Zero-damped modes and nearly extremal horizons

Jason Joykutty

15 December 2022

YTF22

UNIVERSITY OF CAMBRIDGE

Based on arXiv:2112.05669

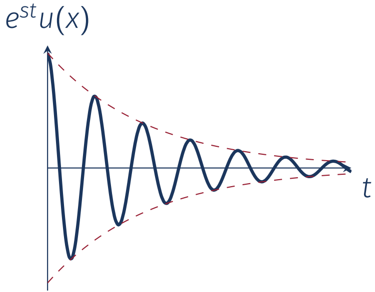
Outline

1. Quasinormal modes
2. Zero-damped modes
3. Potentials in de Sitter
4. Perturbations to the metric

Quasinormal modes

Linear perturbations to spacetimes with horizons exhibit ringdown behaviour, where the gravitational wave signal is dominated by terms of the form $e^{st}u(x)$.

Quasinormal modes

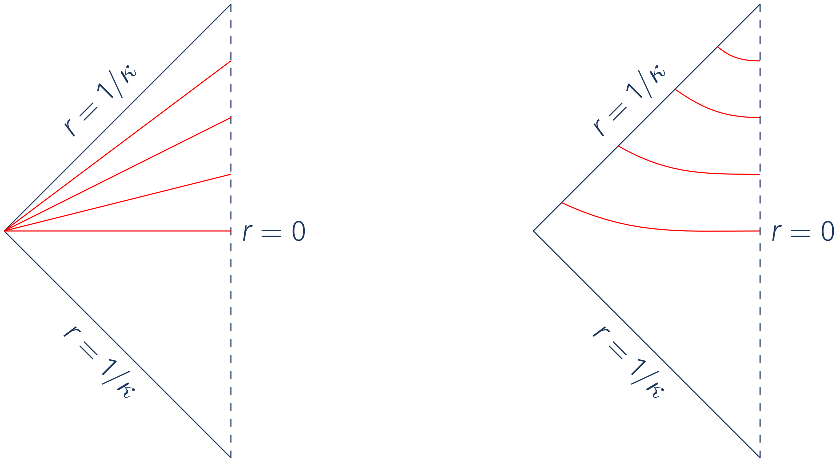


- Tests of general relativity.
- Precise asymptotics:

$$\psi(t,x) \sim \Psi_1 e^{s_1 t} w_1(x) + \Psi_2 e^{s_2 t} w_2(x) + O(e^{-Ct})$$

This was used in the proof of non-linear stability of Kerr-de Sitter (Hintz and Vasy ’18).

Regularity quasinormal modes



Regularity quasinormal modes

Definition

Let t be a time coordinate with hyperbolic level sets and $\mathbf{x}$ be coordinates on $t = \text{const}$. We say s is a *quasinormal frequency* if there exists $u(\mathbf{x})$ smooth at the horizon such that

$$L_s u(\mathbf{x}) := e^{-st}(-\Box_g + 2)(e^{st}u(\mathbf{x})) = 0$$

The corresponding solutions $u(x)$ are *quasinormal modes*.

Co-modes

For a square matrix A,

$$A \quad \text{invertible} \iff A^\dagger \quad \text{invertible}.$$

This holds for 'nice' operators between infinite-dimensional vector spaces too, so

$$L_s \quad \text{invertible at} \quad s \iff L_s^\dagger \quad \text{invertible at} \quad s.$$

We call solutions v to the dual problem $L_s^\dagger v = 0$ *co-modes*, and these naturally live in the space dual to quasinormal modes.

