Novel methods for scattering amplitudes

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THE ANNUAL THEORY CHRISTMAS MEETING, 2022

Introduction & Motivation

In this talk I will only talk about nonperturbative methods for scattering amplitudes. [cf. talk by Claude Duhr]

Old *S*-matrix bootstrap: solve pion scattering.

[many people, many amazing ideas (still to be fully absorbed)]

Novel *S*-matrix bootstrap: explore the space of theories.

[many people, and I am sorry if I forgot some names]

Space of theories

A convenient way to parameterize the space of theories is

$$\mathcal{L} = \mathcal{L}_{\rm IR} + \sum_i g_i \frac{\mathcal{O}_i}{M^{\Delta_i - d}}, \qquad g_i \sim O(1)$$

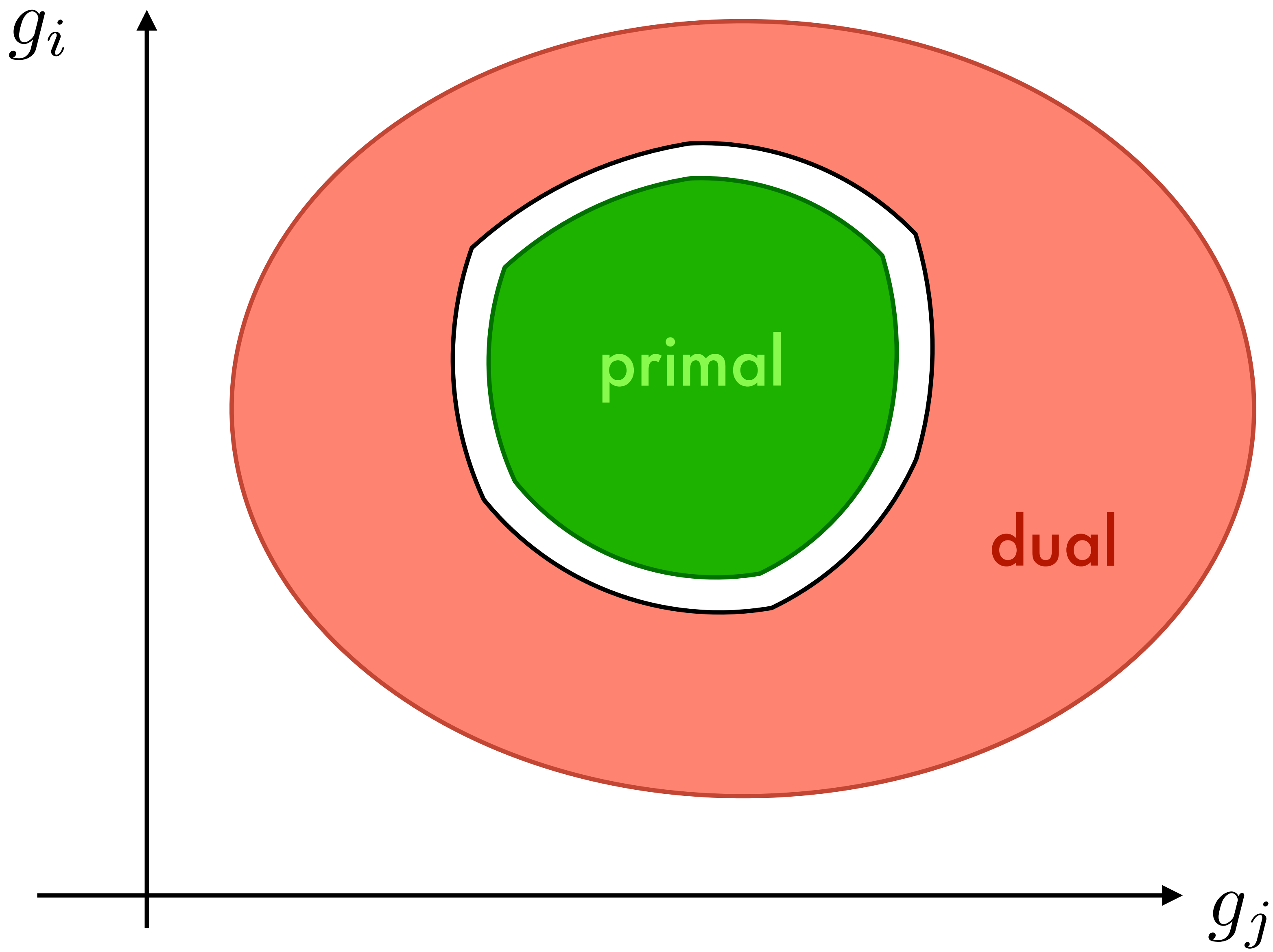
Relatedly, we can directly consider the low-energy expansion of the two-to-two scattering amplitude $T(s,t)$

$$T^{\rm IR}_{\phi\phi\to\phi\phi}(s,t) = 8\pi G_N \left(\frac{su}{t} + \frac{st}{u}\right) + e^2 (\frac{s-u}{t} + \frac{s-t}{u}) - \lambda$$
$$+ g_1 s + g_2 s^2 + g_2' tu + g_3 s^3 + g_3' stu + g_4 s^4 + g_4' s^2 tu + g_4'' (tu)^2$$
$$+ \ldots$$

<u>Example</u>: The SM and GR are leading terms in the EFT expansion.

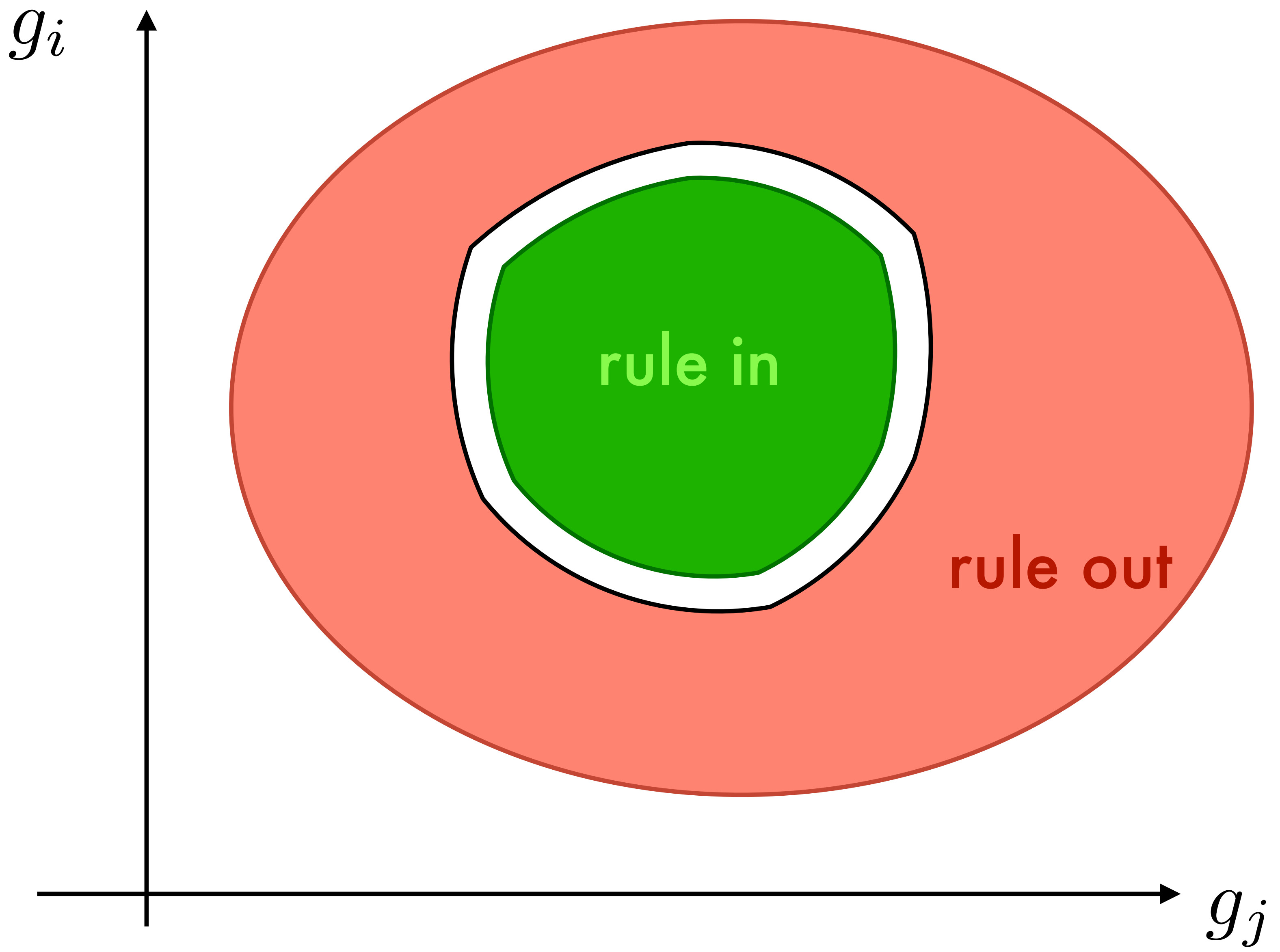
S-matrix bootstraps

Not anything goes



S-matrix bootstraps

Not anything goes



Plan

0. Basic principles (ACU)

1. Dual bootstrap (Dispersion relations)

<u>Task: to rule out!</u>

2. Primal bootstrap

<u>Task: to rule in!</u>

Basic principles:

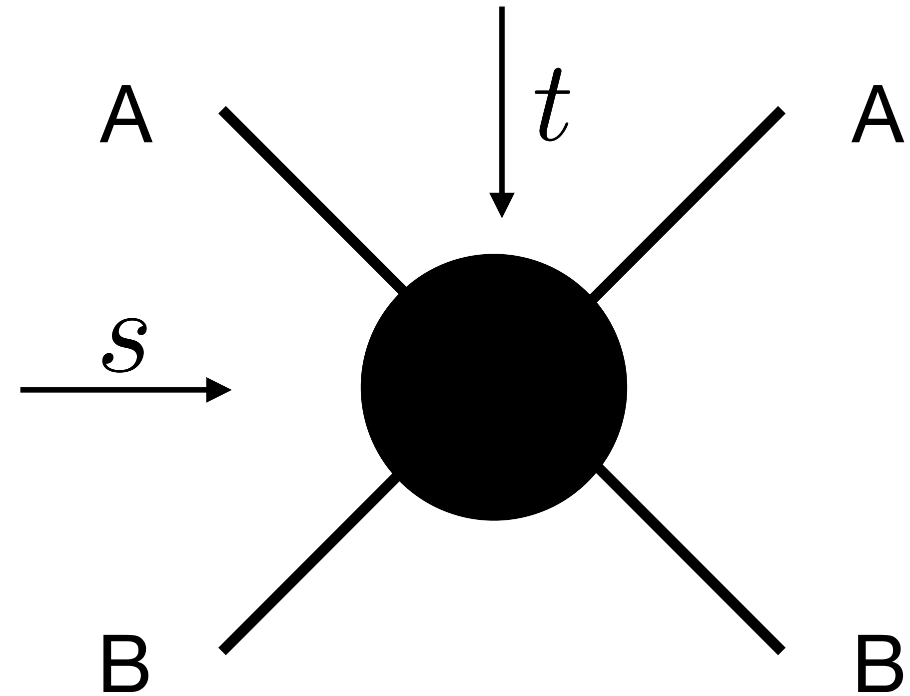
analyticity, unitarity, crossing

The basic observable of interest is the two-to-two scattering relativistic amplitude

$$T(s,t)$$

energy

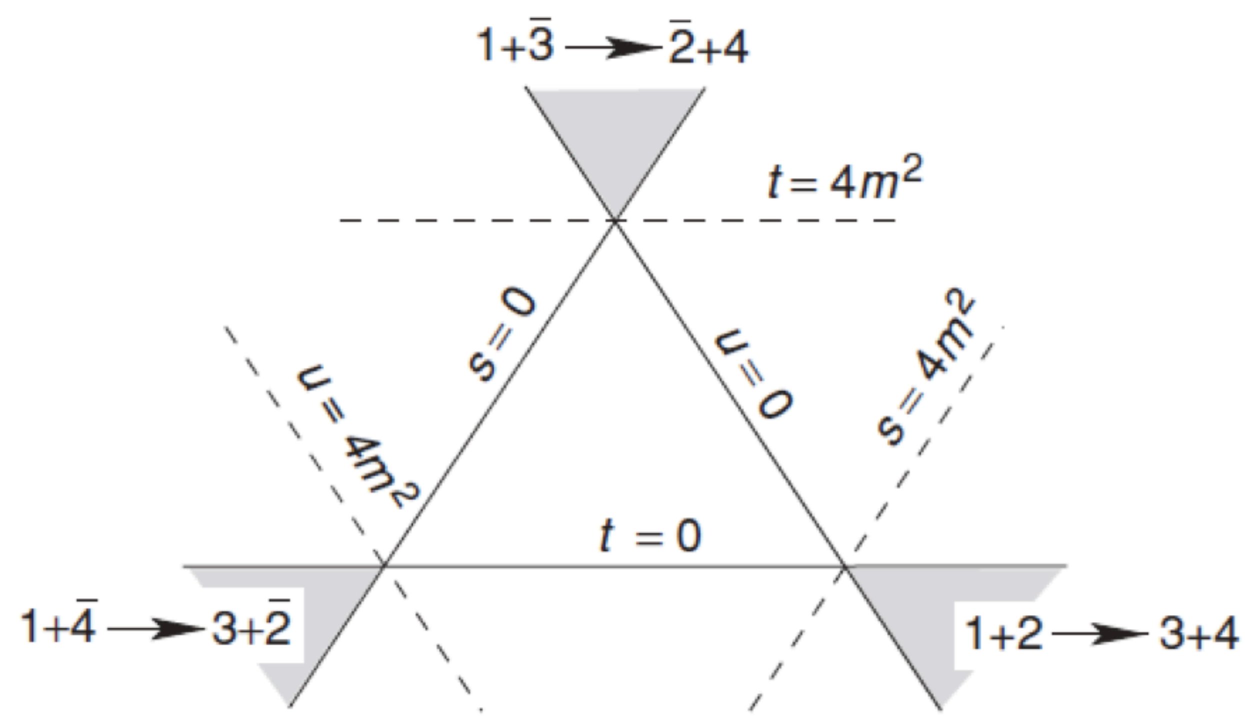
transferred momentum



$$s+t+u=4m^2$$

In the physical experiment the Mandelstam invariants are real.

Complex plane!
(causality)



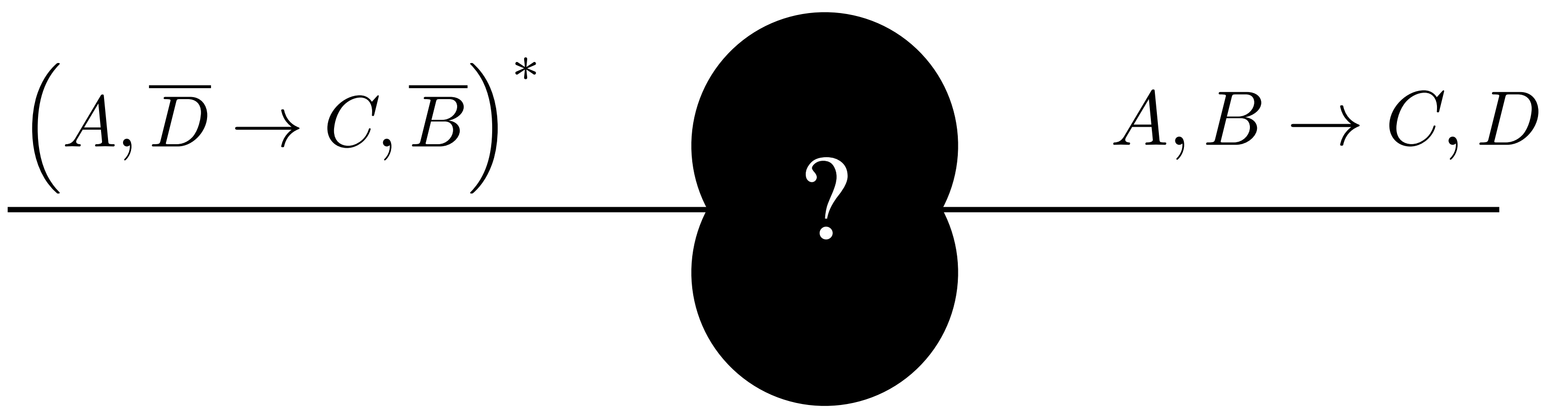
[Dokshitzer]

Basic principles: Analyticity&Crossing

- Axiomatic analyticity

[Bros, Epstein, Glaser, Martin, ...]

