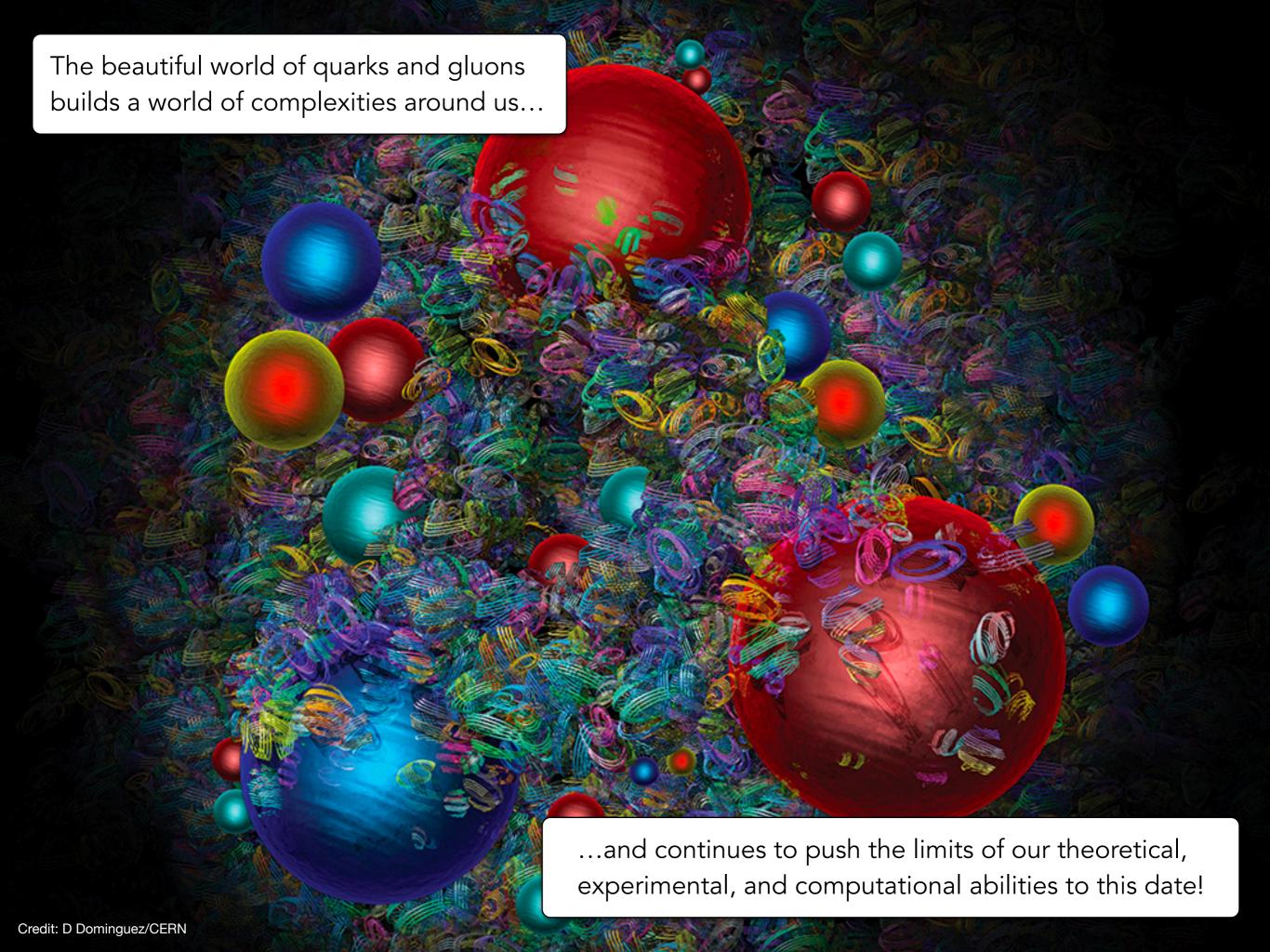
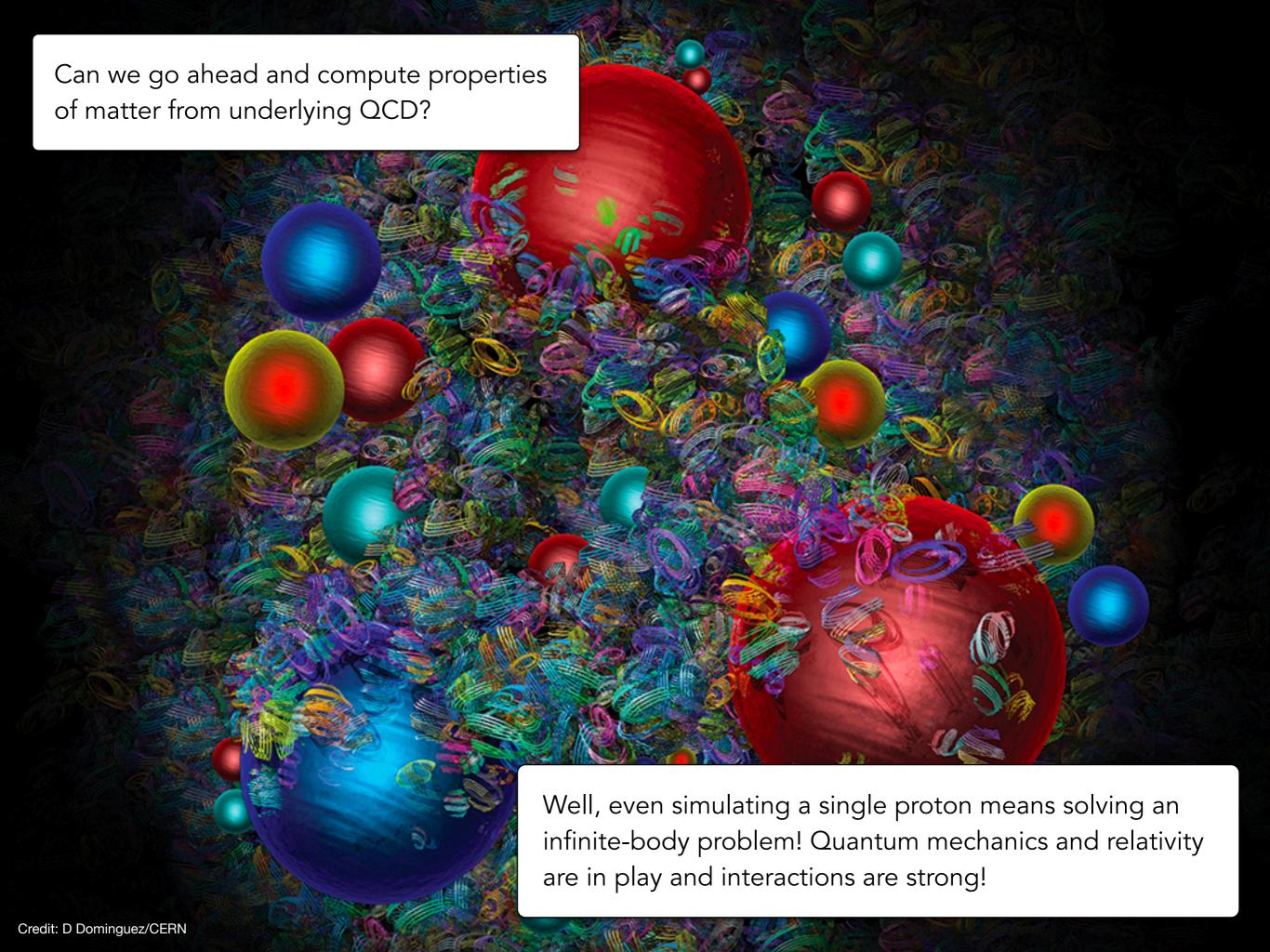


[Remote] Talk @ the UK Annual Theory Meeting Durham University, UK Dec 13-15, 2022

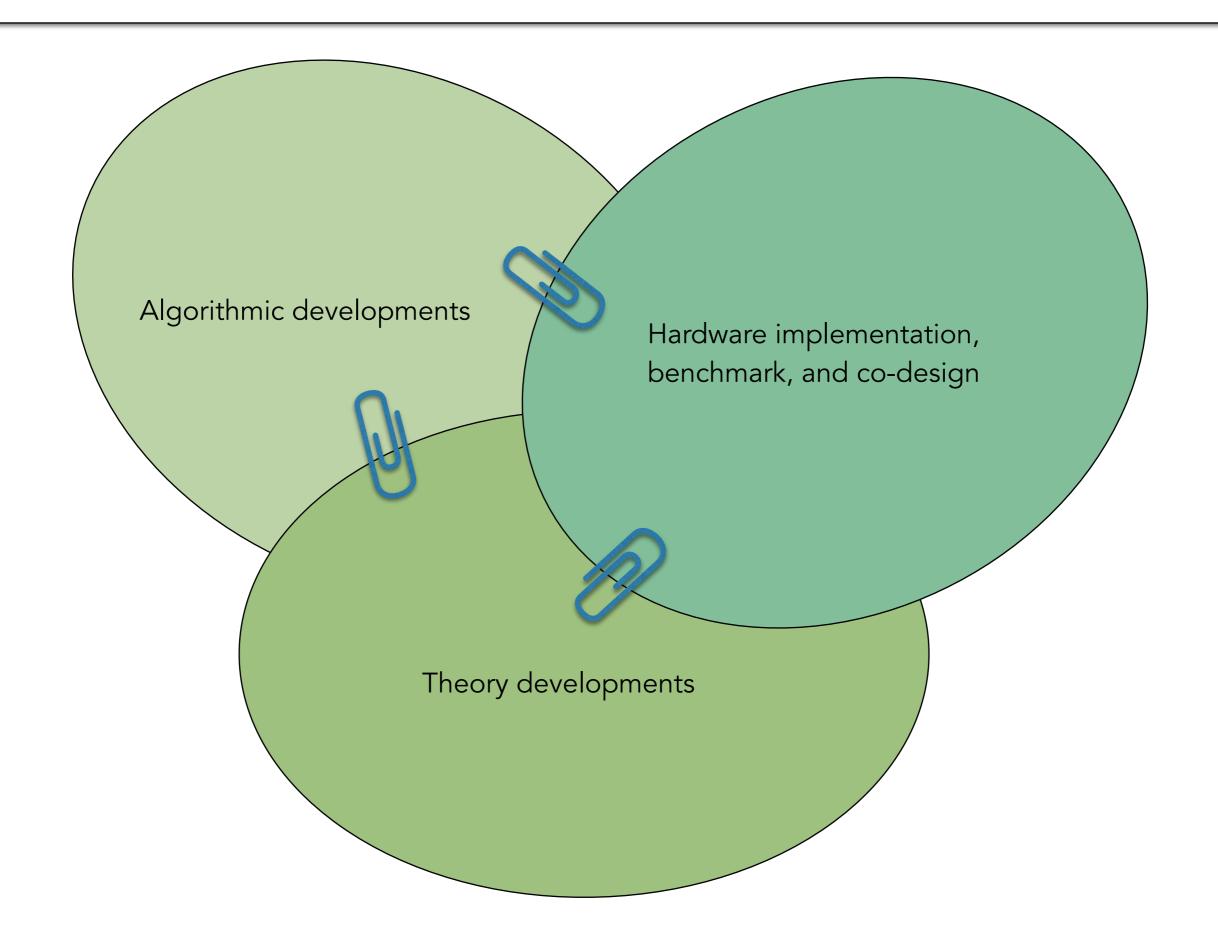
Quantum Simulations for Field Theories

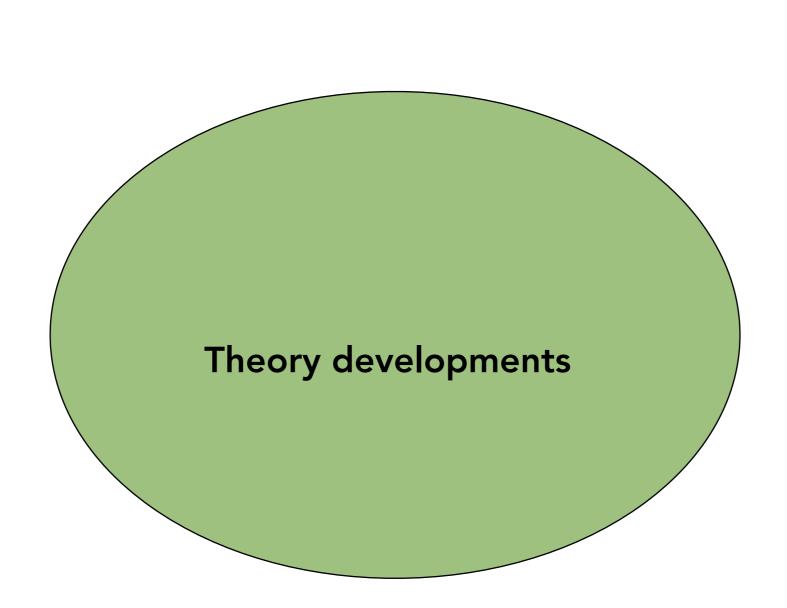
Zohreh Davoudi University of Maryland, College Park





LATTICE QCD: A MULTI-PRONG PROGRAM THAT SIMULATES QCD NON-PERTURBATIVELY





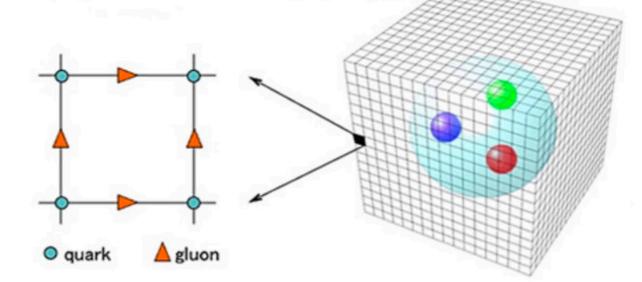


How to define QCD/QED on a finite grid?

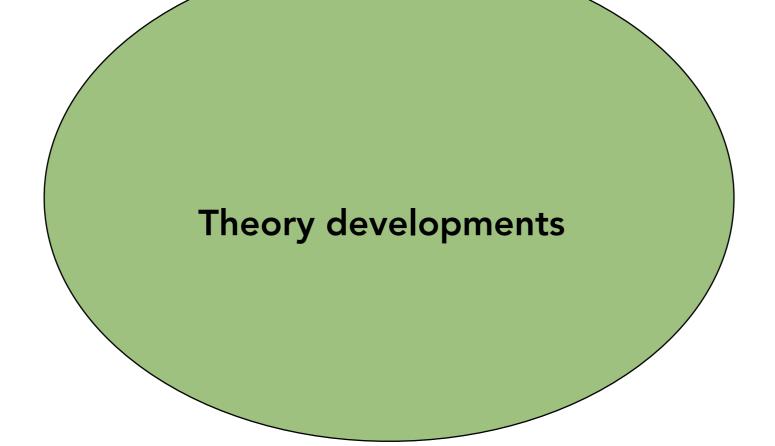


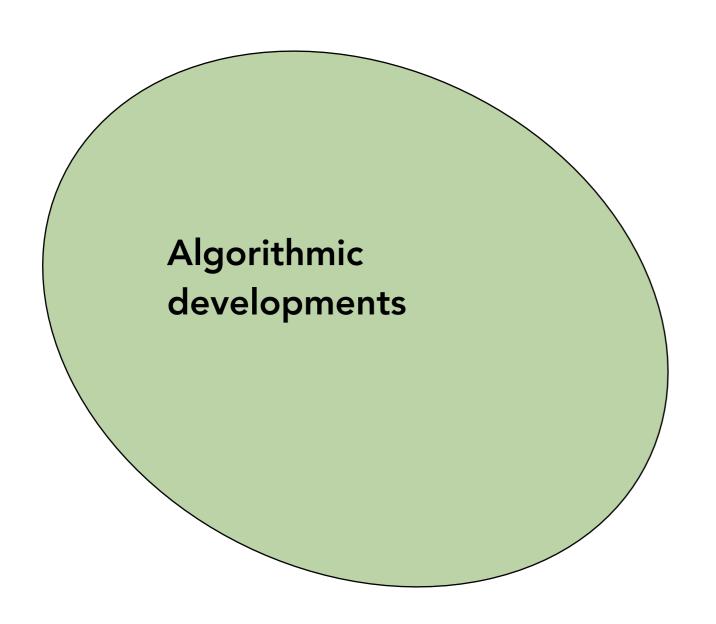
How to preserve/recover symmetries, e.g., gauge symmetry, chiral symmetry, rotational symmetry.

