Hawking Radiation from Asteroid Mass Primordial Black Holes (PBHs): QED Corrections

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Outline of this Talk

- Why is a perturbative O(α) Quantum Electrodynamics (QED) calculation on a curved background necessary?

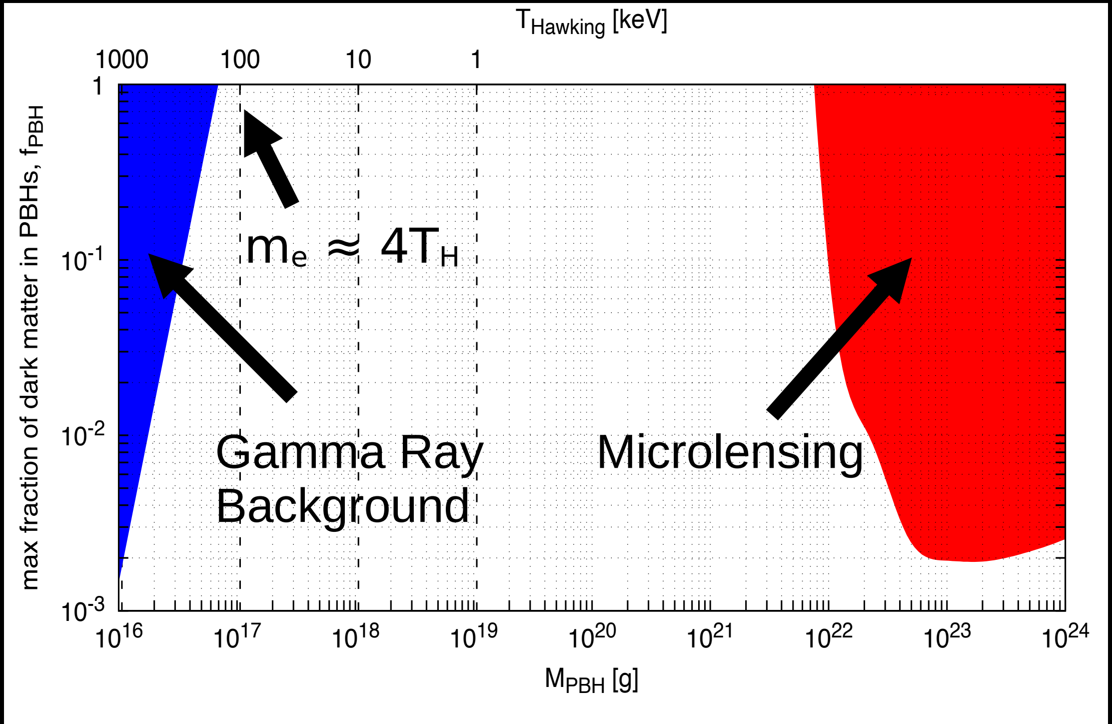
- Develop the formalism to calculate the O(α) change to the spectrum due to dissipative interactions.

- Get a glimpse into the spectra that are currently being numerically calculated.

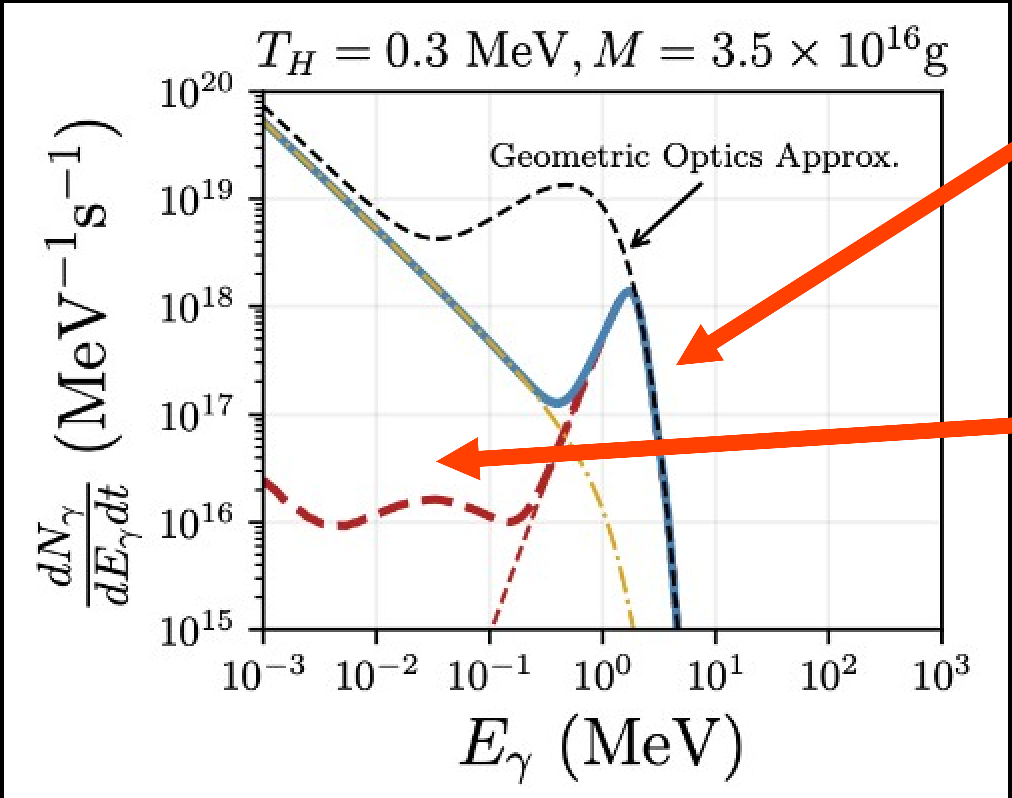
I) Motivation

Asteroid Mass Primordial Black Holes, oh my!

