Multi field inflation:
PBH, GW and loop correction

2205.06065 (SB, I. Zavala)
2306.XXXX (SB, J. Fumagalli, M. Peloso, S. Renaux-Petel)

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New Horizons in Primordial Black Hole Physics
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Motivation

- PBHs can from when large primordial fluctuations from inflation collapse inside the post-inflationary horizon.
- Single field models: inflection points, tiny dips/bumps ... : mechanisms to slow down the inflaton extremely for a range of scales, far away from CMB.
- Multifield inflation: many avenues to lead to enhanced fluctuations: Inflection points can be realised naturally, field-space manifold,...
- Adiabatic and entropic fluctuations are coupled.
- Features in the enhanced inflationary power spectrum: interesting from phenomenological point of view.

$$\boxed{\psi_{\rm PBH}(M) \leftarrow P_R(k) \rightarrow \Omega^{(2)}_{\rm GW}(f)}$$

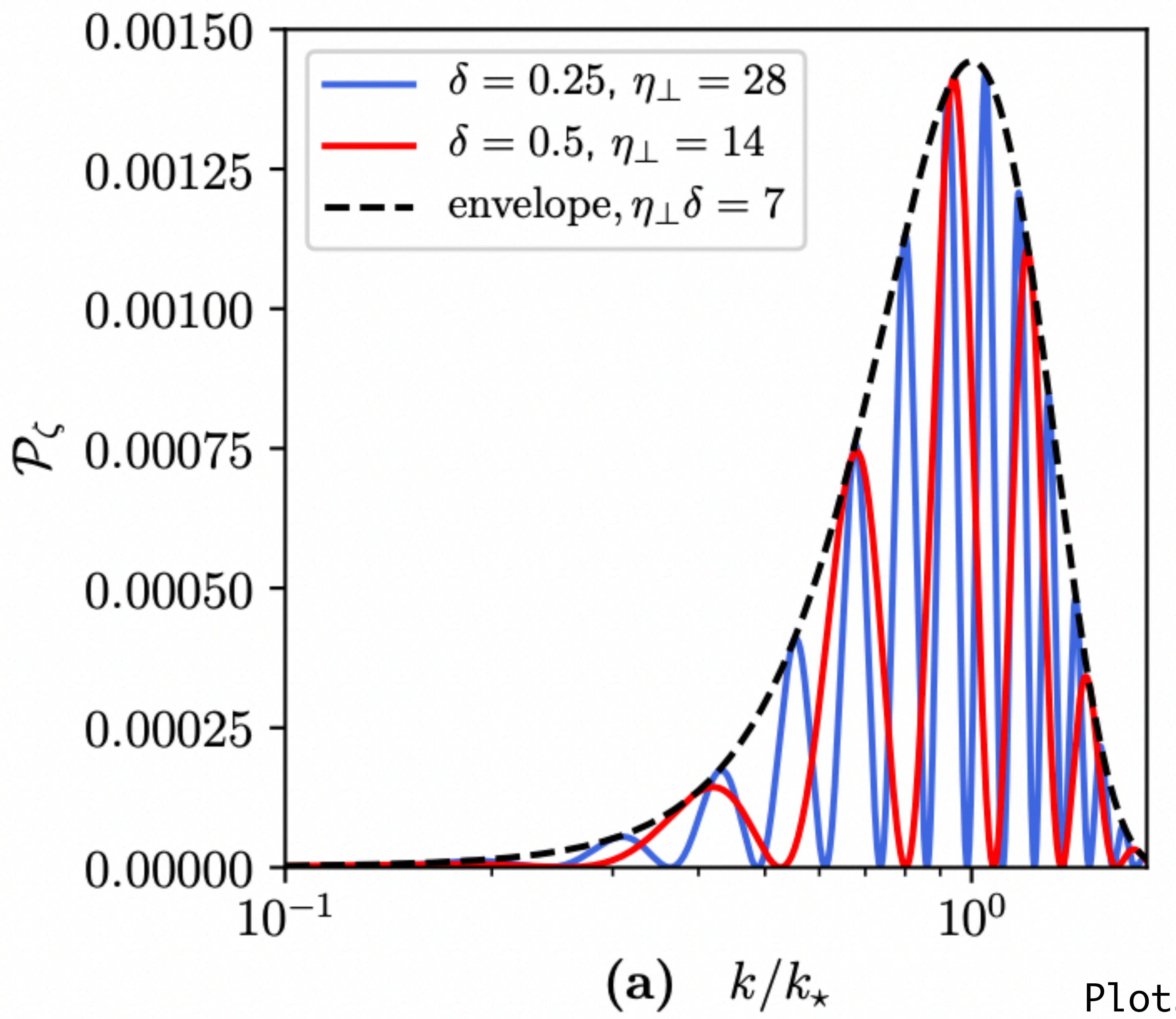
- Oscillatory features in $P_R(k)$ from a multi field inflation model.
- Possible 1-loop correction in such models.

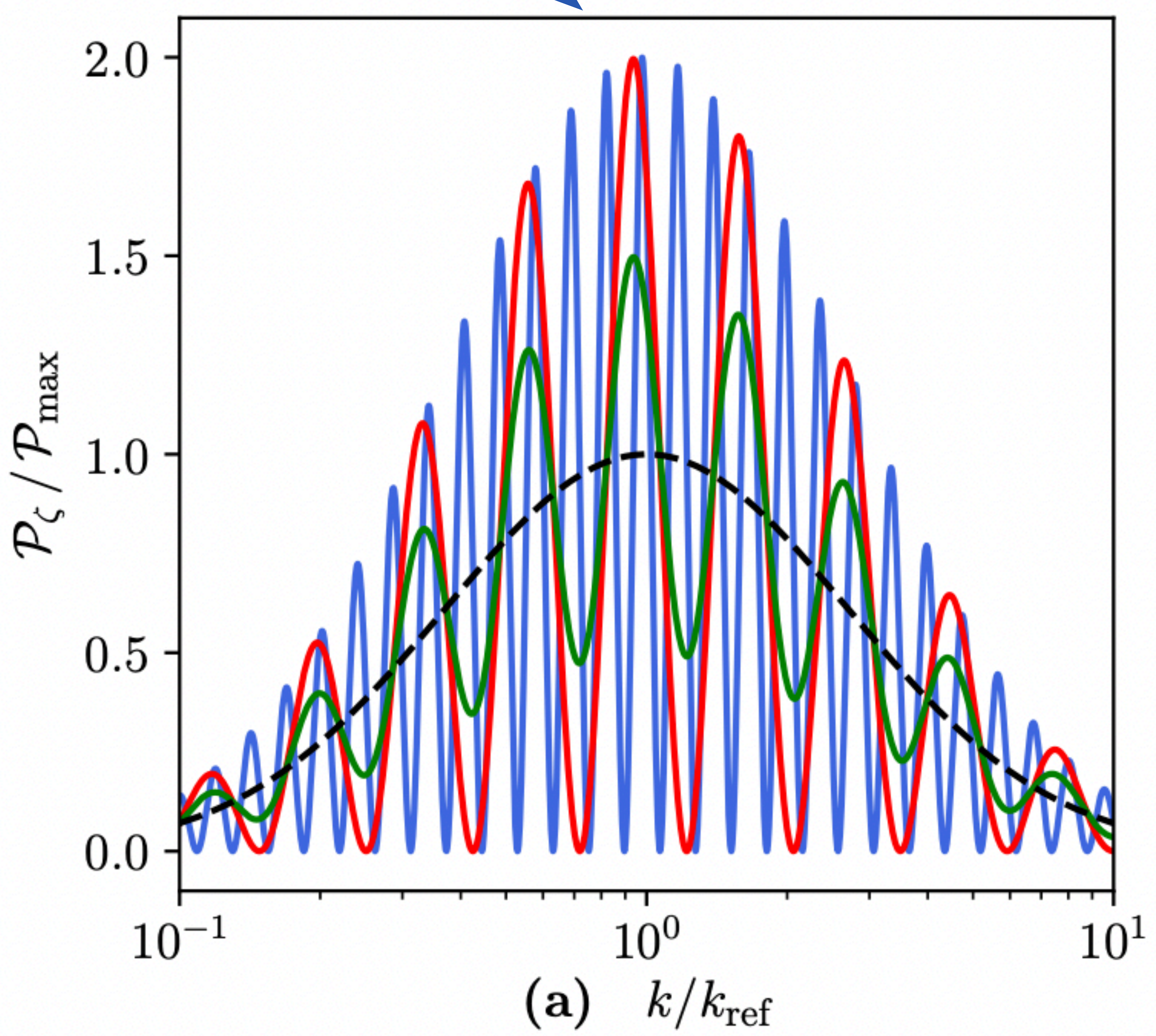
Oscillatory Features

- There can be distinct features in the enhanced power spectrum: Sharp and Resonant (Fumagalli et. al., 2105.06481)

