Should we care about cosmological black holes?

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New Horizons in Primordial Black Hole physics
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Quick outline

1. Motivation
2. Dynamical spacetimes
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    b. Misner-sharp mass
3. Cosmological metrics
    a. Einstein-Strauss
    b. LTB, McVittie (+Kottler),
    c. Generalized McVittie (+Sultana-Dyer)
    d. Thakurta
4. Case study: Thakurta metric
    a. Description
    b. Implementation
    c. Phenomenology
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6. Appendices
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        i. Thakurta cases
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Motivation for cosmological black holes

Lore

- PBHs form in the early universe, at the horizon scale, and surrounded by the thermal bath

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THE HYPOTHESIS OF CORES RETARDED DURING EXPANSION AND THE HOT COSMOLOGICAL MODEL

Ya. B. Zel'dovich and I. D. Novikov

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Original article submitted March 14, 1966

The existence of bodies with dimensions less than $R_g = 2GM/c^2$ at the early stages of expansion of the cosmological model leads to a strong accretion of radiation by these bodies. If further calculations confirm that accretion is catastrophically high, the hypothesis on cores retarded during expansion [3, 4] will conflict with observational data.

BLACK HOLES IN THE EARLY UNIVERSE

B. J. Carr and S. W. Hawking

(Received 1974 February 25)

SUMMARY

The existence of galaxies today implies that the early Universe must have been inhomogeneous. Some regions might have got so compressed that they underwent gravitational collapse to produce black holes. Once formed, black holes in the early Universe would grow by accreting nearby matter. A first estimate suggests that they might grow at the same rate as the Universe during the radiation era and be of the order of 10^{15} to 10^{17} solar masses now. The observational evidence however is against the existence of such giant black holes. This motivates a more detailed study of the rate of accretion which shows that black holes will not in fact substantially increase their original mass by accretion. There could thus be primordial black holes around now with masses from 10^{-5} g upwards.

Lore

- PBHs form in the early universe, at the horizon scale, and surrounded by the thermal bath
- How relevant will these effects be?

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Some quick criteria:

- Embedded in cosmological fluid
- Asymptotically FLRW
- Local definitions for mass, horizons, etc
  - (i.e. ADM won't work, event horizon won't work, etc.)
- Valid when near horizon size
  - Or even just in radiation-domination at all…
- Avoid physical singularities (i.e. that lead to negative energy densities outside the horizon etc)
  - Or naked singularities, etc.

Dynamical Spacetimes

The Kodama foliation

- Lack a killing field
  - Replace with Kodama vector: $k^a \equiv \epsilon_{\perp}^{ab} \nabla_b r$ $\quad k^a \nabla_a r = 0$
  - (Kodama 1980)

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  - Replace with Kodama vector: $k^a \equiv \epsilon_\perp^{ab} \nabla_b r$ $\quad k^a \nabla_a r = 0$
  - (Kodama 1980)
- Defines a unique + natural time coordinate
  - (Abreu & Visser 2010)
- In this 'Kodama foliation', any spherically symmetric metric can be written:

$$ds^2 = e^{-2\phi(R,\tau)} F(R,\tau)\, d\tau^2 - \frac{dR^2}{F(R,\tau)} - R^2 \mathrm{d}\Omega^2$$

$$F(R,t) = \left(1 - \frac{2Gm_{MS}}{R}\right)$$

Misner-Sharp mass

- $m_{\rm MS}$ is naturally the Misner-Sharp mass in this foliation $1 - \frac{2m_{\rm MS}}{R} \equiv \nabla^c R \nabla_c R$
  - (Misner & Sharp 1964)

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- Kodama foliation gives *natural time coordinate* and *local mass* automatically

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Cosmological metrics

The Einstein-Strauss (Swiss-cheese) vacuole

- Stitch a Schwarzschild solution into the background
  - (Einstein & Strauss 1945)

