# Sp($2N_c$) gauge theories for Beyond the Standard Model Physics

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( Based on 1705.05309, 1712.04220, 1909.12662, 1912.06505, 2004.11063, 2010.15781)

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# Outline

- Introduction: the SM and the fine-tuning problem.
- Composite Higgs and Fermion Partial Compositeness.
- The role of $\mathrm{Sp}(2N_c)$ gauge groups.
- Lattice methods and results.
- Conclusion.

# The SM as an Effective Theory

Despite its outstanding success, the Standard Model (SM) has several phenomenological shortcomings:

- Gravity
- Dark Matter
- Origin of ElectroWeak Symmetry Breaking (EWSB)
- ...

there must be some new physics above an energy scale $\Lambda_{\rm SM}$, that addresses at least some of the above.

Where do we place $\Lambda_{\rm SM}$?

- The Higgs boson, responsible for EWSB, was discovered at

$$m_{\rm H} = 125.18(16)\ \text{GeV}, \quad v_{\rm H} \simeq 246\ \text{GeV}\ ,$$

  Then, $\Lambda_{\rm SM} \sim$ TeV?

- Accidental symmetries, neutrino masses, flavour structure,... Then, $\Lambda_{\rm SM} \sim 10^{15}$ GeV?

If $\Lambda_{\rm SM} \gg v_{\rm H}$, then $m_{\rm H}$ with an elementary Higgs would suffer from the fine-tuning problem,

$$\Delta \geq \frac{\delta_{\rm SM} m_{\rm H}^2}{m_{\rm H}^2} \simeq \left(\frac{\Lambda_{\rm SM}}{500\ \text{GeV}}\right)^2 .$$

A way to have $\Lambda_{\rm SM} \gg v_{\rm H}$ and still have a small $m_{\rm H}$ is to "protect" $m_{\rm H}$ from upper scales. Many ideas have been proposed: we will focus on Composite Higgs Models (CHM).

Kaplan and Georgi 1984

# Composite Higgs Models

To stabilize $m_{\mathrm{H}}$, we could mimick QCD,

- New composite sector with gauge symmetry $G_{\mathrm{HC}}$, defined at $\Lambda_{\mathrm{UV}}$.
- At scale $f \ll \Lambda_{\mathrm{UV}}$, spontaneous breaking of global symmetry,

$$\mathcal{G} \longmapsto \mathcal{H}_1 \ ,$$

  leads to appearance of NGbs $\pi^A$.

Symmetry $\mathcal{H}_1$ is explicitly broken by gauging $\mathcal{H}_0 \subset \mathcal{G}$ with external vector bosons

- $\mathcal{H} = \mathcal{H}_1 \cap \mathcal{H}_0$ unbroken gauge group
- A vacuum expectation value (VEV) $v$ for $\pi^A$ is generated radiatively.
- Gauge bosons acquire a longitudinal component.
- NGBs acquire a mass, controlled by $v$.

Kaplan and Georgi 1984

The spectrum is composed by:

- A Higgs multiplet with a mass $m_h \sim g_0 v$, where $g_0$ is a generic gauge coupling.
- Resonances of mass scale $m_\rho \sim g_\rho f$ with $g_\rho$ of order 1.

# Composite Higgs models

## Vacuum misalignement

The parameter

$$\xi = \left(\frac{v}{f}\right)^2$$

controls the deviations from the SM.

- $\xi = 0$ we obtain the SM with an elementary Higgs particle.
- $\xi \sim 1$ we obtain a TechniColor (TC) model.
- $\xi \ll 1$ we have vacuum misalignement, the Higgs stays light and resonances decouple.

So far:

- Not very hard to produce phenomenologically viable models for EW and Higgs physics.
- Very hard to push $\Lambda_{UV}$ to large values and generate SM fermions masses by terms of type

$$\frac{\lambda_t}{\Lambda_{UV}^{d-1}}\, \bar{q}_L \mathcal{O}_S^c t_R + \frac{\lambda_b}{\Lambda_{UV}^{d-1}}\, \bar{q}_L \mathcal{O}_S b_R + \text{h.c.}$$

  where $O_S$ is a composite Lorentz scalar of energy dimension $d$, and $q_L = (t_L, b_L)^T$, $t_R$ and $b_R$ are SM fermions.

Operators with $d = 1 + \varepsilon$ would be dangerously close to reintroducing the fine-tuning problem,

$$d[\mathcal{O}_S^2] < 4$$

# Fermion Partial Compositeness

Introduce resonances $\mathcal{O}^f_{L,R}$ of an additional (extended) composite sector, and couple linearly with the SM fermions,

$$\frac{\lambda_{t_L}}{\Lambda_{UV}^{d^f_L-5/2}}\,\bar{q}_L\mathcal{O}^L_f+\frac{\lambda_{t_R}}{\Lambda_{UV}^{d^f_R-5/2}}\,\bar{t}_R\mathcal{O}^R_f+\text{h.c.}$$

Kaplan 1991

As a consequence:

- Not hard to define operators with $d^f_{L,R}\simeq 5/2$. Flavor hierarchies can be reproduced without fine-tuning.
- These resonances must be charged under $\mathrm{SU}(3)_c$ and must carry the same quantum numbers under $\mathrm{SU}(2)_L\times U(1)_Y$ as their SM "partners"
- The physical states are superpositions of SM fermions with the composite resonances

$$|Phys.\rangle=\alpha|SM_i\rangle+\beta|Composite_i\rangle$$

For example, with an extended composite sector that includes additional (colored) Weyl fermions $\chi$ and $\chi^c$, transforming in the 2-index antisymmetric (2-AS) representation of the gauge group,

$$\mathcal{O}^{f,\alpha ab}=(q^a\chi^\alpha q^b),\quad \mathcal{O}^{c,f,ab}_\alpha=(q^a\chi^c_\alpha q^b)$$

where $\alpha$ is a $\mathrm{SU}(3)_c$ color index, $a$, $b$ global Sp(4) indices.

