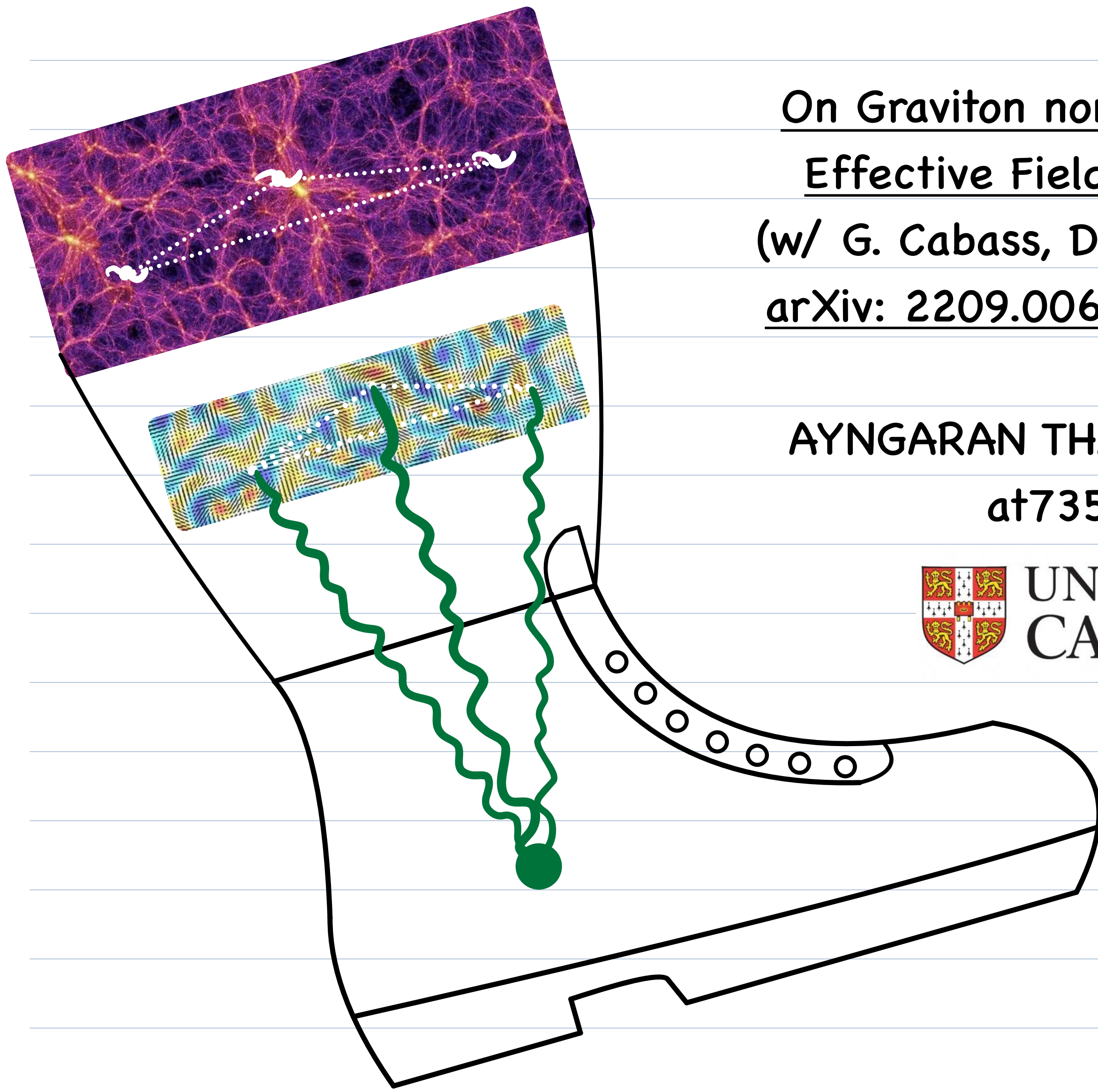

On Graviton non-Gaussianities in the Effective Field Theory of Inflation

(w/ G. Cabass, D. Stefanyszyn, J. Supeł
arXiv: 2209.00677, JHEP10(2022)154)