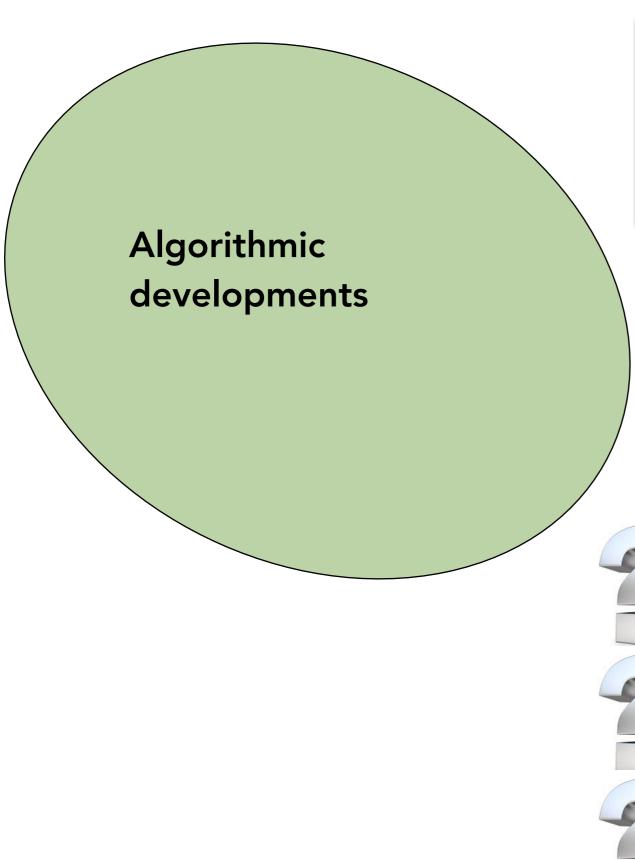
How to take infinite-volume and continuum limits? How to quantify systematics?

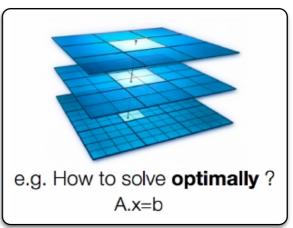


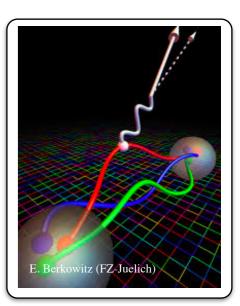
How to obtain scattering amplitudes and decay rates?

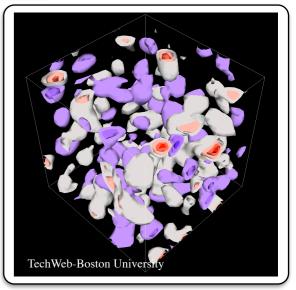








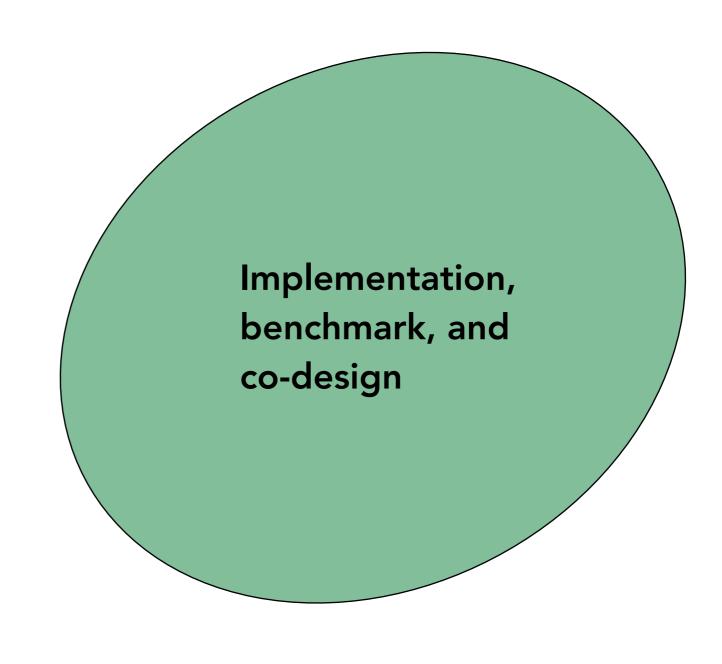


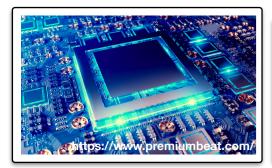


How to importance sample vacuum gauge configurations?

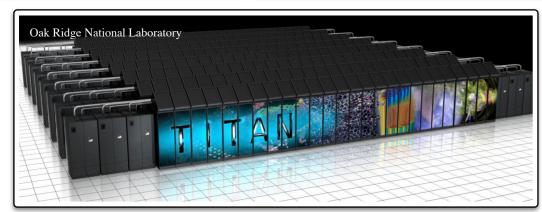
How to evaluate quark propagators (invert large matrices)?

How to contract quarks and form correlation functions efficiently?









Implementation, benchmark, and co-design



Which tasks can be parallelized and which tasks are done in series?



What are the memory requirements and what kind of node connectivity is required?



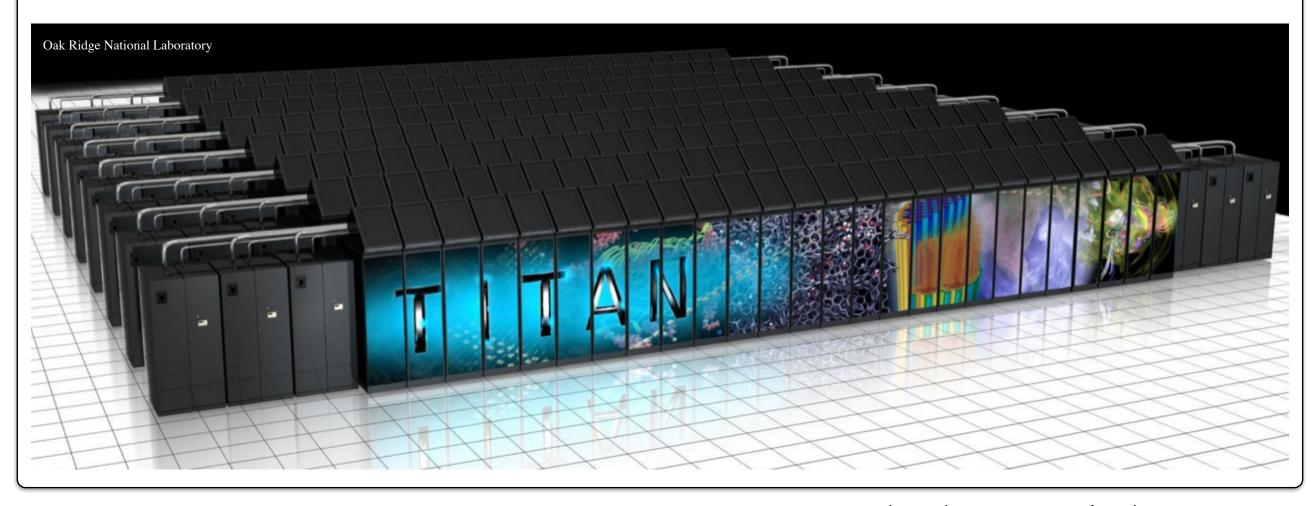
Can we take advantage of GPUs? Which parts of the computations are more suitable for given architecture?

lear Instruments and Methods in Physics Research 222 (1984) 534-539 North-Holland, Amsterdam iclear Instruments and Methods in Physics Research 222 (1984) 534-539 North-Holland, Amsterdam 534 Nuclear Instr ULATOR FOR LATTICE GAUGE THEORY **MULATOR FOR LAT** HARDWARE MATRIX MULTIPLIER / ACCUMULATO than current supercomputers!
Norman H. CHRIST and Anthony E. TERRANO Only few Kbytes, of memory! ~40 years ago! Received 30 September 1983 Received 30 September 1983 TO HOST 12 TO HOS 10 Yes! 4⁴ lattice! ADDER MEMORY MEMORY STRONG COUPLING 4 FROM HOST

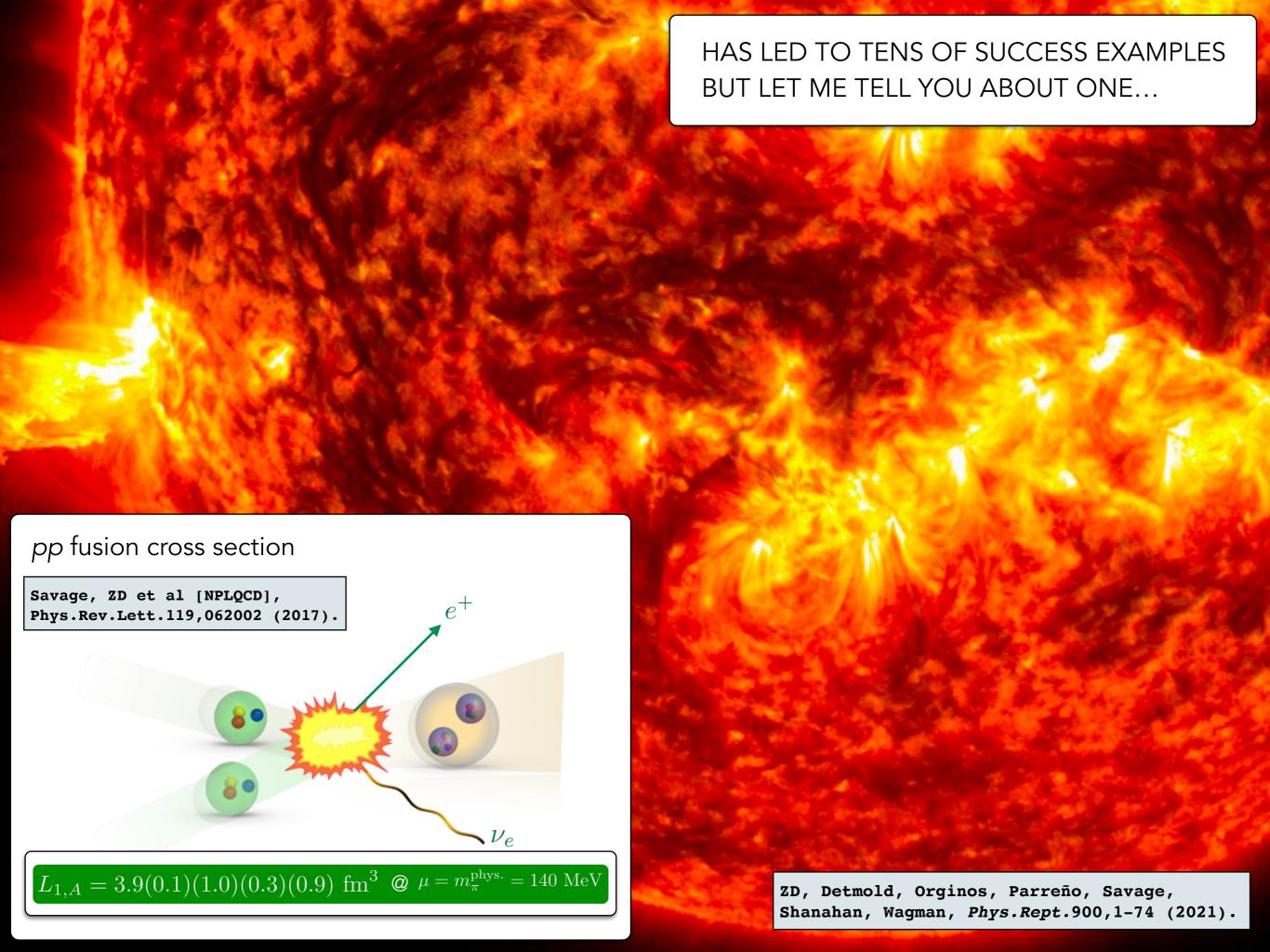
FROM HOST

WEAK COUPLING

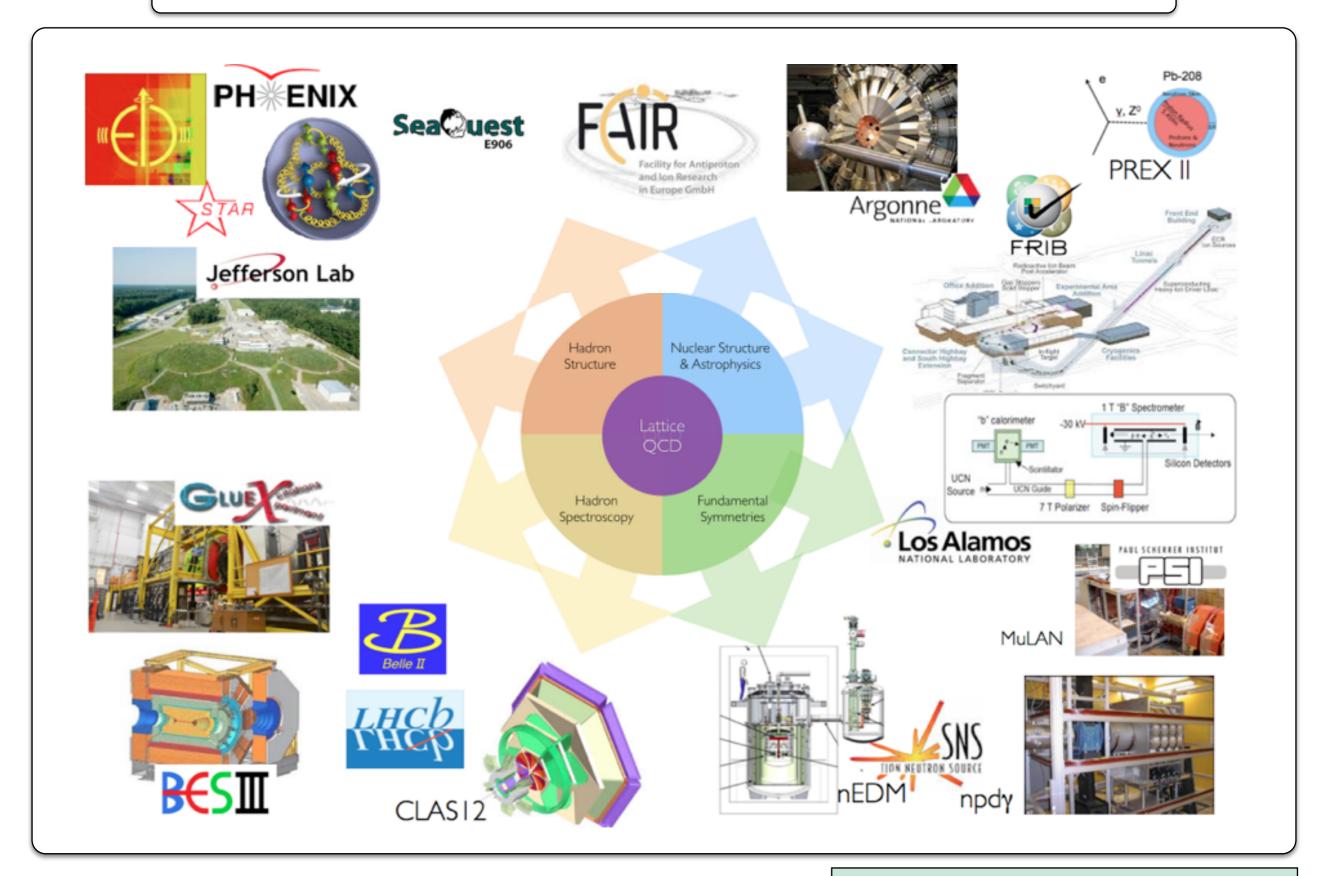
PUTTING ALL THESE HEROIC THEORY, ALGORITHM, AND CO-DESIGN EFFORTS TO WORK AND HAVING ACCESS TO HUNDREDS OF MILLION CPU HOURS ON THE LARGEST SUPERCOMPUTERS IN THE WORLD...