$$\text{Sharp:}\quad \mathcal{P}_\zeta(k) = \mathcal{P}_0(k)\left[1 + A_{\rm lin}\cos\left(\omega_{\rm lin}k + \vartheta_{\rm lin}\right)\right],$$

$$\text{Resonant:}\quad \mathcal{P}_\zeta(k) = \mathcal{P}_0(k)\left[1 + A_{\rm log}\cos\left(\omega_{\rm log}\log(k/k_\star) + \vartheta_{\rm log}\right)\right]$$





Plots from 2105.06481

Axion Monodromy in SUGRA

- $$V = \frac{M^2}{\beta}\left(\rho^2 + \theta^2 + \frac{2\lambda}{M}e^{-b\rho}\left[\theta\cos(b\,\theta) + \rho\,\sin(b\,\theta) + \frac{\lambda}{2M}e^{-b\rho}\right]\right)$$

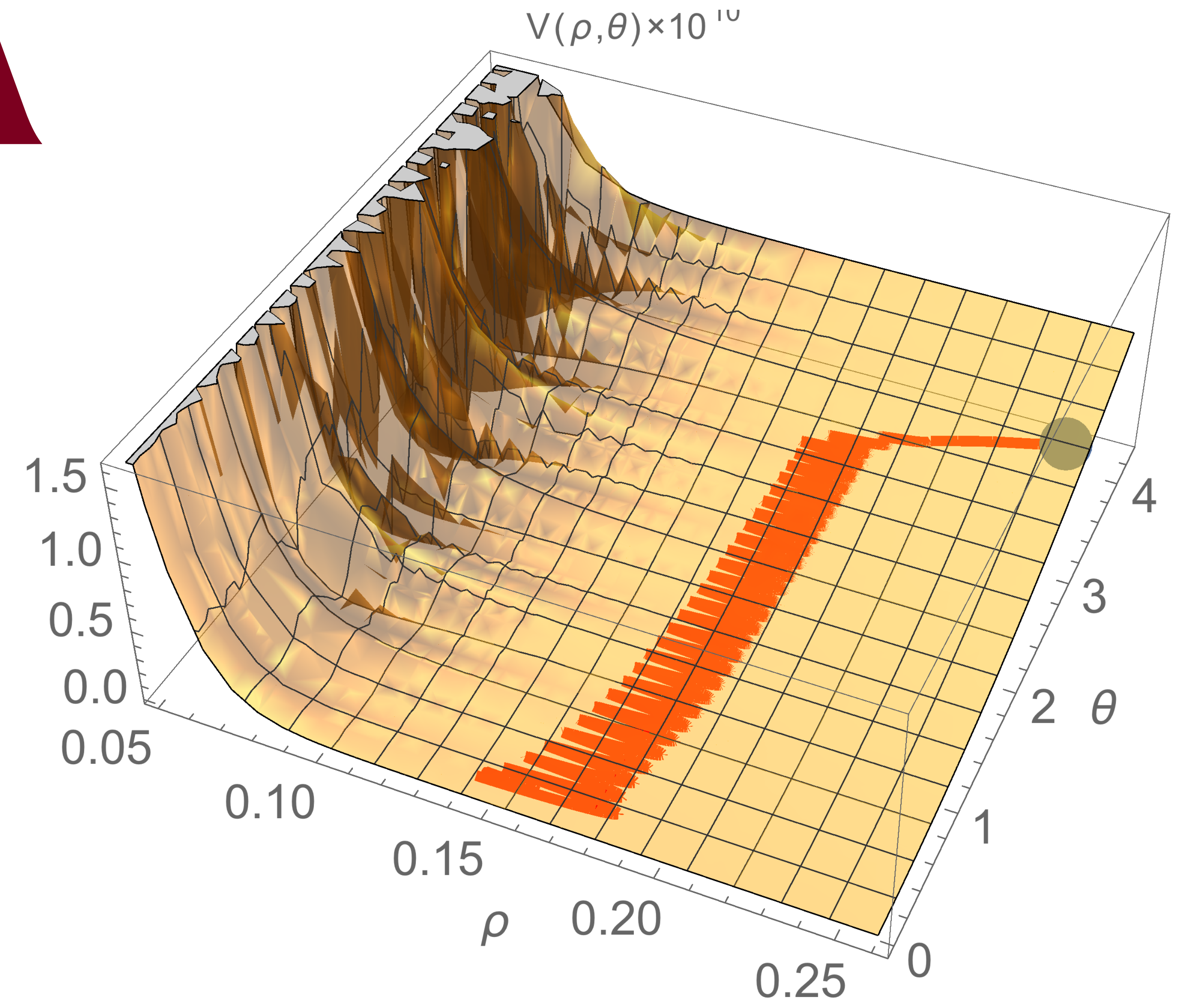
- Multifield inflation model with large and sharp turns via transient slow-roll violations with minimal field space curvature, minimal fine tuning.
(Palma et.al., 2004.06106; Fumagalli et. al., 2004.08369; Anguelova, 2012.03705)

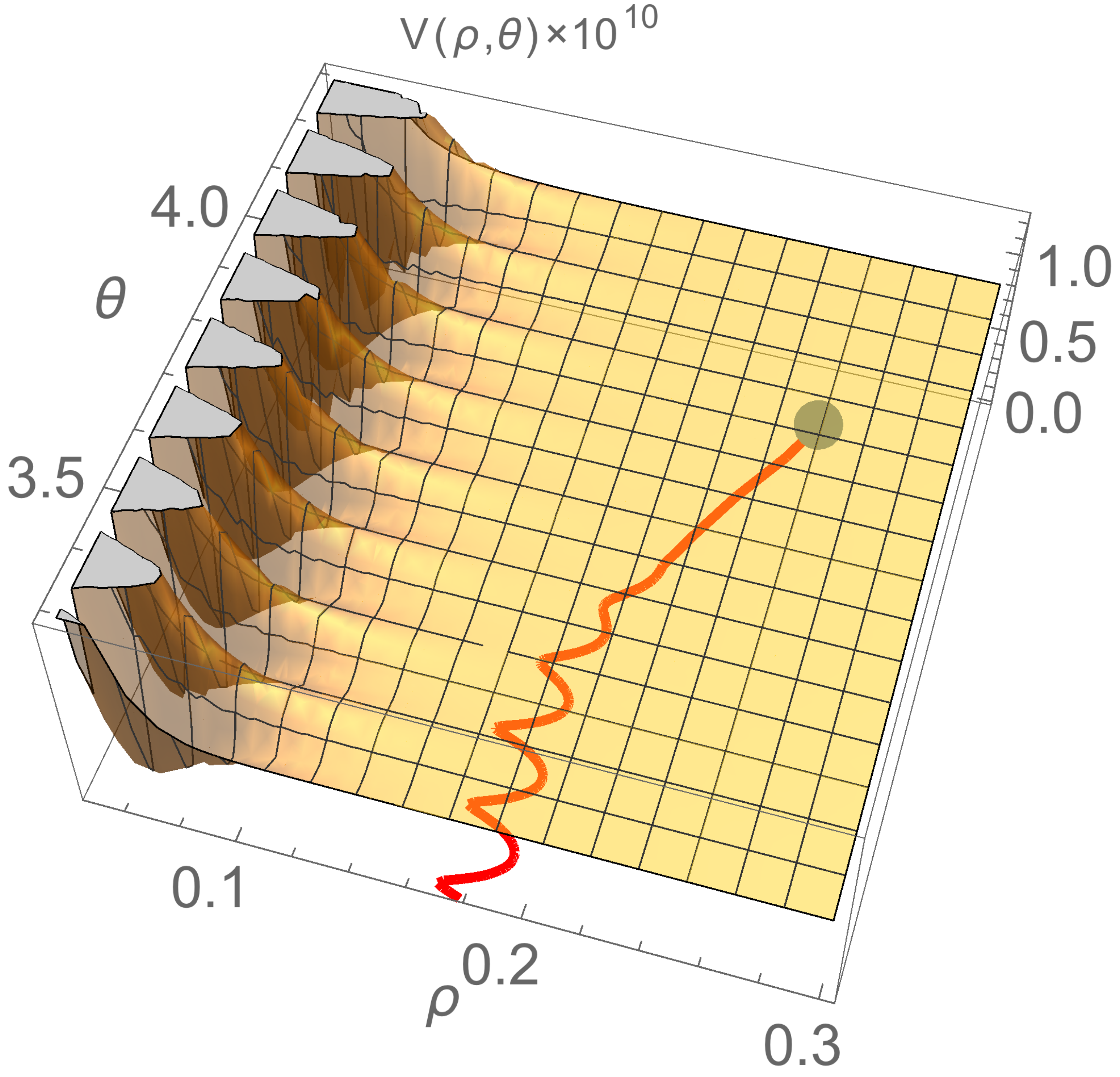
- Growth of fluctuations at small scales with characteristic oscillations.
(Ishikawa & Ketov, 2108.04408).

