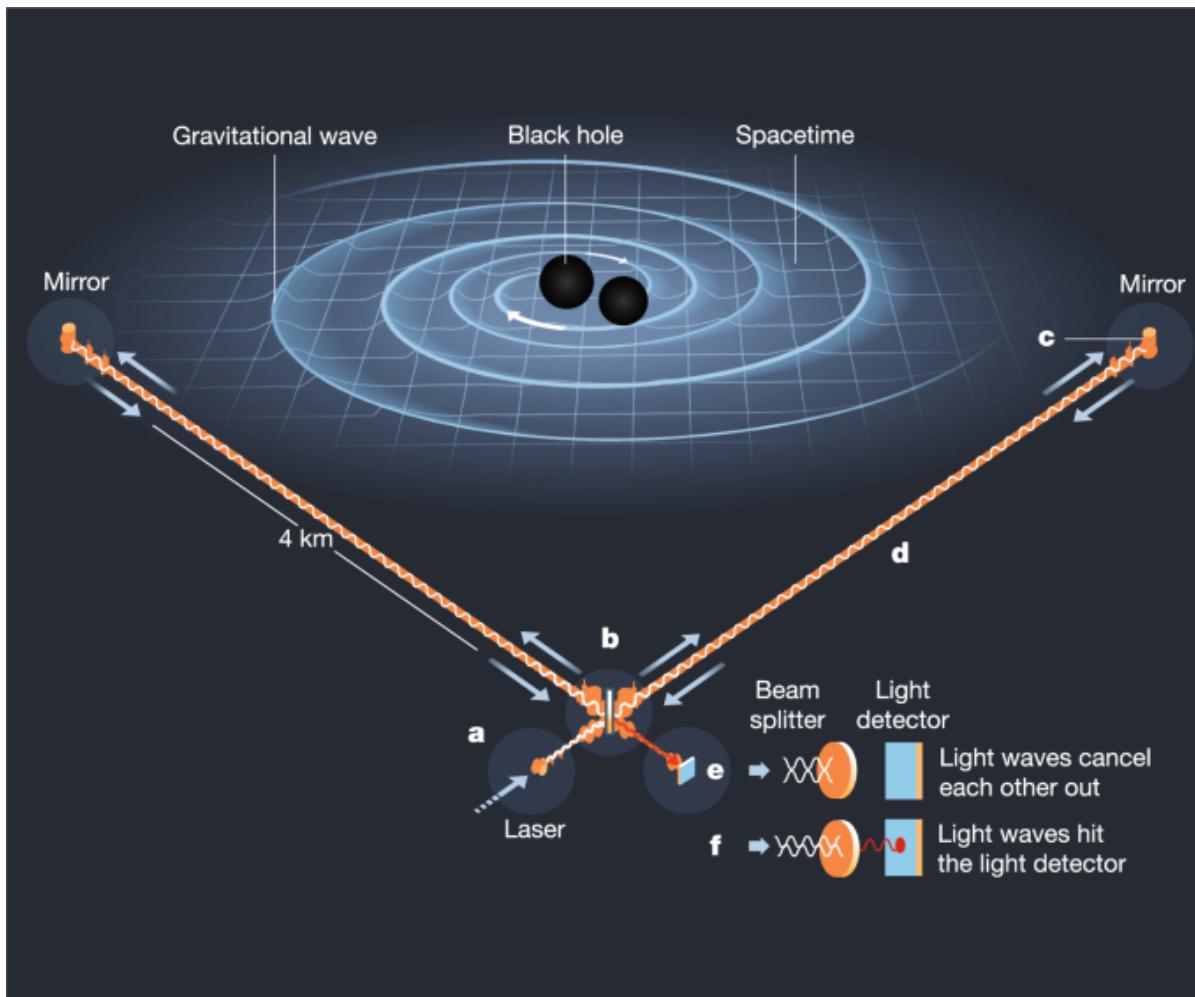


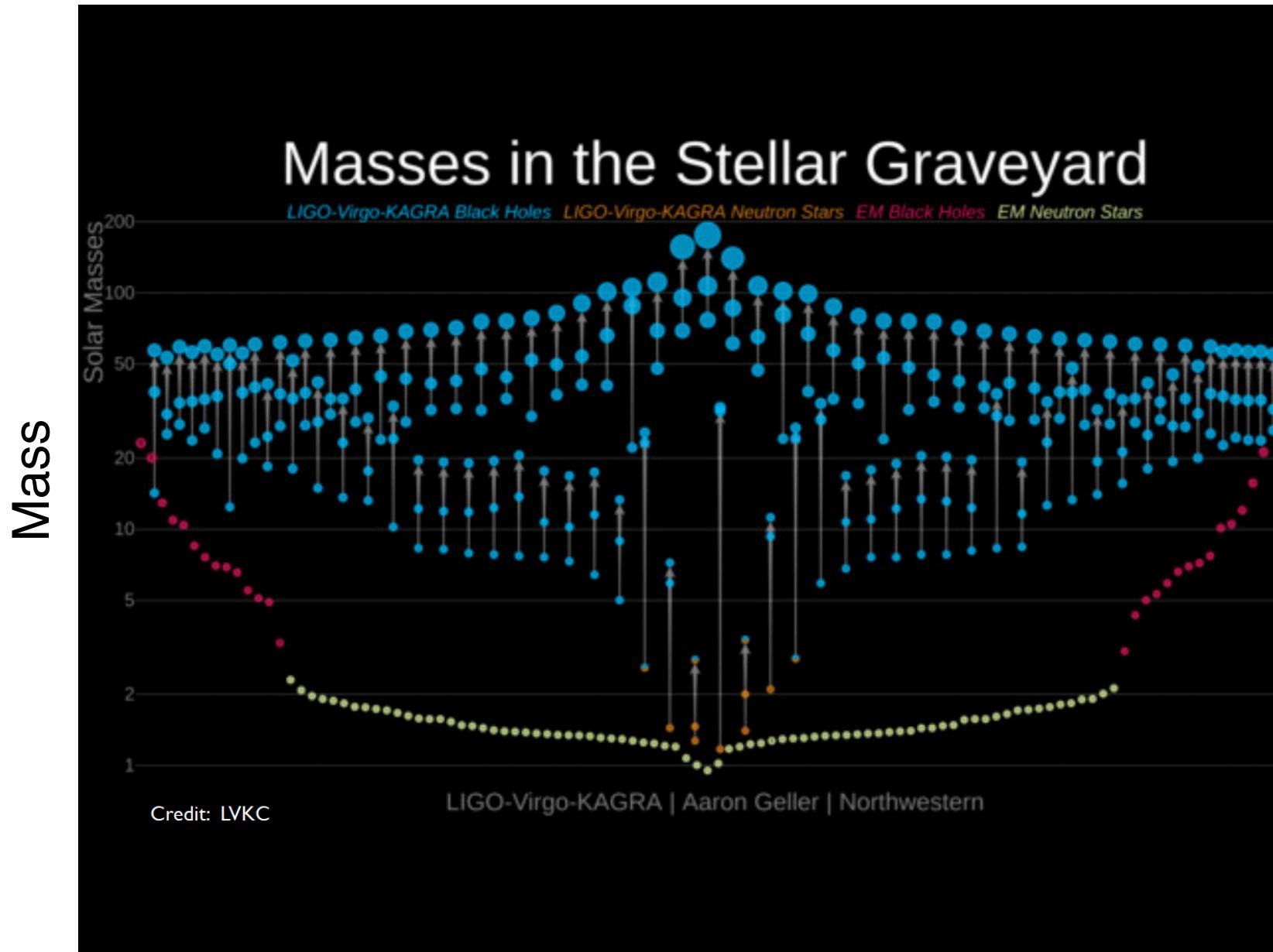
# Primordial Black Holes in the era of Gravitational Wave Astronomy

Antonio Riotto  
University of Geneva

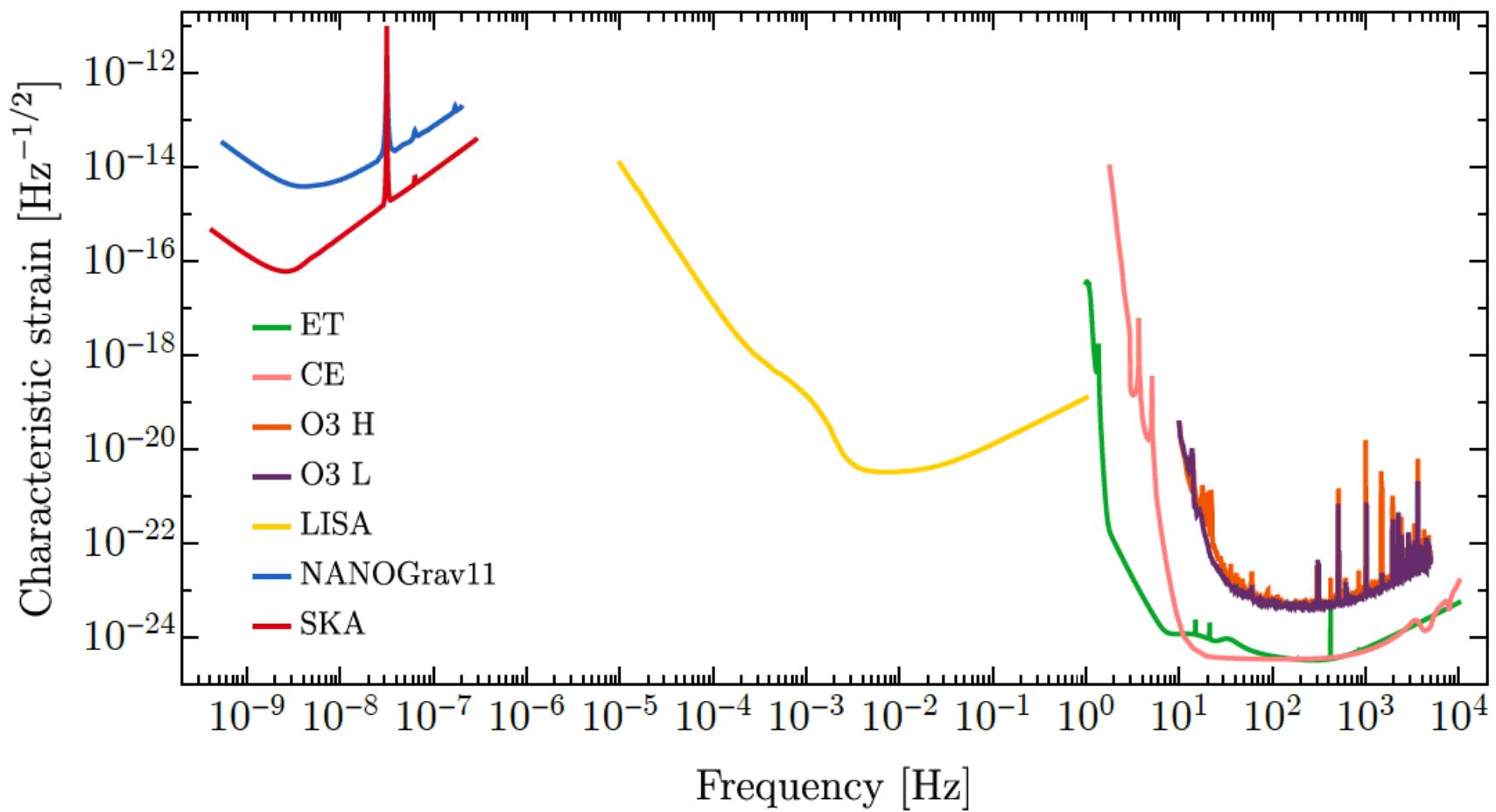
# Gravitational waves and Black Holes are key predictions of General Relativity



# The Era of Gravitational Wave Astronomy



# Current and future sensitivities



# Black Holes

- Astrophysical BHs forms from the gravitational collapse of a star. We know they exist. Their mass must be above the Chandrasekhar limit,

$$M > \mathcal{O}(1) M_{\odot}$$

- PBHs are formed in the early universe. Their mass can be small and they can still be around as long as they do not evaporate within the age of the universe

$$M > 10^{-18} M_{\odot}$$

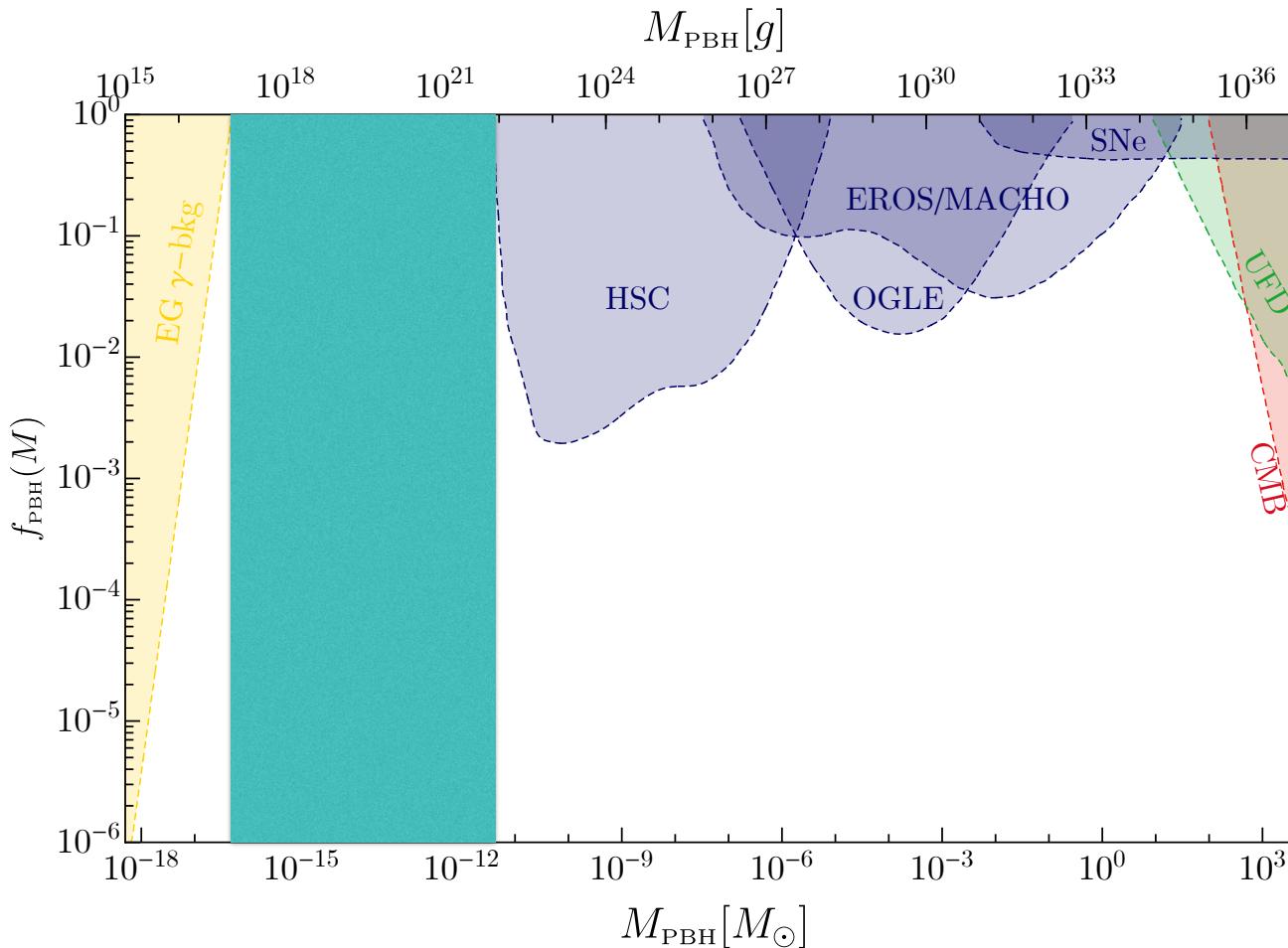
# Key Questions on PBHs in the GW era

- Do (will) PBHs contribute to current (future) GW signals?
- What are the smoking-gun signals of PBHs and how to distinguish them from astrophysical sources?
- Can PBHs account for all the dark matter in the universe?

