

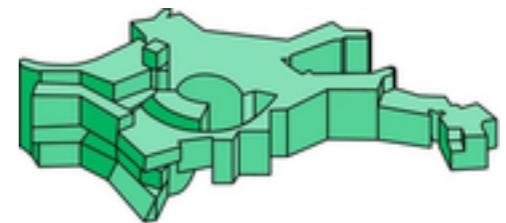
Lattice simulations of axion inflation

Angelo Caravano LMU & MPA @ Munich, Germany

Based on:

A. Caravano, E. Komatsu, K. D. Lozanov, J. Weller arXiv:2204.12874

A. Caravano, M. Peloso arXiv:2307.????? (in prep.)



Max-Planck-Institut
für Astrophysik

@ NEHOP 2023

Axion-U(1) inflation

Interaction between the inflation and a gauge field

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\mathcal{L} \supset \phi F_{\mu\nu} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

Axion-U(1) inflation

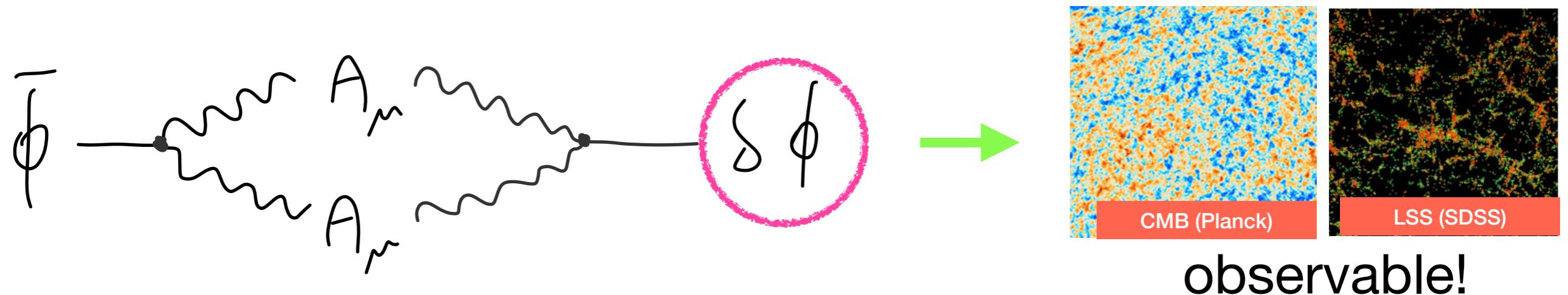
Interaction between the inflation and a gauge field

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\mathcal{L} \supset \phi F_{\mu\nu} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

Observational consequences:

Production of gauge field particles \rightarrow decay into inflaton perturbations



Axion-U(1) inflation

[N. Barnaby, M. Peloso 1011.1500]
[M. Anber, L. Sorbo 0908.4089]

Known results

- Power spectrum:

$$\mathcal{P}_\zeta(k) \simeq \mathcal{P}_{\text{vac}} + \mathcal{P}_{\text{vac}}^2 f_2(\xi) e^{4\pi\xi}$$

The equation is enclosed in a red box. Below the box, two red arrows point to the terms: one points to \mathcal{P}_{vac} with the label "vacuum (single-field)", and another points to $f_2(\xi) e^{4\pi\xi}$ with the label "sourced".

$$\mathcal{P}_{\text{vac}} \simeq \frac{H^4}{(2\pi\dot{\phi})^2}$$
$$\xi = \frac{\alpha\dot{\phi}}{2fH}$$

Axion-U(1) inflation

[N. Barnaby, M. Peloso 1011.1500]
[M. Anber, L. Sorbo 0908.4089]

Known results

- Power spectrum:

$$\mathcal{P}_\zeta(k) \simeq \mathcal{P}_{\text{vac}} + \mathcal{P}_{\text{vac}}^2 f_2(\xi) e^{4\pi\xi}$$

$$\mathcal{P}_{\text{vac}} \simeq \frac{H^4}{(2\pi\dot{\phi})^2}$$

- **Bispectrum:**

$$f_{\text{NL}}^{(\text{equil.})}(\xi) \simeq \frac{f_3(\xi) \mathcal{P}_{\text{vac}}^3 e^{6\pi\xi}}{\mathcal{P}_\xi^2}$$

Assuming constant ξ

Axion-U(1) inflation

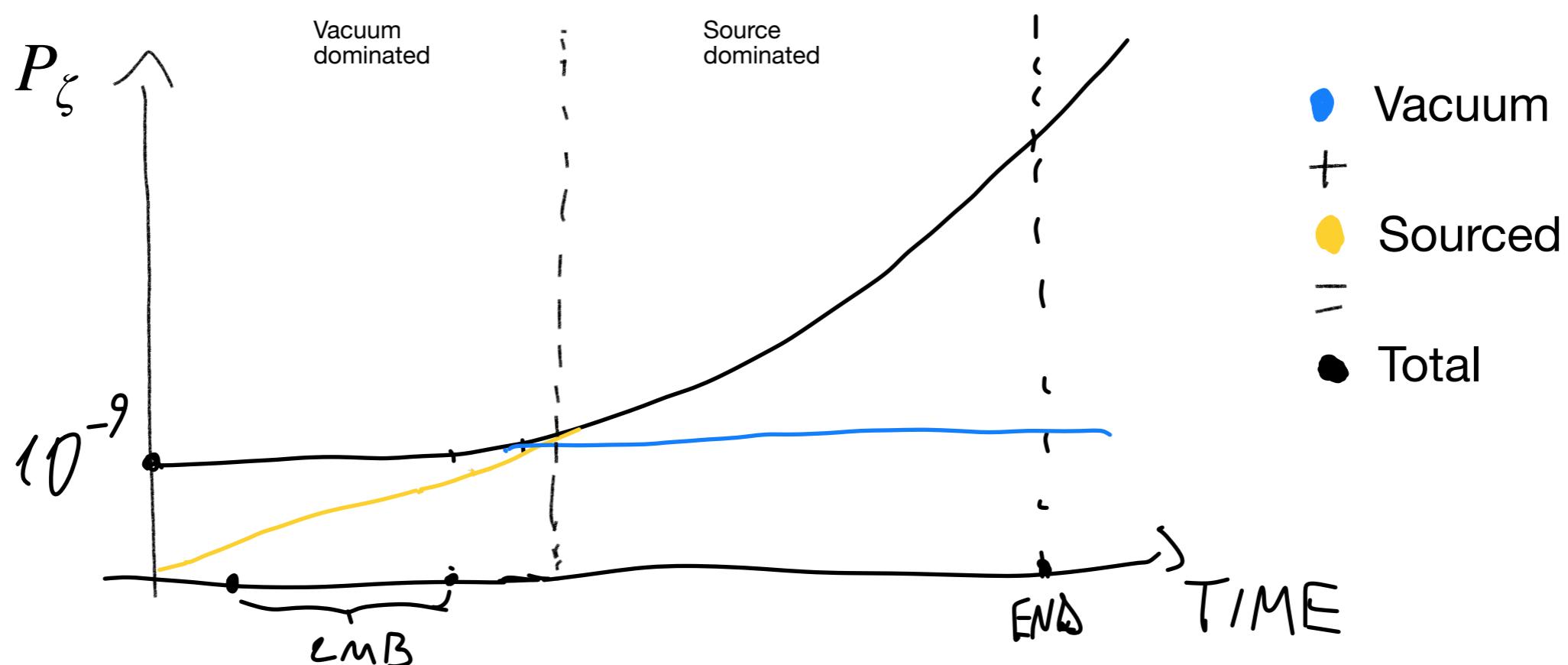
N. Barnaby, M. Peloso 1011.1500
M. Anber, L. Sorbo 0908.4089

Known results:

- $\mathcal{P}_\zeta(k) \simeq \underline{\mathcal{P}_{\text{vac}}} + \underline{\mathcal{P}_{\text{vac}}^2 f_2(\xi)} e^{4\pi\xi}$

$$\xi = \frac{\alpha \dot{\phi}}{2fH}$$

Scalar perturbations naturally grow on small scales



Axion-U(1) inflation

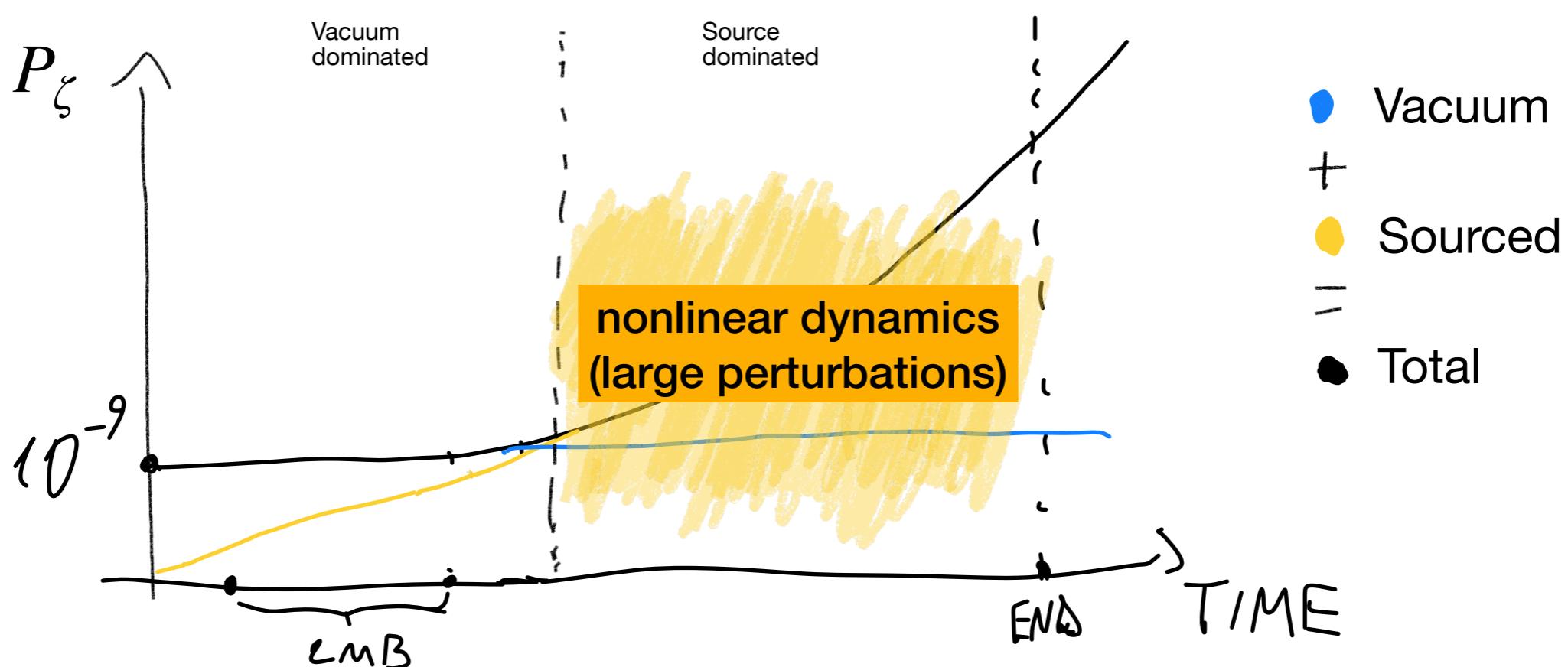
N. Barnaby, M. Peloso 1011.1500
M. Anber, L. Sorbo 0908.4089