Barnard, Gherghetta, and Ray 2014

# UV completions for CHMs

What are the possible UV completions of a phenomenologically viable CHM where FPC can be implemented?

Ferretti and Karateev 2014

Consider Left Handed (LH) Weyl fermions in a representation, $n_1 R_1 \oplus \cdots \oplus n_p R_p$ of $G_{HC}$. Then,

$$\mathcal{G} = SU(n_1) \otimes SU(n_2) \otimes \cdots \otimes SU(n_p) \oplus U(1)^{p-1} \ .$$

The groups $G_{HC}$, $\mathcal{G}$, $\mathcal{H}_1$ and $\mathcal{H}_0$ should be chosen so that:

- $G_{HC}$ is asymptotically free and has no gauge (global) anomalies.
- The Breaking $\mathcal{G} \longmapsto \mathcal{H}_1 \supset \mathcal{H}_{\rm EW}$ should be possible and $\mathcal{G}/\mathcal{H}_1$ can accomodate at least one Higgs multiplet.
- Composite states can be used as partners to SM fermions.

| $G_{HC}$ | $n_1 \times R_1$ | $n_2 \times R_2$ | Restrictions |
|---|---|---|---|
| $\mathrm{Sp}(2N_c)$ | $5 \times Ad$ | $6 \times F$ | $2N_c \geq 12$ |
| $\mathrm{Sp}(2N_c)$ | $5 \times A_2$ | $6 \times F$ | $2N_c \geq 4$ |
| $\mathrm{Sp}(2N_c)$ | $4 \times F$ | $6 \times A_2$ | $2N_c \leq 36$ |
| $\mathrm{SO}(N_c)$ | $5 \times S_2$ | $6 \times F$ | $N_c \geq 55$ |
| $\mathrm{SO}(N_c)$ | $5 \times Ad$ | $6 \times F$ | $N_c \geq 15$ |
| $\mathrm{SO}(N_c)$ | $5 \times F$ | $6 \times Spin$ | $N_c = 7, 9, 10, 11, 13, 14$ |
| $\mathrm{SO}(N_c)$ | $5 \times F$ | $6 \times F$ | $N_c = 7, 9$ |
| $\mathrm{SO}(N_c)$ | $4 \times F$ | $6 \times F$ | $N_c = 11, 13$ |

The minimal realistic cases are $G_{\rm HC} = \mathrm{Sp}(2)$ or $\mathrm{Sp}(4)$ with breaking pattern

$$\mathrm{SU}(4) \otimes \mathrm{SU}(6) \longmapsto \mathrm{Sp}(4) \otimes \mathrm{SO}(6) \ .$$

and $4 \times F$ Weyl and $6 \times A_2$ Weyl fermions.

- $Sp(4) \supset SO(4) \sim SU(2)_L$
- $SO(6) \supset SU(3) \sim SU(3)_c$

This model is known as $M8$ model, we will focus on $N_c = 2$. The case $N_c = 1$ has been explored both analytically and on the lattice.

Barnard, Gherghetta, and Ray 2014
Cacciapaglia, Pica, and Sannino 2020

# The $Sp(2N_c)$ group

## Definition and main properties

The symplectic group $\mathrm{Sp}(2N_c)$ can be defined as a subgroup of $\mathrm{SU}(2N_c)$,

$$\mathrm{Sp}(2N_c) = \left\{ U \in \mathrm{SU}(2N_c) \mid \Omega U \Omega^T = U^* \right\}$$

where $\Omega$ is the Symplectic matrix,

$$\Omega = \begin{bmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{bmatrix}$$

As direct consequences of the definition:

- $\mathrm{Sp}(2) \simeq \mathrm{SU}(2)$.
- The center of the group is $\mathbb{Z}_2$ for every $N_c$
- All representations are pseudo-real and Charge conjugation is trivial.

$$\Omega U \Omega^T = U^\star \longrightarrow \Omega T_R^A \Omega = -\left(T_R^A\right)^\star \ ,$$

## Not only CHMs!

Sp($2N_c$) gauge theories are also interesting for:

- Thermodynamics of gauge theories: as Sp($2N_c$) only have pseudoreal representations, the phase diagram can be explored without a sign problem.
- The center of the group is always $\mathbb{Z}_2$, useful to test the Svetitsky-Yaffe conjecture