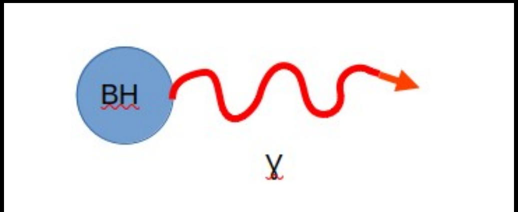
- Nonstellar black holes formed in the early Universe.
- Dark matter (DM) candidate.
- No constraints in the asteroid mass regime.
- Hawking radiation may be observable by future MeV surveys such as AMEGO.
- $t_{evap} \sim 10^{64} \left(\frac{M}{M_\odot}\right)^3 yrs$



Photon Spectra (Primary and Secondary)



Primary



Secondary

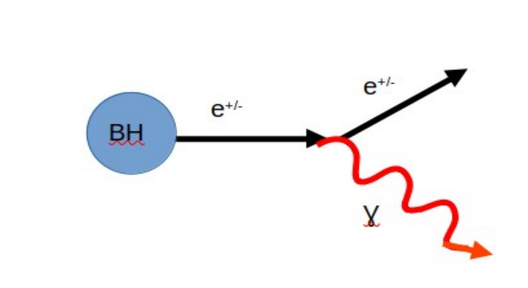
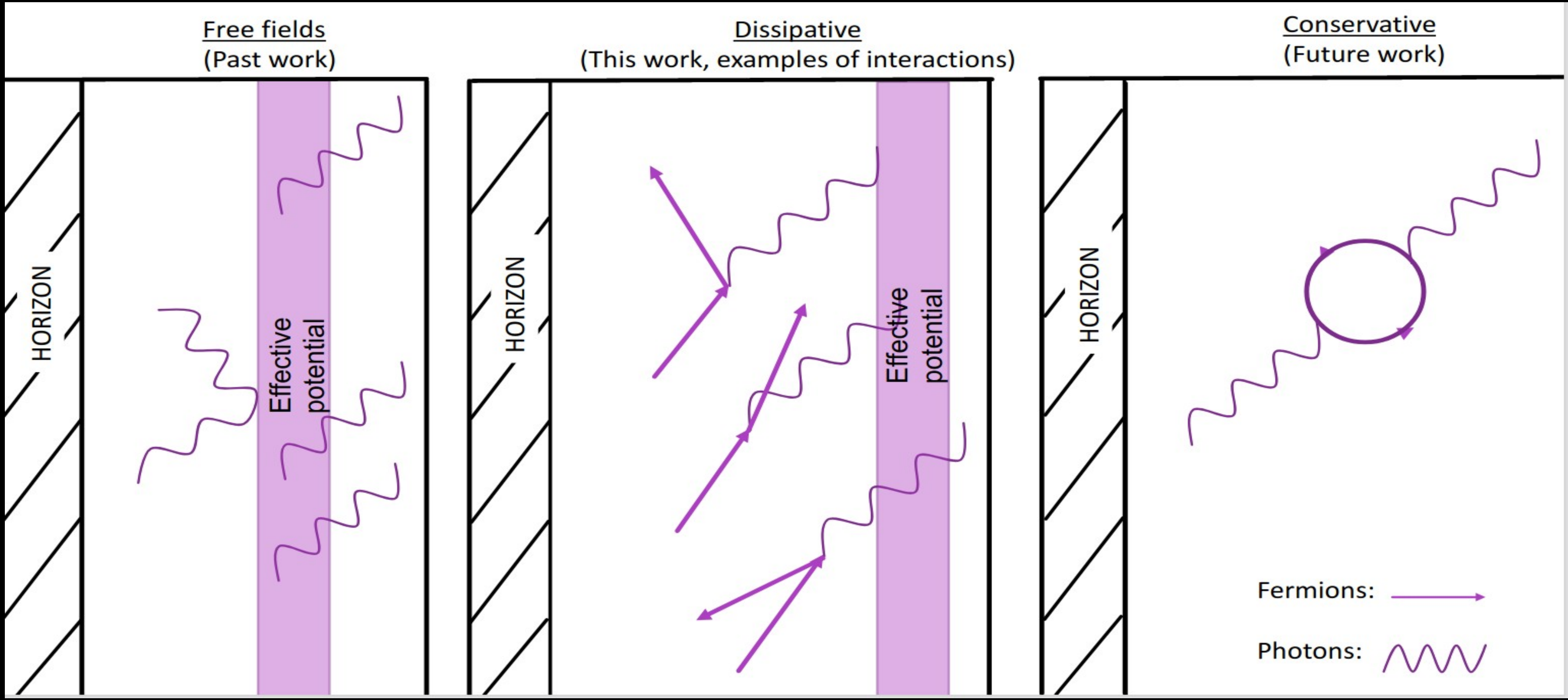


