

# Spectral Reconstruction in NRQCD via the Backus-Gilbert Method

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FASTSUM

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# Non-Relativistic QCD in a nutshell

- NRQCD is an EFT for low-energy quarkonia
- Dynamics higher in energy than the heavy quark mass scale integrated out
- Decoupling of heavy quarks and anti-quarks reduces the system to an initial-value problem

# Spectral Reconstruction in NRQCD

The spectral representation of the Euclidean correlation function  $G_{\Gamma}(\tau, \mathbf{x}) = \langle O_{\Gamma}(\tau, \mathbf{x}) O_{\Gamma}(0, \mathbf{0}) \rangle$  is

$$G(\tau; T) = \int_0^{\infty} \rho(\omega; T) K(\omega, \tau) d\omega \quad (1)$$

where  $K(\omega, \tau; T) = \cosh[\omega(\tau - 1/2T)] / \sinh(\omega/2T) \rightarrow e^{-\omega\tau}$  when  $M \gg T$ .

**Aim:** Invert the Laplace transform

$$G(\tau; T) = \int_0^{\infty} \rho(\omega; T) e^{-\omega\tau} \quad (2)$$

$\implies$  **This is ill-posed!**

**One method:** Backus-Gilbert<sup>1</sup>

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<sup>1</sup>Backus, Gilbert 1968, *Geophysical Journal International*

Construct sampling functions  $A(\omega, \omega_0) = \sum_{\tau} c_{\tau}(\omega_0)K(\omega, \tau)$  which sample  $\rho(\omega)$  at a particular value  $\omega_0$ :

$$\begin{aligned}\hat{\rho}(\omega_0) &= \sum_{\tau} c_{\tau}(\omega_0)G(\tau) \\ &= \int A(\omega, \omega_0)\rho(\omega)d\omega\end{aligned}\tag{3}$$

$c_{\tau}(\omega_0)$  chosen to minimise the 'width' of  $A(\omega, \omega_0)$  (i.e. maximise resolving power)<sup>2</sup>:

$$J(\omega_0) = \int [A(\omega, \omega_0) - \delta(\omega - \omega_0)]^2 d\omega\tag{4}$$

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<sup>2</sup>Oldenburg 1984, IEEE GE-22 vol. 6

Laplace shift transform:

$$e^{\Delta \cdot \tau} G(\tau) \xrightarrow{\mathcal{L}} \rho(\omega + \Delta) \quad (5)$$

Under (5):

$$\hat{\rho}(\omega) \xrightarrow{\mathcal{L}} \hat{\rho}(\omega + \Delta) \quad (6)$$

$$A(\omega, \omega_0) \xrightarrow{\mathcal{L}} A(\omega, \omega_0) \quad (7)$$

# Laplace-Shifting: An example

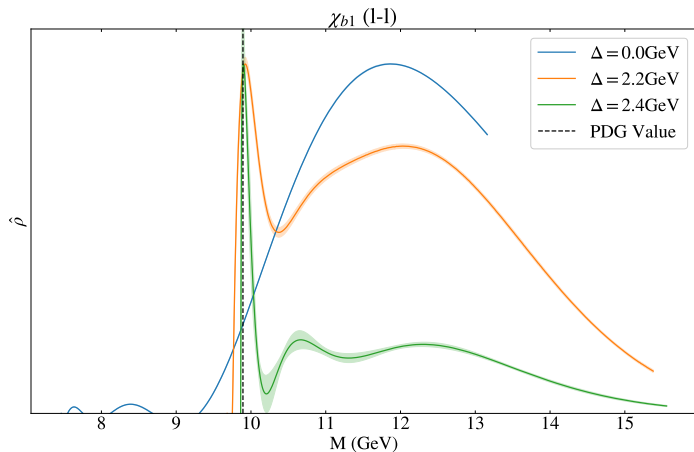
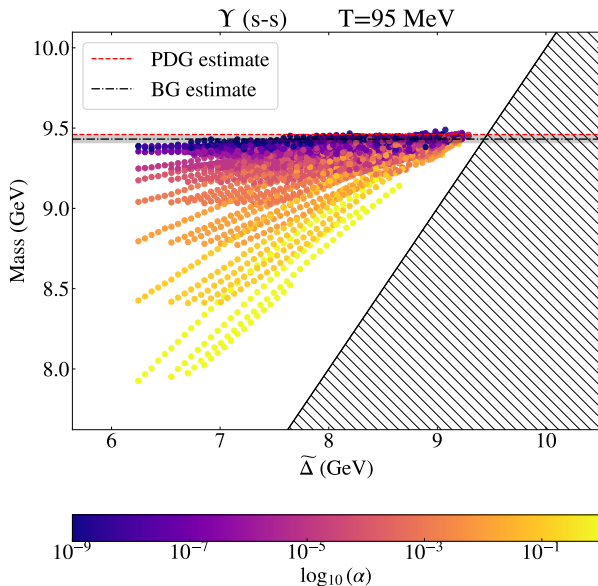


Figure: Laplace-shift in action using correlation functions for the  $\chi_{b1}$ .

# Results from the lattice



Thank you for your time.

Questions are welcome!

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