Holland, Pepe, and Wiese 2004

- Large-N Physics: Sp($2N_c$) symmetric gauge theories can be shown to have the same $N \to \infty$ of the SU($N_c$) and SO($N_c$) cases, with a $O(N_c^{-1})$ difference in the approach to $N = \infty$,

$$\frac{m}{\sqrt{\sigma}}(N_c) = \begin{cases} \frac{m}{\sqrt{\sigma}}(N_c = \infty) + \frac{c_{N_c}}{N_c}, & \mathrm{Sp}(2N_c), \mathrm{SO}(N_c) \\ \frac{m}{\sqrt{\sigma}}(N_c = \infty) + \frac{c_{N_c}}{N_c^2}, & \mathrm{SU}(N_c) \end{cases}$$

Lovelace 1982

- Quantities related to the glueball spectrum may be computed in alternative to SU($N_c$) and SO($N_c$) gauge theories: Casimir scaling and the ratio of the tensor to glueball masses.

E. Bennett, Holligan, et al. 2020; Hong et al. 2017

- The SIMP miracle: the $2 \to 2$ scattering producing WIMP miracle is supplemented by a $3 \to 2$ (strong) annihilation. This is capable of producing Tev mass DM and both credible and (nearly) testable predictions with Sp($2N_c$) gauge group.

Hochberg et al. 2015

# The breaking of SU(4)

The Lagrangian for 2 Dirac fermions coupled to a Sp(4) gauge field is,

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\ V_{\mu\nu}V^{\mu\nu} \ + \ i\bar{Q}^i_a\gamma^\mu(D_\mu Q^i)^a - m\bar{Q}^i_a Q^{ia} \ , \quad D_\mu = \partial_\mu + igV^A_\mu T^A_R$$

with $T^A_R \in \mathfrak{sp}(4)$.

From the pseudo-real nature of the gauge group, $\Omega T^A_R \Omega = -\left(T^A_R\right)^\star$, and in terms of LH Weyl fermions $q^{ia}$,

$$\mathcal{L} = iq^{ak\dagger}\bar{\sigma}^\mu(D_\mu q^k)^a - m\Omega_{kn}\Sigma^{kn} \ , \quad \Sigma^{kn} = \Omega_{ab}\left(q^{kbT}\tilde{C}q^{na}\right) \ ,$$

where now $k, n = 1, \ldots, 4$.

For $m \to 0$, the Lagrangian above enjoys a SU(4) symmetry. The condensation,

$$\langle\Sigma\rangle = \Omega \neq 0 \ ,$$

drives the breaking $\mathrm{SU}(4) \longmapsto \mathrm{Sp}(4)$.

# The description of the broken phase

The coset SU(4)/Sp(4) is parametrized by the NGbs $\pi^A(x)$ transforming in the 5 representation of Sp(4),

$$\Sigma(x) = e^{\frac{2i}{f}\pi^A \hat{T}^A}\Omega \ ,$$

where $\hat{T}^A$ are the broken generators and $f$ has the dimensions of energy.

At Leading Order (LO),

$$\mathcal{L}_{p^2} = \frac{f^2}{4}\mathrm{Tr}\left\{\partial_\mu \Sigma \partial_\mu \Sigma^\dagger\right\} - \frac{v^3}{4}\mathrm{Tr}\left\{M\Sigma + \Sigma^\dagger M^\dagger\right\} \ ,$$

where $v$ is the magnitude of the condensate and the transformation properties of $\Sigma$ and of the spurion $M = m\Omega$ are

$$M \to U^\star M U^\dagger, \quad \Sigma \to U \Sigma U^T \ , \quad U \in \mathrm{SU}(4) \ .$$

The above EFT treatment can be extended:

- To Include "light" mesons, like the $\rho$ and the $a_1$, along the lines of Hidden Local Symmetry (HLS).
- To consider fermions un multiple representations to implement FPC. Then one can describe the coset

$$\frac{SU(4) \times SU(6)}{Sp(4) \times SO(6)}$$

with the condensates $\Sigma_6$ and $\Sigma_{21}$.

# Lattice discretizations

## Definition

On a lattice of spacing $a$,

$$S\left[U,\bar{\psi},\psi\right] = \beta \sum_{x,\,\mu>\nu} \left(1 - \frac{1}{2N}\Re\,\mathrm{Tr}\,P_{\mu\nu}(x)\right) \;+\; a^4 \sum_x \bar{\psi}(x) D^f_m \psi(x) \;+\; a^4 \sum_x \bar{\Psi}(x) D^{as}_m \Psi(x)\;,$$

where $\beta = 2N_c/g_0^2$ and

$$D^{f,as}_{m,y,x} = \left(\frac{4}{a} + m_0^{f,as}\right)\delta_{y,x} - \frac{1}{2a}\sum_\mu \left\{(1-\gamma_\mu)U^{f,as}_\mu(x)\delta_{y,x+\mu} + (1+\gamma_\mu)U^{f,as}_\mu(x-\mu)\delta_{y,x-\mu}\right\}\;,$$

is the unimproved Wilson-Dirac operator, with $m_0^f$ and $m_0^{as}$ the bare quark mass parameter for fundamental (F) and 2-index antisymmetric (2AS) fermions.

