Cauchy Slice Holography

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Based on work with Goncalo Araujo-Regado and Aron Wall [ArXiv: 2204.00591]

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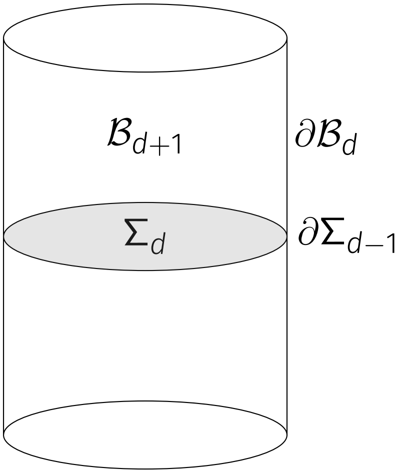
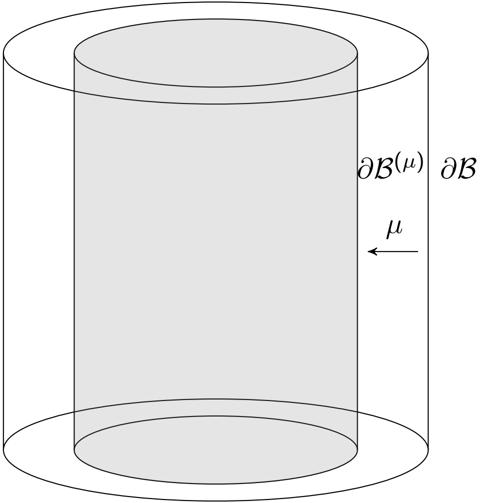
Motivational Questions

Holographic Principle: AdS/CFT correspondence, Finite cutoff holography





Is CQG holographic?

Reformulate canonical theory of quantum gravity as a holographic theory?

Hilbert space of CQG?

$\mathcal{H}_{\text{CQG}}$ isomorphic to some $\mathcal{H}_{\text{QFT}}$?

What is that isomorphism (a.k.a dictionary)?

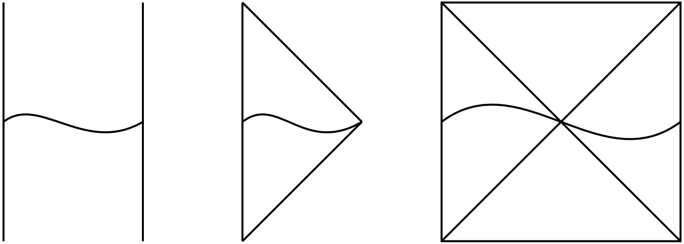
Is the time evolution consistent?

Right inner product of CQG?

Reformulate holographic principle on Cauchy slices?

Generalise holographic principle to other spacetimes? Especially, dS has no spatial boundary and so how would one do holography there?

Reformulate holographic principle on Cauchy slices? (hence the name Cauchy slice holography).



$\Lambda < 0$ $\quad$ $\Lambda = 0$ $\quad$ $\Lambda > 0$

Manifestly Background Independent “effective” Quantum Gravity?

In quantum gravity, the background is the very thing that is being quantised.

Hence we need a background independent theory.

Quantum physics allows us to take superpositions of two largely different spacetime geometries. Such states will not have one background and it is not clear if quantising the perturbations of metric around a fixed background will teach us about such states.

Can we at least get a effective theory of quantum gravity that is still manifestly background independent?

Manifestly Background Independent UV complete Quantum Gravity???

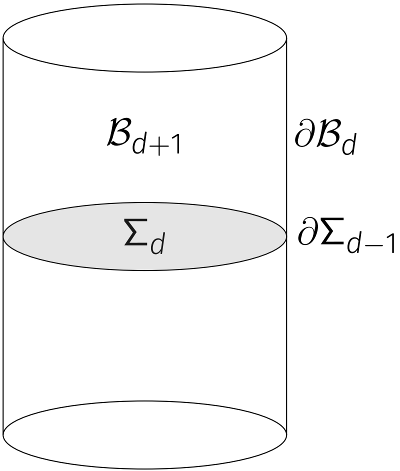
UV completing QG is hard.

Preserving manifest background independence while UV completing is much harder.

How can one find or define a UV complete manifestly background independent theory of quantum gravity?