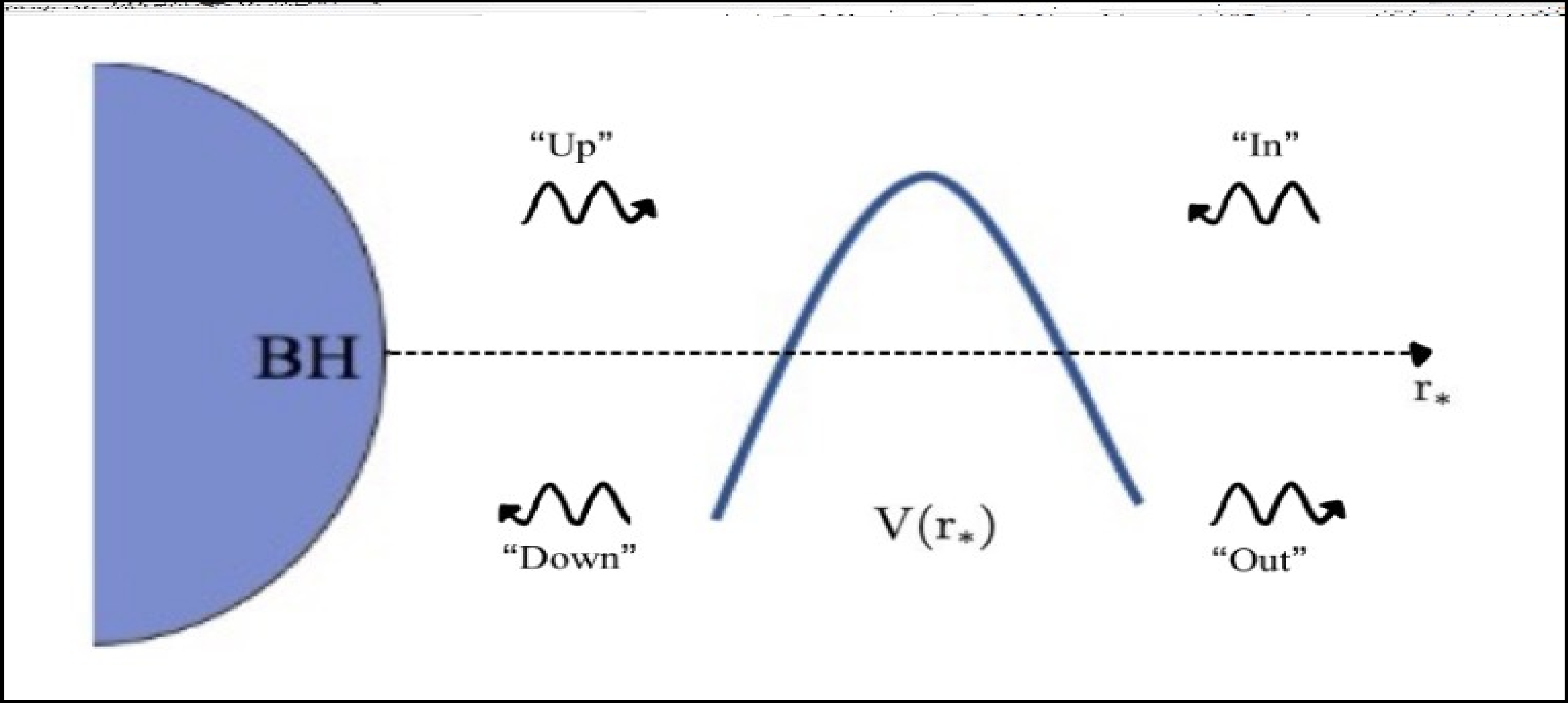
Fig. 2, Coogan et. al 2019

II) Formalism

Types of QED Interactions



Basis States



$$
\left.\frac{dN_\gamma^{(1)}}{dt\,d\omega}\right|_{\rm diss} = \frac{1}{2\pi}\sum_{\ell m_\gamma p}\frac{d}{dt}\langle \hat a^\dagger_{{\rm out},\ell m_\gamma\omega(p)}\hat a_{{\rm out},\ell m_\gamma\omega(p)}\rangle_{\rm diss}
$$