$\lfloor s \qquad |t| \leq R$

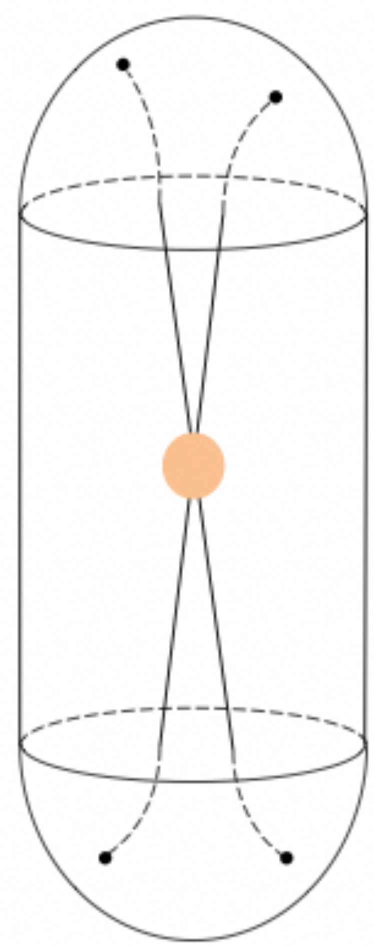


[gap]

- AdS analyticity

[Hijano '19]
[Komatsu, Paulos, van Rees, Zhao '20]
[Cordova, He, Paulos '22]
[van Rees, Zhao '22]

[gap+no gap]



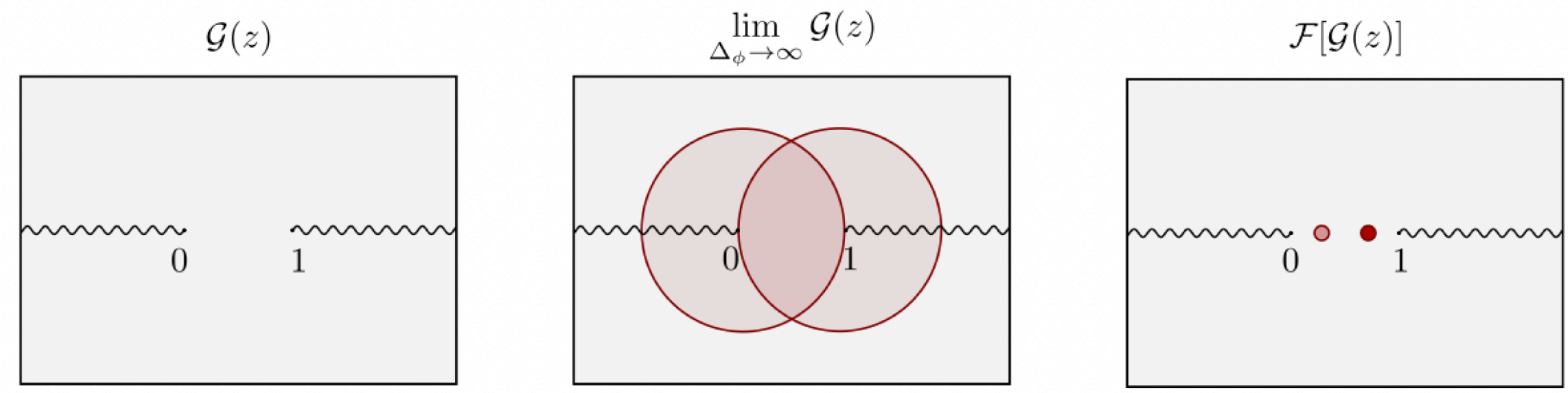
$$\xrightarrow{R_{\text{AdS}} \to \infty} \quad T(s,t)$$

"QFT in AdS instead of LSZ"

AdS Analyticity

[Cordova, He, Paulos '22]

[van Rees, Zhao '22]



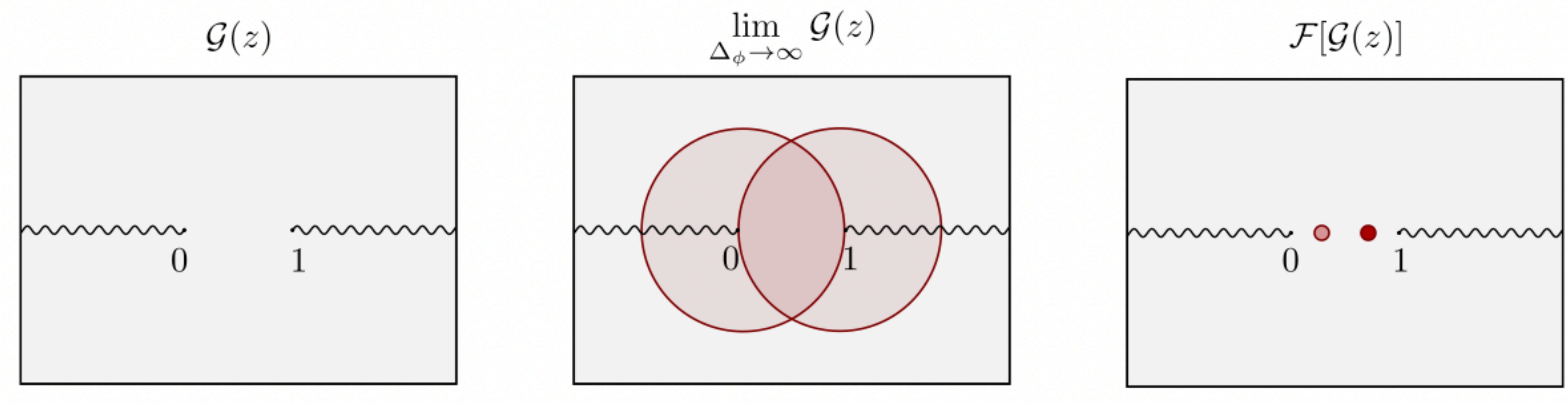
- Axiomatic analyticity follows

$$
\begin{aligned}
S(s) &= 1 + \int_{s_0}^{4m^2} \mathrm{d}s' \, \widetilde{K}(s,s') \, \tilde{\rho}(s') - \int_{4m^2}^{\infty} \mathrm{d}s' \, K(s,s') \, \rho(s') \\
\rho(s) &= \lim_{\Delta_\phi \to \infty} {\Delta_\phi}^{-\alpha} \sum_{\Delta \in B_\alpha(s)} \left(\frac{a_\Delta}{a_\Delta^{\text{free}}} \right) 4 \sin^2 \left[\frac{\pi}{2} (\Delta - 2\Delta_\phi) \right] \\
\tilde{\rho}(s) &= \lim_{\Delta_\phi \to \infty} {\Delta_\phi}^{-\alpha} \sum_{\Delta \in B_\alpha(s)} \left(\frac{a_\Delta}{\tilde{a}_\Delta^{\text{free}}} \right) .
\end{aligned}
$$

$$s \sim (\Delta / \Delta_\phi)^2$$

AdS Analyticity

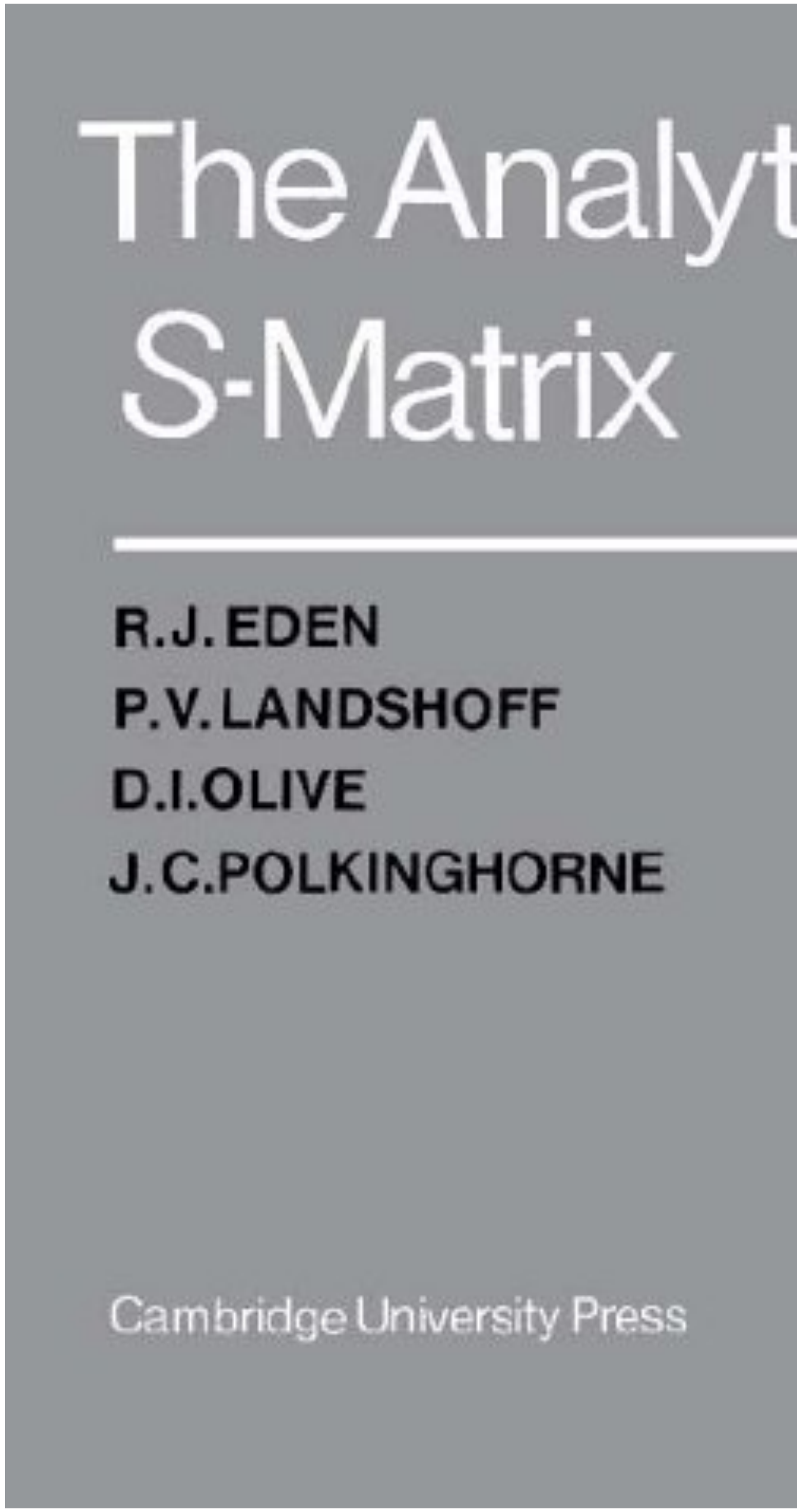
[Cordova, He, Paulos '22]



- Extended unitarity follows $(2m^2 < s < 4m^2)$

- Anomalous thresholds? $(s < 2m^2)$ $\quad a = 4m^2 - m^4/\mu^2$

spurious solutions to crossing/unboundedness of OPE



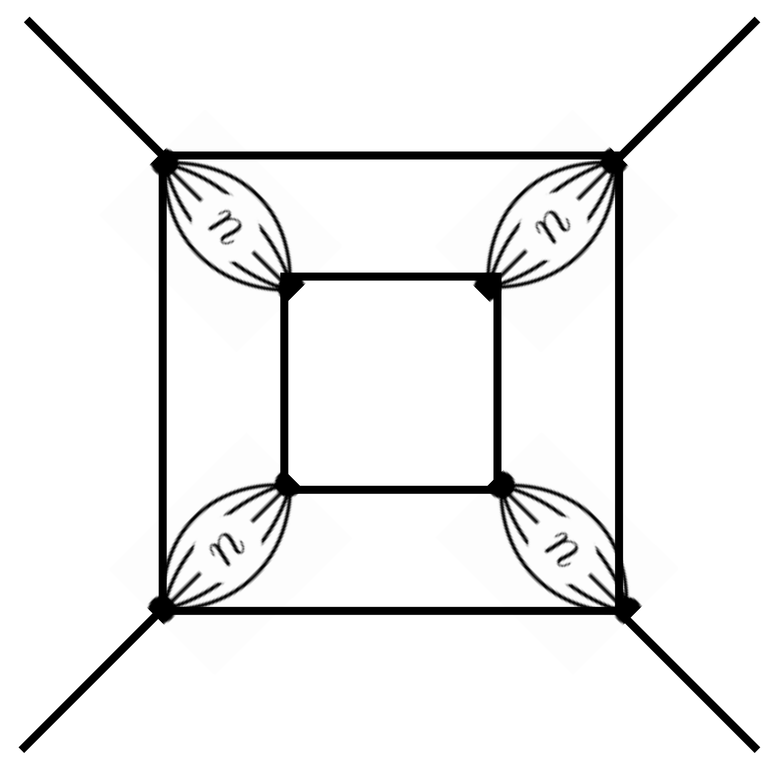
Maximal analyticity for lightest particles:

$$F(s,t,u) = P + \frac{1}{\pi^2}\iint \frac{\rho_{st}(s',t')}{(s'-s)(t'-t)}\,ds'\,dt'$$

$$+\frac{1}{\pi^2}\iint_{4m^2} \frac{\rho_{tu}(t',u')}{(t'-t)(u'-u)}\,dt'\,du' + \frac{1}{\pi^2}\iint_{4m^2} \frac{\rho_{us}(s',u')}{(u'-u)(s'-s)}\,du'\,ds', \tag{1.3.33}$$

where P denotes the pole terms, and $s'+t'+u' = 4m^2$. This is believed to be valid for the scattering of spinless particles in an equal mass theory on the assumption that F tends to zero as the variables tend to infinity in any direction in the complex plane. It has however not been fully proved even within the framework of perturbation theory.

Remarkable conjecture!
(assumed in some papers)



Unitarity: $SS^\dagger = 1$

Consider the partial wave expansion of the amplitude

$$T(s,t) = s^{\frac{4-d}{2}} \sum_{J=0}^{\infty} n_J^{(d)} f_J(s) P_J^{(d)}(\cos\theta)$$

- Positivity $\quad \mathrm{Im} f_J(s) \geq 0$
- Nonperturbative unitarity $\quad 2 \geq \mathrm{Im} f_J(s) \geq |f_J(s)|^2 \geq 0$

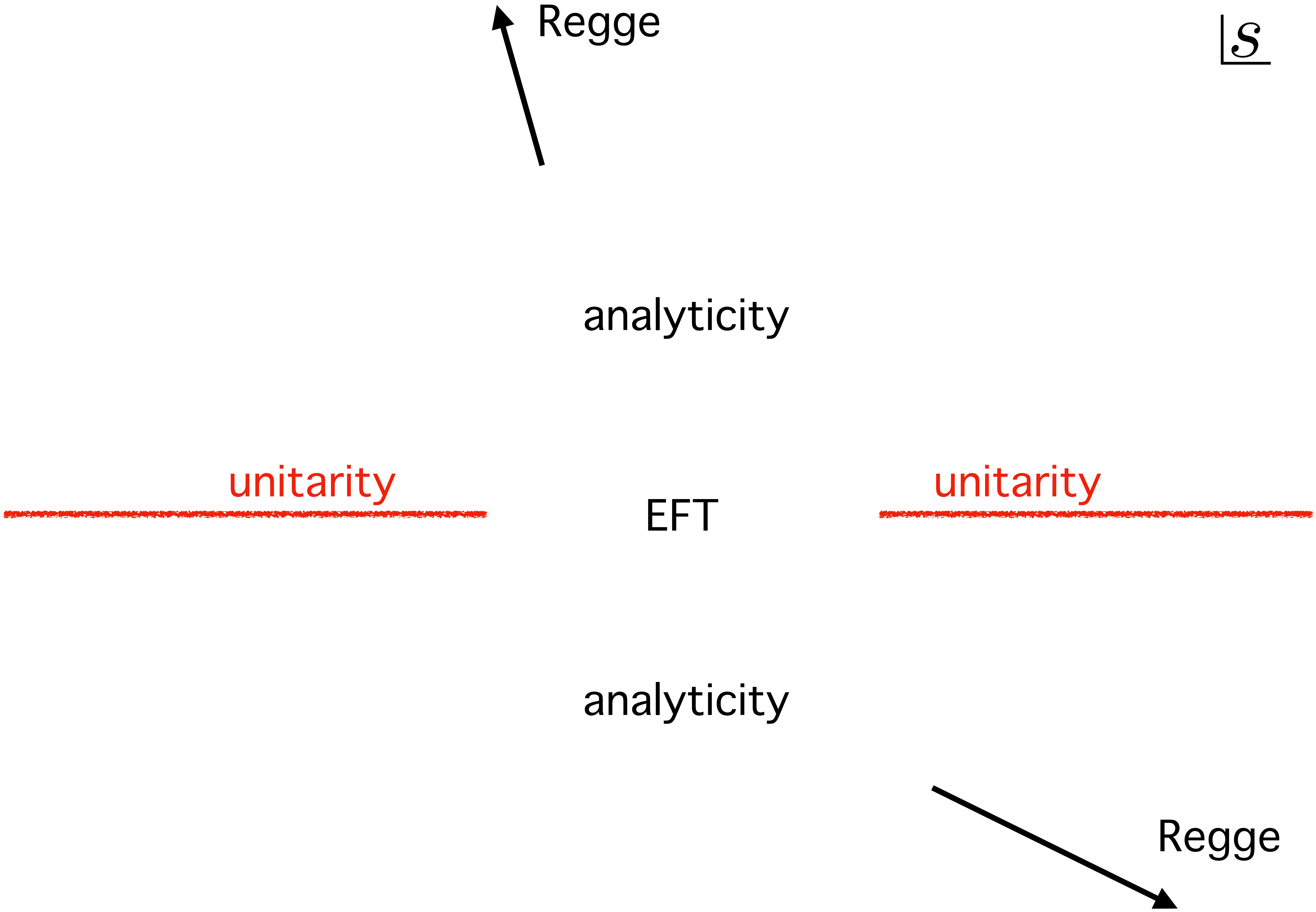
$$\text{NP unitarity} \subset \text{Positivity}$$

- Elastic unitarity $\quad \mathrm{Im} f_J(s) = |f_J(s)|^2 \qquad s_{\mathrm{MP}} > s \geq 4m^2$

$$\text{Elastic unitarity} \subset \text{NP unitarity}$$

- Multi-particle unitarity

$$\text{MP unitarity} \subset \text{Elastic unitarity} + \text{NP unitarity}$$


Regge
s
analyticity
unitarity
EFT
unitarity
analyticity
Regge

Regge bound (gap)

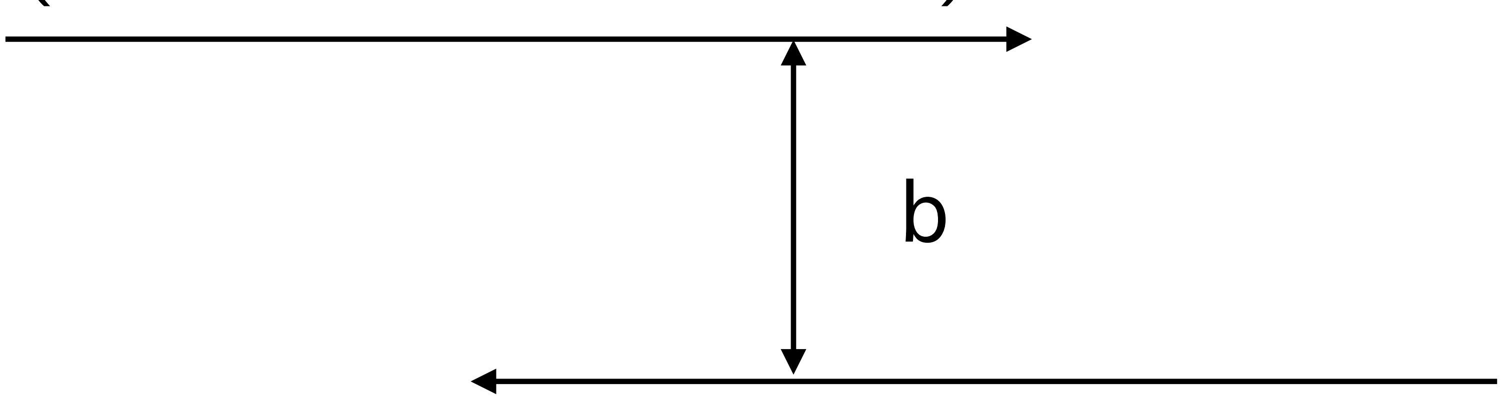
Importantly, the amplitudes do not grow too fast at infinity

$$\lim_{|s|\to\infty} \frac{T_{\rm QFT}(s,t)}{|s|^2} = 0, \quad t < m_{\rm gap}^2$$

[Froissart, Martin, Jin]

The standard derivation of this result relies on the basic physical principles (unitarity, analyticity, crossing, subexponentiality) and existence of the mass gap.

There is an intuitive way to state the bound above: effective radius of interaction does not grow with energy faster than $\log s$ (cf. the Froissart bound).



$$\delta(s,b) \sim s^N e^{-m_{\rm gap} b}, \quad b \gtrsim \log s$$

[Adams, Arkani-Hamed, Dubovsky, Rattazzi `06]

As such it is the statement about large impact parameters.