# PBHs

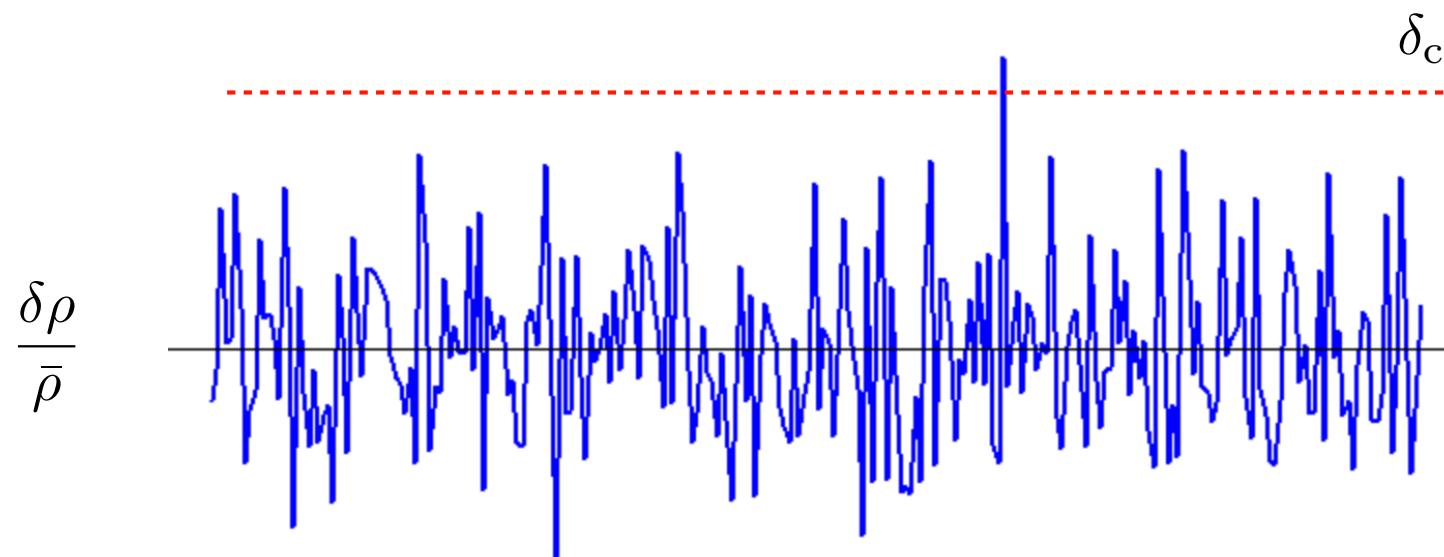
Primordial black holes can compose  
all the dark matter (or a fraction of it)

$$f_{\text{PBH}} = \Omega_{\text{PBH}} / \Omega_{\text{DM}}$$



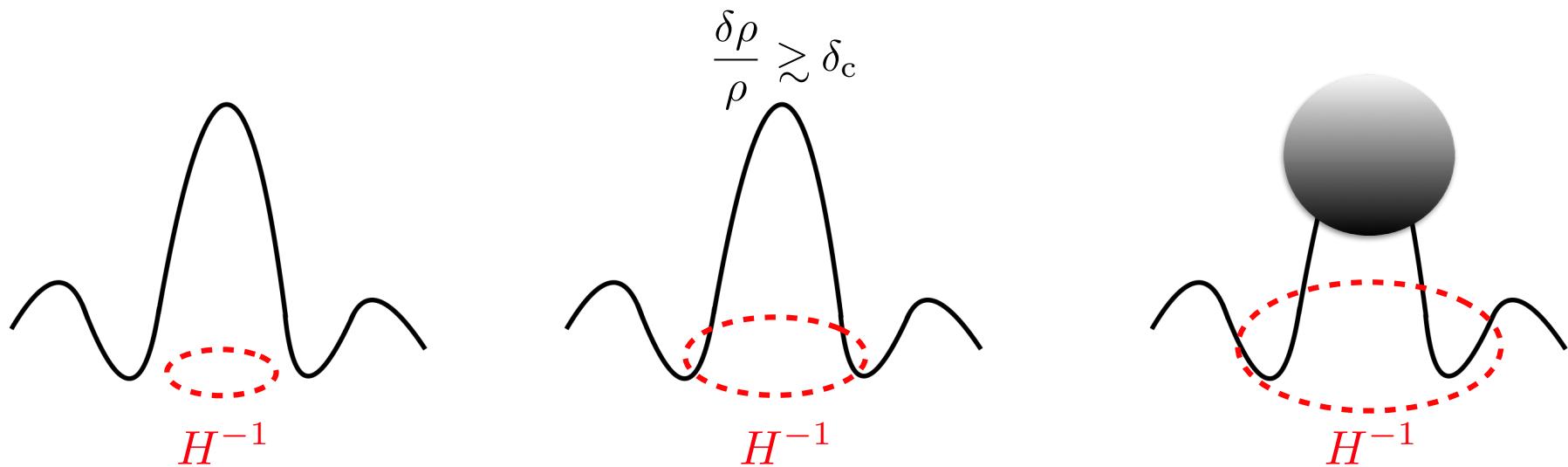
# Where the PBHs may come from?

PBHs may be originated from peaks of the density perturbations generated in the early universe



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PBHs may be originated from peaks of the density perturbations generated in the early universe

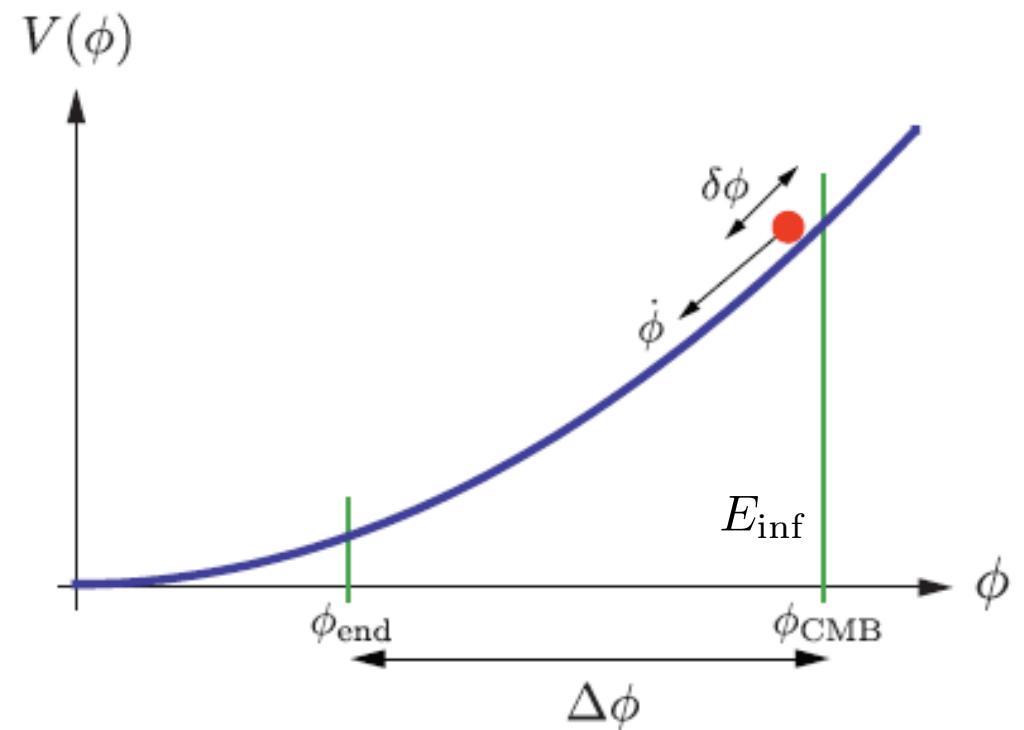
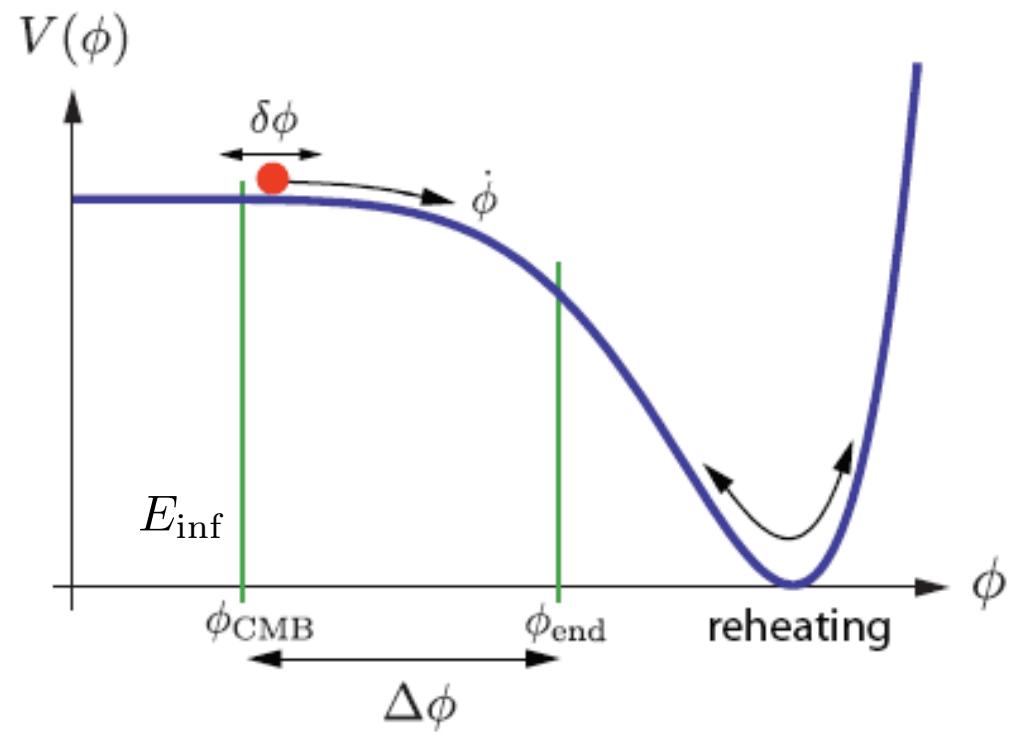


$$M_{\text{PBH}} \sim M_{\text{H}}$$

PBHs are rare events, tail of the distribution

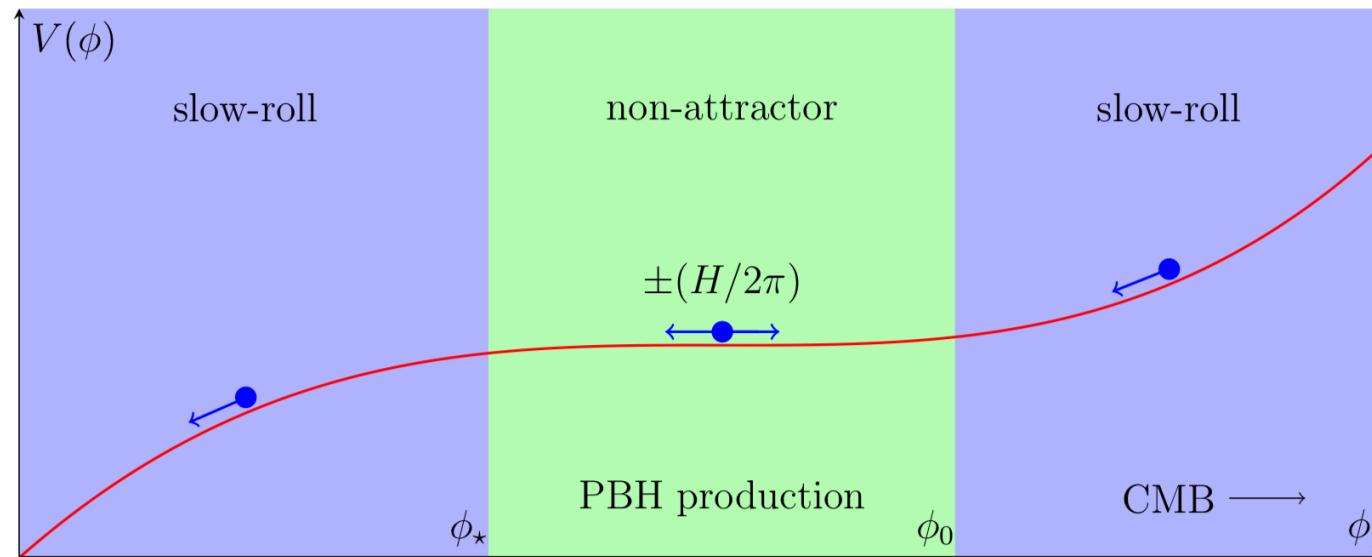
One possible mechanism: large fluctuations from inflation

# Inflation



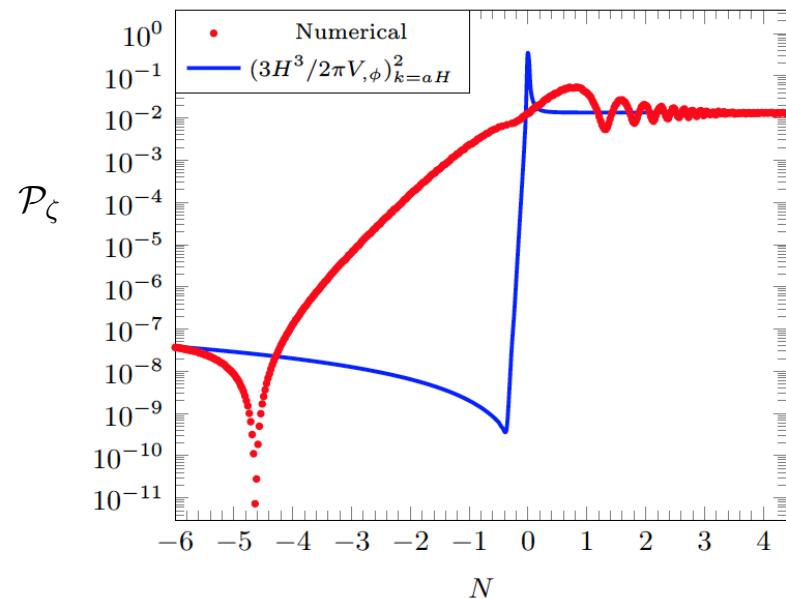
$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{\rho}{3m_P^2} = \frac{E_{\text{inf}}^4}{3m_P^2} \Rightarrow a(t) \sim e^{Ht}$$