Known results:

- $\mathcal{P}_\zeta(k) \simeq \underline{\mathcal{P}_{\text{vac}}} + \underline{\mathcal{P}_{\text{vac}}^2 f_2(\xi)} e^{4\pi\xi}$

$$\xi = \frac{\alpha \dot{\phi}}{2fH}$$

Scalar perturbations naturally grow on small scales



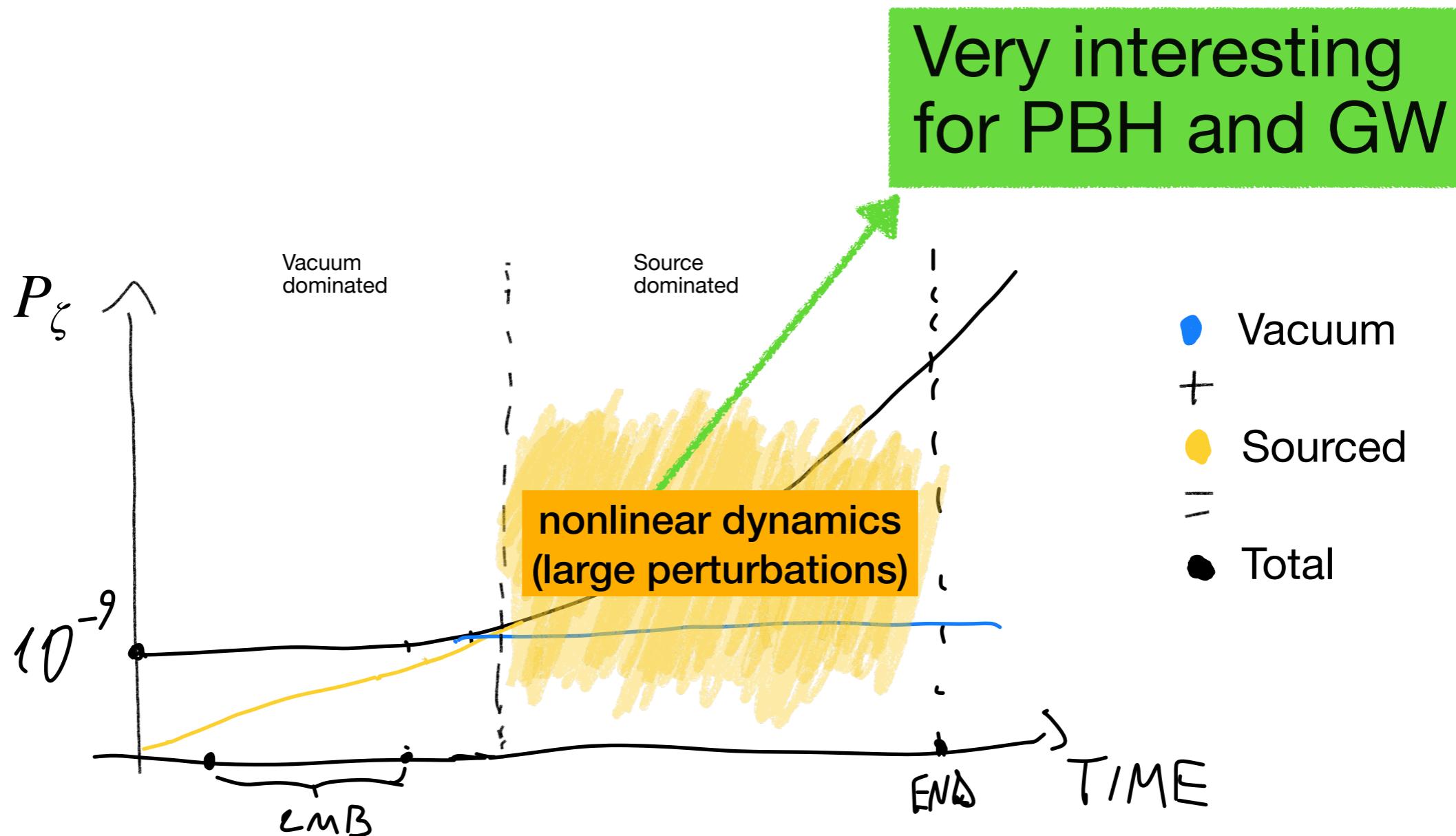
Axion-U(1) inflation

N. Barnaby, M. Peloso 1011.1500
M. Anber, L. Sorbo 0908.4089

Known results:

- $\mathcal{P}_\zeta(k) \simeq \underline{\mathcal{P}_{\text{vac}}} + \underline{\mathcal{P}_{\text{vac}}^2 f_2(\xi)} e^{4\pi\xi}$

$$\xi = \frac{\alpha \dot{\phi}}{2fH}$$



Axion-U(1) inflation

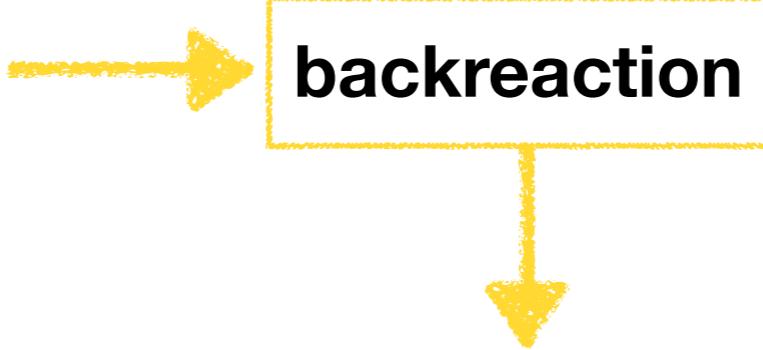
N. Barnaby, M. Peloso 1011.1500

M. Anber, L. Sorbo 0908.4089

More quantitatively:

$$\partial_\tau^2 \bar{\phi} + 2\mathcal{H}\partial_\tau \bar{\phi} + a^2 V'(\bar{\phi}) = a^2 \frac{a}{f} \langle F_{\mu\nu} \tilde{F}^{\mu\nu} \rangle$$

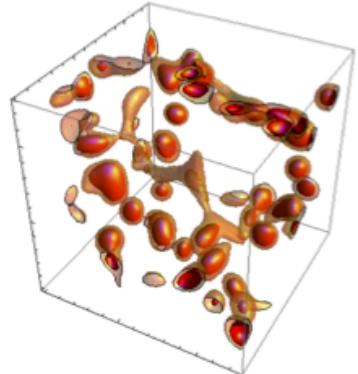
If these terms become comparable



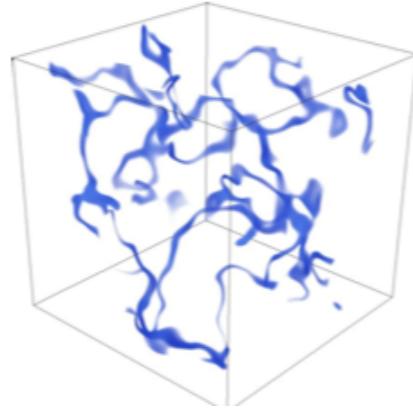
Some sort of **extra friction**,
but not so simple (as we
will see)

Lattice simulations

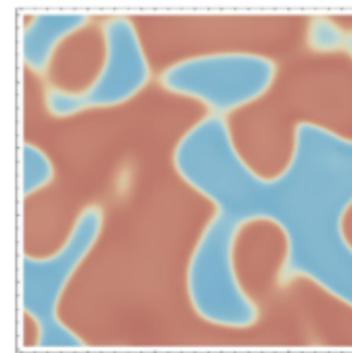
- Numerical tool to study **non-linear** cosmological phenomena.
- Typically associated with the **reheating phase** after inflation.



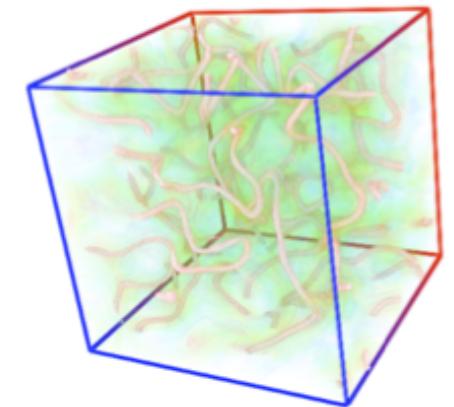
[M. A. Amin, R. Easther, H. Finkel, arXiv:1009.2505]



[A. V. Frolov, arXiv:1004.3559]



[M. A. Amin, J. Fan, K. D. Lozanov, M. Reece, arXiv:1802.00444]

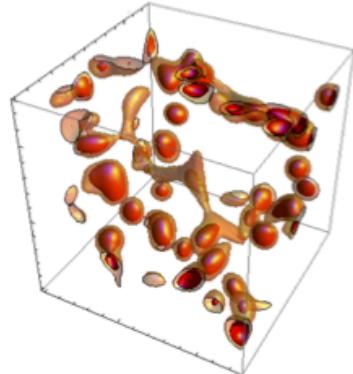


[J. Dufaux, D.G. Figueroa, J. Garcia-Bellido, arXiv:1006.0217]

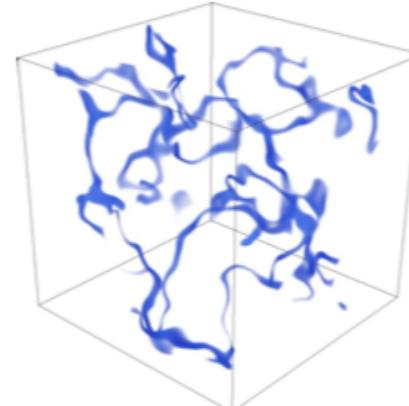
Lattice simulations

[A. Caravano 2209.13616 (PhD thesis)]

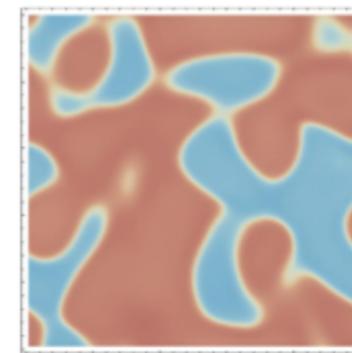
- Numerical tool to study **non-linear** cosmological phenomena.
- Typically associated with the **reheating phase** after inflation.