Canonical Quantum Gravity

ADM Formalism



- $(d+1)$-dim asymptotically AdS spacetime.
- $ds^2 = -N^2dt^2 + g_{ab}(dx^a + N^a dt)(dx^b + N^b dt)$.
- Lapse N and shift N_a are gauge freedom.
- $(g_{ab}, \Pi^{ab}, \phi, \Pi_\phi) \longrightarrow$ Phase space
- Diffeomorphism invariance constrains this phase space.

Classical Constraints

Hamiltonian constraint (coming from temporal diffeomorphism)

$$\mathcal{H}(x) = 16\pi G_N G_{abcd}\Pi^{ab}\Pi^{cd} - \frac{\sqrt{g}}{16\pi G_N}(R - 2\Lambda) + \mathcal{H}_{\text{matter}} = 0$$

$$G_{abcd} := \frac{1}{2\sqrt{g}}\left(g_{ac}g_{bd} + g_{ad}g_{bc} - \frac{2}{d-1}g_{ab}g_{cd}\right).$$

Momentum constraint (coming from spatial diffeomorphism)

$$\mathcal{D}_a = -2\nabla_b \Pi_a^b + \partial_a\phi\,\Pi_\phi = 0$$

These are constraints on the classical phase space

Canonical Quantum Gravity

Canonical quantisation in the metric representation

$$\Pi^{ab} = -i\frac{\delta}{\delta g_{ab}}, \quad \Pi_\phi = -i\frac{\delta}{\delta \phi}$$

$$[g_{ab}(x), \Pi^{cd}(y)] = i\delta^{ab}_{cd}(x-y), \quad [\phi(x), \Pi_\phi(y)] = i\delta(x-y)$$

And states are superposition of metric and matter fields on Σ_d

$$\Psi[g, \phi]$$

But there are constraints to impose!

Wheeler-DeWitt equation

Imposing Hamiltonian constraint gives us the Wheeler-DeWitt (WDW) equation:

$$\mathcal{H}(x)\Psi = \left\{-16\pi G_N G_{abcd}\frac{\delta^2}{\delta g_{ab} g_{cd}} - \frac{\sqrt{g}}{16\pi G_N}(R - 2\Lambda) + \mathcal{H}_{\mathrm{matter}}\right\}\Psi[g,\phi] = 0.$$

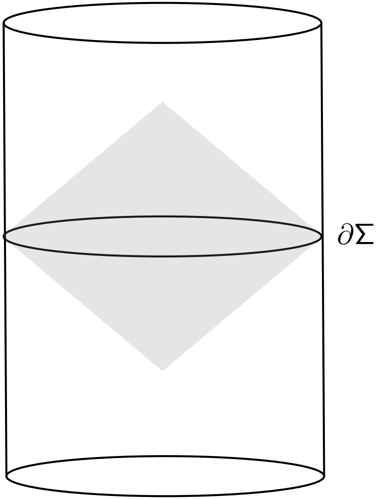
And imposing the momentum (a.k.a diff) constraints:

$$\mathcal{D}_a\Psi = -i\left\{-2g_{ac}\nabla_b\frac{\delta}{\delta g_{bc}} + \partial_a\phi\,\frac{\delta}{\delta\phi}\right\}\Psi[g,\phi] = 0.$$

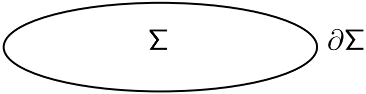
Diff constraints $\implies$ Ψ is invariant under change of coordinates on Σ_d.

Any Ψ satisfying the above constraints is called a WDW state: Ψ_{WDW}.

Where do WDW states live?



- No background a priori, so no Cauchy slices.
- Only $\partial\Sigma$ is meaningful.
- WDW states live on abstract Cauchy slices Σ whose boundary is $\partial\Sigma$.
- Equivalently, one can think of WDW state living on all Cauchy slices anchored to $\partial\Sigma$ and it encodes the causal diamond.



Abstract Cauchy Slice

Solving the WDW equation

$\Psi_{\text{WDW}} = Z_{\text{QFT}}$?

Consider a euclidean QFT on Σ with background sources being (g_{ab}, ϕ).

The QFT partition function $Z_{\text{QFT}}[g, \phi]$ is of the same mathematical form as the WDW state $\Psi_{\text{WDW}}[g, \phi]$.