# Ultra-slow-roll during inflation

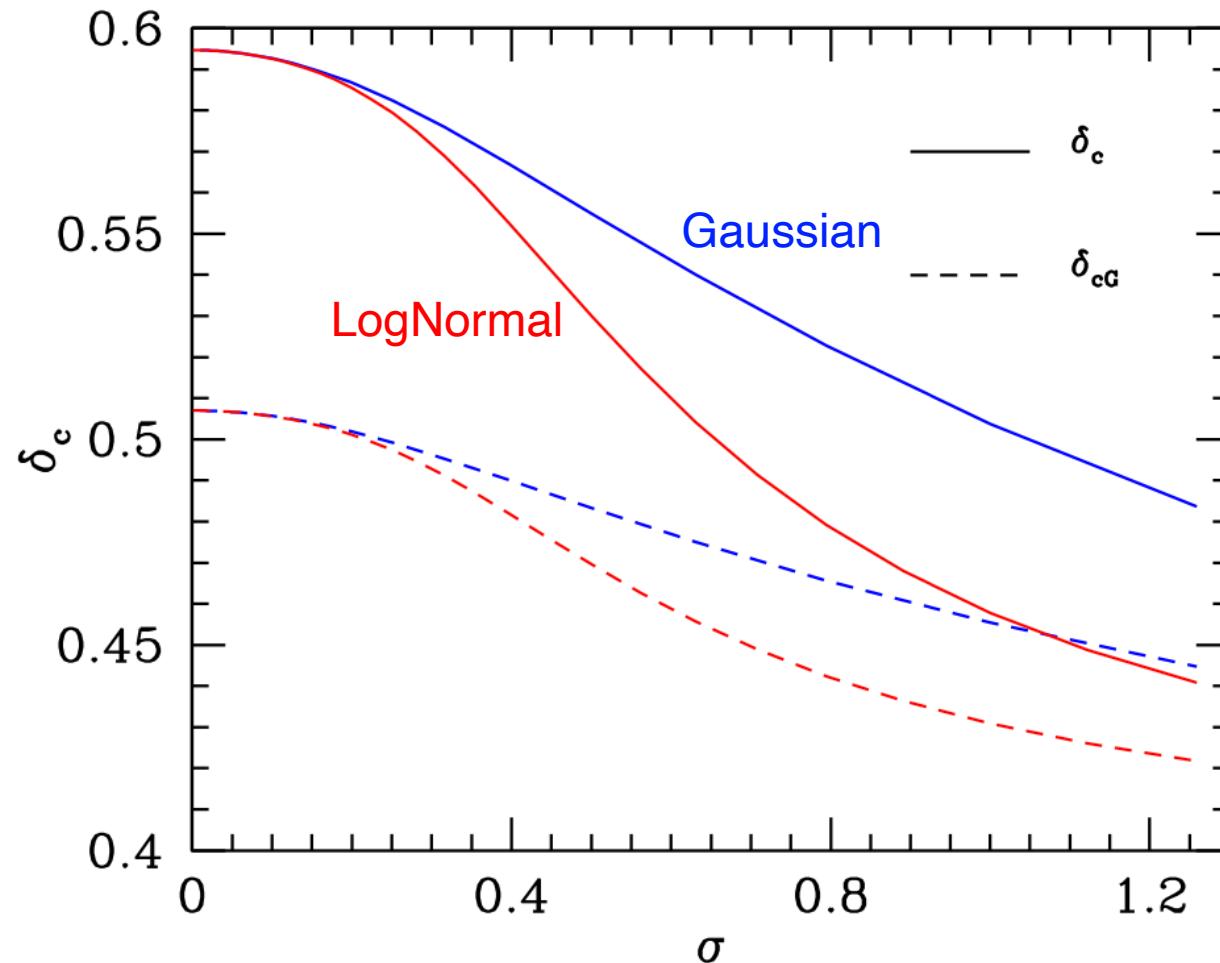


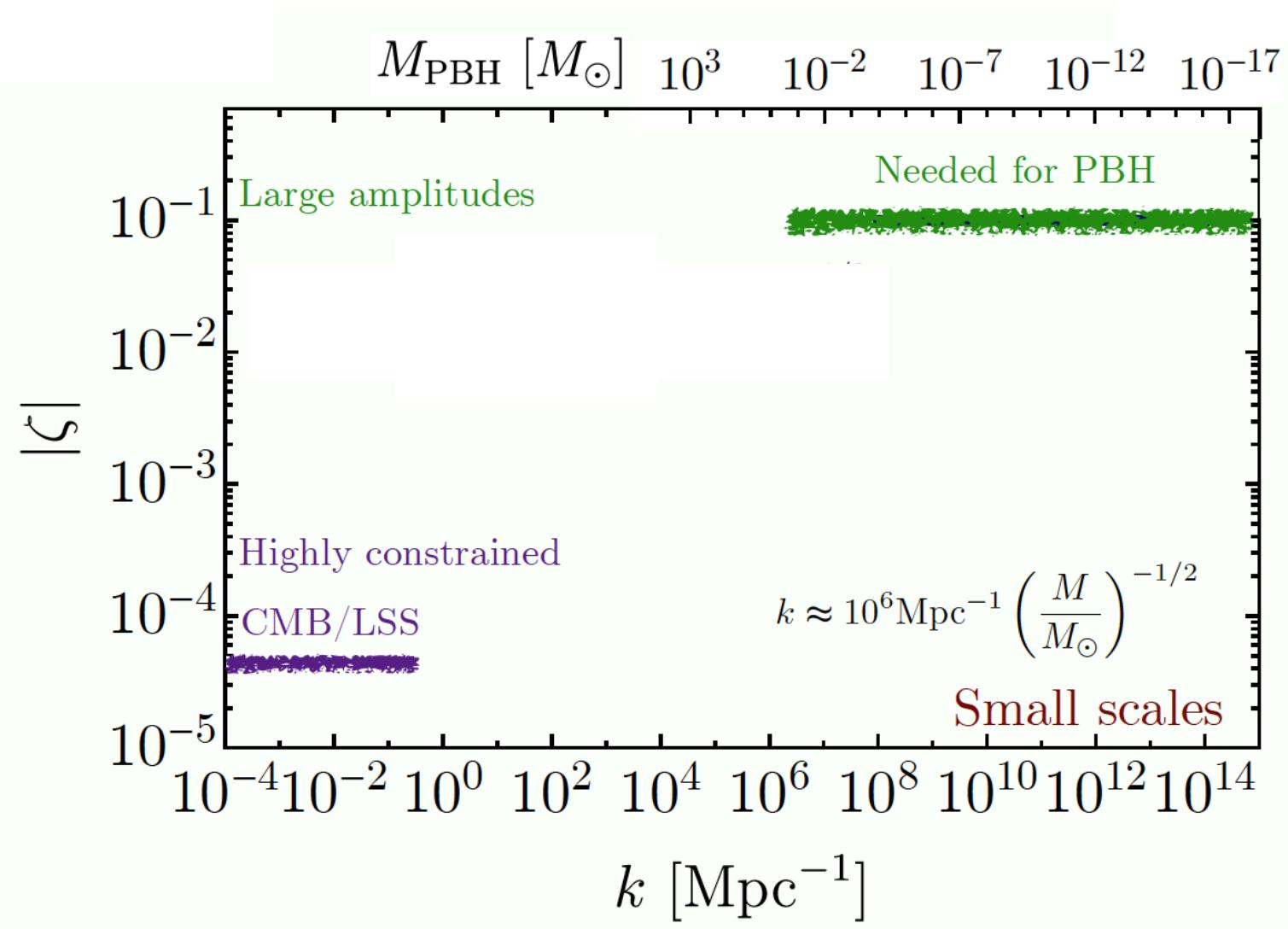
$$\mathcal{P}_\zeta^{1/2} = \frac{H^2}{2\pi |\dot{\phi}|}$$

$$\frac{d\phi}{dN} \sim e^{-3N} \Rightarrow \mathcal{P}_\zeta^{1/2} \sim e^{3N}$$



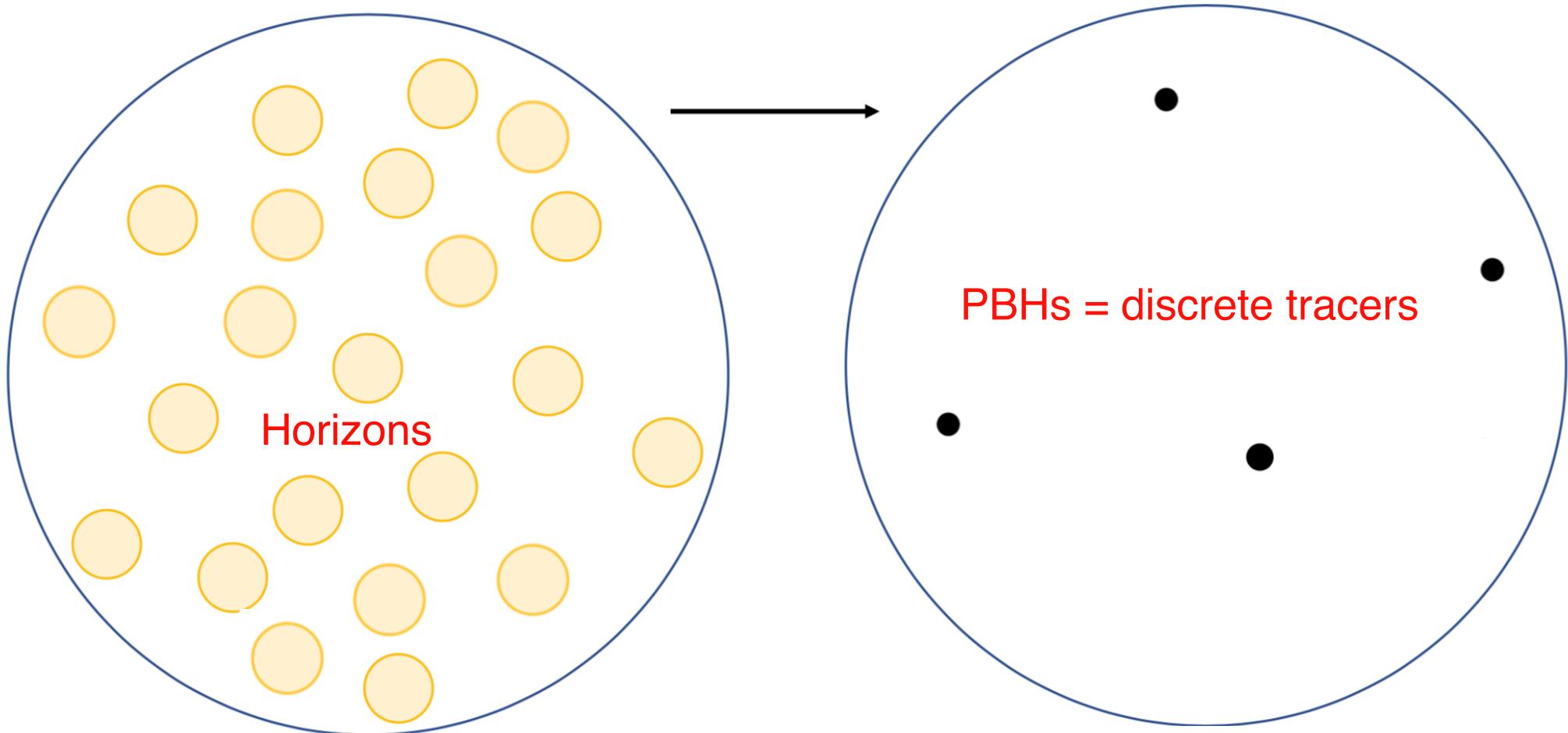
The PBH threshold is NOT a universal number,  
it does depend on the shape of the curvature perturbation





# Properties of PBHs at formation

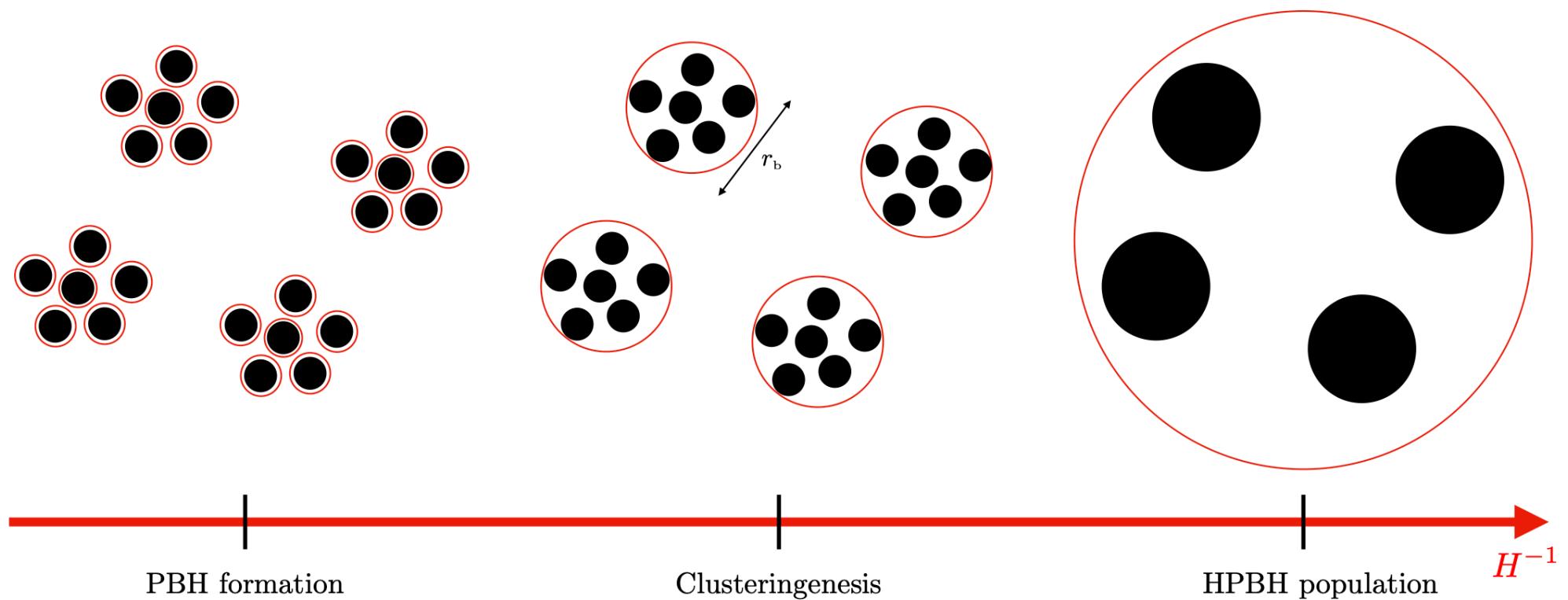
# PBHs are not clustered at formation in the absence of small-large scale correlation



$$\left\langle \frac{\delta\rho_{\text{PBH}}(\vec{x}, z)}{\bar{\rho}_{\text{DM}}} \frac{\delta\rho_{\text{PBH}}(0, z)}{\bar{\rho}_{\text{DM}}} \right\rangle = \frac{f_{\text{PBH}}^2}{n_{\text{PBH}}} \delta_{\text{D}}(\vec{x}) + \xi(x, z)$$