- Slow Roll parameters: $\epsilon \equiv -\frac{\dot{H}}{H^2}$; $\eta \equiv \frac{\dot{\epsilon}}{H\epsilon}$; $\delta_\varphi \equiv \frac{\ddot{\varphi}}{H\dot{\varphi}}$

- $\omega \equiv \frac{\Omega}{H} \equiv \frac{-N_a(D_t T^a)}{H} = \frac{V_N}{H\dot{\varphi}}$: Turning rate

- $\nu \equiv \frac{\dot{\omega}}{H\omega}$: Sharpness of turns





SB, I. Zavala, 2205.06065

Transient SR violation

- Benchmark point: $\lambda/M = 80,\ b = 50$

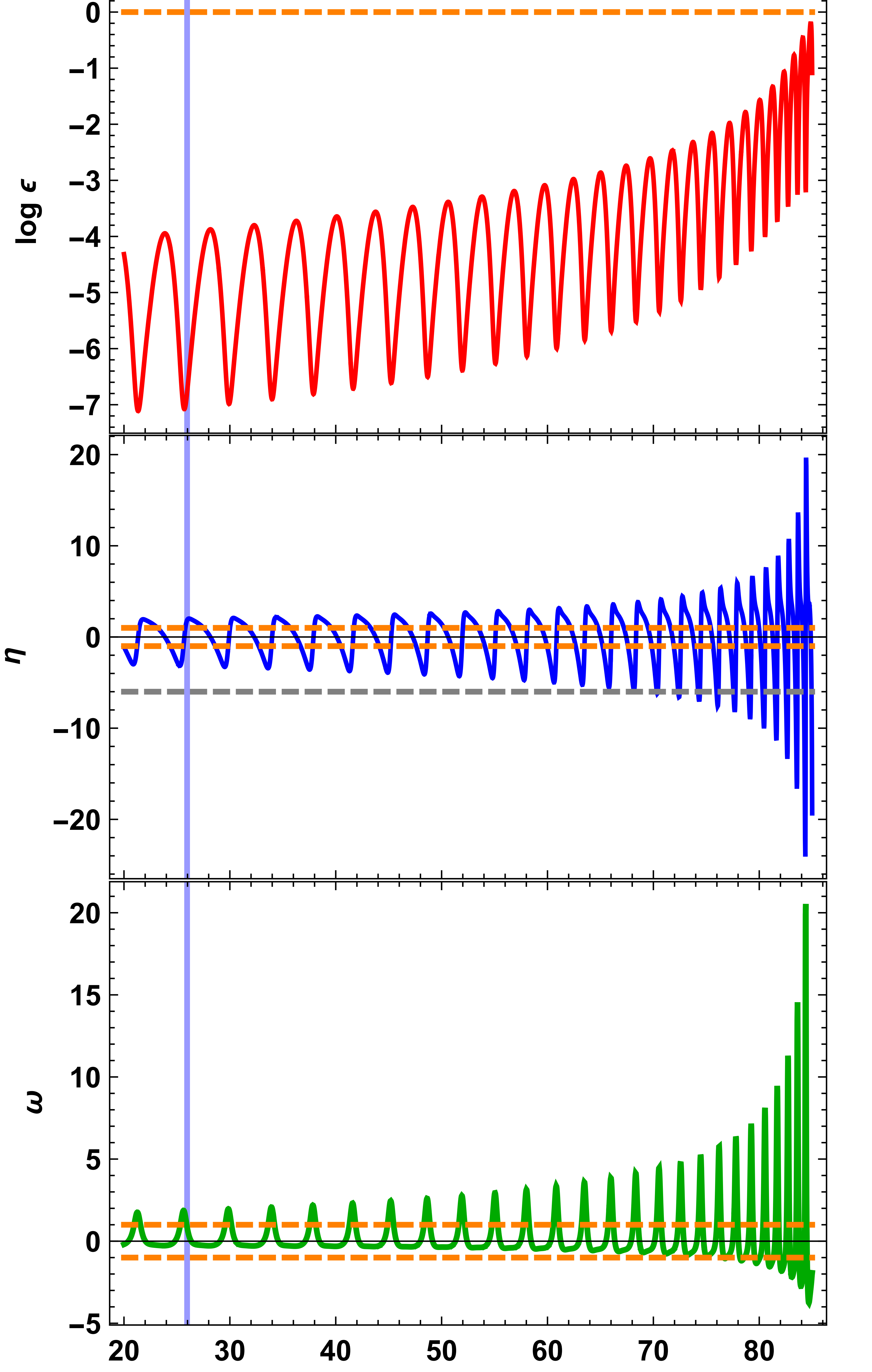
- Oscillations with decreasing period.

