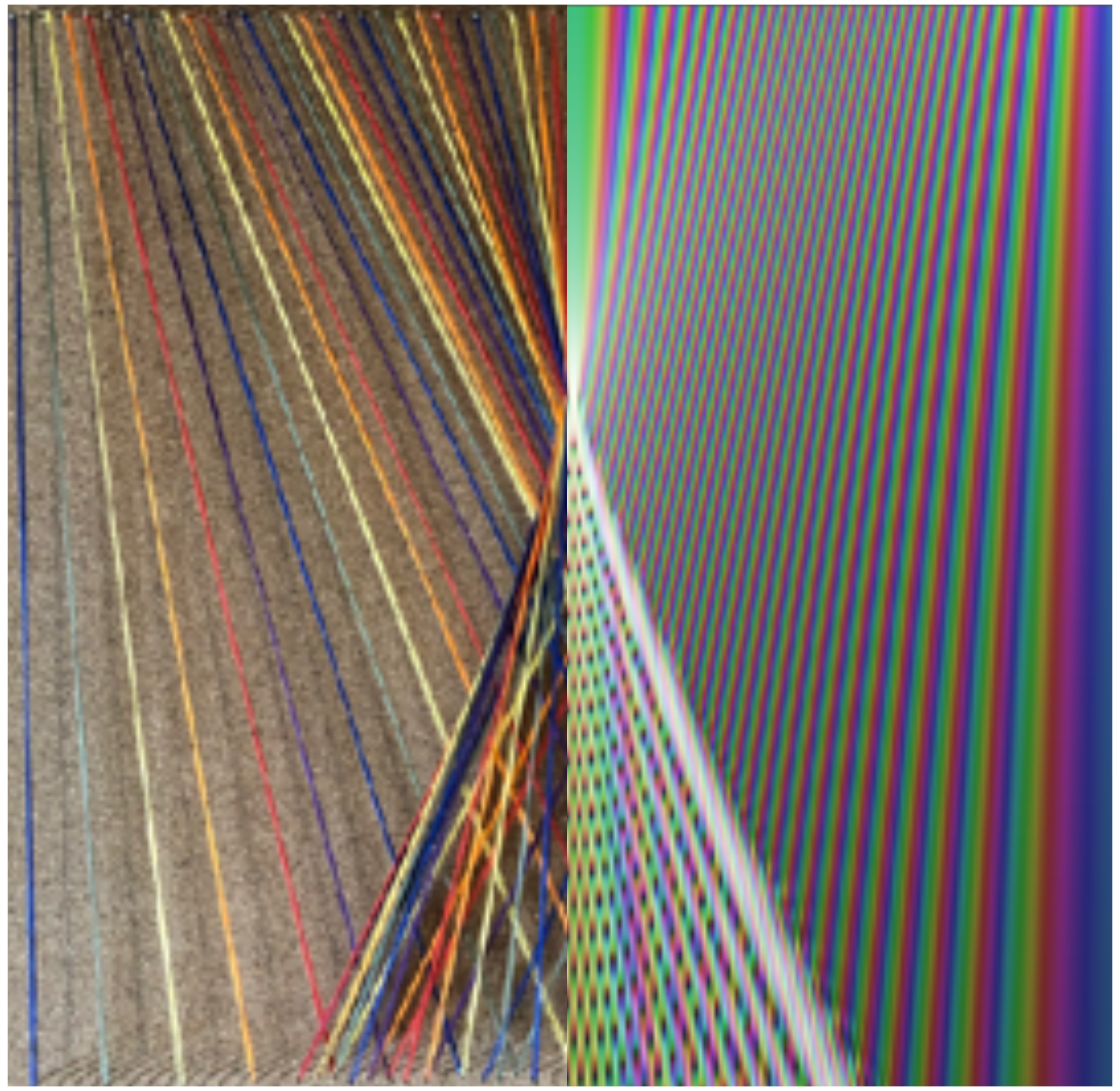
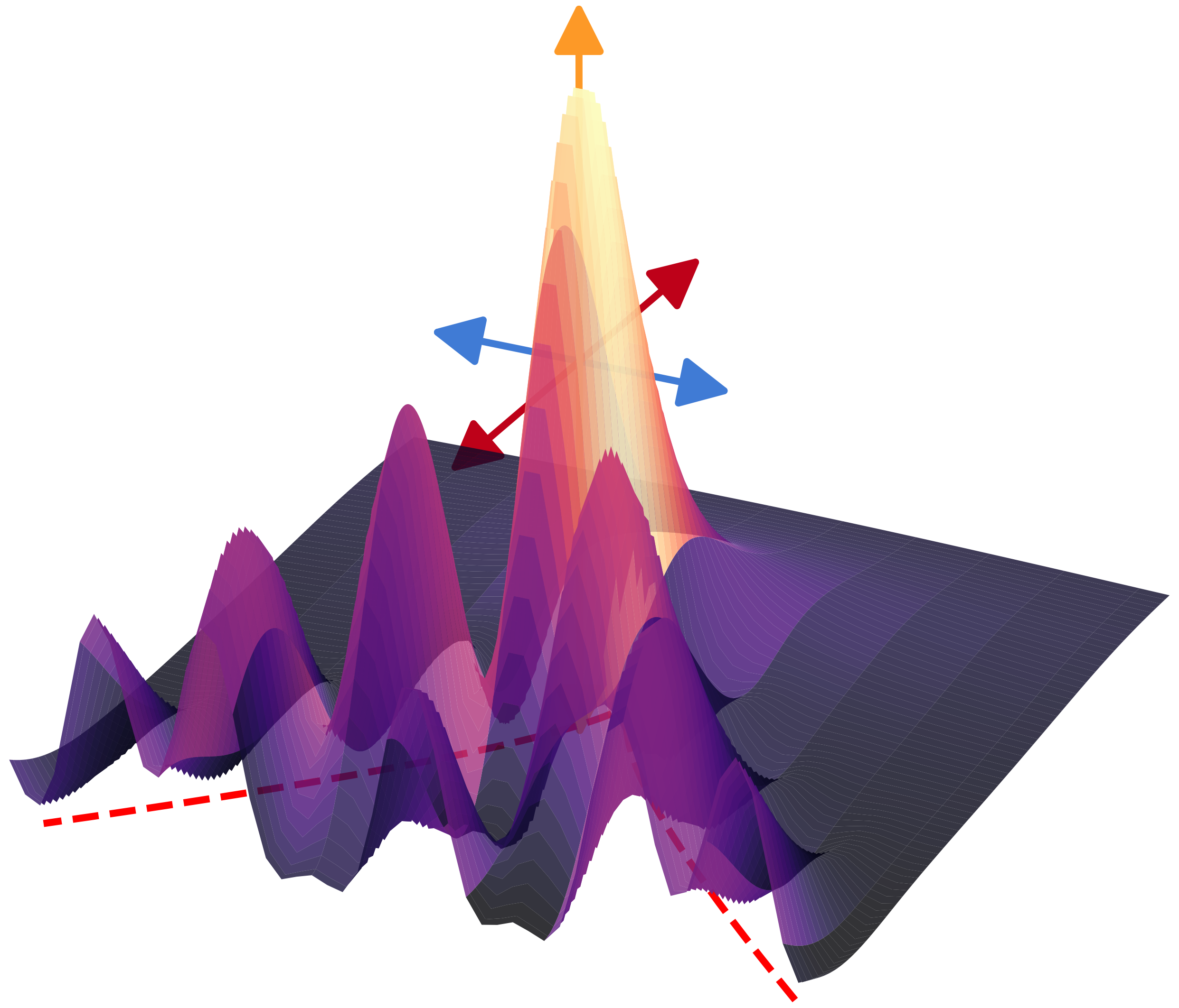
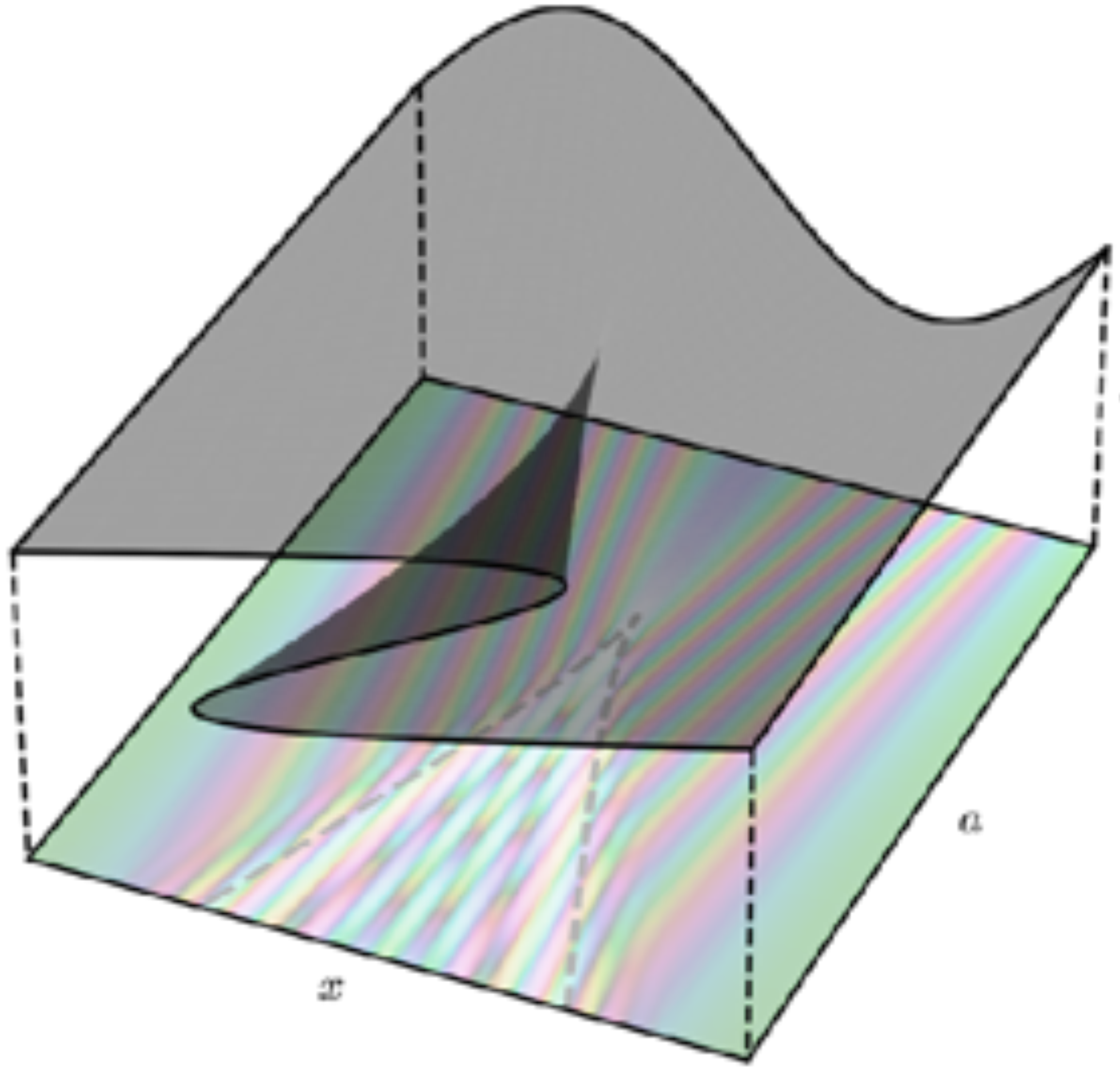
The complexity of cosmic large-scale structure encoded in a single wavefunction

Gough & Uhlemann 2022 (OJAp); (2206.11918)







YTF22

15-16 Dec 2022

Alex Gough

(they/them)

with Cora Uhlemann

Newcastle University

Newcastle University PhD studentships

Applied Mathematics & Theoretical Physics in 2022-2023

https://www.ncl.ac.uk/postgraduate/fees-funding/search-funding/?code=S0000007

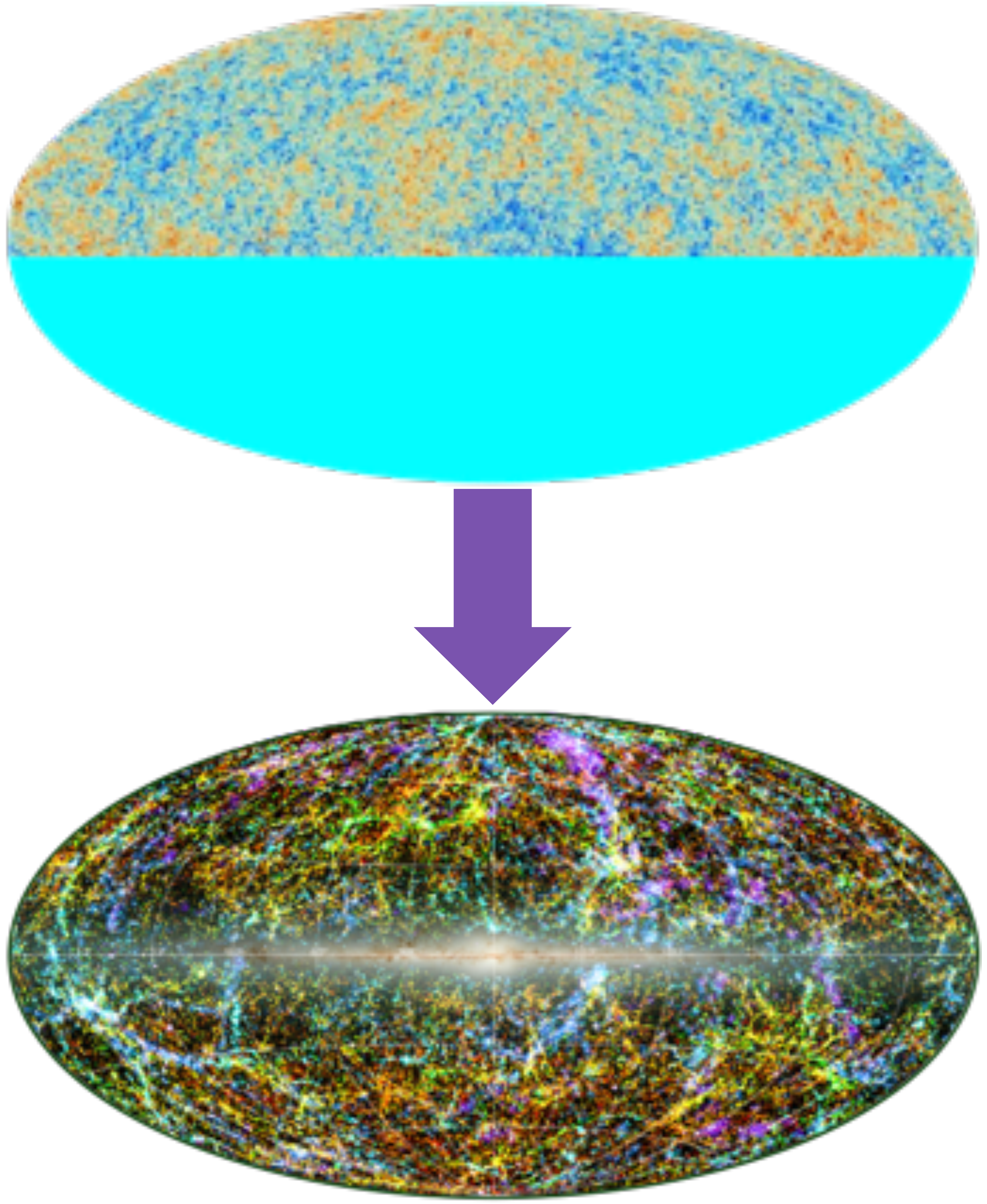
- cosmology & quantum gravity
- observational astronomy
- astro- & geophysical MHD
- quantum matter
- math biology

Robinson Cosmology

NUdata CDT data-intensive astrophysics

STFC & EPSRC for their respective areas

Big questions



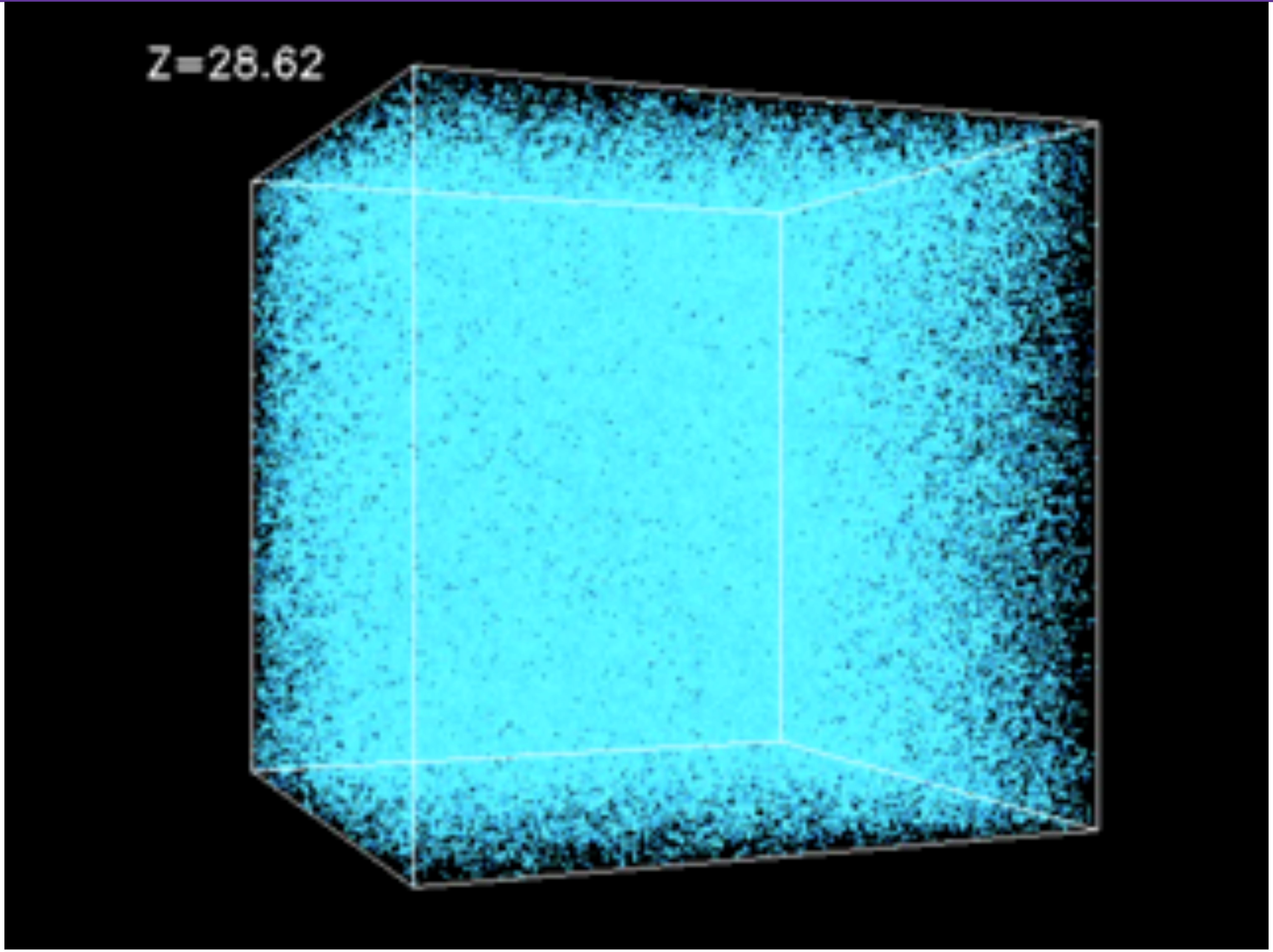
Afterglow
of the early universe

nearly uniform

Cosmic web
of galaxies

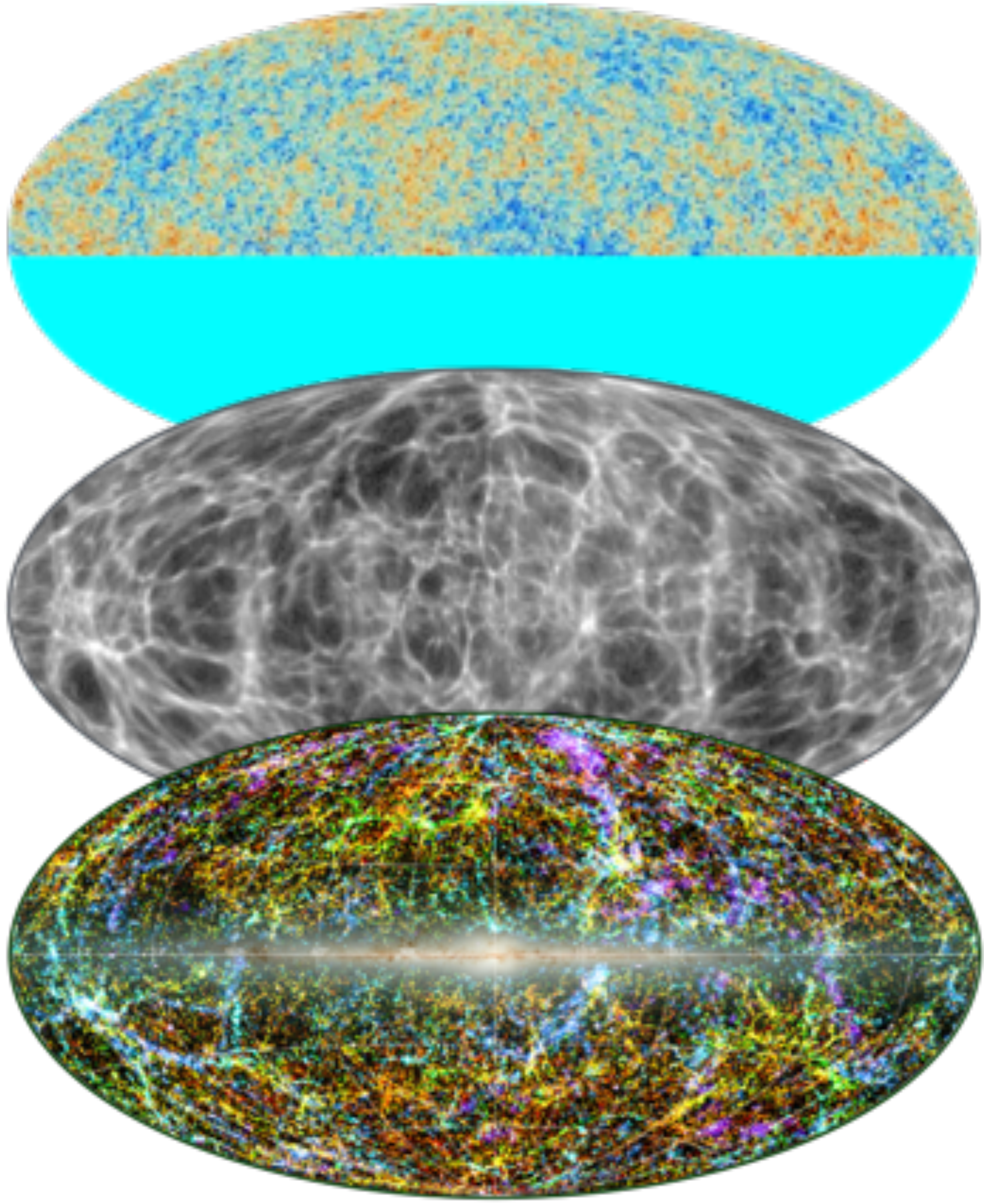
rich structure

Dark matter: a piece of the puzzle



Center for Cosmological Physics (A. Kravtsov & A. Klypin)