The masses $m_M$ and decay constants $f_M^{bare}$ are obtained from

$$C_{\mathcal{O}_M}(t) = \sum_{\vec{x}} \langle 0|\mathcal{O}_M(\vec{x},\,t)\mathcal{O}^\dagger_M(\vec{0},\,0)|0\rangle \quad \longmapsto_{t\to\infty} \quad \frac{|\langle 0|\mathcal{O}_M|M\rangle|^2}{2m_M} e^{-m_M t}\;,$$

where $\mathcal{O}_M = \bar{\psi}_1\Gamma_M\psi_2$, and $f_M^{bare}$ obtained from $\langle 0|\mathcal{O}_M|M\rangle$.

Discretization effects mildened with

$$f_M = Z_M f_M^{\rm bare}\;.$$

where $Z_M$ computed with lattice PT at 1-loop.

# Observables

- The multiplets obtained in the Sp(4) $\times$ SO(6) theory can be classified with $J^P$ quantum numbers and the number of states in the (unbroken group) multiplet.

| Label (M) | $\mathcal{O}_M$ | Meson | $J^P$ | Sp(4) |
|---|---|---|---|---|
| PS | $\bar{\psi}_1\gamma_5\psi_2$ | $\pi$ | $0^-$ | 5 |
| S | $\bar{\psi}_1\psi_2$ | $a_0$ | $0^+$ | 5 |
| V | $\bar{\psi}_1\gamma_\mu\psi_2$ | $\rho$ | $1^-$ | 10 |
| T | $\bar{\psi}_1\gamma_0\gamma_\mu\psi_2$ | $\rho$ | $1^-$ | 10 |
| AV | $\bar{\psi}_1\gamma_5\gamma_\mu\psi_2$ | $a_1$ | $1^+$ | 5 |
| AT | $\bar{\psi}_1\gamma_5\gamma_0\gamma_\mu\psi_2$ | $b_1$ | $1^+$ | 10 |

| Label (M) | $\mathcal{O}_M$ | Meson | $J^P$ | SO(6) |
|---|---|---|---|---|
| ps | $\bar{\Psi}_1\gamma_5\Psi_2$ | $\pi$ | $0^-$ | 20 |
| s | $\bar{\Psi}_1\psi_2$ | $a_0$ | $0^+$ | 20 |
| v | $\bar{\Psi}_1\gamma_\mu\Psi_2$ | $\rho$ | $1^-$ | 15 |
| t | $\bar{\Psi}_1\gamma_0\gamma_\mu\Psi_2$ | $\rho$ | $1^-$ | 15 |
| av | $\bar{\Psi}_1\gamma_5\gamma_\mu\Psi_2$ | $a_1$ | $1^+$ | 20 |
| at | $\bar{\Psi}_1\gamma_5\gamma_0\gamma_\mu\Psi_2$ | $b_1$ | $1^+$ | 15 |

- Note that $T$ and $V$ source the same states: only 5 independent measurements.
- For Chimera Baryons (CB), we have, schematically,

$$\mathcal{O}^k_{CB,\Lambda} = [\overline{Q^{1a}}\gamma_5 Q^{2b}]\Omega_{bc}\Psi^{k,ca}, \quad \mathcal{O}^k_{CB,\Sigma} = \imath[\overline{Q^{1a}}\gamma_\mu Q^{2b}]\Omega_{bc}\Psi^{k,ca}$$

These source states analogous to $\Lambda(J=1/2)$, $\Sigma(J=1/2)$ and $\Sigma^\star(J=3/2)$ in QCD, after Spin and Parity projections.

The Sp($N_c$) gauge group was accomodated into the Hirep code

Del Debbio, Patella, and Pica 2010

In particular:

- For quenched systems, the Cabibbo-Marinari technique was adapted to Sp($N_c$).
- For dynamical fermions simulation, HMC and Rational HMC were implemented for fermions in multiple representations of Sp($N_c$)

The bare parameters $\beta$, $m_0^f$, $m_0^{as}$ must be chosen in order to:

- avoid bulk phase transitions.
- be free of finite size effects at moderate values of $L/a$.

Masses and decay constants of flavoured mesons were computed for various matter contents

- $\omega_0$ defined from the Gradient flow was used to set the scale of the lattice

$$\hat{m}_M = m_M w_0, \qquad \hat{f}_M = f_M w_0$$

Borsanyi et al. 2012

- WIlson $\chi$-PT was used to parameterize the systematical errors due to finite $a$ and finite $m_{PS}$.

$$\hat{f}_M^2 = \hat{f}_M^{2,\chi}(1 + L_M^f \hat{m}_{PS}^2) + W_M^f \hat{a}, \qquad \hat{m}_M^2 = \hat{f}_M^{2,\chi}(1 + L_M^m \hat{m}_{PS}^2) + W_M^m \hat{a},$$

where $\hat{a} = a/w_0$ and $L_M^{f,m}$, $W_M^{f,m}$ are coefficients.