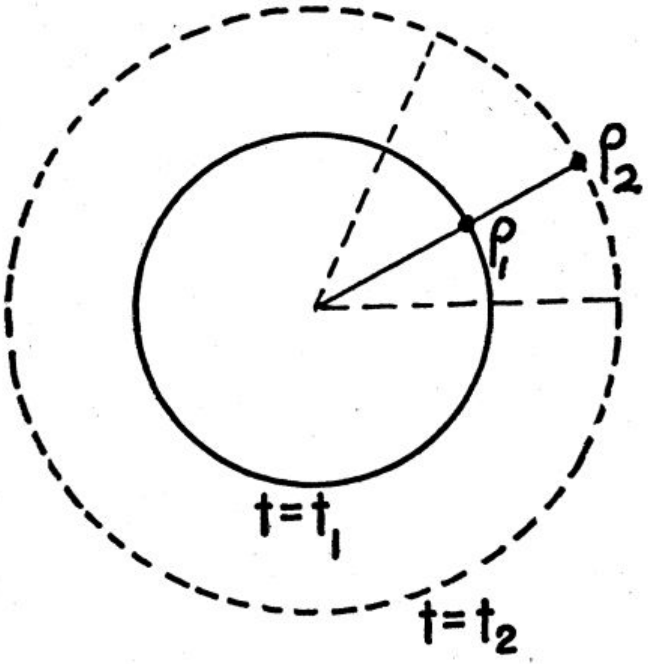

FIG. 1.

The Einstein-Strauss (Swiss-cheese) vacuole

- Stitch a Schwarzschild solution into the background
  - (Einstein & Strauss 1945)
- Matching conditions require zero pressure (i.e. matter domination)
  - And dynamic stitching

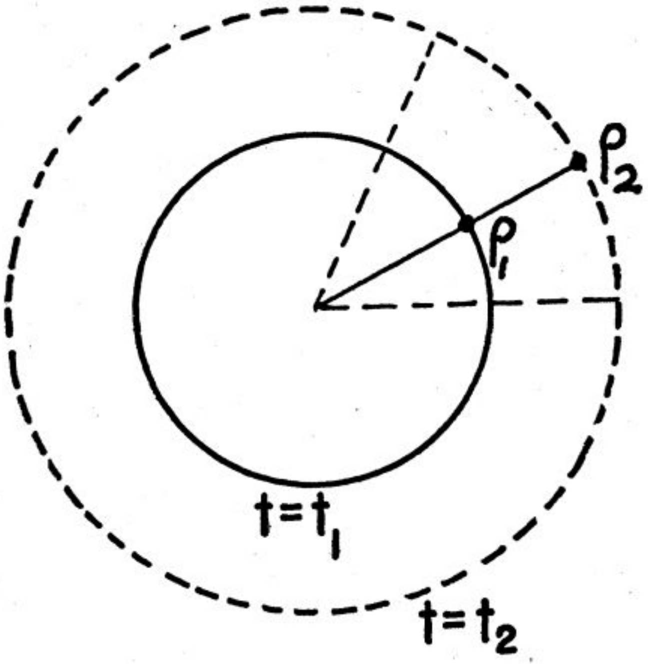

FIG. 1.

Lemaitre-Tolman-Bondi spacetimes

- Gravitational collapse spacetimes
- Also require dust backgrounds
- Have shell-crossing singularities or result in naked singularities
  - (Joshi, Malafarina 2015)

$$ds^2 = -dt^2 + \frac{R'^2}{1+f}dr^2 + R^2 d\Omega^2$$

McVittie and Einstein-de Sitter/Kottler spacetimes

- Central inhomogeneity in FLRW universe
    - (McVittie 1933)

$$ds^2 = e^{\zeta(r,t)} dt^2 - \frac{e^{\nu(r,t)}}{c^2}\{dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\}.$$

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- Stationary, spacelike singularity at *R=2m*
- Perfect fluid as source
    - Divergent pressure at horizon

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  - Divergent pressure at horizon
- Einstein-de Sitter spacetime is special case
  - (de Sitter 1917, Kottler 1918)

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Generalized McVittie metrics