- $\eta \gg 1$

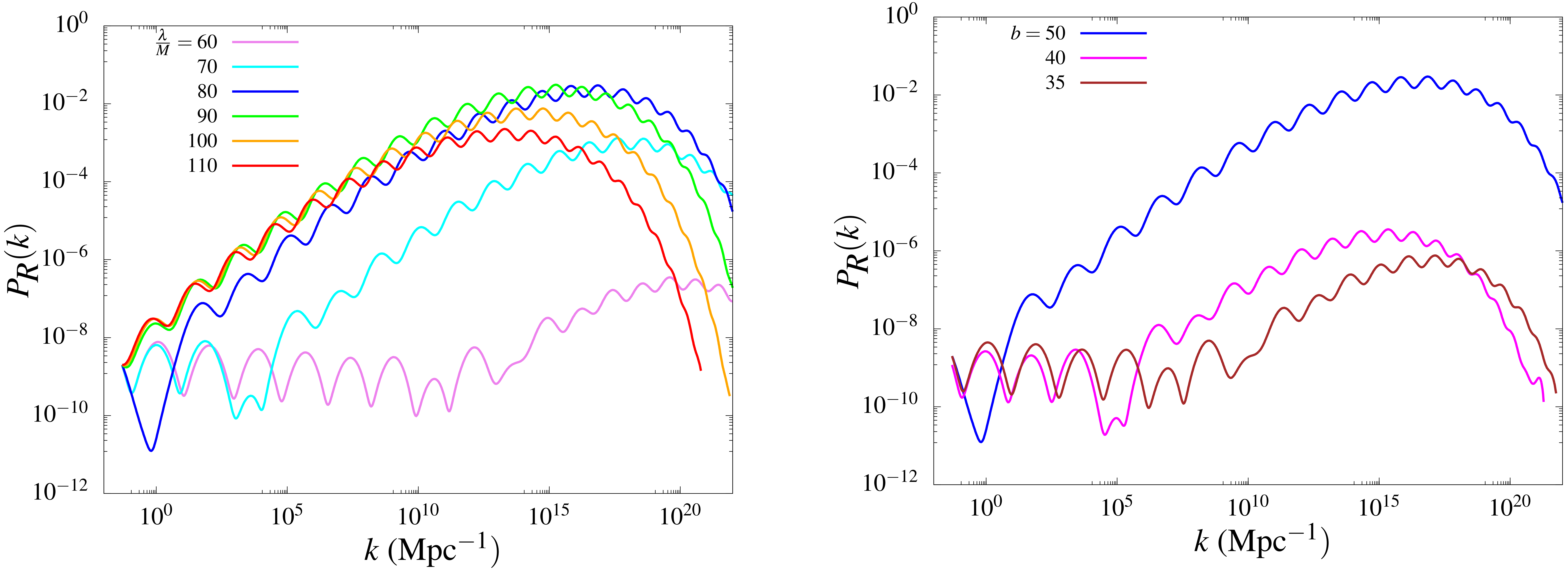
- $\dfrac{V_{TT}}{3H^2} = \boxed{\dfrac{\omega^2}{3}} + 2\epsilon \boxed{-\dfrac{\eta}{2}} - \dfrac{\xi_\varphi}{3}$

- $\dfrac{V_{TN}}{3H^2} = \omega\left(1 - \epsilon + \boxed{\dfrac{\eta}{3}} + \boxed{\dfrac{\nu}{3}}\right)$

- $\omega,\ |\nu| \gg 1.$



Adiabatic power spectra



- $V = \frac{M^2}{\beta}\left(\rho^2 + \theta^2 + \frac{2\lambda}{M} e^{-b\rho}\left[\theta\,\cos(b\,\theta) + \rho\,\sin(b\,\theta) + \frac{\lambda}{2M} e^{-b\rho}\right]\right)$

- $N_{\rm inf} \sim 55 - 65,\ r \sim 0.010 - 0.024,\ V_{\rm inf}^{1/4} \sim 0.003 - 0.03 M_{\rm Pl}$

- For CMB normalization at pivot, $\beta = 1,\ M \sim 10^{-8} - 10^{-6} M_{\rm Pl}$

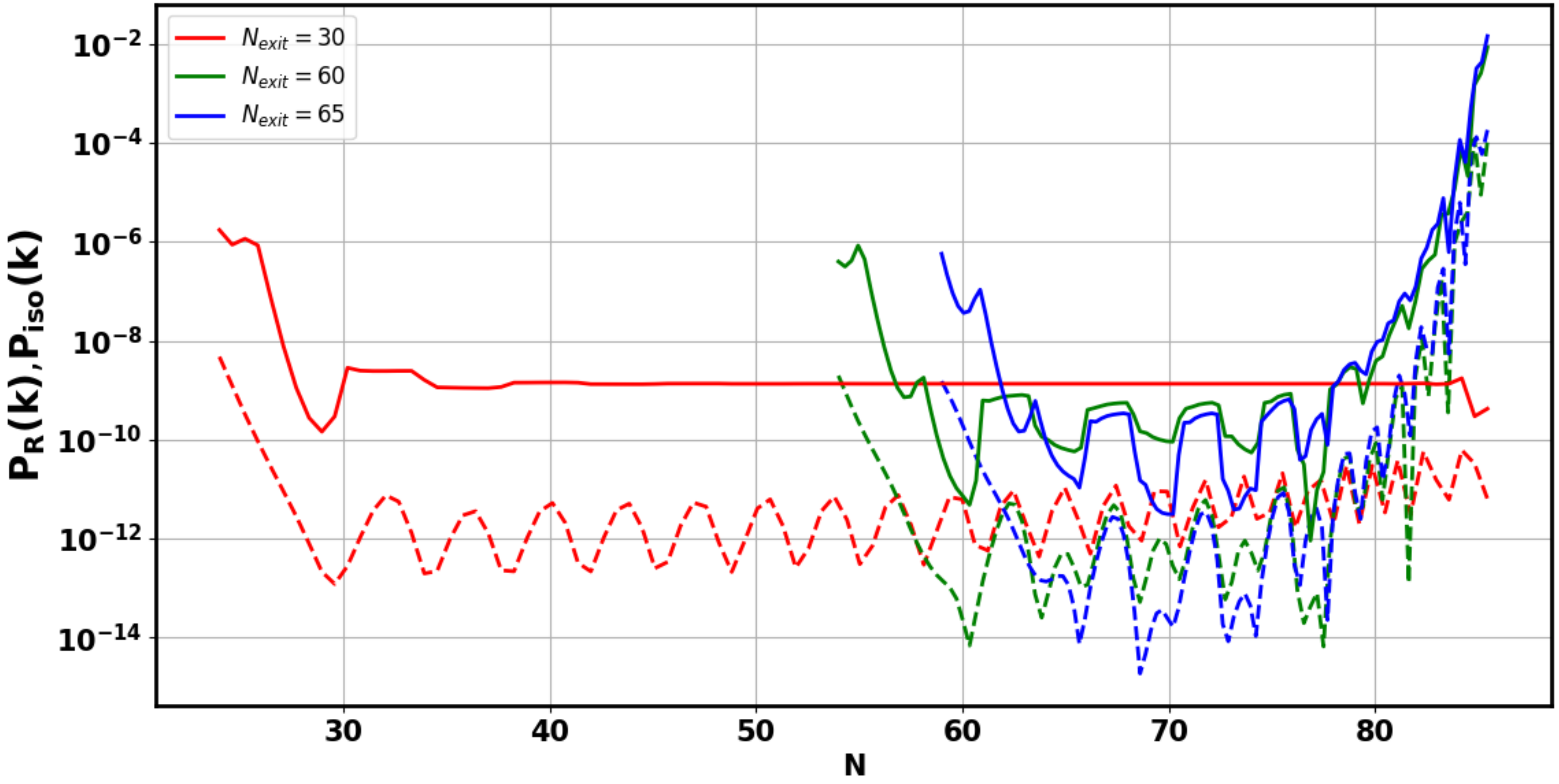
Growth of perturbations

$$\ddot{Q}_T + 3H\dot{Q}_T + \left(\frac{k^2}{a^2} + m_T^2\right) Q_T = \left(2\omega H Q_N\right)^{\cdot} - \left(\frac{\dot{H}}{H} + \frac{V_T}{\dot{\varphi}}\right) 2\omega H Q_N$$

$$\ddot{Q}_N + 3H\dot{Q}_N + \left(\frac{k^2}{a^2} + m_N^2\right) Q_N = -\,2\,\omega\dot{\varphi}\dot{R}$$

$$\frac{m_T^2}{H^2} \equiv -\frac{3}{2}\eta - \frac{1}{4}\eta^2 - \frac{1}{2}\epsilon\eta - \frac{1}{2}\frac{\dot{\eta}}{H}$$

$$\frac{m_N^2}{H^2} = \frac{V_{NN}}{H^2} + M_{\rm Pl}^2\epsilon\mathbb{R} - \omega^2$$



- $Q_T = \dot{\varphi}R/H$

Growth of perturbations

$$\ddot{Q}_T + 3H\dot{Q}_T + \left(\frac{k^2}{a^2} + m_T^2\right) Q_T = \left(2\omega H Q_N\right)^{\cdot} - \left(\frac{\dot{H}}{H} + \frac{V_T}{\dot{\varphi}}\right) 2\omega H Q_N$$