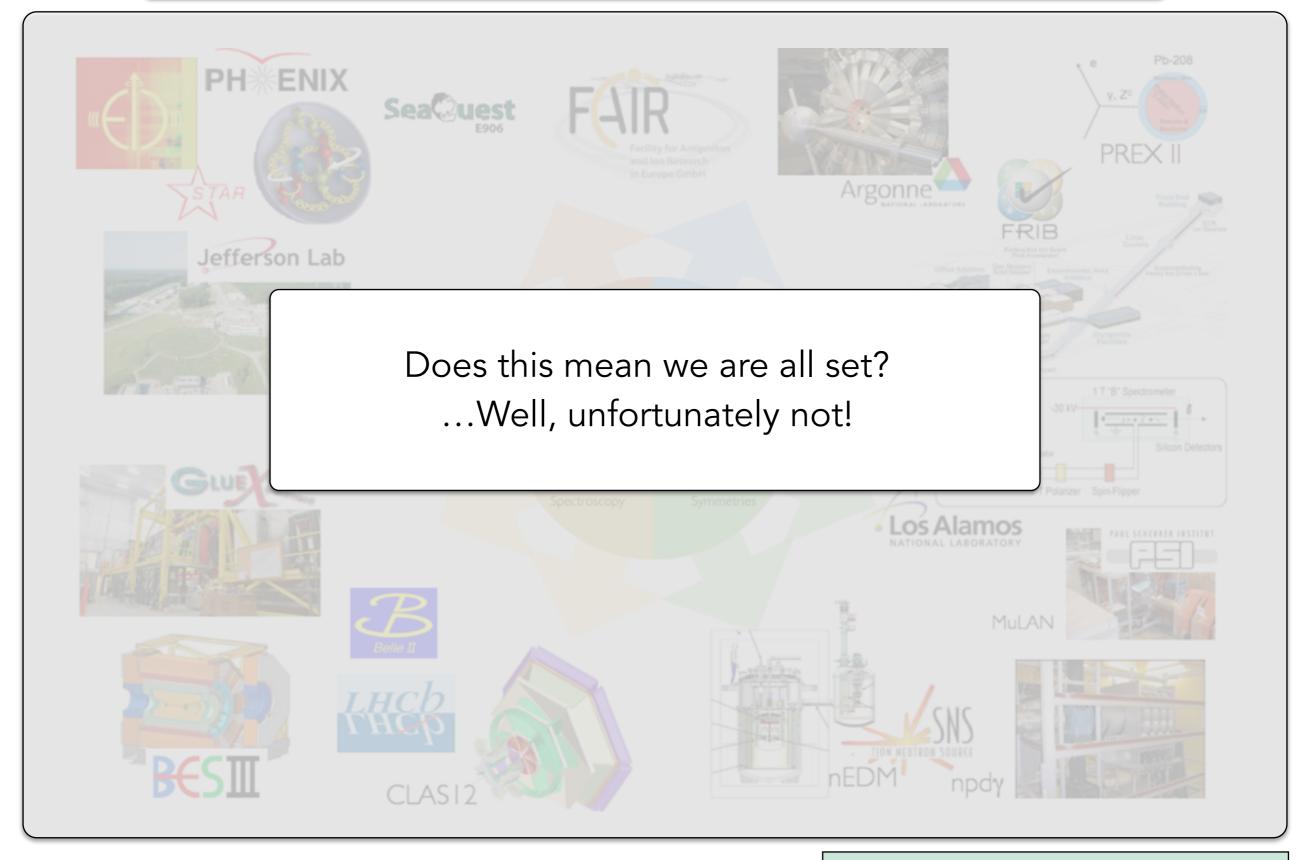
Titan supercomputer, Oak Ridge National Laboratory, USA



LATTICE QCD IS SUPPORTING A MULTI-BILLION DOLLAR EXPERIMENTAL PROGRAM!



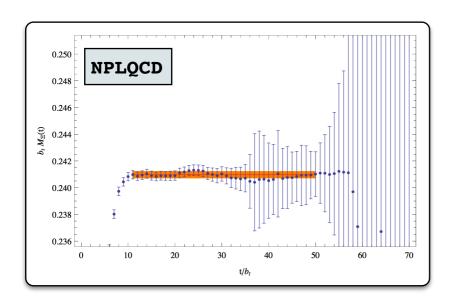
LATTICE QCD IS SUPPORTING A MULTI-BILLION DOLLAR EXPERIMENTAL PROGRAM!



THREE FEATURES MAKE LATTICE QCD CALCULATIONS OF NUCLEI HARD:

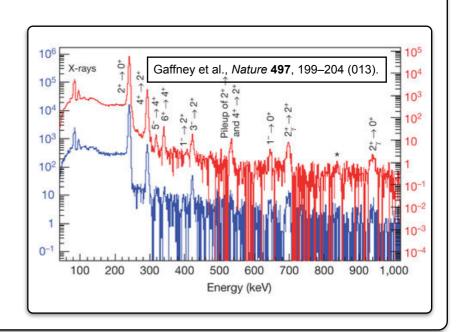
i) The complexity of systems grows factorially with the number of quarks.





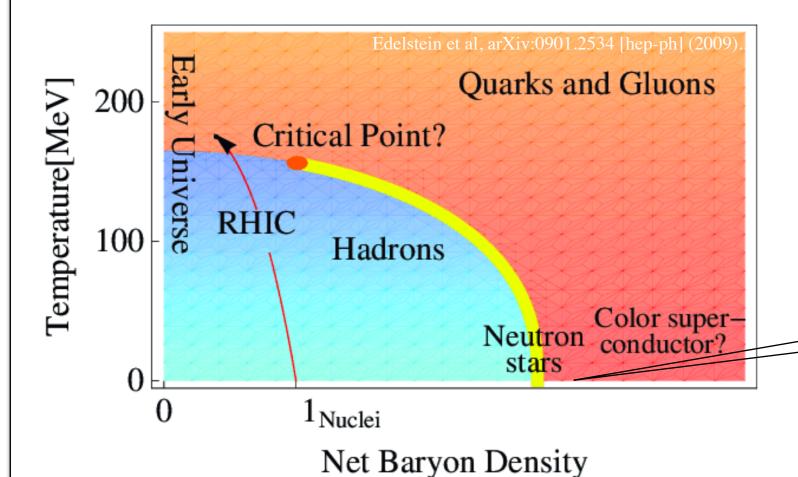
ii) There is a severe signal-to-noise degradation.

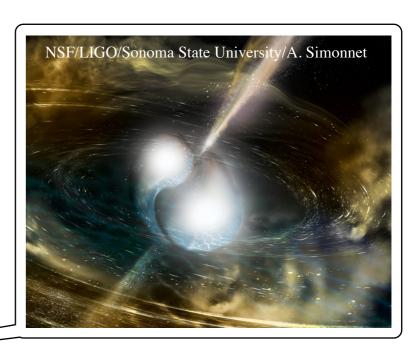
iii) Excitation energies of nuclei are much smaller than the QCD scale.



ADDITIONALLY THE SIGN PROBLEM FORBIDS:

i) Studies dense matter such as interior of neutron stars and phase diagram of QCD





Path integral formulation:

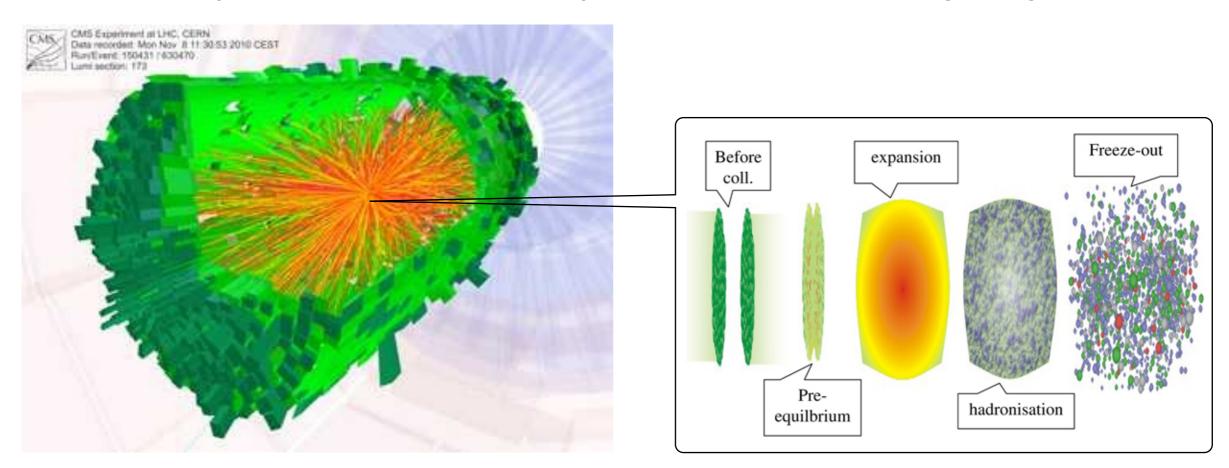
$$e^{-S[U,q,\bar{q}]}$$

with a complex action:

$$\mathcal{L}_{\mathrm{QCD}} \to \mathcal{L}_{\mathrm{QCD}} - i\mu \sum_{f} \bar{q}_{f} \gamma^{0} q_{f}$$

ADDITIONALLY THE SIGN PROBLEM FORBIDS:

ii) Real-time dynamics of matter in heavy-ion collisions or after Big Bang...



...and a wealth of dynamical response functions, transport properties, parton distribution functions, and non-equilibrium physics of QCD.

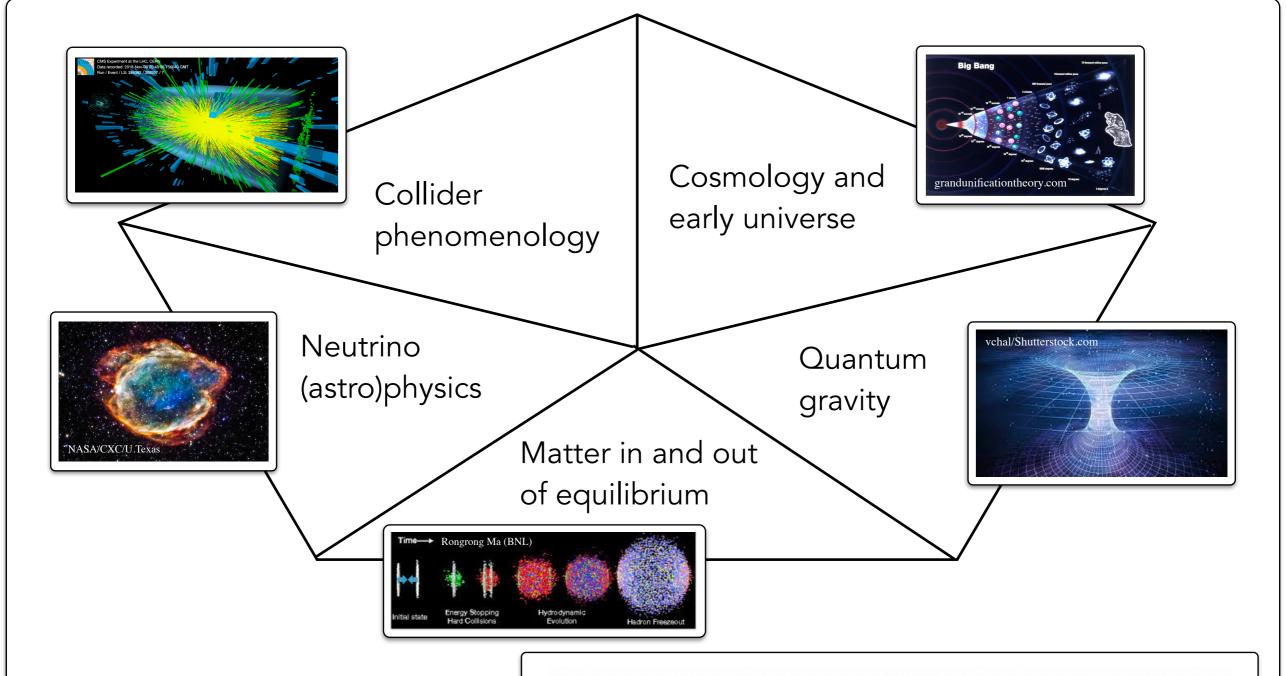
Path integral formulation:

$$e^{iS[U,qar{q}]}$$

Hamiltonian evolution:

$$U(t) = e^{-iHt}$$

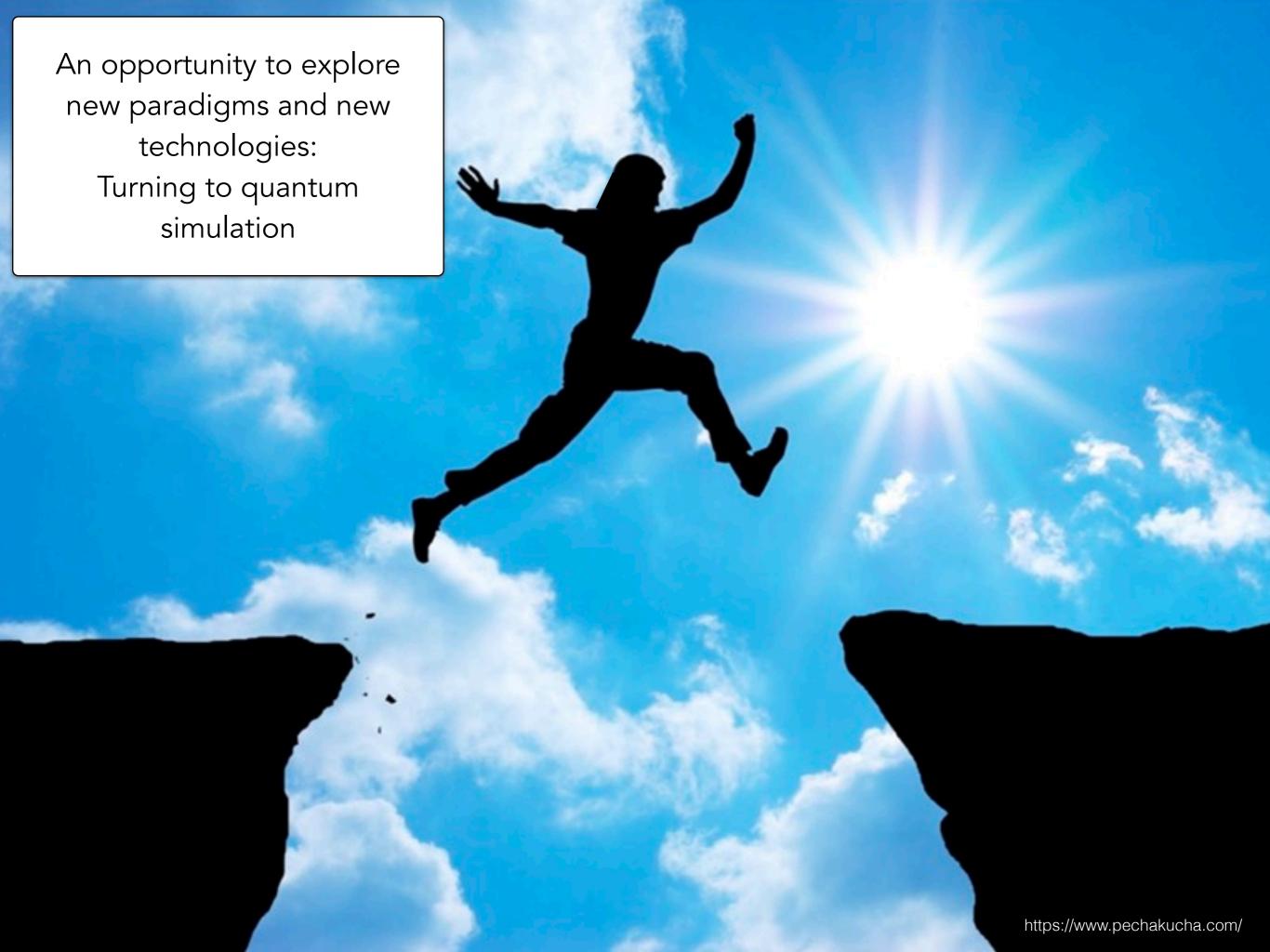
PLUS MANY INTRACTABLE QUESTIONS IN HIGH ENERGY PHYSICS AS WELL...



Bauer, ZD, et al,
arXiv:2204.03381 [quant-ph].