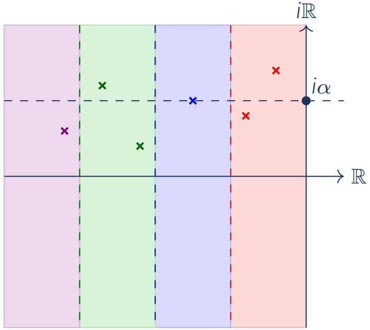
Quasinormal modes in de Sitter

$$-\Box_{\mathrm{dS}}\psi + \frac{2\Lambda}{3}\psi = 0.$$

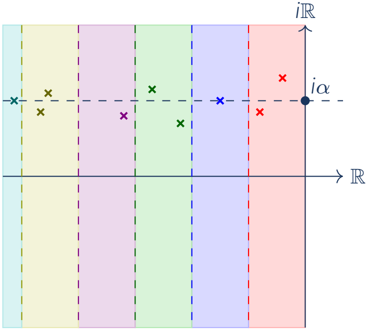
- The quasinormal frequencies are $-\kappa, -2\kappa, -3\kappa, \ldots$ where $\kappa = \sqrt{\Lambda/3}$.
- The modes are polynomials (suitable coordinates).
- The co-modes are concentrated on the cosmological horizon.
- Similar results hold for the wave equation (but not for generic KG masses).

Zero-damped modes

Zero-damped modes



$\kappa = 1$



$\kappa = 3/4$

Zero-damped modes

- These frequencies determine the decay of perturbations for spacetimes with asymptotically de Sitter ends.
- Understanding the spectral gap is tied to the strong cosmic censorship conjecture for spacetimes with positive cosmological constant (Dias et al. '18, 19', Cardoso et al. '18, Destounis '19, Liu et al. '19).
- There are heuristic arguments that these frequencies contribute to the polynomial tails observed for extremal horizons/asymptotically flat ends.

Zero-damped modes (Kerr)

- Method of matched asymptotics for Teukolsky (Hod '08, Yang et al. '13) in near extremal limit:

$$s = -\kappa\left(n + \frac{1}{2}\right) - im\Omega + O(\kappa)$$

  this holds for co-rotating modes.
- WKB analysis for Teukolsky (Yang et al. '13) in eikonal limit $l \gg 1$.

Previous work

- Schwarzschild-de Sitter:
  - Sá Barreto and Zworski '97

$$\frac{\sqrt{1-9\Lambda M^2}}{3\sqrt{3}M}\left(-\left(k+\frac{1}{2}\right)\pm i\left(l+\frac{1}{2}\right)\right) \text{ for } l \gg 1.$$

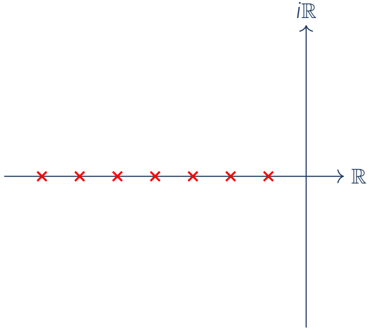
  - Hintz and Xie '22

$$-(l+k+1)\sqrt{\frac{\Lambda}{3}} \text{ as } M \to 0.$$

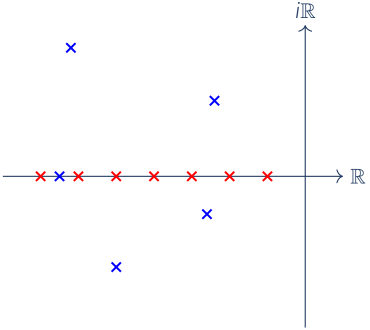
- Kerr-de Sitter:
  - Dyatlov '12: similar expression to above in $l \gg 1$ regime.
  - Hintz '21.

Potentials in de Sitter

Potentials supported away from the horizon



$-\Box_{dS} + 2$

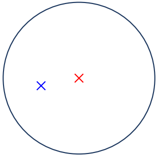


$-\Box_{dS} + 2 + W$

General smooth potentials

Theorem (J. ’22)

Given W smooth up to the horizon, $\delta > 0$, and $m \in \mathbb{N}$, take ϵ sufficiently small. Then there exists a quasinormal frequency s_0 of $L_s + \epsilon W$ such that $|s_0 + m| < \delta$.