$$
\begin{aligned}
&= \frac{e^2}{2\pi}\sum_{\ell=1}^{\infty}\sum_p\int\frac{dh}{2\pi}\sum_{kk'}\Delta(j,j',\ell)\delta_{ss'(-1)^{k+k'+\ell},(-1)^p}\\
&\times\Big[|R_{1,\ell,\omega}|^2\Big\{\frac{2}{e^{8\pi M(\omega+h)}+1}|[\![I^{-+}_{{\rm in},k,{\rm up},k',{\rm in},\ell,(p)}(h,\omega+h,\omega)]\!]|^2\\
&+\frac{1}{(e^{8\pi Mh}+1)(e^{8\pi M(\omega-h)}+1)}|[\![I^{++}_{{\rm up},k,{\rm up},k',{\rm in},\ell,(p)}(h,\omega-h,\omega)]\!]|^2\\
&+\frac{2e^{8\pi Mh}}{(e^{8\pi Mh}+1)(e^{8\pi M(\omega+h)}+1)}|[\![I^{-+}_{{\rm up},k,{\rm up},k',{\rm in},\ell,(p)}(h,\omega+h,\omega)]\!]|^2\Big\}\\
&+\frac{|T_{1,\ell,\omega}|^2}{e^{8\pi M\omega}-1}\Big\{-|[\![I^{++}_{{\rm in},k,{\rm in},k',{\rm up},\ell,(p)}(h,\omega-h,\omega)]\!]|^2+\frac{2e^{8\pi M\omega}}{e^{8\pi M(\omega+h)}+1}|[\![I^{-+}_{{\rm in},k,{\rm up},k',{\rm up},\ell,(p)}(h,\omega+h,\omega)]\!]|^2\\
&-\frac{2}{e^{8\pi Mh}+1}\Big(e^{8\pi Mh}|[\![I^{++}_{{\rm up},k,{\rm in},k',{\rm up},\ell,(p)}(h,\omega-h,\omega)]\!]|^2+|[\![I^{-+}_{{\rm up},k,{\rm in},k',{\rm up},\ell,(p)}(h,\omega+h,\omega)]\!]|^2\Big)\Big\}+\\
&+{\rm Re}\,T^*_{1,\ell,\omega}R_{1,\ell,\omega}\Big[\frac{2(2e^{8\pi M\omega}-1)}{(e^{8\pi M(\omega+h)}+1)(e^{8\pi M\omega}-1)}[\![I^{-+}_{{\rm in},k,{\rm up},k',{\rm up},\ell,(p)}(h,\omega+h,\omega)]\!][\![I^{-+*}_{{\rm in},k,{\rm up},k',{\rm in},\ell,(p)}(h,\omega+h,\omega)]\!]\\
&+\frac{1}{(e^{8\pi Mh}+1)(e^{8\pi M(\omega-h)}+1)}[\![I^{++}_{{\rm up},k,{\rm up},k',{\rm up},\ell,(p)}(h,\omega-h,\omega)]\!][\![I^{++*}_{{\rm up},k,{\rm up},k',{\rm in},\ell,(p)}(h,\omega-h,\omega)]\!]\\
&+\frac{2e^{8\pi Mh}}{(e^{8\pi Mh}+1)(e^{8\pi M(\omega+h)}+1)}[\![I^{-+}_{{\rm up},k,{\rm up},k',{\rm up},\ell,(p)}(h,\omega+h,\omega)]\!][\![I^{-+*}_{{\rm up},k,{\rm up},k',{\rm in},\ell,(p)}(h,\omega+h,\omega)]\!]\\
&-\frac{1}{e^{8\pi M\omega}-1}\Big([\![I^{++}_{{\rm in},k,{\rm in},k',{\rm up},\ell,(p)}(h,\omega-h,\omega)]\!][\![I^{++*}_{{\rm in},k,{\rm in},k',{\rm in},\ell,(p)}(h,\omega-h,\omega)]\!]\\
&+\frac{2e^{8\pi Mh}}{e^{8\pi Mh}+1}[\![I^{++}_{{\rm up},k,{\rm in},k',{\rm up},\ell,(p)}(h,\omega-h,\omega)]\!][\![I^{++*}_{{\rm up},k,{\rm in},k',{\rm in},\ell,(p)}(h,\omega-h,\omega)]\!]\\
&+\frac{2}{e^{8\pi Mh}+1}[\![I^{-+}_{{\rm up},k,{\rm in},k',{\rm up},\ell,(p)}(h,\omega+h,\omega)]\!][\![I^{-+*}_{{\rm up},k,{\rm in},k',{\rm in},\ell,(p)}(h,\omega+h,\omega)]\!]\Big)\Big]\Big],
\end{aligned}
$$

- $O(\alpha)$ correction to Hawking spectrum
- Photons escaping BH
- It's a lot…

$$\left.\frac{dN_\gamma^{(1)}}{dt\,d\omega}\right|_{\rm diss} = \frac{1}{2\pi}\sum_{\ell m_\gamma p}\frac{d}{dt}\langle \hat a^\dagger_{{\rm out},\ell m_\gamma \omega(p)}\hat a_{{\rm out},\ell m_\gamma\omega(p)}\rangle_{\rm diss}$$

$$= \frac{e^2}{2\pi}\sum_{\ell=1}^{\infty}\sum_p\int\frac{dh}{2\pi}\sum_{kk'}\Delta(j,j',\ell)\delta_{ss'(-1)^{k+k'+\ell},(-1)^p}$$

$$\times\Big[|R_{1,\ell,\omega}|^2\Big\{\frac{2}{e^{8\pi M(\omega+h)}+1}|[\![I^{-+}_{{\rm in},k,{\rm up},k',{\rm in},\ell,(p)}(h,\omega+h,\omega)]\!]|^2$$
$$+\frac{1}{(e^{8\pi Mh}+1)(e^{8\pi M(\omega-h)}+1)}|[\![I^{++}_{{\rm up},k,{\rm up},k',{\rm in},\ell,(p)}(h,\omega-h,\omega)]\!]|^2$$
$$+\frac{2e^{8\pi Mh}}{(e^{8\pi Mh}+1)(e^{8\pi M(\omega+h)}+1)}|[\![I^{-+}_{{\rm up},k,{\rm up},k',{\rm in},\ell,(p)}(h,\omega+h,\omega)]\!]|^2\Big\}$$