- Remove perfect fluid requirement
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  - Fixes singularity problems
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- Unique late-time attractor solution: *Thakurta* metric
  - (Thakurta 1981)
  - Also GR limit of Brans-Dicke gravity with cosmological fluid
    - (Clifton, Mota & Barrow 2005)

Case study: the Thakurta metric

Thakurta metric

- Relatively simple spacetime: $\mathrm{d}s^2 = a^2 \mathrm{d}s^2_{schw.}$
- In cosmological coordinates:

$$\mathrm{d}s^2 = f(R)\left(1 - \frac{H^2 R^2}{f^2(R)}\right)\mathrm{d}t^2 + \frac{2HR}{f(R)}\mathrm{d}t\,\mathrm{d}R - \frac{\mathrm{d}R^2}{f(R)} - R^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right)$$

$$f(R) = 1 - 2Gma(t)/R$$

- Source is imperfect fluid, with radial *energy* flow:

$$T_{\mu\nu} = (\rho + P)\, u_\mu u_\nu + g_{\mu\nu} P + q_{(\mu} u_{\nu)}$$
$$q_\mu = (0, q, 0, 0)\ ,$$
$$u_\mu = (u, 0, 0, 0)\ .$$

Mass and apparent horizons

- In the Kodama foliation:

$$ds^2 = e^{-2\phi(R,\tau)} F(R,\tau)\, d\tau^2 - \frac{dR^2}{F(R,\tau)} - R^2 d\Omega^2 \qquad F(R,t) = \left(1 - \frac{2Gm_{MS}}{R}\right)$$

$$\boxed{m_{\mathrm{MS}} = ma(t) + \frac{H^2 R^3}{2Gf(R)}}$$

- Horizons:

$$R_{\mathrm{C}} = \frac{1}{2H}\left(1 + \sqrt{1 - 8HGma}\right) \approx 1/H$$

$$R_{\mathrm{BH}} = \frac{1}{2H}\left(1 - \sqrt{1 - 8HGma}\right) \approx 2ma$$

Interpreting the Misner-Sharp mass

- In the Kodama foliation:

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$$\boxed{m_{\mathrm{MS}} = ma(t) + \frac{H^2 R^3}{2Gf(R)}}$$

- Should be somewhat careful:

$$m_{\mathrm{MS}} = ma + \frac{4\pi}{3}\rho(R,t)R^3 \qquad \rho(R,t) \equiv T_{\mu\nu}v^\mu v^\nu = \frac{3H^2}{8\pi f(R,t)}$$

$$f(R) = 1 - 2Gma(t)/R$$

Physical implementation

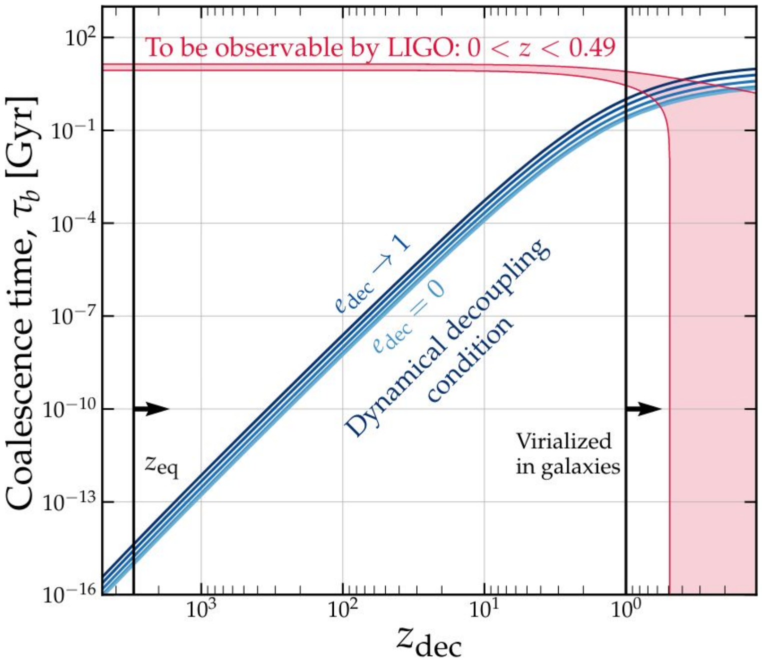
Technically these metrics are eternal, but we need a more realistic PBH model:

$$m_{\rm MS} = ma(t) + \frac{H^2 R^3}{2Gf(R)}$$

1. Form from overdensity of mass *m*
2. Stay as Thakurta black holes, until—
3. Their local environment becomes dominated by anything other than the imperfect fluid
    a. Other black holes, structure formation, etc
4. Spacetime should transition back to ~Schwarzschild black hole of mass *m*

Possible phenomenological consequences

- Binary formation is greatly suppressed
  - No LIGO signatures from coalescing binaries today
  - (Boehm, Kobakhidze, O'Hare, ZP, Sakellariadou 2020)
- Hawking radiation may be significantly higher
  - 'Larger' black holes evaporate rapidly
  - (ZP 2021)



Are Thakurta black holes realistic?

Pros:

- No spacelike singularities
- Valid in radiation domination (only one??)
- Simple…

Cons:

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Pros:

- No spacelike singularities
- Valid in radiation domination (only one??)
- Simple…

Cons:

- Question about white hole vs black hole horizon
- Unrealistic temperature gradients?
- Multi-black hole solution/ global energy density
- ‘Synge’ procedure
- Doesn’t follow the ‘lore’
- Simple…

Conclusions

- We probably do need a clear, reliable cosmological black hole metric
  - Esp. for radiation-domination and just after formation

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  - Esp. for radiation-domination and just after formation
- It is *possible* that this metric may have larger phenom. consequences than we are willing to admit
  - Case study: Thakurta metric

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  - Esp. for radiation-domination and just after formation
- It is *possible* that this metric may have larger phenom. consequences than we are willing to admit
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- …I think we probably should care, but I can’t say I definitely have the right answers

Conclusions

- We probably do need a clear, reliable cosmological black hole metric
  - Esp. for radiation-domination and just after formation
- It is *possible* that this metric may have larger phenom. consequences than we are willing to admit
  - Case study: Thakurta metric

- …I think we probably should care, but I can’t say I definitely have the right answers

Thanks for listening!

Appendices: apparent horizons

- Null radial geodesics:

$$\ell^\mu \ell_\mu = n^\mu n_\mu = 0 \,,$$
$$\ell^\mu n_\mu = 1 \,.$$

- Expansion scalars:

$$\theta_\ell = \nabla_\mu \ell^\mu + \ell^\mu \ell_\nu \nabla_\mu n^\nu \,,$$
$$\theta_n = \nabla_\mu n^\mu + n^\mu n_\nu \nabla_\mu \ell^\nu \,.$$

Appendices: Thakurta horizons

Cosmological foliation:

$$\frac{dr}{dt} = \pm\frac{f}{a},$$

$$\ell^\mu = (1, f/a, 0, 0) \ ,$$
$$n^\mu = \frac{1}{2f}(1, -f/a, 0, 0)$$

$$\theta_\ell = \frac{2}{R}(HR + f(R,t)) \ ,$$
$$\theta_n = \frac{1}{Rf(R,t)}(HR - f(R,t)) \ .$$

Kodama foliation:

$$\frac{dR}{d\tilde{\tau}} = -c\sqrt{1-F} \pm c \ ;$$

$$\ell^\mu = \frac{1}{c}\left(1, c - c\sqrt{1-F}, 0, 0\right) \ ,$$
$$n^\mu = \frac{1}{2c}\left(1, -c - c\sqrt{1-F}, 0, 0\right)$$

$$\theta_\ell = \frac{2}{R}\left(1 - \sqrt{1-F}\right) \ ,$$
$$\theta_n = -\frac{1}{R}\left(1 + \sqrt{1-F}\right) \ .$$