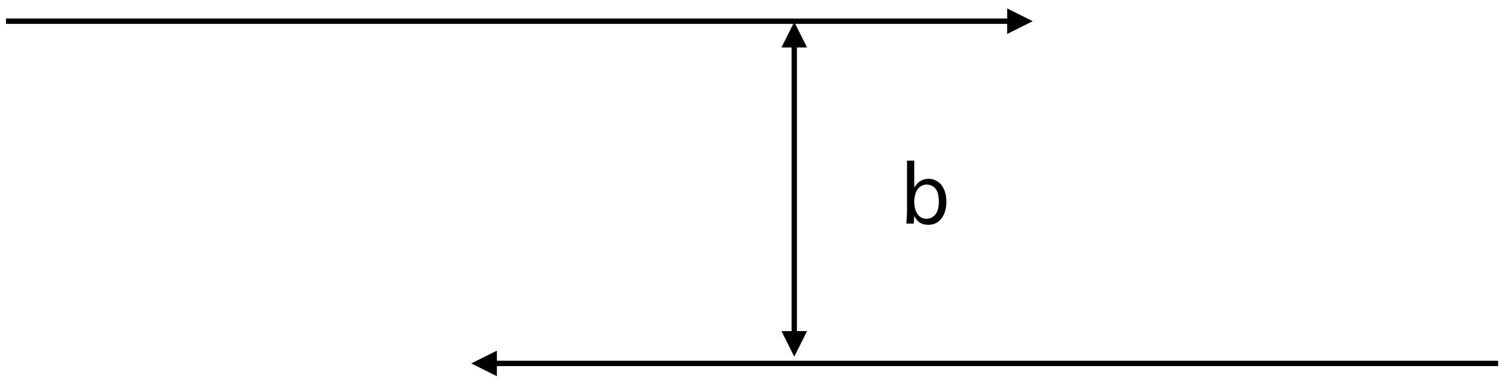
Regge bound (no gap)

[Caron-Huot, Mazac, Rastelli, Simmons-Duffin'21]
[Häring, AZ '22]

The same bound naturally arises in gravity (d>4)

$$\lim_{|s|\to\infty} \frac{T(s,t)}{s^2} = 0, \quad t < 0$$

The bound can be again derived using analyticity unitarity, crossing and the large impact parameter physics.



$$\delta(s,b) \sim \frac{G_N s}{b^{d-4}}$$

$$b \gtrsim b_{GR}(s)$$

['t Hooft, Amati, Ciafaloni, Veneziano, Verlinde, Verlinde, Kabat, Ortiz, ...]

Or by taking the flat space limit from AdS.

Open problems I

- Axiomatic analyticity (how optimal are the bounds? gravity?)

- AdS analyticity (beyond axiomatic analyticity? gravity!)

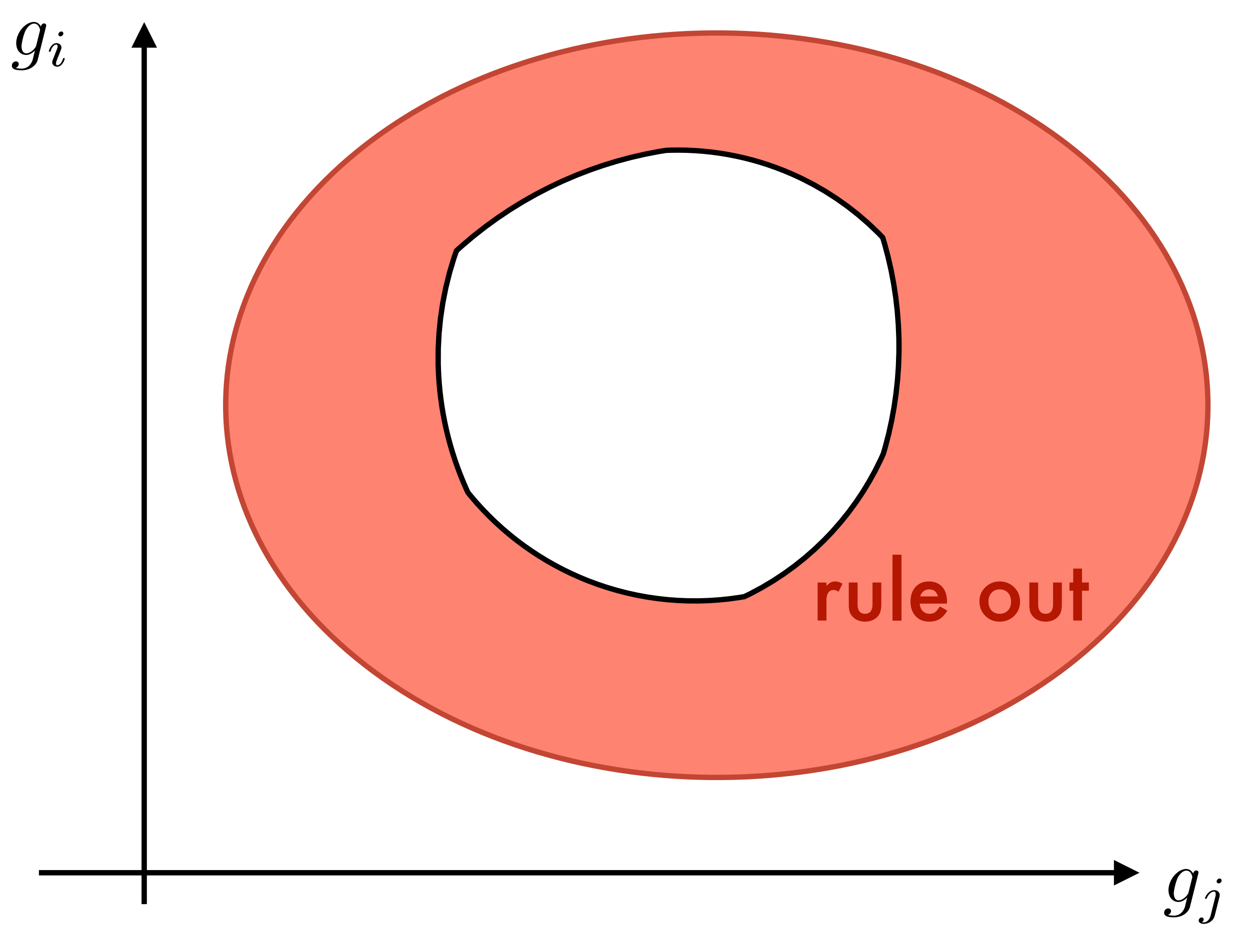
- Beyond lightest particles (extended analyticity, anomalous thresholds, second sheet, $i\epsilon$, unstable particles) [Hannesdottir, Mizera '22; Mizera '22; Aoki '22; Correia '22]

- Maximal analyticity (perturbative proof?)
- Crossing symmetry (anyons; electric and magnetic charges) [Mehta, Minwalla, Patel, Prakash, Sharma '22; Csaki, Shirman, Telem, Terning]

- Multi-particle amplitudes and multi-particle unitarity (how to go beyond 2-2?)

Next we will use tension between analyticity, unitarity and crossing to derive some interesting predictions about the space of theories.

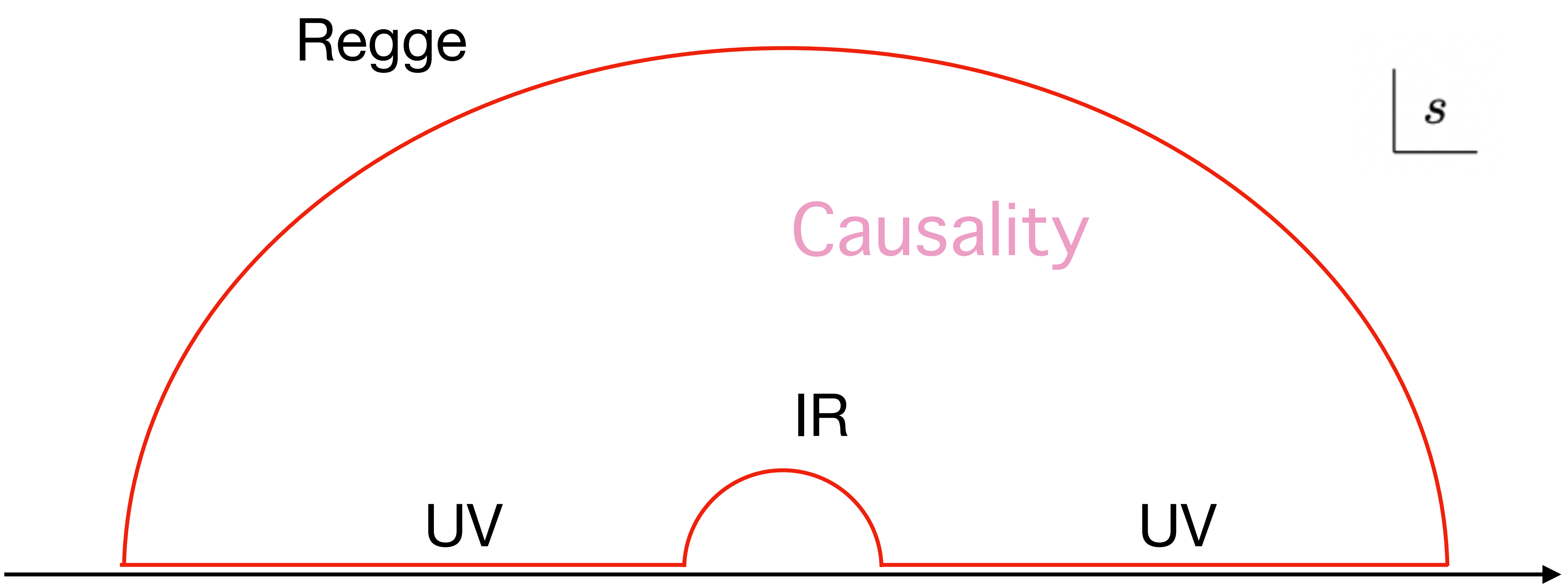
Dispersive Sum Rules



Dispersive sum rule (aka Cauchy theorem)

$$\mathrm{IR} + \mathrm{UV} = 0$$

Bound on Regge



$$\oint \frac{ds}{2\pi i} f(s,t) T(s,t) = 0$$

Example 1: $\lambda(\partial\phi)^4$

[Adams Arkani-Hamed Dubovsky Nicolis Rattazzi `06]

Let us consider an EFT of a scalar with $\lambda(\partial\phi)^4$ interaction.
It induces the following term in the scattering amplitude

$$T(s,t) = \lambda(s^2 + t^2 + u^2) + \ldots$$

We can write this

$$\lambda = \frac{1}{2\pi i}\oint_{|s'|=\epsilon} \frac{ds'}{2\pi i}\frac{T(s',0)}{(s')^3} = \frac{2}{\pi}\int_{m^2_{\rm gap}}^{\infty} \frac{ds'}{(s')^2}\sigma_{\rm tot}(s') \geq 0 \ .$$

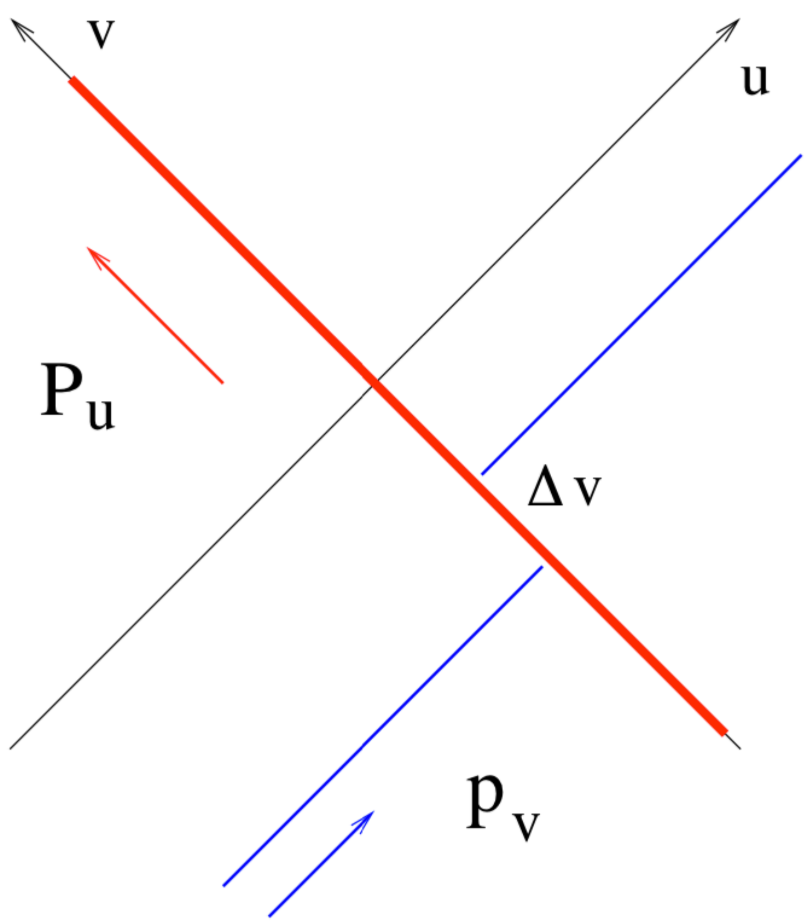
<u>Result</u>: In an EFT that admits a UV completion that satisfies ACU $\lambda \geq 0$.

It can be also related to causality on a nontrivial background.

Example 2: Graviton three-point coupling

[Camanho Edelstein Maldacena AZ '14]

$$S = M_{Pl}^2 \int d^4x \sqrt{-g} \left(R + \cancel{g_2 R^2} + g_3 R^3 + \ldots \right)$$



By scattering polarized graviton we find:

$$\Delta v = \Delta t_{\mathrm{GR}} \left(1 \pm \frac{g_3}{b^4 \log b} \right)$$

Two sided-bound from causality:

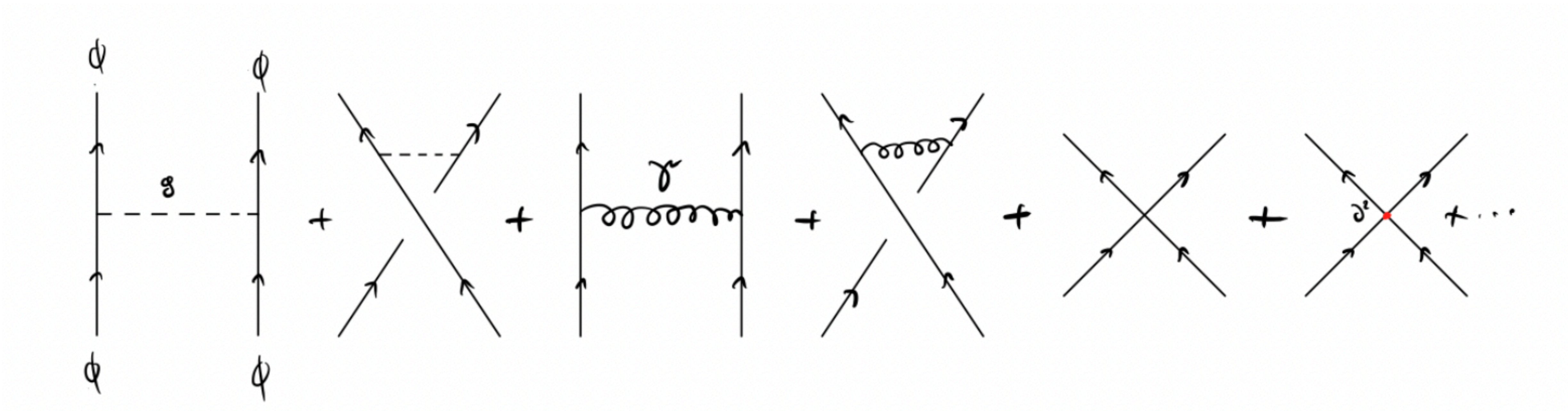
$$|g_3| \lesssim \frac{1}{M_{\mathrm{str}}^4}$$

Question: Can we derive the two-sided bounds from ACU?

Answer: Yes! Move away from forward scattering (t=0).

Fixed t (no logs+weak coupling)

Let's consider scattering of massless scalars in a weakly-coupled gravitational theory



$$T^{\mathrm{IR}}_{\phi\phi\to\phi\phi}(s,t) = 8\pi G_N\left(\frac{su}{t}+\frac{st}{u}\right) + e^2(\frac{s-u}{t}+\frac{s-t}{u}) - \lambda$$
$$+ g_1 s + g_2 s^2 + g_2' tu + g_3 s^3 + g_3' stu + g_4 s^4 + g_4' s^2 tu + g_4''(tu)^2$$

For identical particles (extra crossing $T(s,t)=T(s,u)$):

$$g_1 = g_2 + g_2' = g_3 = g_4' + 2g_4 = g_4'' - g_4 = 0$$

Dispersive sum rules (identical scalars, no gravity)

[Caron-Huot van Duong '20]

Consider instead the following set of sum rules

$$B_k(t) \equiv \oint_\infty \frac{ds}{2\pi i} \frac{1}{s} \frac{T(s,t)}{\left[s(s+t)\right]^{k/2}} = 0, \qquad (t<0, k=2,4,6,...)$$

$$B_2: \quad 2g_2 - g_3 t + \underline{8g_4 t^2} + \ldots = \sum_{J\text{even}} \int_{M^2}^{\infty} dm^2 \rho_J(m^2) F_2(J,m^2,t)$$

$$B_4: \quad \underbrace{\underline{4g_4} + \ldots}_{\text{IR}} = \underbrace{\sum_{J\text{even}} \int_{M^2}^{\infty} dm^2 \rho_J(m^2) F_4(J,m^2,t)}_{\text{UV}}$$

Null constraint:

$$\partial_t^2 B_2(t) - 4B_4(t)\Big|_{t=0}: \quad \sum_{J\text{even}} \int_{M^2}^{\infty} dm^2 \rho_J(m^2) \frac{\mathcal{J}^2\left(2\mathcal{J}^2-(5d-4)\right)}{m^8} = 0$$

Linear optimization problem

The EFT space can be then carved out as follows

$$v(m^2, J) \equiv \big(g_2(m^2, J),\ M^2 g_3(m^2, J),\ n_4(m^2, J),\ n_5(m^2, J),\ \ldots\big)\,.$$

Task

$$\text{maximize:} \quad A$$
$$\text{subject to:} \quad 0 \leq \big(-A, 1, c_4, c_5, \ldots\big) \cdot v(m^2, J) \qquad \forall\ m \geq M,\ \forall J = 0, 2, 4, \ldots$$