$$+\frac{|T_{1,\ell,\omega}|^2}{e^{8\pi M\omega}-1}\Big\{-|[\![I^{++}_{{\rm in},k,{\rm in},k',{\rm up},\ell,(p)}(h,\omega-h,\omega)]\!]|^2+\frac{2e^{8\pi M\omega}}{e^{8\pi M(\omega+h)}+1}|[\![I^{-+}_{{\rm in},k,{\rm up},k',{\rm up},\ell,(p)}(h,\omega+h,\omega)]\!]|^2$$
$$-\frac{2}{e^{8\pi Mh}+1}\Big(e^{8\pi Mh}|[\![I^{++}_{{\rm up},k,{\rm in},k',{\rm up},\ell,(p)}(h,\omega-h,\omega)]\!]|^2+|[\![I^{-+}_{{\rm up},k,{\rm in},k',{\rm up},\ell,(p)}(h,\omega+h,\omega)]\!]|^2\Big)\Big\}+$$

$$+{\rm Re}\,T^*_{1,\ell,\omega}R_{1,\ell,\omega}\Big[\frac{2(2e^{8\pi M\omega}-1)}{(e^{8\pi M(\omega+h)}+1)(e^{8\pi M\omega}-1)}[\![I^{-+}_{{\rm in},k,{\rm up},k',{\rm up},\ell,(p)}(h,\omega+h,\omega)]\!][\![I^{-+*}_{{\rm in},k,{\rm up},k',{\rm in},\ell,(p)}(h,\omega+h,\omega)]\!]$$
$$+\frac{1}{(e^{8\pi Mh}+1)(e^{8\pi M(\omega-h)}+1)}[\![I^{++}_{{\rm up},k,{\rm up},k',{\rm up},\ell,(p)}(h,\omega-h,\omega)]\!][\![I^{++*}_{{\rm up},k,{\rm up},k',{\rm in},\ell,(p)}(h,\omega-h,\omega)]\!]$$
$$+\frac{2e^{8\pi Mh}}{(e^{8\pi Mh}+1)(e^{8\pi M(\omega+h)}+1)}[\![I^{-+}_{{\rm up},k,{\rm up},k',{\rm up},\ell,(p)}(h,\omega+h,\omega)]\!][\![I^{-+*}_{{\rm up},k,{\rm up},k',{\rm in},\ell,(p)}(h,\omega+h,\omega)]\!]$$
$$-\frac{1}{e^{8\pi M\omega}-1}\Big([\![I^{++}_{{\rm in},k,{\rm in},k',{\rm up},\ell,(p)}(h,\omega-h,\omega)]\!][\![I^{++*}_{{\rm in},k,{\rm in},k',{\rm in},\ell,(p)}(h,\omega-h,\omega)]\!]$$
$$+\frac{2e^{8\pi Mh}}{e^{8\pi Mh}+1}[\![I^{++}_{{\rm up},k,{\rm in},k',{\rm up},\ell,(p)}(h,\omega-h,\omega)]\!][\![I^{++*}_{{\rm up},k,{\rm in},k',{\rm in},\ell,(p)}(h,\omega-h,\omega)]\!]$$
$$+\frac{2}{e^{8\pi Mh}+1}[\![I^{-+}_{{\rm up},k,{\rm in},k',{\rm up},\ell,(p)}(h,\omega+h,\omega)]\!][\![I^{-+*}_{{\rm up},k,{\rm in},k',{\rm in},\ell,(p)}(h,\omega+h,\omega)]\!]\Big)\Big]\Big],$$

$|R_{1,\ell,\omega}|^2$ reflection to horizon

$|T_{1,\ell,\omega}|^2$ transmission to infinity (escapes BH)

$Re\{R_{1,\ell,\omega}T^\star_{1,\ell,\omega}\}$ interference of both processes

Pair production and bremsstrahlung interactions

Dissipative Interactions

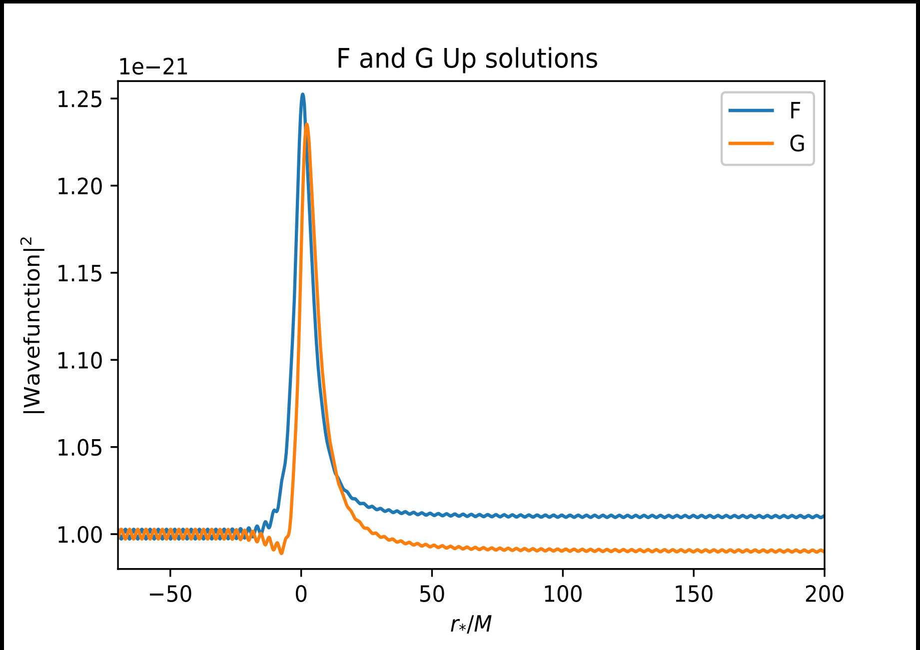
Numerical results coming soon…

III) Numerical Results

What Ingredients Do We Need?