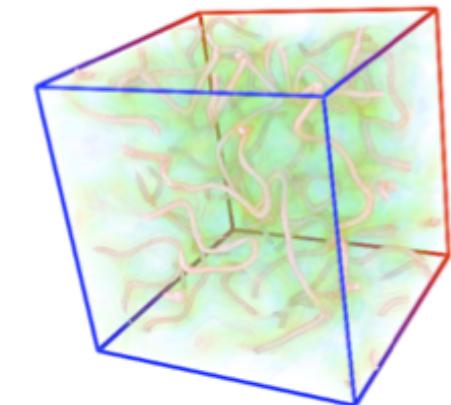
[M. A. Amin, R. Easther, H. Finkel, arXiv:1009.2505]



[A. V. Frolov, arXiv:1004.3559]



[M. A. Amin, J. Fan, K. D. Lozanov, M. Reece, arXiv:1802.00444]



[J. Dufaux, D.G. Figueroa, J. Garcia-Bellido, arXiv:1006.0217]

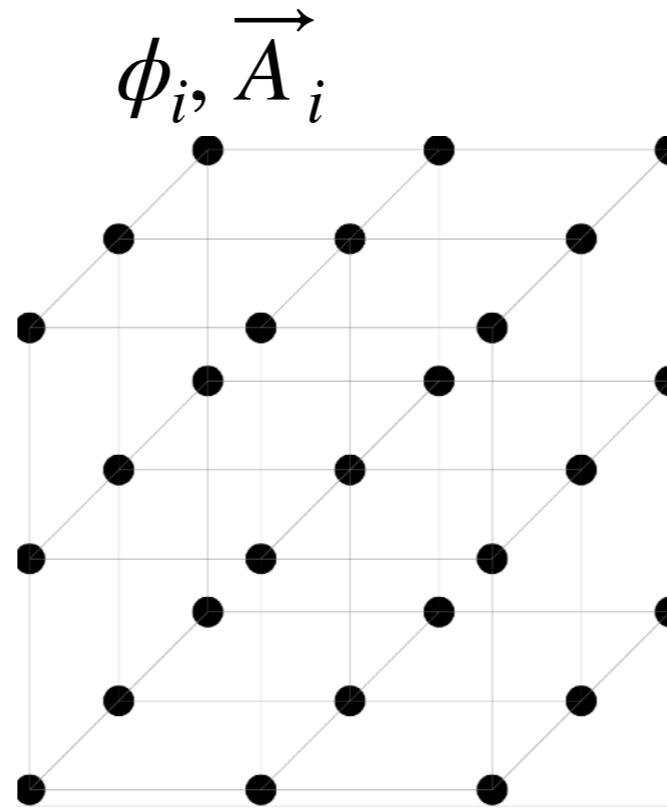
Our goal:

Develop lattice techniques for inflation

A. Caravano, E. Komatsu, K. D. Lozanov, J. Weller arXiv:2102.06378
arXiv:2110.10695
arXiv:2204.12874

In this talk: **focus on axion-U(1) model.**

Lattice approach



Solve numerically for all lattice points:

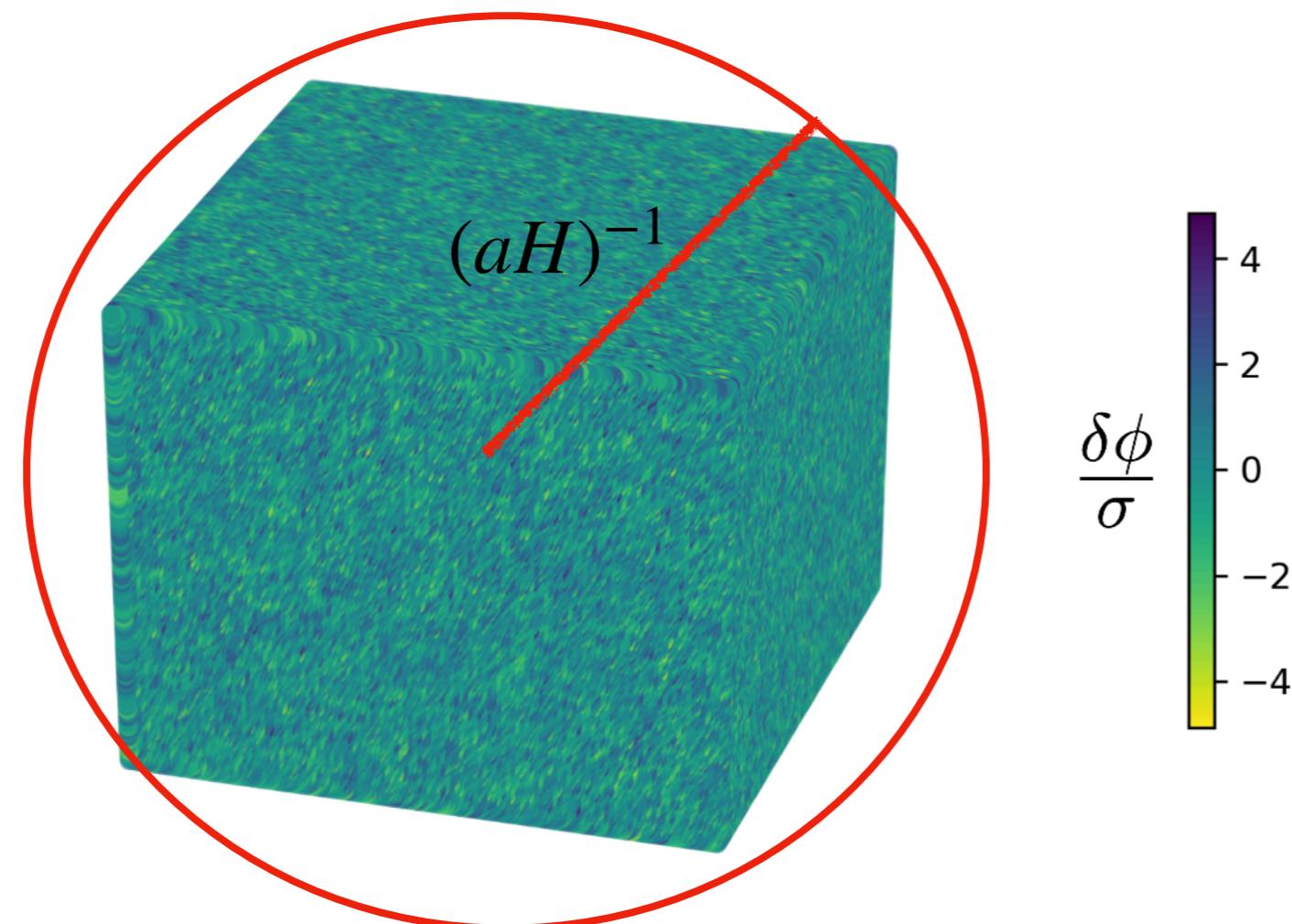
$$\phi'' + 2H\phi' - \partial_j \partial_j \phi + a^2 \frac{\partial V}{\partial \phi} = -a^2 \frac{\alpha}{4f} F_{\mu\nu} \tilde{F}^{\mu\nu},$$

$$A_0'' - \partial_j \partial_j A_0 = \frac{\alpha}{f} \epsilon_{ijk} \partial_k \phi \partial_i A_j,$$

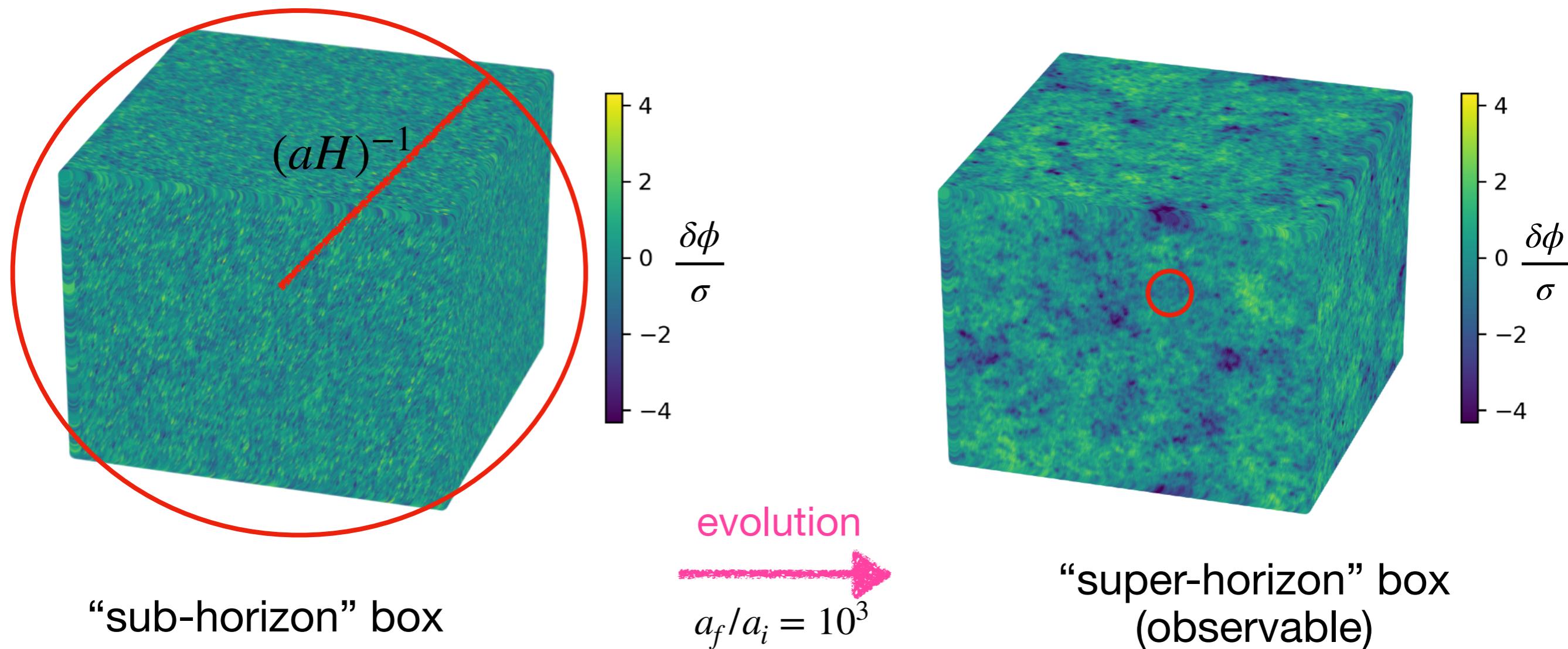
$$A_i'' - \partial_j \partial_j A_i = \frac{\alpha}{f} \epsilon_{ijk} \phi' \partial_j A_k - \frac{\alpha}{f} \epsilon_{ijk} \partial_j \phi (A'_k - \partial_k A_0)$$

Lattice approach

Start with a sub-horizon box



Lattice approach



Results of the simulation:

1. Large scales

$$\text{small } \xi = \frac{\alpha \dot{\phi}}{2fH}$$

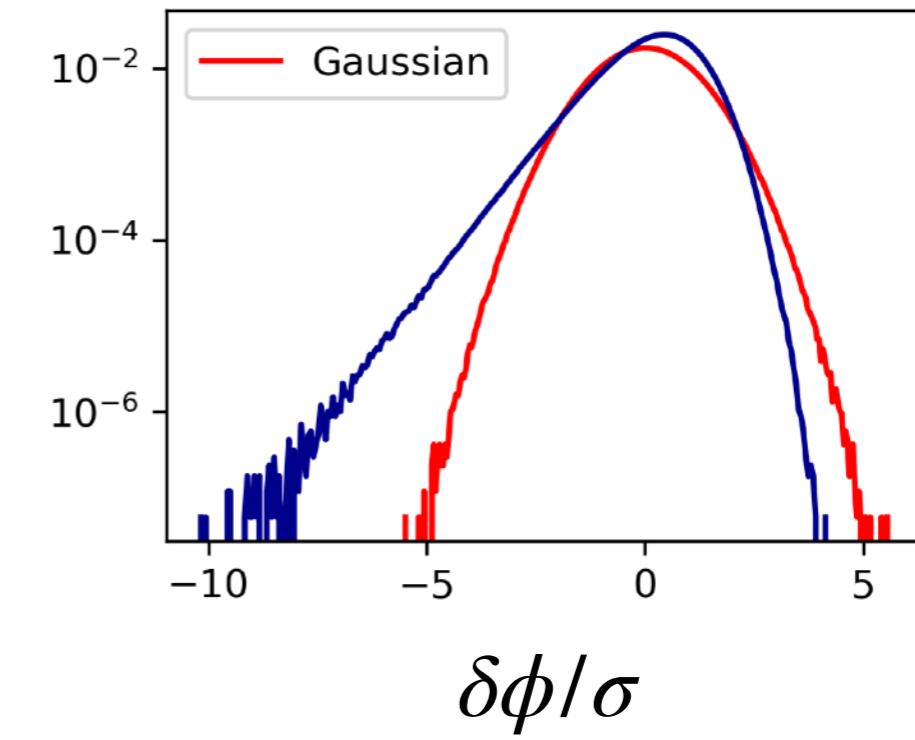
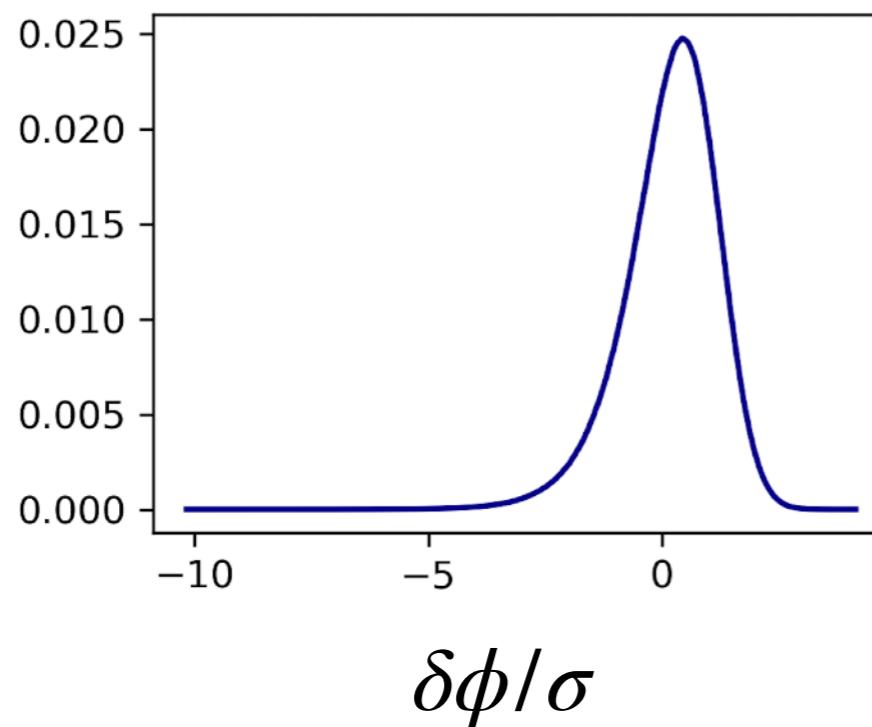
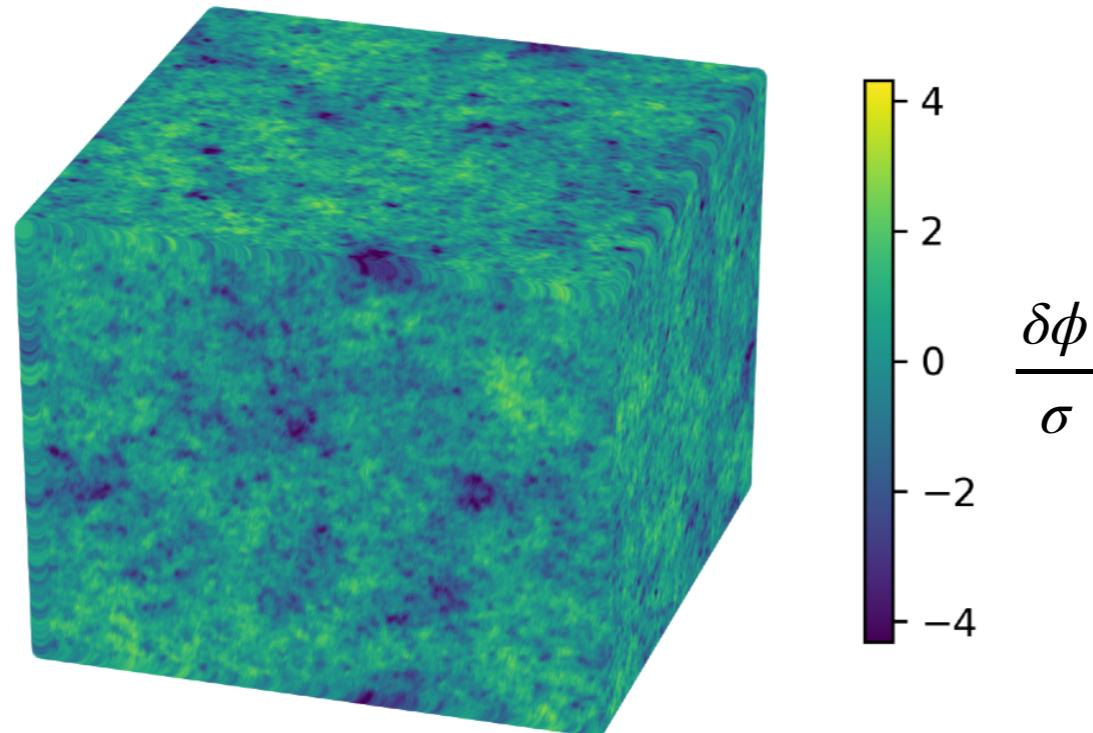
2. Small scales

$$\text{large } \xi = \frac{\alpha \dot{\phi}}{2fH}$$

Linear regime (large scales)

A. Caravano et al. arXiv:2204.12874

Thanks to the lattice,
we know the full $\delta\phi(\mathbf{x})$ in real space!



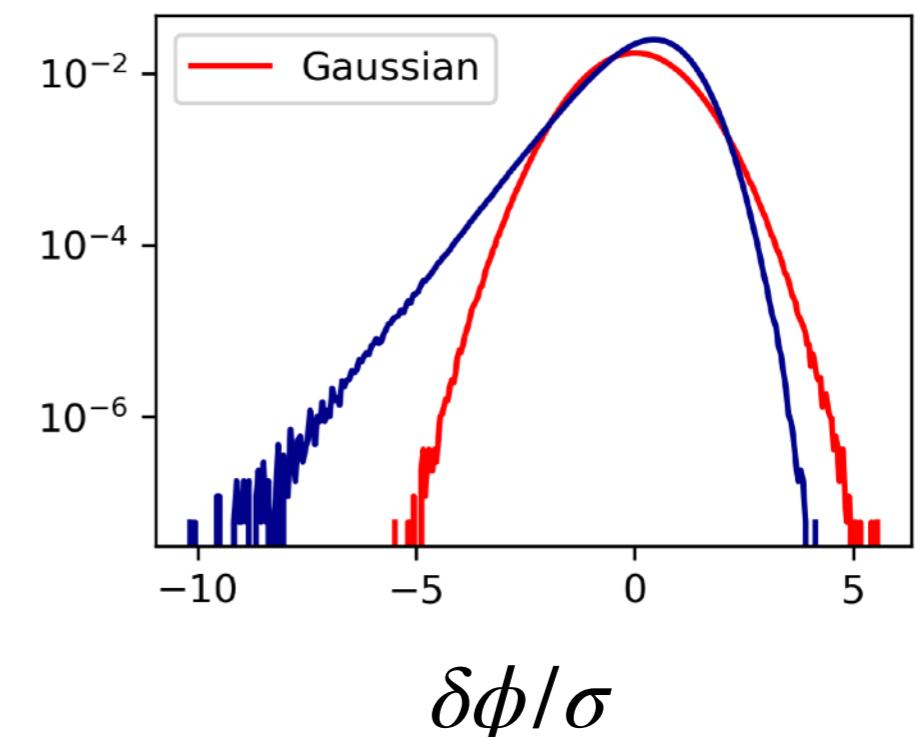
Linear regime (large scales)

A. Caravano et al. arXiv:2204.12874

Define cumulants:

$$\kappa_n = \frac{\langle \delta\phi^n \rangle_c}{\sigma^n}$$

κ_3 “skewness”, κ_4 “kurtosis”, etc.



Linear regime (large scales)

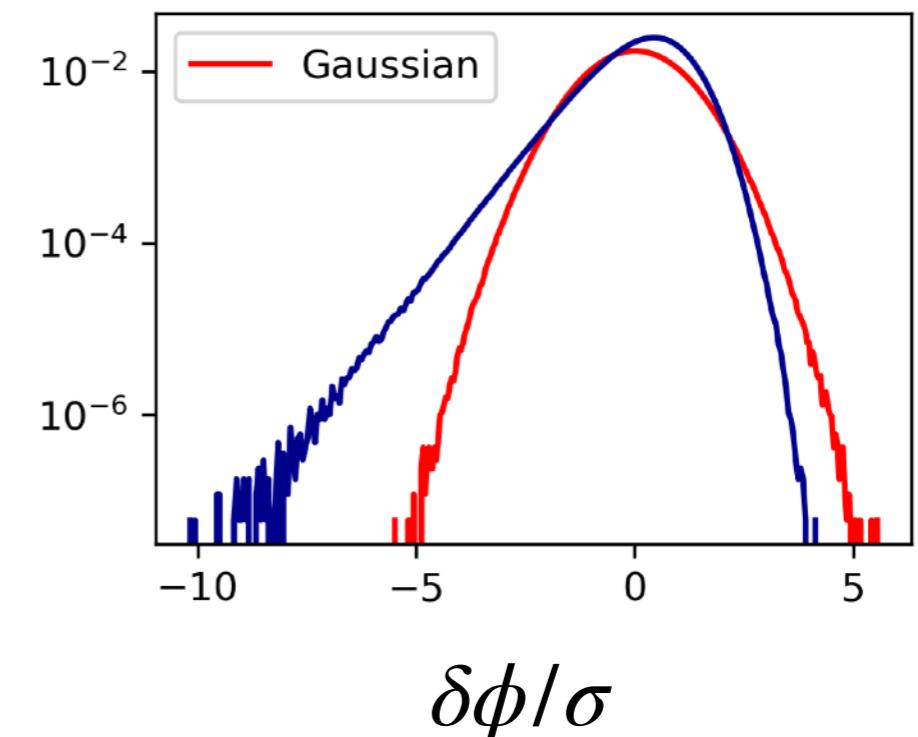
A. Caravano et al. arXiv:2204.12874

Define cumulants:

$$\kappa_n = \frac{\langle \delta\phi^n \rangle_c}{\sigma^n}$$

κ_3 “skewness”, κ_4 “kurtosis”, etc.