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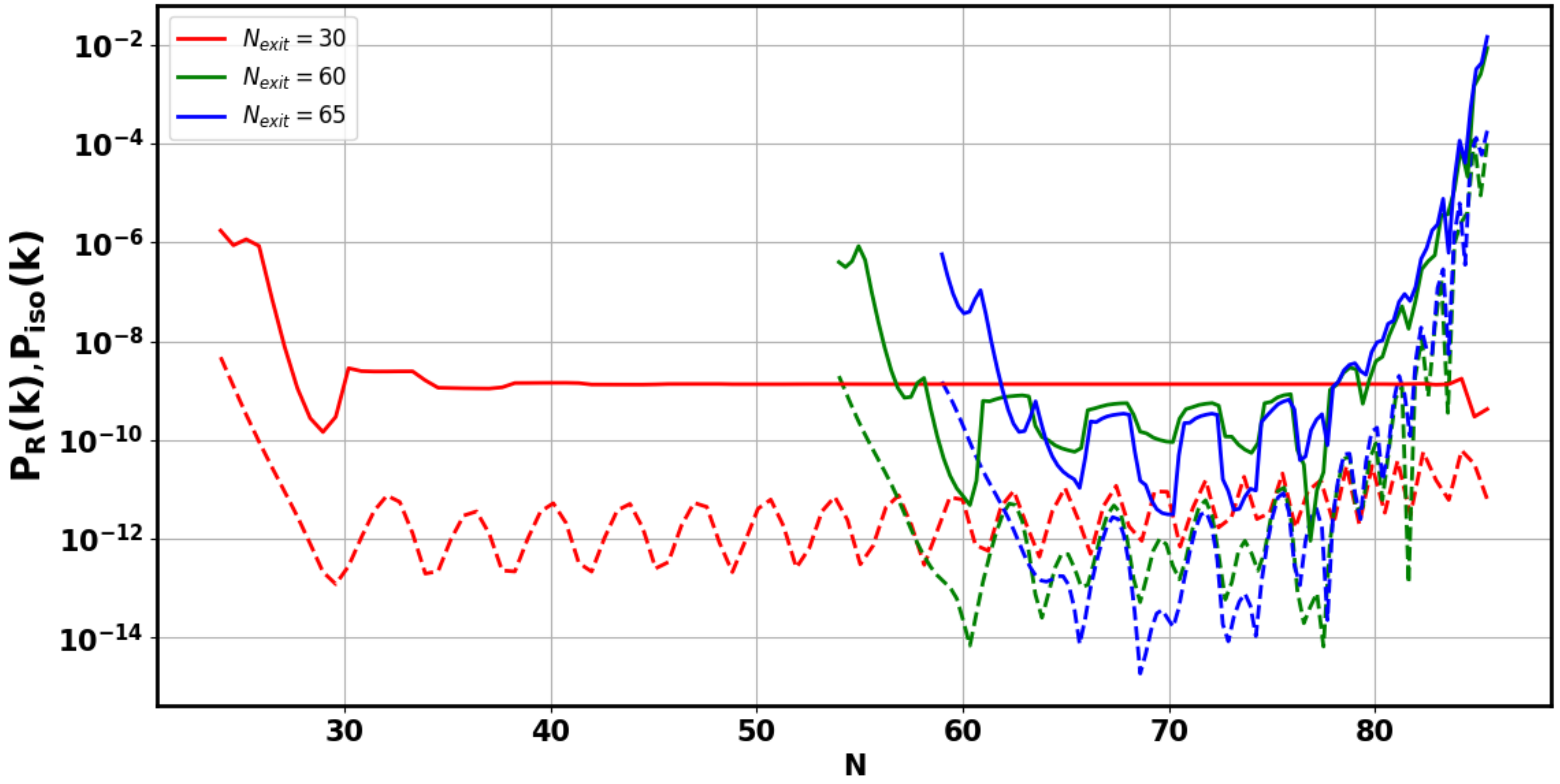
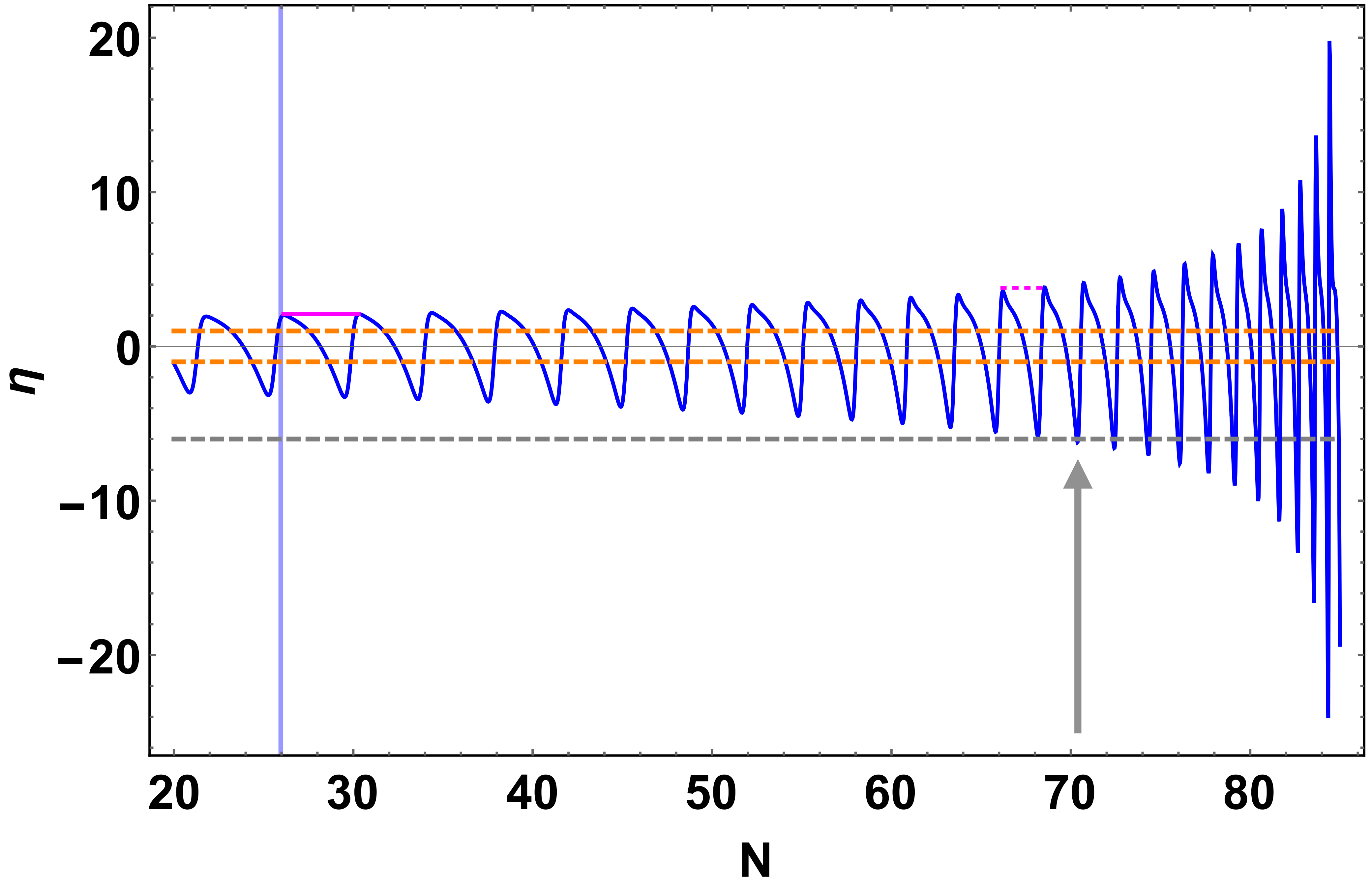