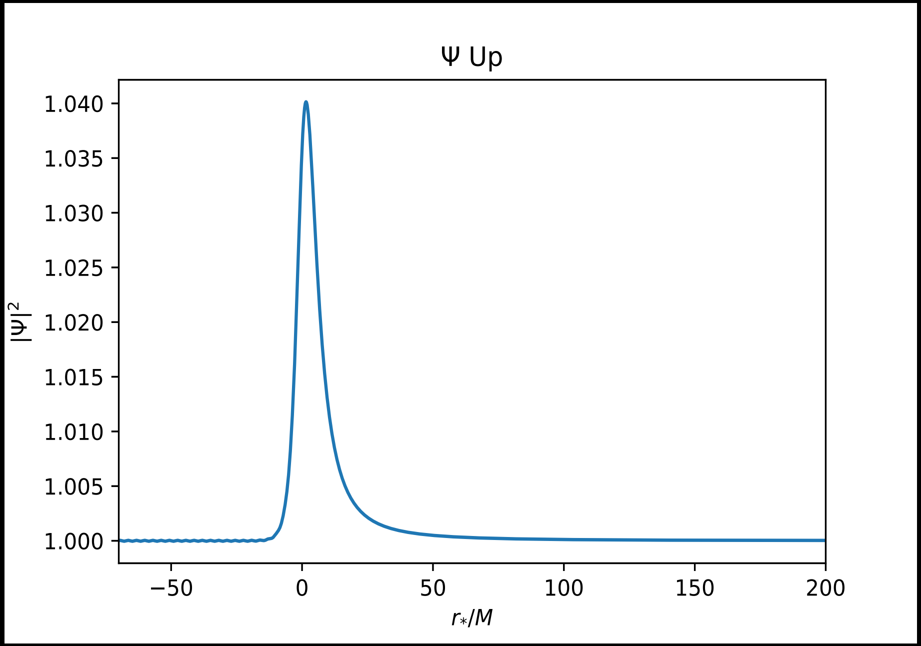
- We want a spectra!
- Electron/Positron wave functions.
- Photon wave functions.
- Integrate the mode functions (I integrals) that are functions of the electron, positron, and photon wave functions.
- Shout out to Emily Koivu for spending many CPU hours to make this feat of coding possible!

Numerics: Electron and Photon Wavefunctions



Electron Wavefunction (Up Basis)

$M=10^{21}\ M_{Planck}$, $k=3$, $h=10^{-21}\ M_{Planck}$

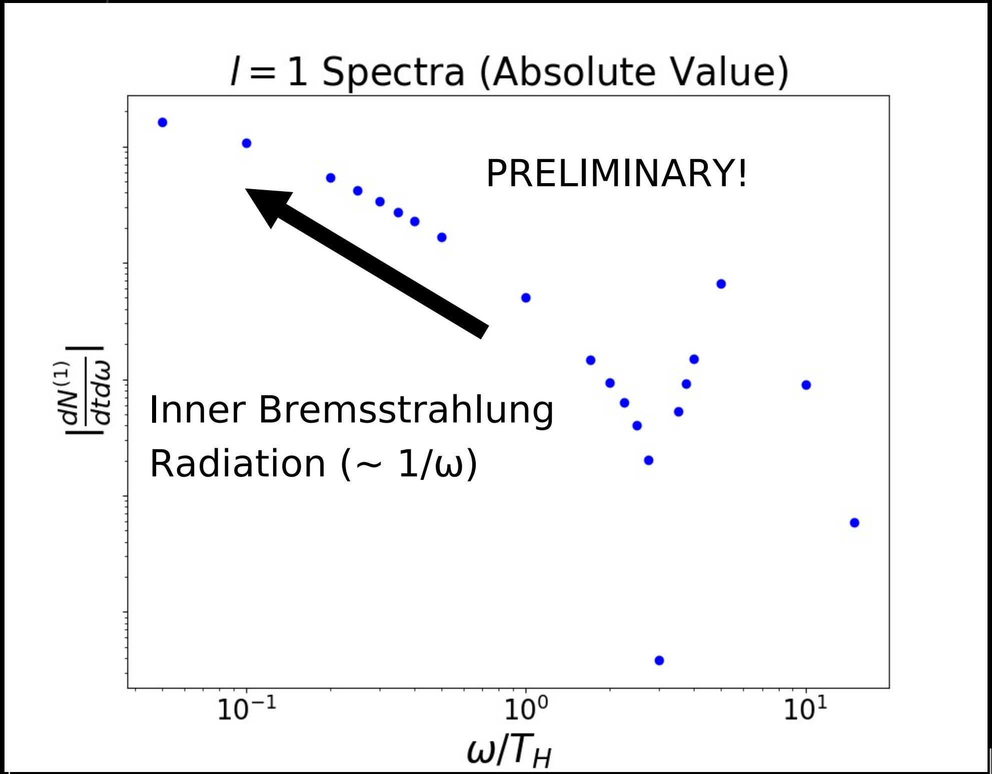


Photon Wavefunction (Up Basis)