$$\dots > \kappa_6 > \kappa_5 > \kappa_4 > \kappa_3$$



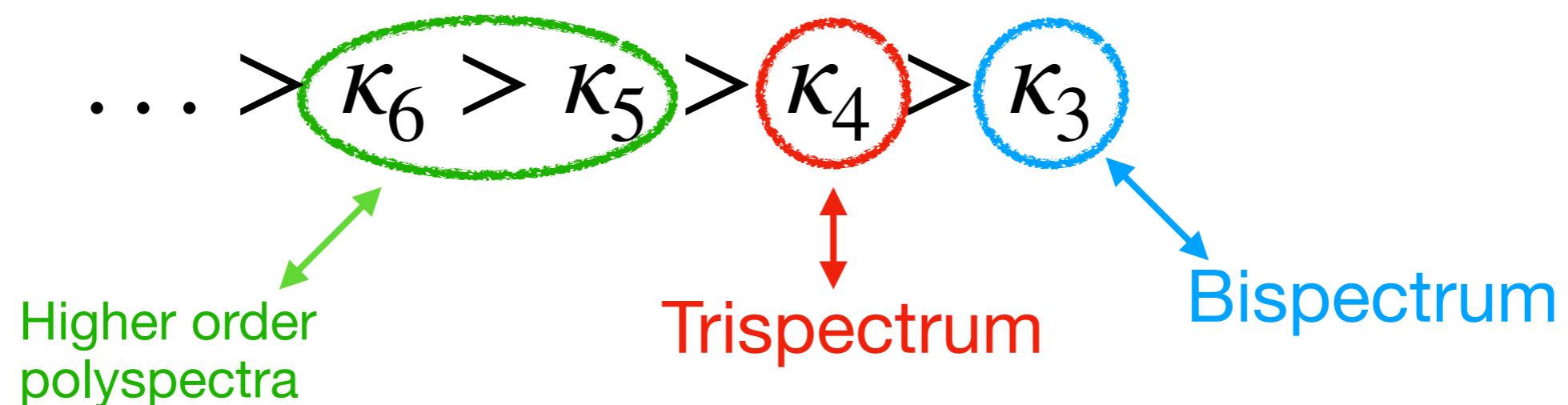
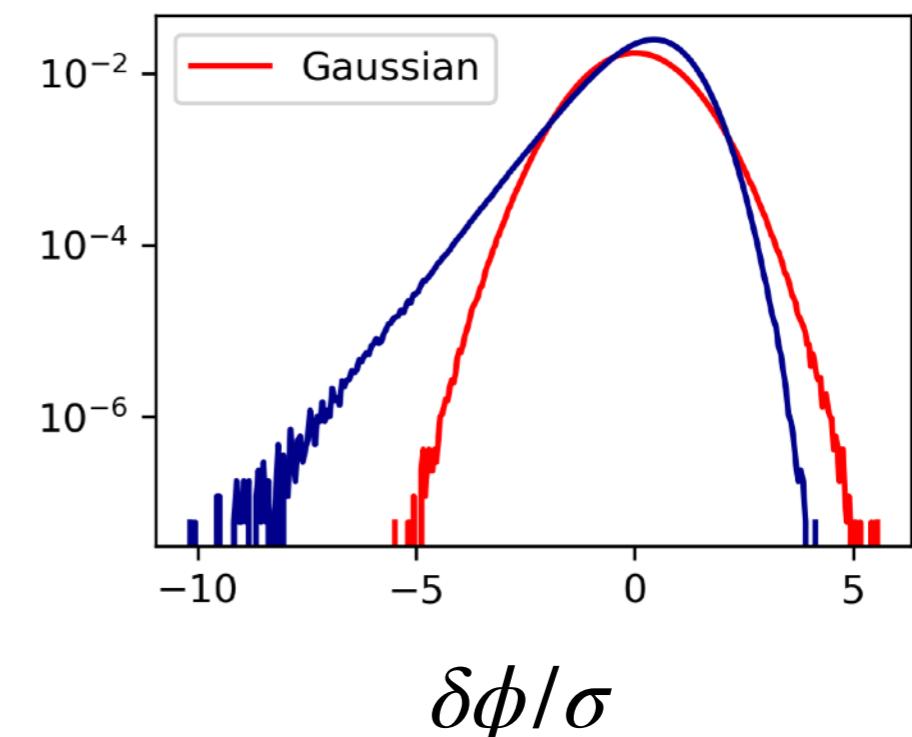
Linear regime (large scales)

A. Caravano et al. arXiv:2204.12874

Define cumulants:

$$\kappa_n = \frac{\langle \delta\phi^n \rangle_c}{\sigma^n}$$

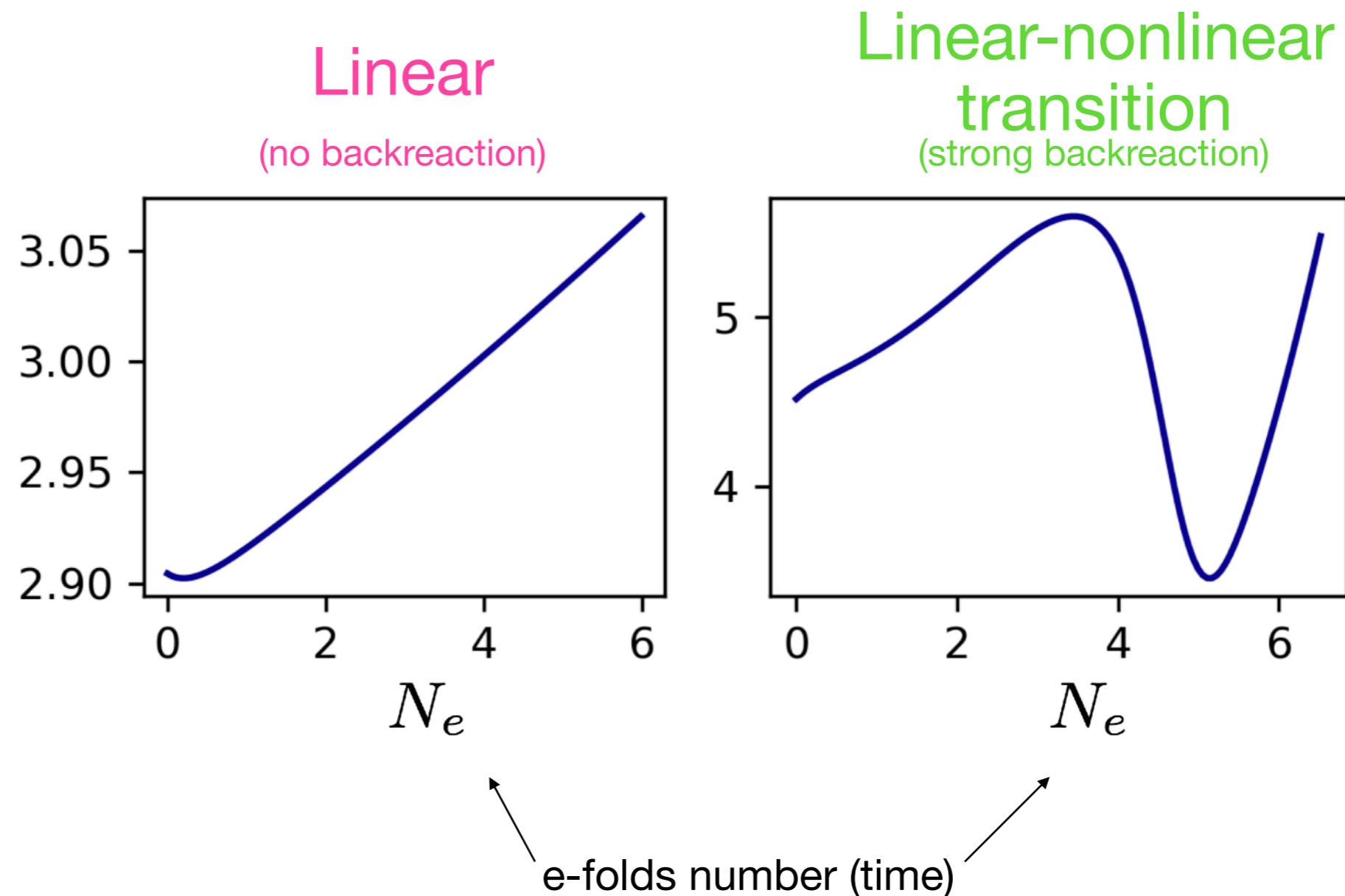
κ_3 “skewness”, κ_4 “kurtosis”, etc.



Strong backreaction (small scales)