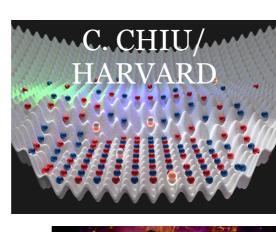
Quantum Simulation for High-energy Physics

Christian Bauer,^{1, a} Zohreh Davoudi,^{2, b} A. Baha Balantekin,³ Tanmoy Bhattacharya,⁴ Marcela Carena,^{5,6,7} Wibe A. de Jong,¹ Nate Gemelke,⁸ Dmitri Kharzeev,⁹ Henry Lamm,⁵ Ying-Ying Li,⁵ Yannick Meurice,¹⁰ Christopher Monroe,^{11,12,13,14} Benjamin Nachman,¹ Guido Pagano,¹⁵ John Preskill,¹⁶ Alessandro Roggero,^{17,18} David I. Santiago,^{19,20} Martin J. Savage,²¹ Irfan Siddiqi,^{19,20,22} George Siopsis,²³ Yukari Yamauchi,² and Kübra Yeter-Aydeniz²⁴



A RANGE OF QUANTUM SIMULATORS WITH VARING CAPACITY AND CAPABILITY IS AVAILABLE!

- Atomic systems (trapped ions, cold atoms, Rydbergs)
- Condensed matter systems
 (superconducting circuits,
 dopants in semiconductors
 such as in Silicon, NV centers
 in diamond)
- Laser-cooled polar molecules
- Optical systems (cavity quantum electrodynamics)

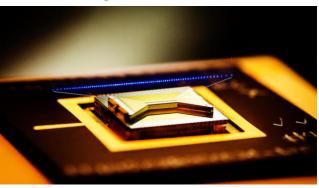




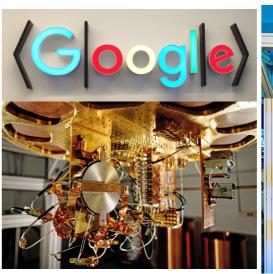


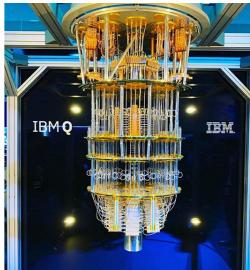












QUANTUM SIMULATION OF QCD?

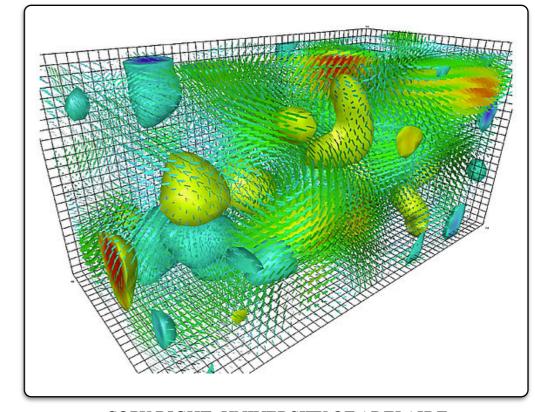
A controlled quantum system



CREDIT: EMILY EDWARDS, UNIVERSITY OF MARYLAND

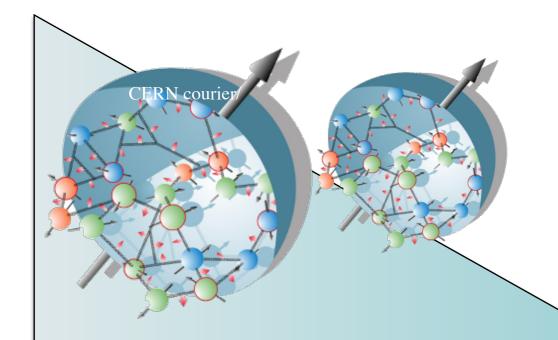


Strong-interaction physics



COPY RIGHT: UNIVERSITY OF ADELAIDE

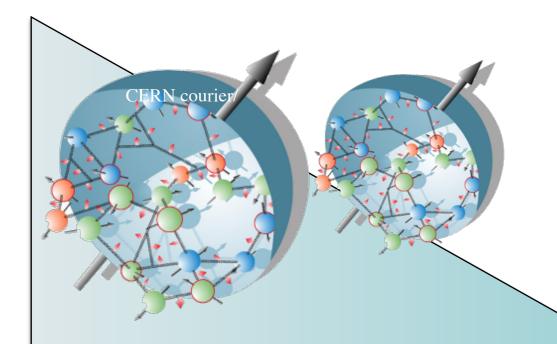
DIFFERENT FROM QUANTUM-CHEMISTRY SIMULATIONS



Starting from the Standard Model

Both bosonic and fermionic DOF are dynamical and coupled, exhibit both global and local (gauge) symmetries, relativistic hence particle number not conserved, vacuum state nontrivial in strongly interacting theories.

DIFFERENT FROM QUANTUM-CHEMISTRY SIMULATIONS

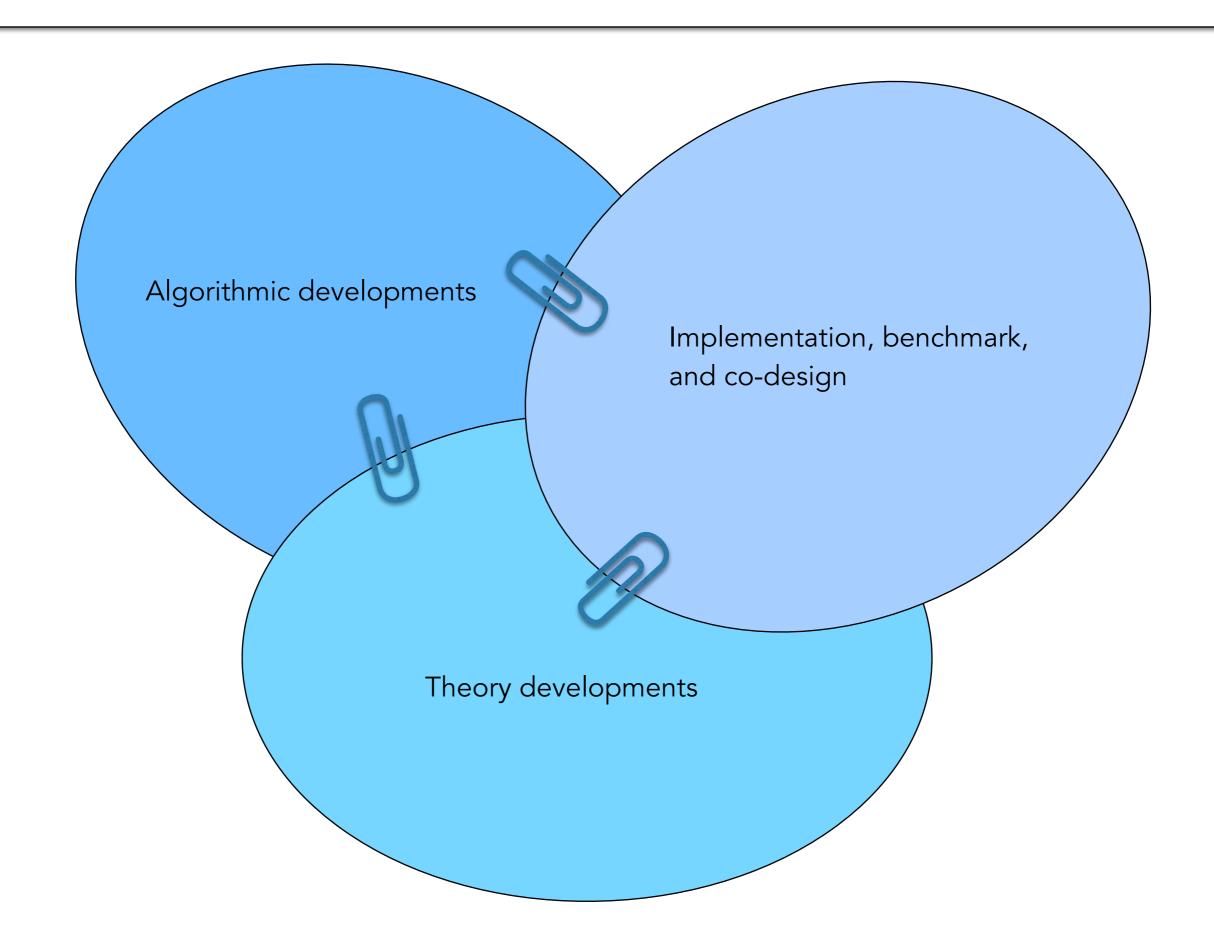


Starting from the Standard Model

Both bosonic and fermionic DOF are dynamical and coupled, exhibit both global and local (gauge) symmetries, relativistic hence particle number not conserved, vacuum state nontrivial in strongly interacting theories.

Attempts to cast QFT problems in a language closer to quantum chemistry and NR simulations: Kreshchuk, Kirby, Goldstein, Beauchemin, Love, arXiv:2002.04016 [quant-ph], Kreshchuk, Jia, Kirby, Goldstein, Vary, Love, Entropy 2021, 23, 597, Liu, Xin, arXiv:2004.13234 [hep-th], Barata, Mueller, Tarasov, Venugopalan (2020)

QUANTUM SIMULATION OF QUANTUM FIELD THEORIES: A MULTI-PRONG EFFORT





How to formulate QCD in the Hamiltonian language?

What are the efficient formulations? Which bases will be most optimal toward the continuum limit?

How to preserve the symmetries? How much should we care to retain gauge invariance?

How to quantify systematics such as finite volume, discretization, boson truncation, time digitization, etc?

Theory developments

Hamiltonian formalism maybe more natural than the path integral formalism for quantum simulation/computation:

Kogut and Susskind formulation:

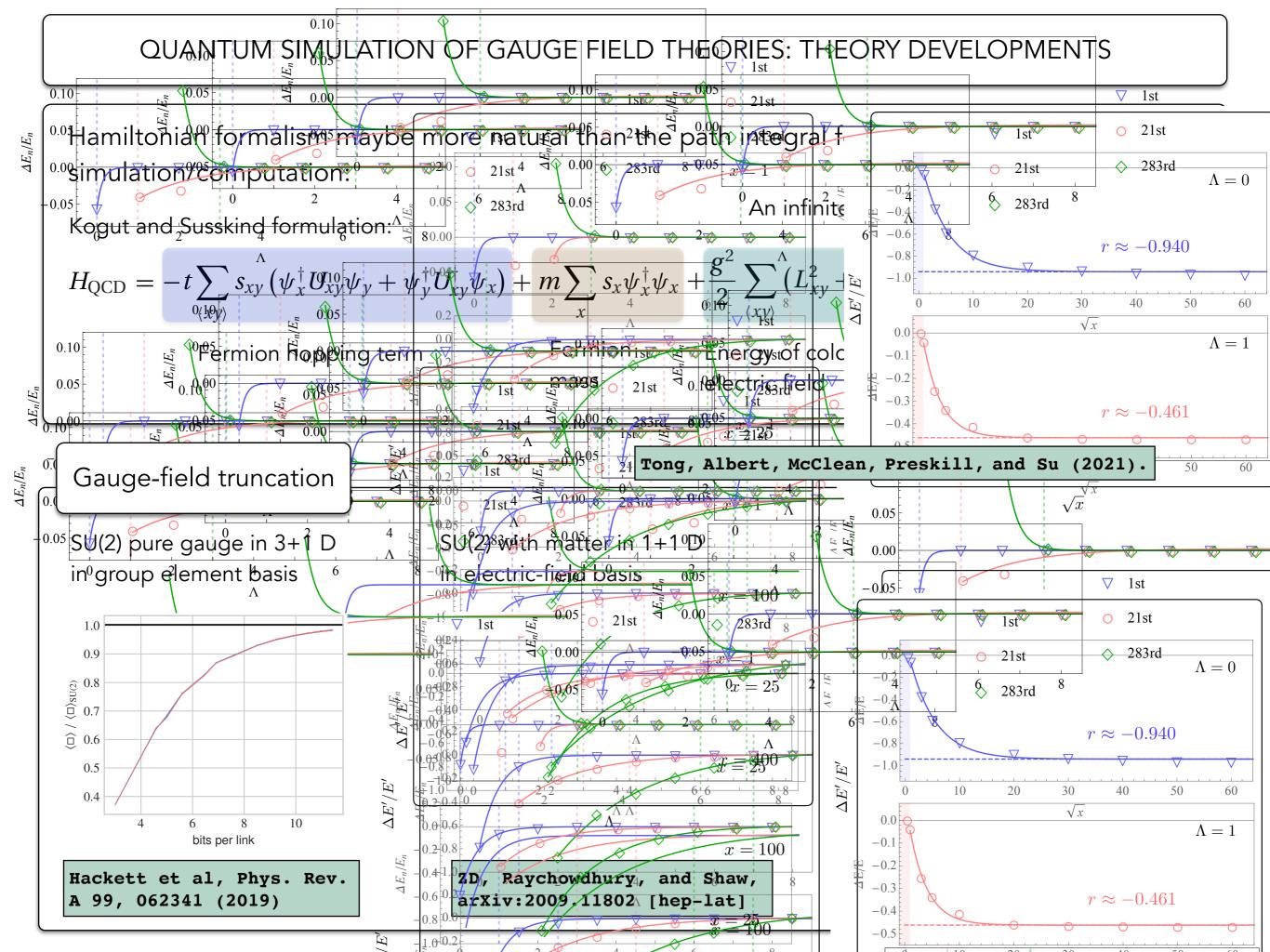
$$H_{\text{QCD}} = -t \sum_{\langle xy \rangle} s_{xy} \left(\psi_x^{\dagger} U_{xy} \psi_y + \psi_y^{\dagger} U_{xy}^{\dagger} \psi_x \right) + m \sum_{x} s_x \psi_x^{\dagger} \psi_x + \frac{g^2}{2} \sum_{\langle xy \rangle} \left(L_{xy}^2 + R_{xy}^2 \right) - \frac{1}{4g^2} \sum_{\square} \text{Tr} \left(U_{\square} + U_{\square}^{\dagger} \right).$$

Fermion hopping term

Fermion mass

Energy of color electric field

Energy of color magnetic field



Hamiltonian formalism maybe more natural than the path integral formalism for quantum simulation/computation:

Kogut and Susskind formulation:

$$H_{\text{QCD}} = -t \sum_{\langle xy \rangle} s_{xy} \left(\psi_x^{\dagger} U_{xy} \psi_y + \psi_y^{\dagger} U_{xy}^{\dagger} \psi_x \right) + m \sum_{x} s_x \psi_x^{\dagger} \psi_x + \frac{g^2}{2} \sum_{\langle xy \rangle} \left(L_{xy}^2 + R_{xy}^2 \right) - \frac{1}{4g^2} \sum_{\square} \text{Tr} \left(U_{\square} + U_{\square}^{\dagger} \right).$$

Fermion hopping term

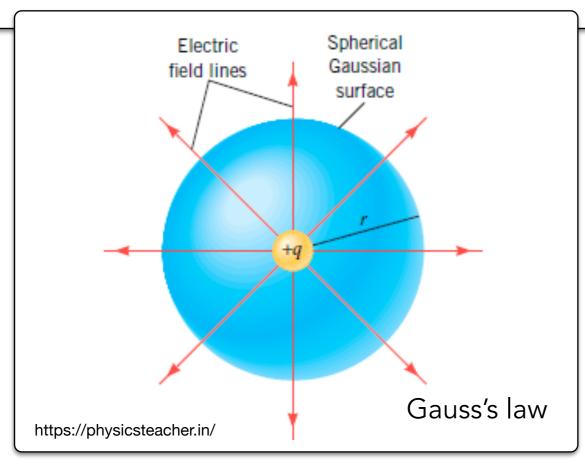
Fermion

mass

Energy of color electric field

Energy of color magnetic field

Generator of infinitesimal $G_x^a = \psi_x^{i\dagger} \lambda_{ij}^a \psi_x^j + \sum_k \left(L_{x,x+\hat{k}}^a + R_{x-\hat{k},x}^a \right) \quad \square \rangle \quad G_x^i |\psi(\{q_x^{(i)}\})\rangle = q_x^{(i)} |\psi(\{q_x^{(i)}\})\rangle$ gauge transformation



Hamiltonian formalism maybe more natural than the path integral formalism for quantum simulation/computation:

Kogut and Susskind formulation:

$$H_{\text{QCD}} = -t \sum_{\langle xy \rangle} s_{xy} \left(\psi_x^{\dagger} U_{xy} \psi_y + \psi_y^{\dagger} U_{xy}^{\dagger} \psi_x \right) + m \sum_{x} s_x \psi_x^{\dagger} \psi_x + \frac{g^2}{2} \sum_{\langle xy \rangle} \left(L_{xy}^2 + R_{xy}^2 \right) - \frac{1}{4g^2} \sum_{\square} \text{Tr} \left(U_{\square} + U_{\square}^{\dagger} \right).$$

Fermion hopping term

Fermion

mass

Energy of color electric field

Energy of color magnetic field

Generator of infinitesimal
$$G_x^a = \psi_x^{i\dagger} \lambda_{ij}^a \psi_x^j + \sum_k \left(L_{x,x+\hat{k}}^a + R_{x-\hat{k},x}^a \right) \quad \square \rangle \quad G_x^i |\psi(\{q_x^{(i)}\})\rangle = q_x^{(i)} |\psi(\{q_x^{(i)}\})\rangle$$
 gauge transformation

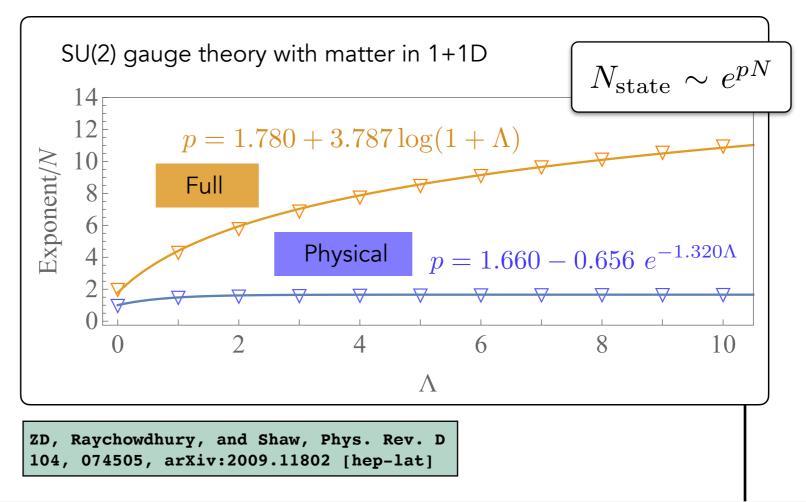
$$q_x = 0$$

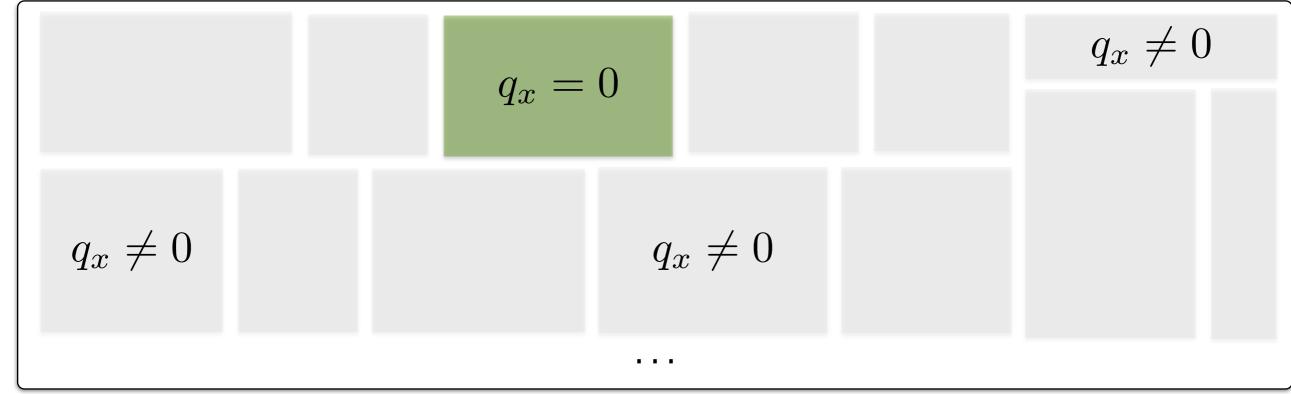
 $q_x \neq 0$

$$q_x \neq 0$$

$$q_x \neq 0$$

. . .





MANY HAMILTONIAN FORMULATIONS OF GAUGE THEORIES EXIST, BUT WHICH ONE TO PICK?

Gauge-field theories (Abelian and non-Abelian):

Group-element representation Zohar et al; Lamm et al

Prepotential formulation Mathur, Raychowdhury et al Loop-String-Hadron basis Raychowdhury and Stryker, ZD, Shaw

Link models Chandrasekharan, Wiese et al

Light-front quantization Kreshchuk, Love, Goldstien, Vary et al.; Ortega at al Fermionic basis Hamer et al; Martinez et al; Banuls et al

Local irreducible representations Byrnes and Yamamoto; Ciavarella, Klco, and Savage Bosonic basis Cirac and Zohar

Manifold lattices Buser et al

Dual plaquette (magnetic) basis Bender, Zohar et al; Kaplan and Styker; Unmuth-Yockey; Hasse et al; Bauer and Grabowska

Spin-dual representation Mathur et al

Scalar field theory

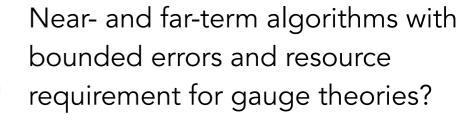
Field basis Jordan, Lee, and Preskill Continuous-variable basis Pooser, Siopsis et al

Harmonic-oscillator basis Klco and Savage

Single-particle basis
Barata , Mueller, Tarasov, and Venugopalan.

Algorithmic developments [Digital]

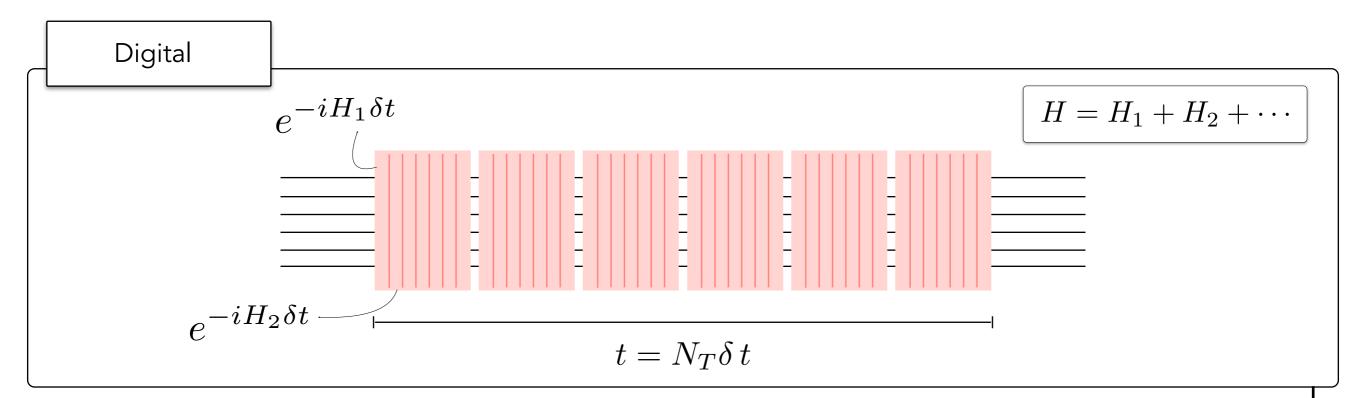
Simulators will not be covered here.

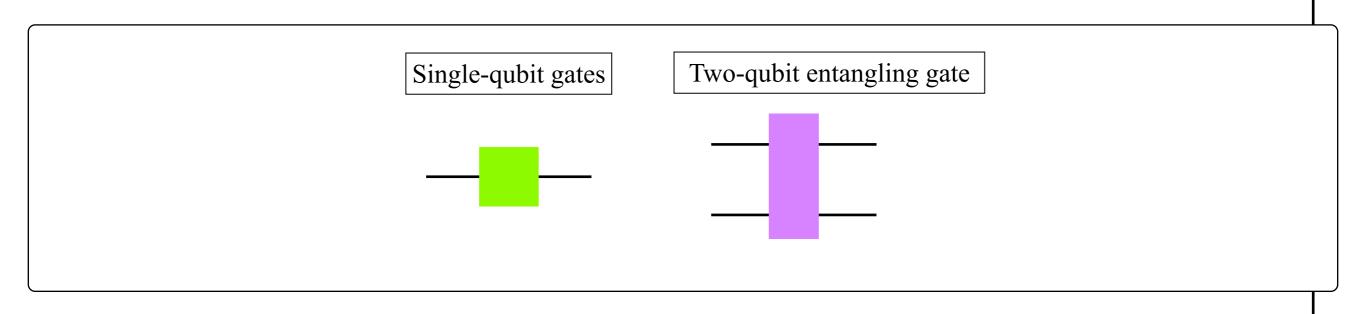


Can given formulation/encoding reduce qubit and gate resources?

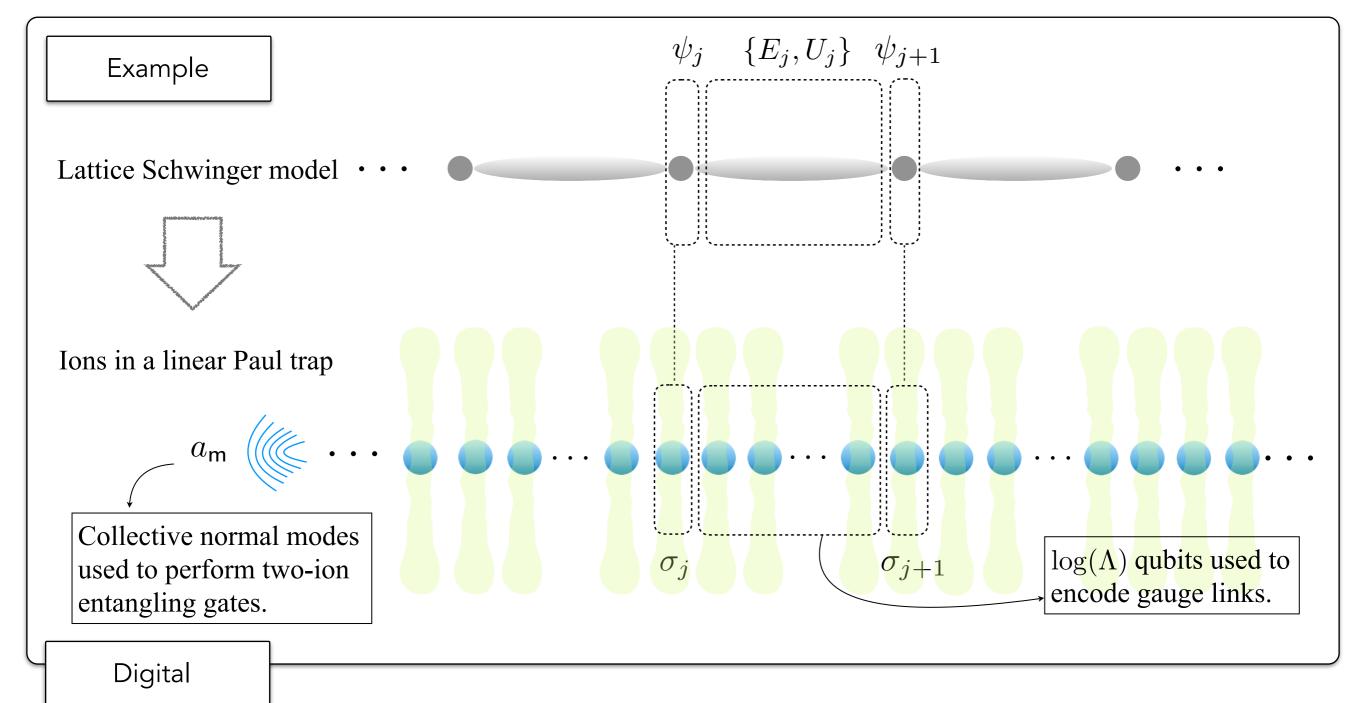
Should we develop gauge-invariant simulation algorithms?

How do we do state preparation and compute observables like scattering amplitudes?





How many qubits and gates are required to achieve accuracy ϵ in a given observables? Are there algorithms that scale optimally?

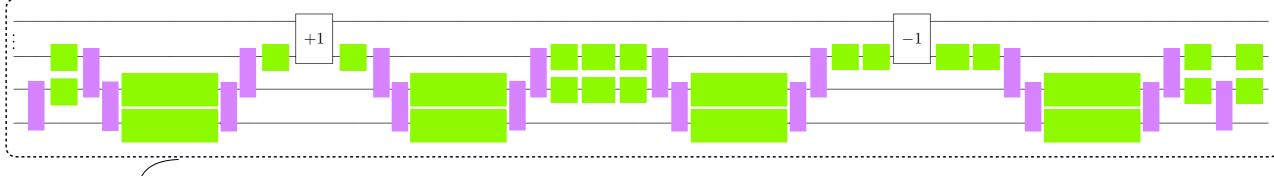


$$H = -ix \sum_{n=1}^{N-1} \left[\psi_n^{\dagger} U_n \psi_{n+1} - \text{h.c.} \right] + \sum_{n=1}^{N-1} E_n^2 + \mu \sum_{n=1}^{N} (-1)^n \psi_n^{\dagger} \psi_n$$

Interesting algorithmic progress for SU(2), and SU(3) theories can be found in: Ciavarella, Klco, and Savage, Phys. Rev. D 103, 094501 (2021), arXiv:2101.10227 [quant-ph]. Kan and Nam, arXiv:2107.12769 [quant-ph].

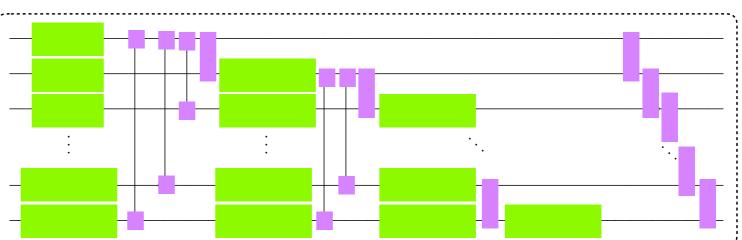


Shaw, Lougovski, Stryker, Wiebe, Quantum 4, 306 (2020).