Bar and Golterman 2014; Rupak and Shoresh 2002

# Lattice studies of $\mathrm{Sp}(2N_c)$ gauge theories

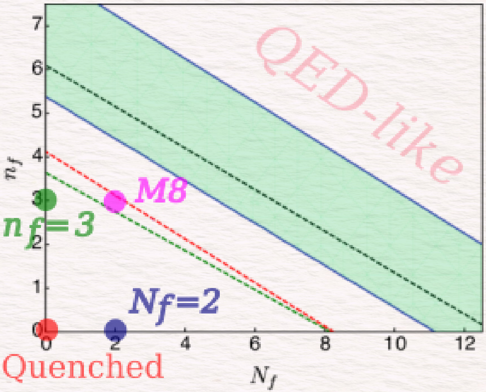



Figure: Dashed lines correspond to various analytical estimates of the location of the conformal window. Taken from Kim, Hong, and Lee 2020

So far, for $N_c = 1, 2, 3, 4$,

- Quenched Glueball spectrum and Topological susceptibility at $N_c \leq 4$
  Bennett, Holligan, et al. 2020; Bennett, Hong, Lee, et al. 2022

For $N_c = 2$, Meson spectrum and decay constants of

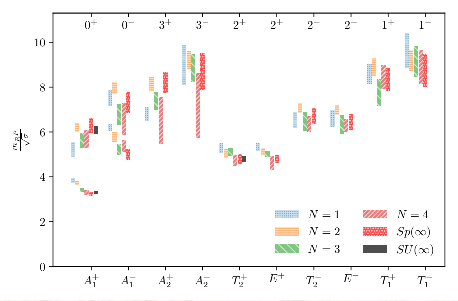
- Dynamical $2 \times F$ fermions.
  E. Bennett, Hong, Lee, C.-J. David Lin, et al. 2019
- Quenched $2 \times F$ and $3 \times 2AS$ fermions.
  E. Bennett, Hong, Lee, Chi-Jen David Lin, et al. 2020
- Dynamical $2 \times F + 3 \times 2AS$ fermions.
  Bennett, Hong, Hsiao, et al. 2022

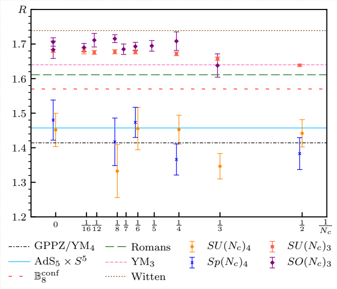
WIP:

- Quenched meson spectrum at $N_c > 2$.
- (Quenched) Chimera Baryon spectrum at $N_c = 2$, for $N_f = 2$, $n_f = 3$.
- Continuum limit for dynamical $2 \times F + 3 \times 2AS$ theory.
- Iso-singlet mesons at $N_c = 2$.
  Zierler et al. 2022

# The quenched theory

## The glueball spectrum




Universality: different families of gauge groups have a common $N_c \to \infty$ limit.

- The quenched $\mathrm{Sp}(2N_c)$ glueball spectrum was obtained for $N_c = 1, 2, 3, 4$ and extrapolated to $N_c \to \infty$. The limits in $\mathrm{SU}(N_c)$ and $\mathrm{Sp}(2N_c)$ are compatible.

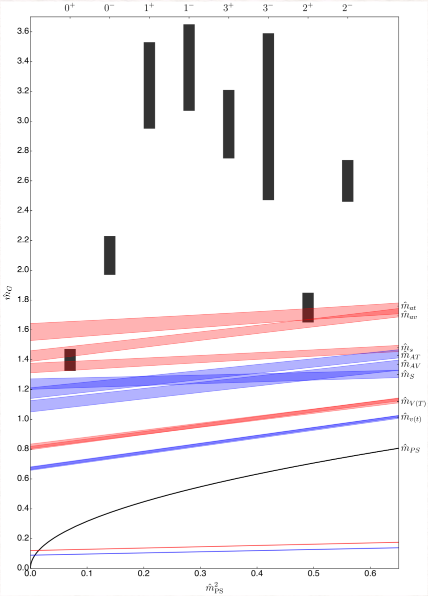
- The quantity

$$R = \frac{m(2^{++})}{m(0^{++})}$$

is a quantity that can be defined in many different theories. Is it universal?

# The quenched theory

## Summary of meson and glueball spectrums




- $F$ quenched mesons (red)
- $2AS$ quenched mesons (blue)
- quenched Glueballs (black)

- Finite size effects negligible.
- Continuum limit.

# $N_f = 2$ dynamical fermions

## The GMOR relation

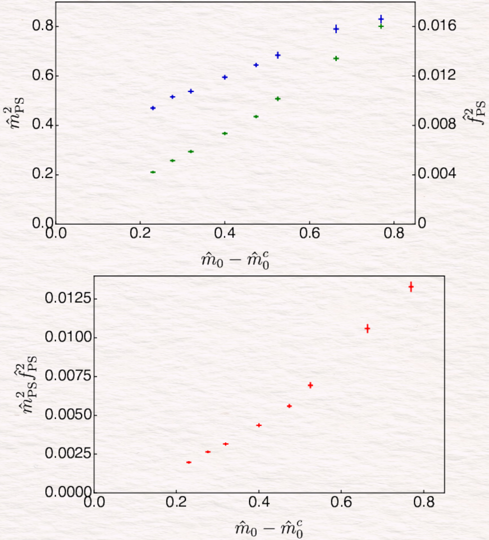



Figure: $\hat{m}^2_{PS}$, $\hat{f}^2_{PS}$ and their product as a function of $\hat{m}_0 - \hat{m}_c$ for $\beta = 7.2$.