Also, $Z_{\text{QFT}}[g, \phi]$ manifestly satisfies the diff constraints because it is covariant on Σ.

$$\Psi_{\text{WDW}}[g, \phi] \stackrel{?}{=} Z_{\text{CFT}}[g, \phi]$$

Dynamical variables

Fixed background sources on Σ

Not quite true because Z_{CFT} satisfies the Weyl-Anomaly equation:

$$\big(\mathcal{W}(x) - i\mathcal{A}(x)\big)\, Z_{CFT} = 0,$$

Friedel's Asymptotic Analysis

By an infinite uniform rescaling of the metric $\tilde{g} = g/\mu^{2/d}$, the WDW equation simplifies to the Weyl-Anomaly equation (upto counter terms):

$$\mathcal{H}\Psi_{WDW} \xrightarrow{\mu\to 0} \left(\mathcal{W} - i\mathcal{A}\right)\Psi_{WDW}.$$

This implies all WDW states looks like CFT partition (upto counter terms) functions on large scales:

$$\lim_{\mu\to 0} \Psi\left[\tilde{g},\mu\right] = e^{iS[\tilde{g},\mu]} Z_{CFT}[\tilde{g}].$$

So we can write the WDW state as a deformation of a CFT:

$$Z^{(\mu)}_{T^2} = e^{\mathrm{CT}(\mu)} \left(\mathrm{P}\exp\int_0^{\mu} \frac{d\lambda}{\lambda}\, O(\lambda)\right) Z_{\mathrm{CFT}}.$$

The T^2 deformation

$$
\begin{tikzcd}[column sep=small]
Z^{\mu}_{T^2} = & e^{CT(\mu)} & \left(\mathrm{P}\exp \int_0^{\mu} \frac{d\lambda}{\lambda} O(\lambda)\right) & Z_{CFT} \\
\mathcal{H} \arrow[u] \arrow[r, leftrightarrow] & \tilde{\mathcal{H}} \arrow[u] \arrow[rr, leftrightarrow] & & \mathcal{W} - i\mathcal{A} \arrow[u]
\end{tikzcd}
$$

- Start from any general set of constraints that closes.
- $\tilde{\mathcal{H}}$ cannot have relevant terms and its only marginal term must be the weyl-anomaly operator. This uniquely fixes $CT(\mu)$.
- Requiring $\tilde{\mathcal{H}}$ becoming weyl-anomaly operator when commuted past the path ordered exponential uniquely fixes $O(\lambda)$ to just be irrelevant terms in $\tilde{\mathcal{H}}$ which comes from $\mathcal{H}$.
- This also fixes uniquely the relations between free parameters on both sides.

Example: Real Scalar Field Coupled to Gravity

For scalar field coupled to gravity in $(3+1)$-dim and $\Delta_\phi \in \left(\frac{3}{4}, \frac{3}{2}\right)$.

$$\mathrm{CT} = \int d^d y \sqrt{g} \left(-2 \frac{d-1}{16\pi G_N} \mu^{-1/d} + - \frac{1}{(d-2)16\pi G_N} \mu^{1/d}\, R + - \frac{\Delta_\phi}{2} \mu^{-1/d}\, \Phi^2 \right).$$

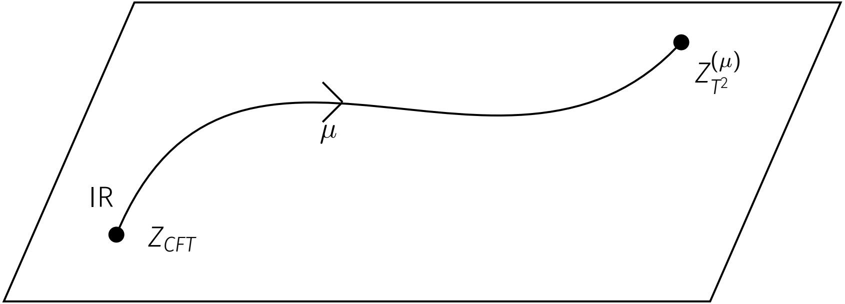
$$O(\lambda) = \int d^3x \Bigg[-\frac{1}{3}\lambda^{1-2\Delta_\phi/3} \frac{1}{2\sqrt{g}} \Pi_\phi^2 + i \frac{\Delta_\phi}{12\alpha} \lambda^{2\Delta_\phi/3} \phi^2 \Pi - \frac{\Delta_\phi^2}{32\alpha} \lambda^{4\Delta_\phi/3-1} \sqrt{g} \phi^4$$
$$- \frac{1}{3}\lambda^{(2\Delta_\phi-1)/3} \frac{\sqrt{g}}{2} \left(g^{ab} \nabla_a \phi \nabla_b \phi + \frac{\Delta_\phi}{4} R \phi^2 \right) - \frac{1}{3} \lambda^{1/3} \sqrt{g} V\left(\lambda^{(\Delta_\phi - 1)/3} \phi, R\right)$$
$$- \frac{\lambda}{3\alpha} \frac{1}{\sqrt{g}} : \left(\Pi^{ab}\Pi_{ab} - \frac{1}{2}\Pi^2 \right) : + i \frac{2}{3} \lambda^{2/3} \left(G_{ab}\Pi^{ab} - \frac{1}{2} G\Pi \right) + \frac{\alpha}{3} \lambda^{1/3} \sqrt{g} \left(G^{ab}G_{ab} - \frac{1}{2} G^2 \right) \Bigg].$$