Sample gauge-fermion interaction block

Part of electric field interactions acting on gauge DOF registers



Near term cost

	$\delta_g = 10^{-3}$		$\delta_g = 10^{-4}$		$\delta_g = 10^{-5}$		$\delta_g = 10^{-6}$		$\delta_g = 10^{-7}$	
	$\widetilde{\epsilon}^2$	CNOT	$ ilde{\epsilon}^2$	CNOT	$ ilde{\epsilon}^2$	CNOT	$ ilde{\epsilon}^2$	CNOT	$ ilde{\epsilon}^2$	CNOT
$x = 10^{-2}$		7.3e4	_	1.6e5		3.4e5		7.3e5	5.6e-2	1.6e6
$x = 10^{-1}$		1.6e4		$3.5\mathrm{e}4$	_	7.5e4	5.9e-2	1.6e5	2.7e-3	3.5e5
x = 1		4.6e3		9.9e3	1.0e-1	2.1e4	4.7e-3	4.6e4	2.2e-4	9.9e4
$x = 10^{2}$		2.8e3	8.3e-1	6.1e3	3.8e-2	1.3e4	1.8e-3	2.8e4	8.2e-5	$6.0\mathrm{e}4$

$$H = -ix \sum_{n=1}^{N-1} \left[\psi_n^{\dagger} U_n \psi_{n+1} - \text{h.c.} \right] + \sum_{n=1}^{N-1} E_n^2 + \mu \sum_{n=1}^{N} (-1)^n \psi_n^{\dagger} \psi_n$$



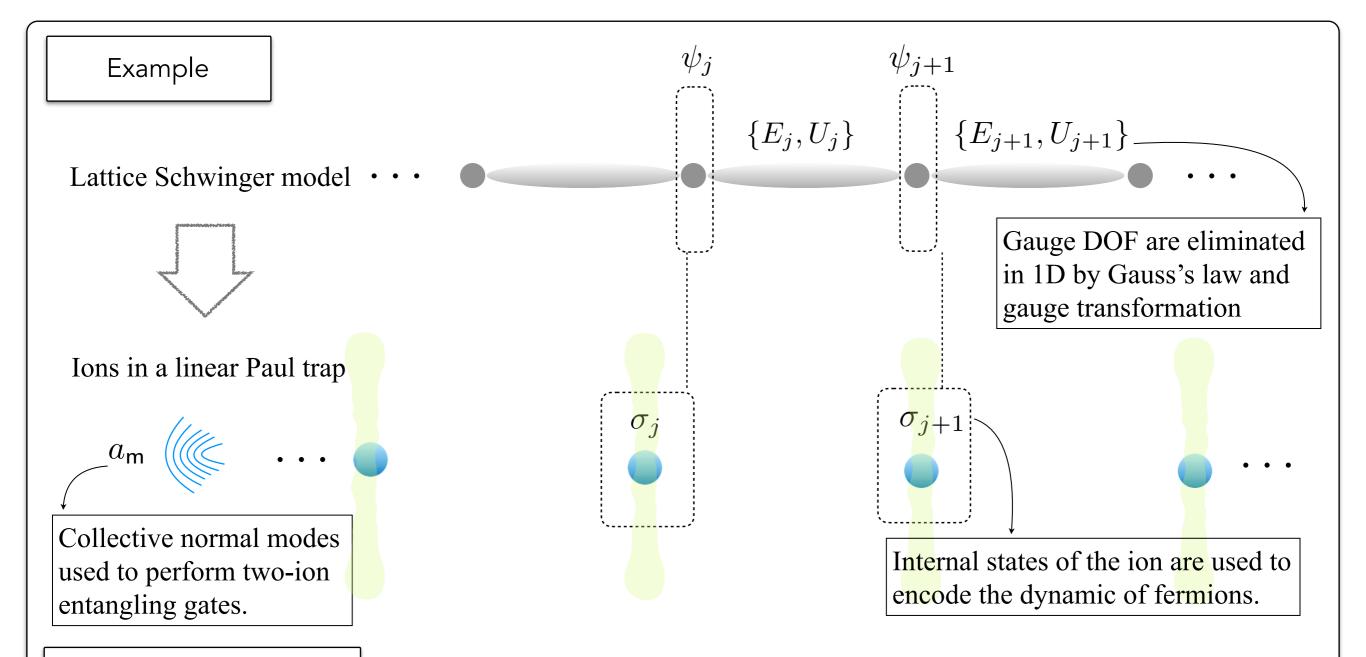
What is the capability limit of the hardware for gauge-theory simulations so far?

What is the nature of noise in hardware and how can it best be mitigated?

Can we co-design dedicated systems for gauge-theory simulations?

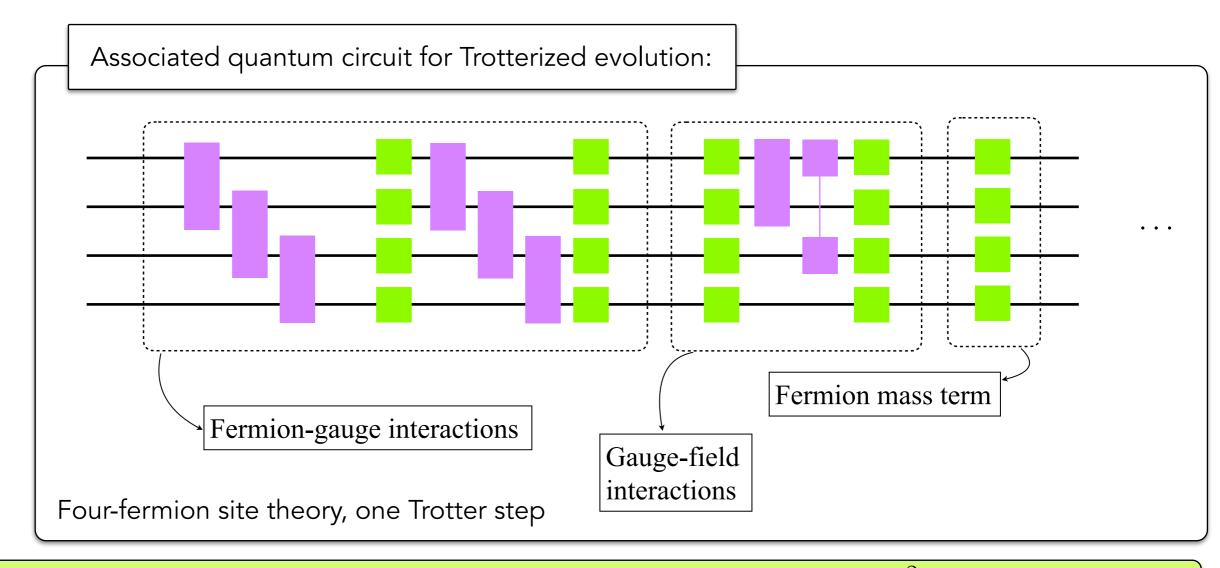
Can digital and analog ideas be combined to facilitate simulations of field theories?



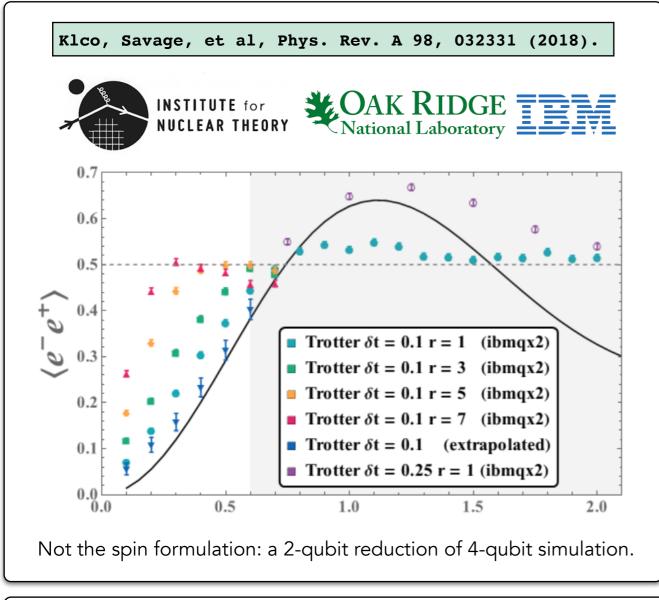


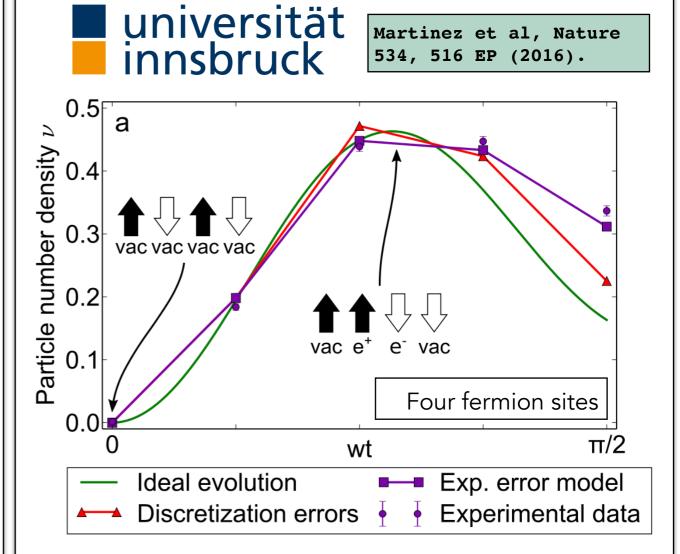
Digital (No gauge DOF)

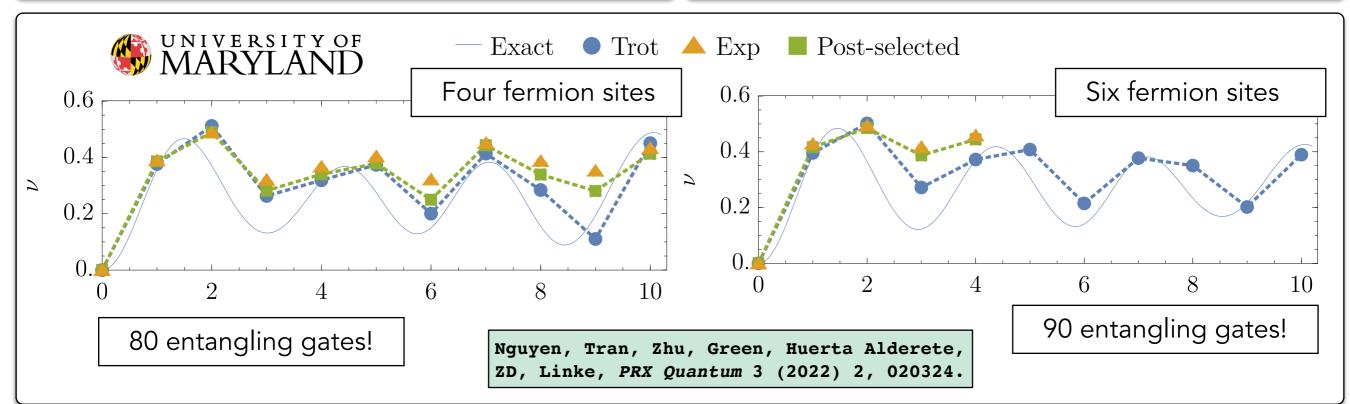
$$H = x \sum_{n=1}^{N-1} \left[\sigma_{+}^{(n)} \sigma_{-}^{(n+1)} + \sigma_{+}^{(n+1)} \sigma_{-}^{(n)} \right] + \sum_{n=1}^{N-1} \left[\epsilon_{0} + \frac{1}{2} \sum_{m=1}^{n} \left(\sigma_{z}^{(m)} + (-1)^{m} \right) \right]^{2} + \frac{\mu}{2} \sum_{n=1}^{N} (-1)^{n} \sigma_{z}^{(n)}$$

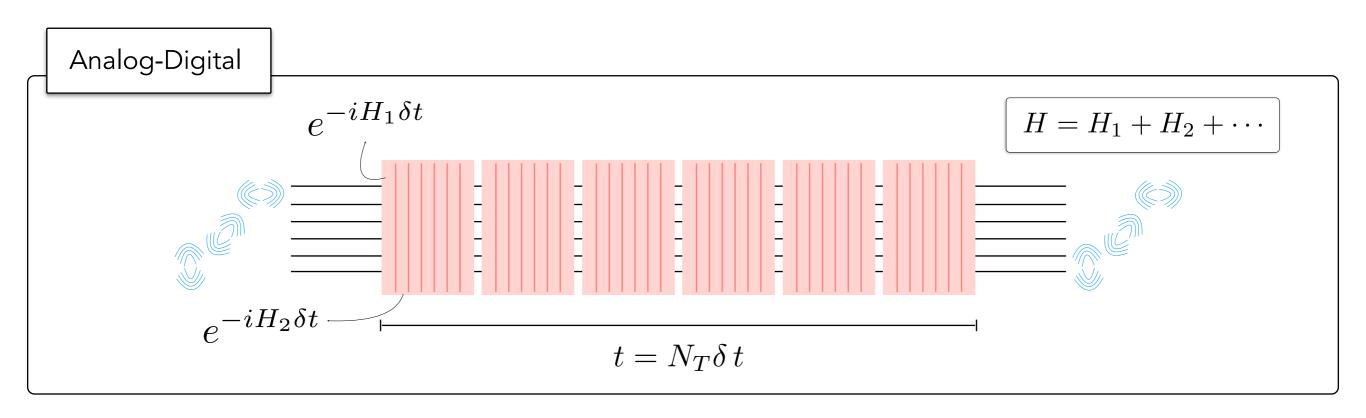


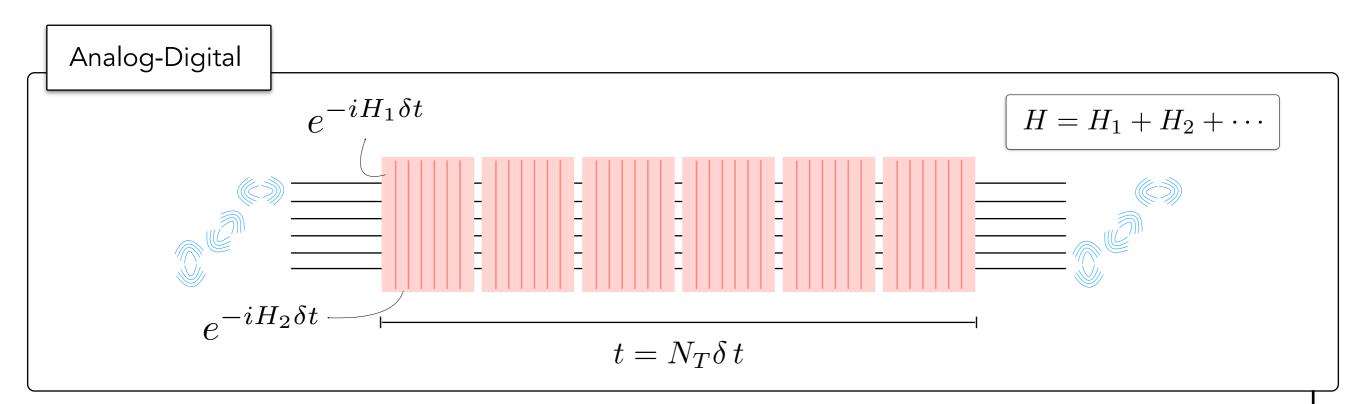
$$H = x \sum_{n=1}^{N-1} \left[\sigma_{+}^{(n)} \sigma_{-}^{(n+1)} + \sigma_{+}^{(n+1)} \sigma_{-}^{(n)} \right] + \sum_{n=1}^{N-1} \left[\epsilon_{0} + \frac{1}{2} \sum_{m=1}^{n} \left(\sigma_{z}^{(m)} + (-1)^{m} \right) \right]^{2} + \frac{\mu}{2} \sum_{n=1}^{N} (-1)^{n} \sigma_{z}^{(n)}$$

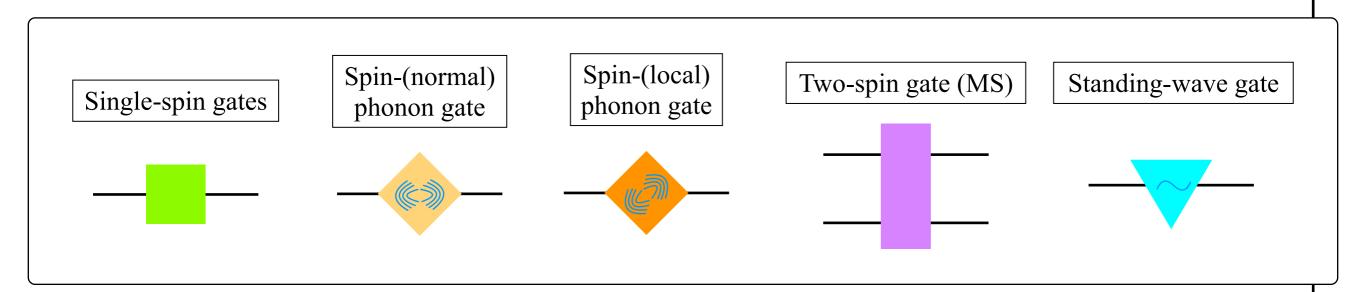






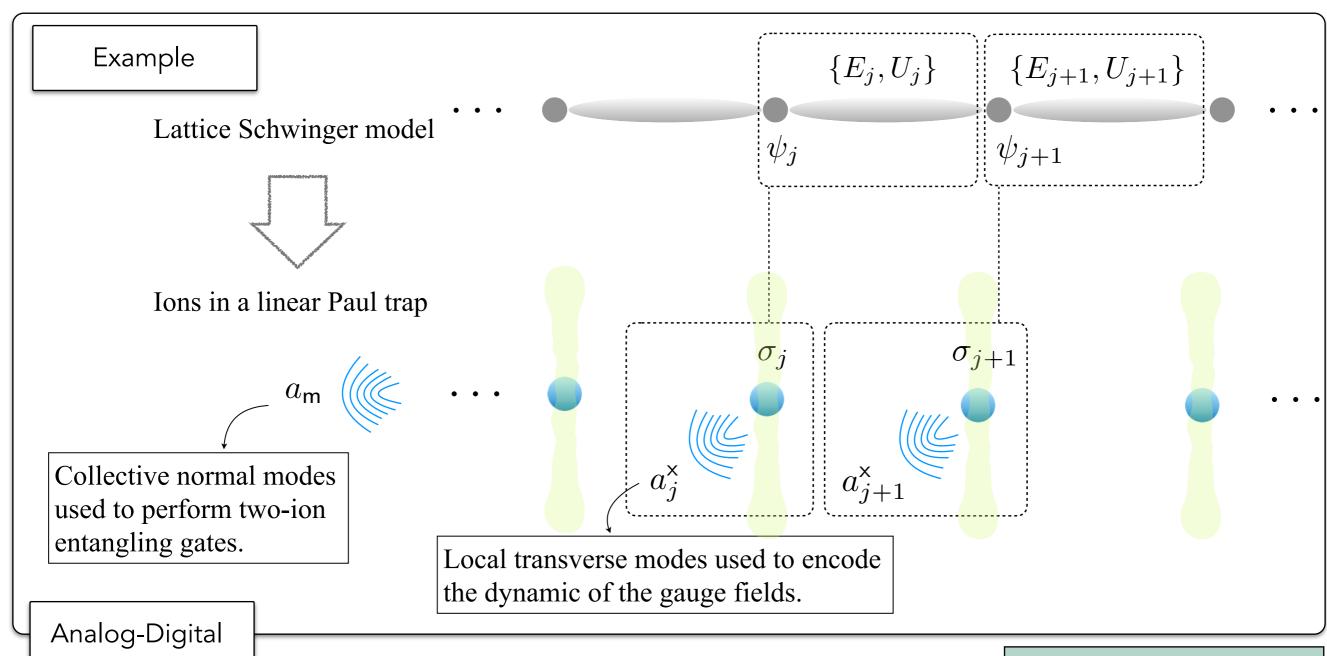






ZD, Linke, Pagano, Phys. Rev. Research 3, 043072 (2021).