- No evidence of a bulk phase transition was found.
- PS mesons are the lightest as expected.
- $m_{PS}$ large: no $V$ decays.
- Finite volume effects negligible for

$$m_{PS}L \gtrsim 7.5, \quad f_{PS}L \gtrsim 1.5$$

The parameter $\hat{m}_0$ was replaced with $\hat{m}_{PS}$, using the LO relation

$$\hat{m}^2_{PS} = 2B\hat{m}_f$$

where $B$ is a LEC.

Are our simulations in this regime? Yes, provided $\hat{m}^2_{PS} \lesssim 4$.

# $N_f = 2$ dynamical fermions

## continuum extrapolations

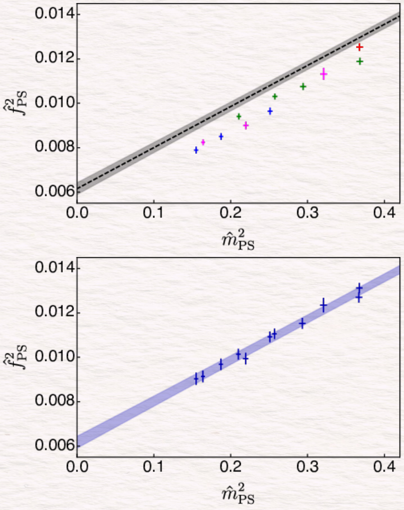



Figure: Top: $\hat{f}_{PS}$ as a function of $\hat{m}_{PS}$, each color corresponds to a different $\beta$ value. Bottom: Continuum limit of same quantities and result of global fit.

NLO Wilson $\chi$PT was used to describe systematical errors introduced by finite quark mass and finite lattice spacing,

$$\hat{f}^2_M = \hat{f}^{2,\chi}_M (1 + L^f_M \hat{m}^2_{PS}) + W^f_M \hat{a} \ ,$$

$$\hat{m}^2_M = \hat{f}^{2,\chi}_M (1 + L^m_M \hat{m}^2_{PS}) + W^m_M \hat{a} \ .$$

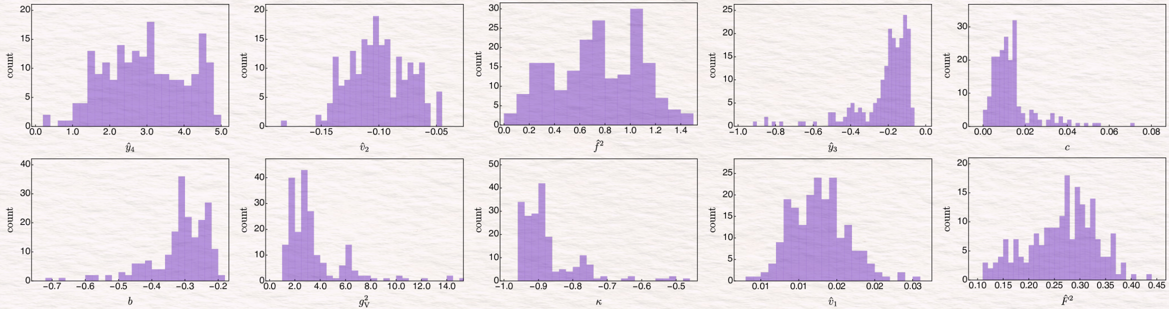
for $M = S,\ PS,\ V,\ AV,\ T,\ AT$

The continuum limit was taken at fixed $\hat{m}^2_{PS} (\leq 0.4)$, at LO, considering the ensembles such that $\hat{a} \leq 1$

A global fit was then used to determine the value of the LECs in units of $w_0$.

# The LECs of $N_f = 2$

- We expand the dependence of masses and decay constants on $\hat{m}_{PS}^2$ and truncate at linear order
- We consider ensembles for which $\hat{m}_{PS}^2 \leq 0.4$
- 10 parameters (constrained[1]) are to be determined from 5 measurements.



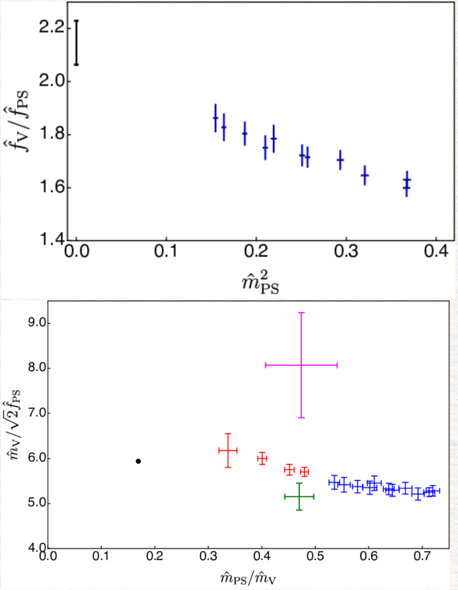

Caution:

- The quark mass parameters are not small, $V$ is stable: $1.39 \leq \hat{m}_V / \hat{m}_{PS} \leq 1.87$.
- $g_{VPP} (\neq g_V)$ is not small, $\hat{g}_{VPP} \sim 6$.
- However, a global (resampled) fit yields $\chi^2 / N_{\text{d.o.f.}} \sim 0.4$

[1] by unitarity

# $N_f = 2$ dynamical fermions

## Sample results - KSRF relations




We can check the validity of the KSRF relations,

$$g_{VPP} = \frac{m_V}{\sqrt{2} f_{PS}} \,, \quad f_V = \sqrt{2} f_{PS} \,.$$

to the same quantities in real-world QCD and in other lattice models.