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YTF 22

On Graviton non-Gaussianities in the Effective Field Theory of Inflation

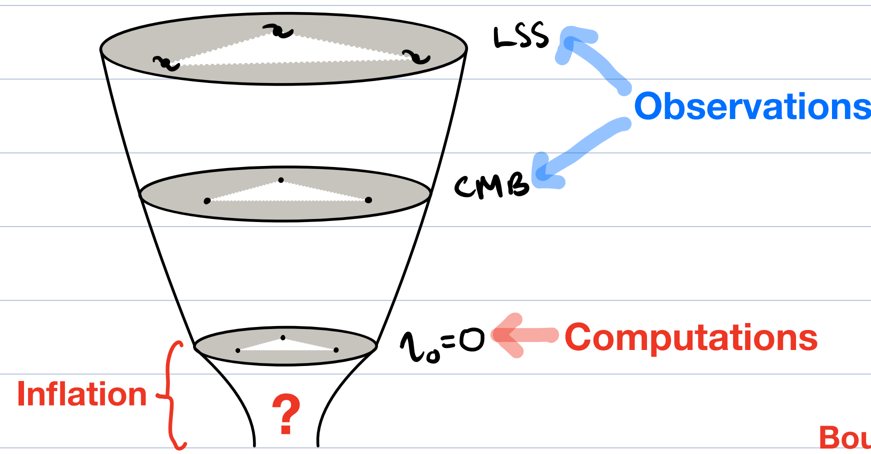
(w/ G. Cabass, D. Stefanyszyn, J. Supeł **arXiv: 2209.00677, JHEP10(2022)154**)

- We derived all tree-level graviton 3-pt functions (bispectra) in the EFToI to all orders in derivatives.
- Giving observers an idea of what to look for in the (very far) future.
- Key words: WFU, cosmological correlators, EFToI.

$\langle \bar{\phi}(\underline{x}_1) \dots \bar{\phi}(\underline{x}_n) \rangle$

$\to \langle \bar{\phi}(\underline{k}_1) \dots \bar{\phi}(\underline{k}_n) \rangle'$

n-pt correlator of massless spin-0 and spin-2 fields



boundary at end of inflation

Boundary $\Rightarrow$ holography? dS/CFT?

We have measured $\overset{\text{CMB}}{\langle \zeta\zeta \rangle}$ and hope to measure $\langle \gamma\gamma \rangle, \langle \zeta\zeta\gamma \rangle, \langle \zeta\gamma\gamma \rangle, \langle \gamma\gamma\gamma \rangle$

We are interested in $\underline{\underline{\langle \gamma\gamma\gamma \rangle}}$

Wavefunction of the Universe (WFU)

QFT in dS path integral

$$\Psi[\bar{\phi}; \eta_0] = \int_{\phi(-\infty)=\Omega_{BD}}^{\phi(\eta_0=0)=\bar{\phi}} D\phi \, e^{iS[\phi]} \propto \exp\left[-\sum_{n=2}^{\infty}\frac{1}{n!}\int\left(\prod_{a=1}^{n}\bar{\phi}_{\underline{k}_a} d^3\underline{k}_a\right)\psi_n(\underline{k}_a)\right]$$

any spinning field

where $\psi_n = \psi'_n(\underline{k}_1, \dots, \underline{k}_n)\,(2\pi)^3\,\delta^{(3)}\left(\sum \underline{k}_a\right)$

$$\frac{\partial}{\partial\eta}\left(\frac{\delta L_2}{\delta\phi'} + \frac{\delta L_{int}}{\delta\phi'}\right) - \left(\frac{\delta L_2}{\delta\phi} + \frac{\delta L_{int}}{\delta\phi}\right) = 0$$

quadratic/free part

for massless fields $\psi'_n \propto |\underline{k}|^3 = k^3$ by dimensional analysis and/or scale invariance