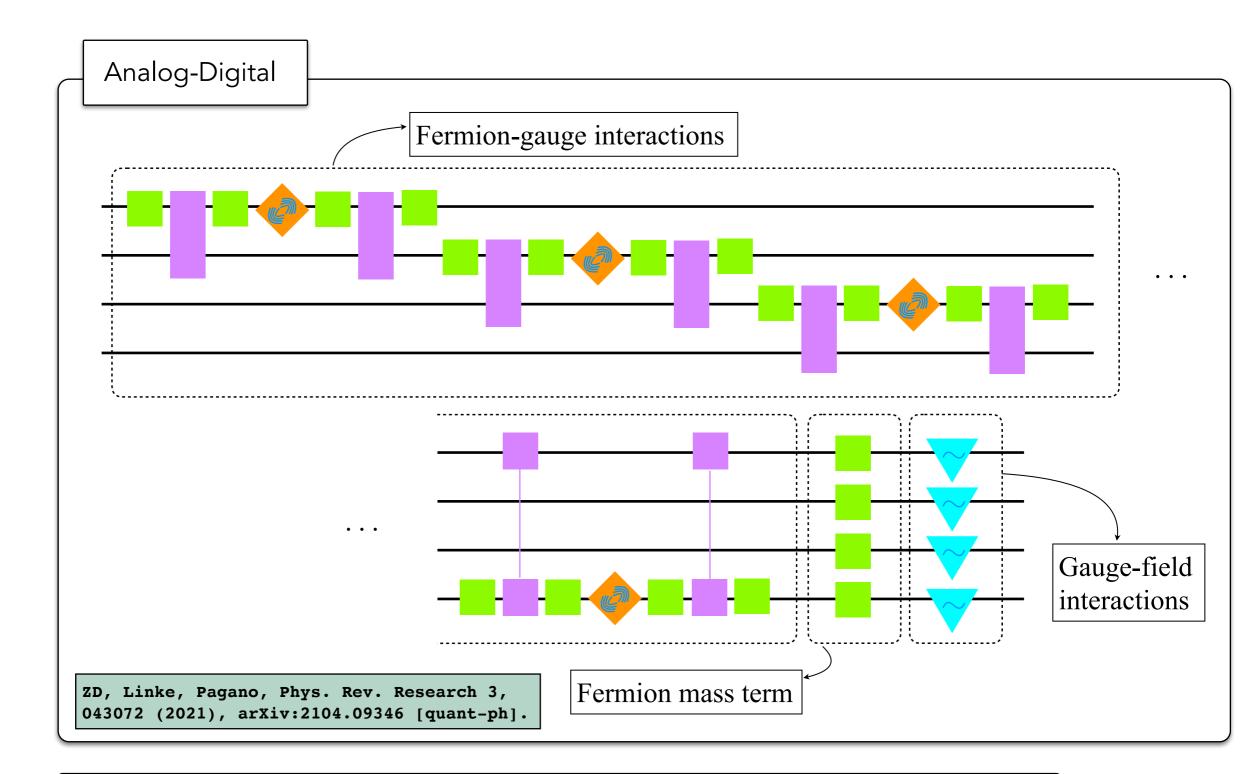
How many qubits and gates are required to achieve accuracy ϵ in a given observables? Are there algorithms that scale optimally?



ZD, Linke, Pagano, Phys. Rev. Research 3, 043072 (2021).

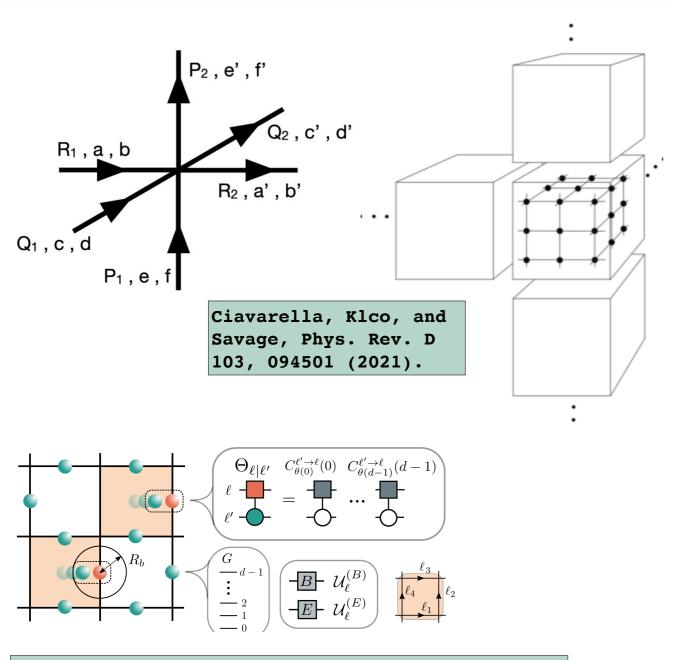
See also Casanova et al, Phys. Rev. Lett. 108, 190502 (2012), Lamata et al, EPJ Quant. Technol. 1, 9 (2014), and Mezzacapo et al, Phys. Rev. lett. 109, 200501 (2012) for analog-digital approaches to other interacting fermion-boson theories.

$$H = -ix \sum_{n=1}^{N-1} \left[\psi_n^{\dagger} U_n \psi_{n+1} - \text{h.c.} \right] + \sum_{n=1}^{N-1} E_n^2 + \mu \sum_{n=1}^{N} (-1)^n \psi_n^{\dagger} \psi_n$$



$$H = -ix \sum_{n=1}^{N-1} \left[\psi_n^{\dagger} U_n \psi_{n+1} - \text{h.c.} \right] + \sum_{n=1}^{N-1} E_n^2 + \mu \sum_{n=1}^{N} (-1)^n \psi_n^{\dagger} \psi_n$$

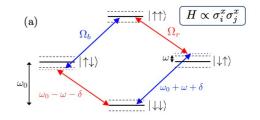
More co-design examples: Multi-dimensional local Hilbert spaces and multi-mode interactions

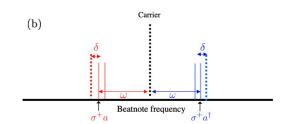


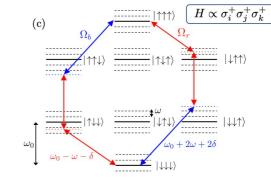
González-Cuadra, Zache, Carrasco, Kraus, Zoller,
arXiv:2203.15541 [quant-ph].

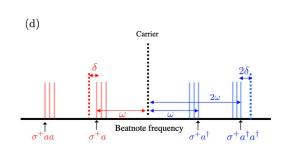










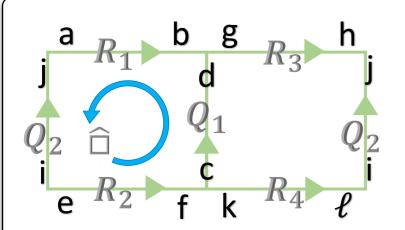


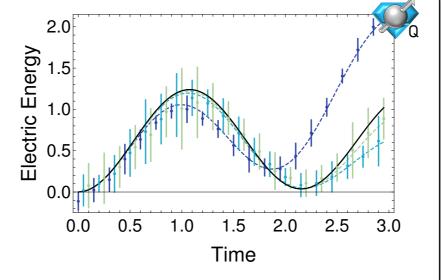
Andrade, ZD, Grass, Hafezi, Pagano, Seif, arXiv:2108.01022 [quant-ph], Bermudez et al, Pays.Rev.A79, 060303 R (2009), Katz, Centina, Monroe, arXiv:2202.04230 [quant-ph].

4

Finally a few more examples showcasing progress in hardware implementation of a range of QCD-inspired problems...

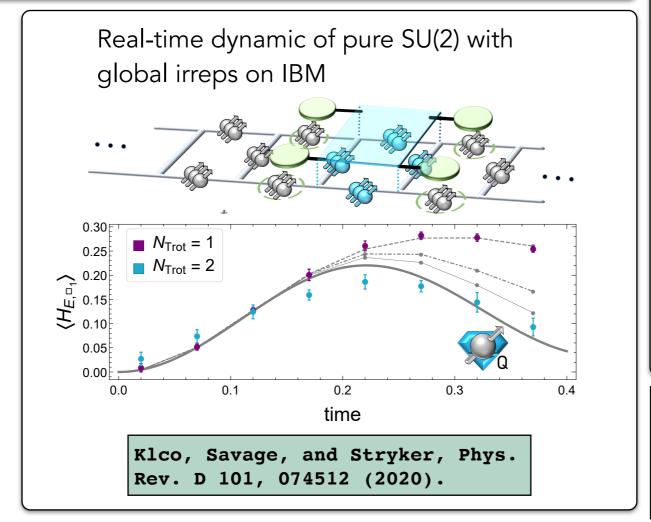
DIGITAL COMPUTATIONS OF NON-ABELIAN LGTs





Real-time dynamic of pure SU(3) with global irrupts on IBM

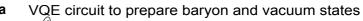
Ciavarella, Klco, and Savage, Phys. Rev. D 103, 094501 (2021).

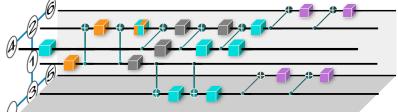


Low-lying spectrum of SU(2) with matter in 1+1 D on IBM

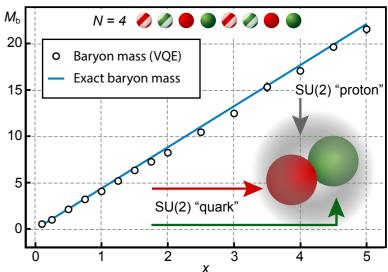
0000

0000





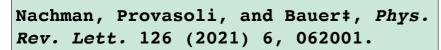
VQE preparation of the baryon mass

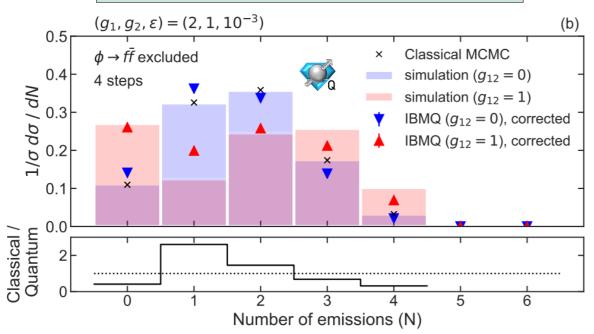


Atas et al, Nature Communications 12, 6499 (2021). SU(3) example: Atas et al: arXiv:2207.03473 [quant-ph].

See also studies on D-wave annealers: Rahman et al, Phys. Rev. D 104, 034501 (2021), Illa and Savage, arXiv:2202.12340 [quant-ph], Farrel et al, arXiv:2207.01731 [quant-ph].

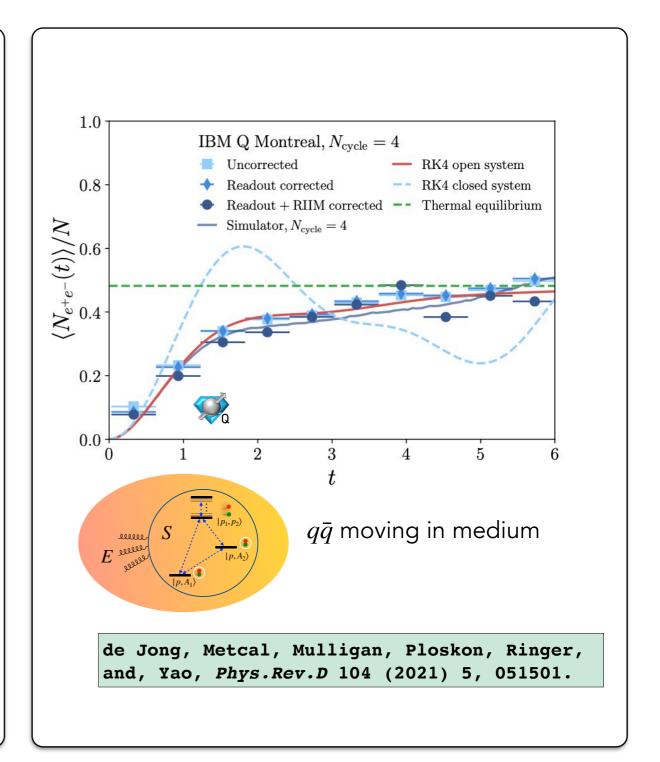
PARTON SHOWER ALGORITHMS AND HEAVY QUARKONIA MOTION IN QGP



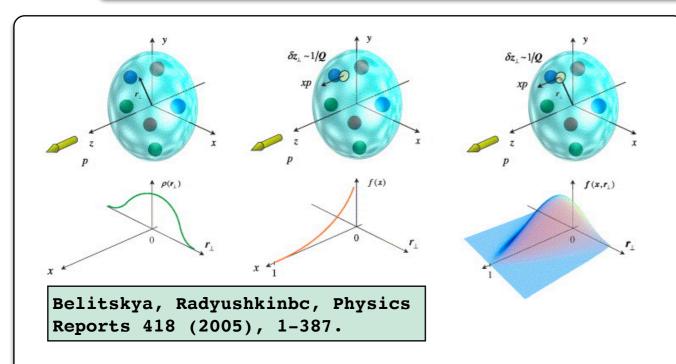


A polynomial time quantum final state shower algorithm that accurately models the effects of intermediate spin states similar to those present in electroweak showers.

See also Bepari, Malik, Spannowsky, Williams, Phys. Rev. D 103, 076020 (2021), Williams, Malik, Spannowsky, Bepari, Phys. Rev. D 106 (2022) 056002, Gustafson, Prestel, Spannowsky, Williams, J. High Energ. Phys. 2022, 35 (2022).