Appendices: Thakurta decoupling

'Standard:'

$$\ddot{R} = -\frac{Gm}{R^2} + \frac{\ddot{a}}{a}R\,.$$

$$\frac{m}{V} \gg \frac{3}{4\pi G}\left|\frac{\ddot{a}}{a}\right|$$

'Dynamical':

$$\frac{\dot{R}}{R} \sim -\frac{\dot{E}}{E} + 2H$$

$$\dot{E} \sim -\frac{32}{5}\frac{G^4M^3\mu^2a^5}{\mathfrak{a}^5(1-e^2)^{7/2}}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right) = \dot{E}_{\text{schw.}}a^5$$

Appendices: Hawking radiation

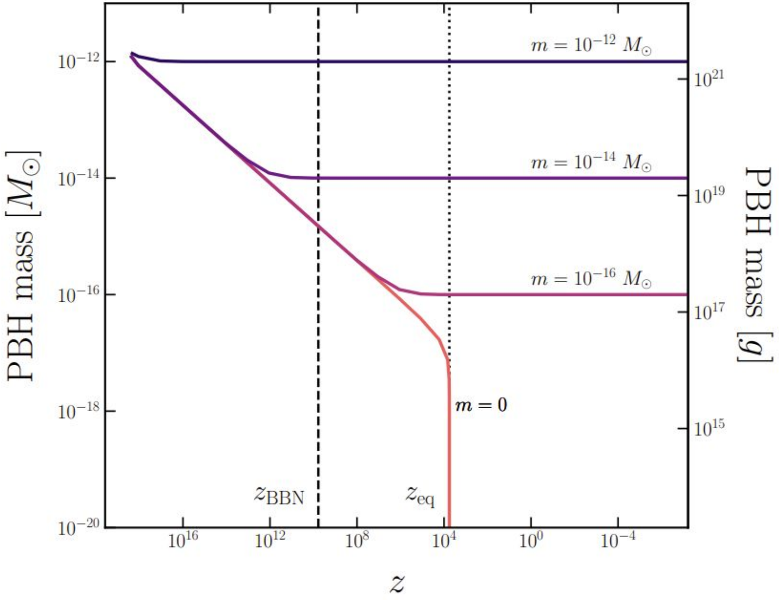
- Hayward's surface gravity:

$$\kappa_{\mathrm{h}} = \frac{1 - 2m'_{\mathrm{MS}}}{2r_{\mathrm{h}}}$$

- Follow 'usual' recipe:

$$\kappa \sim \frac{1}{2gma} - \frac{3\delta}{Gma} \sim 2\kappa_{\mathrm{Schw.}}/a \qquad \delta \equiv HGma < \frac{1}{8}$$

$$\frac{\mathrm{d}U}{d\tau} = -\sigma T^4 A + 2\delta \qquad \frac{\mathrm{d}m}{\mathrm{d}\tau} \sim -\frac{1}{1920\pi G^2 m^2 a^2} \sim \frac{8}{a^2}\frac{\mathrm{d}m}{\mathrm{d}\tau}\bigg|_{\mathrm{Schw.}}$$



Appendices: Farrah et al.

- Main differences:
  - We only assume relevance at early times, transition to ~standard BH after decoupling
  - They are more worried about spin when interpolating between locally Kerr and FLRW
  - No specific metric given in their paper
  - Different mass dependence:
    - k=3

$$M(a) = M(a_i)\left(\frac{a}{a_i}\right)^k$$

Observational Evidence for Cosmological Coupling of Black Holes and its Implications for an Astrophysical Source of Dark Energy

Duncan Farrah[1,2], Kevin S. Croker[2], Michael Zevin[3,4], Gregory Tarlé[5], Valerio Faraoni[6], Sara Petty[7,8], Jose Afonso[9,10], Nicolas Fernandez[11], Kurtis A. Nishimura[2], Chris Pearson[12,13,14], Lingyu Wang[15,16], David L Clements[17], Andreas Efstathiou[18], Evanthia Hatziminaoglou[19], Mark Lacy[20], Conor McPartland[21,22], Lura K Pitchford[23,24], Nobuyuki Sakai[25], and Joel Weiner[26]