Result

$$0 \leq -A g_2 + M^2 g_3 + 0$$

Introducing dimensionless ratios

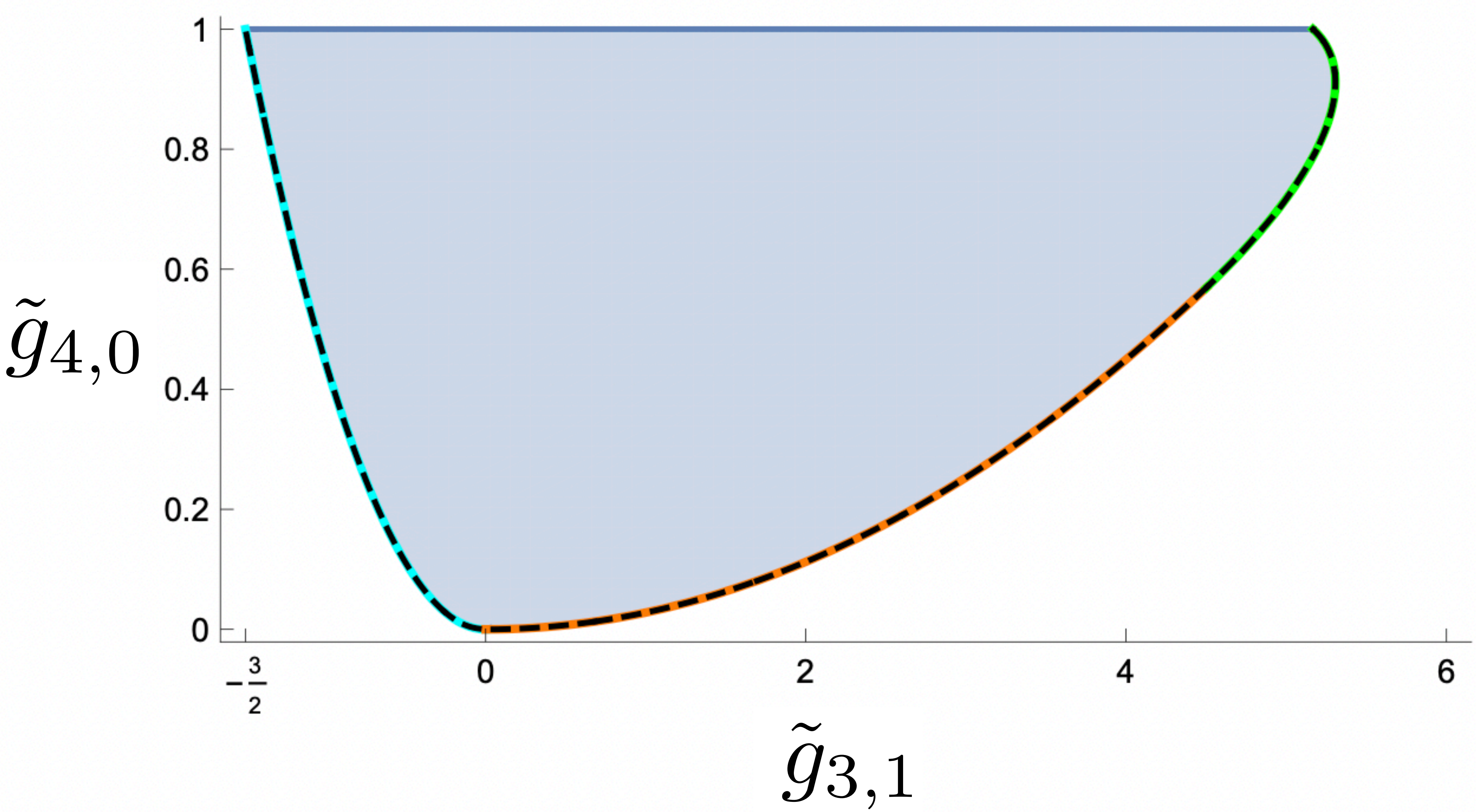
$$\tilde{g}_n = \frac{g_n M^{2(n-2)}}{g_2}$$

One gets two-sided bounds

$$A \leq \tilde{g}_3 \leq B$$

Two-sided bounds

$$\tilde{g}_{n,q} = \frac{g_{n,q} M^{2(n-2)}}{g_2}$$



[Tolley Wang Zhou]

[Caron-Huot van Duong]

[Chiang Y-t Huang Li Rodina Weng]

EFT-hedron

[Arkani-Hamed T-C Huang Y-t Huang]
[Chiang Y-t Huang Li Rodina Weng]

Performing a linear transformation on the Wilson coefficients we can get to the following basis

2d moment problem: $$a_{k,q} = \sum_{J=0}^{\infty} \int_{M^2}^{\infty} dm^2 \rho_J(m^2) \frac{1}{m^{2k+2}} \mathcal{J}^{2q}$$

(energy, spin)

Moment matrix:

truncated moment problem

$$\forall v \qquad v^T M(a) v \geq 0 : \qquad M(a) = \begin{pmatrix} \boxed{\begin{matrix} a_{0,0} & a_{1,0} & a_{0,1} \\ a_{1,0} & a_{2,0} & a_{1,1} \\ a_{0,1} & a_{1,1} & a_{0,2} \end{matrix}} & \begin{matrix} a_{2,0} & \\ a_{3,0} & \\ a_{2,1} & \cdots \end{matrix} \\ \begin{matrix} a_{2,0} & a_{3,0} & a_{2,1} \\ & & \vdots \end{matrix} & \begin{matrix} a_{4,0} & \\ & \ddots \end{matrix} \end{pmatrix} \succcurlyeq 0$$

Crossing symmetry is an extra linear constraint among a's.

Treatment of the graviton pole

[Caron-Huot Mazáč Rastelli Simmons-Duffin '21]

Certain sum rules cannot be expanded around the origin t=0

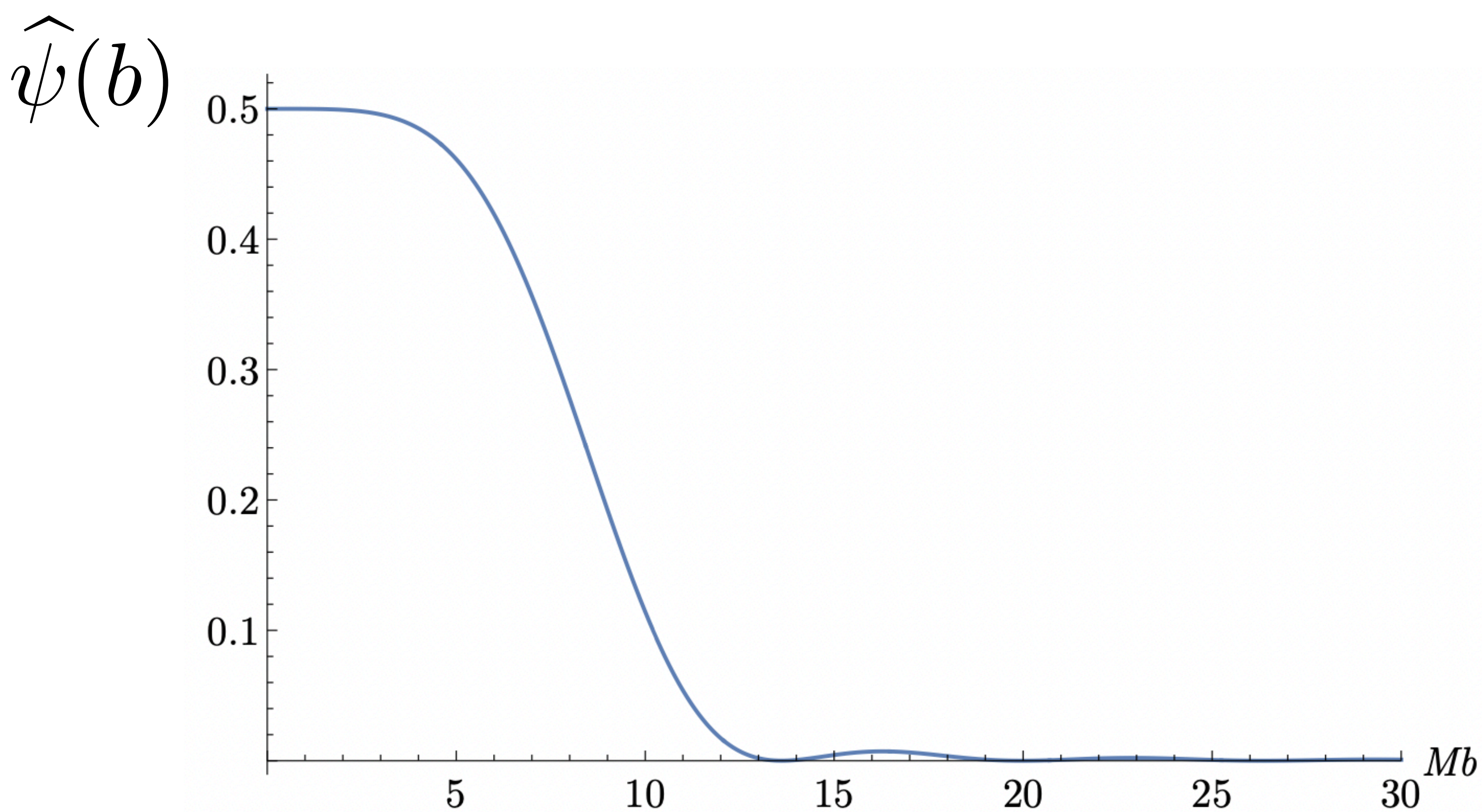
$$B_2: \quad -\frac{8\pi G_N}{t} + 2g_2 - g_3 t + 8g_4 t^2 + \ldots = \sum_{J\text{even}} \int_{M^2}^{\infty} dm^2 \rho_J(m^2) F_2(J, m^2, t)$$

The resolution is simply to consider smeared amplitudes

$$\int_0^M dq \; \psi(q) T(s, -q^2) \qquad\qquad \psi(q) \quad \text{polynomial}$$

Physically

$$b \lesssim \frac{1}{M}$$

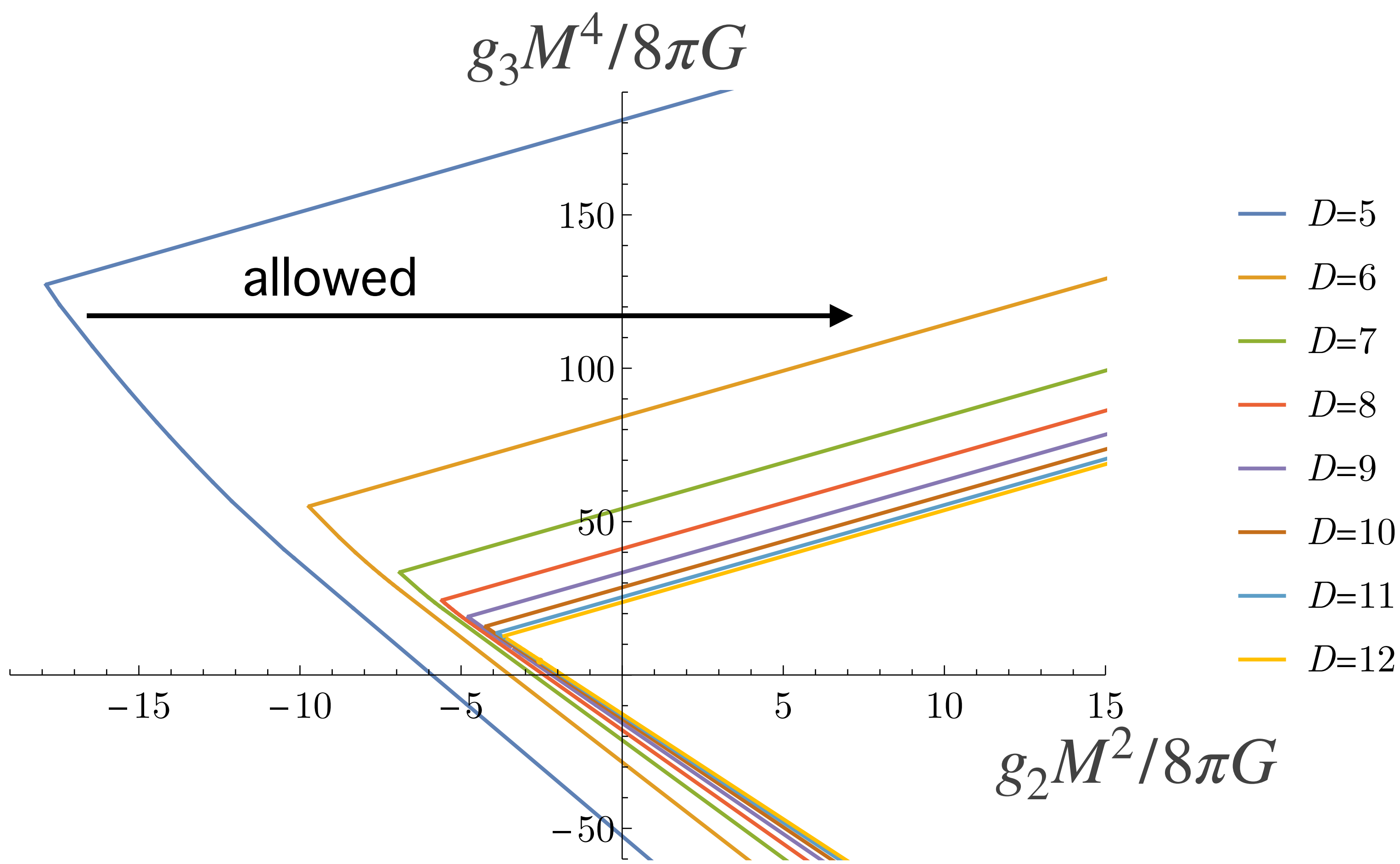


CRG conjecture

[Chowdhury Gadde Gopalka Halder Janagal Minwalla '19]
[Chandokar Chowdhury Kundu Minwalla '21]
[Häring AZ '22]

(locally) $T(s,t) \lesssim s^2, \quad t \leq 0$

EFT bounds with gravity



$$g_2(\partial\phi)^4$$

$$\mathrm{QFT}: \quad g_2 \geq 0$$

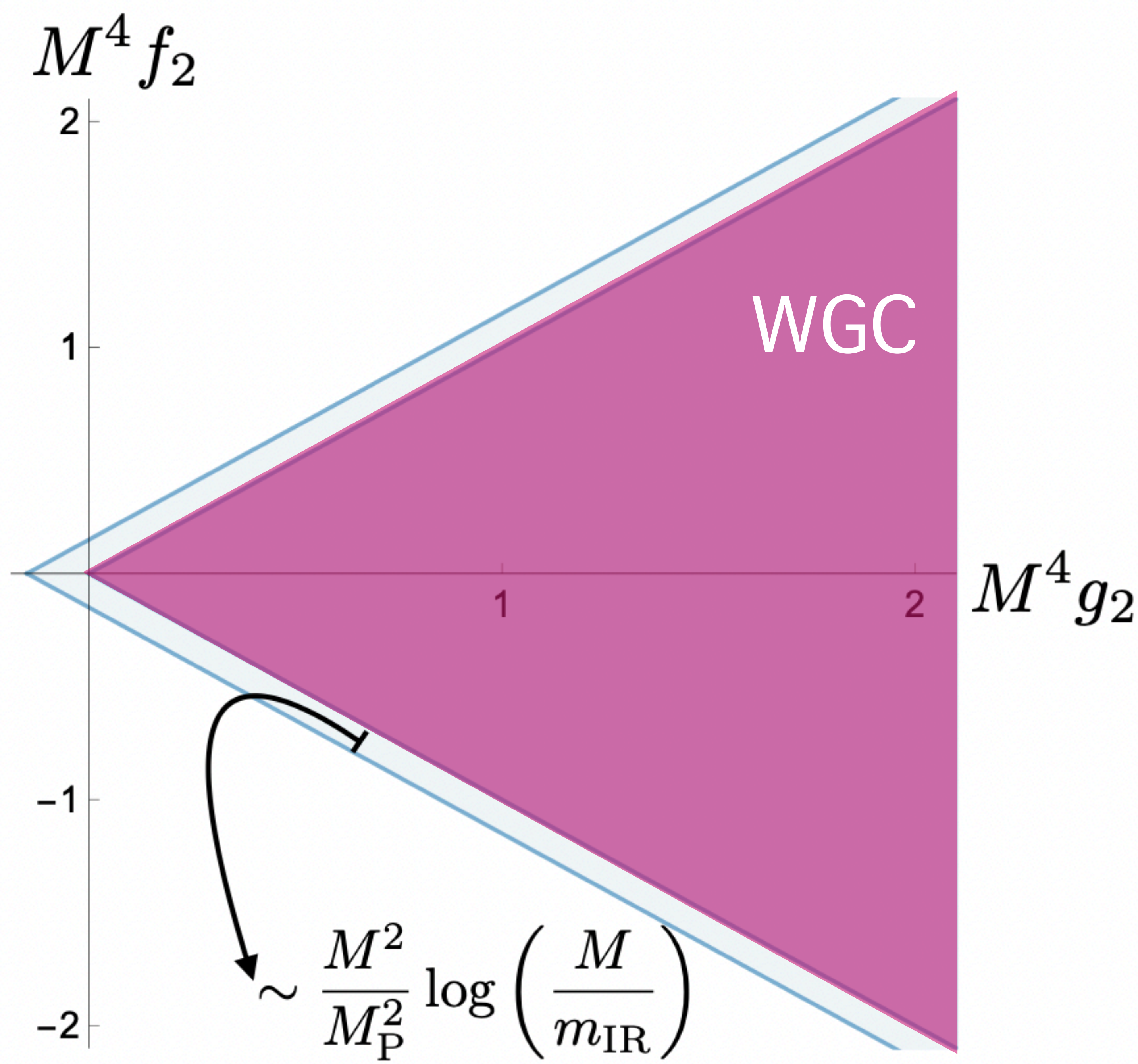
[Adams et al `06]

[Caron-Huot Mazáč Rastelli Simmons-Duffin '21]

Black hole WGC

[Henriksson McPeak Russo Vichi '22]

$$\mathcal{L} = \sqrt{-g}\Big(\frac{M_{\rm P}^2}{2}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \alpha_1(F_{\mu\nu}F^{\mu\nu})^2 + \alpha_2(F_{\mu\nu}\tilde{F}^{\mu\nu})^2 + \beta W_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} + \dots\Big)\,.$$



[Kats, Motl, Padi '07]

$$\frac{Q}{M} \geq \frac{Q}{M}\Big|_{\rm extr}:$$

$$16\alpha_{1,2} = g_2 \pm f_2$$

WGC:

$$g_2 \geq |f_2| \geq 0$$

AdS generalization

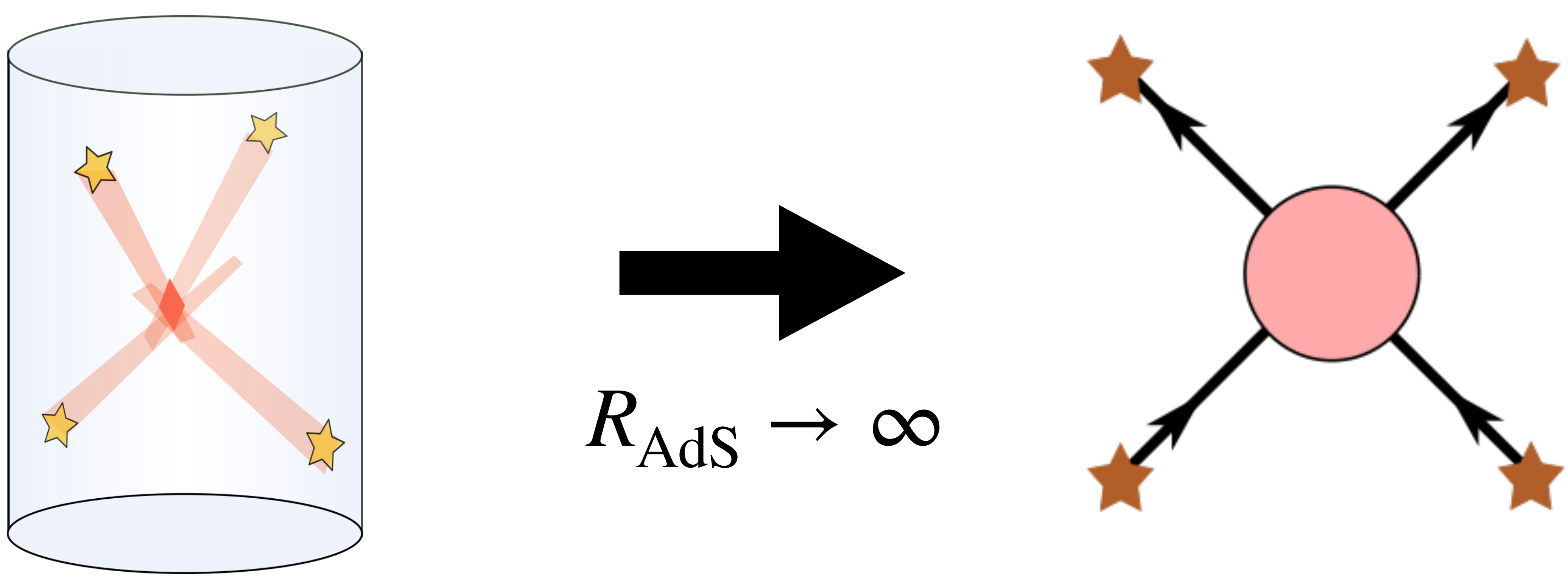
The same techniques apply in AdS but kinematics is more complicated: CFT dispersion relations.