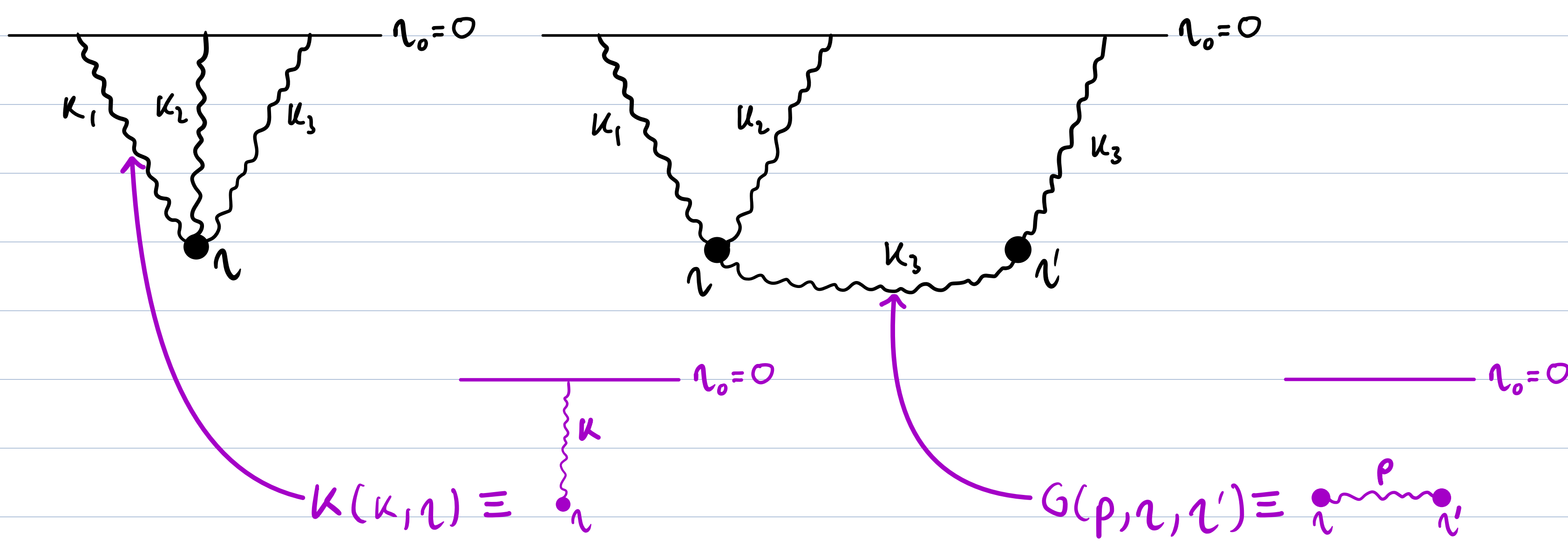
WFU (cont.)

Solve using method of Green's functions (propagators) to compute $\Psi_n \sim \langle \phi_1, \dots, \phi_n \rangle$

at **TREE LEVEL** (no loops) in perturbation theory.

Obtain $K(k,\eta) \propto (1-ik\eta)e^{ik\eta}$ and $G(p,\eta,\eta')$

which are the Feynman-Witten diagram propagators.