Kawarabayashi and Suzuki 1966; Riazuddin and Fayyazuddin 1966

- In the case of $Sp(4)$ gauge theory, the first KSRF relation is not satisfied

  $$\hat{f}_V / \hat{f}_{PS} \sim 2.1 \neq \sqrt{2} \,,$$

  while the second holds,

  $$\hat{m}_V / \sqrt{2} \hat{f}_{PS} = 5.72(18) \simeq 6.0(4) = g_{VPP} \,.$$

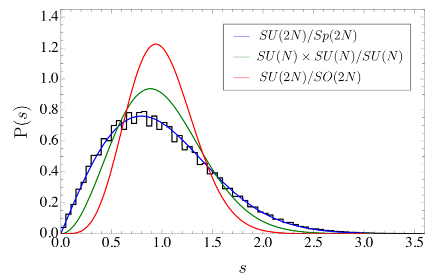
- $\hat{m}_V / \sqrt{2} \hat{f}_{PS}$ can be computed on the lattice for
  - $SU(2)$ (purple, Arthur et al. 2016)
  - $SU(3)$ (red, ETM 2009)
  - $SU(4)$ (green, TACO 2017)

  and compared to $Sp(4)$ (blue).

E. Bennett, Hong, Lee, C.-J. David Lin, et al. 2019

# Quenched $2 \times F + 3 \times 2AS$ theory

## Global symmetry breaking



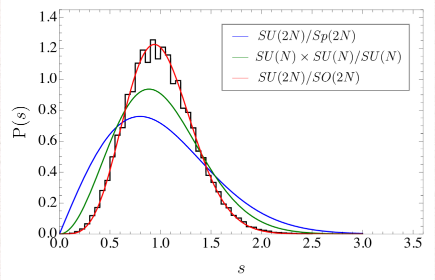




Figure: Taken From Bennett, Hong, Hsiao, et al. 2022.

The (unfolded) density of spacing $s$ of eigenvalues of the Dirac operator is related to the global symmetries of the system.

$$P(s) = N_{\bar{\beta}} s^{\bar{\beta}} e^{-c_{\beta} s^2}$$

where $N_{\bar{\beta}}$ and $c_{\bar{\beta}}$ are indep. of $s$. The value of $\bar{\beta}$ depends on the breaking pattern.

According to ChRMT, the breaking pattern

$$\mathrm{SU}(2N_f) \longrightarrow \mathrm{Sp}(2N_f) \,,$$

yields matrices in the chiral Gaussian Symplectic Ensemble, with $\bar{\beta} = 1$, while the breaking pattern

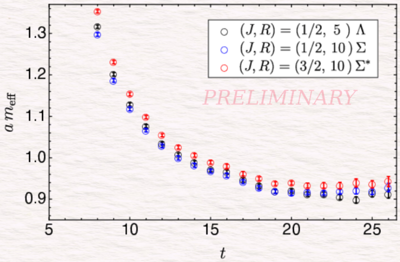
$$\mathrm{SU}(2N_f) \longrightarrow \mathrm{SO}(2N_f) \,,$$

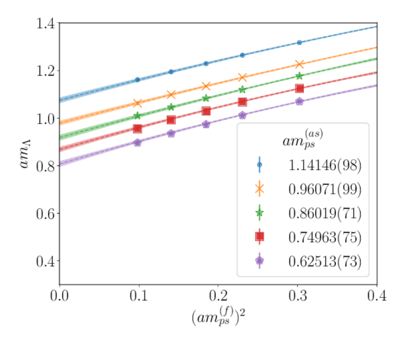
yields matrices in the chiral Gaussian Orthogonal Ensemble, with $\bar{\beta} = 4$

Cossu et al. 2019; Verbaarschot and Wettig 2000

# Quenched $2 \times F + 3 \times 2AS$ theory - WIP

## Spectrum of Chimera Baryons




Combine two fundamental fermions $Q$ and one AS fermion $\Psi$,

$$\mathcal{O}^{ijk} = \left(Q^{i,a}\Gamma^1 Q^{j,b}\right)\ \Omega_{ac}\ \Omega_{bd}\Gamma^2\ \Psi^{k,cd}$$

where $\Gamma^{1,2}$ combinations of Dirac matrices , $ijk$ flavour indices, $abc$ gauge indices.

A natural top partner is the state

$$\Lambda:\ \ (J,\ R) = (1/2,\ \mathbf{5})$$

where $J$ spin, $R$ representation under flavour Sp(4).