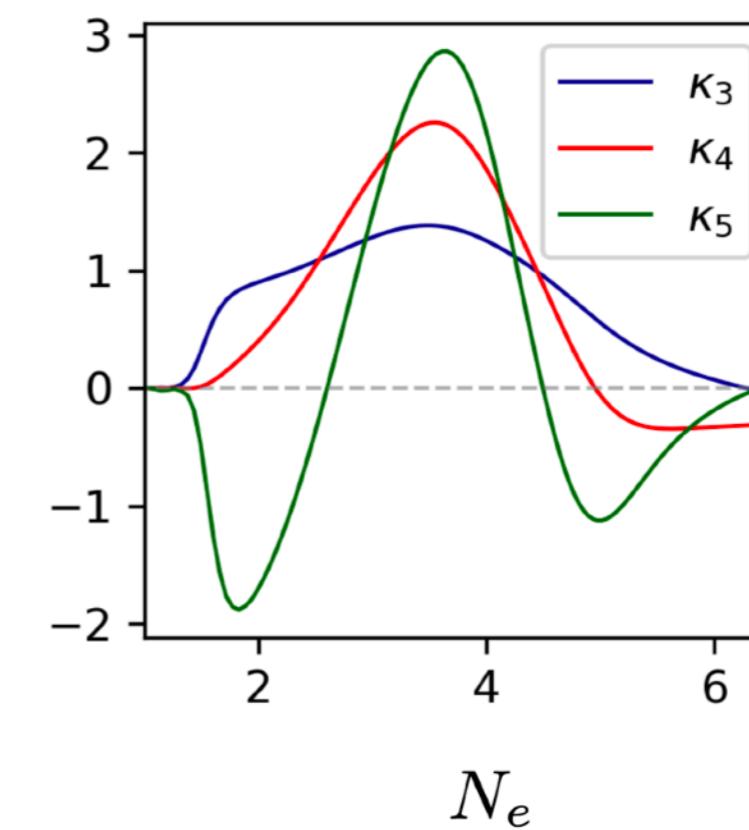
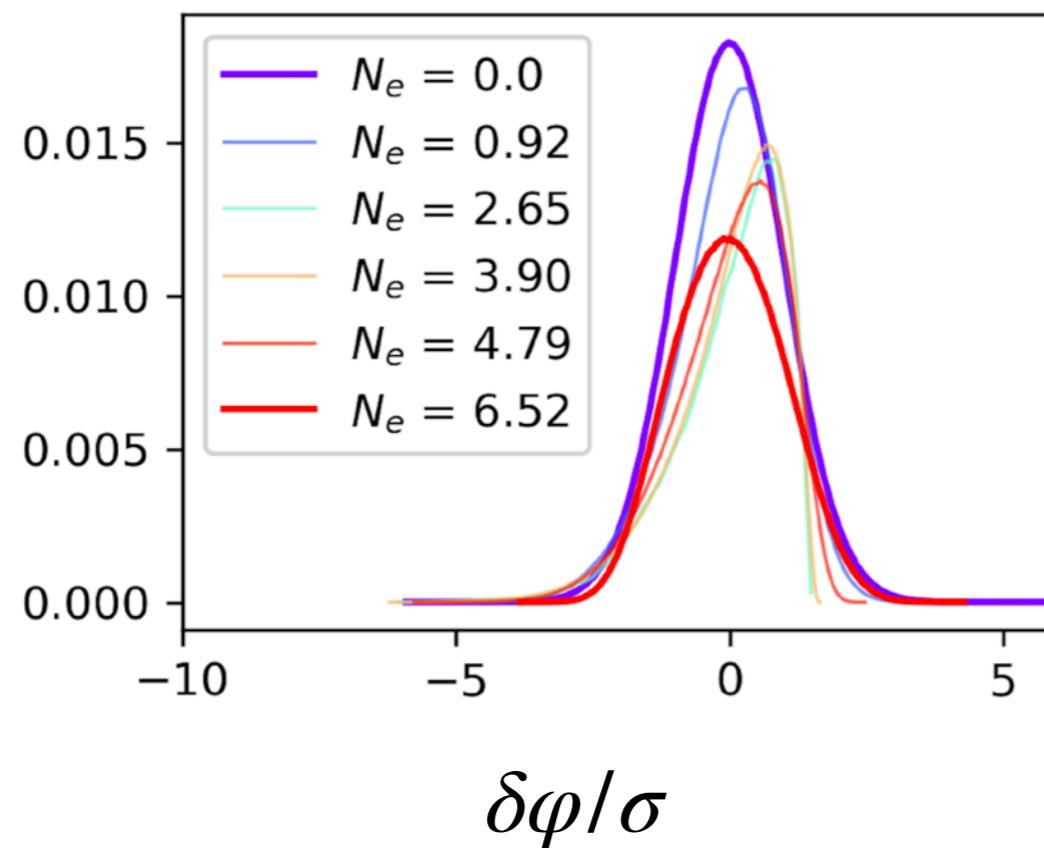
Study transition linear \longrightarrow nonlinear

$$\xi = \frac{\alpha \dot{\phi}}{2fH}$$



Strong backreaction (small scales)

Non-Gaussianity is suppressed
in the nonlinear regime!



Why? Central limit theorem

Suppression of non-Gaussianity

A. Linde, S. Mooij, E. Pajer,
arXiv:1212.1693

J. Garcia-Bellido, M. Peloso,
C. Unal, arXiv:1610.03763

etc...

Before our study, it was believed that:

Large ξ \longrightarrow large non-Gaussianity

Suppression of non-Gaussianity

A. Linde, S. Mooij, E. Pajer,
arXiv:1212.1693

J. Garcia-Bellido, M. Peloso,
C. Unal, arXiv:1610.03763

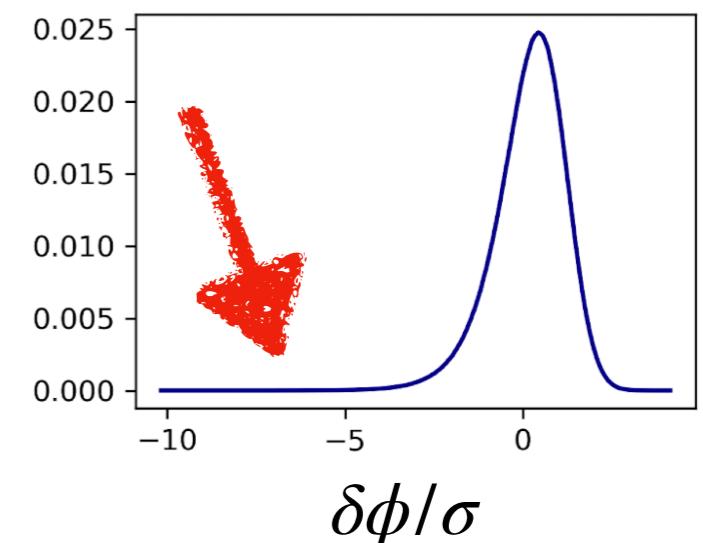
etc...

Before our study, it was believed that:

Large ξ \longrightarrow large non-Gaussianity



Very efficient production of Primordial BH



Suppression of non-Gaussianity

A. Linde, S. Mooij, E. Pajer,
arXiv:1212.1693

J. Garcia-Bellido, M. Peloso,
C. Unal, arXiv:1610.03763

etc...

Before our study, it was believed that:

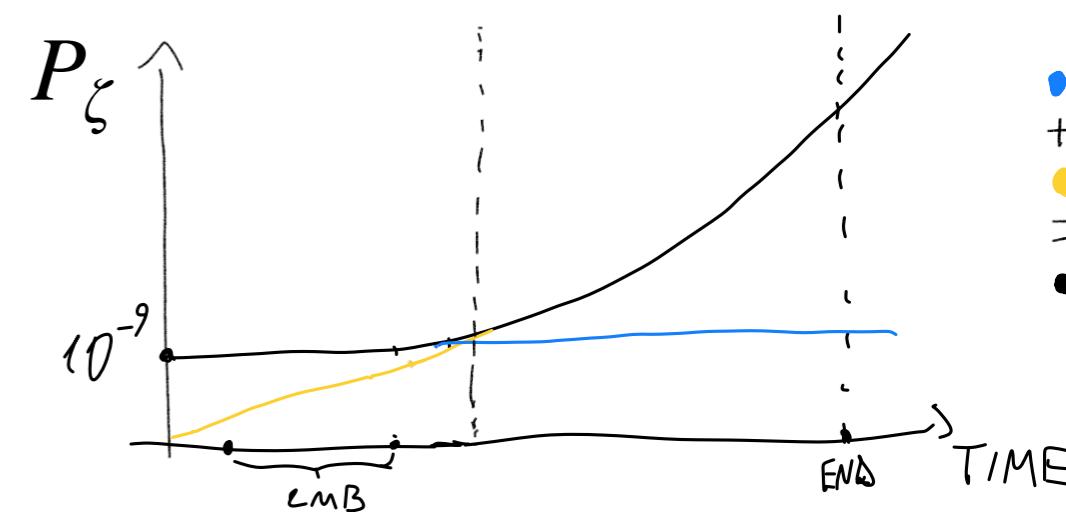
Large ξ \longrightarrow large non-Gaussianity



Very efficient production of Primordial BH



ξ has to remain small at all times



Suppression of non-Gaussianity

A. Linde, S. Mooij, E. Pajer,
arXiv:1212.1693

J. Garcia-Bellido, M. Peloso,
C. Unal, arXiv:1610.03763

etc...

Before our study, it was believed that:

Large ξ \longrightarrow large non-Gaussianity



Very efficient production of Primordial BH



ξ has to remain small at all times



No effects at “larger” scales (i.e. CMB, GW interferometers)

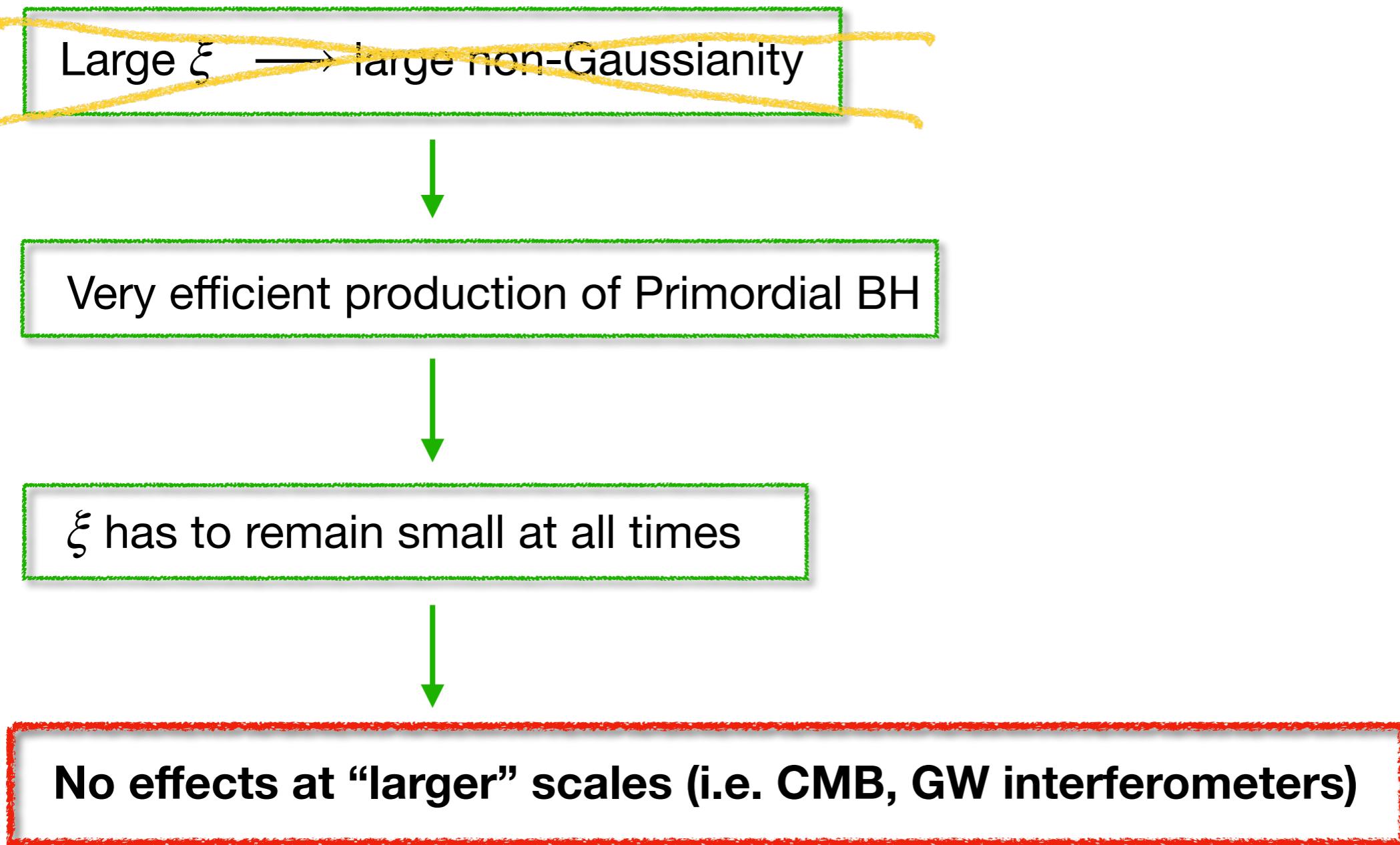
Suppression of non-Gaussianity

A. Linde, S. Mooij, E. Pajer,
arXiv:1212.1693

J. Garcia-Bellido, M. Peloso,
C. Unal, arXiv:1610.03763

etc...