Bootstrap

Physical principles $\longrightarrow$ Theory Lagrangian $\longrightarrow$ Observables

Bootstrap

$$\begin{tikzcd} \text{Physical principles} \arrow[r] \arrow[rr, bend right, "\text{Bootstrap}"'] & \cancel{\text{Theory Lagrangian}} \arrow[r] & \text{Observables} \end{tikzcd}$$

Bootstrap

Physical principles $\longrightarrow$ ~~Theory Lagrangian~~ $\longrightarrow$ Observables

Bootstrap

	Unitarity		
Flat space	Locality	$\lambda\phi^3 + \dots$	Amplitudes
	Causality		
	Lorentz invariance		

Bootstrap

Physical principles $\longrightarrow$ ~~Theory Lagrangian~~ $\longrightarrow$ Observables

Bootstrap

Flat space

Unitarity
Locality
Causality
Lorentz invariance

$\cancel{\lambda\phi^4 + \dots}$

Amplitudes

S-matrix bootstrap (1960's)

Bootstrap

$$\text{Physical principles} \longrightarrow \cancel{\text{Theory Lagrangian}} \longrightarrow \text{Observables}$$

$$\text{Physical principles} \xrightarrow{\text{Bootstrap}} \text{Observables}$$

Flat space

Unitarity

Locality

Causality

Lorentz invariance

$\cancel{\lambda\phi^3 + \ldots}$

Amplitudes

S-matrix bootstrap (1960's)