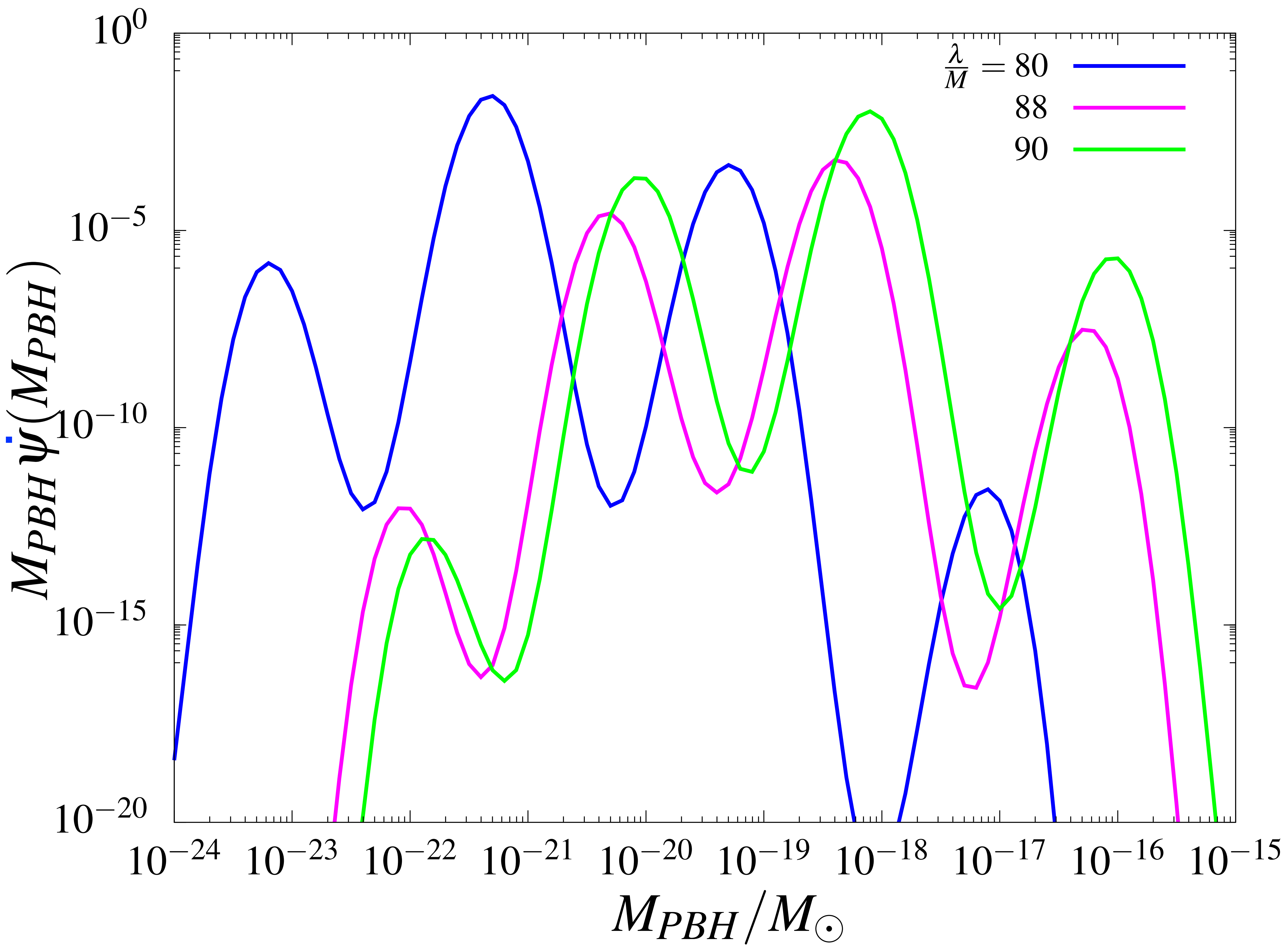


- Ultra Slow-Roll for
  $N \gtrsim 70$ with $\eta < -6,\ V_T = 0$

PBH

$$\psi_{\rm PBH}(M) \simeq \frac{\gamma\beta(M)}{M}\left(\frac{M_{\rm eq}}{M}\right)^{1/2}\left(\frac{\Omega_m h^2}{\Omega_c h^2}\right); \qquad \beta(M) = \int_{\delta_c}^{\infty} d\delta P(\delta)$$

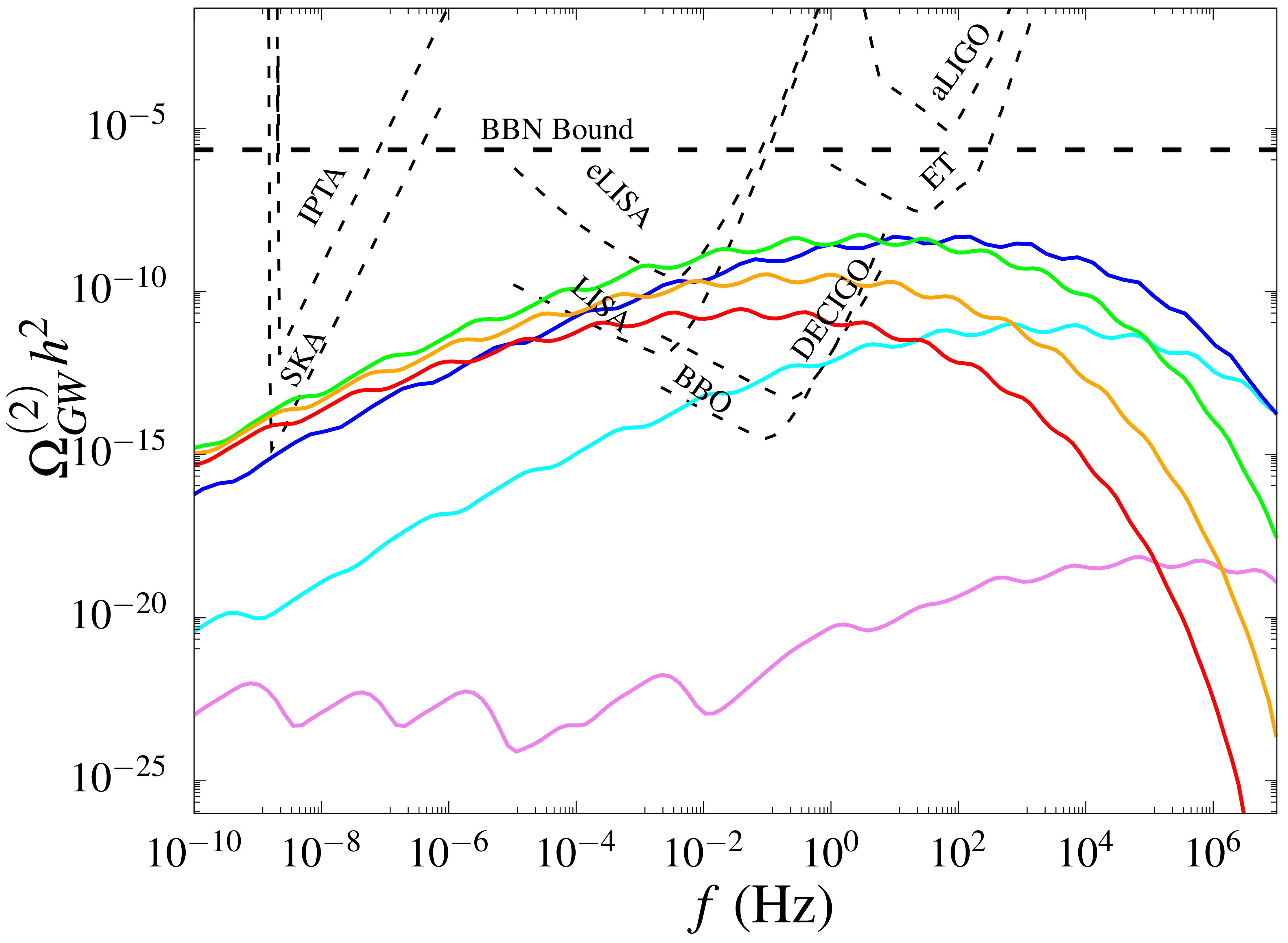
- $M_{\rm PBH} \sim k^{-2}$ in RD $\longrightarrow$ light PBHs.
- PBH abundance $f_{\rm PBH} \equiv \dfrac{\Omega_{\rm PBH}}{\Omega_{\rm DM}} \sim 10^{-3} - 10^{-2}$
- Some target masses are produced more than the others.
- BBN constraints: $10^{-24}M_\odot \lesssim M_{\rm PBH} \lesssim 10^{-20}M_\odot$;
  CMB and γ-ray constraints:
  $10^{-20}M_\odot \lesssim M_{\rm PBH} \lesssim 10^{-17}M_\odot$ (Carr et.al., 2002.12778, 2110.02821)
- Constraints also possible from PBH induced GW if these light PBHs dominate the energy density after inflation.
  (Papanikolaou et.al., 2010,11573, 2207.11041)