[Carmi Caron-Huot, Mazáč, Mazáč Paulos, Mazáč Rastelli Zhou, Penedones Silva AZ]

Formulated simplest in Mellin space

$$T(s,t) \to M(s,t)$$

Dispersive sum rules and bounds match the flat space ones as expected

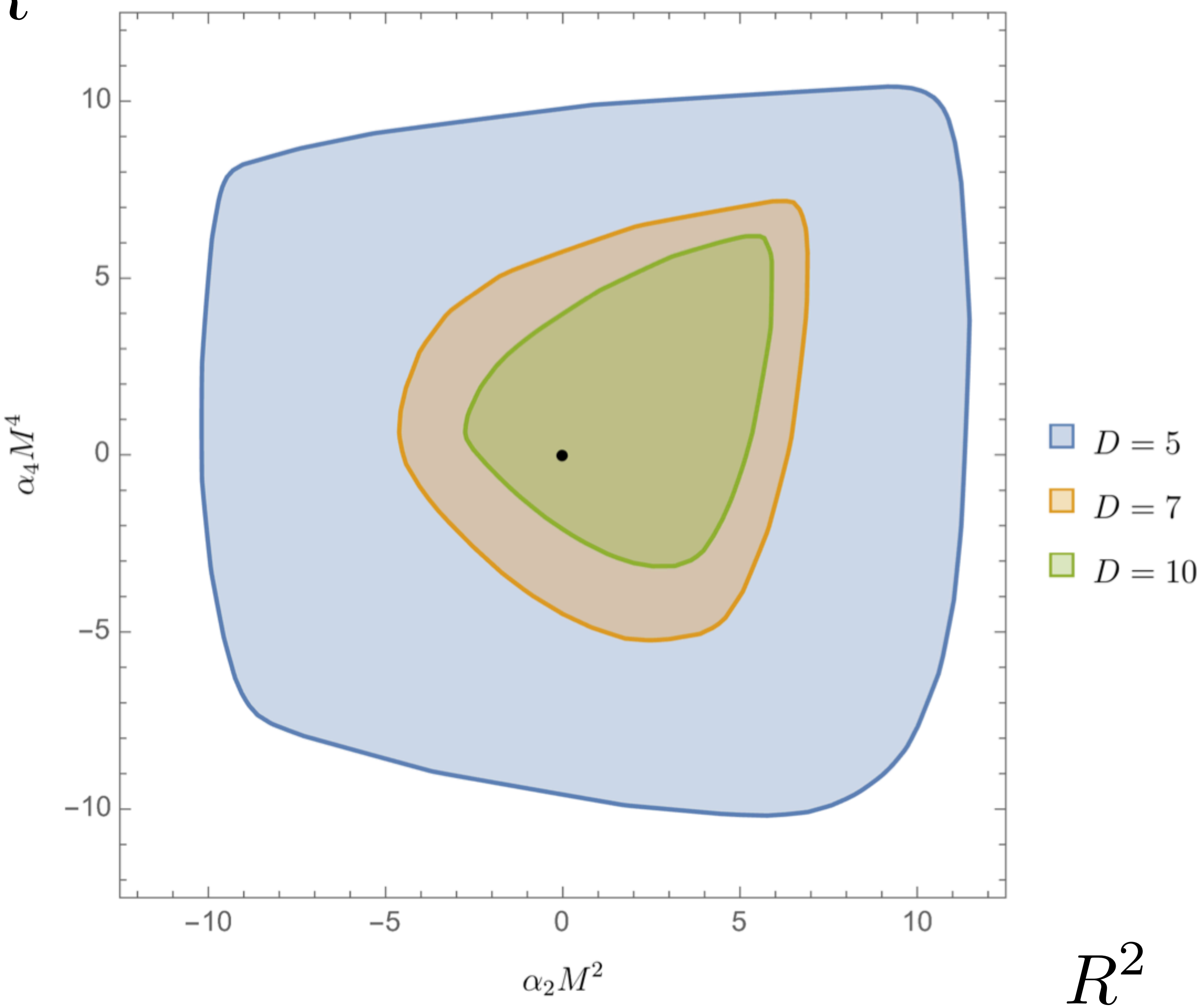


[Caron-Huot Mazáč Rastelli Simmons-Duffin]

Graviton 3-point function

[Caron-Huot Li Parra-Martinez Simmons-Duffin '22]

R^3



R^2

$$\mathrm{AdS}_5/\mathrm{CFT}_4 :$$

$$\left|\frac{a-c}{c}\right| \leq \frac{\#}{\Delta_{\mathrm{gap}}^2}$$

Dispersive sum rules

$$\mathrm{IR} + \mathrm{UV} = 0$$

Bound on Regge

- forward scattering

[Adams Arkani-Hamed Dubovsky Nicolis Rattazzi `06] [Bellazzini de Rham Melville Tolley Zhou ...]

- non-forward scattering, crossing [2019-now]

[Arkani-Hamed T-C Huang Y-t Huang][Tolley Wang Zhou] [Caron-Huot van Duong] [Sinha Zahed '20]

- graviton pole [Caron-Huot Mazáč Rastelli Simmons-Duffin '21]
- spinning particles

[Arkani-Hamed T-C Huang Y-t Huang] [Bern, Kosmopolous, AZ] [Henriksson McPeak Russo Vichi]
[Chiang Y-t Huang Li Rodina Weng] [Caron-Huot Li Parra-Martinez Simmons-Duffin '22]

- AdS

[Carmi Caron-Huot] [Mazáč, Mazáč Paulos, Mazáč Rastelli Zhou] [Penedones Silva AZ '19]
[Caron-Huot Mazáč Rastelli Simmons-Duffin '21]

- quantum corrections

[Bellazzini Mirò Rattazzi Riembau Riva '20][Bellazzini Riembau Riva '21]

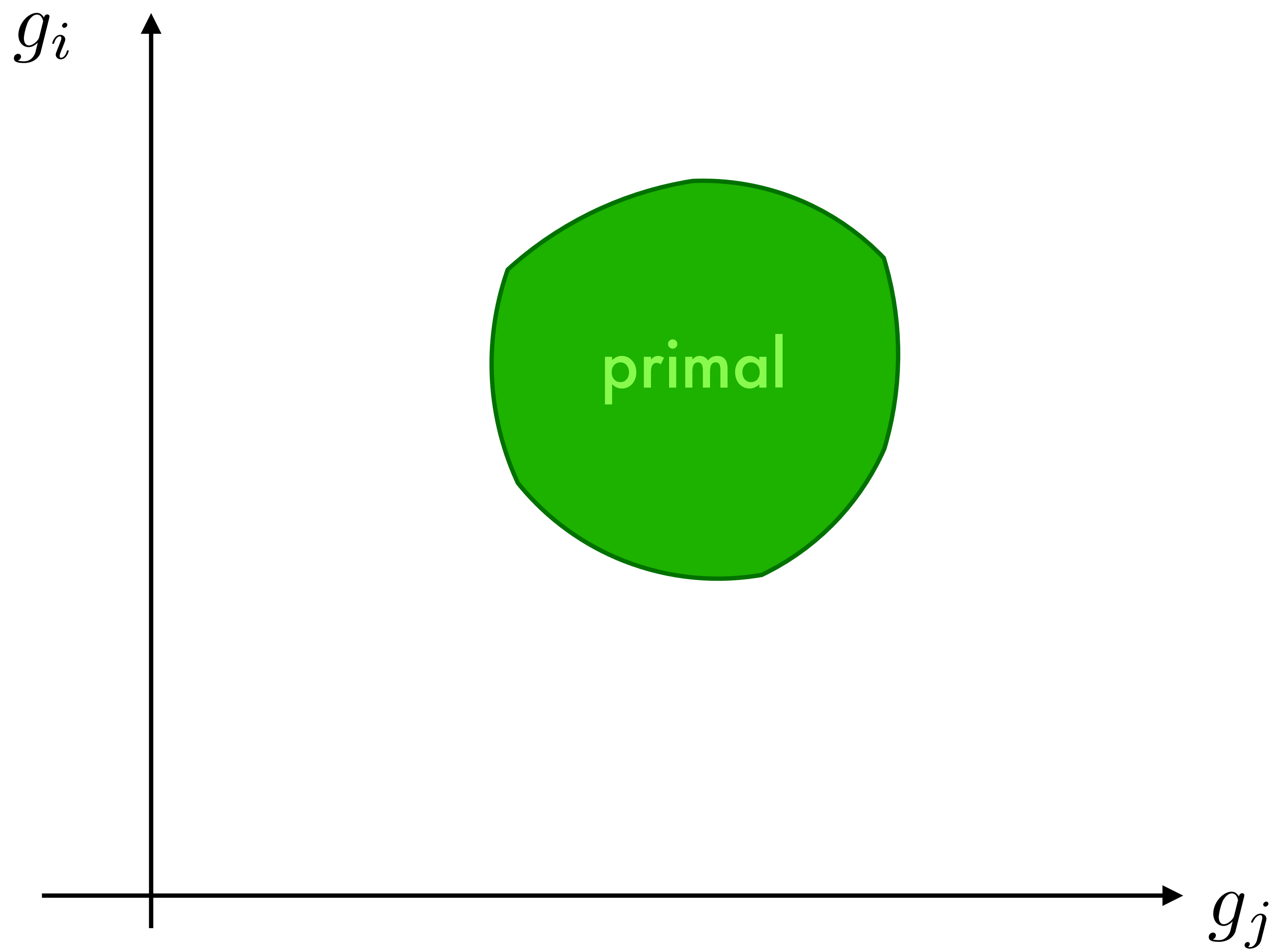
- nonperturbative unitarity

[Guerrieri Sever '21][He Kruczsenki '21]

Open problems II

- Precise relation to causality? [local time machines]
- Constraints from the IR on the UV? [not rigorous]
- Including nonperturbative unitarity? [used positivity]
  Bounds on the dimension 6 operators ? [cannot drop the arc]
- IR divergencies in $d = 4$ [OK in AdS]
- Treatment of logarithmic loops [use arcs]
- Theories with accumulation points in the spectrum
  [tend to saturate bounds]
- Higher-point amplitudes [stronger constraints?]

Primal Bootstrap



Basic Idea

[Paulos, Penedones, Toledo, van Rees, Vieira '17]

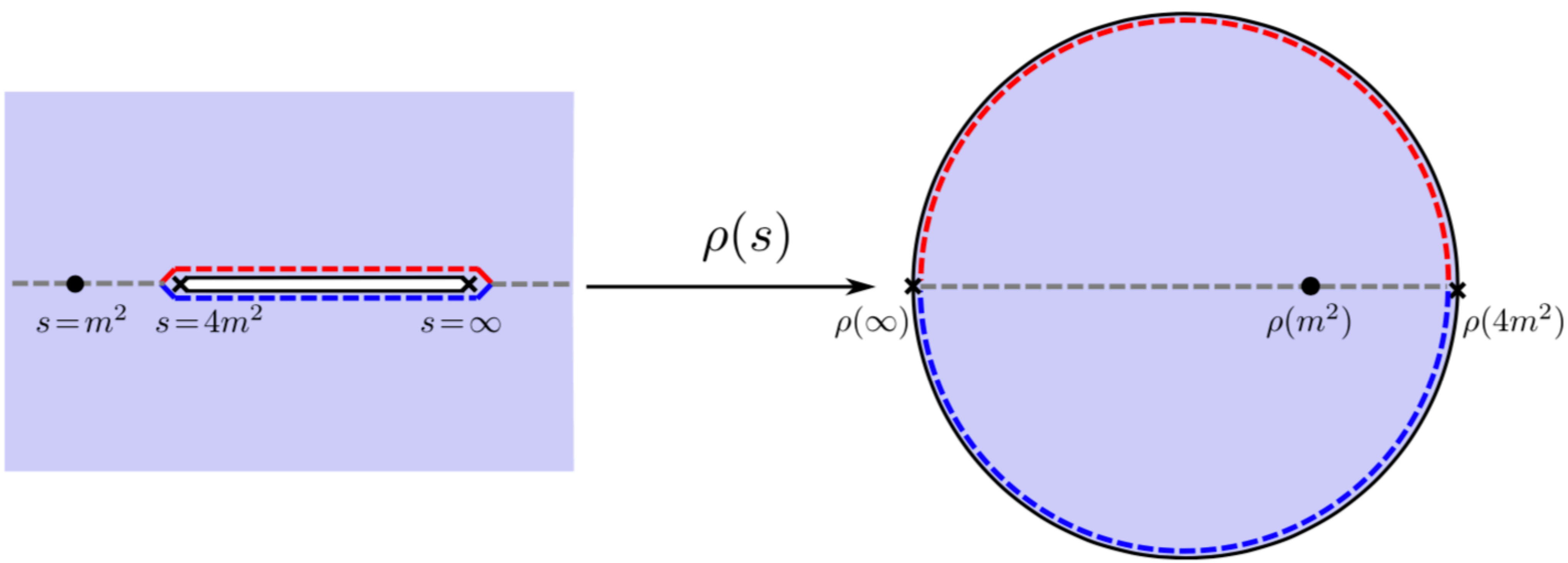
We would like to populate the space of amplitudes by explicitly constructing them.

$$T(s,t) = \sum_{a,b,c=0} \alpha_{abc}\rho_s^a \rho_t^b \rho_u^c + \text{extra}\big|_{u=4m^2-s-t}$$

simple and complete basis

$$\rho_s \equiv \frac{\sqrt{4m^2-s_0}-\sqrt{4m^2+s}}{\sqrt{4m^2-s_0}+\sqrt{4m^2+s}}$$

$$a+b+c \leq N_{\max}$$



The ρ ansatz

[Paulos, Penedones, Toledo, van Rees, Vieira '17]

$$T(s,t) = \sum_{a,b,c=0} \alpha_{abc}\rho_s^a \rho_t^b \rho_u^c + \text{extra}\Big|_{u=4m^2-s-t}$$

☑ (Maximal) Analyticity ☑ Crossing ☐ Unitarity (numerically)

$$\alpha_{abc} = \alpha_{bac} = \ldots$$

Unitarity implementation:

$$\left.\begin{array}{l} \bullet\ \text{Impose nonperturbative unitarity on a set of lattice points } s_i \text{ for } J \le J_{\max}(N_{\max}) \\ \bullet\ \text{Maximize some Wilson coefficient} \end{array}\right\} \text{linear in the } \alpha_{abc}\text{'s}$$

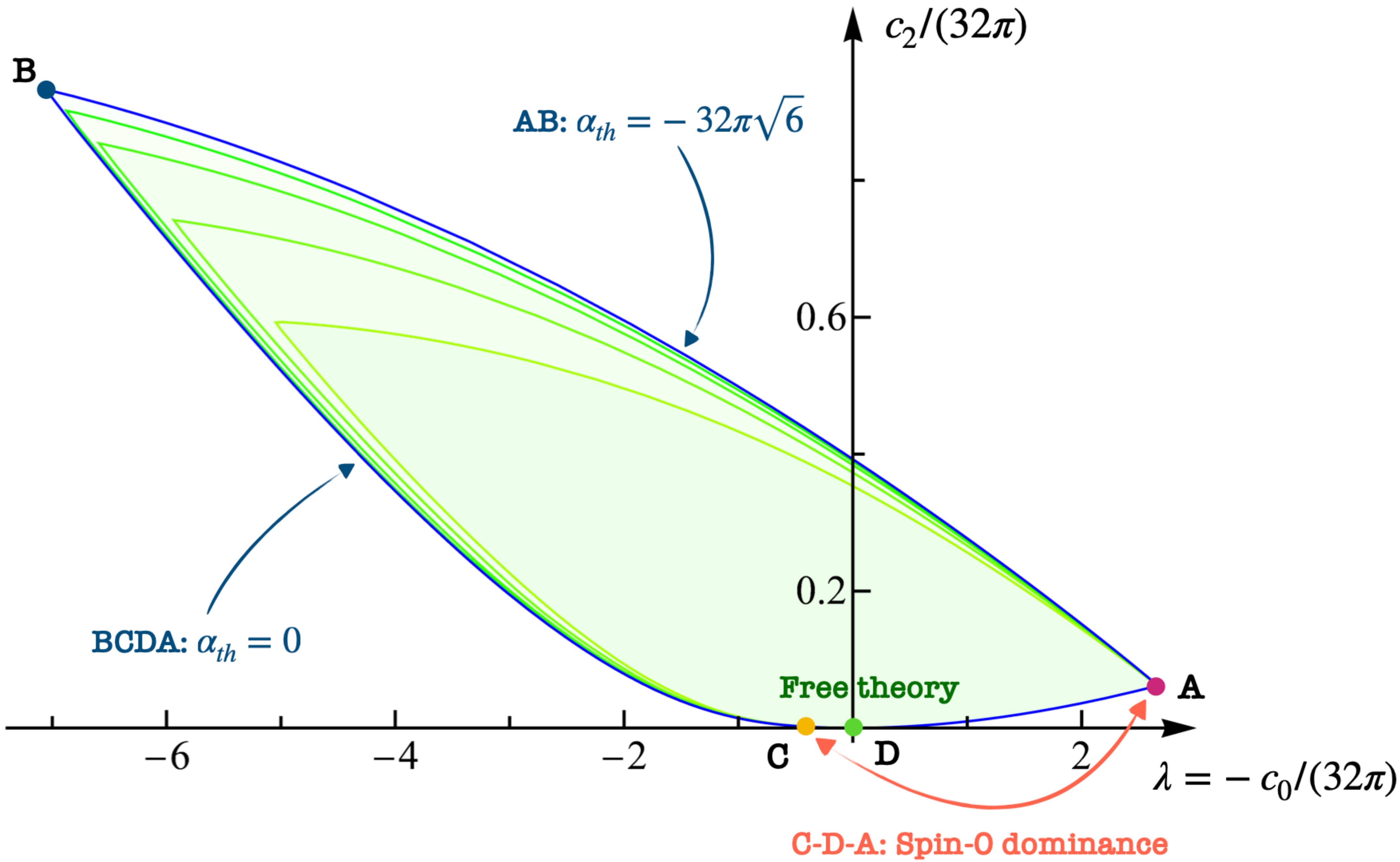
- Extrapolate $N_{\max} \to \infty$ (and hope for the better)

ϕ^4

[Chen, Fitzpatrick, Karateev '22]
[Elias Miro, Guerrieri, Gümüs '22]

Consider a theory with a single stable particle and Z2 symmetry.

