\mathbf{k}} = \mathbf{k}/k$.

- At the end, defining $\Pi_B(k,\eta)$ as $\langle \Pi_{ij}(\mathbf{k},\eta)\Pi_{ij}(\mathbf{q},\eta)\rangle \equiv \Pi_B(k,\eta)\delta(\mathbf{k},\mathbf{q})$, $\Pi_B(k,\eta)$ can be related with the magnetic field power spectrum as

$$\Pi_B(k,\eta) = \int \mathrm{d}^3\mathbf{q} P_B(q,\eta) P_B(|\mathbf{q}-\mathbf{k}|,\eta)(1+\gamma^2)(1+\beta^2), \quad \beta = \hat{k}\cdot\hat{p}, \quad \gamma = \hat{k}\cdot\widehat{k-p}. \quad (13)$$

GWs from magnetised PBHs

- Having derived before $\Pi_B(k,\eta)$, one can extract the respective equation of motion for the tensor perturbations reading as [Caprini & Durrer - 2006]

$$h^{s,''}_{\mathbf{k}} + 2\mathscr{H} h^{s,'}_{\mathbf{k}} + k^2 h^{s}_{\mathbf{k}} = \frac{8\pi G}{a^2}\sqrt{\Pi_B(k,\eta)}\,. \quad (14)$$

- Solving the above mentioned equation, we can extract the tensor power spectrum and the GW signal which will read as follows:

$$\mathscr{P}_h(\eta,k) \equiv \frac{k^3\,|\,h_{\mathbf{k}}\,|^2}{2\pi^2}, \quad \Omega_{\rm GW}(\eta,k) = \frac{1}{24}\left[\frac{k}{aH}\right]^2 \overline{\mathscr{P}}_h(\eta,k)\,. \quad (15)$$

- At the end, at leading order in $k/k_{\rm UV} \ll 1$ one gets that

$$\Omega_{\rm GW}(k,\eta_0) \simeq 6\times 10^{-85}\left(\frac{k}{\rm Mpc^{-1}}\right)\left(\frac{10^{10}M_\odot}{M}\right)^4 q^4 \ell_R^8 \Omega^7_{\rm PBH,f} \leq 1.5\times 10^{-13} \quad (16)$$

$$\text{since} \quad \ell_R < 10^{11}, \quad q < 1, \quad M > 10^{10}M_\odot \quad \text{and} \quad \Omega_{\rm PBH,f} < 10^{-4}\left(\frac{M}{10^{10}M_\odot}\right)^{1/2}.$$

GWs from magnetised PBHs

- One should account for **galactic** and **turbulent** dynamo **MF amplification** mechanisms present during LSS formation ⇒ MHD simulations.

- Avoiding MHD simulations, we adopt an effective power-law toy-model for the MF amplification which reads as