Spherically symmetric potentials

Theorem

Let W be a smooth, spherically symmetric potential on de Sitter. Then for ϵ sufficiently small, there exist quasinormal frequencies s_n of $L_s + \epsilon W$ such that

$$s_n = -n + \sum_{m=1}^{\infty} S_m \epsilon^m,$$

where

$$S_m = \frac{(-1)^m}{2\pi i} \operatorname{tr} \oint_\Gamma z L_z' L_z^{-1} (W L_z^{-1})^m \, dz.$$

The extremal limit

Consider the family of metrics

$$g_\kappa = -(1-\kappa^2 r^2)dt^2 - 2\kappa r dt dr + dr^2 + r^2 g_{S^2}$$

and take the limit as $\kappa \to 0$.

The extremal limit

Theorem (J. '22)

Let V be a smooth function on $\mathbb{R}^3$ such that

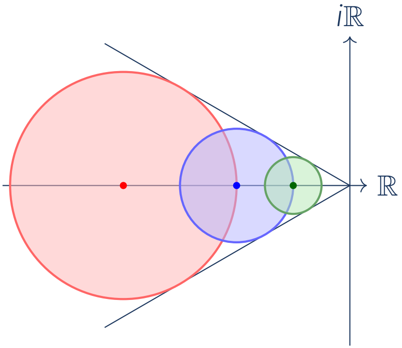
$$|\mathbf{x}|^{|\alpha|+2}\partial^\alpha V(\mathbf{x}) \to 0 \quad as \quad |\mathbf{x}| \to \infty$$

for each multi-index α. Then the equation

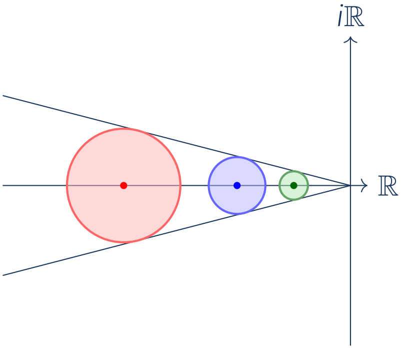
$$-\Box_{g_\kappa}\psi + 2\kappa^2\psi + V\psi = 0$$

exhibits zero-damped quasinormal frequencies.

The extremal limit



$s = -\kappa$



$s = -2\kappa$

Perturbations to the metric

A class of static, spherically symmetric metrics

We consider metrics of the form:

$$g = -f_\Lambda(r)dt^2 + \frac{dr^2}{f_\Lambda(r)} + r^2 g_{S^2}$$

where

$$f_\Lambda(r) = 1 + w_\Lambda(r) + \frac{\Lambda}{3}\alpha_\Lambda r - \frac{\Lambda}{3}r^2.$$

A class of spherically symmetric metrics

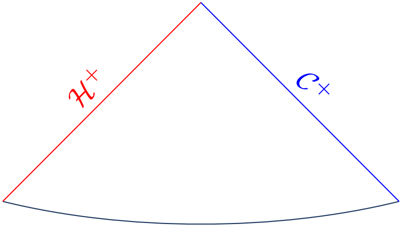
Theorem (J. '22)

Consider the conformal Klein-Gordon equation on the background described previously:

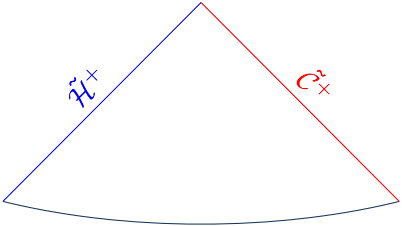
$$-\Box_g \psi + \frac{R}{6}\psi = 0.$$

In the extremal limit $\Lambda \to 0$, this equation exhibits zero-damped quasinormal frequencies.

Reissner-Nordström-de Sitter



Reissner-Nordström-de Sitter



Transformed spacetime

Theorem (J. '22)

Consider the conformal Klein-Gordon equation on a Reissner-Nordström-de Sitter black hole background:

$$-\Box_g \psi + \frac{R}{6}\psi = 0.$$

In the extremal limit where the event and Cauchy horizons coalesce, this equation exhibits zero-damped quasinormal frequencies.

Summary

- Zero-damped modes are important for understanding asymptotics of perturbations for asymptotically de Sitter spacetimes.
- They have been observed in many spacetimes where a horizon becomes extremal: it is conjectured they are a generic feature.
- It can be proved for a class of static, spherically symmetric spacetimes that they are present.