Big questions



Afterglow
of the early universe

Skeleton
of dark matter

Cosmic web
of galaxies

nearly uniform

rich structure

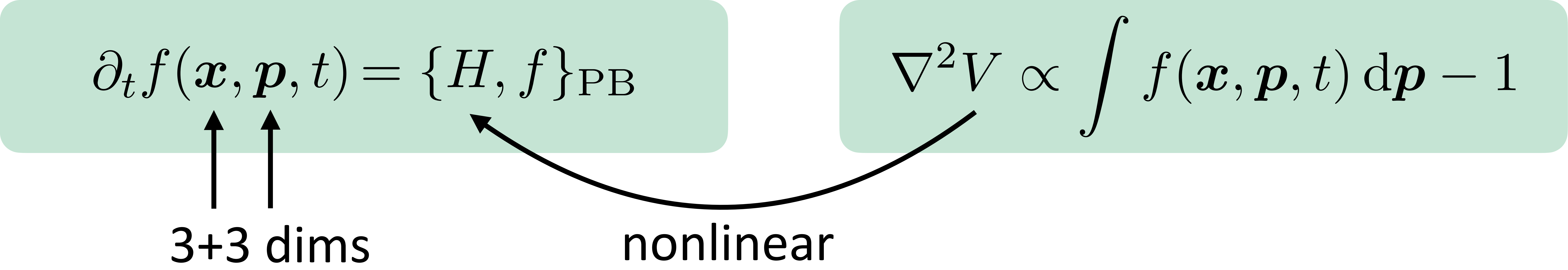
Challenges to modelling

Numerical	**Analytical**
N particles	2 fields
computational power	perturbative fluid
limited sampling	limited features
large-scale accuracy	small-scale accuracy

Key problem

(Cold) Dark Matter Dynamics

Vlasov-Poisson equation (collisionless Boltzmann, long range force)



$$\partial_t f(\boldsymbol{x},\boldsymbol{p},t) = \{H, f\}_{\mathrm{PB}}$$

$$\nabla^2 V \propto \int f(\boldsymbol{x},\boldsymbol{p},t)\,\mathrm{d}\boldsymbol{p} - 1$$

3+3 dims

nonlinear

7 dimensional, non-linear, integro-differential equation…

One nice thing

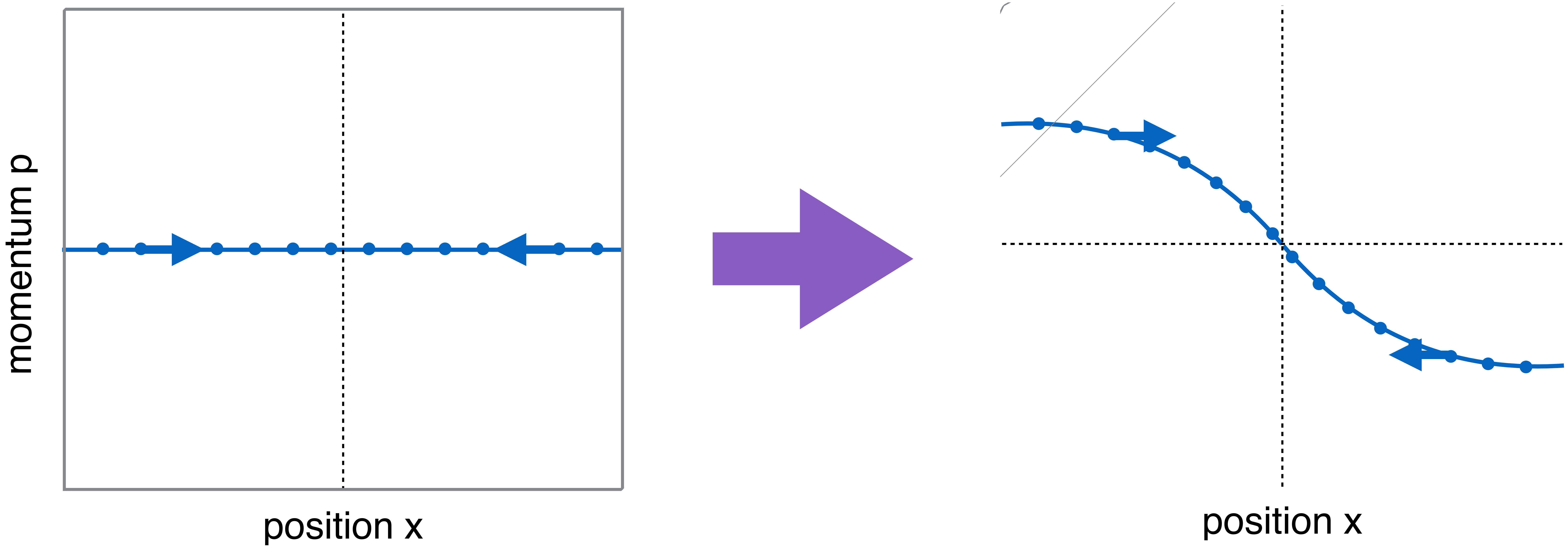
simple “cold” initial conditions: flat sheet

Gravitational collapse

(Cold) Dark Matter Dynamics

Perfect fluid: single stream

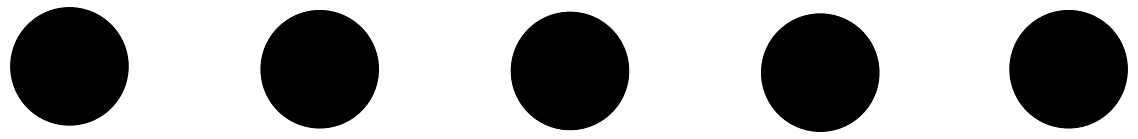
$$f_{\text{fluid}}(\boldsymbol{x},\boldsymbol{p}) = \rho(\boldsymbol{x})\delta_D(\boldsymbol{p} - m\boldsymbol{\nabla}\phi(\boldsymbol{x}))$$



How do waves come in?

Cold Dark Matter

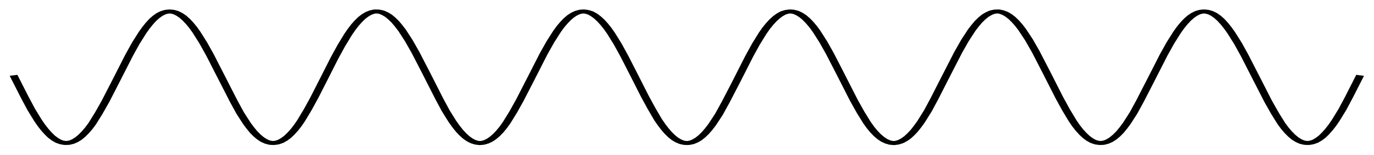
Particles/Fluids



2 fields: density & velocity

Wave Dark Matter

Waves



1 wavefunction: $\psi(\boldsymbol{x})$

$$\psi = \sqrt{\rho}\exp[i\phi/\hbar]$$

$$\rho(\boldsymbol{x}) = |\psi(\boldsymbol{x})|^2$$

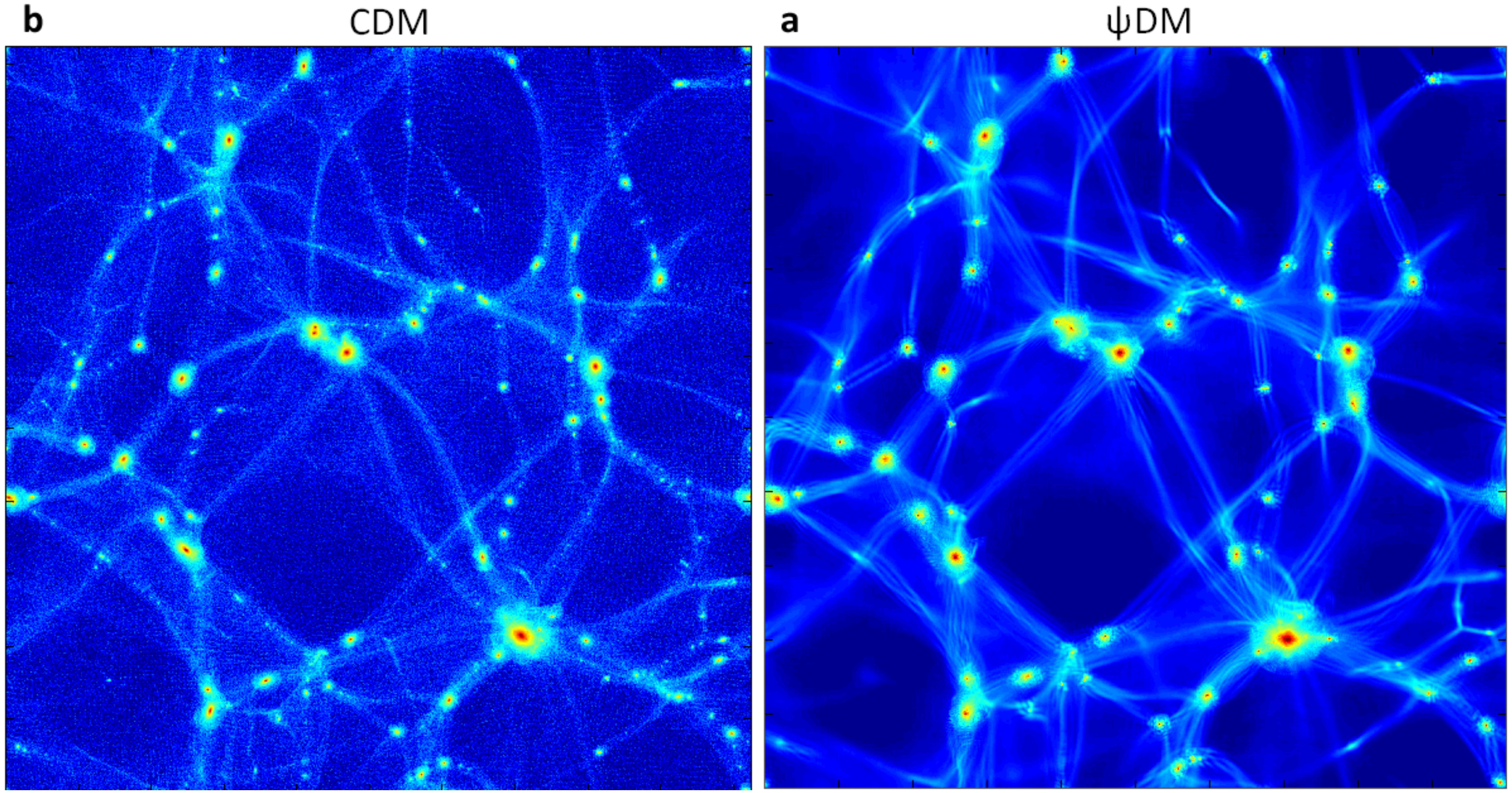
$$\boldsymbol{v} = \frac{i\hbar}{2}\frac{\psi\boldsymbol{\nabla}\bar{\psi} - \bar{\psi}\boldsymbol{\nabla}\psi}{|\psi|^2} (= \boldsymbol{\nabla}\phi)$$

not necessarily potential

Wave dark matter

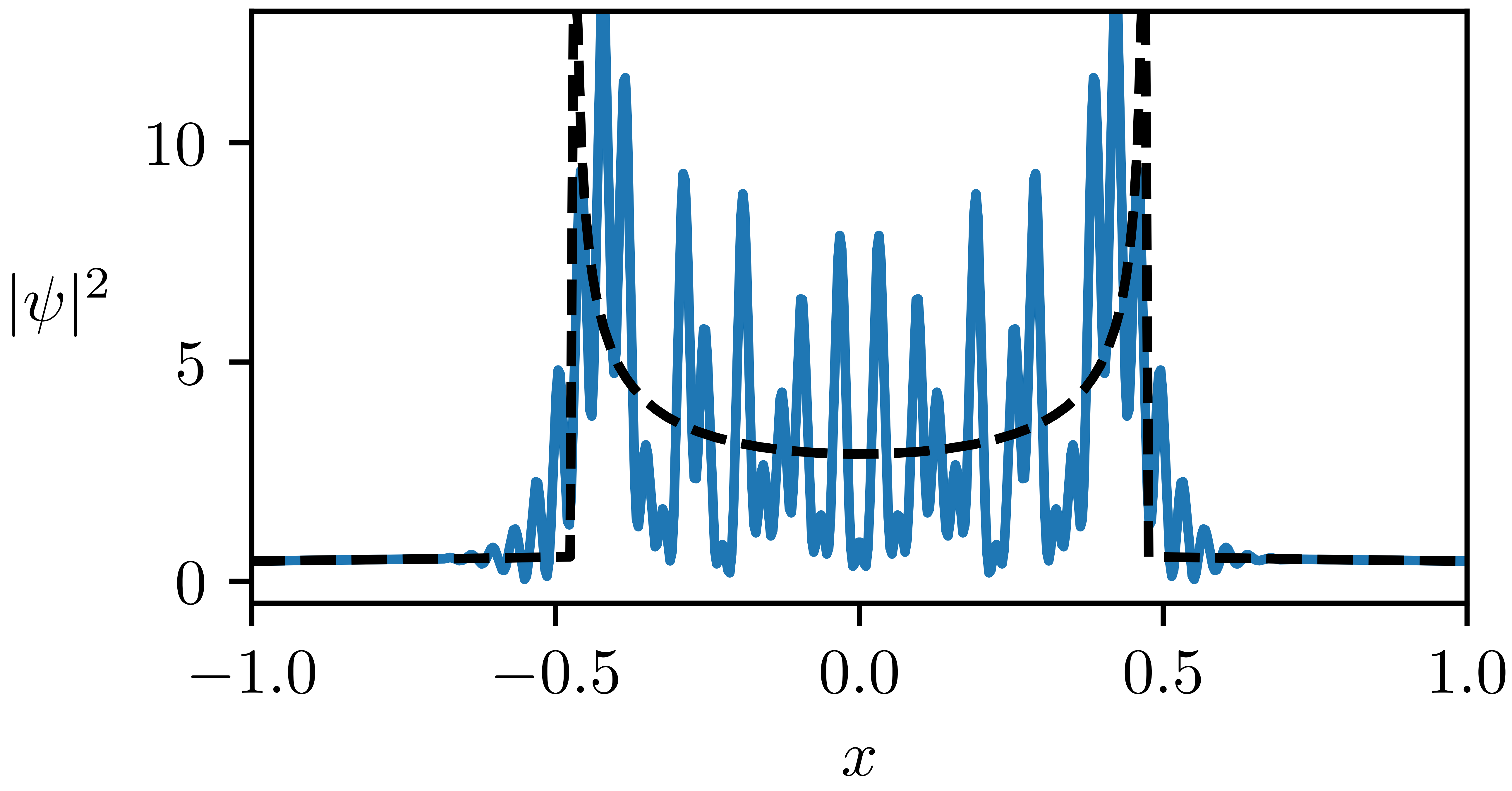
Spot the difference

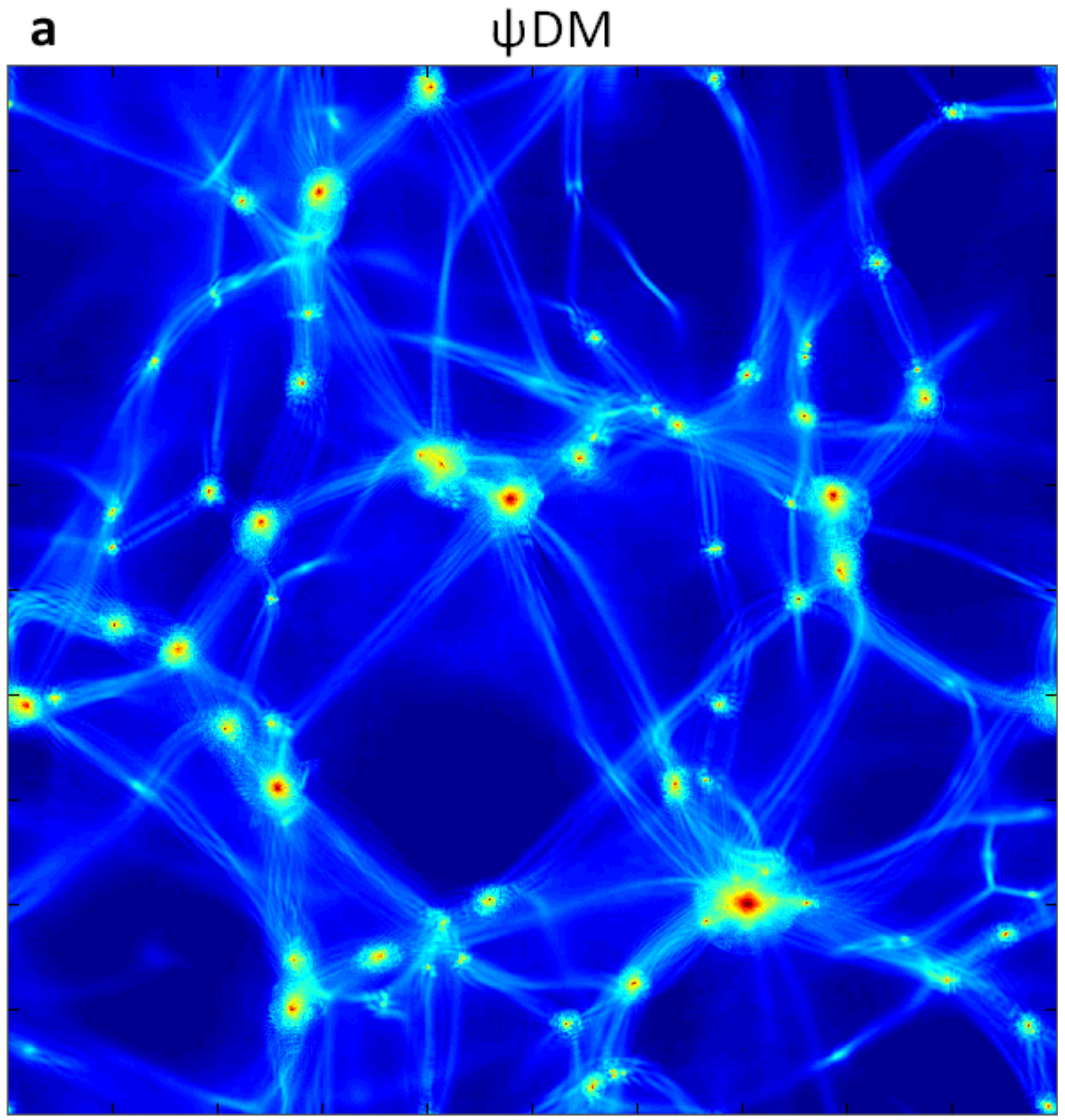
- Same large scale network as CDM
- Wave interference “decorates” the cosmic web



Schive ++ Nature Phys. Lett, `15
astrophysical imprints: Hui, Ostriker, Tremaine & Witten `17, Hui `21

Wave dark matter





Schive ++ Nature Phys. Lett, `15

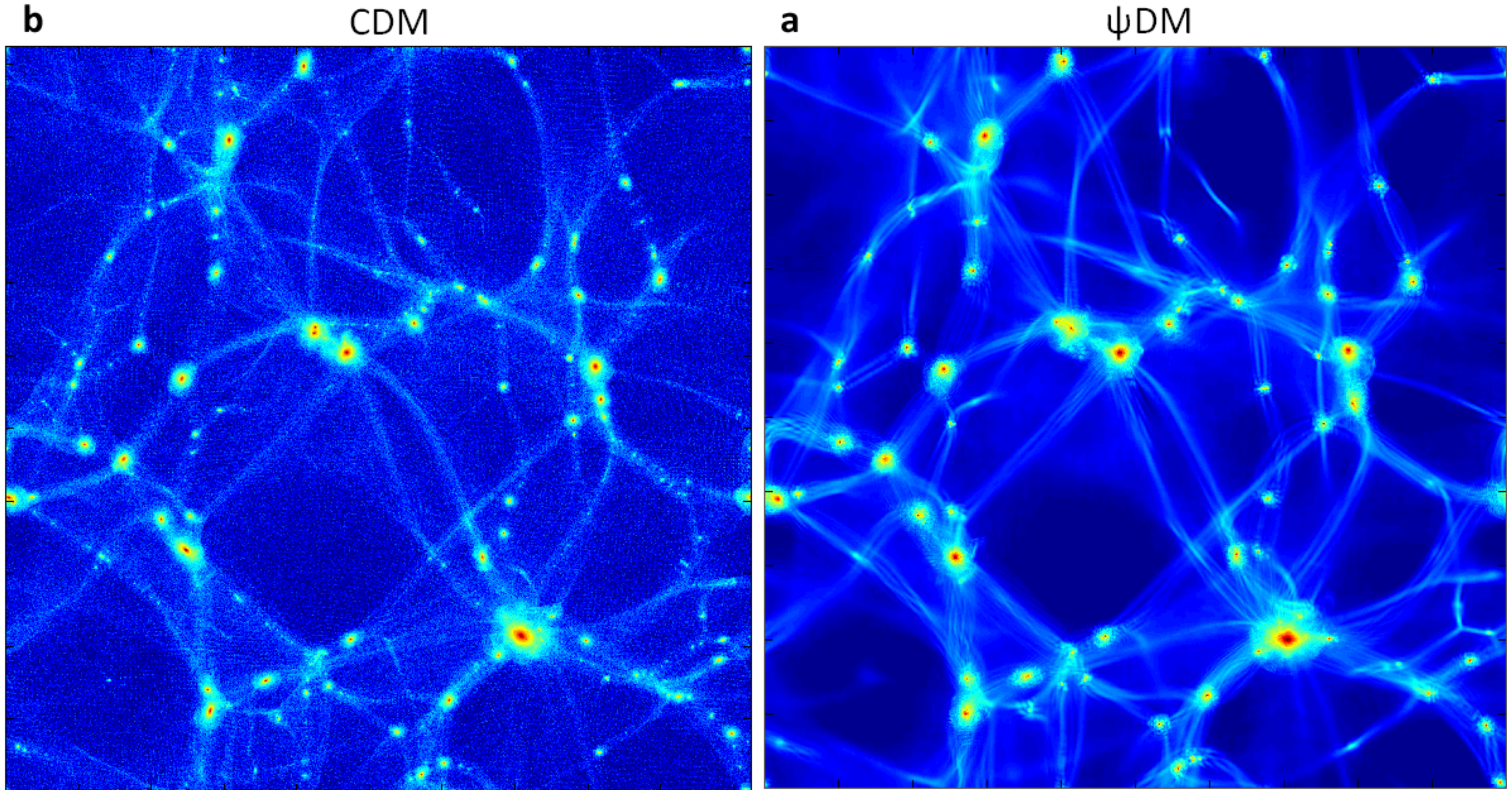
astrophysical imprints: Hui, Ostriker, Tremaine & Witten `17, Hui `21

Wave dark matter

Why do we care?