Before our study, it was believed that:



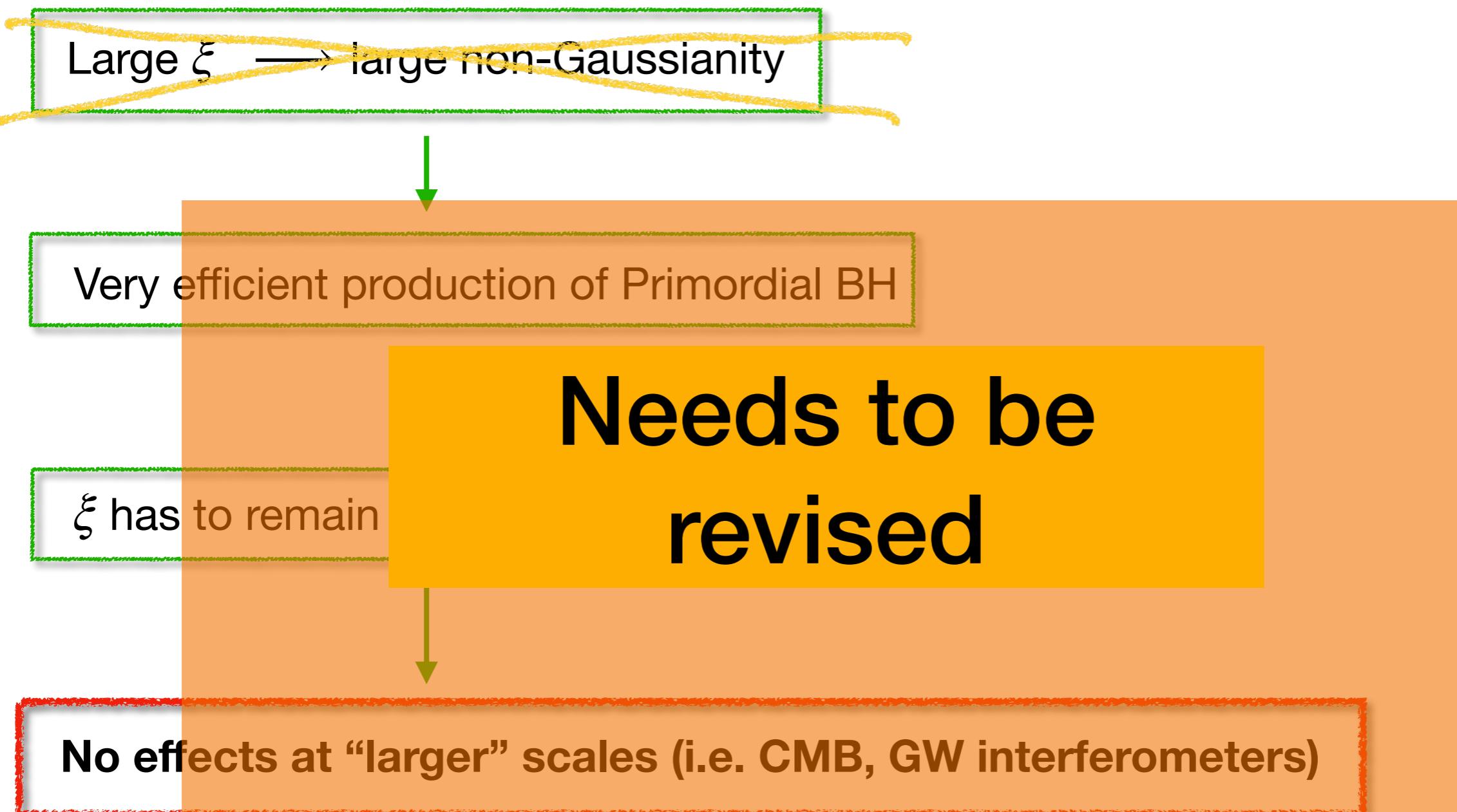
Suppression of non-Gaussianity

A. Linde, S. Mooij, E. Pajer,
arXiv:1212.1693

J. Garcia-Bellido, M. Peloso,
C. Unal, arXiv:1610.03763

etc...

Before our study, it was believed that:



Strong backreaction (small scales)

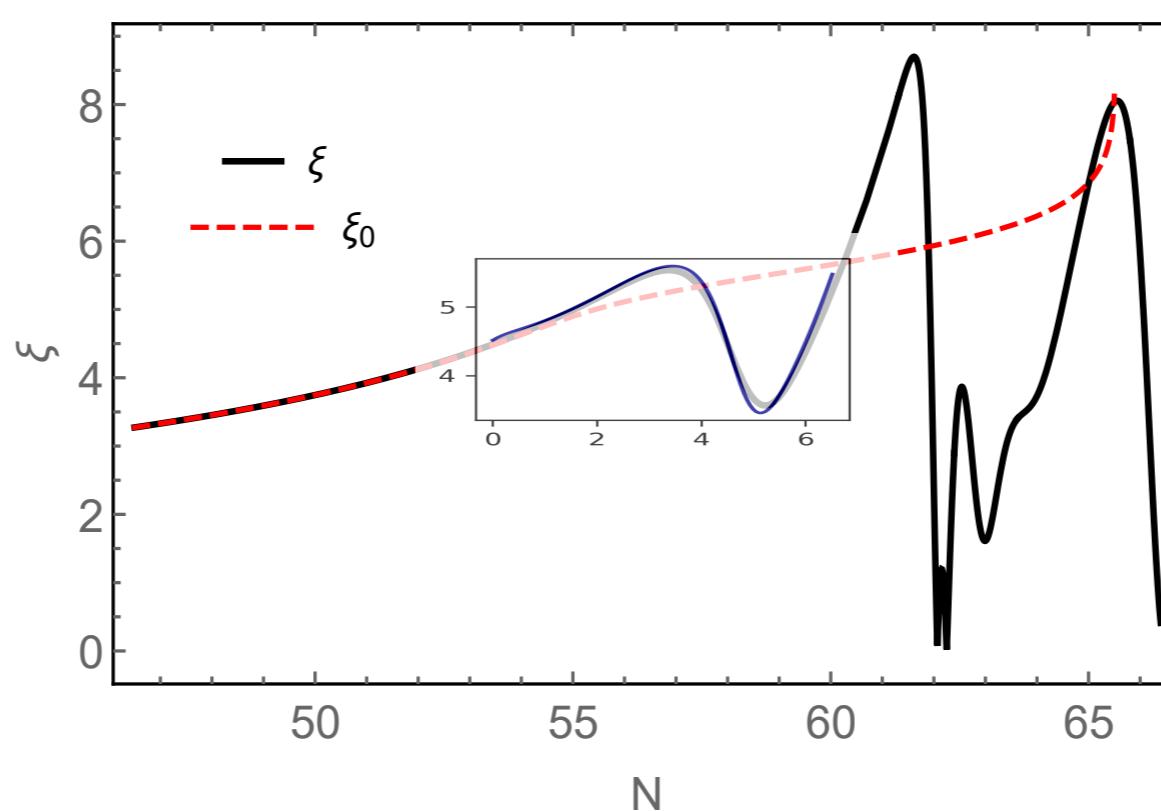


Figure from 2002.02952
[courtesy of V. Domcke]

Confirms the semi-analytical results of:

[V. Domcke, V. Guidetti, Y.
Welling, A. Westphal
arXiv:2002.02952]

[E.V. Gorbar, K. Schmitz, O. O.
Sobol, S. I. Vilchinskii
arXiv:2109.01651]

Strong backreaction (small scales)

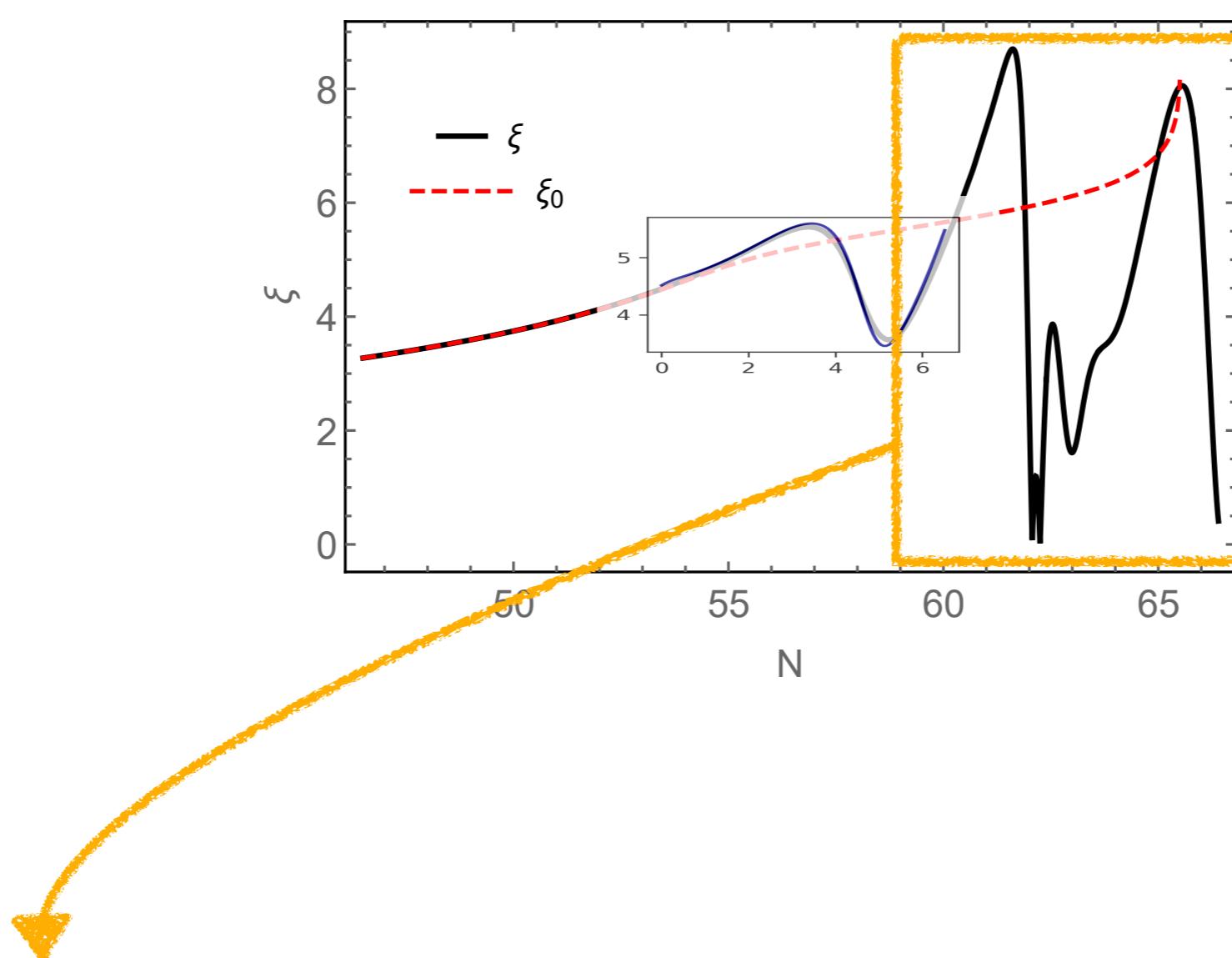


Figure from 2002.02952
[courtesy of V. Domcke]