$$m^2 = \Delta_\phi(\Delta_\phi - d)\mu^{-2/d}, \quad \Lambda = -\frac{d(d-1)}{2}\mu^{-2/d}, \quad 16\pi G_N = \frac{1}{\alpha}\mu^{\frac{d-1}{d}}.$$

RG Flow and WDW states as "effective" QG states

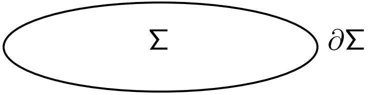
The deformation without the counter term satisfies a Callan-Symanzik equation

$$\left(\int d^d x\, \mathcal{W}(x) - id\, \mu \frac{\partial}{\partial \mu}\right)\left(\mathrm{P}\exp \int_0^{\mu} \frac{d\lambda}{\lambda} O(\lambda)\right) Z_{\mathrm{CFT}}[g, \phi, \{\chi\}] = 0.$$



So CQG is an manifestly background independent "effective" theory of QG.

Boundary $\longrightarrow$ Bulk Dictionary: The CSH Dictionary



Abstract Cauchy Slice

The T^2 theory lives on an open manifold Σ with a boundary $\partial\Sigma$. Hence its partition function $Z_{T^2}[g;\{\chi\}]$ depends on the boundary conditions $\{\chi\}$. The CFT state living on $\partial\Sigma$ naturally supplies the required boundary condition.

$$\mathcal{H}_{\text{CFT}} \longrightarrow \mathcal{C} := \{\Psi_{\text{WDW}}\}$$

$$\boxed{\Psi_{\text{WDW}} = \int d\{\chi\}\, Z_{T^2}[g;\{\chi\}]\, \psi_{\text{CFT}}[\{\chi\}]}$$

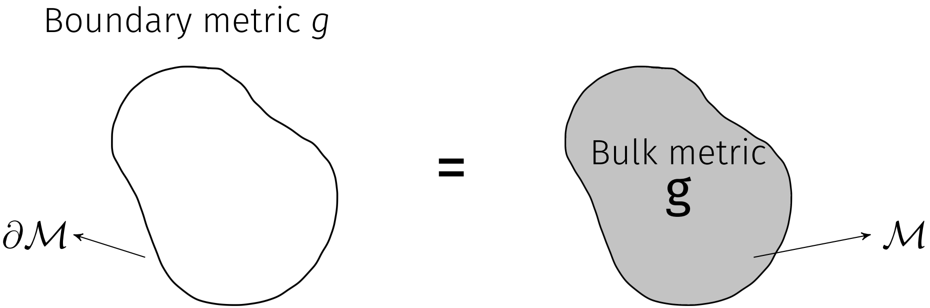
Holographic CQG

Bulk $\longrightarrow$ Boundary Dictionary

$$\begin{tikzcd} & \mathcal{D}(\mathcal{S}) \arrow[dl, "{\psi_{\text{CFT}}[\{\chi\}] = Z_{T^2}[g_0, \{\chi\}]}"'] & \\ \mathcal{H}_{\text{CFT}} \arrow[rr, "{\Psi_{\text{WDW}} = \int d\{\chi\}\, Z_{T^2}[g;\{\chi\}]\, \psi_{\text{CFT}}[\{\chi\}]}"'] & & \mathcal{C} = \{\Psi_{\text{WDW}}\} \end{tikzcd}$$

The Generalized Holographic Principle (GHP)