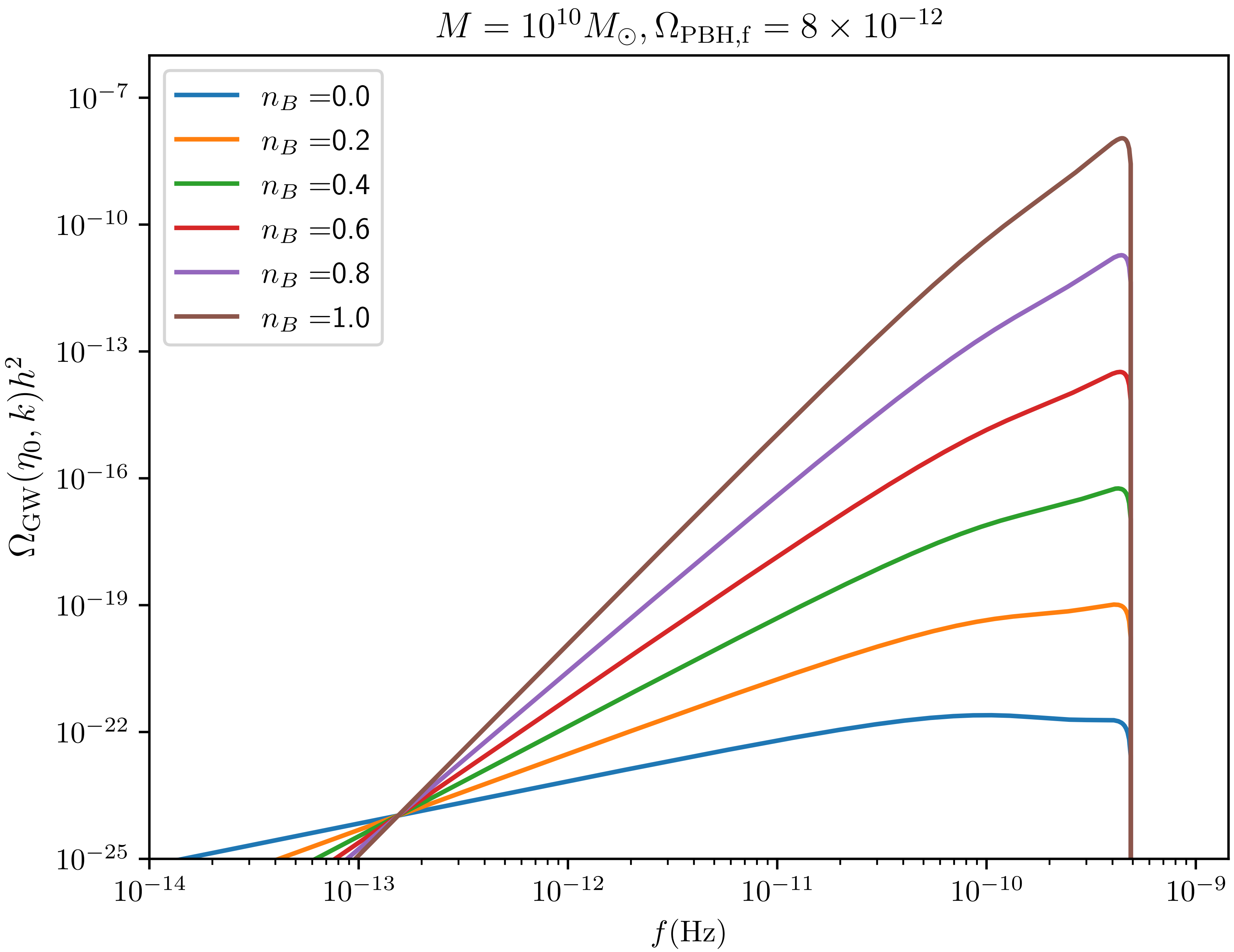
$$\alpha(k) \equiv \frac{B^{\rm ampl.}(k)}{B^{\rm non-ampl.}(k)} = \alpha(k_*)\left(\frac{k}{k_*}\right)^{n_B}, \quad \text{with} \quad k_* = 100\mathrm{Mpc}^{-1}, \quad n_B \geq 0 \quad \text{and}$$

$$\alpha(k_* = 100\mathrm{Mpc}^{-1}) = \frac{10^{-18}}{10^{-30} q \left(\frac{\ell_{\rm R}}{10^6}\right)^2 \left(\frac{M_{\rm PBH}}{10^{14} M_\odot}\right)^{5/2}}\,. \quad (17)$$

- Since $P_B(k) \propto B_k^2$ and $\Omega_{\rm GW} \propto \int\int P_B^2$ one gets that $\Omega_{\rm GW}^{\rm ampl.} \propto \alpha^4(k)\Omega_{\rm GW}^{\rm non-ampl.}$. At the end one gets that

$$\Omega_{\rm GW}(k,\eta_0) \simeq 6 \times 10^{51-8n_B}\left(\frac{k}{\mathrm{Mpc}^{-1}}\right)^{4n_B+1}\left(\frac{10^{10}M_\odot}{M}\right)^{14}\Omega^7_{\rm PBH,f}\,. \quad (18)$$