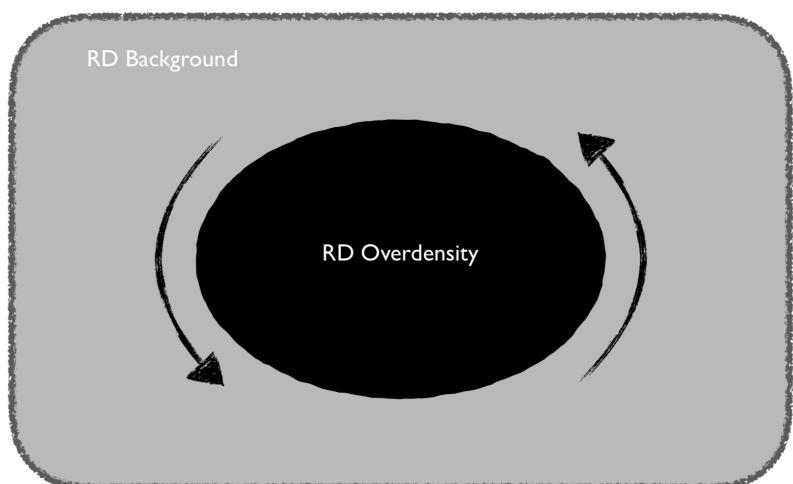
with Desjacques (2018)

# Heavy PBHs from light PBHs in the presence of small-large scale correlation



# The spin of PBHs at formation is small

- PBHs originate from peaks, that is from *maxima* of the local density contrast.
- The spin results from the action of the torques generated by the gravitational tidal forces upon horizon crossing



$$\vec{\chi} = \vec{S}/G_N M_{\text{PBH}}^2$$

$$\chi_i \sim 10^{-2} \sqrt{1 - \gamma^2}$$

Shape of the density power spectrum

# The PBH mass function at formation

Mass distribution dependent on the overdensity perturbation spectrum and statistical properties

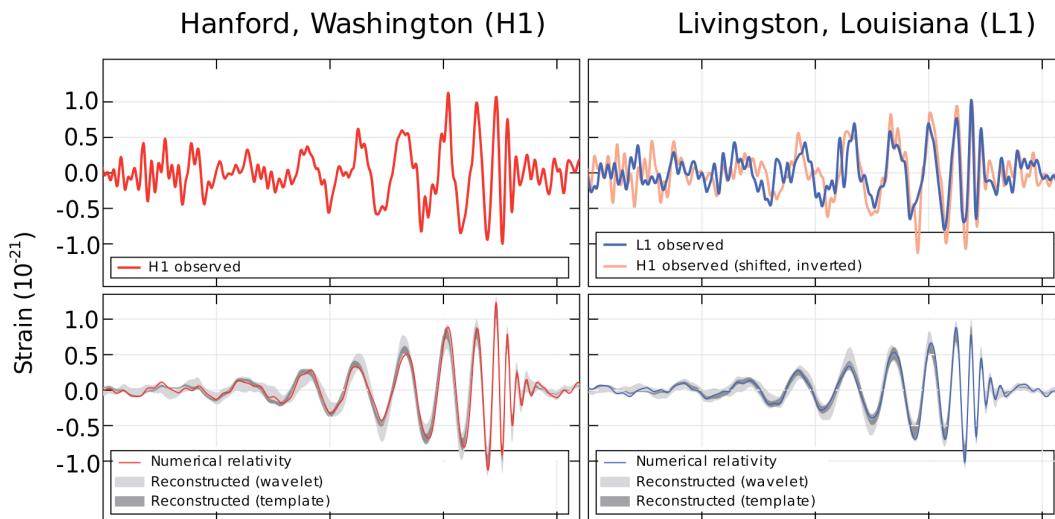
Standard parametrisation

$$\psi(M_{\text{PBH}}) = \frac{1}{\sqrt{2\pi}M_{\text{PBH}}} \exp\left(-\frac{\ln^2(M_{\text{PBH}}/M_c)}{2\sigma^2}\right)$$

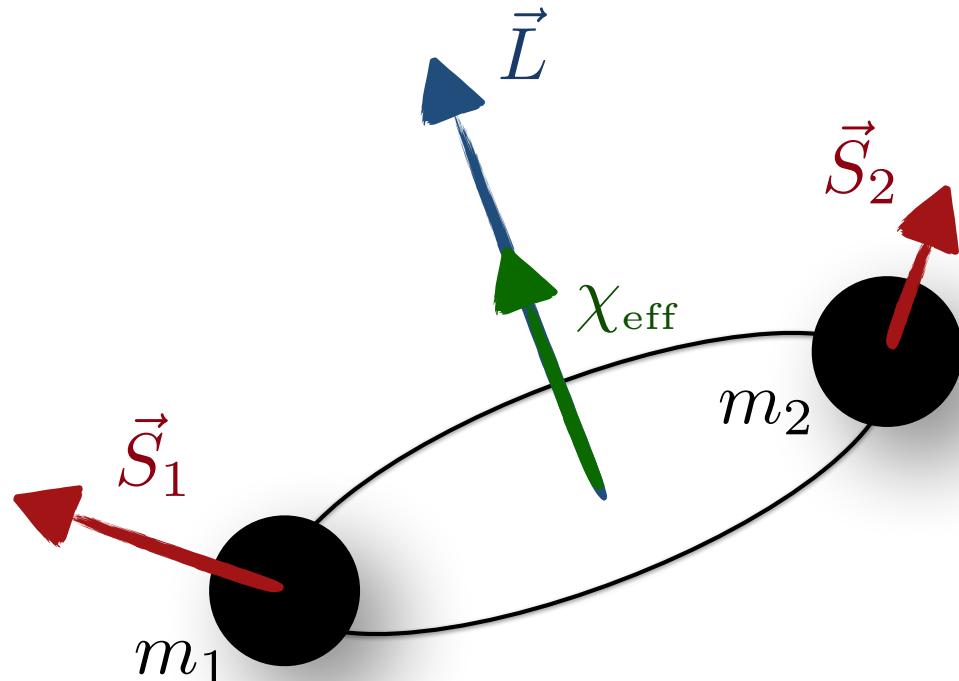
# Key Questions on PBHs in the GW era

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# BH binary



GW150914, LIGO (2016)



Waveforms dependent  
on the binary event parameters

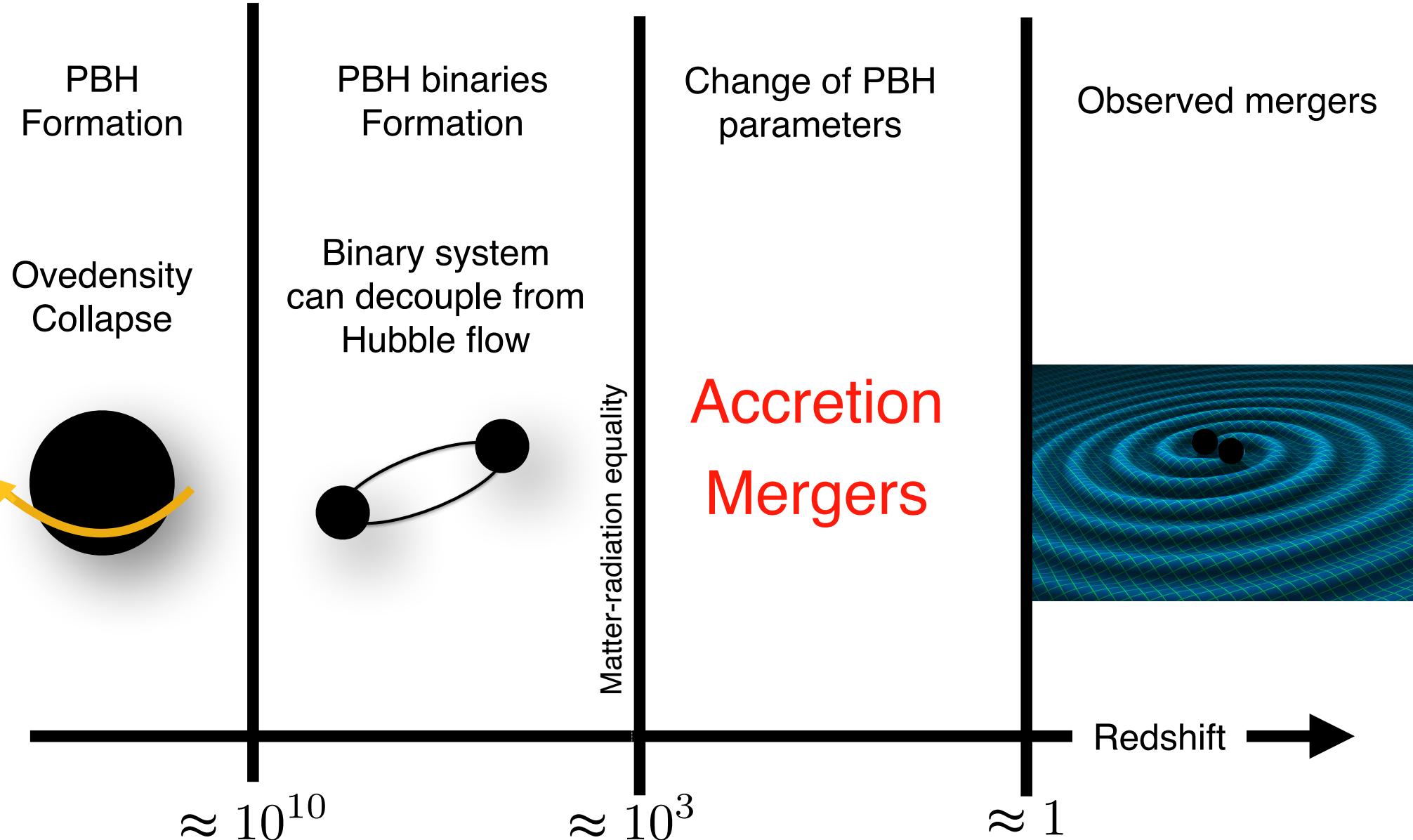
$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

$$q = m_2/m_1$$

$$\chi_{\text{eff}} = \frac{\vec{S}_1/m_1 + \vec{S}_2/m_2}{m_1 + m_2} \cdot \hat{L}$$

...

# PBH evolution



# Accretion onto isolated PBHs

For  $f_{\text{PBH}} < 1$  PBHs coexist with another DM component in the universe

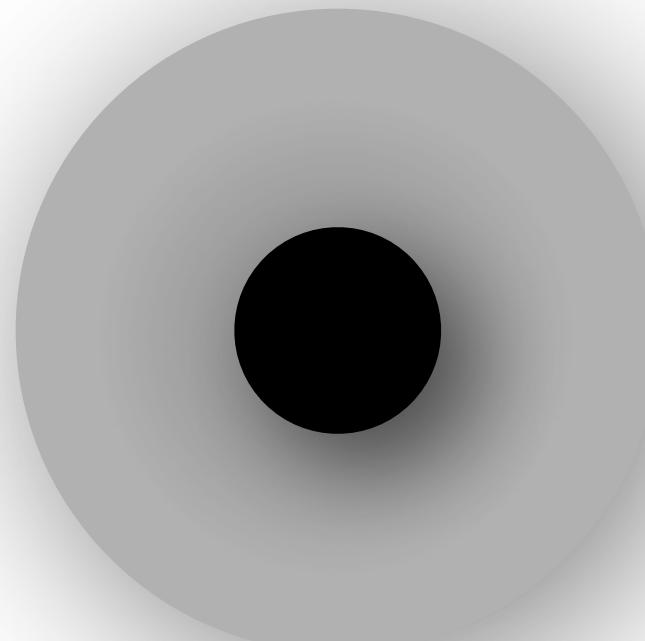
A DM halo builds up around the PBHs  
enhancing accretion

(larger gravitational potential well)

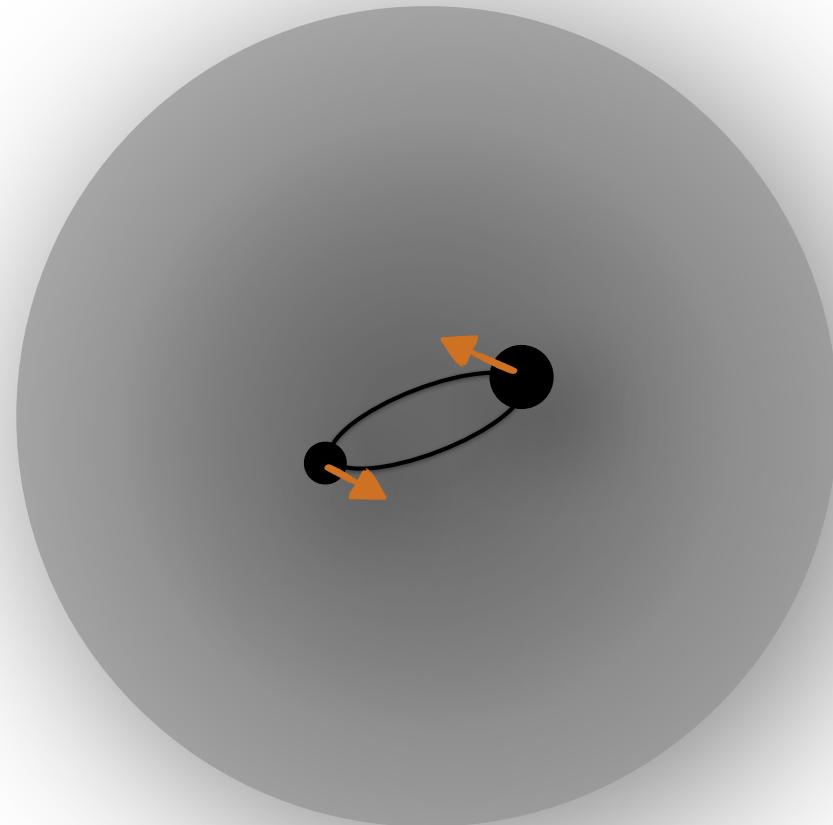
$$M_h(z) \approx 3M_{\text{PBH}} \left( \frac{1000}{1+z} \right)$$