$M=10^{21}\ M_{Planck}$, $l=1$, $\omega=10^{-21}\ M_{Planck}$

Numerics: Photon Spectrum

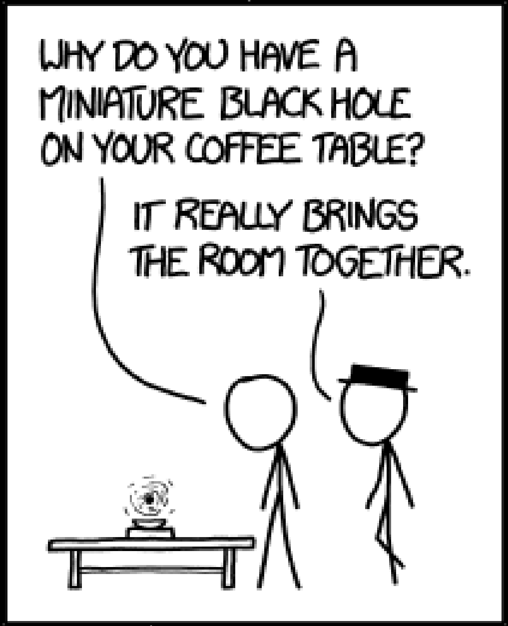
- Want to determine the O(α) correction due to dissipative interactions.
- $M = 10^{21}\ M_{planck}$ ($M \approx 10^{16}$ grams)
- Low resolution, but with more data points we can distinguish interesting features.
- Expected inner bremsstrahlung behavior!
- Not complete, but is a spoiler of what comes next!



Conclusions and Future Work

- Develop the formalism for $O(\alpha)$ dissipative QED interactions for a Schwarzchild PBH.
- We are numerically calculating spectra and will compare our results to that of in Coogan et. al 2019. In progress!
- Account for conservative interactions (vacuum polarization) which requires renormalization on a curved background. Also in progress!
- Develop the formalism for spinning (Kerr) PBHs. Not started yet.

Thank You! Questions?



Hawking Radiation

- Hawking showed that black holes have a temperature and radiate particles (Hawking, 1975)

- Hawking temperature: $$T_H = \frac{1}{8\pi M}$$

- M < 10^{17} g (T_H > 100 keV) allows for the emission of electron/positron pairs.

- Current and future MeV telescopes may be able to set bounds.