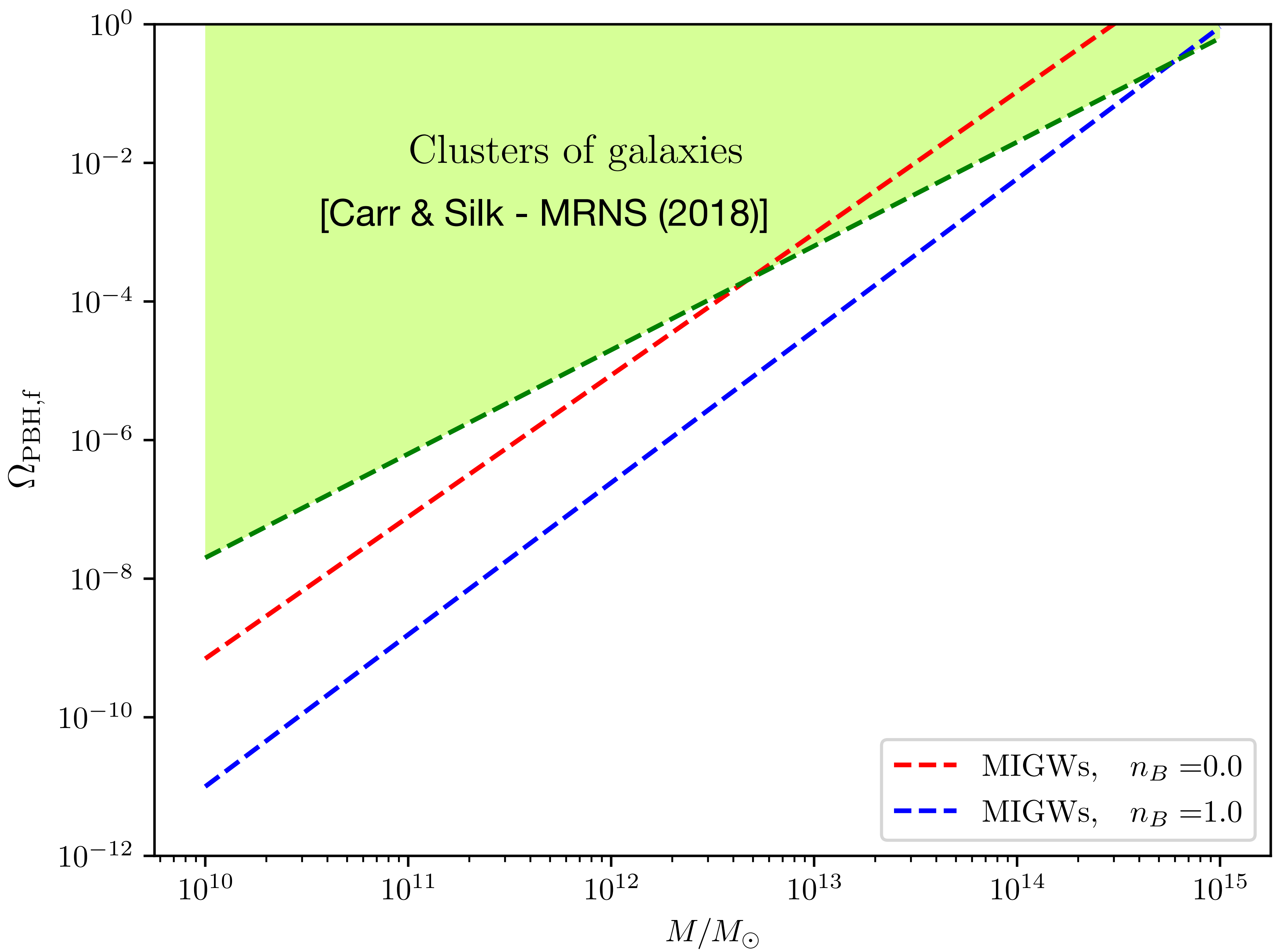
GWs from magnetised PBHs



$$f = \frac{k}{2\pi} < \frac{k_{UV}}{2\pi} = 10^5 \frac{M_\odot}{M} \Omega_{\rm PBH,f}^{1/3} = 5 \times 10^{-11} \left(\frac{10^{10} M_\odot}{M} \right)^{5/6} \leq 5 \times 10^{-7} (\rm Hz)$$

Constraints on $\Omega_{\mathrm{PBH,f}}$

$$\Delta N_{\mathrm{eff}} < 0.3 \quad \text{[Planck 2015]} \Rightarrow \Omega_{\mathrm{GW}}(\eta_0) < 10^{-6} \quad \longrightarrow \quad \Omega_{\mathrm{PBH,f}} \leq 10^{-\frac{3(67+28n_B)}{22+4n_B}} \left(\frac{M}{10^{10} M_\odot}\right)^{\frac{3(15+4n_B)}{22+4n_B}}$$



Conclusions

- **Primordial magnetic fields can naturally arise** à la Biermann **from accretion disks around supermassive PBHs** with masses $M > 10^{10}M_{\odot}$.

- A population of magnetised PBHs can induce a **stochastic GW background at low frequencies** $f_{\rm GW} < 5\times 10^{-7}{\rm Hz}$ and with $\Omega_{\rm GW} < 1.5\times 10^{-13}$.

- Accounting for the galactic/turbulent dynamo MF amplification mechanisms through an effective model, we set **conservative constraints on** $\Omega_{\rm PBH,f}$ **being tighter compared to that coming from LSS probes.**

- One needs to perform **MHD simulations** in order to have an **accurate answer** regarding the **MF amplification** and the effect on the **GW signal.**

- **The formalism developed here for the derivation of the MIGWs is quite generic** and can be applied to any population of "magnetised" PBHs, e.g. **PBHs with magnetic charge** [Maldacena - 2021] or **Kerr-Newmann PBHs** [Hooper et al. - 2023, See Krnjaic’s Talk], promoting thus the portal of MIGWs to a **new GW counterpart associated to PBHs, potentially detectable by future GW detectors.**

Thank you for your attention and your time!