Bondi-Hoyle accretion from the  
surrounding baryonic fluid

$$\dot{M} = 4\pi\lambda m_H n_{\text{gas}} v_{\text{eff}}^{-3} M^2$$



# Accretion onto PBH binaries



Accretion on the system enhances the gas density around the PBH binary

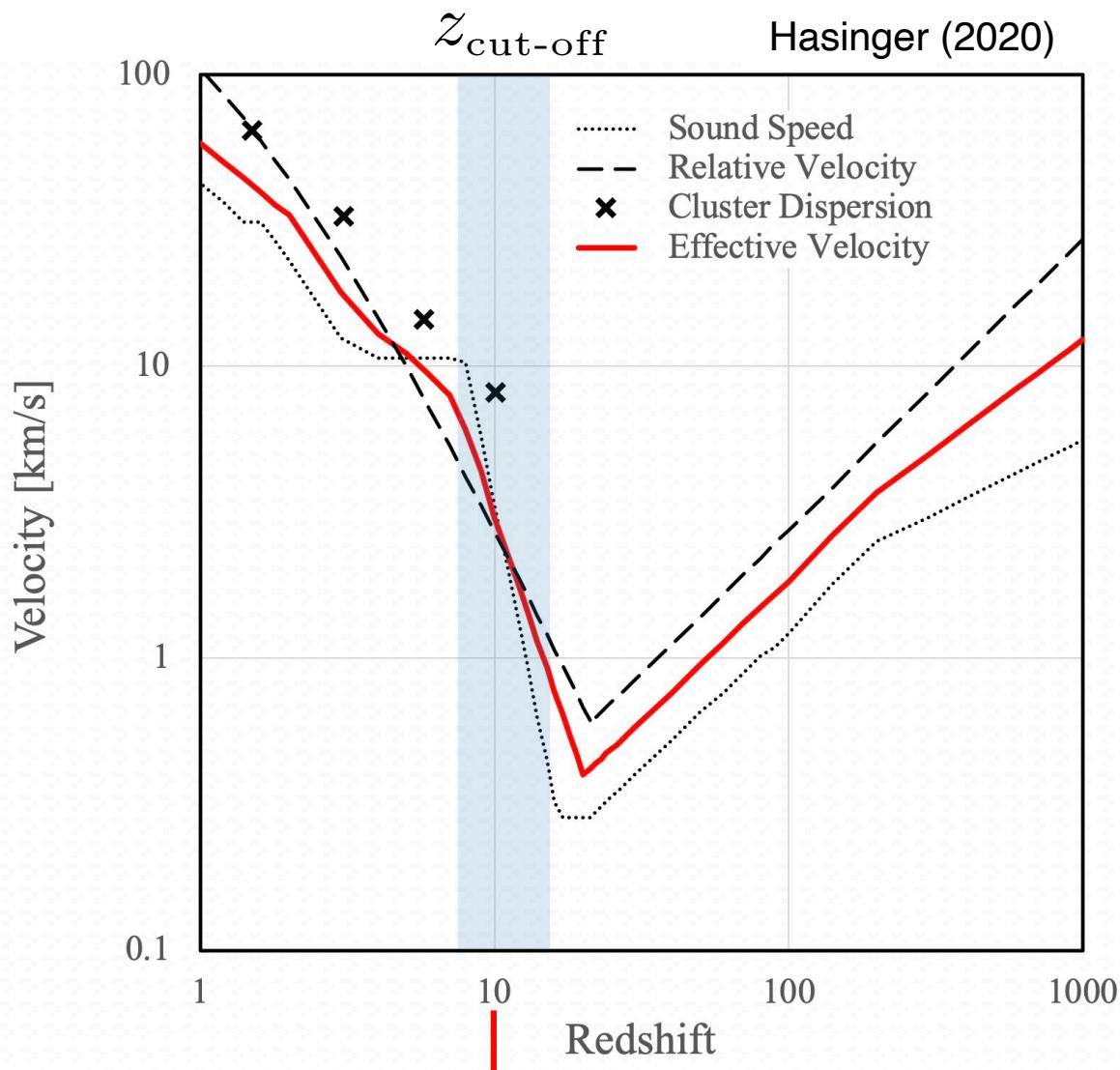
Accretion on the single PBH modulated by masses and orbital velocities

$$\dot{M}_1 = \dot{M} \frac{1}{\sqrt{2(1+q)}}$$

$$\dot{M}_2 = \dot{M} \sqrt{\frac{q}{2(1+q)}}$$

$$q = 1 \quad \text{fixed point}$$

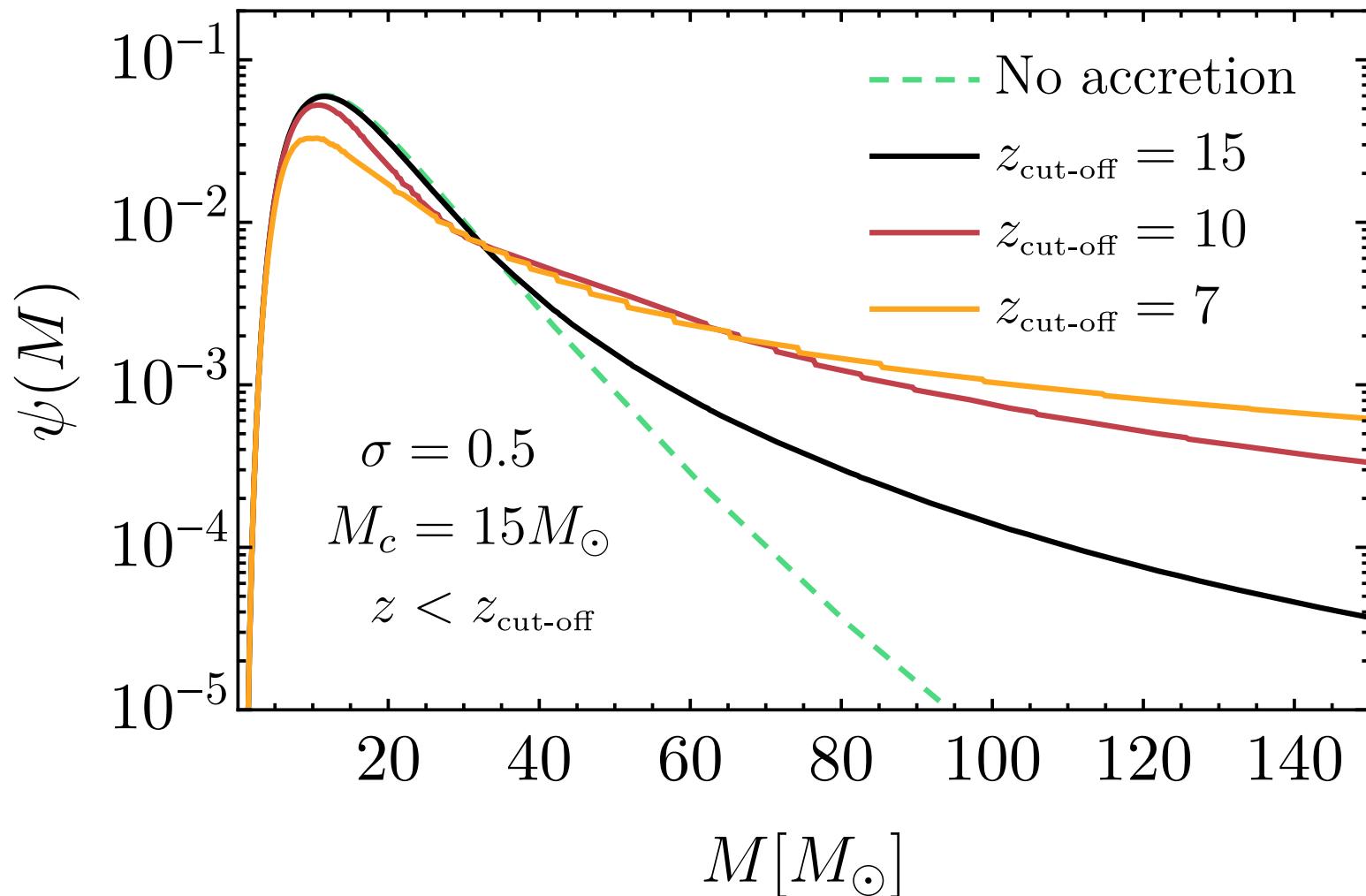
- The smaller PBH always experiences a larger relative accretion
- PBH can experience accretion for  $M \gtrsim \mathcal{O}(10)M_{\odot}$



- Virialised velocities
  - Higher temperatures
- $$\dot{M} \approx (v_{\text{rel}}^2 + c_s^2)^{-3/2}$$
- Strong suppression around  $z_{\text{cut-off}}$
- Uncertainties in the accretion model accounted for by varying the cut-off

Structure formation  
reionization epoch

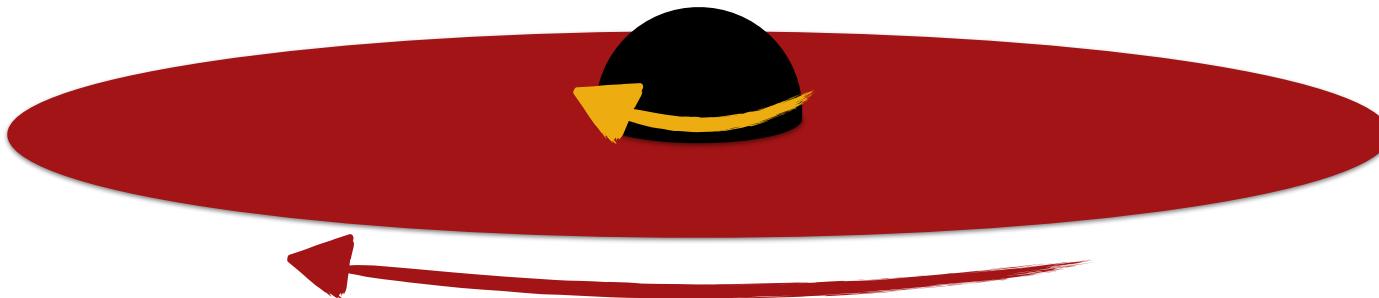
# PBH mass function evolution



Non-linear mass evolution enhances large-mass tails

# PBH spin evolution

If matter angular momentum is large enough, an accreting disk forms, leading to a spin growth



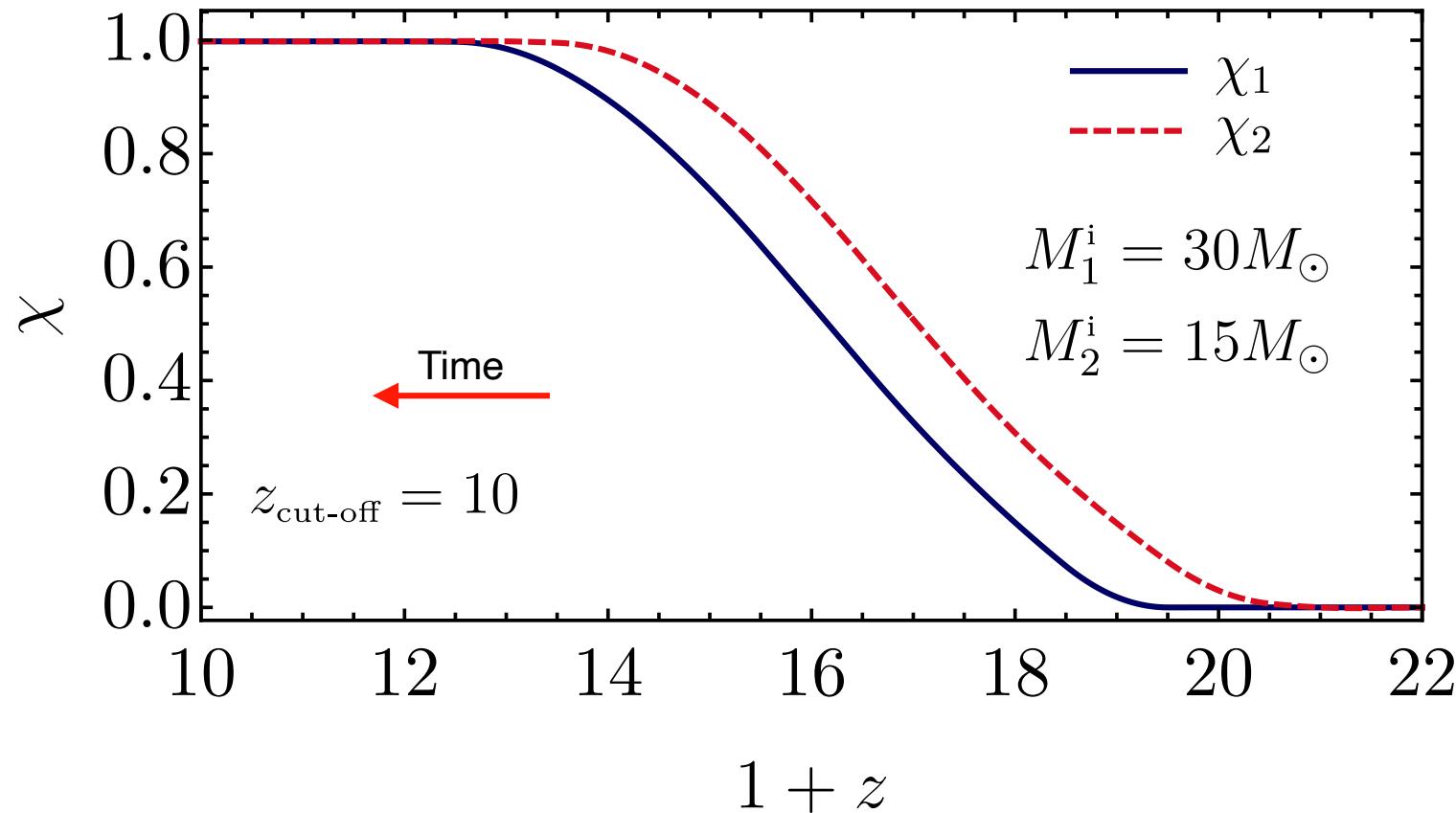
Angular momentum transfer  
between gas and PBH

$$\dot{\chi} = g(\chi) \frac{\dot{M}}{M}$$

by solving the geodesic model of disk accretion

Bardeen et al. (1972)

# Spins pushed towards extremality



- Uncorrelated spin orientation
- Effective spin spreads around zero
- Accretion: low/large mass - low/large spin correlation