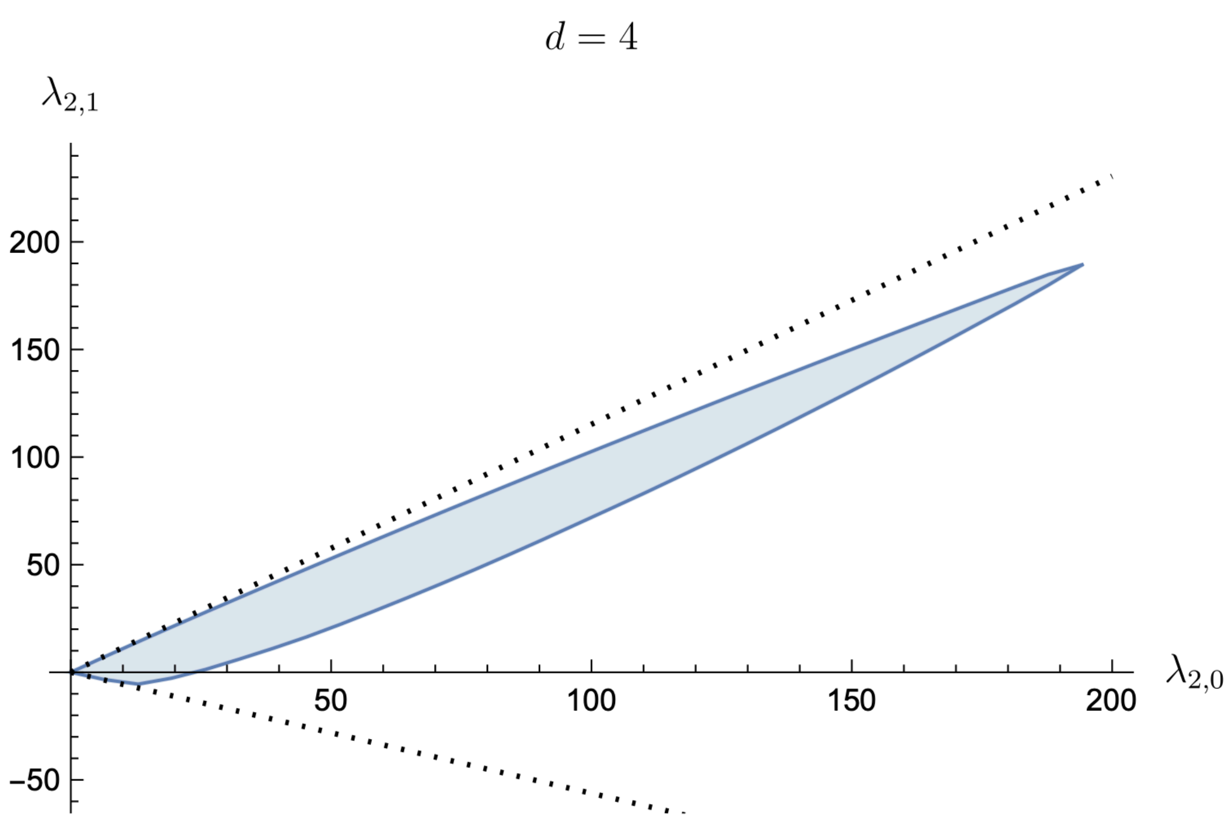
$$T(s,t) = -c_0 + c_2(\overline{s}^2 + \overline{t}^2 + \overline{u}^2) + \ldots \qquad \overline{s} = s - \frac{4m^2}{3}$$

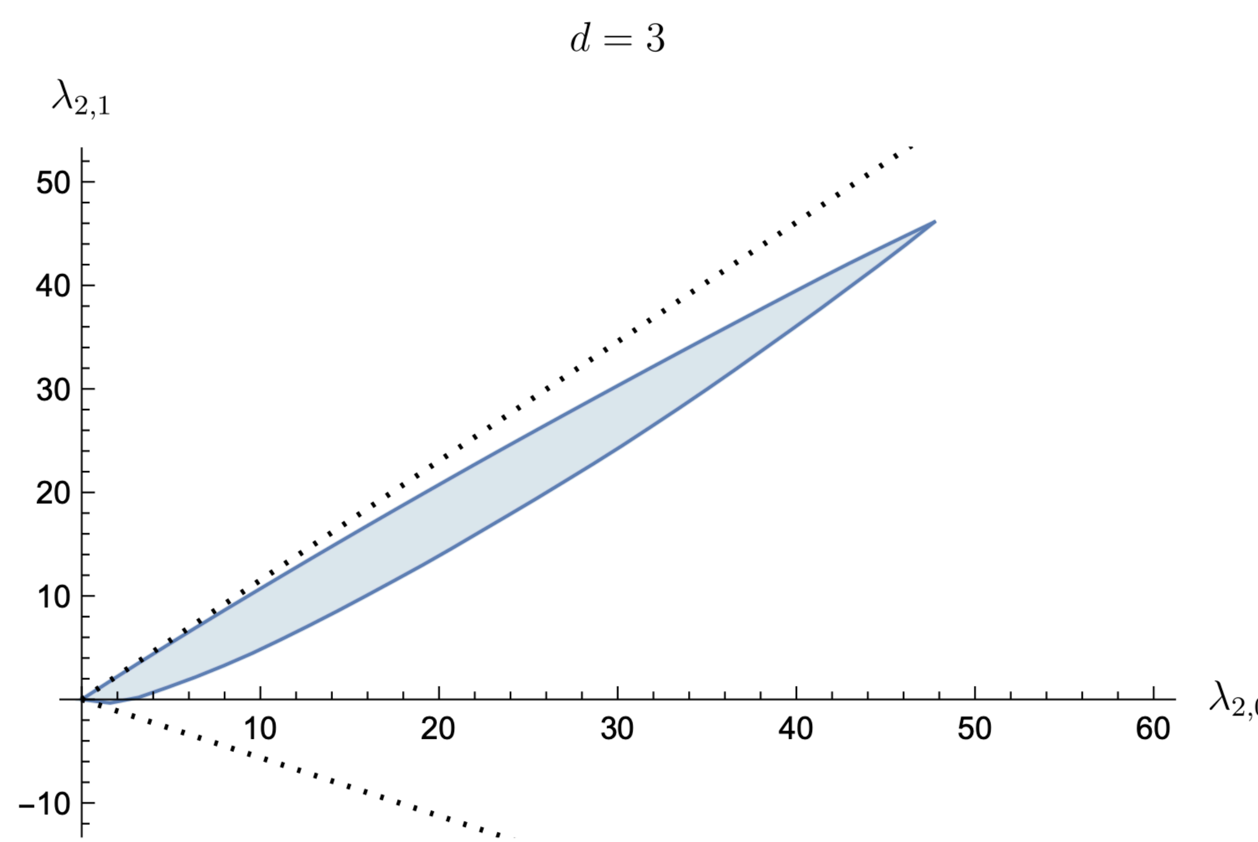


Positivity vs full unitarity

[Chen, Fitzpatrick, Karateev '22]

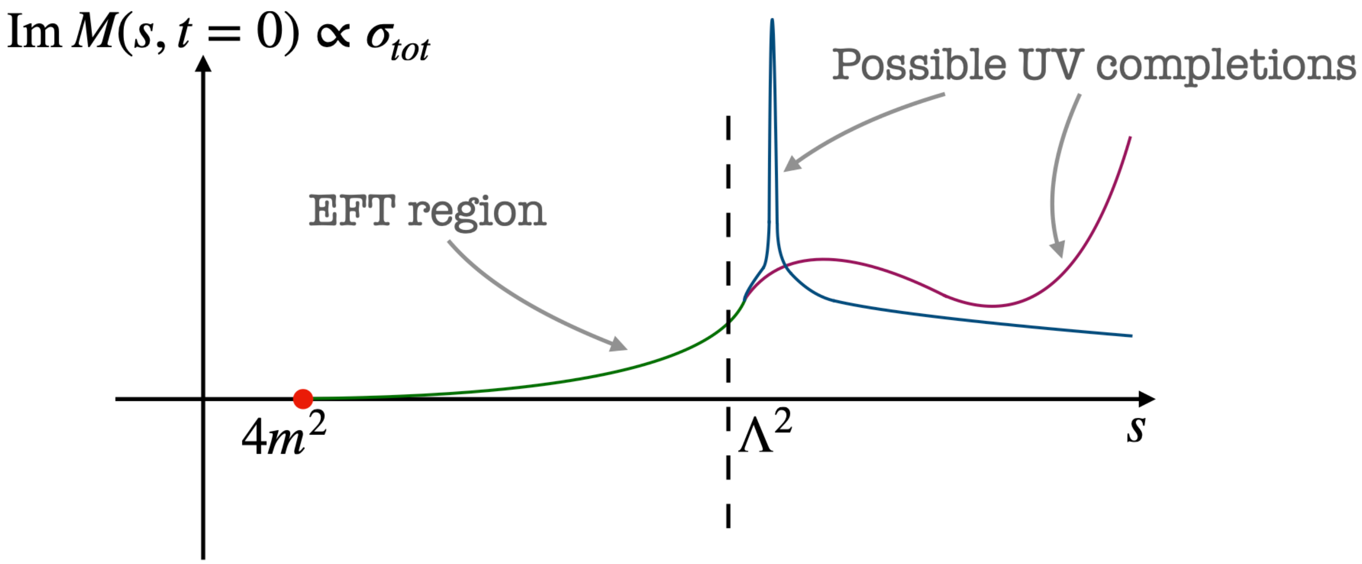
- Positivity $$\mathrm{Im} f_J(s) \geq 0$$
- Nonperturbative unitarity $$2 \geq \mathrm{Im} f_J(s) \geq |f_J(s)|^2 \geq 0$$

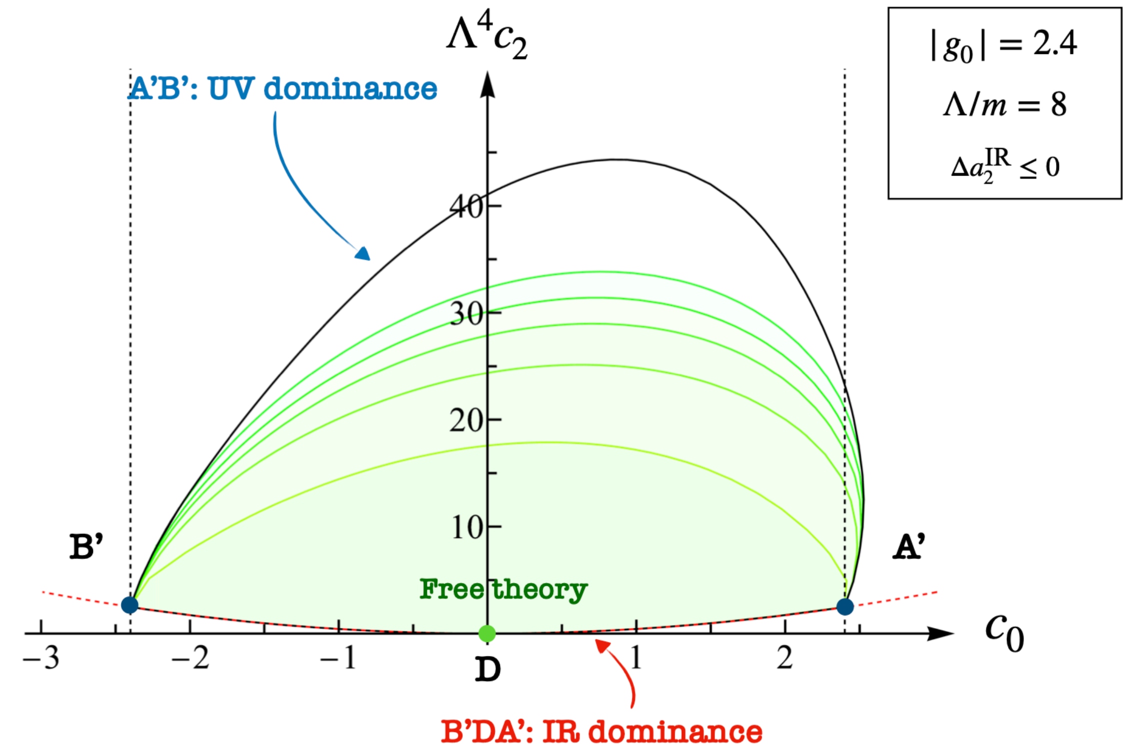


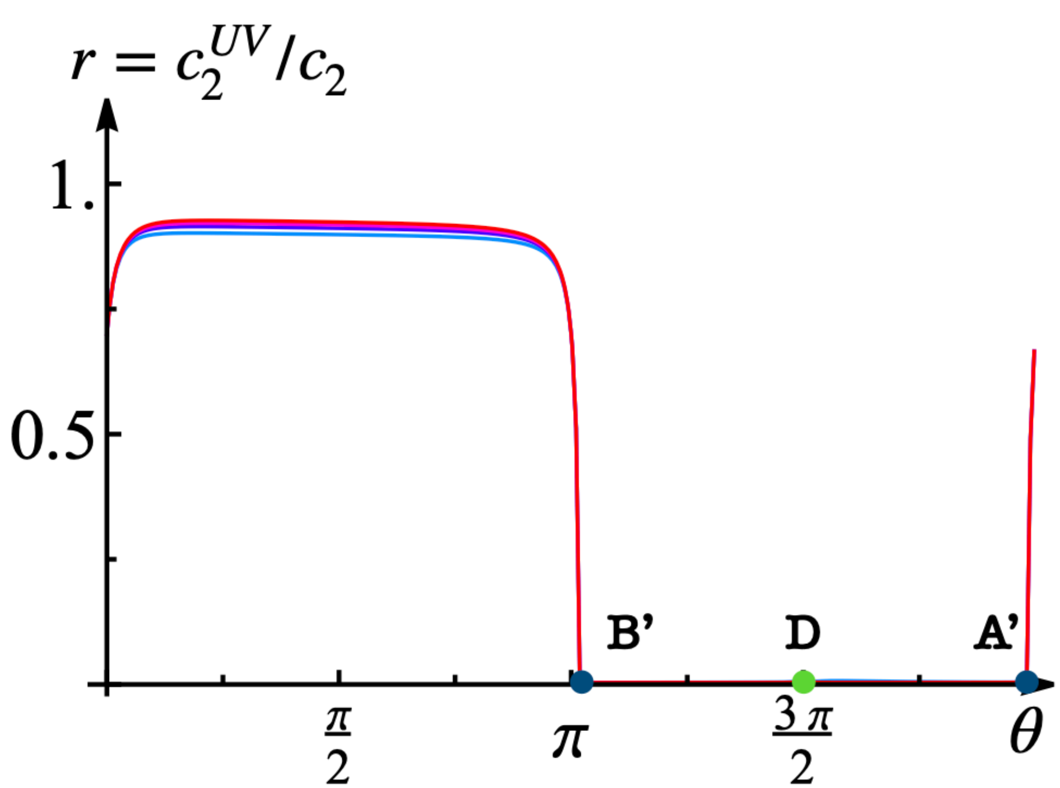


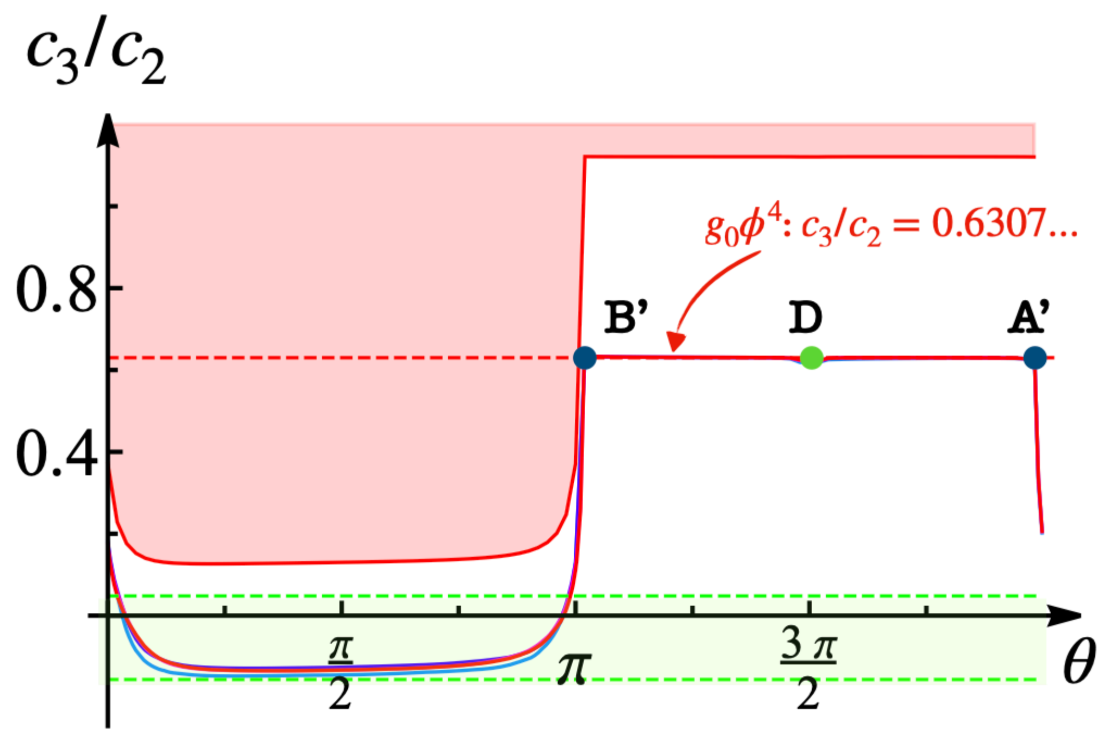
EFT vs full unitarity

[Elias Miro, Guerrieri, Gümüs '22]







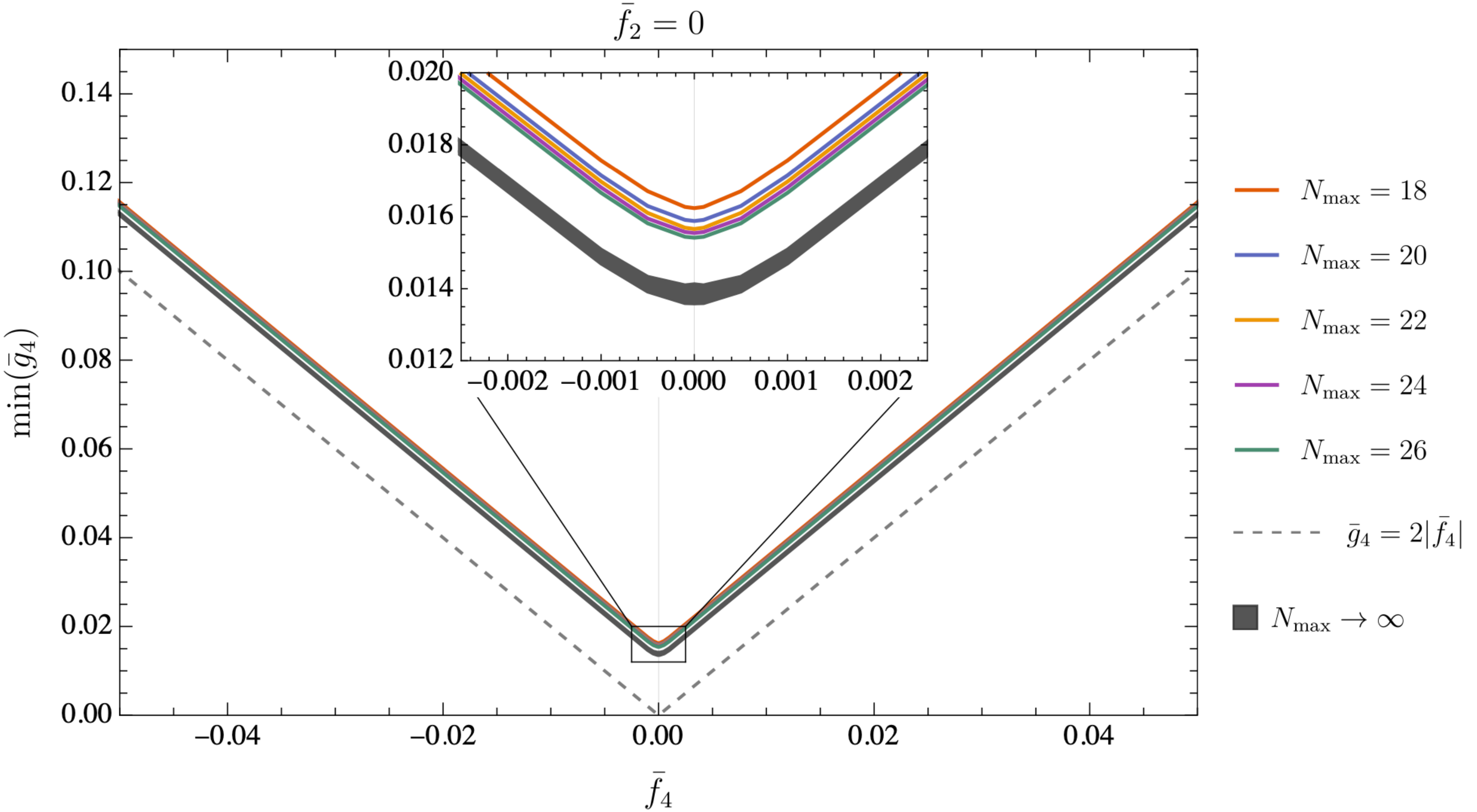


Photon scattering

[Häring, Hebbar, Karateev, Meineri, Penedones '22]