- True wavelike dark matter (e.g. axions)

- Rich phenomenology
- Universal features (tool even for CDM)



Schive ++ Nature Phys. Lett, `15

astrophysical imprints: Hui, Ostriker, Tremaine & Witten `17, Hui `21

Simple models

Wave Dark Matter

Waves

Evolution: Propagation

Simple model: free Sch

$$i\hbar\partial_a\psi = -\frac{\hbar^2}{2}\nabla^2\psi$$

$$\hbar \sim \frac{\hbar^{\text{phys}}}{m}$$

semiclassical parameter

Cold Dark Matter

Particles

Evolution: Displacement

Simple model: Zel'dovich approximation

$$\boldsymbol{x} = \boldsymbol{q} - a\boldsymbol{\nabla}\varphi_g^{(\text{ini})}$$

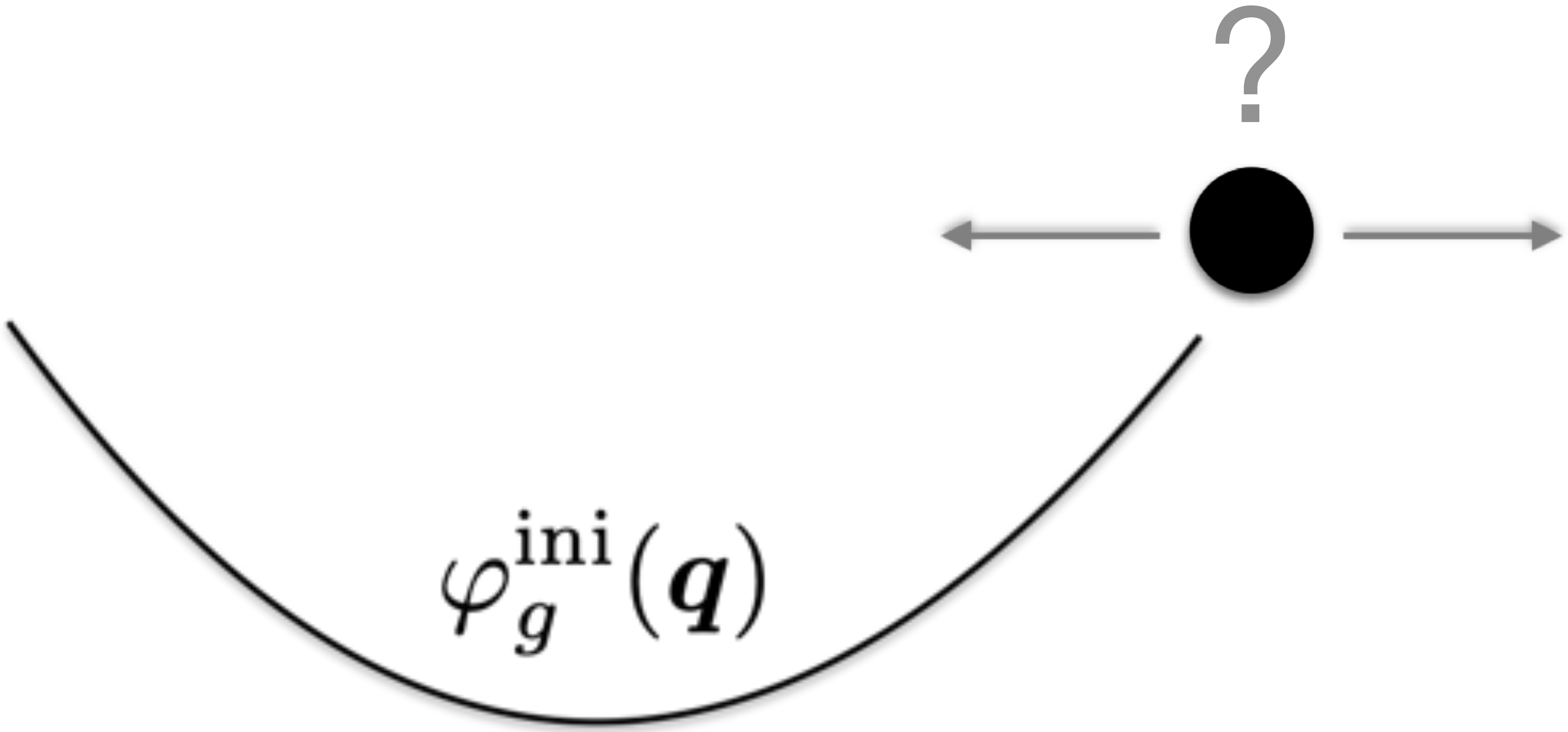
Widrow & Kaiser APJ `93
Coles `02, Uhelmann ++ `19

Simple models

Approximate: shoot particles following initial potential

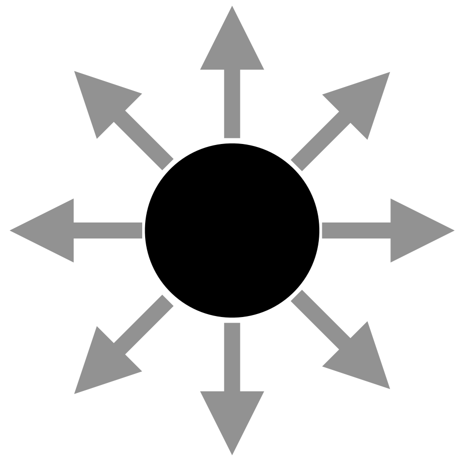
$$\boldsymbol{v}(\boldsymbol{q}, a) = -\boldsymbol{\nabla}\varphi_g^{(\mathrm{ini})}(\boldsymbol{q})$$

$$\boldsymbol{x}(\boldsymbol{q}, a) = \boldsymbol{q} - a\boldsymbol{\nabla}\varphi_g^{(\mathrm{ini})}(\boldsymbol{q})$$ Zel’dovich approximation*

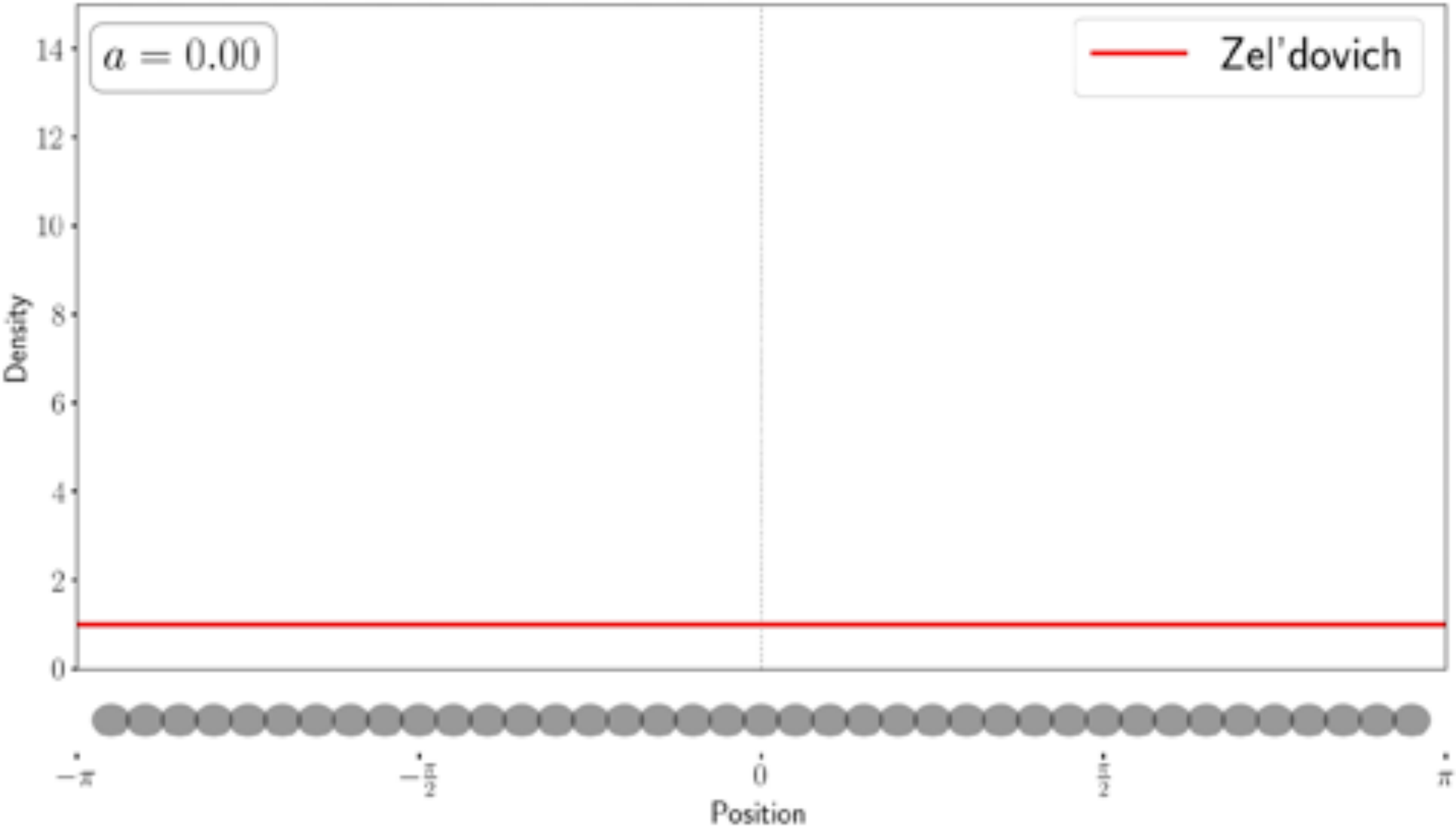


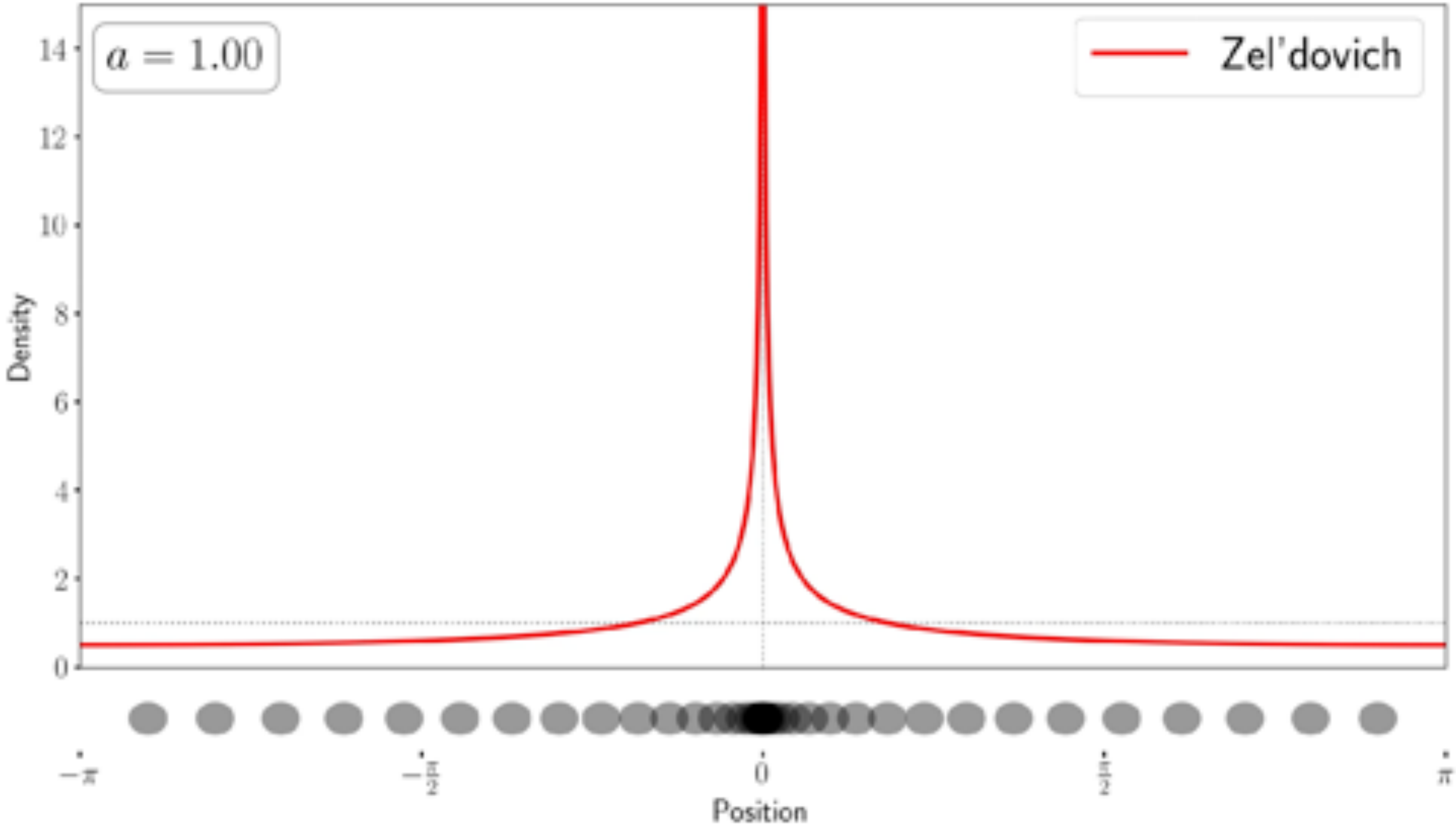
*exact in 1D before shell crossing

*(Lagrangian) perturbation theory:
ZA + tidal effects



Zel’dovich A&A 1970


$a = 0.00$
Zel'dovich
Density
0
2
4
6
8
10
12
14
$-\pi$
$-\frac{\pi}{2}$
0
$\frac{\pi}{2}$
π
Position


$a = 1.00$
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Density
14
12
10
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6
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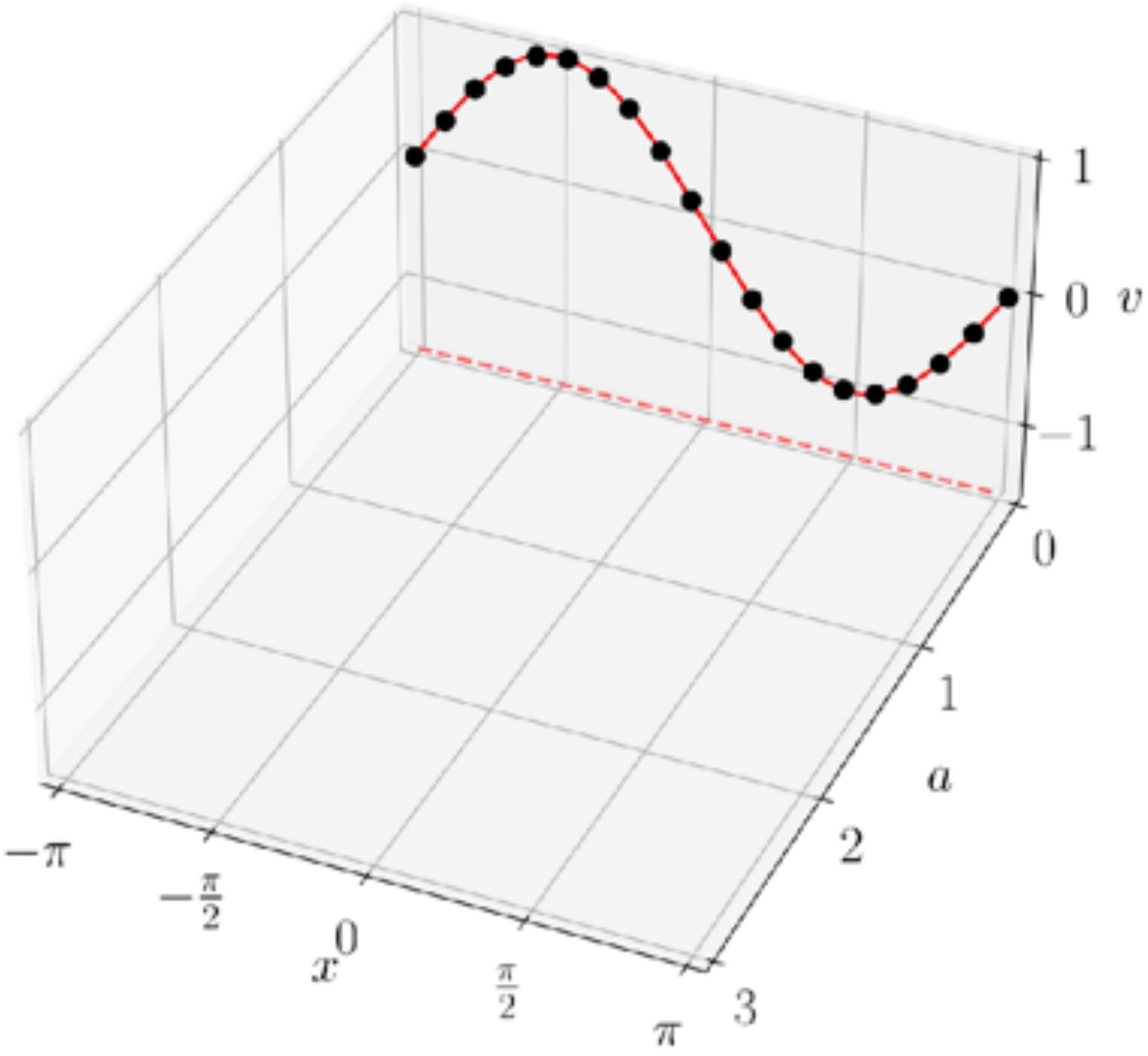
2
0
$-\pi$
$-\frac{\pi}{2}$
0
$\frac{\pi}{2}$
π
Position