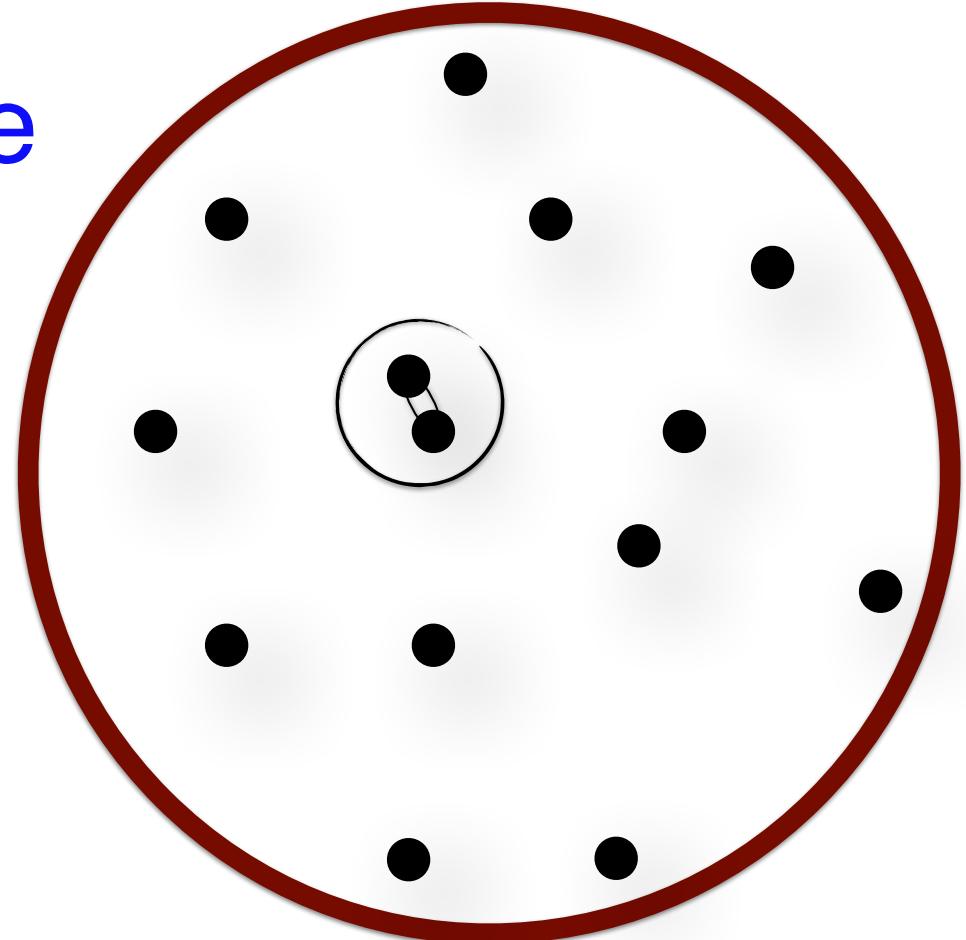
# Merger rate

- Initial spatial Poisson distribution
- Random decoupling of binary systems



Compute probability of decoupling  
and the binary initial geometry

- Semi-major axis
- Eccentricity



Raidal et al (2018)

$$\frac{dR}{dm_1 dm_2} = \frac{1.6 \times 10^6}{\text{Gpc}^3 \text{yr}} f_{\text{PBH}}^{\frac{53}{37}} \eta^{-\frac{34}{37}} \left( \frac{t}{t_0} \right)^{-\frac{34}{37}} \left( \frac{M_{\text{tot}}}{M_\odot} \right)^{-\frac{32}{37}} S(M_{\text{tot}}, f_{\text{PBH}}) \mathcal{A}_{\text{acc}}(m_j) \psi(m_1) \psi(m_2)$$

- Accretion hardens the binaries
- Larger masses leads to shorter mergers



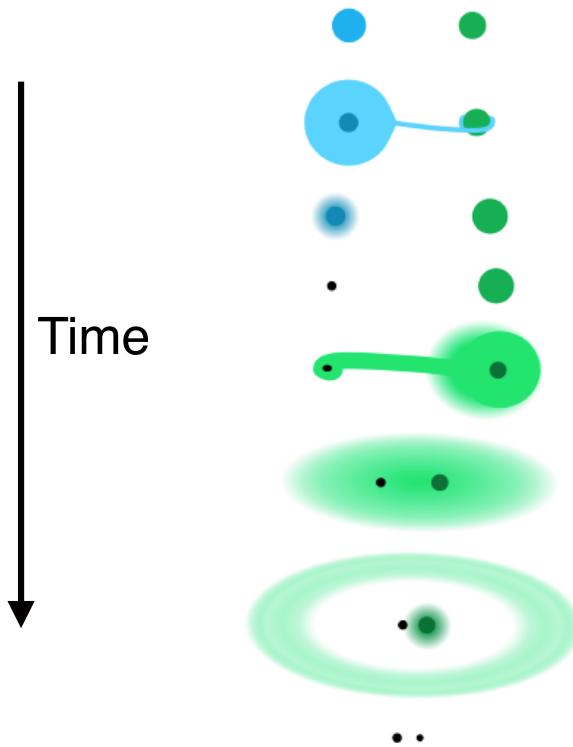
De Luca et al. (2020)

# Astrophysical populations

Zevin et al. (2021)

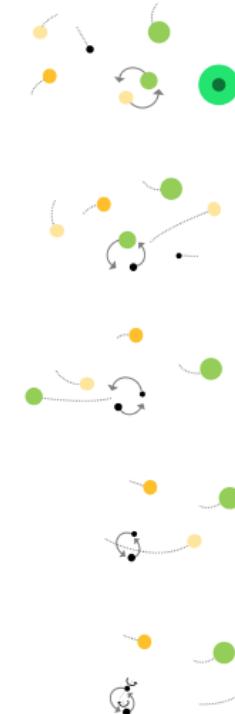
## Isolated formation

Binary formation in galactic fields  
through a Stable Mass Transfer (SMT)  
or Common-Envelope (CE) phase



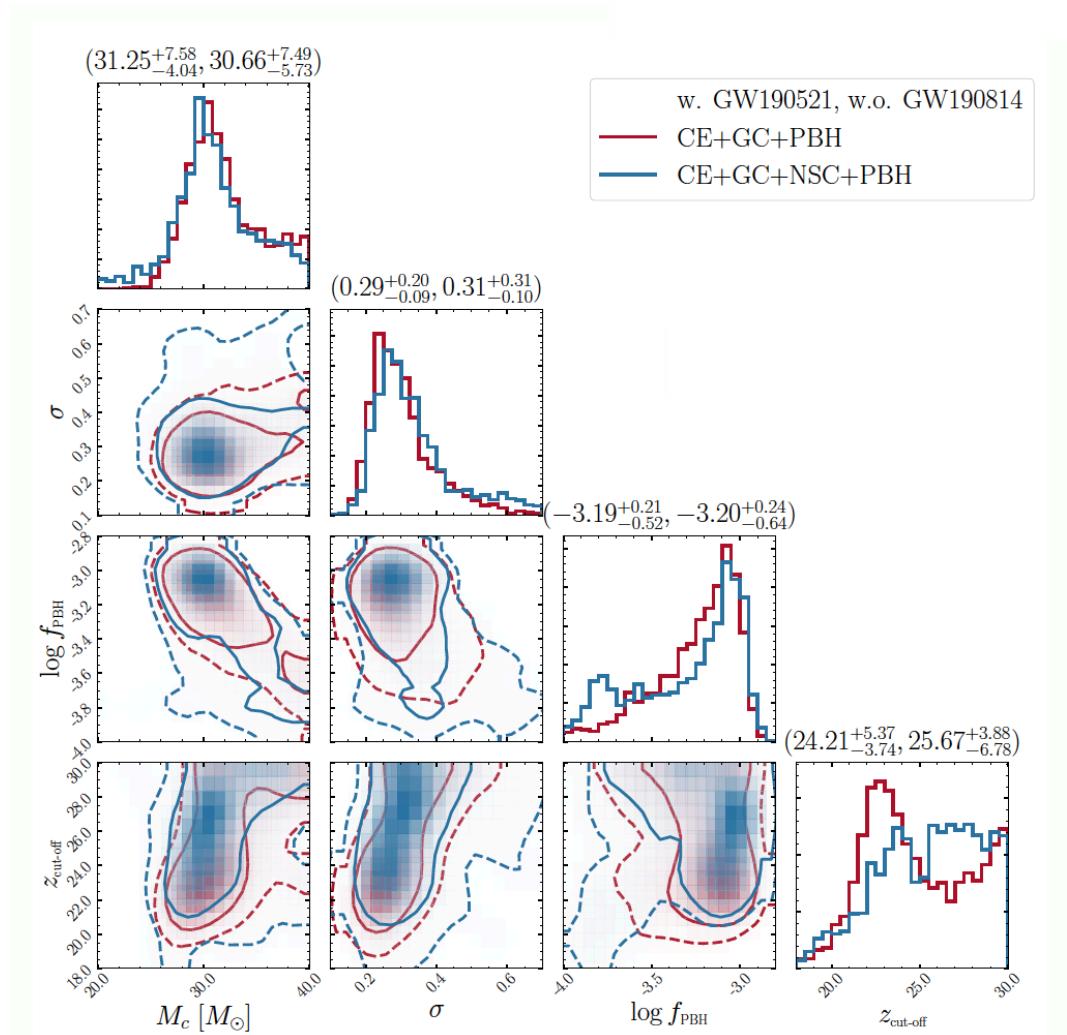
## Dynamical formation

Binary formation in Globular Cluster (GC)  
or Nuclear Star Clusters (NCS)  
through encounters and GW captures



For a review, Mandel and Farmer (2018)

# Population posterior distributions



$$M_c \simeq 30 M_\odot$$

$$\sigma \simeq 0.3$$

$$f_{\text{PBH}} \simeq 6 \cdot 10^{-4}$$

$$z_{\text{cut-off}} \simeq 25$$

PBH not the dark matter

Moderate accretion

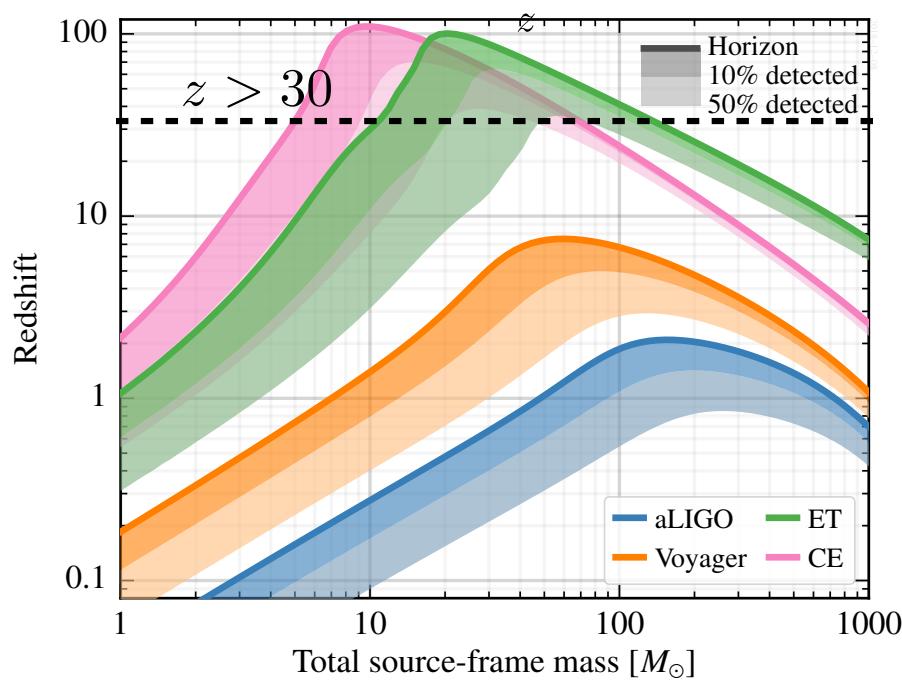
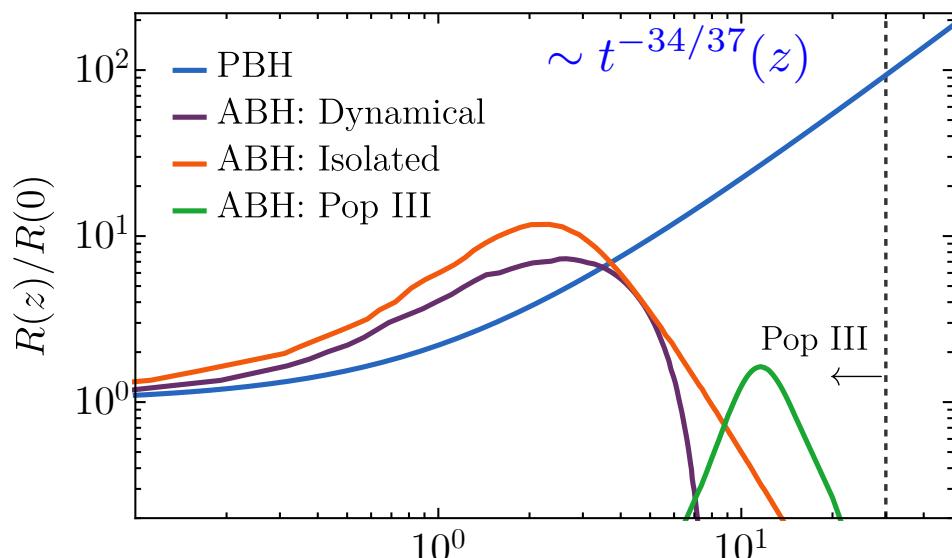
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# Smoking-gun signals of PBHs

- Merger rate time evolution at high redshifts
- Spin of PBHs (large spins for large masses)
- Stochastic GW background from PBHs at high redshifts

K. Ng et al. (2020)

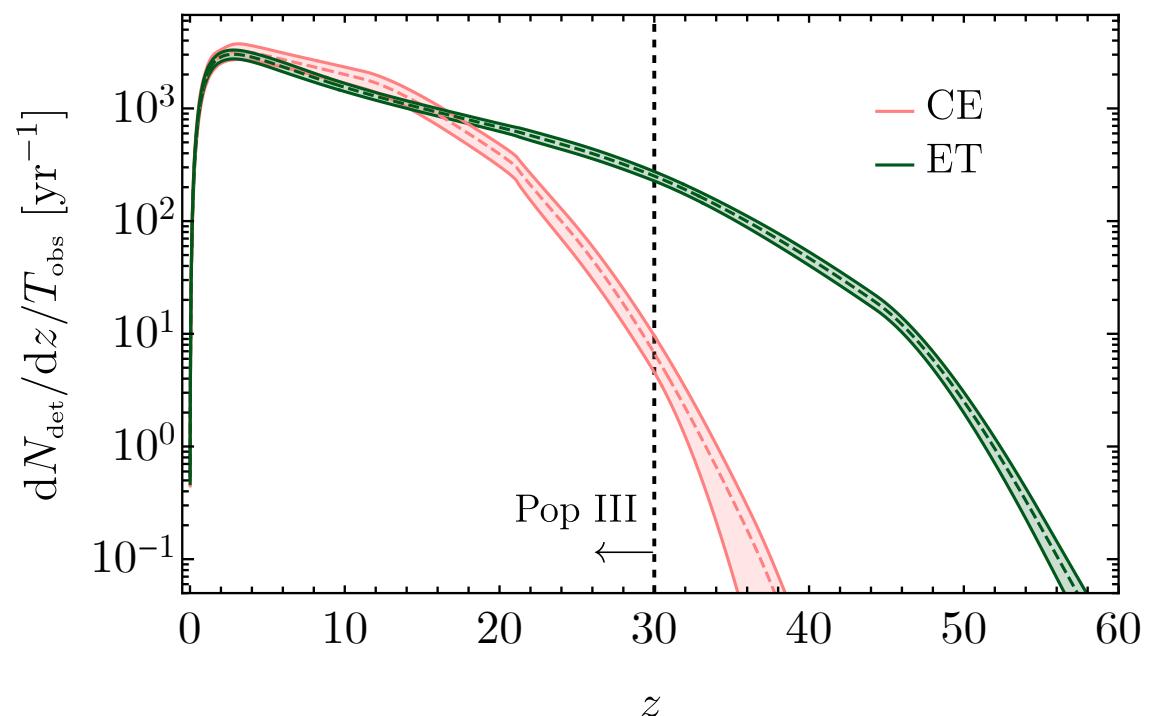


- The PBH population would imply high-redshift observations:

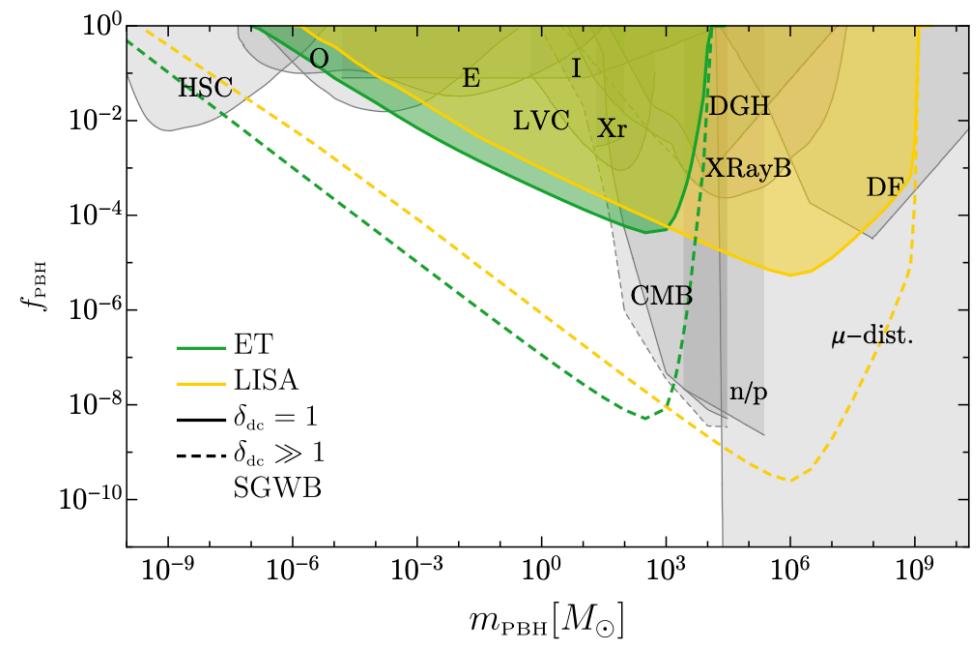
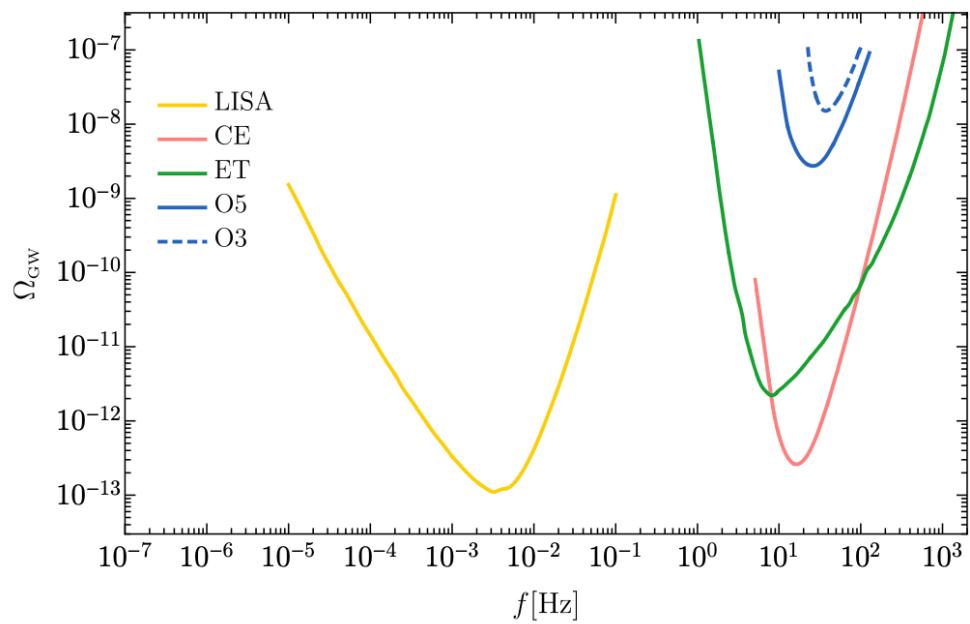
$$N_{\text{det}}^{\text{ET}}(z > 30) = 1315^{+305}_{-168} / \text{yr}$$

No astrophysical contamination

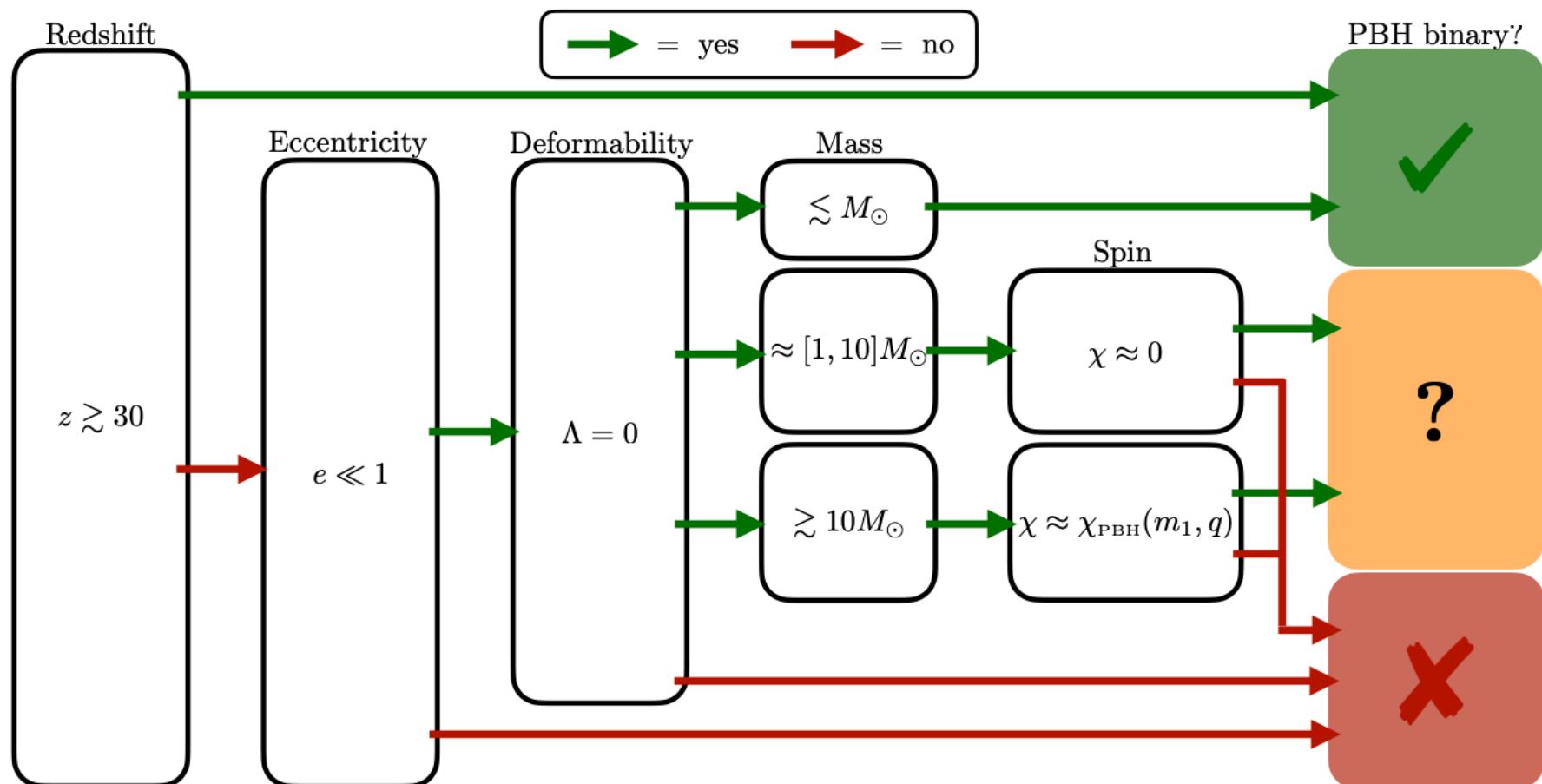
V. De Luca et al. (2021)



# The minimum testable PBH abundance



# Flowchart to assess PBHs



# Key Questions on PBHs in the GW era

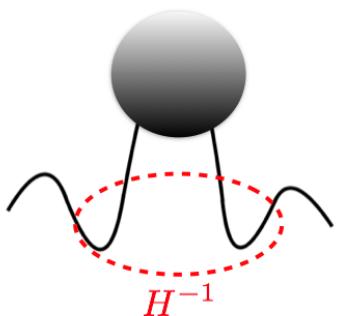
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# PBHs and the stochastic background of GWs

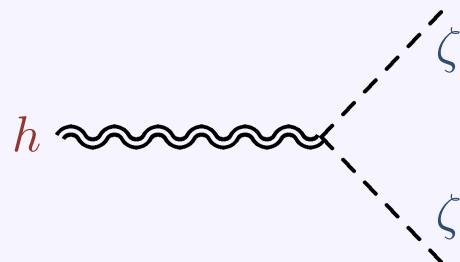
# GWs from PBHs

The same curvature perturbations giving rise to PBHs are unavoidably a source for GWs at *second-order* in perturbation theory

$$\frac{\delta\rho}{\bar{\rho}} \sim \frac{\nabla^2\zeta}{a^2 H^2}$$



$$h''_{ij} + 2\mathcal{H}h''_{ij} - \nabla^2 h_{ij} = \mathcal{O}(\partial_i \zeta \partial_j \zeta)$$



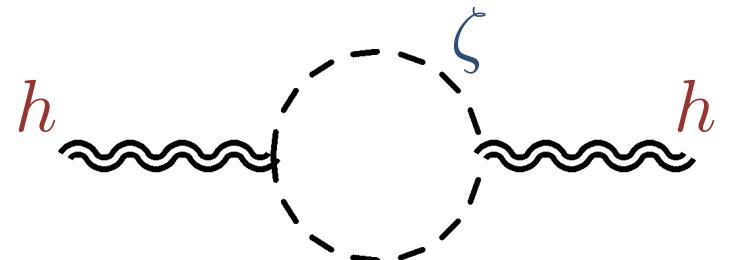
Potentially observable at current and future GW observatories



# GW Power Spectrum

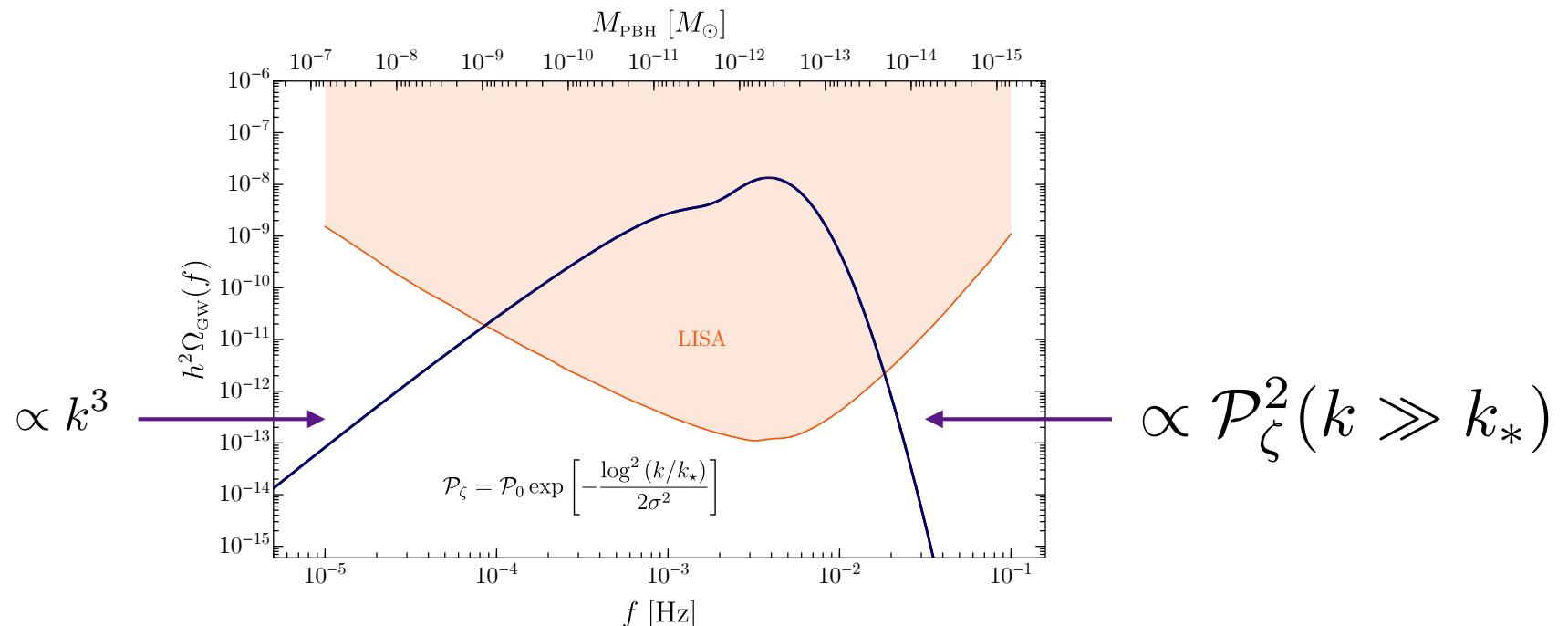
Power spectrum of GWs:

$$\left\langle h^{\lambda_1}(\eta, \vec{k}_1) h^{\lambda_2}(\eta, \vec{k}_2) \right\rangle' \approx \mathcal{P}_\zeta \mathcal{P}_\zeta$$