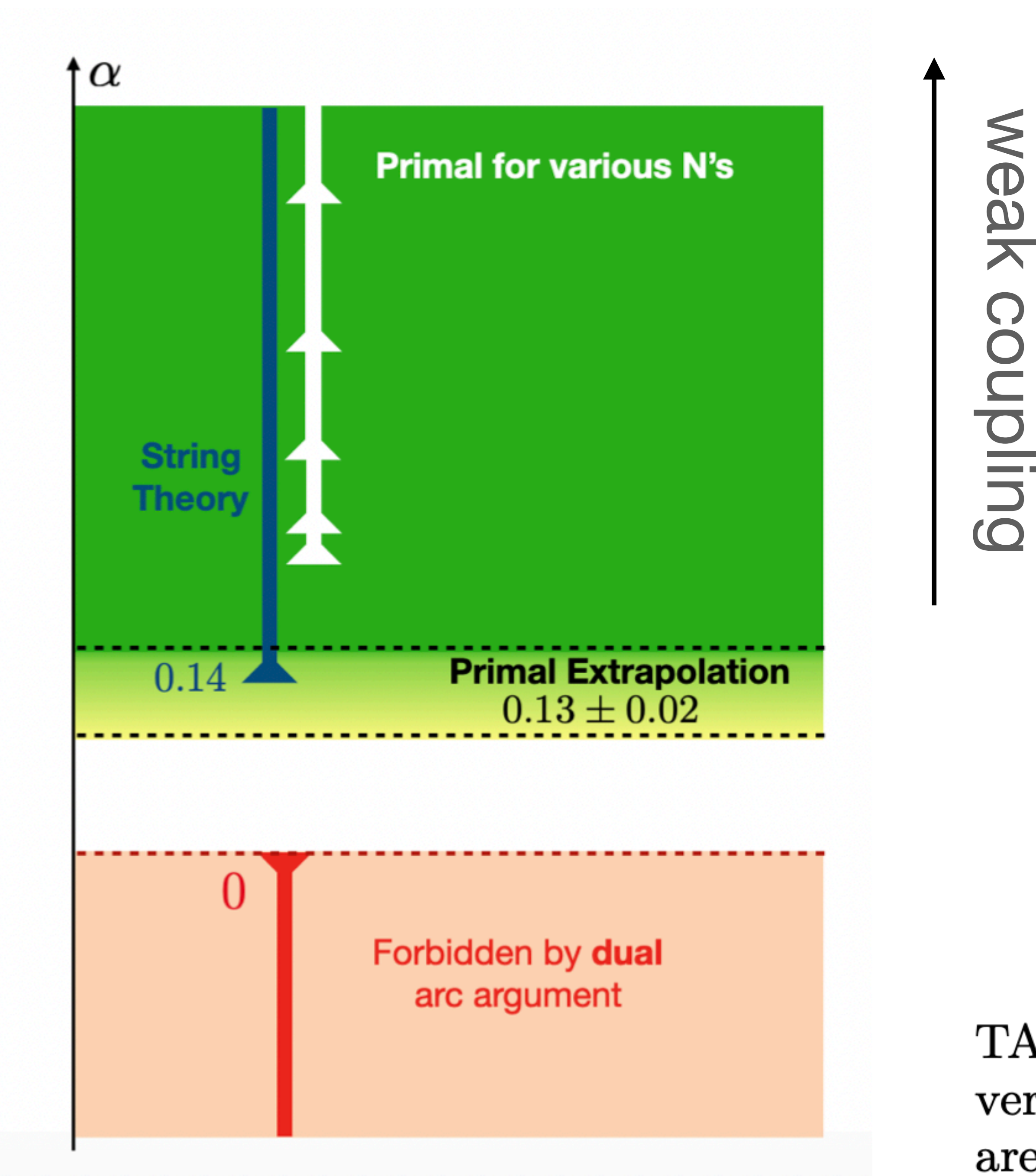
$$\gamma(\lambda_1)\gamma(\lambda_2) \to \gamma(\lambda_3)\gamma(\lambda_4)$$

$$\bar{g}_4 \pm 2\bar{f}_4 \geq 0 + \underbrace{\frac{42\bar{f}_2^2 \pm 50\bar{f}_2 + 21}{480\pi^2}}_{>0} \log(\hat{s}\sqrt{g_2}) + \mathcal{O}(\hat{s}\sqrt{g_2})$$



10d maximal sugra: graviton scattering

Scalars from the graviton multiplet



$$T_{\text{Sugra}} = \frac{8\pi G_N}{stu} + g_0 + \ldots$$

$$g_0 \sim \frac{\alpha}{M_{\text{Pl}}^6}$$

Dimension	Bootstrap	String/M–Theory
9	0.223 ± 0.002	0.241752
10	0.124 ± 0.003	0.138949
11	0.101 ± 0.005	0.102808

TABLE I. The bootstrap minimal value estimates for α are very close to the minima attained by String/M-theory. They are slightly smaller; that small gap between the two might be due to unaccounted inelasticity effects as discussed below.

[Guerrieri Penedones Vieira '21-22]

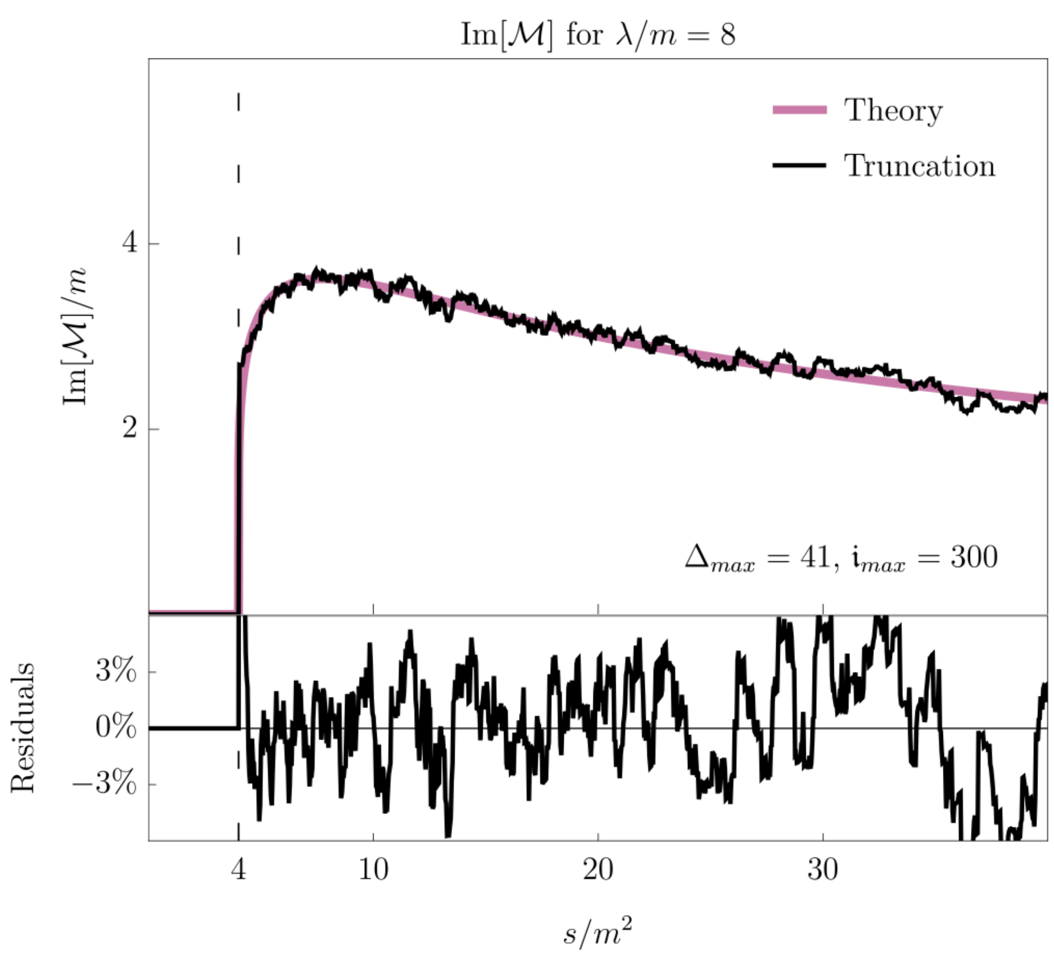
Hamiltonian truncation

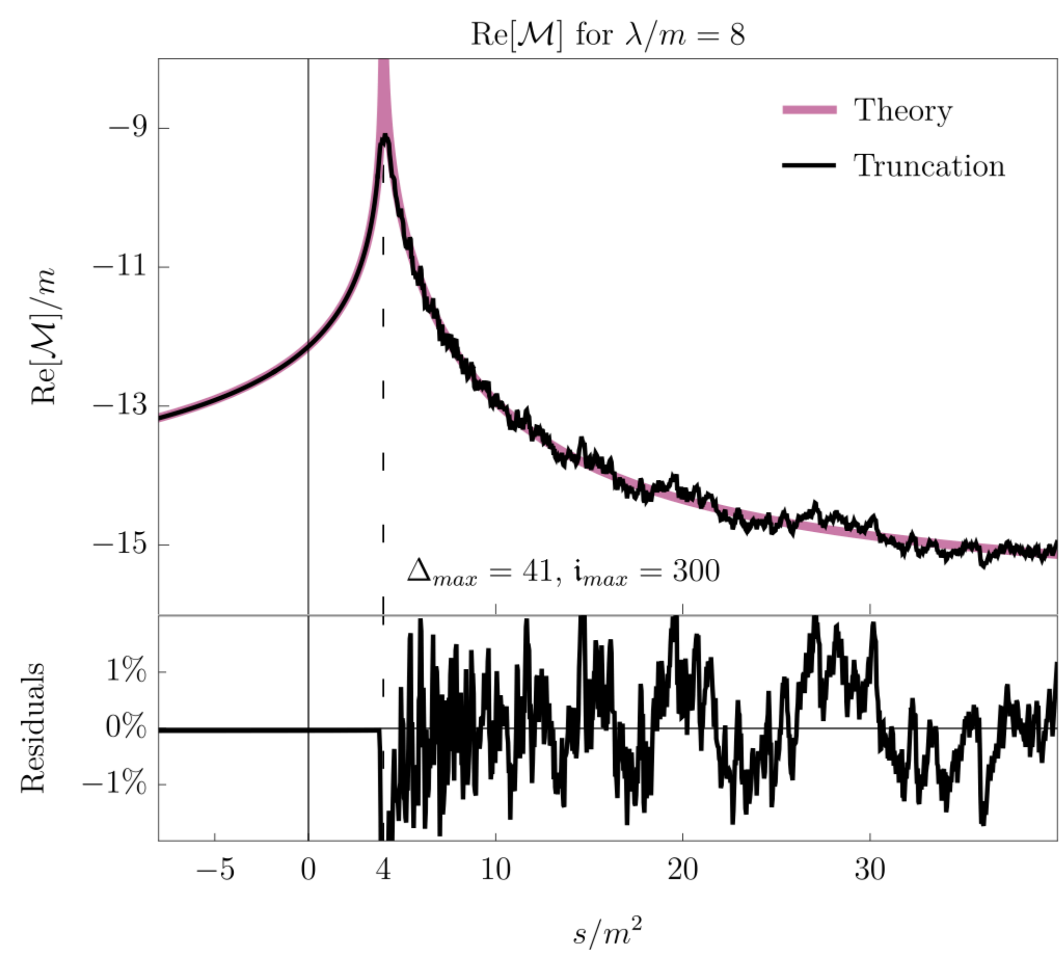
[Henning, Murayama, Riva, Thompson '22]

- Nonperturbative recipe for directly computing the S-matrix
- Discretize, truncate, diagonalize energy eigenstates

$$H_{\mathrm{QFT}} = H_{\mathrm{CFT}} + V, \qquad V = g_i \int d^{d-1}x\, \mathcal{O}_i(x).$$

$$\langle \mathbf{p}_3, \mathbf{p}_4; \mathrm{out} | \mathbf{p}_1, \mathbf{p}_2; \mathrm{in} \rangle \overset{\text{LSZ}}{\Longrightarrow} \langle \mathbf{p}_4 | \phi_3 \phi_2 | \mathbf{p}_1 \rangle \overset{\text{Schwinger-Dyson}}{\Longrightarrow} \langle \mathbf{p}_4 | J_3 J_2 | \mathbf{p}_1 \rangle \overset{\mathbb{1} \simeq \sum_\alpha |M_\alpha^2\rangle\langle M_\alpha^2|}{\Longrightarrow} \langle \mathbf{p}_4 | J_3 | M_\alpha^2 \rangle \langle M_\alpha^2 | J_2 | \mathbf{p}_1 \rangle$$





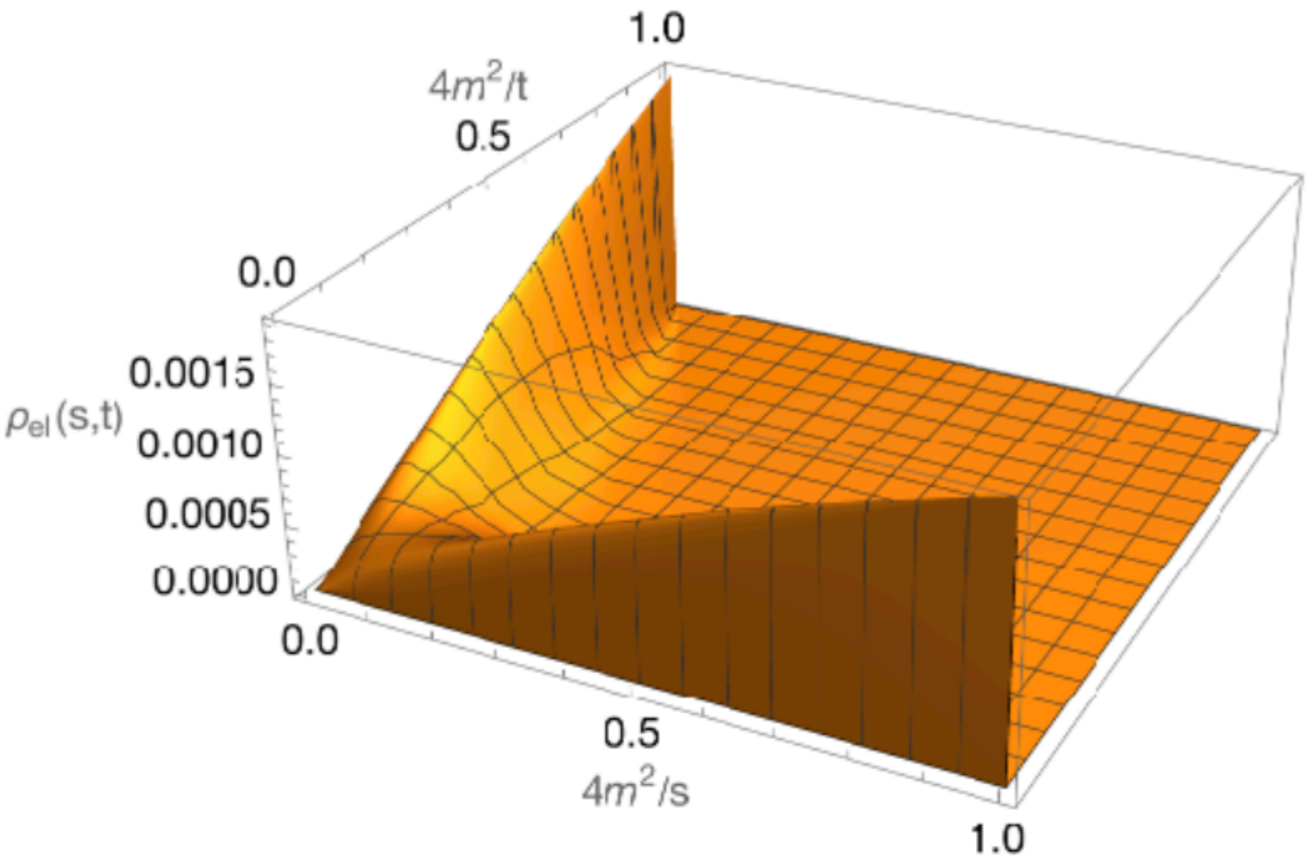
[d=3 O(N)]

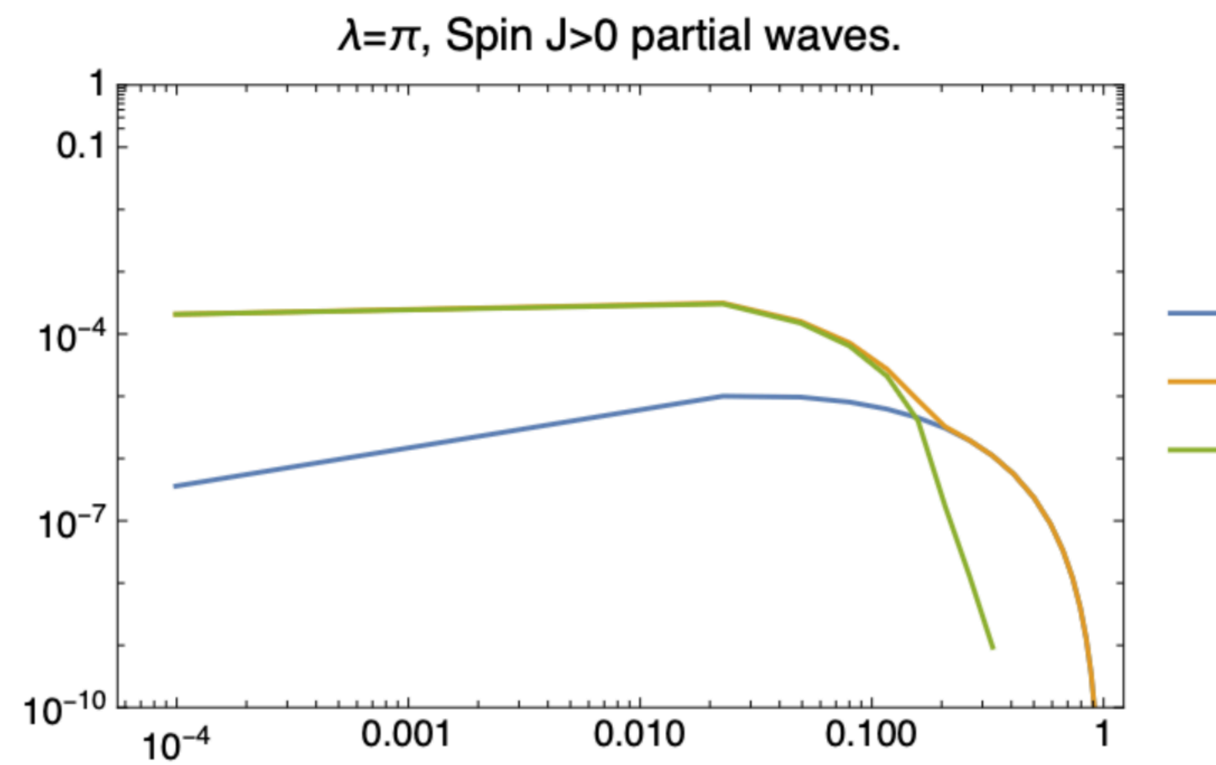
Scattering from dispersive iterations

[Atkinson '68]
[Tourkine AZ (to appear)]

- ☑ (Maximal) Analyticity
- ☑ Crossing
- ☐ Unitarity (numerically)

We start with the Mandelstam representation. There is a way to impose elastic unitarity by iterating it and searching for the fixed point of iterations.