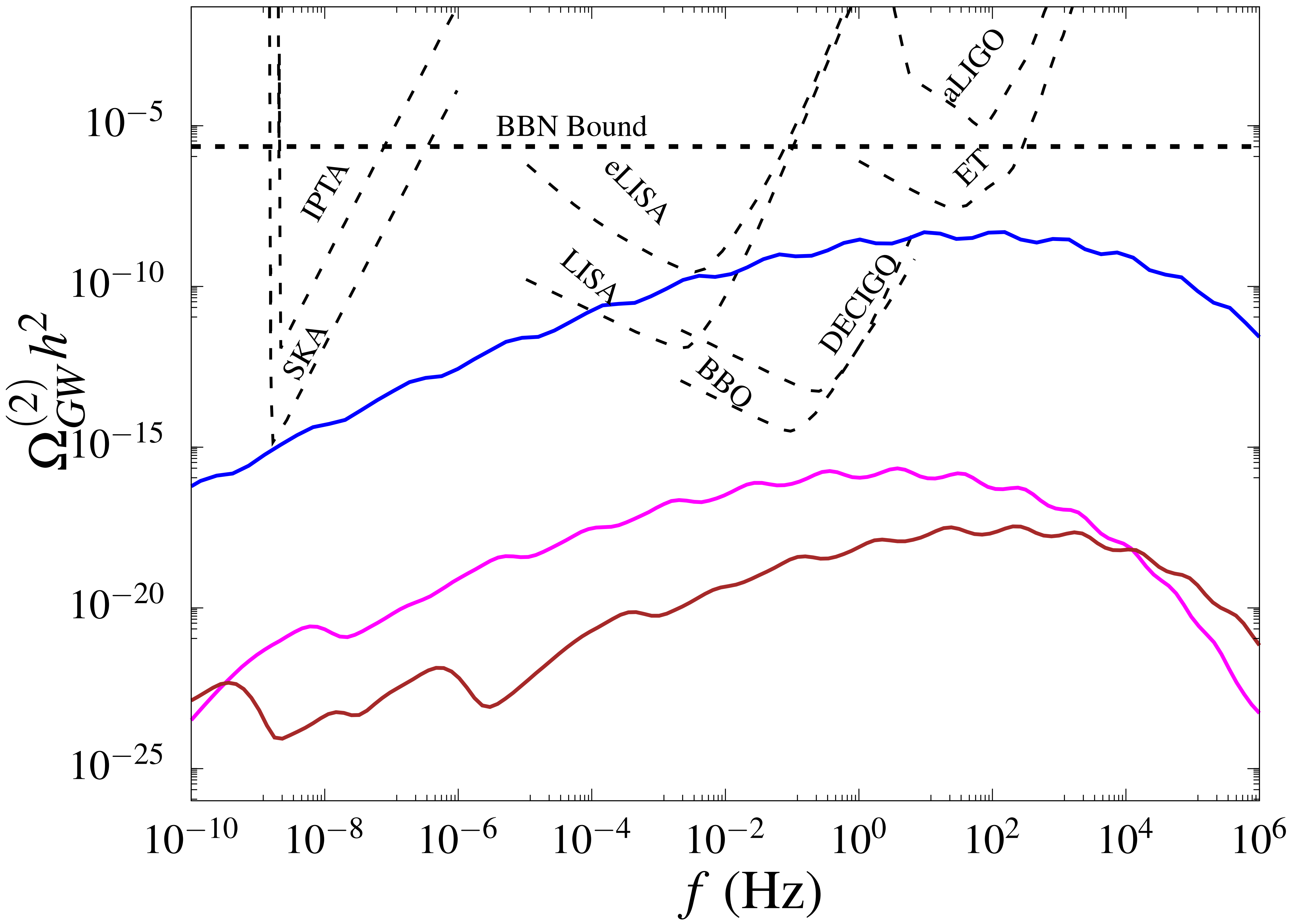


Induced GW

$$P_h(k,\tau) \propto \int du \int dv \mathcal{I}_{\rm RD} P_{\mathscr{R}}(ku) P_{\mathscr{R}}(kv)$$

$$\Omega^{(2)}_{\rm GW}(k,\tau) \sim k^2\tau^2 P_h(k,\tau)\,.$$





- Wide and large GW spectra, Spectral shape contains characteristic resonant oscillations from power spectra. Possible to probe with LISA (Fumagalli et. al., 2105.06481; Braglia et.al., 2012.05821; Caprini et.al., 1906.09244).

1-loop corrections

2306.XXXX
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- In presence of a spectator field, it is important to understand their effect at the level of perturbations.
- Adiabatic perturbations (loop-level) sourced by entropic perturbations (tree-level): $\propto \mathscr{F}\mathscr{F}\mathscr{R}$
- Consider a much more simplified scenario: $(\eta_\perp, R_{\rm fs})$ are negligible.

$$\boxed{(\partial_\tau^2 + 2aH(1+\eta/2)\partial_\tau + k^2)\hat{\mathcal{R}}(\boldsymbol{k},\tau) \equiv \mathcal{O}_k\,\hat{\mathcal{R}}(\boldsymbol{k},\tau) = \underline{\hat{S}}(\boldsymbol{k},\tau),}$$

$$\mathcal{F}(p,\tau) = \frac{H}{\sqrt{2p^3}}\left[\alpha_{\mathcal{F}}(p)\mathcal{U}(p\tau) + \beta_{\mathcal{F}}(p)\mathcal{U}^*(p\tau)\right] \equiv \frac{H}{\sqrt{2p^3}}\widetilde{\mathcal{F}},$$