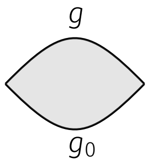
$$\boxed{Z_{T^2}^{(\partial\mathcal{M})}[g] = \sum_{\mathcal{M}} \int_{\mathbf{g}|_{\partial\mathcal{M}}=g} \frac{D\mathbf{g}}{\mathrm{Diff}(\mathcal{M})} e^{iI_{\mathrm{grav}}[\mathbf{g}]}}$$



$$
\begin{tikzcd}
 & \mathcal{D}(\mathcal{S}) \arrow[dl] \arrow[dr] & \\
\mathcal{H}_{\text{CFT}} \arrow[rr] & & \mathcal{H}_{\text{QG}}
\end{tikzcd}
$$

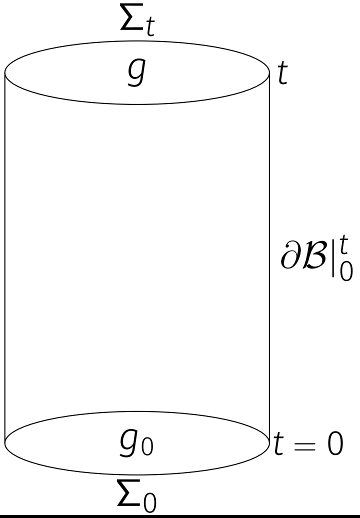
$$\psi_{\text{CFT}}[\{\chi\}] = Z_{T^2}[g_0, \{\chi\}]$$

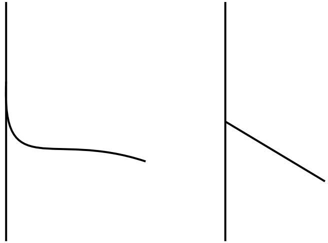
$$\Psi_{\text{WDW}}[g] =$$



$$\Psi_{\text{WDW}} = \int d\{\chi\}\; Z_{T^2}[g; \{\chi\}]\; \psi_{\text{CFT}}[\{\chi\}]$$

Asymptotic Time Evolution





$$\boxed{\mathrm{H}_{\mathrm{CFT}} = \mathrm{H}_{\mathrm{ADM}}}$$

$$i\frac{\partial}{\partial t}\Psi[g,t] = H_{\mathrm{ADM}}\Psi[g,t]$$

Future Work and Summary

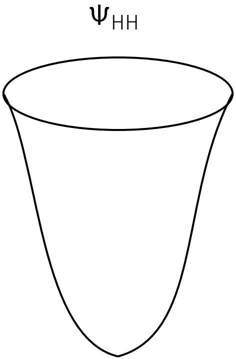
UV Complete Manifestly Background Independent theory of Quantum Gravity

Hard

$$\begin{tikzcd} \Psi_{\mathrm{UV}}[?] \arrow[r, squiggly, -] \arrow[d, "\mathrm{RG}"'] & Z_{\mathrm{UV}}[?] \arrow[d, "\mathrm{RG}"] \\ \Psi_{\mathrm{WDW}}[g,\phi] \arrow[r, squiggly, -] & Z_{T^2}[g,\phi] \end{tikzcd}$$

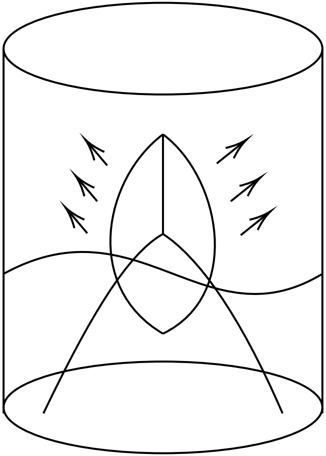
Easy

Holographic Cosmology/ dS Holography



- Similar deformation also works for $\Lambda > 0$.
- New proposal for the wavefunction of the universe.
- Contour problem in quantum gravity.
- UV completing this would give us a theory of quantum gravity in dS.

BH Information Paradox



- Cauchy slices probe BH interiors.
- A calculational framework to address the black hole information paradox.
- More on this was presented in this talk: Emergent Semiclassical Gravity from WheelerDeWitt Wavefunction & Black Hole Info Paradox Youtube: **https://youtu.be/UsG0CjDvccE**

Summary

- CQG is reformulated as a holographic theory.
- Also, the holographic principle is reformulated on Cauchy slices.
- The Hilbert spaces of CQG and CFT are the same, and the Hamiltonians are also the same.
- The GHP provides us with a definition for the gravitational path integral.
- The CSH dictionary maps any state of the CFT to a WDW state. Also we have bulk to boundary maps. These are new AdS/CFT dictionaries.
- By UV completing this T^2 QFT, one could get a manifestly background independent, UV complete theory of quantum gravity.

Thank You

Questions?