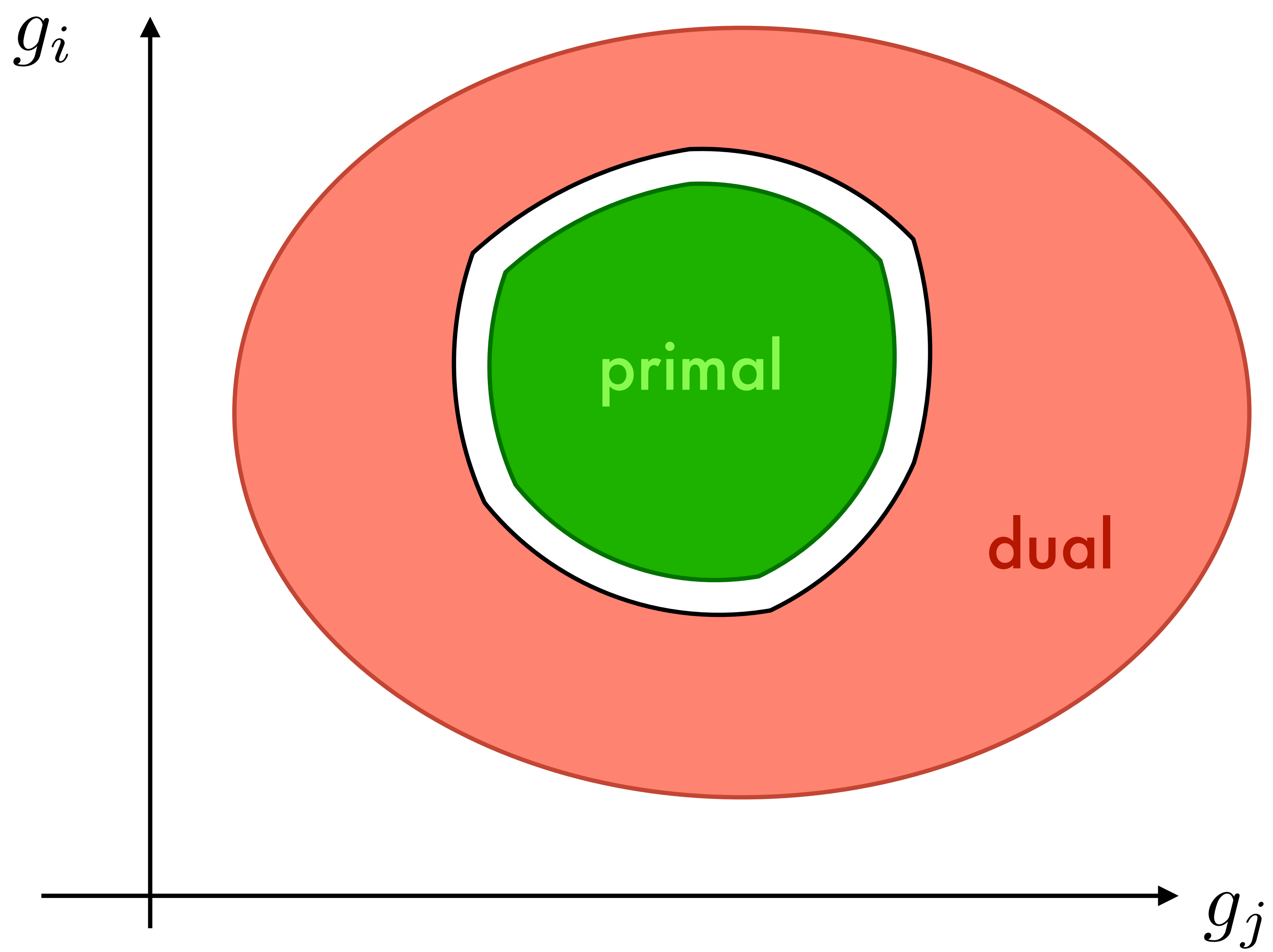




These amplitudes exhibit particle production (Aks theorem).

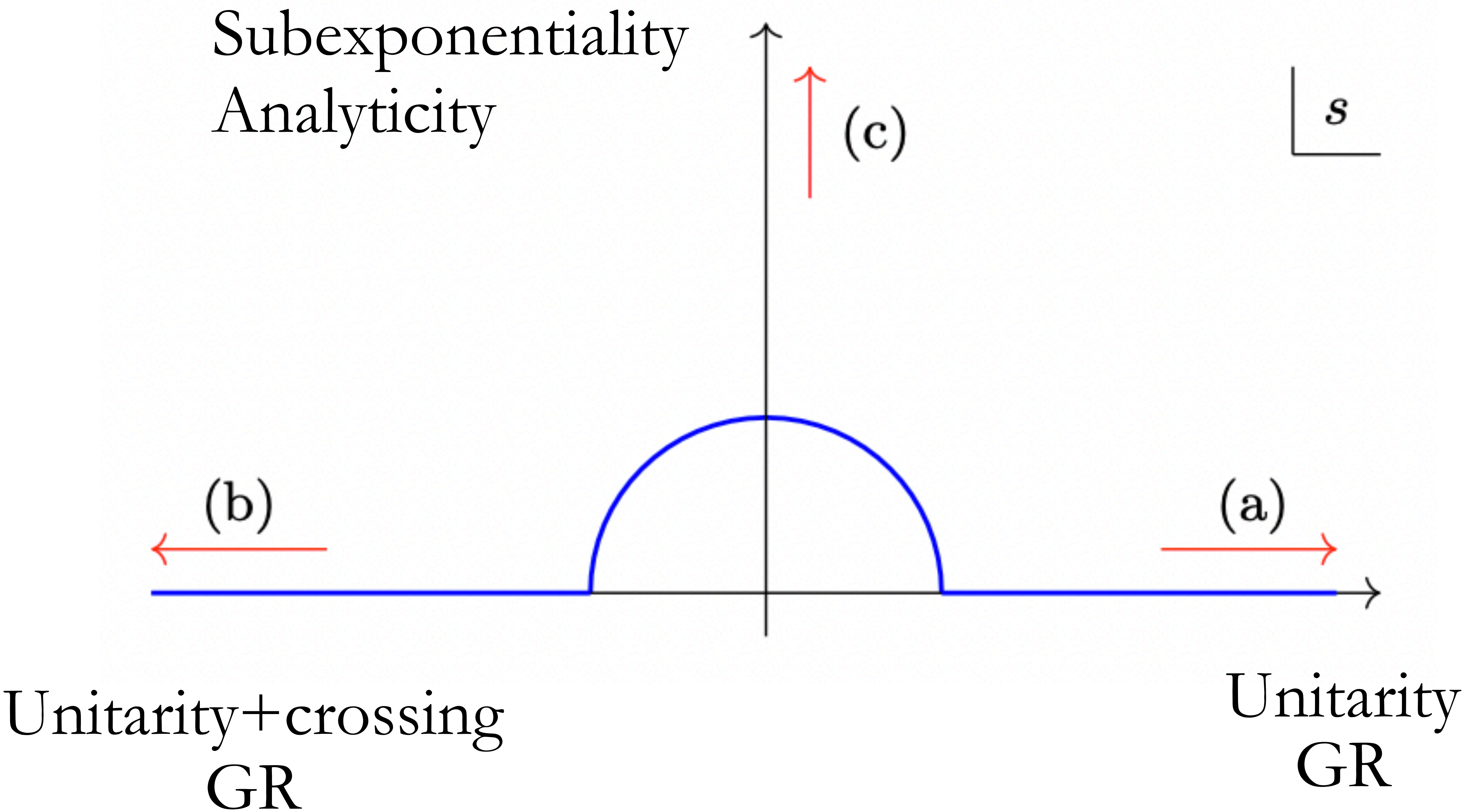
Open problems

- Dimension 5&6 operators? [relation to phenomenology]
- How do we put in particle production?
- Solvable amplitudes? [beyond d=2]
- Other observables? [Regge limit]
- Why does it work so well?
- What are the interesting targets? [saturation of the Froissart bound]
- Pion scattering?



Thank you for listening!

Strategy



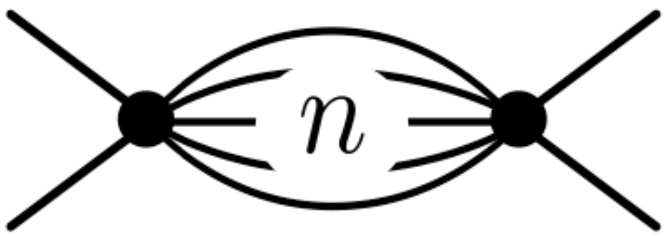
High energy $\neq$ Short distance

maximum modulus principle
(Pragmen-Landelof)

Unitarity & Non-analyticity

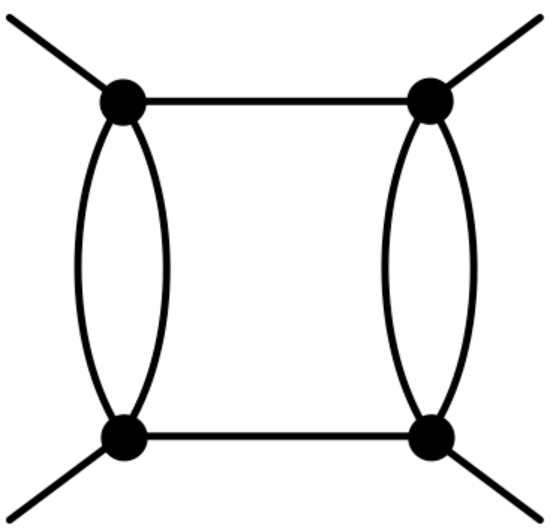
Scattering amplitudes have singularities/non-analyticities.

[normal thresholds]



$s = (nm)^2$

[Karplus/Landau curves]



$$\mathrm{dDisc}T: \quad (s-4m^2)(t-16m^2)-64m^2=0$$

Could be probed using the Landau equations.

[Mizera, Telen, Hannesdottir, McLeod, Schwartz, Vergu, Correia, Sever, AZ '21-22]

Regge bound (no gap)

[Martin '66]
[CMRS '21]

Similarly, it is useful to consider a smeared amplitude

$$T_{\psi_{a,b}}(s) \equiv \int_0^{q_0} dqq[\psi_{a,b}(q)T(s,-q^2)]$$

$$\psi_{a,b}(q) \overset{q\to 0}{\sim} q^a, \quad a>0$$ ⟵ graviton pole

$$\psi_{a,b}(q) \overset{q\to q_0}{\sim} (q_0-q)^b, \quad b\geq 0.$$

Model	Pointwise	Smeared
Born	$\lvert s\rvert^{2-\frac{d-7}{2(d-4)}}$	$\lvert s\rvert^{2-\min(1,\frac{\mathrm{a}}{d-4},\frac{\mathrm{b}+\frac{d-5}{2}}{d-4})}$
$\text{Eikonal+tidal}\big\rvert_{d>5}$	$\lvert s\rvert^{2-\frac{d-4}{2(d-3)}}$	$\lvert s\rvert^{2-\min(1,\frac{\mathrm{a}}{d-4},\frac{\mathrm{b}+\frac{d-1}{2}}{d-2})}$
$\text{Eikonal+GW}\big\rvert_{d=5}$	$\lvert s\rvert^{2-\frac{1}{5}}$	$\lvert s\rvert^{2-\min(1,\mathrm{a},\frac{2\mathrm{b}+3}{5})}$

$$\lim_{|s|\to\infty}\frac{T_\psi(s)}{|s|^2}=0, \quad d\geq 5$$

[CMRLi '22]
[Häring, AZ '22]

Comments

[Caron-Huot,Li,Parra-Martinez,Simmons-Duffin '22]

- Our results agree with the flat space limit of the AdS/CFT analysis.

This can be taken as an indirect argument supporting our assumptions.

[talk by Leonardo]

- Upon further constraint on the smearing function we get

$$\lim_{|s|\to\infty} |T_{\psi_{a\geq d-4,b\geq\frac{d-3}{2}}}(s)| \leq C|s|.$$

- Black hole formation recuses the number of subtractions to one

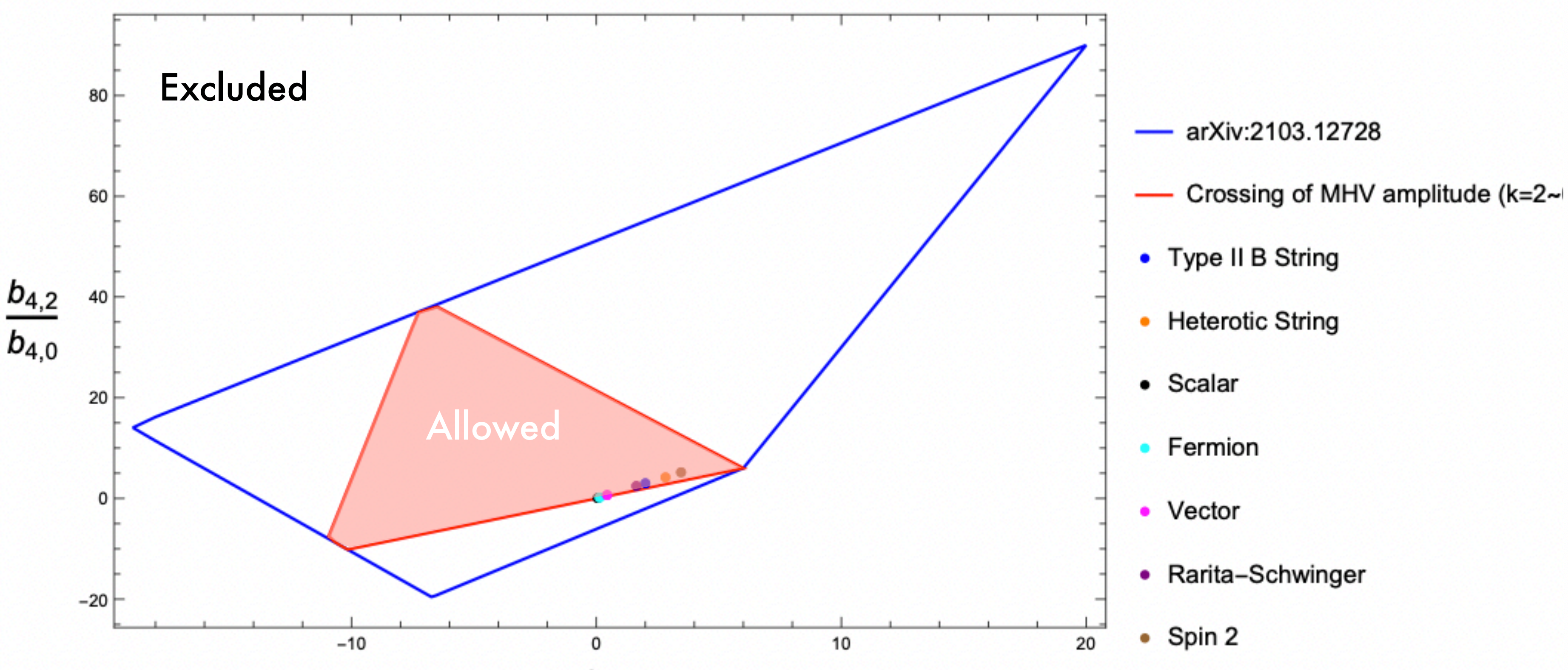
$$\mathrm{BH}: \qquad \lim_{|s|\to\infty} \frac{|T_{\psi_{a>d-4,b>\frac{d-3}{2}}}(s)|}{|s|} = 0$$

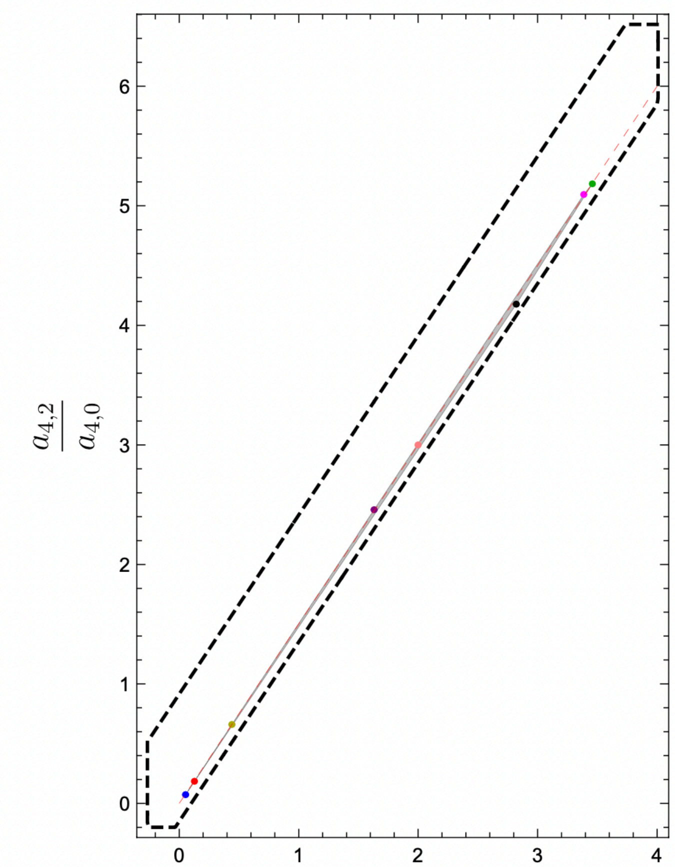
Low spin dominance

[Arkani-Hamed T-C Huang Y-t Huang]
[Bern Kosmopolous AZ]
[Chiang Y-t Huang Li Rodina Weng]

It is interesting to fill in the theory space with known theories:

- tree-level string theories
- one-loop matter





$$\text{Low-spin dominance } (\alpha\text{-factor}) : \quad \langle \rho_4^{+-} \rangle_k \geq \alpha \langle \rho_{J>4}^{+-} \rangle_k , \quad \langle \rho_0^{++} \rangle_k \geq \alpha \langle \rho_{J>0}^{++} \rangle_k \, .$$

Infinitely many bounds are controlled by the lightest lowest spin exchanged particle.

(Related to species bound?)