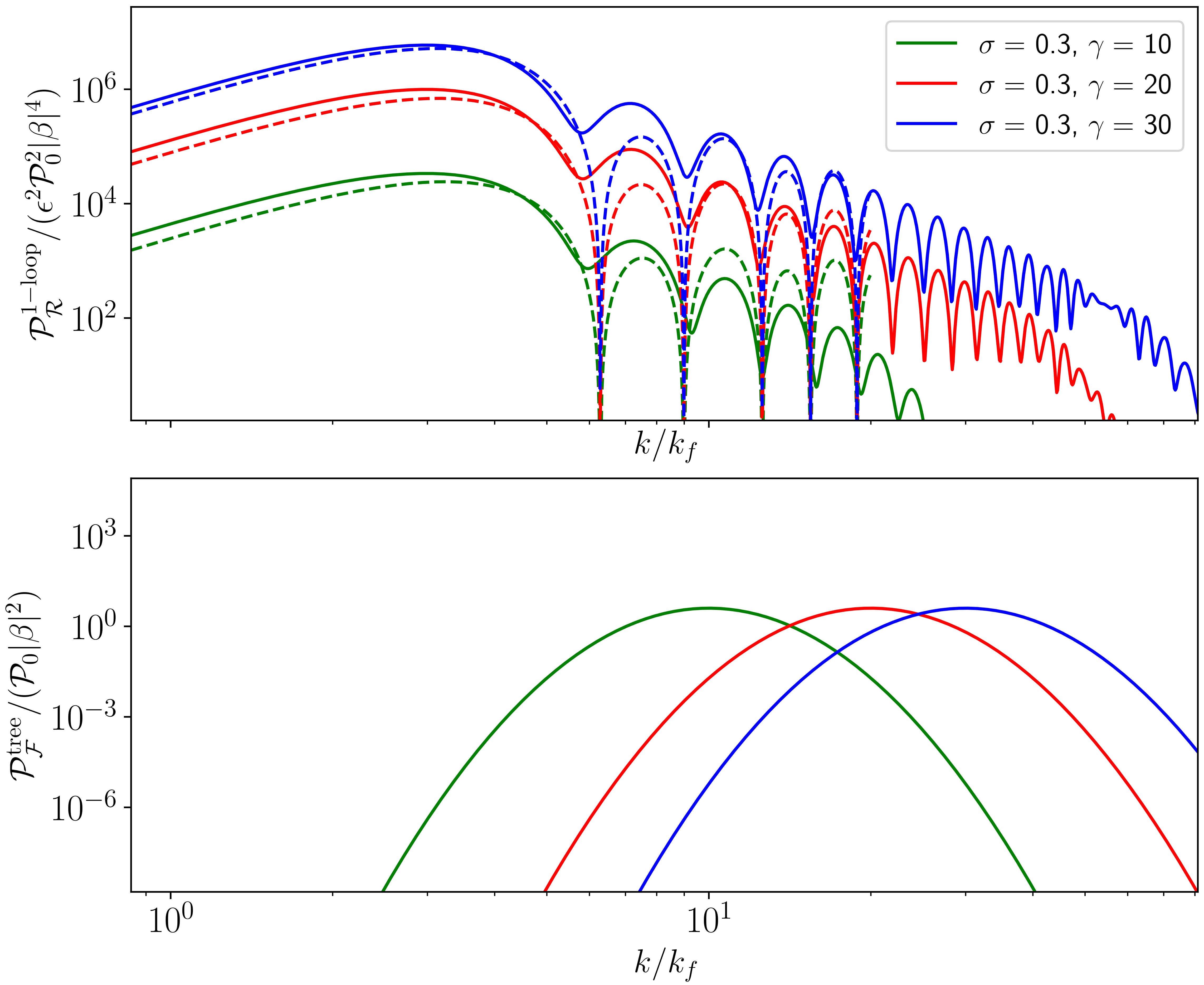
$$\boxed{\mathcal{P}_{\mathcal{R}}^{1-\mathrm{loop}}(k) = \epsilon^2\,\mathcal{P}_0^2 \int_0^\infty \mathrm{d}x \int_{|1-x|}^{1+x} \mathrm{d}y\, \mu_{\mathcal{F}}^2(x,y)\cdot\left|\sum_{\mathrm{s}_{1,2}=\pm} \alpha^{\mathrm{s}_1}(kx)\alpha^{\mathrm{s}_2}(ky)\mathcal{G}(\mathrm{s}_1 x, \mathrm{s}_2 y, z_{\mathrm{out}})\right|^2}$$

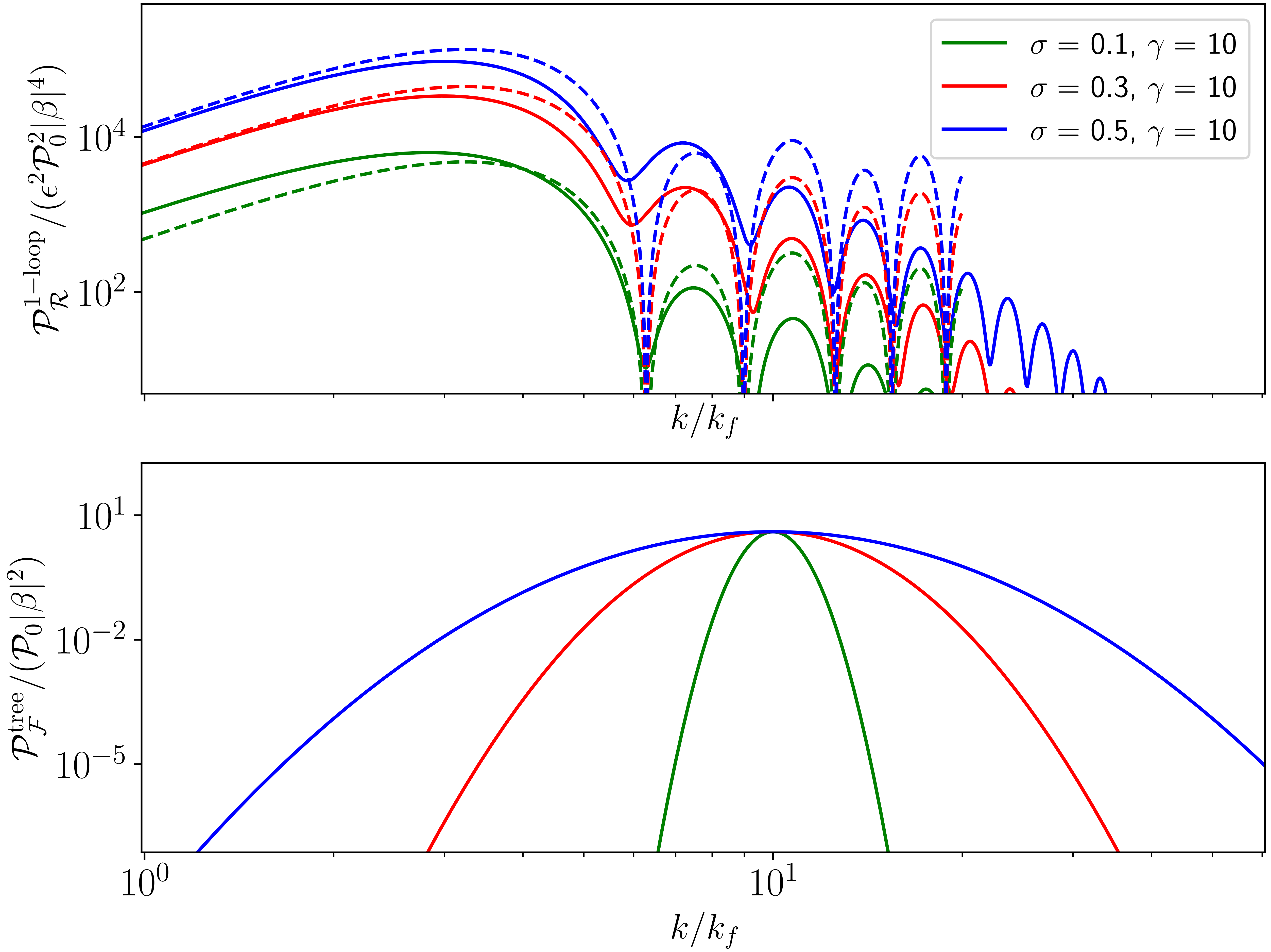
$$\mathcal{P}_0 = \frac{H^2}{8\pi^2 M_{\rm Pl}^2\,\epsilon}$$

Source: Entropic fluctuations

$$\beta_k = |\beta| \exp\left(-\frac{1}{2\sigma^2}\ln^2(k/k_*)\right).$$

$$\gamma = k_\star/k_{\rm out}$$





Summary

- Large and sharp turns originate from periodic violations of slow-roll without requiring large $\mathbb{R}$: Novel mechanism.
- First concrete, theoretically motivated example of a model where resonant oscillations are realised.
- Adiabatic power spectrum has resonant oscillations in log k. Interesting PBH and GW results.
- CMB consistency not achieved. Slow growth of oscillations in k is required: tuning is required to reach consistent model.
- Light PBHs formed, some more abundantly than others. Induced GW contains features.

- Large 1-loop contribution is possible, sourced by entropic perturbations, especially when modes deep in the subhorizon are excited: sharply peaked β for spectator field case, $\gamma \gtrsim 30$.
- Peak positions depend on γ.
- (β, γ) depend on model parameters: in a realistic model, possible to give an estimate of the viable parameter range: to produce abundant PBH, not to overproduce PBHs, check perturbative limits.