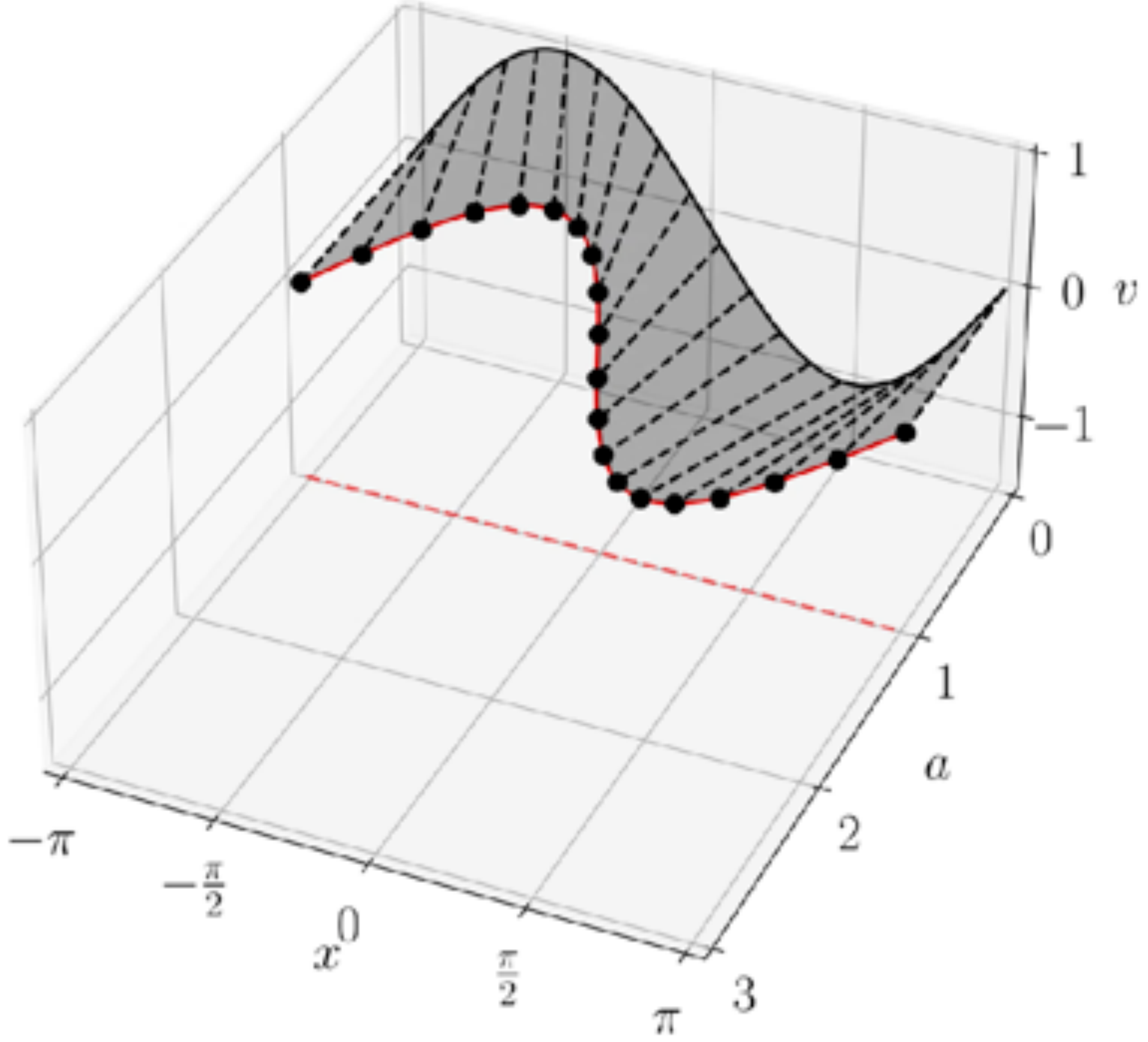
Multi-streaming



animation on wikimedia commons

Gough & Uhlemann 2022

Multi-streaming



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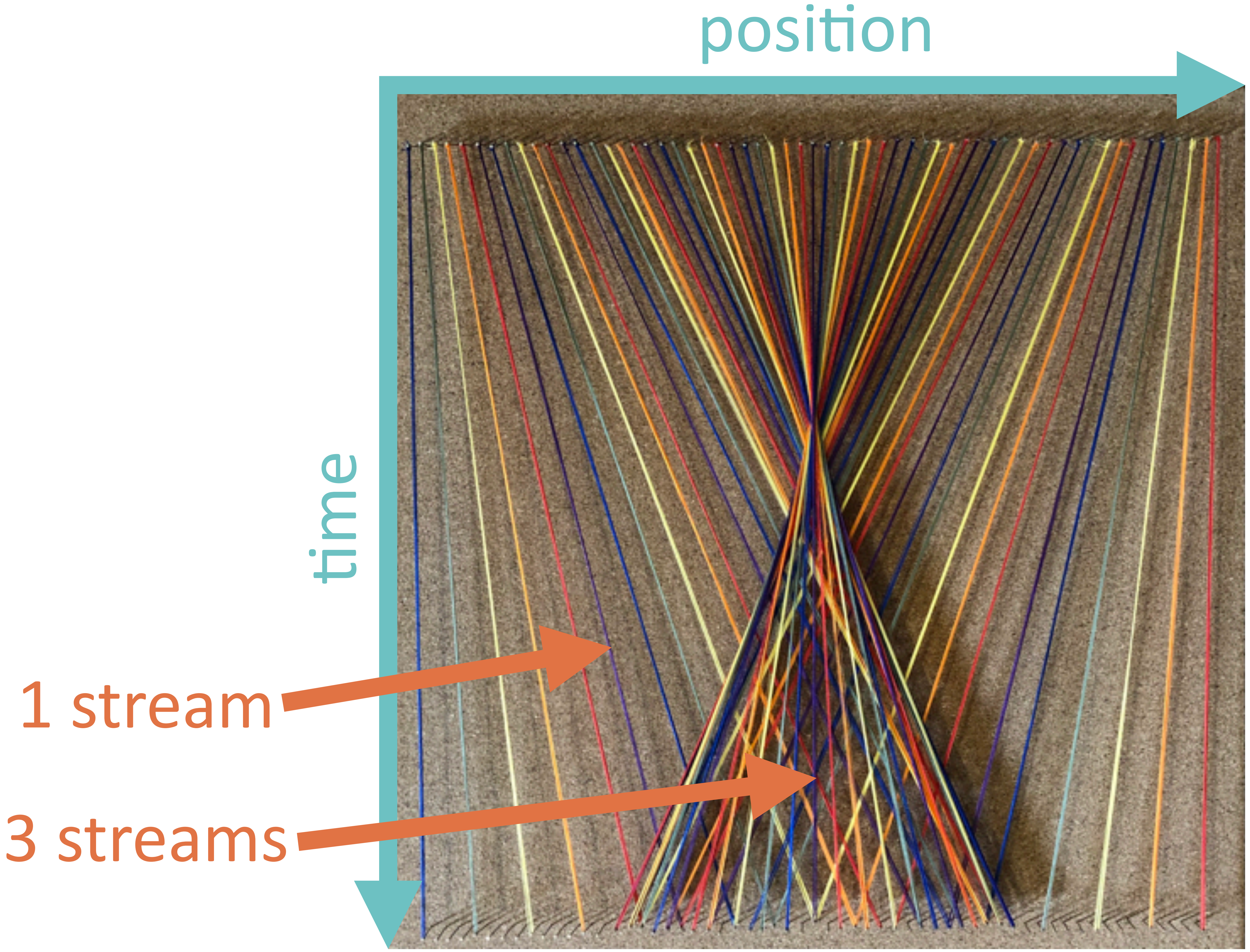
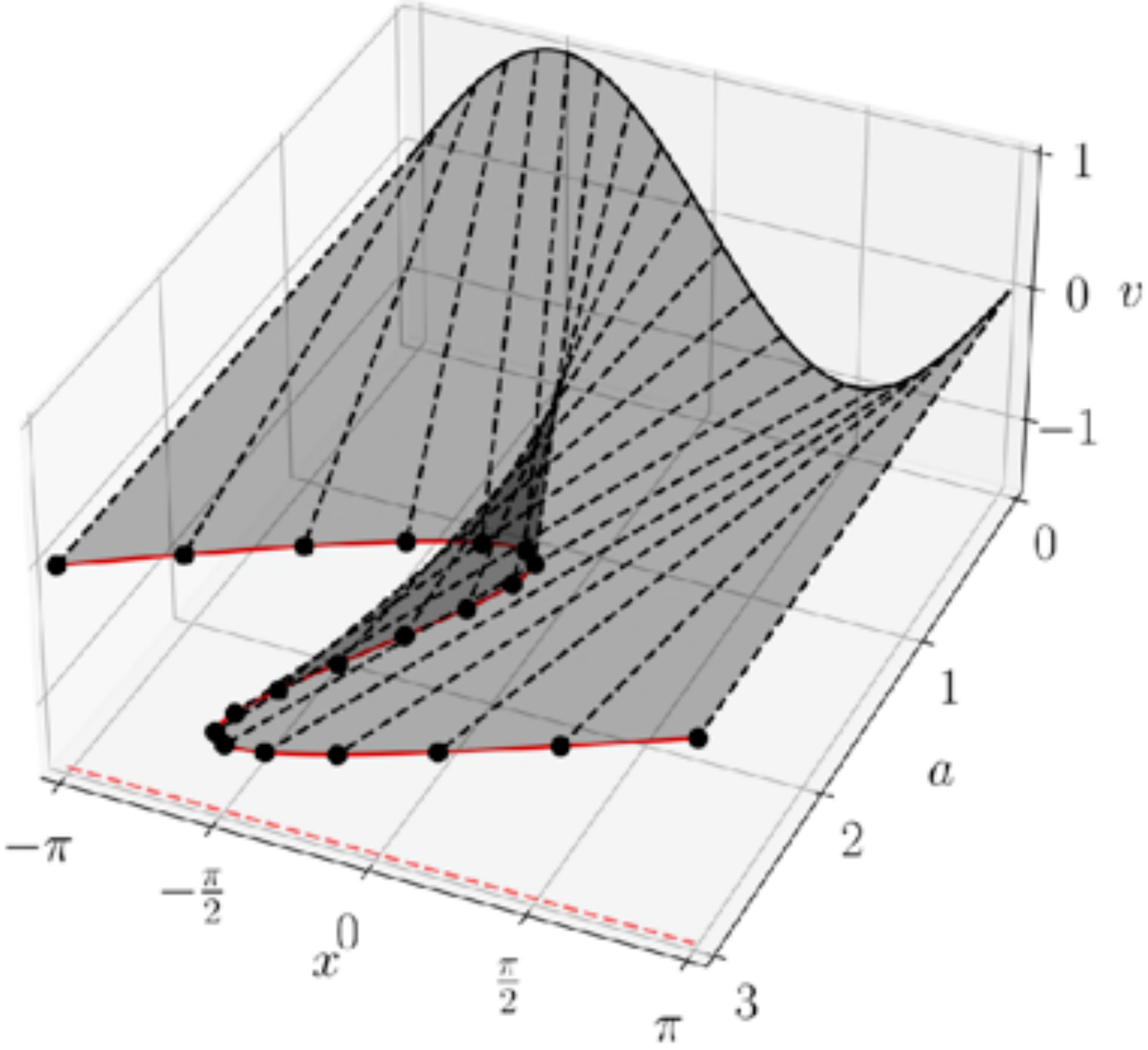
Gough & Uhlemann 2022

Multi-streaming





ZA produces multi-streaming, no secondary infall

animation on wikimedia commons
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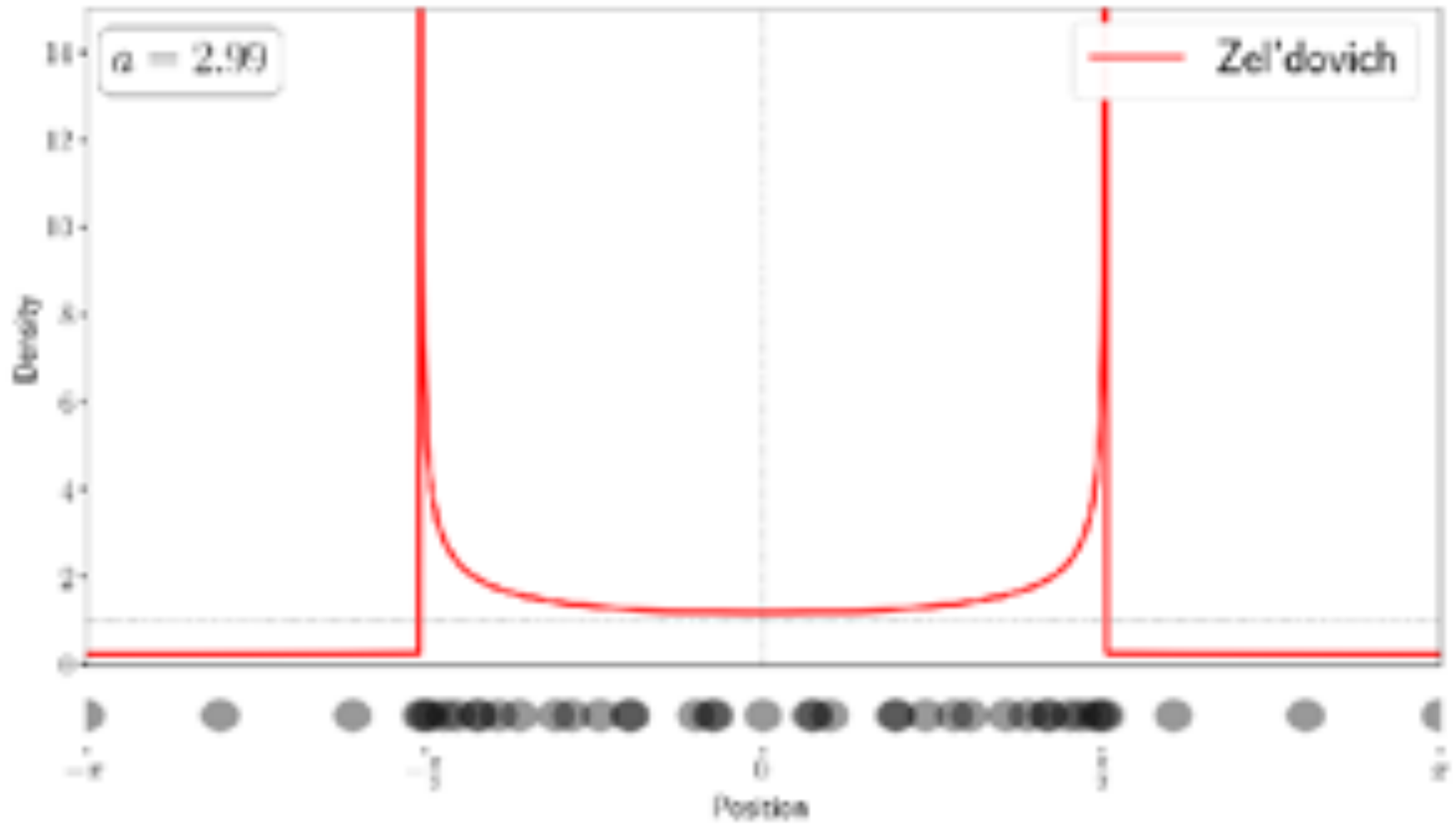
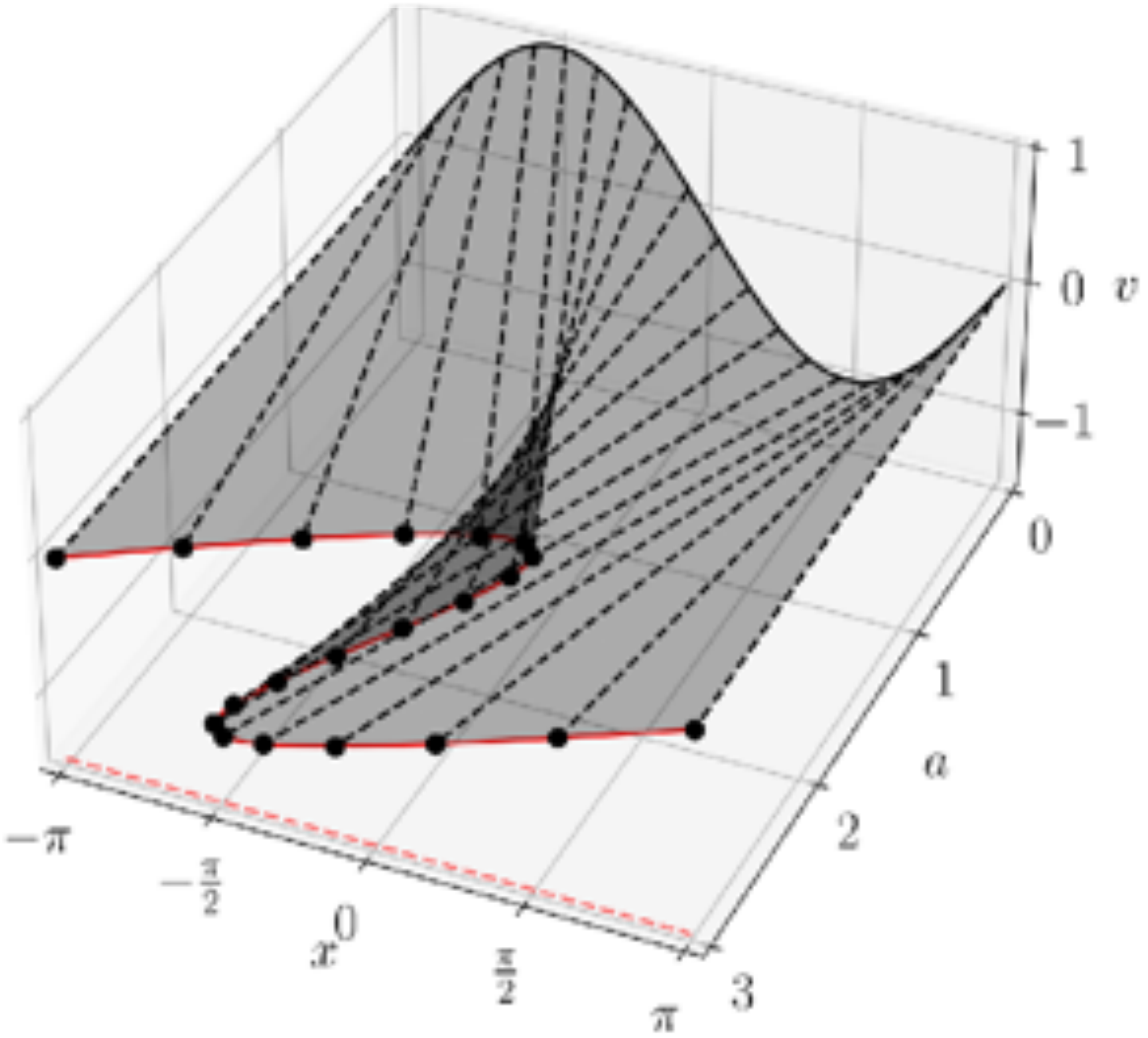
Particles to waves

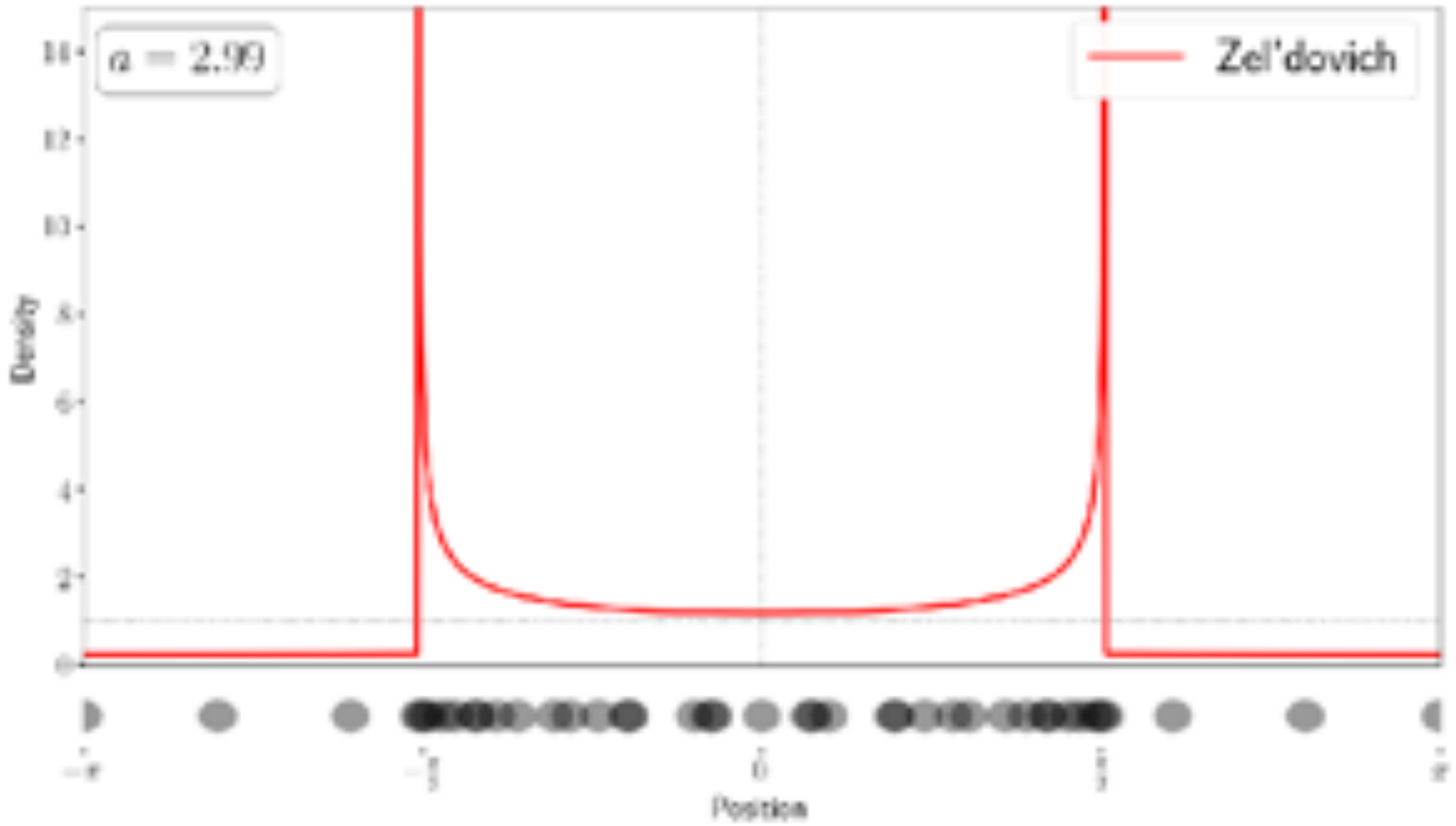


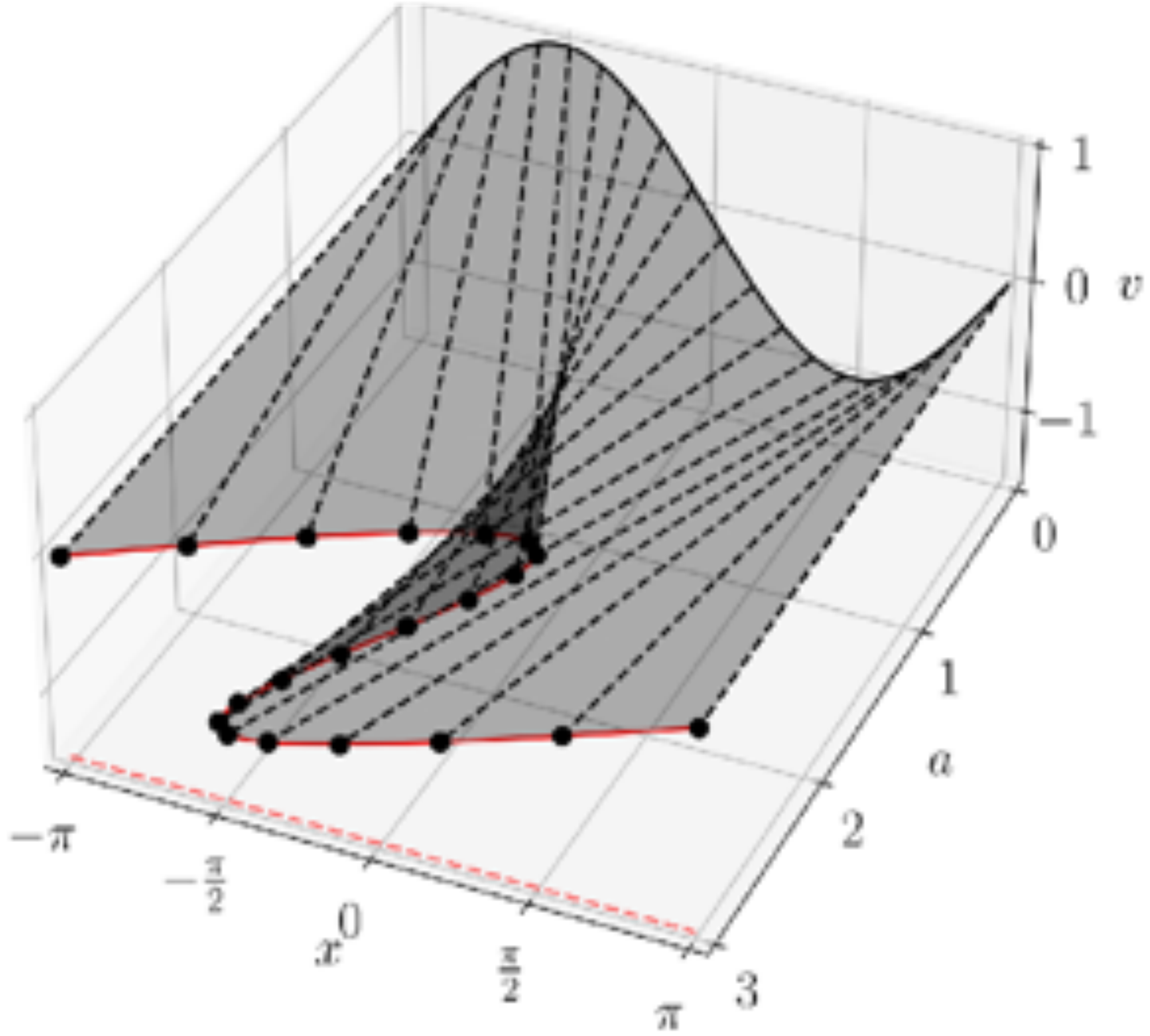


?