dS space

Unitarity

Locality

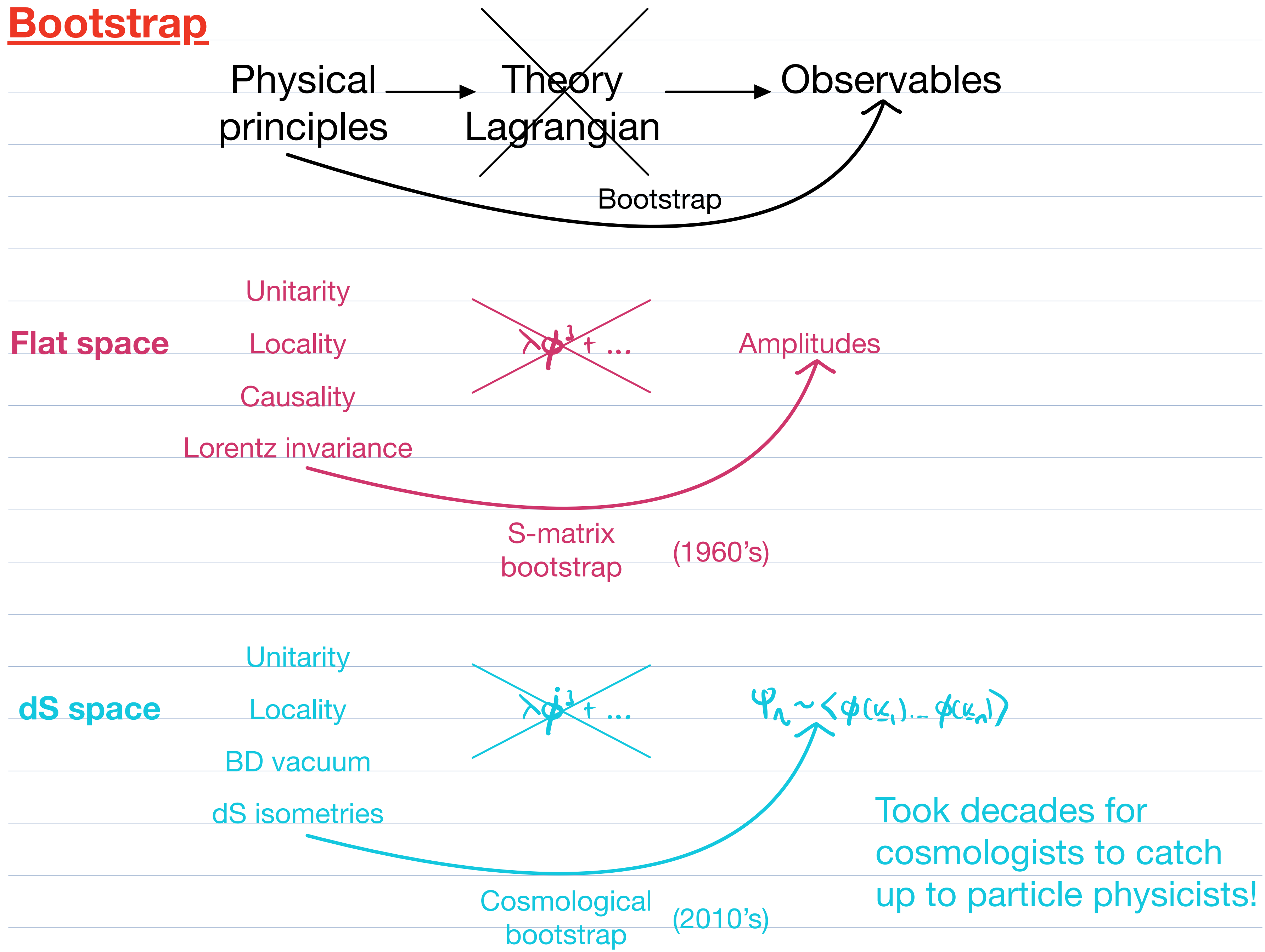
BD vacuum

dS isometries

$\lambda\dot{\phi}^3 + \ldots$

$\Psi_n \sim \langle \phi(\underline{k}_1) \ldots \phi(\underline{k}_n) \rangle$

Bootstrap


Physical principles
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λφ³ + ...
ψn ~ ⟨φ(k1)...φ(kn)⟩
Cosmological bootstrap
(2010's)
Took decades for cosmologists to catch up to particle physicists!

Bootstrap (cont.)

But why do we think we should be able to "bootstrap" in dS?

$$\lim_{k_T \to 0} \psi_n \sim \frac{A_n}{k_T^{p}}$$

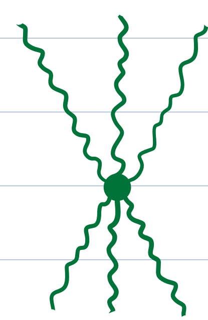
Flat-space Amplitudes live in Cosmological Correlators!

ESP: $k_T = k_1 + k_2 + k_3$; $e_2 = k_1 k_2 + k_1 k_3 + k_2 k_3$; $e_3 = k_1 k_2 k_3$

in this high energy limit modes do not feel universe's expansion

equivalent to pushing interactions to infinite past where they don't "see" the boundary so

becomes $\langle in | out \rangle$ amplitude



interactions no longer
"see" the boundary

Previous results for Graviton 3-pt functions $\langle\gamma\gamma\gamma\rangle$

$\epsilon_{i_1 i_2}(\underline{k}_1)\epsilon_{i_3 i_4}(\underline{k}_2)\epsilon_{i_5 i_6}(\underline{k}_3)$ (parity-even)

$\varepsilon_{i_1 i_2 i_3}\epsilon_{i_4 i_5}(\underline{k}_1)\epsilon_{i_6 i_7}(\underline{k}_2)\epsilon_{i_8 i_9}(\underline{k}_3)$ (parity-odd)

dS_4	SO(4,1)	2 parity-even $\langle\gamma\gamma\gamma\rangle$
(dS_{3+1})	rotations, translations, dilations,	0 parity-odd $\langle\gamma\gamma\gamma\rangle$
	dS boosts (or SCT)	arXiv:1104.2846

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dS_4 (dS_{3+1})	SO(4,1) rotations, translations, dilations, dS boosts (or SCT)	2 parity-even $\langle\gamma\gamma\gamma\rangle$ 0 parity-odd $\langle\gamma\gamma\gamma\rangle$ arXiv:1104.2846
dS_4 w/o dS boosts (observed symmetries)	ISO(3) w/ dilations + Unitarity + Locality	∞ parity-even $\langle\gamma\gamma\gamma\rangle$ 3 parity-odd $\langle\gamma\gamma\gamma\rangle$ 1 parity-odd in EFToI arXiv:2109.10189