PARTON DISTRIBUTION FUNCTIONS, DECAY AMPLITUDES

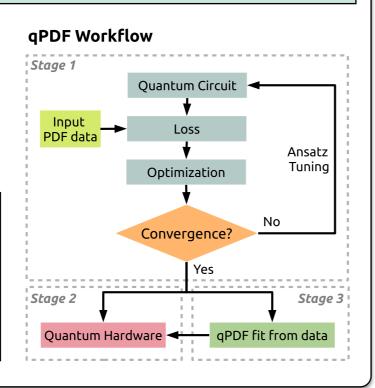


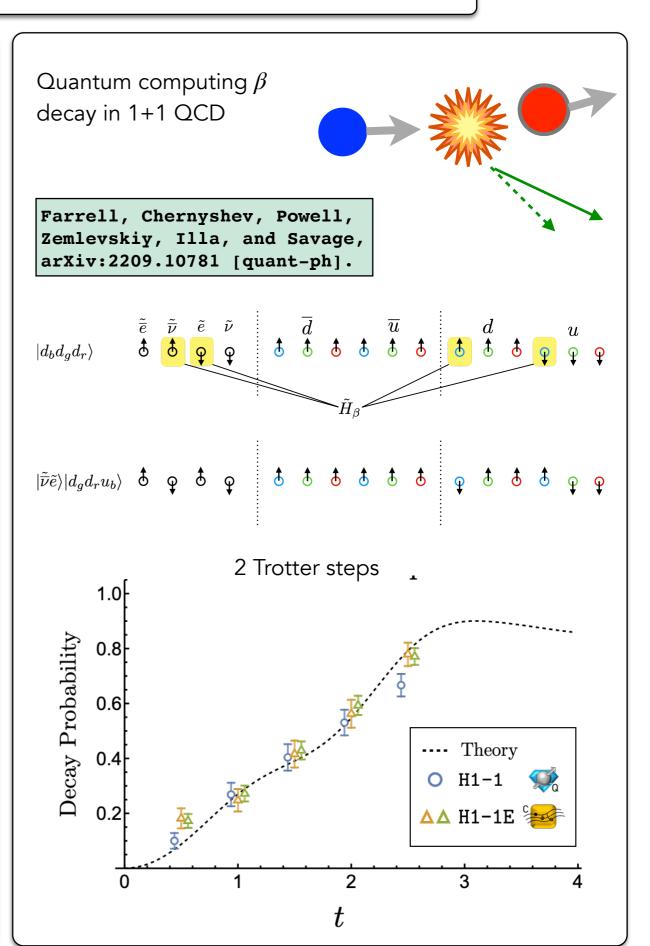
Either calculate PDFs directly since non-equal time amplitudes are possible on quantum computers...

Mueller, Tarasov, and Raju Venugopalan, PRD 102, 016007 (2020), Lamm, Lawrence, and Yamauchi, Phys. Rev. Res. 2, 013272 (2020), Echevarria, Egusquiza, Rico, and G Schnell, PRD 104, 014512 (2021).

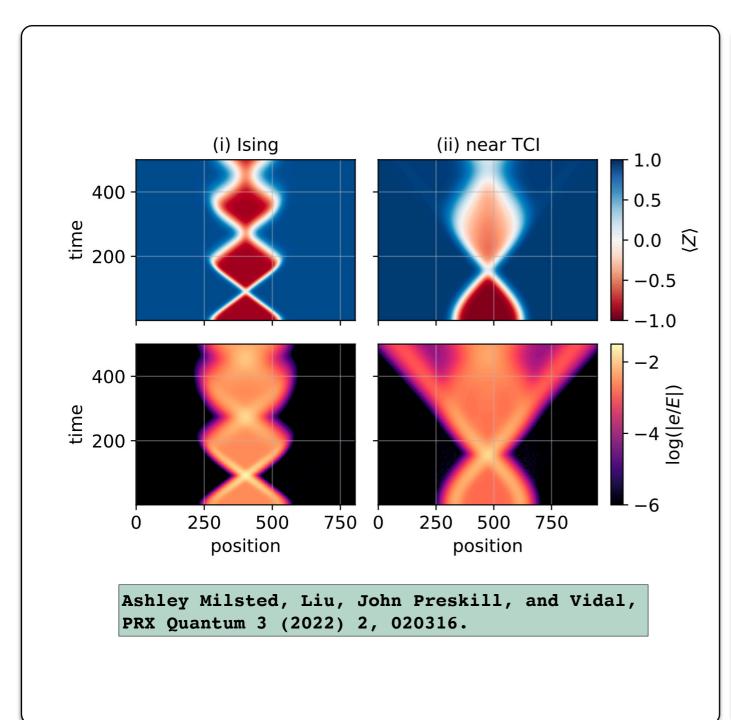
...or expedite global fitting of PDFs with variational quantum eigensolvers...

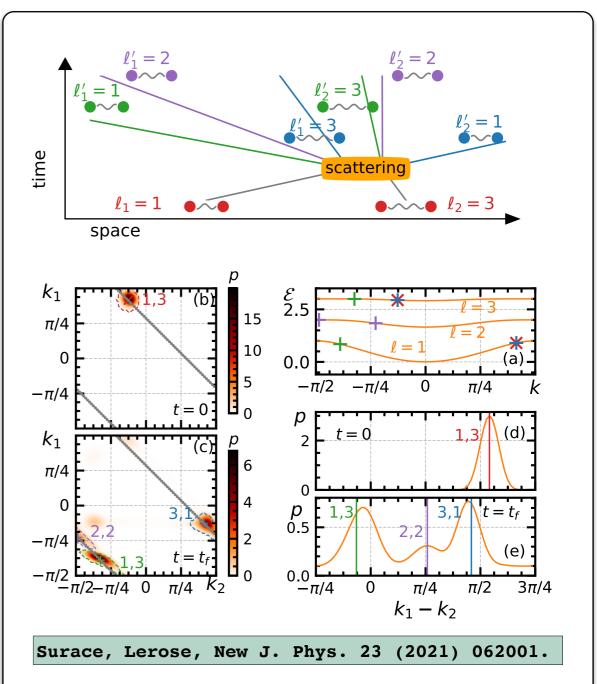
Perez-Salinas, Cruz-Martinez, Alhajri, and Carrazza, PRD 103, 034027 (2021), Qian, Basili, Pal, Luecke, and Vary, arXiv:2112.01927 (2021).





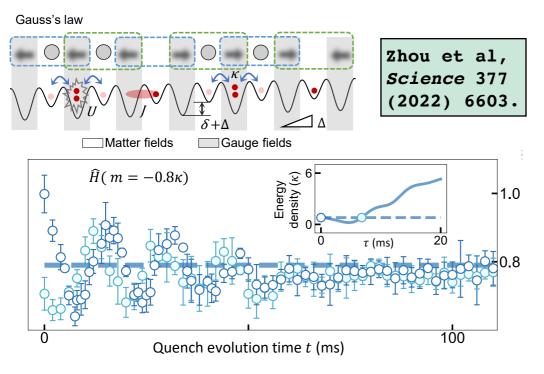
FIRST STEPS TOWARD SCATTERING IN SPIN SYSTEMS — NUMERICAL SIMULATIONS —





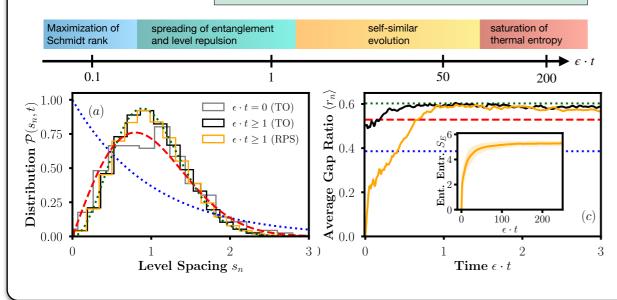
THERMALIZATION AND NON-EQUILIBRIUM PROPERTIES

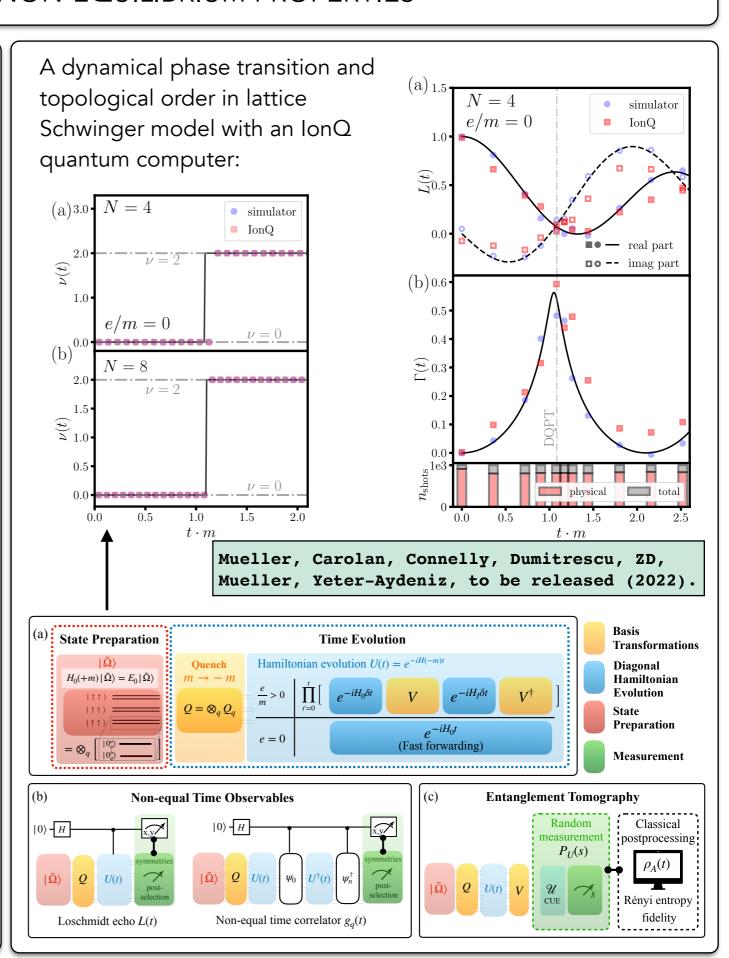
Thermalization dynamics of U(1) Quantum Link Model in a 71-site analog simulator



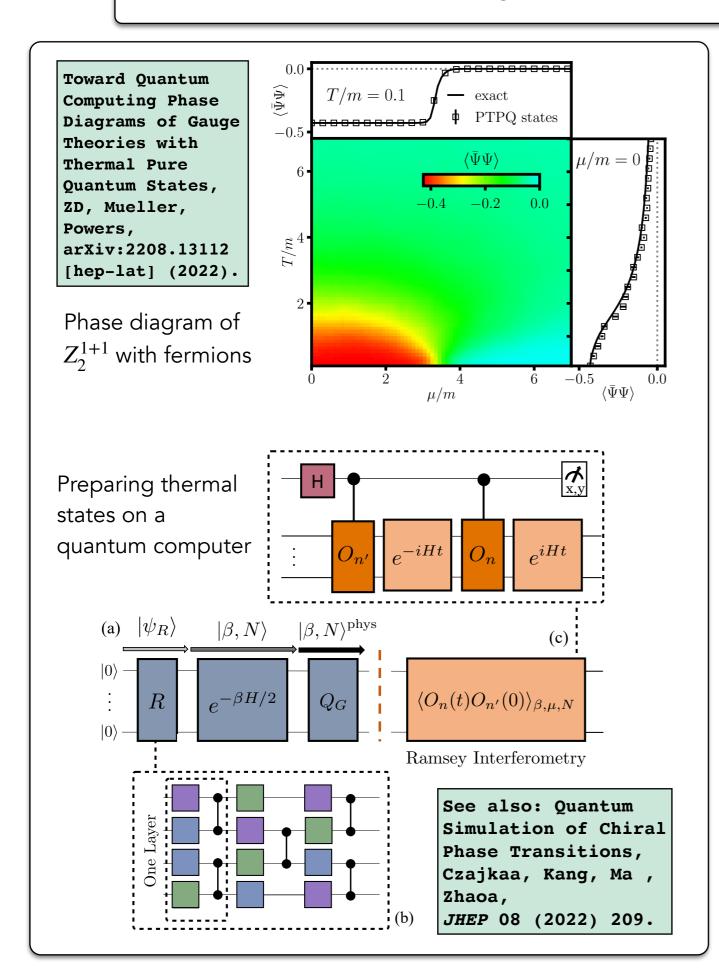
Stages of thermalization dynamics of Z2 LGT in 2+1 D from entanglement spectrum

Mueller, Zache, Ott, Phys. Rev. Lett. 129, 011601 (2022).



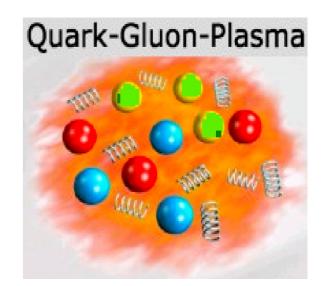


FINITE TEMPERATURE AND FINTIE DENSITY PHASE DIAGRAM

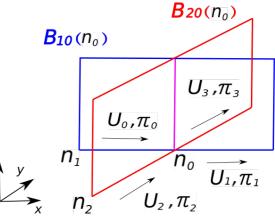


Transport coefficients from real-time correlators of energy momentum tensor

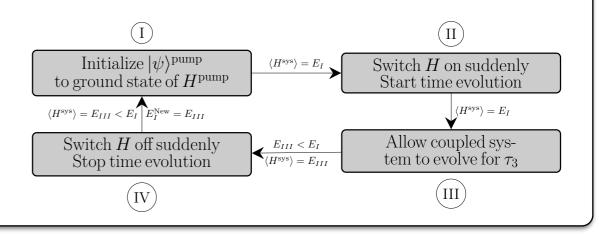
Cohen, Lamm, Lawrence, and Yamauchi, Phys. Rev. D 104, 094514 (2021).



How to define energymomentum tensor in Hamiltonian formulation

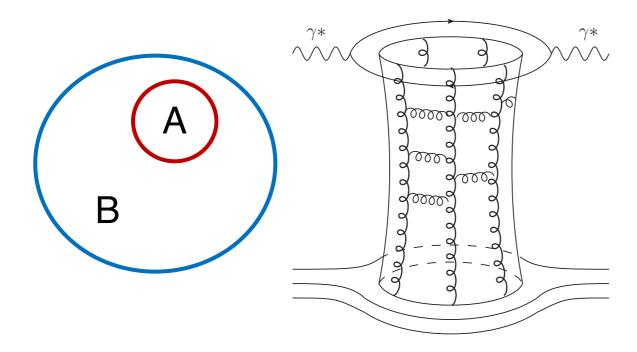


How to prepare a proton state? [Generally not developed sufficiently.]

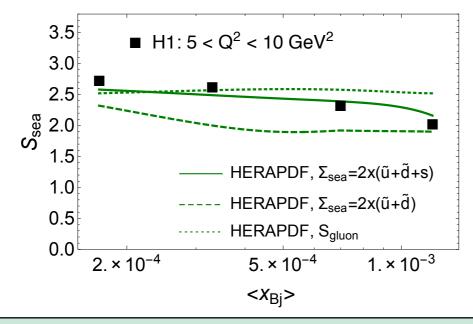


QUANTUM ENTANGLEMENT IN HIGH- AND LOW-ENERGY NUCLEAR PHYSICS

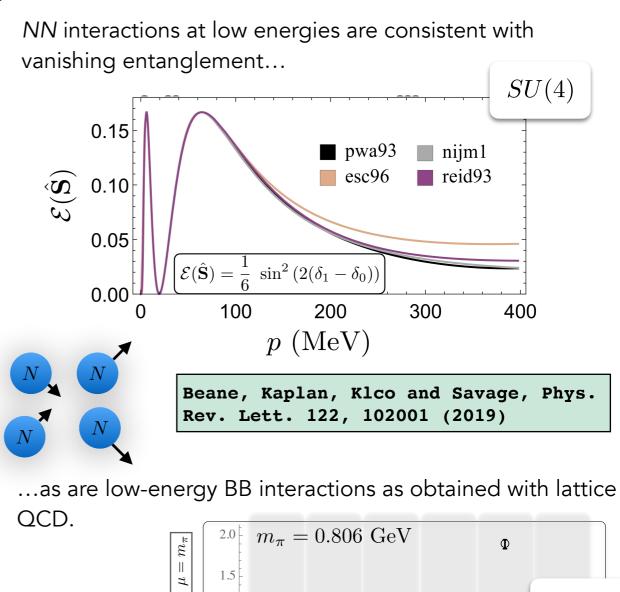
Deep inelastic scattering as a probe of entanglement?



Entropy of hadrons derived from PDFs can be related to entanglement entropy.



Kharzeev and Levin,, Phys. Rev. D 95, 114008 (2017), Zhang, Hao, Kharzeev, and Korepin, Phys. Rev. D 105, 014002 (2022).



 $m_{\pi}=0.806~{
m GeV}$ Φ SU(16) SU(16) C_1 C_2 C_3 C_4 C_5 C_6 C_6

Orginos, Savage, Shanahan (NPLQCD), Phys.

Rev. D 96, 114510 (2017)

We've got a long way to go to get to **QCD** but we know what to do! If one thing we learned from the successful conventional lattice-QCD program is that **theory/ algorithm/experiment** collaborations will be the key. It is even more important in the quantum-computing era since our computers are themselves physical systems!