$$\psi(x, a)$$

Particles to waves





Initial conditions

$$\psi = \sqrt{\rho} e^{i\phi_v/\hbar}$$

Fluid variables

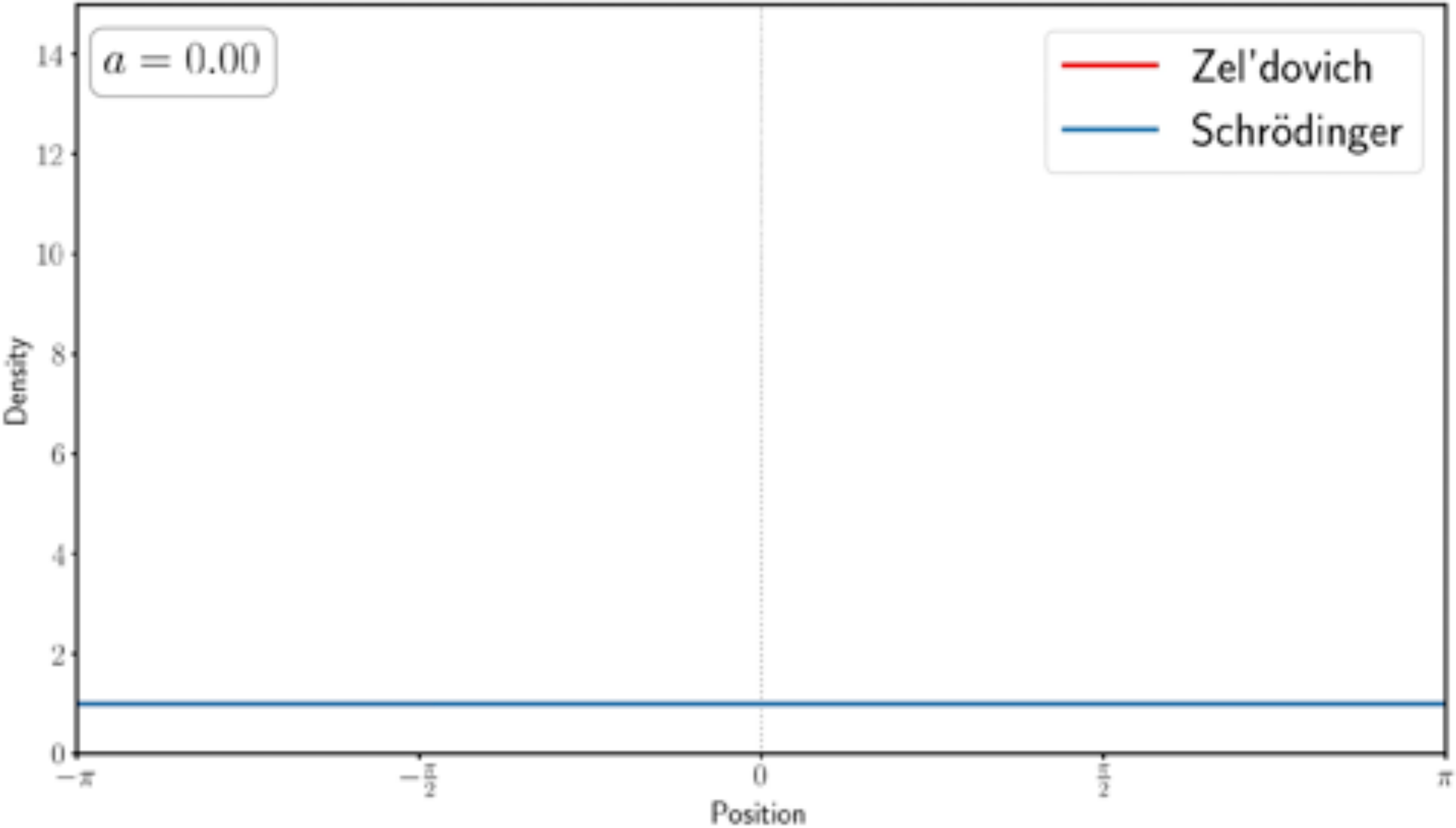
Uniform density

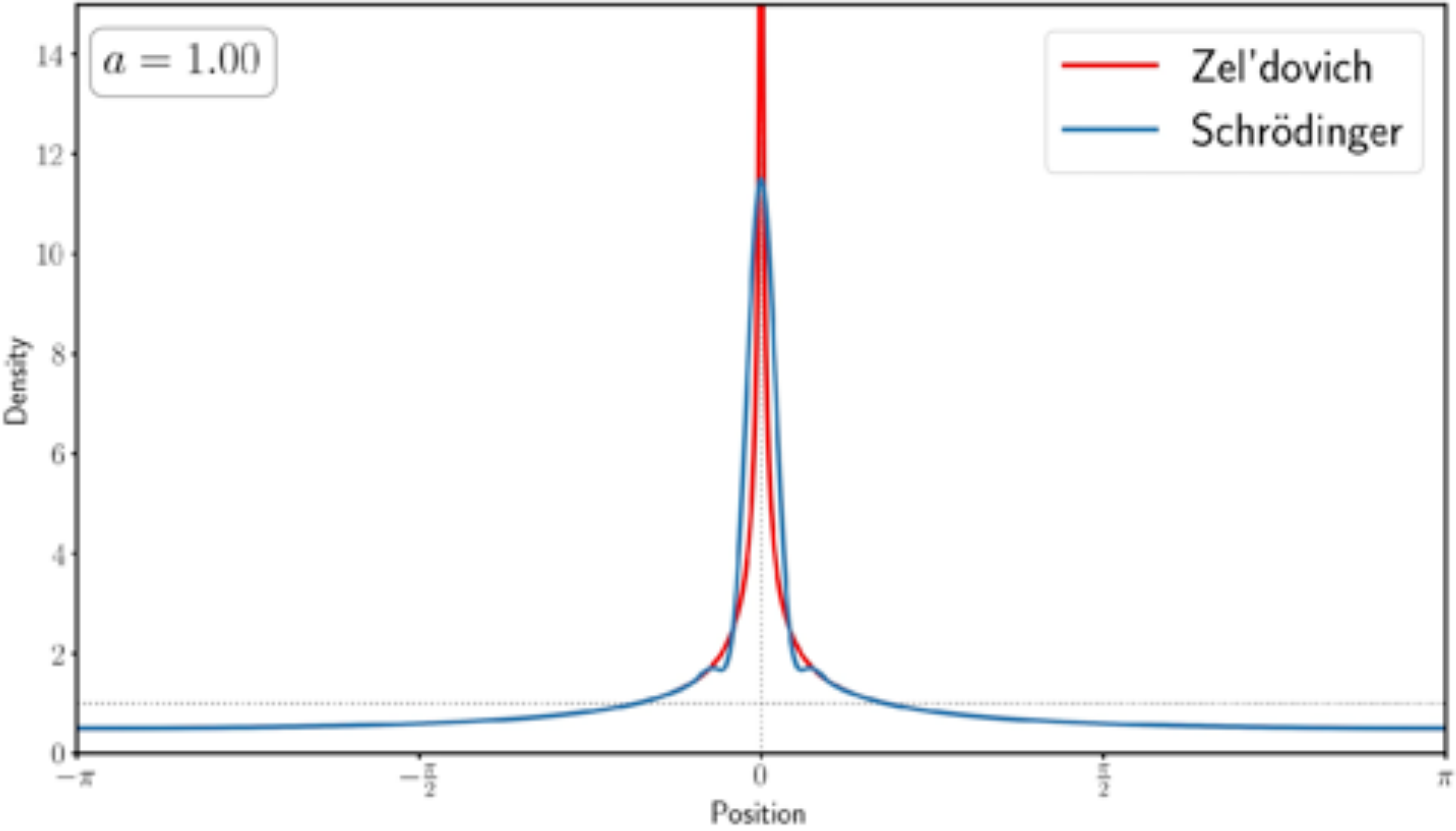
Sinusoid velocity

$$\psi^{(\mathrm{ini})}(q) = \exp\left(\frac{i}{\hbar}\cos(q)\right)$$

$$i\hbar\partial_a\psi = -\frac{\hbar^2}{2}\partial_x^2\psi$$

Toy Model


$a = 0.00$
Zel'dovich
Schrödinger
Density
14
12
10
8
6
4
2
0
$-\pi$
$-\frac{\pi}{2}$
0
$\frac{\pi}{2}$
π
Position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Zel'dovich
Schrödinger
Density
Position
14
12
10
8
6
4
2
0
$-\pi$
$-\frac{\pi}{2}$
0
$\frac{\pi}{2}$
π

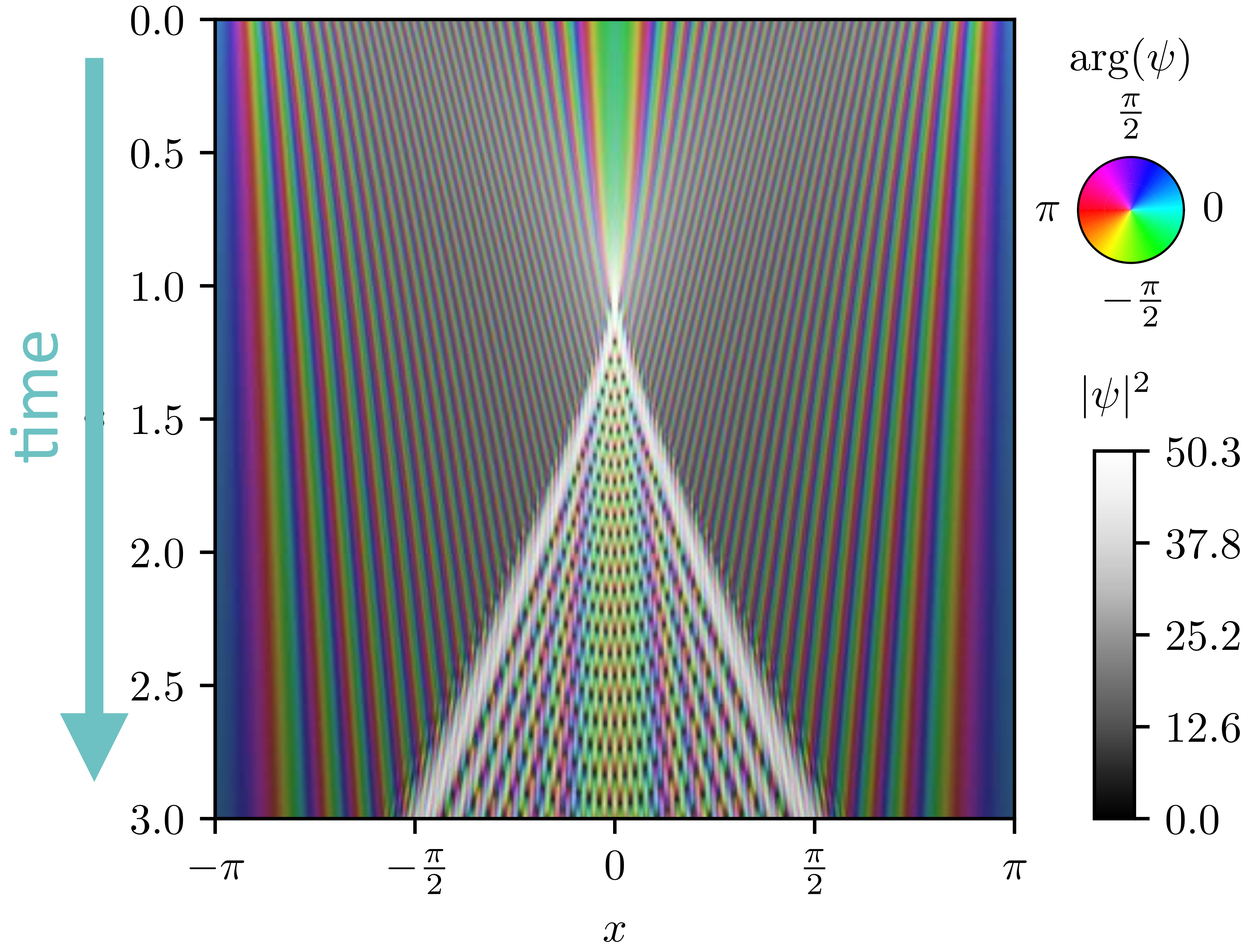
Free wave evolution

Amplitude: brightness

Phase: colour

Features

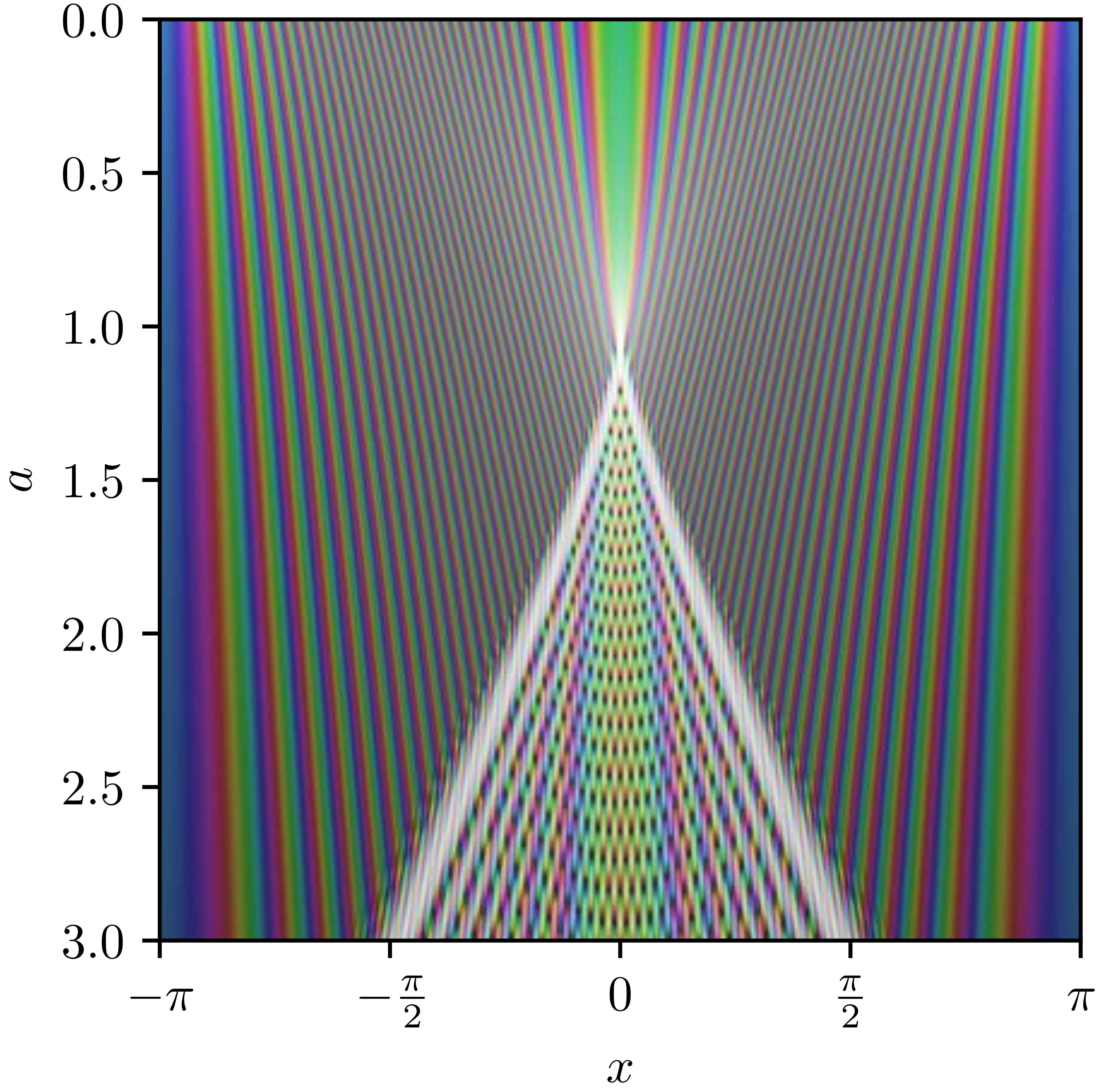
- Interference
  - what is interfering?
- Regularised caustic
  - how bright?
  - how wide?