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dS_4 (dS_{3+1})

SO(4,1)

rotations, translations, dilations, dS boosts (or SCT)

2 parity-even $\langle\gamma\gamma\gamma\rangle$

0 parity-odd $\langle\gamma\gamma\gamma\rangle$

arXiv:1104.2846

dS_4 w/o dS boosts

(observed symmetries)

ISO(3) w/ dilations
+
Unitarity
+
Locality

∞ parity-even $\langle\gamma\gamma\gamma\rangle$

3 parity-odd $\langle\gamma\gamma\gamma\rangle$

1 parity-odd in EFToI

arXiv:2109.10189

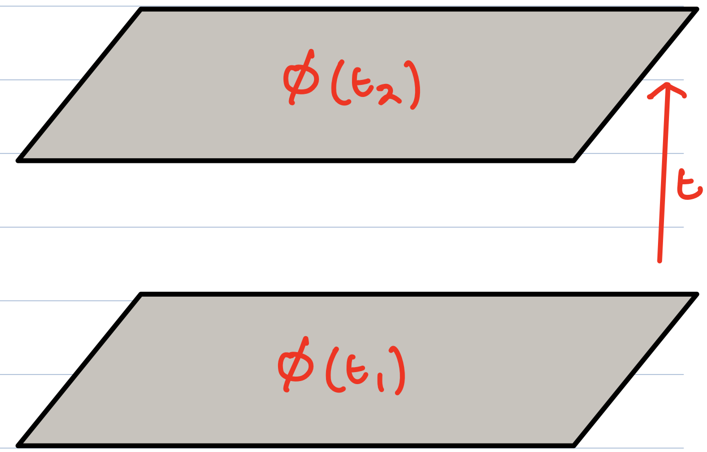
which belong to EFToI?

In our paper we use a combination of bulk computations and bootstrap tools to identify which parity-even $\langle\gamma\gamma\gamma\rangle$ belong to the EFToI.

Ways to break dS boosts (or SCT)

EFToI: dS boosts broken by $\phi(t)$ with t-dependent VEV

Time diffs. non-linearly realised by $\phi(t)$,
i.e. $\phi(t_0) \equiv \phi_0$ is a "clock" in unitary gauge



Spatial diffs. unbroken $\phi(t, \vec{x}) \equiv \phi_0(t)$

EFToI is the most general theory of
fluctuations around a quasi-dS background
for single field models

Solid inflation: Spatial diffs. broken

1 parity-odd $\langle \gamma\gamma\gamma \rangle$ in EFToI
BUT what about parity-even $\langle \gamma\gamma\gamma \rangle$?

EFToI

ADM formalism $ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$

Unitary gauge $h_{ij} = a^2(t) e^{2\zeta} (e^{\gamma})_{ij}$ $\quad \gamma_{ii} = 0 = \partial_i \gamma_{ij}$

$\zeta = 0$ $\because$ only care about γ_{ij} traceless transverse

Resulting metric is $ds^2 = -dt^2 + a^2(t)(e^{\gamma})_{ij} dx^i dx^j$

In unitary gauge the most general action we can write is:

$$S = \int d^4x \sqrt{-g}\, F(\overbrace{R_{\mu\nu\rho\sigma}, g^{00}, K_{\mu\nu}, \nabla_\mu}^{\text{these are the building blocks e.g. } f(t)\, g^{00} \nabla_\mu R^{\mu}{}_{0\nu\rho} K^{\rho}{}_{0}}, t)$$

arXiv:0709.0293

i.e. a theory which is invariant under spatial diffs. and time diffs.

non-linearly realised by a $\phi(t, \vec{x}) = \phi_0(t)$ which acts as a "clock"

EFToI (cont.)