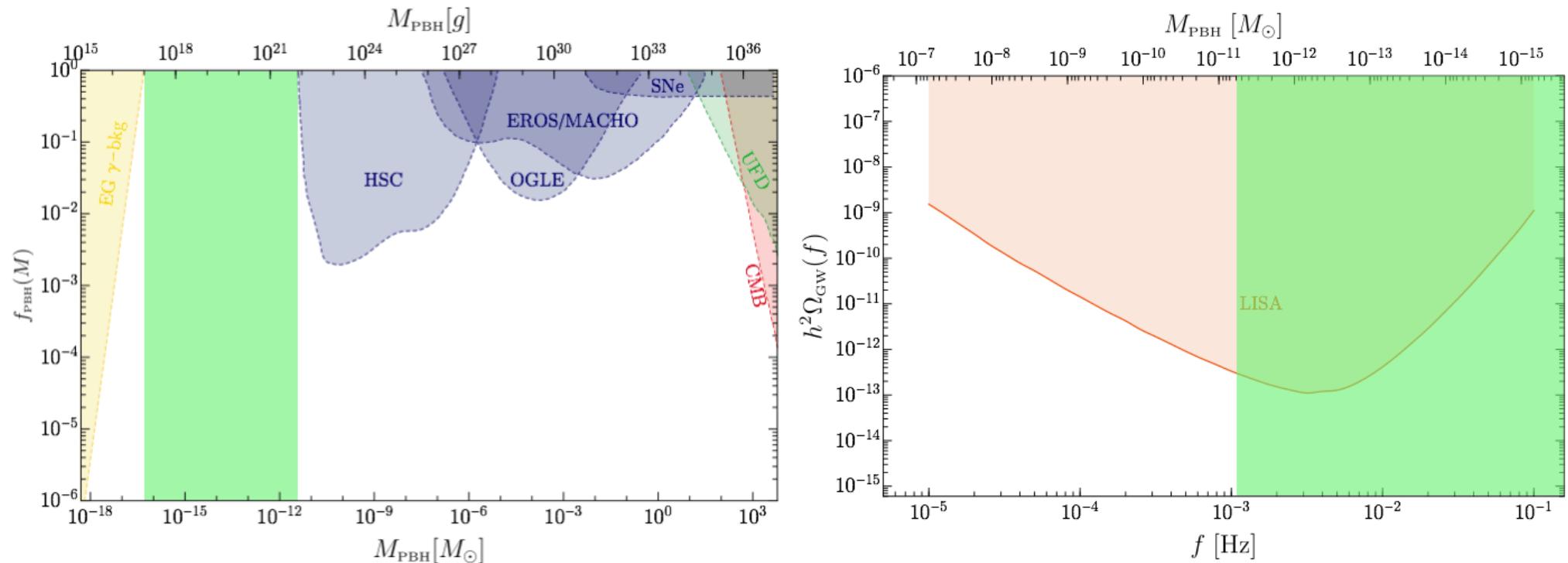


At second order in comoving curvature perturbation, after averaging over the fast oscillating pieces

$$\Omega_{\text{GW}}(\eta, k) = \frac{\pi^2}{243\mathcal{H}^2\eta^2} \int \frac{d^3 p}{(2\pi)^3} \frac{p^4 [1 - \mu^2]^2}{p^3 |\vec{k} - \vec{p}|^3} \boxed{\mathcal{P}_\zeta(p) \mathcal{P}_\zeta(|\vec{k} - \vec{p}|)} \boxed{\mathcal{I}^2(\vec{k}, \vec{p})}$$



# The PBH dark matter-LISA serendipity



$$M \simeq 10^{-12} M_{\odot} \left( \frac{f_{\text{LISA}}}{f} \right)^2$$

$$f_{\text{LISA}} = 3.4 \text{ mHz}$$

$$M \approx 10^{-12} M_{\odot}$$

Bartolo et al. PRL (2019)

# Nano-Grav 12.5 year



Millisecond pulsars whose signal sensitive to the stochastic GW background

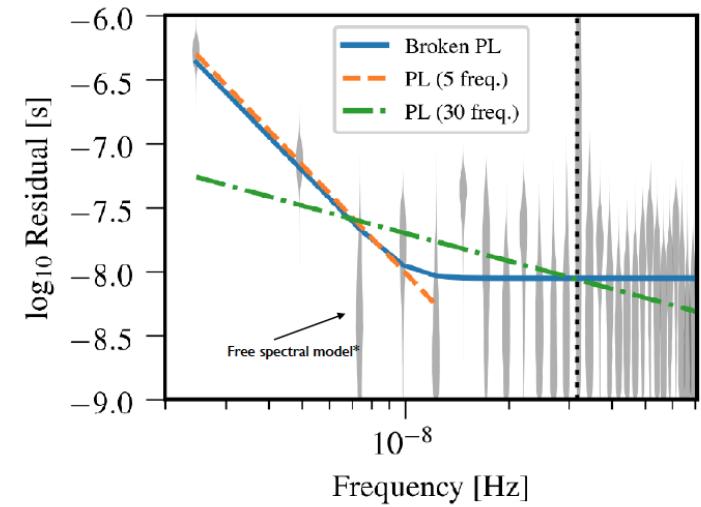
Cross-correlation of  
timing residuals

$$S_{ab} = \Gamma_{ab} \frac{h_c^2}{12\pi^2 f^3}$$

# Nano-Grav 12.5 year

Strong evidence for a stochastic common process across 45 pulsars

$$\Omega(f) = \frac{2\pi^2}{3H_0^2} A^2 f_{\text{yr}}^2 \left( \frac{f}{f_{\text{yr}}} \right)^{5-\gamma}$$

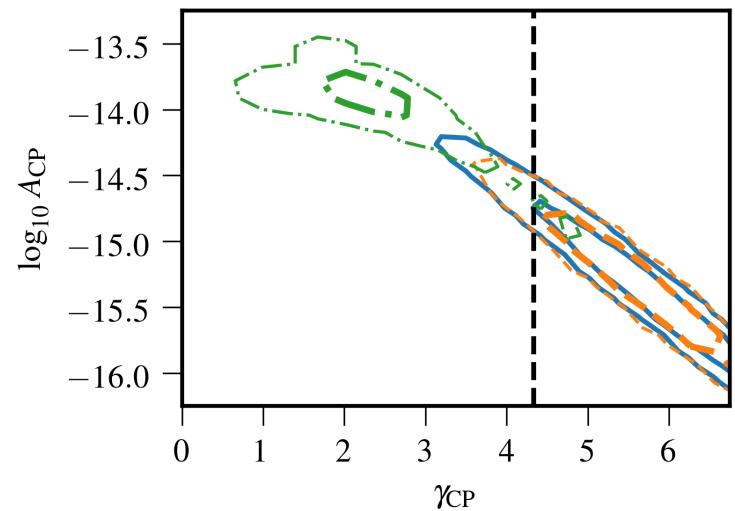


Possible flat spectrum with amplitude  $\Omega(f) \sim 5 \cdot 10^{-10}$

# Nano-Grav 12.5 year

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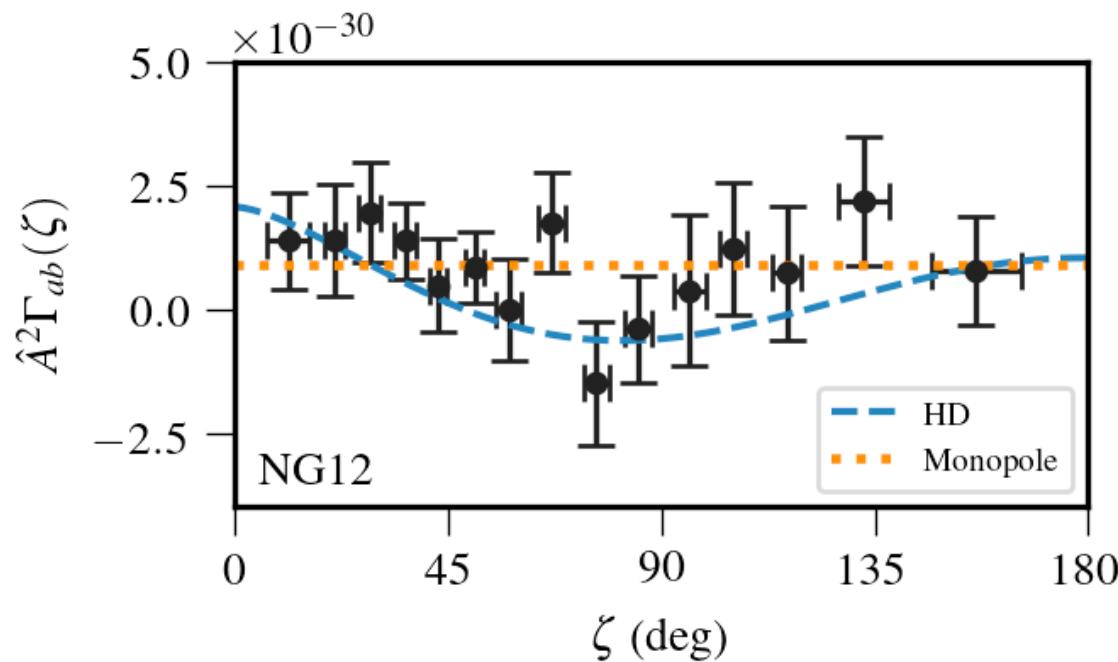
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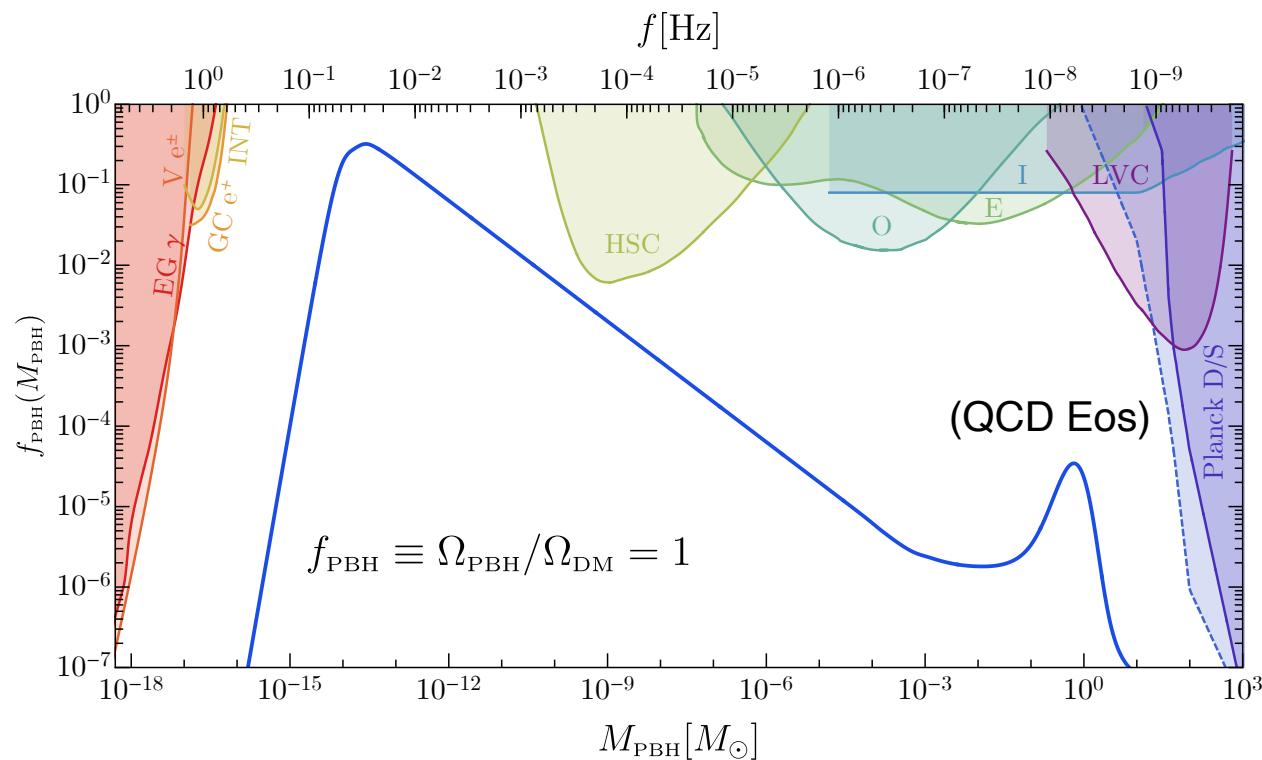
# Nano-Grav 12.5 year

Non-conclusive evidence for quadrupolar Hellings-Downs (HD) correlation pattern (GW footprint)



Need to wait for more data (soon!)

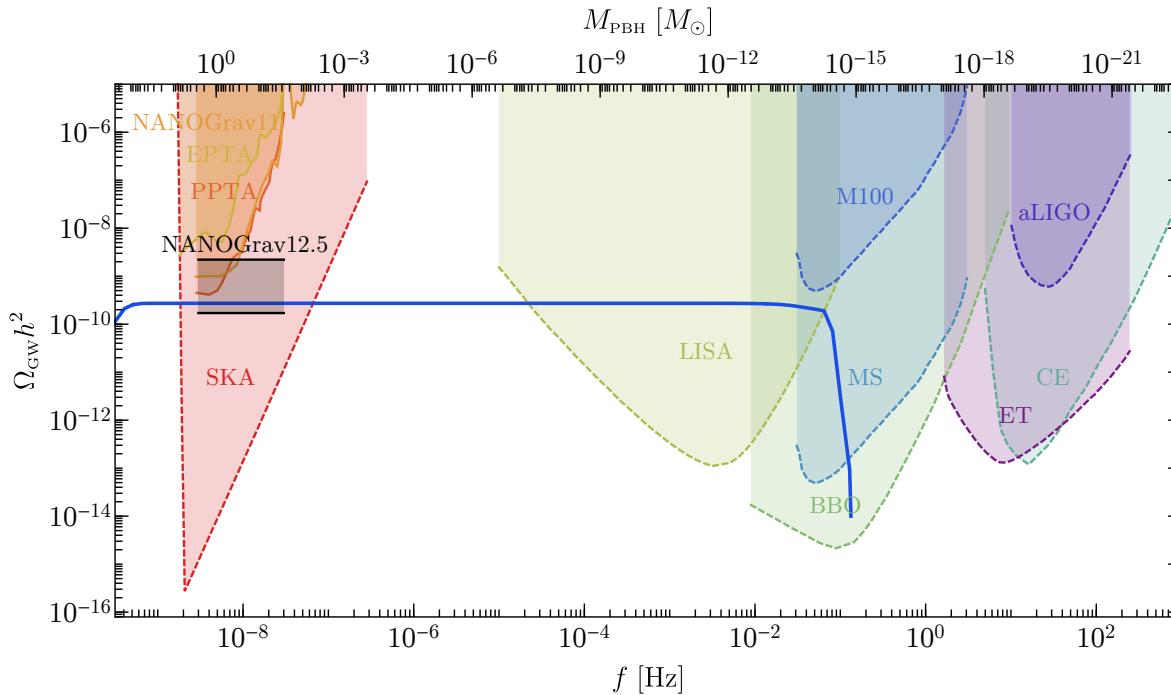
# Can be consistent with a PBH = DM scenario



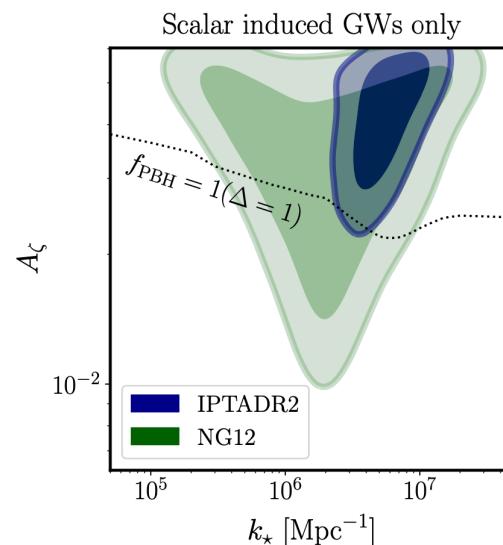
From flat curvature power spectrum

$$\mathcal{P}_\zeta(k) = A_\zeta \Theta(k_s - k) \Theta(k - k_l) \quad k_s \gg k_l$$

# Can be consistent with a PBH = DM scenario



De Luca et al. PRL (2021)



Dandoy et al. 2023)

# Conclusions

- The era of gravitational wave astronomy has begun opening a new window into fundamental physics and cosmology
- PBHs may exist and comprise the totality of the dark matter, future data will tell us