Uhlemann++ PRD 2019
Gough & Uhlemann 2022

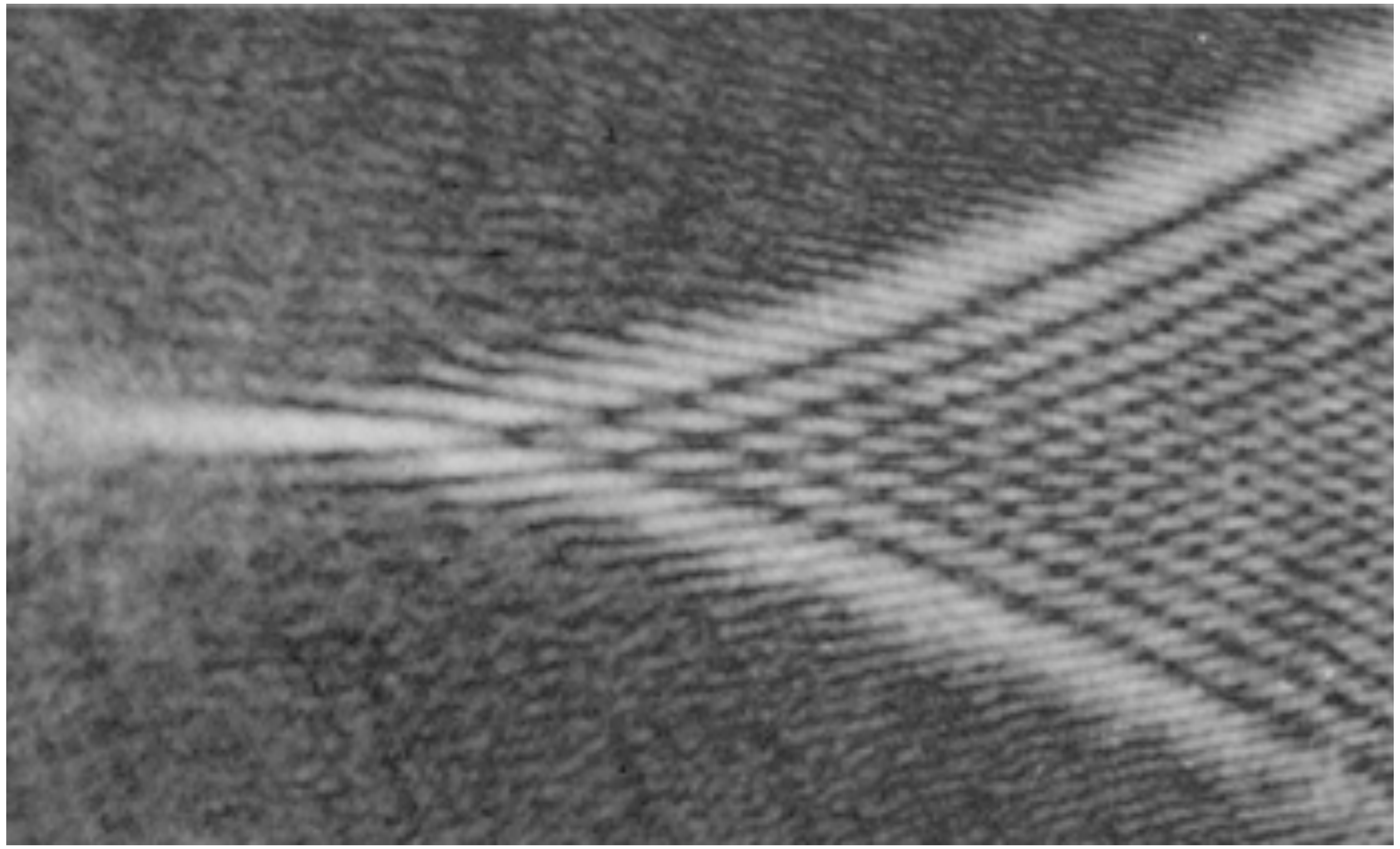
Optics analogy

Dark matter



🤔

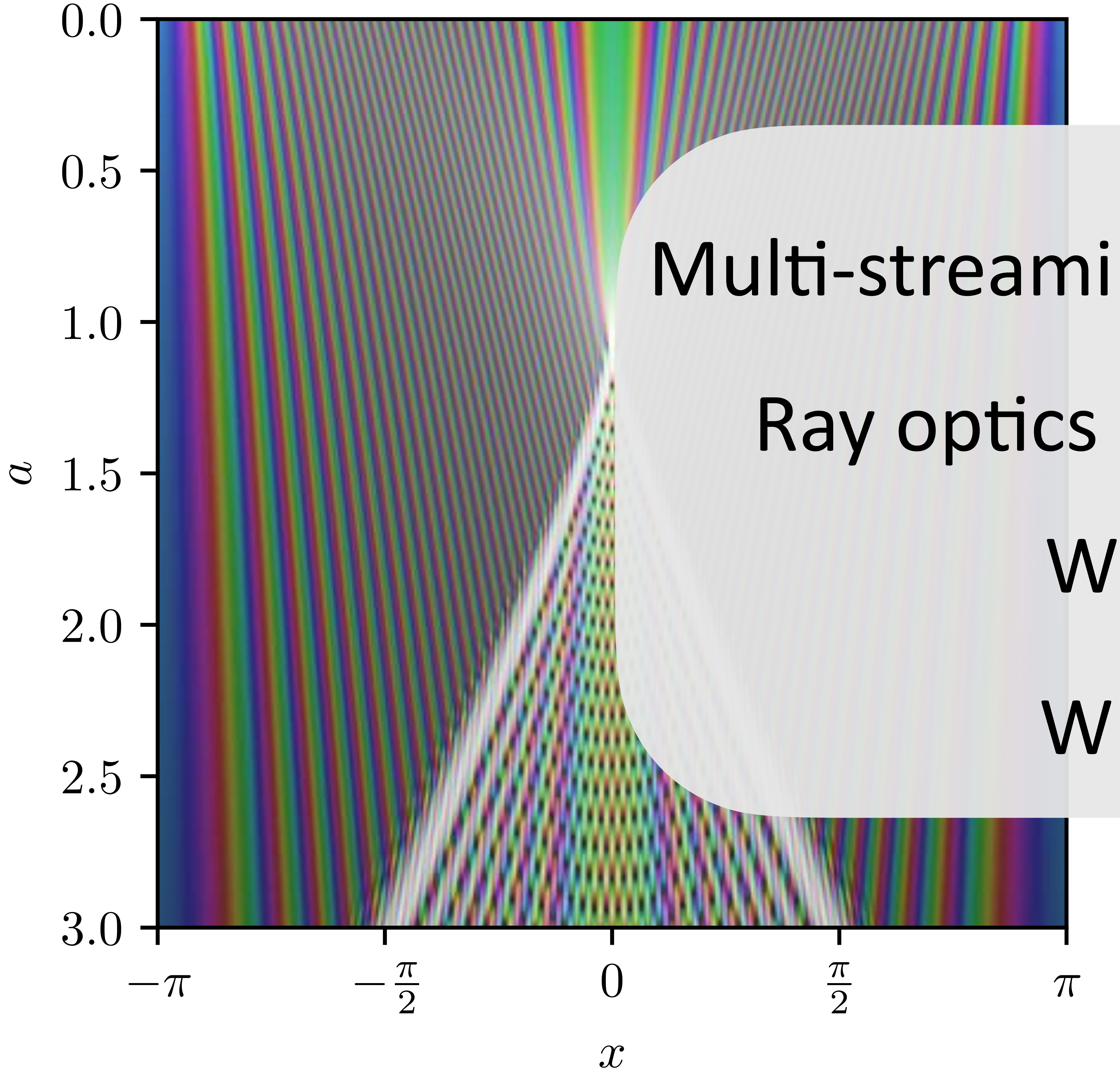
Optics



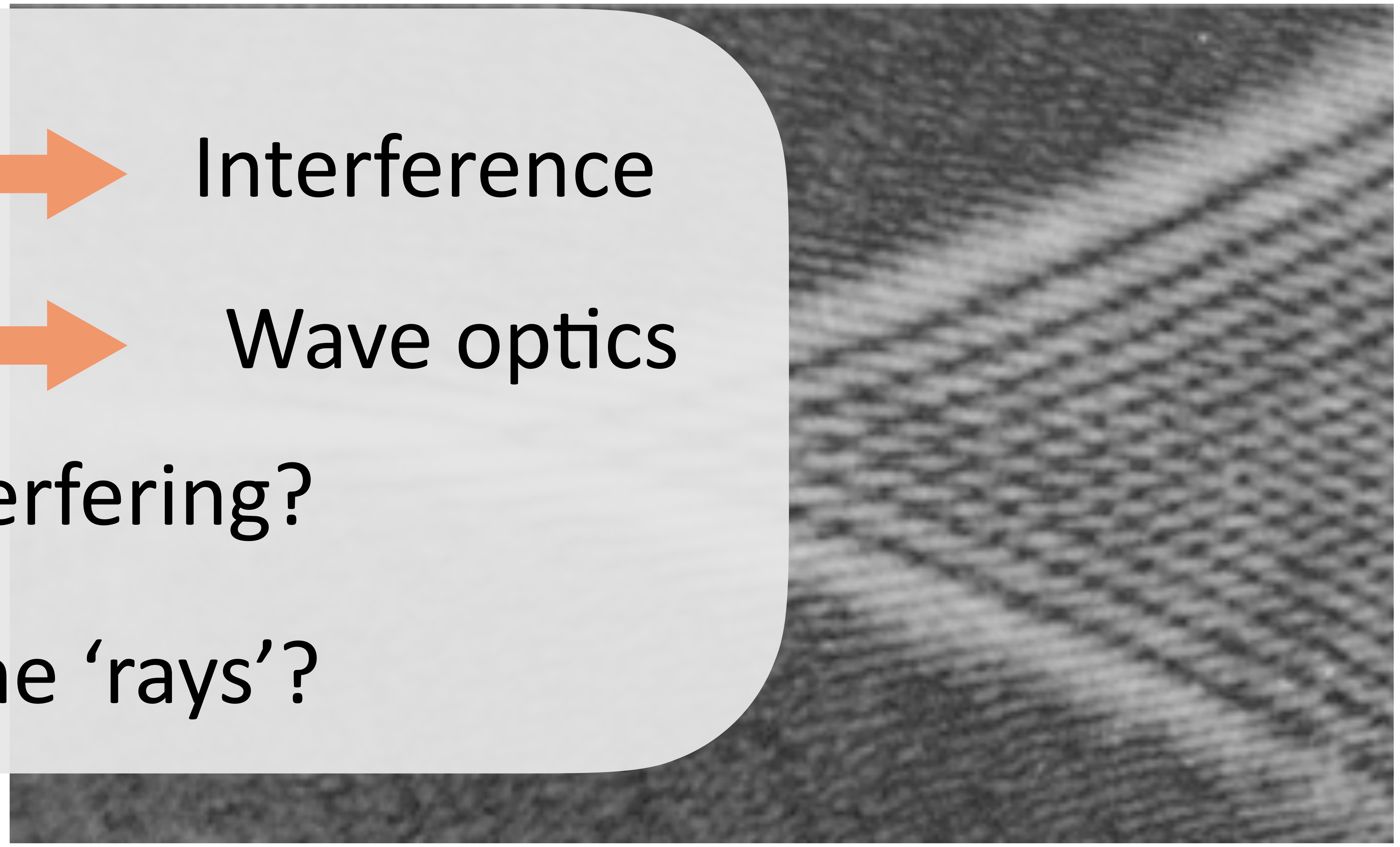
Berry, Nye, Wright `79

Optics analogy

Dark matter



Optics



Multi-streaming ⟷ Interference

Ray optics ⟷ Wave optics

What is interfering?

What are the ‘rays’?

Propagator formalism

Solving the wavefunction

- Useful to write solution in certain form

initial position *transition amplitude*

$$\psi(x,a) \sim \int \mathrm{d}q\, \underbrace{K_0(q;x,a)\psi^{(\mathrm{ini})}(q)}_{\exp\left[\frac{i}{\hbar}\zeta(q;x,a)\right]}$$

- $\zeta(q;x,a)$ contains the *action* and the *initial conditions*

$$\psi(x,a) \sim \int \mathrm{d}q\, \exp\left(\frac{i}{\hbar}\left[S_0(q;x,a) + \varphi_g^{(\mathrm{ini})}(q)\right]\right)$$

Unweaving the wavefunction

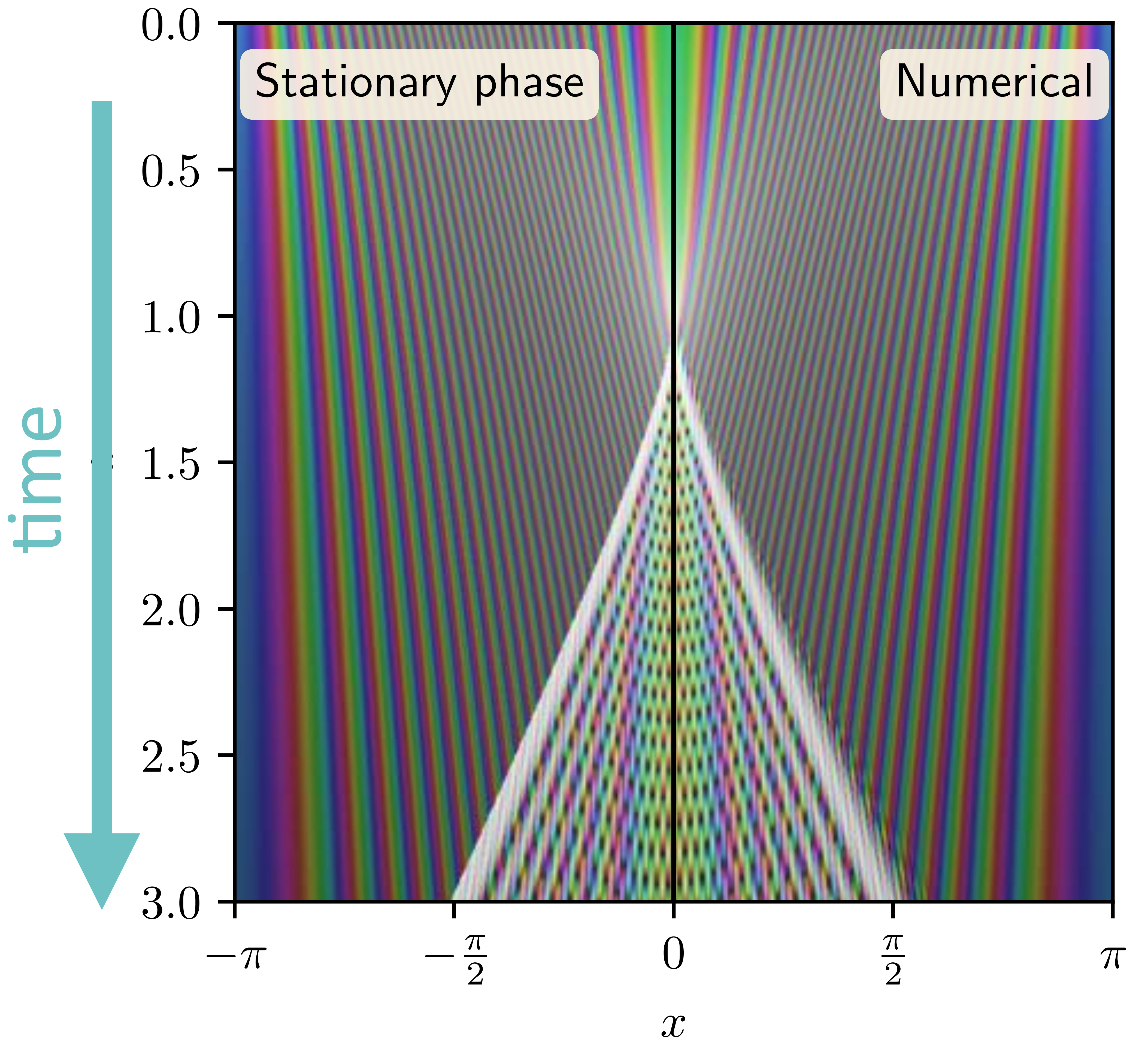
$$\psi(x,a) \sim \int \mathrm{d}q \, \exp\left(\frac{i}{\hbar}\zeta(q;x,a)\right)$$

- $\hbar$ small → integrand oscillatory
- where oscillations slow dominate integral

Stationary Phase Approximation

q where $\zeta'(q) = 0$ dominate integral

(quantum amplitude dominated by classical path as $\hbar \to 0$)



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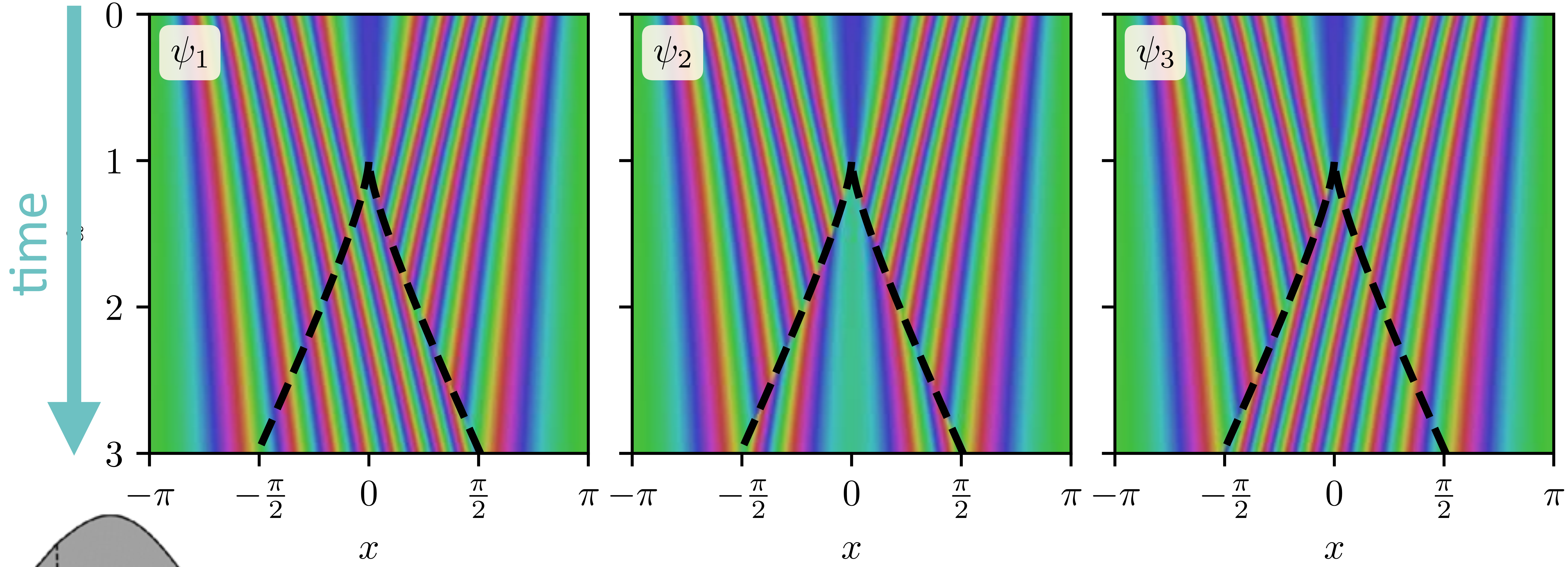
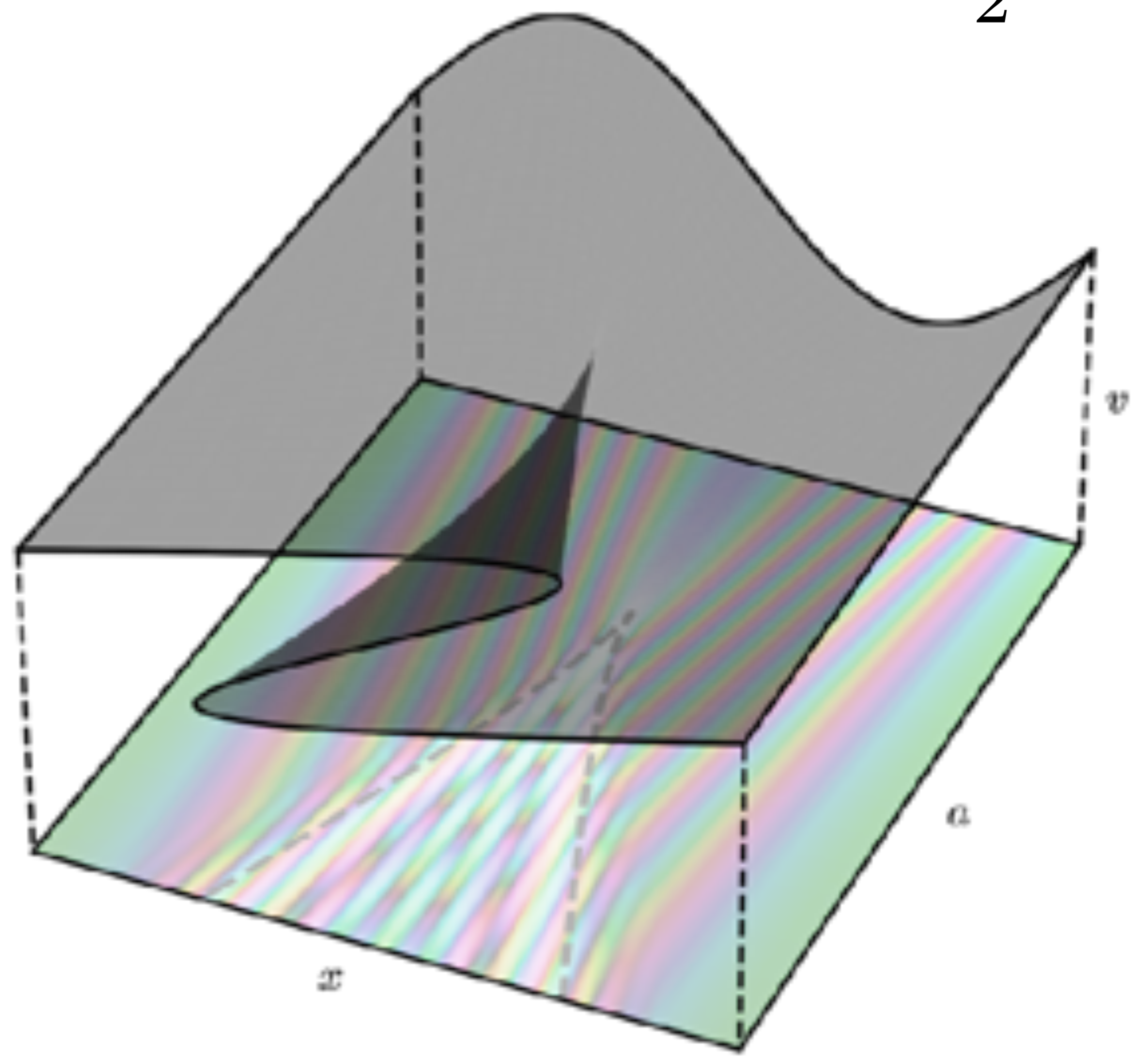
Stream wavefunctions





- It is the classical trajectories which interfere!
- Recover classical information from just interference, no need for phase space!

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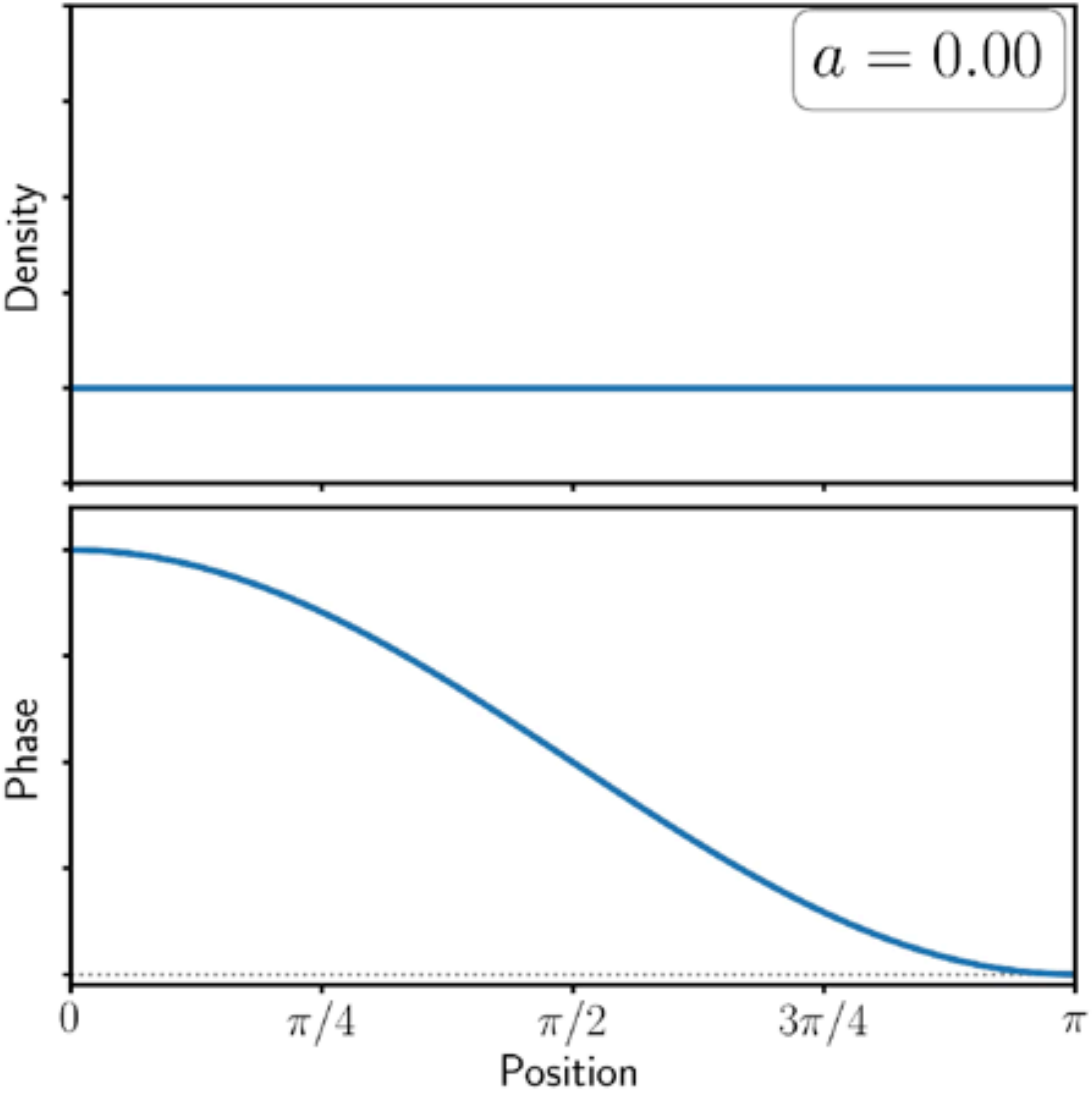
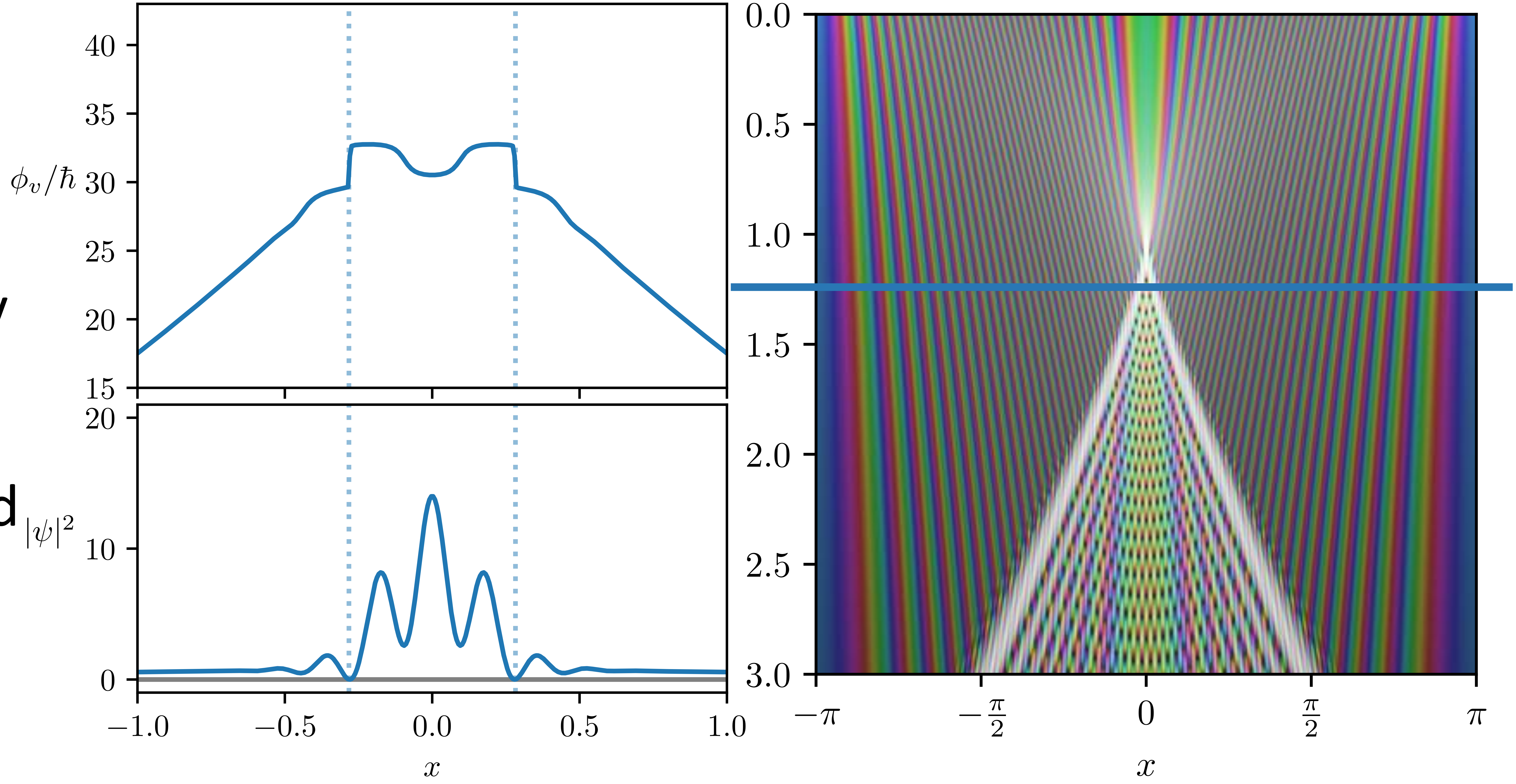
$$\boldsymbol{v} = \boldsymbol{\nabla}\phi?$$