$$S = S_0 + M_{pl}^2 \int d^4x \sqrt{-g} \left[\delta K^{ij} \mathcal{O}_{(0)} \delta K_{ij} + \mathcal{O}(\delta K^3) \right]$$

where $\mathcal{O}_{(i)} = \frac{1}{M_{pl}^{(i)}} \sum_{m,n} b_{m,n} \nabla_0^m \bar{\nabla}^{2n}$ $\quad [b_{m,n}] = -(m+2n)$

Showed for the <u>FIRST TIME</u> using field redefinitions and geometric identities that we only need <u>ONE</u> building block δK_{ij} to construct cubic interactions which contribute to $\langle \gamma\gamma\gamma \rangle$

$$S_0 = S_{\gamma, GR} = \frac{M_{pl}^2}{8} \int dt\, d^3x\, a^3(t) \left[\dot{\gamma}_{ij} \dot{\gamma}_{ij} - a^{-2} \partial_k \gamma_{ij} \partial_k \gamma_{ij} + a^{-2} (2\gamma_{ik}\gamma_{jl} - \gamma_{ij}\gamma_{kl}) \partial_k \partial_l \gamma_{ij} \right] + \mathcal{O}(\gamma^4)$$

for γ_{ij} operators indices raised+lowered by δ_{ij}

Convert to flat-slicing (Poincaré) co-ordinates $\quad ds^2 = a^2(\eta)(-d\eta^2 + dx^2) \quad a(\eta) = -\frac{1}{H\eta}$

Parity-even $\langle\gamma\gamma\gamma\rangle$ in EFToI

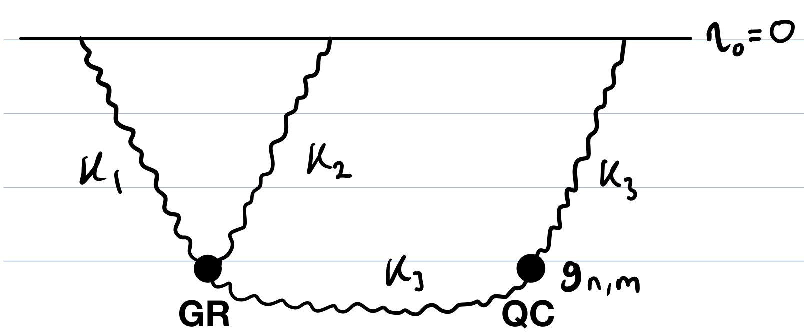
$$S = S_{EH} + \int d^4x \sqrt{-g}\, F\left(\gamma'_{ij} + O(\gamma^2), \nabla_0, \bar{\nabla}_i\right)$$

Type-I from $(\delta K)^2$ operators

$$\delta K^i{}_j\, O_{(0)}\, \delta K^j{}_i = (\gamma' + O(\gamma^2))^i{}_j (\gamma' + O(\gamma^2))^j{}_i$$

Both Quadratic $O(\gamma^2)$ and Cubic $O(\gamma^3)$ contributions

At leading order in field theory couplings the QC $\Rightarrow$ single exchange process



GR vertex has no $g_{n,m}$

No k_3 poles $\because$ too many derivatives in EFToI

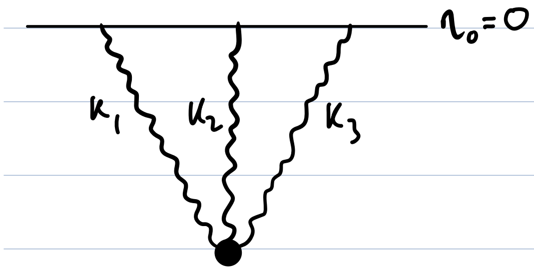
General MASTER TIME INTEGRAL computed using Meltzer method arXiv:2107.10266

$$M(\alpha,\beta) = \int d\eta\, d\eta'\, a^2(\eta) K(k_1,\eta) K(k_2,\eta) G(k_3,\eta,\eta') a^{\alpha}(\eta') K^{(\beta)}(k_3,\eta')$$

$$\alpha \leq 0 \qquad \beta \geq 1$$

Type-I (cont.) from $(\delta K)^2$ operators

Type-I operators also have Cubic $\mathcal{O}(\gamma^3)$ contribution $\Rightarrow$ contact diagram