What happens here is still
under investigation. See e.g.:

**D. Figueroa, J. Lizarraga,
A. Urió, J. Urrestilla**
2303.17436

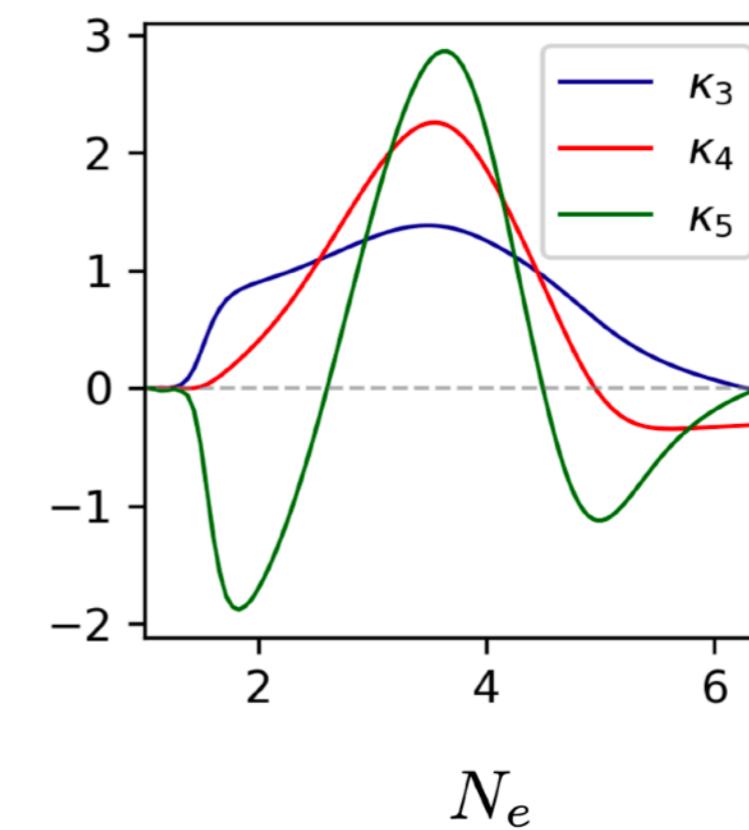
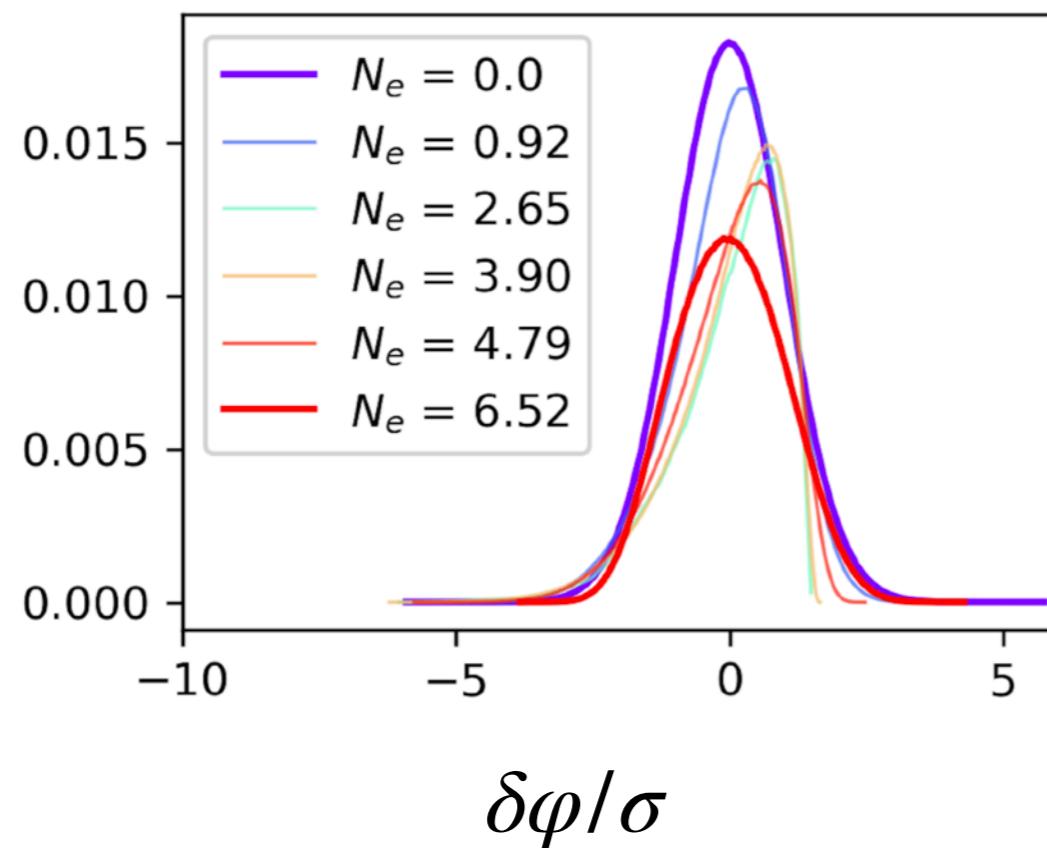
Lattice simulation

M. Peloso, L. Sorbo
2209.08131

Purely analytical

Strong backreaction (small scales)

Non-Gaussianity is suppressed
in the nonlinear regime!



Why? Central limit theorem

Spectator axion-gauge sector

A. Caravano, M. Peloso
arXiv:23???.???? (in prep.)

Ongoing work with Marco Peloso (University of Padova)

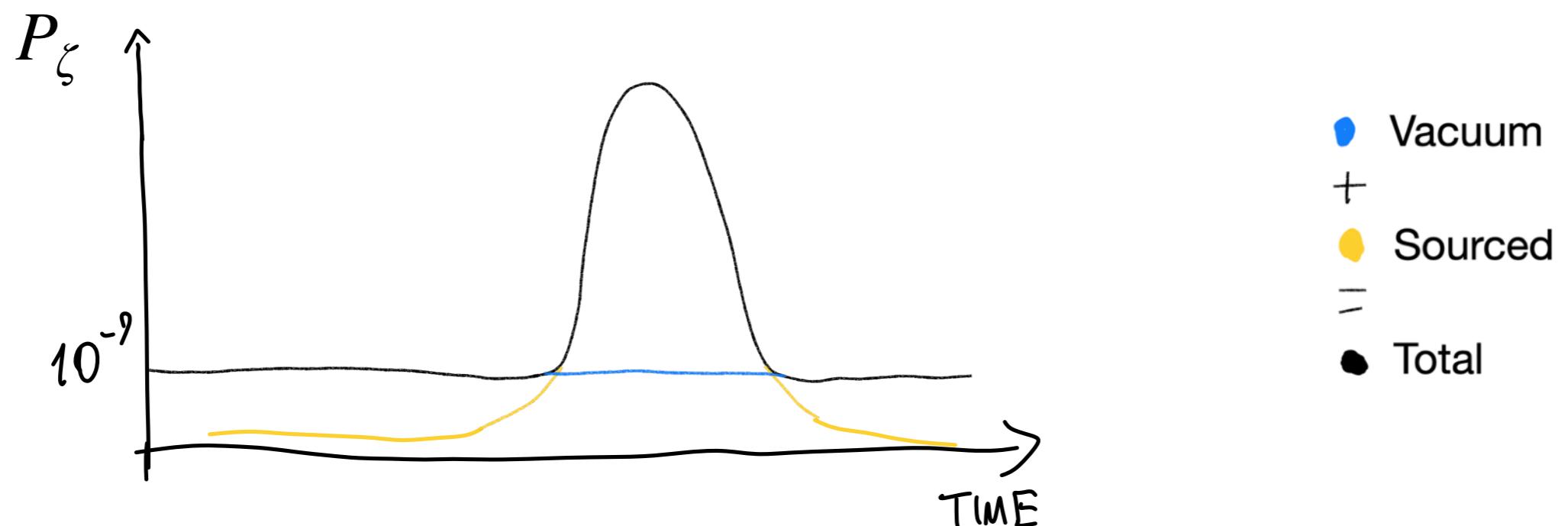
$$\mathcal{L} \supset -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V(\sigma) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\alpha}{4f}\sigma F_{\mu\nu}\tilde{F}^{\mu\nu}$$

↓ Inflaton ↓ Spectator axion-gauge

Tachyonic production of A_μ

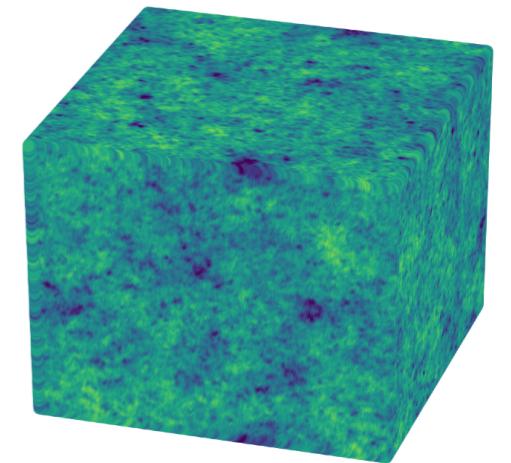
growth of $\delta\sigma$

Sources $\delta\phi$ via gravitational interaction



Summary:

- First simulation of an axion-gauge model during inflation

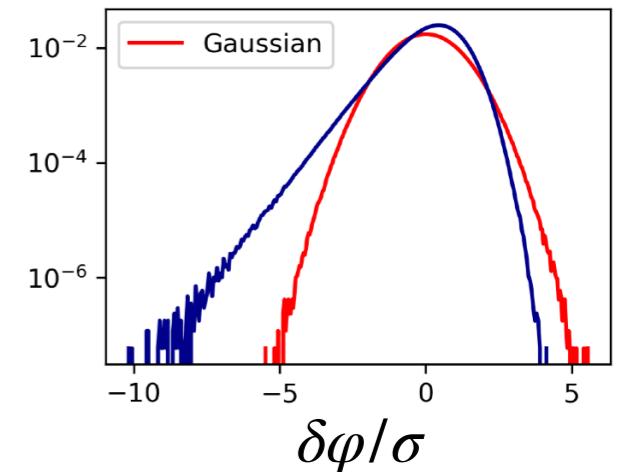


We studied both:

- Linear regime (large scales):

Full characterisation of $\delta\phi$ and its non-Gaussianity

$$\longrightarrow \dots > \kappa_6 > \kappa_5 > \kappa_4 > \kappa_3 > 1$$



- Nonlinear regime (small scales):

Backreaction and its consequences on PBH and GW

→ Confirms nontrivial background dynamics

→ perturbations become Gaussian, due to nonlinearity