Non-potential velocity

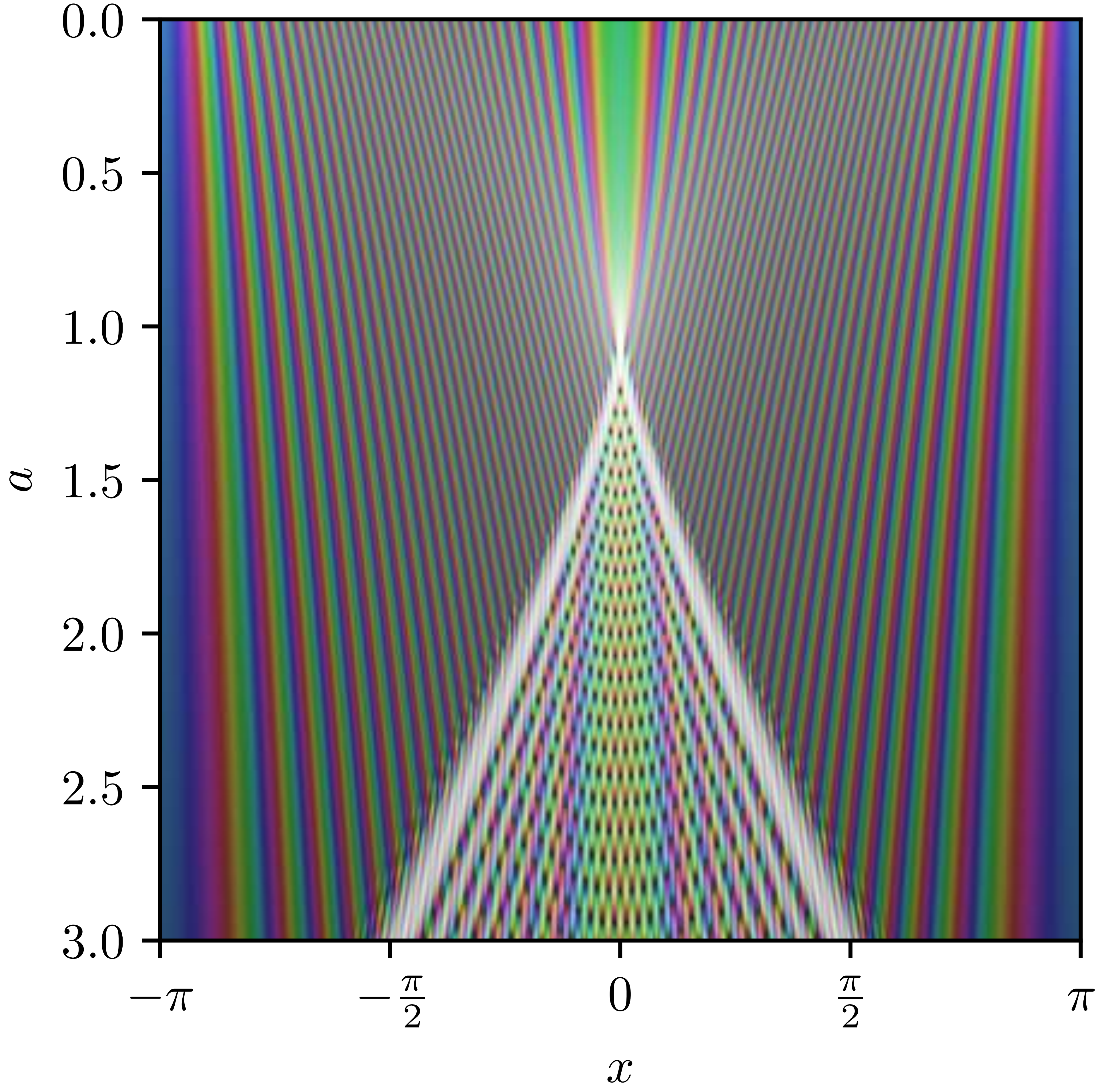
- Phase jumps correspond to zeros in the density
- ψ encodes information beyond a perfect fluid!



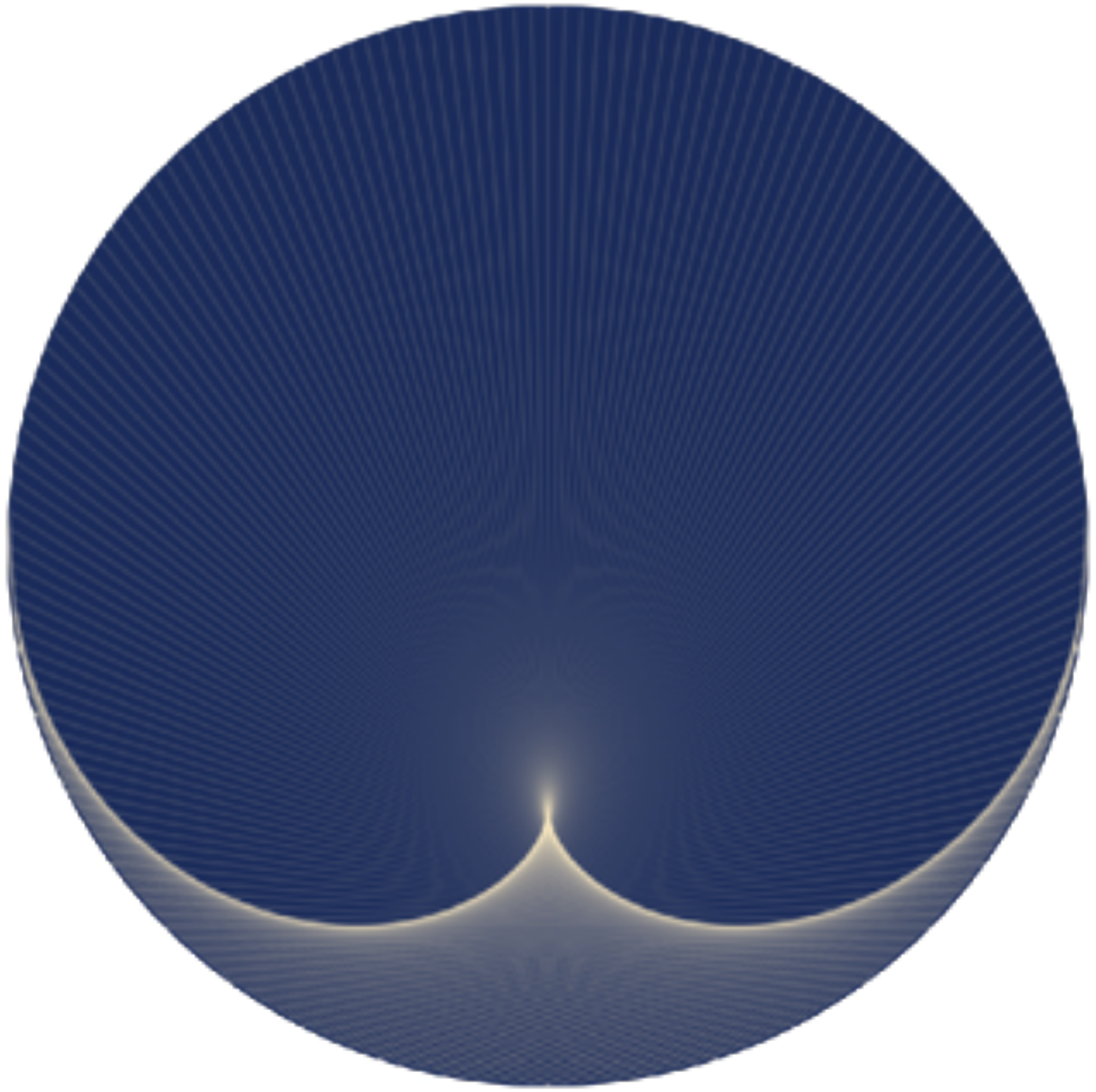
Get effect of stream averaging without explicit dissection of streams!

Optics analogy (caustics)

Dark matter



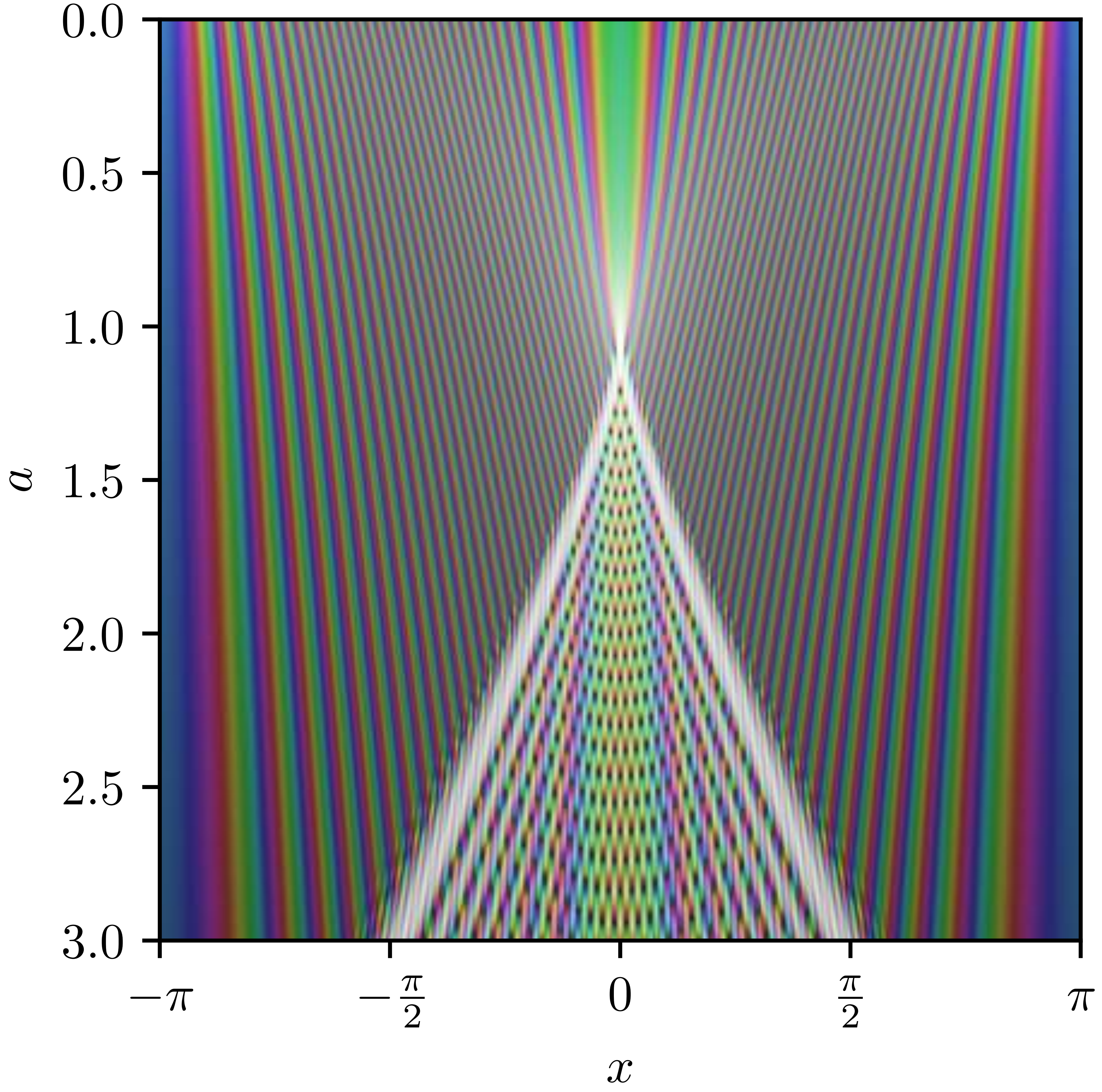
Optics



“Coffee cup caustic”

Optics analogy (caustics)

Dark matter



Optics

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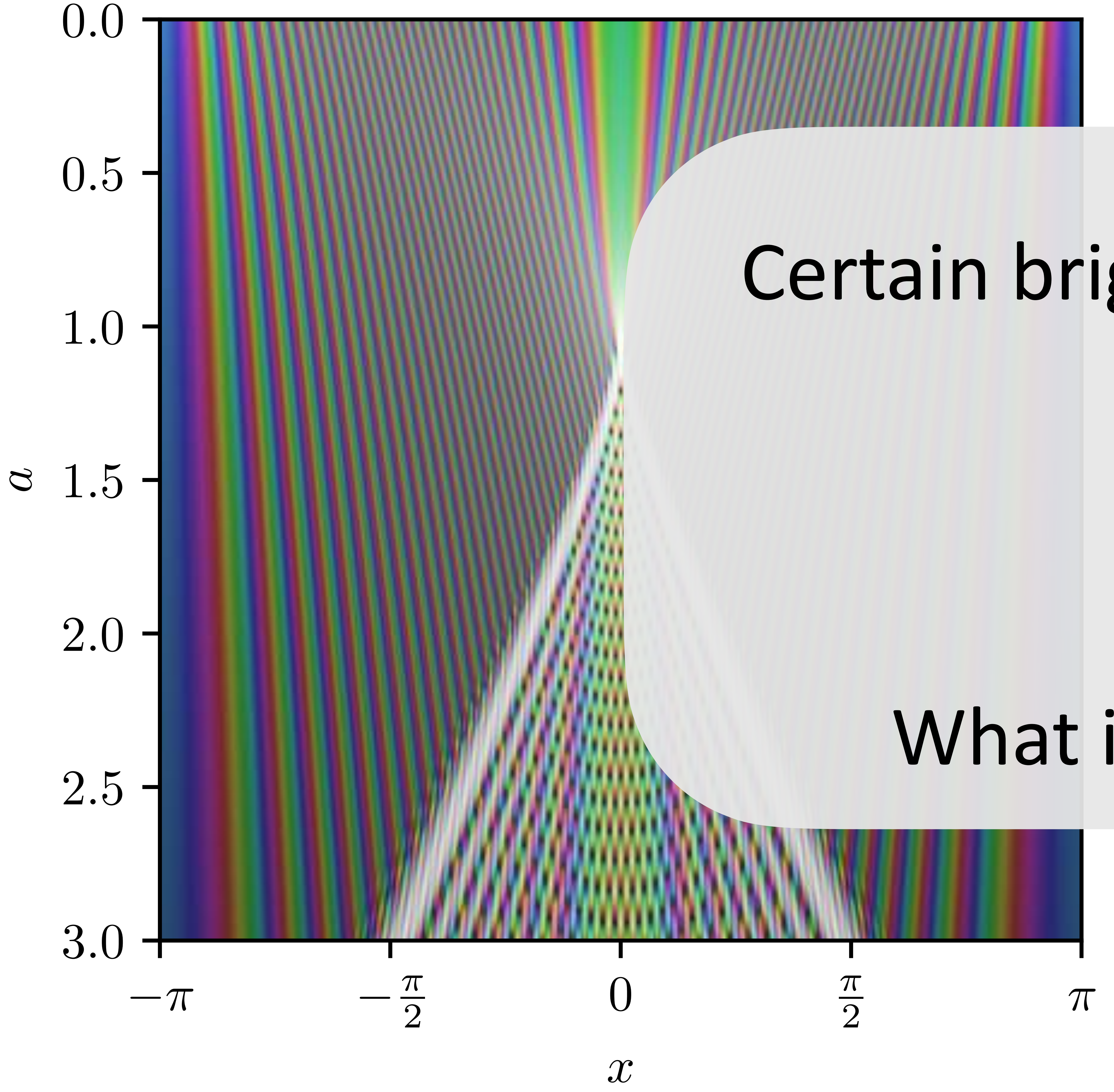
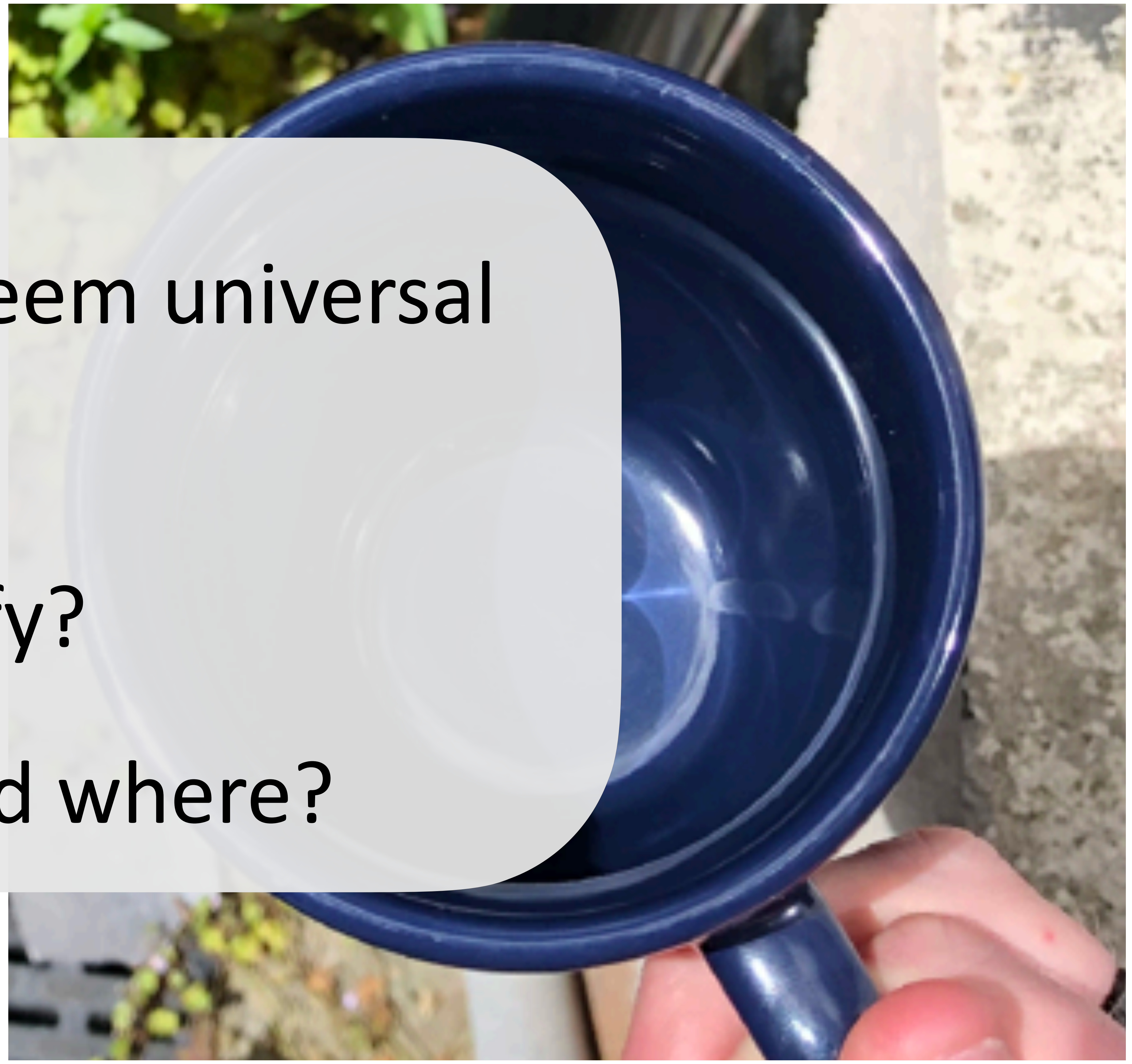


Optics



Certain bright patterns seem universal

Can we classify?

What is universal and where?