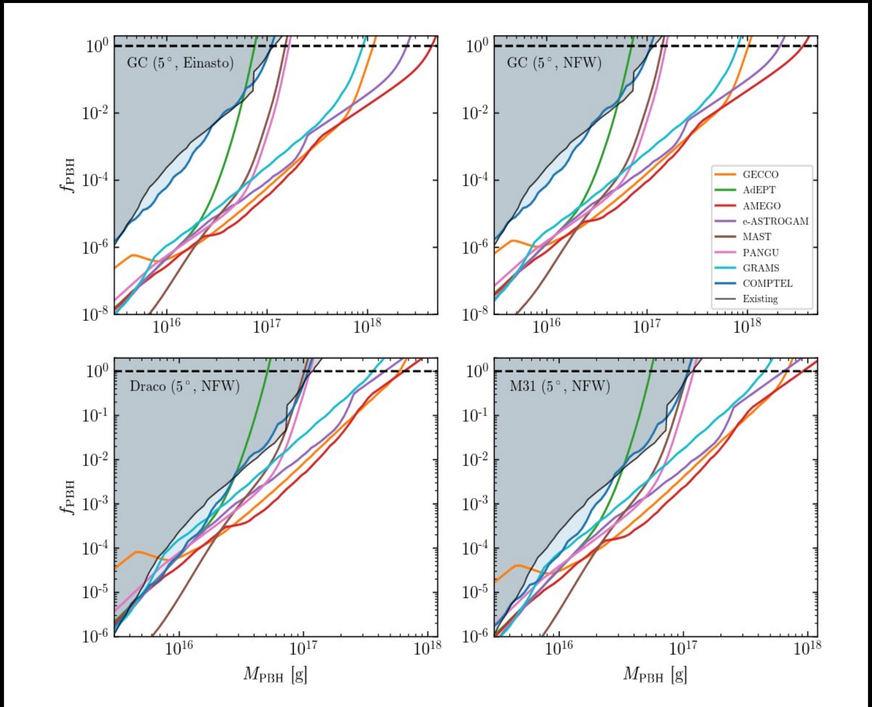
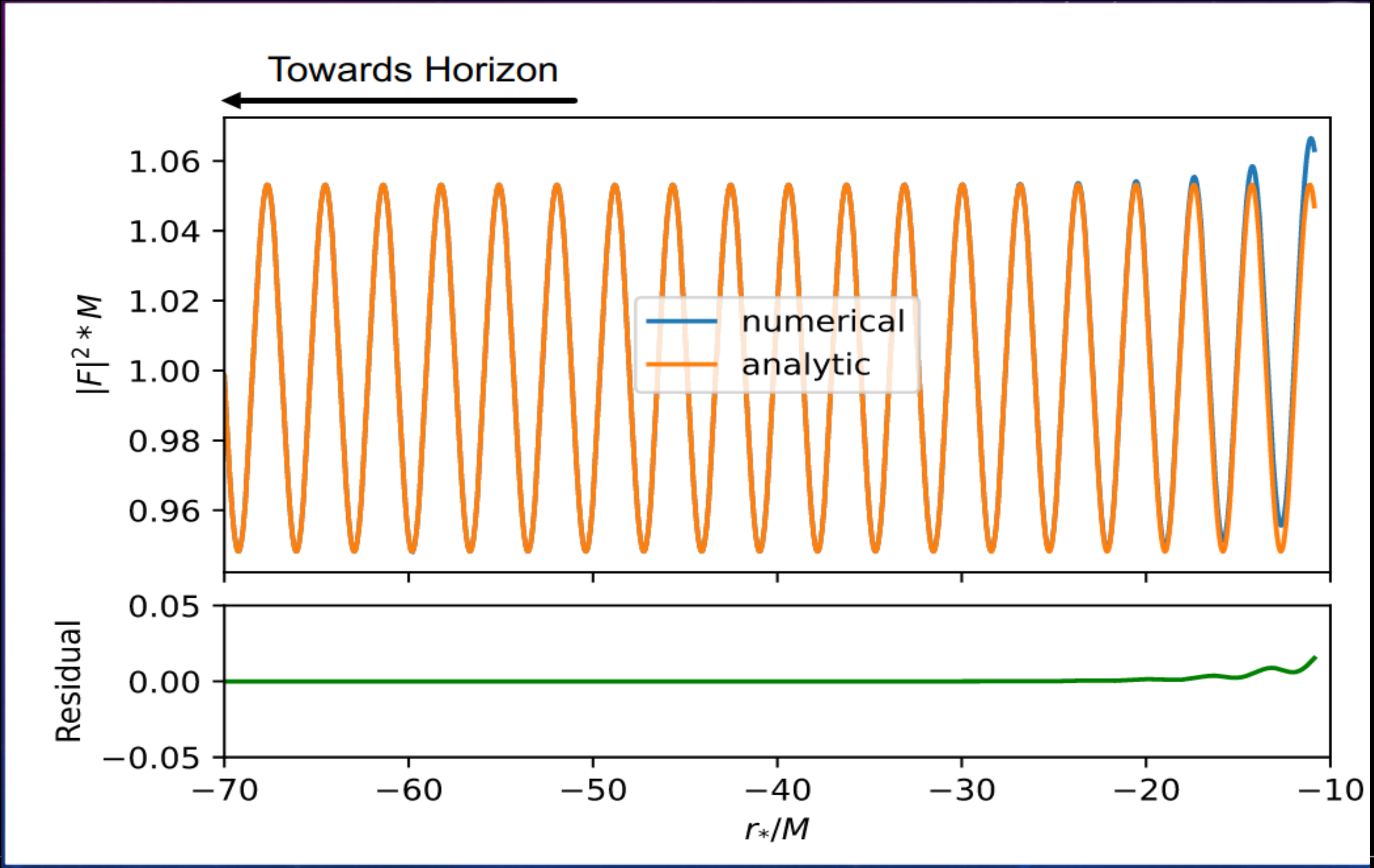
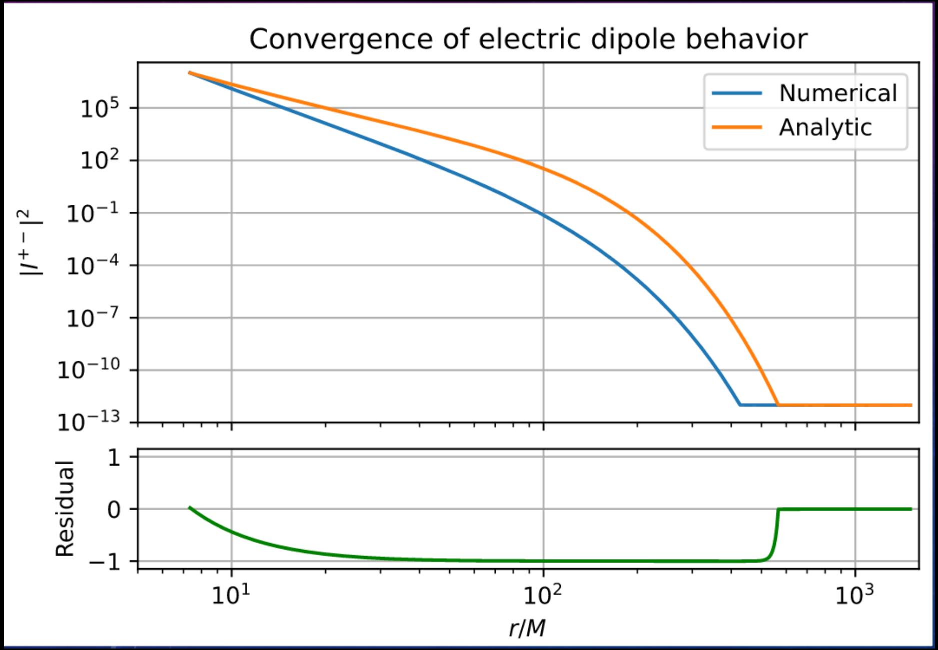


Fig. 4, Coogan et. al 2019

Checks: Limiting Behavior of Wavefunctions



Checks: Electric Dipole



- Numerical result vs. Flat spacetime electric dipole result.
- Bound state: $|h - \mu| << \mu$
- Far field: $r >> M$
- Large wavelength: $\omega << 1/r$
- $e_{up} \rightarrow e_{up} + \gamma_{in}$