Banerjee, Franzosi, and Ferretti 2022

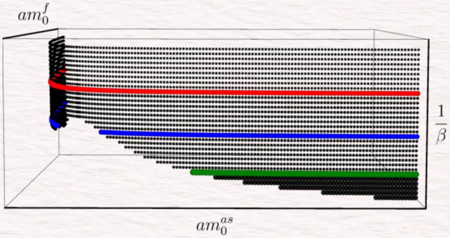
For

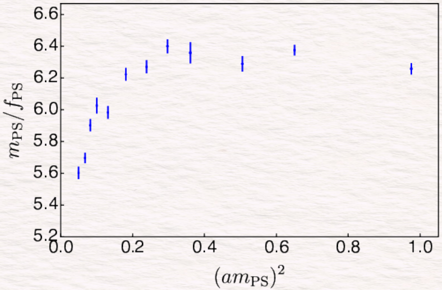
$$\frac{m_{ps}^{as}}{m_{ps}^{f}} \gtrsim 4.46$$

this is indeed the lightest (stable) state.

Hsiao et al. 2022

# $2 \times F + 3 \times 2AS$ Dynamical fermions - WIP




- Line of first order bulk phase transition was found in $(\beta,\, m_0^f,\, m_0^{as})$ bare parameter space.

  Bennett, Hong, Hsiao, et al. 2022

- Meson spectrum and decay constants were computed at fixed $a$, at
  $$am_0^f = -0.71, \quad am_0^{as} = -1.01, \quad \beta = 6.5$$
  to analyze Finite Size effects.

The spectrum and decay constants of PS, V, AV, S states were obtained for $\beta \gtrsim 6.5$ and $m_{PS}L \gtrsim 8$.

The sharp drop of $m_{PS}/f_{PS}$ as $am_{PS} \to 0$ at $\beta = 6.7$ shows that we are in the broken phase.

# $2 \times F + 3 \times 2AS$ Dynamical fermions

## Spectrum of Chimera Baryons

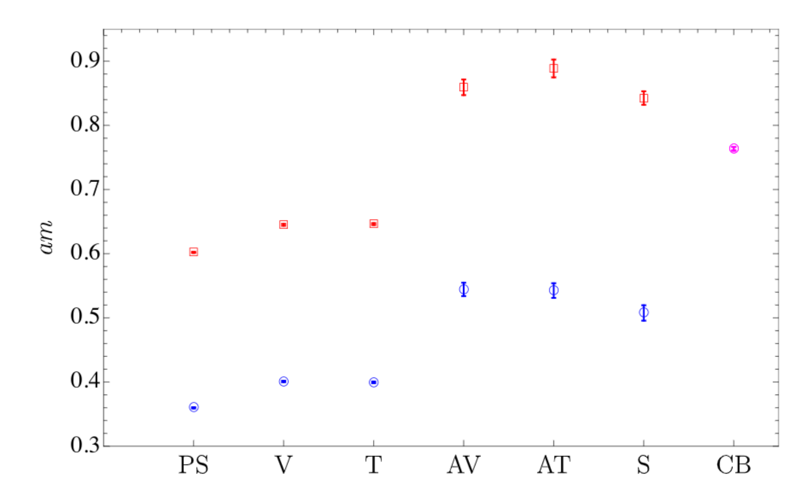



Figure: Spectrum of F mesons (red), 2AS mesons (blue) and chimera baryons (magenta), at $\beta = 6.5$, $m_0^f = -0.71$, $m_0^{as} = -1.01$.

# Conclusion

The gauge theories based on $\mathrm{Sp}(2N_c)$ are relevant in realizations of UV complete composite higgs models and in the study of the large-$N_c$ regime of YM theories.

- The properties of the quenched models were thorougly explored at $N_c = 2, 3, 4$.
- The meson spectrum and decay constants of $N_c = 2$, $N_f = 2$ were determined.
- The meson spectrum and decay constants of $N_c = 2$, $n_f = 3$ is WIP.
- The Chimera Baryons in the quenched $M8$ model is WIP.

We are slowly but steadily approaching the full $M8$ model.

In the future:

- Non-perturbative improvement.
- Spectrum at $N_c > 2$.
- Iso-singlet mesons.

Thank you for your attention!

# Scale setting & Topology

The gradient flow $B_\mu(x,\,t)$ is defined by

$$\frac{\mathrm{d}B_\mu(x,\,t)}{\mathrm{d}t} = D_\nu G_{\nu\mu}(x,\,t), \qquad G_{\mu\nu}(t) = [D_\mu,\,D_\nu]\,, \qquad D_\mu \equiv \partial_\mu + [B_\mu,\,\cdot\,]$$

where the independent variable $t$ is known as *flow time*, and $B_\mu(x,\,0) = A_\mu(x)$.

Lüscher 2010, 2014

The Gradient Flow has some very useful properties:

- $B_\mu(x,t)$ is a renormalized field.
- $B_\mu(x,\,t)$ is a smoothening of $A_\mu(x)$.

Hence, the computation of $B_\mu(x,\,t)$ will allow us to

- Set the scale $w_0$ by computing $E(t) = \frac{1}{4}\mathrm{Tr}G_{\mu\nu}(t)G_{\mu\nu}(t)$ and setting

$$\mathcal{W}(t_0) = \left[t\frac{d}{dt}\left(t^2 E(t)\right)\right]_{w_0^2=t_0} = \mathcal{W}_0\;.$$

- Compute the topological charge $Q$ on smoothened configurations, and check for topological freezing.

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