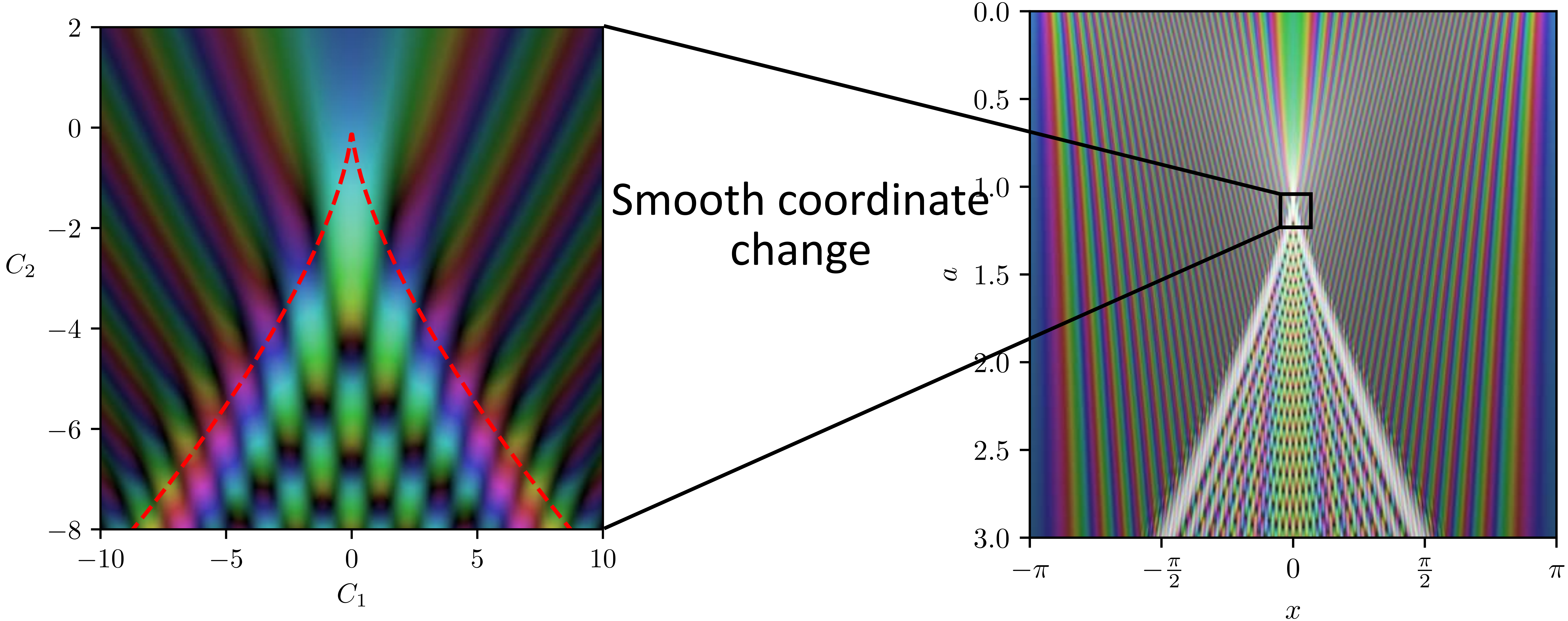
Local behaviour

Canonical cusp ($\lambda = 1$)

$$\zeta_{\text{cusp}}(s; C_1, C_2) = \frac{s^4}{4} + C_2 \frac{s^2}{2} + C_1 s$$

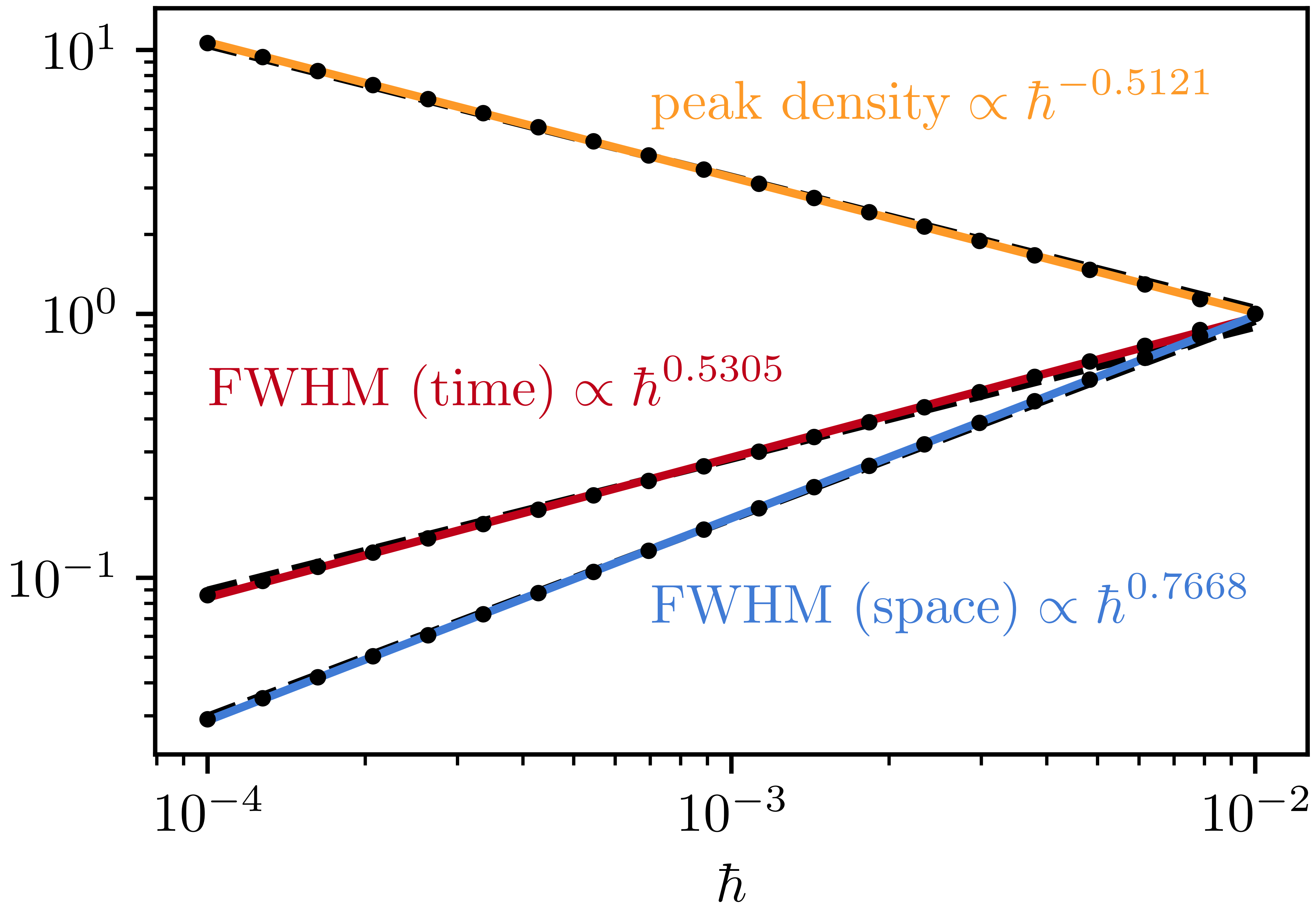
Wave model ($\hbar = 0.01$)

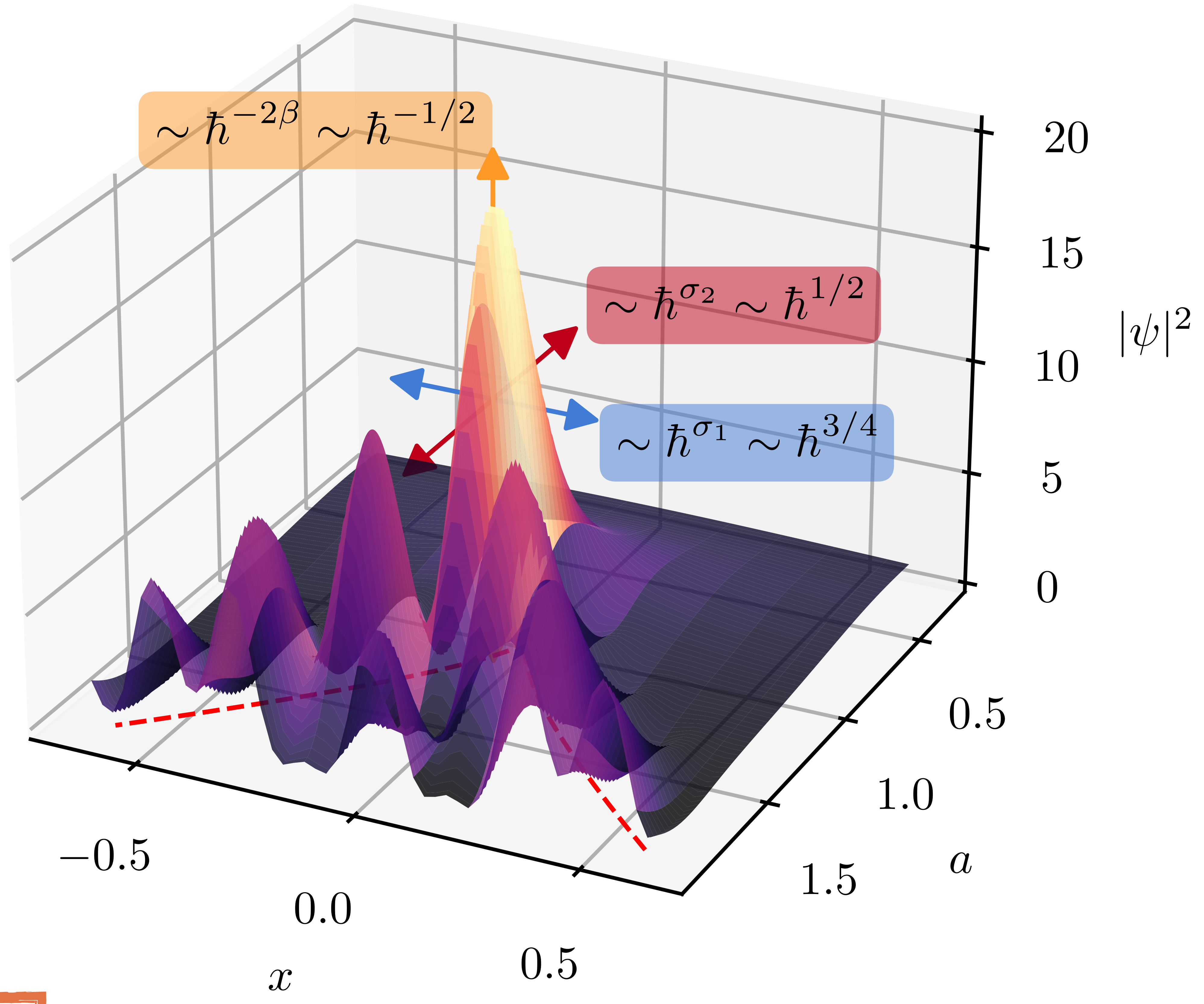
$$\zeta_{\text{DM}}(q; x, a) = \frac{(x-q)^2}{2a} + \cos(q)$$



Smooth coordinate change

Universal properties





Justifies looking at simple models!

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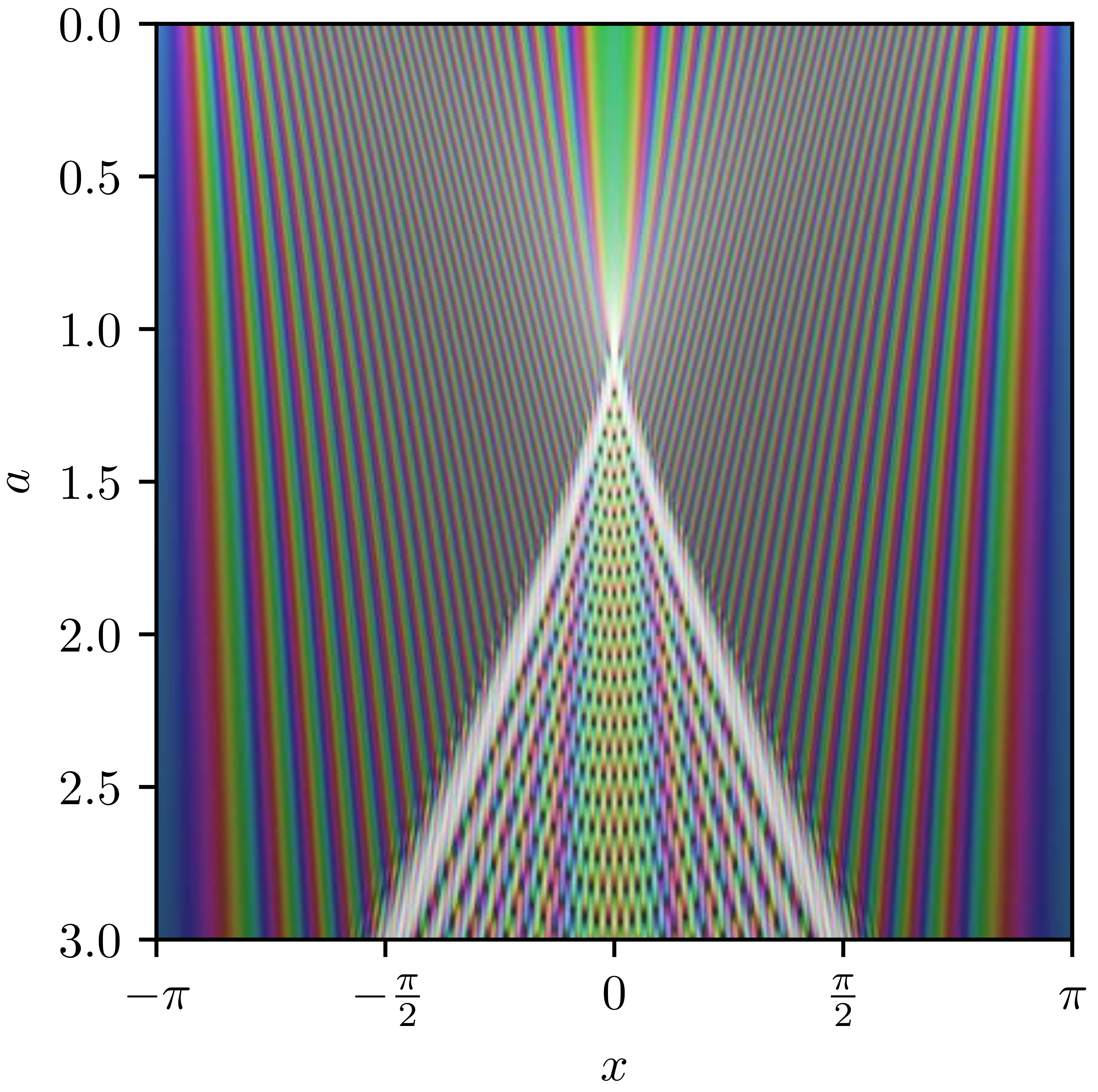
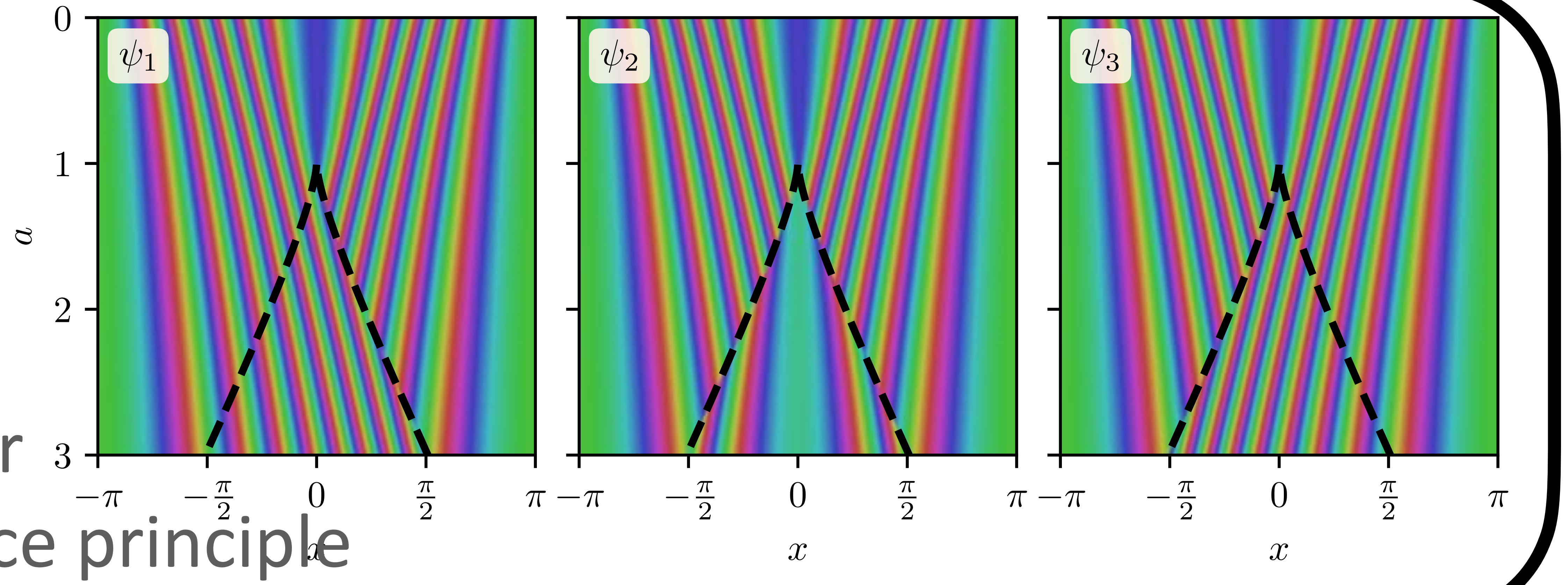
SPA + caustics



Stream components

fluid behaviour

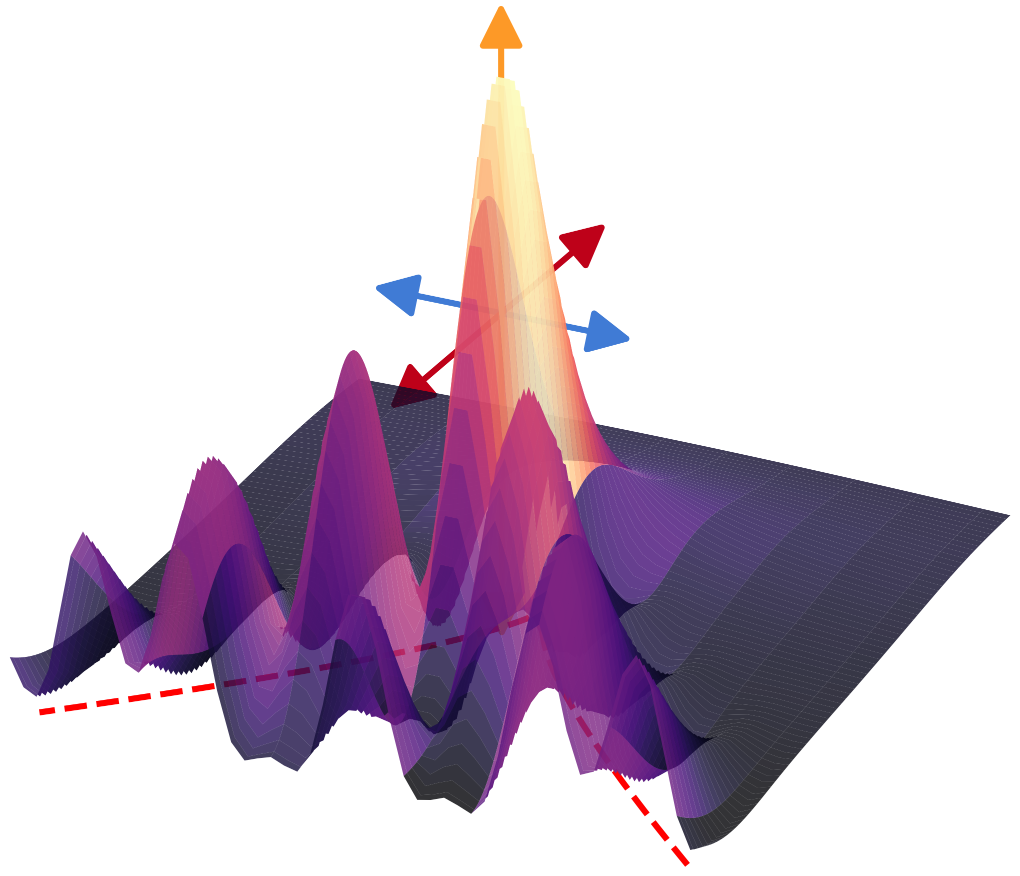
correspondence principle



Caustic classification

divergence & fringe scalings (in $\hbar$)

universal features



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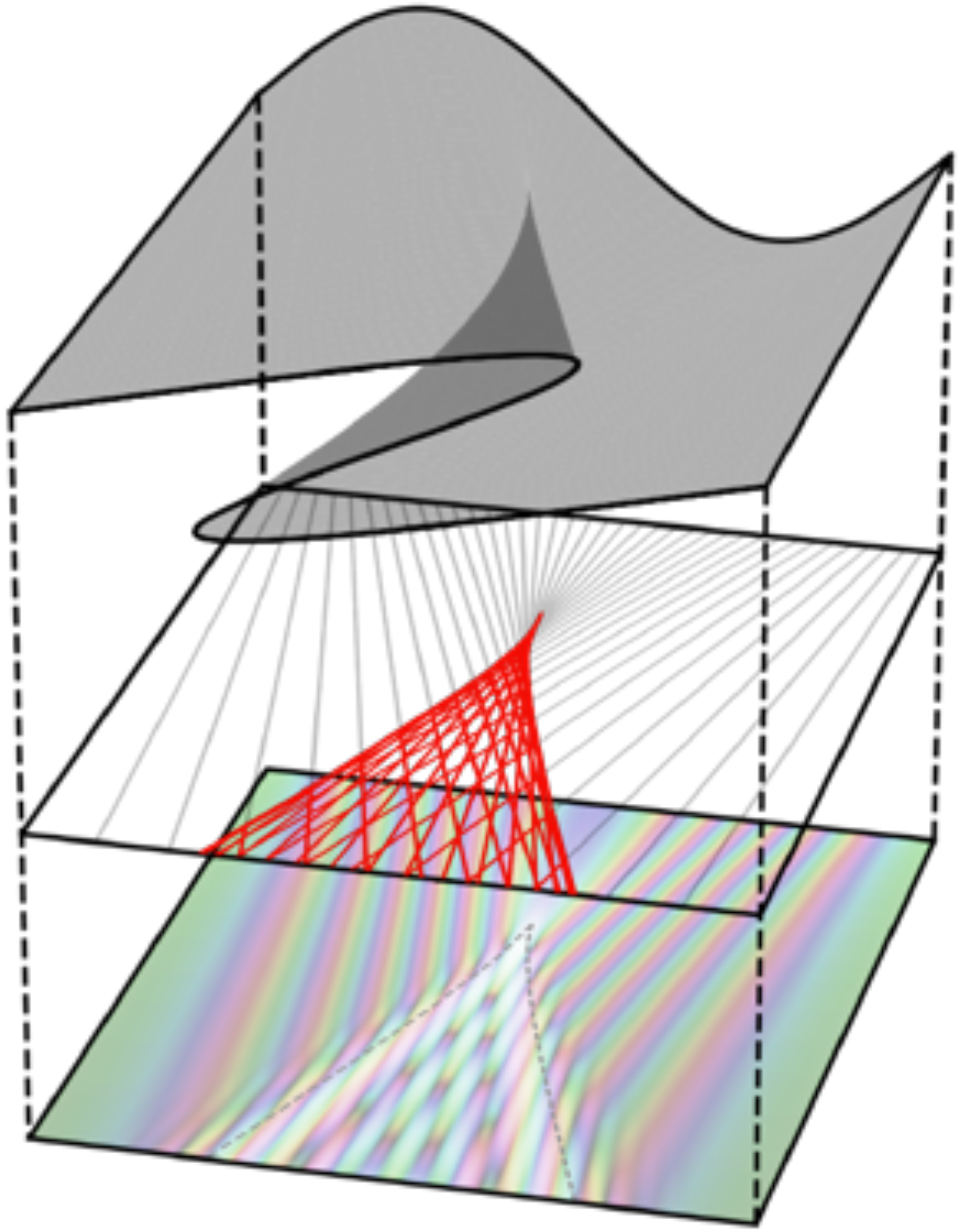
Takeaways

Wave DM presents rich phenomenology, decorating the cosmic web

- universal caustic structures (fully classified)
- interference ~ multi-streaming
- oscillations/phase jumps ~ beyond perfect fluid

Wave models of CDM efficiently capture information beyond fluid models

- prospects for analytic modelling and complementing numerics



Gough & Uhlemann 2022

arXiv: 2206.11918