Comparable size to single-exchange diagram $\therefore$

they are needed to satisfy EFToI consistency relations

Time integral computed

$$I_{n,m}(k_1,k_2,k_3) = \int d\eta \, \eta^{n+2m-2} K^{(n+1)}(k_1,\eta) K'(k_2,\eta) K(k_3,\eta)$$

Tensor structures computed separately using S-H formalism

Combine Contact + Single-Exchange diagrams to obtain Type-I B_{2BG}^{+++}, B_{2BG}^{++-} etc.

Checked that these satisfied the EFToI consistency relations

Take soft limit $k_3 \to 0$ of $\Psi_{3,\text{total}} = \Psi_{2BB,\text{exchange}} + \Psi_{2BB,\text{contact}}$

Type-II from $(\delta K)^3$ operators

$$\delta K_i^{\ j}\,\delta K_j^{\ k}\,\delta K_k^{\ i} = (\gamma' + O(\gamma^2))_i^{\ j}(\gamma' + O(\gamma^2))_j^{\ k}(\gamma' + O(\gamma^2))_k^{\ i}$$

$$\gamma'_{ij} \propto k^2 \Rightarrow (\gamma')^3 \propto k_1^2 k_2^2 k_3^2 = e_3^2$$

$$\psi_3^{h_1 h_2 h_3} = \epsilon_{ij}^{h_1}(\underline{k}_1)\,\epsilon_{kl}^{h_2}(\underline{k}_2)\,\epsilon_{mn}^{h_3}(\underline{k}_3)\,\psi_{3,\text{trimmed}}$$

$\delta K_{ij} \propto \gamma'_{ij}$ α related to no. of ∂_k

$$\psi_n \propto k^3 \qquad \psi_{3,\text{trimmed}} \propto \frac{e_3^2}{k_T^p}\,\text{Poly}_{p-3-\alpha}(k_T, e_2, e_3)$$

BD-vacuum   polynomial of degree $p-3-\alpha$

no logs $\because$ $2n\partial_\eta + n\partial_i > 3$ in EFToI

MLT $\left.\dfrac{\partial \psi_{n,\text{trimmed}}}{\partial k_a}\right|_{k_a=0} = 0$ this is trivially satisfied $\because$ of e_3^2

Combine with tensor structures to obtain full $\psi_3^{+++} \longrightarrow B_{3BB}^{+++}$

B_{3BB}^{++-} etc. in paper obtained by $k_3 \to -k_3$ of h_α

$\psi_{3,\text{trimmed}}$ left unchanged

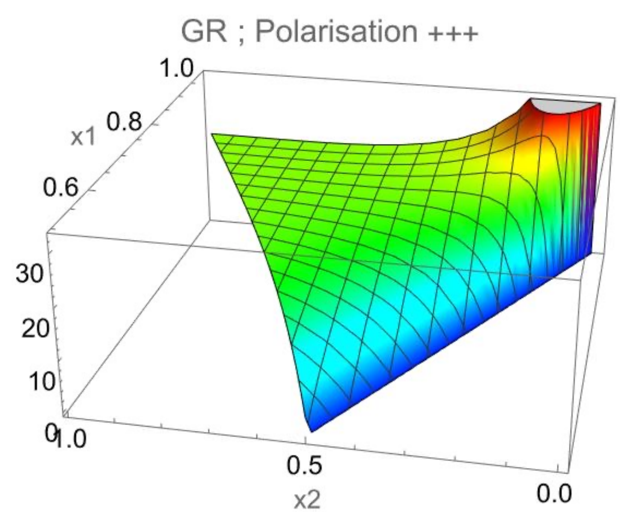
Look at paper for $\alpha = 2, 4, 6$

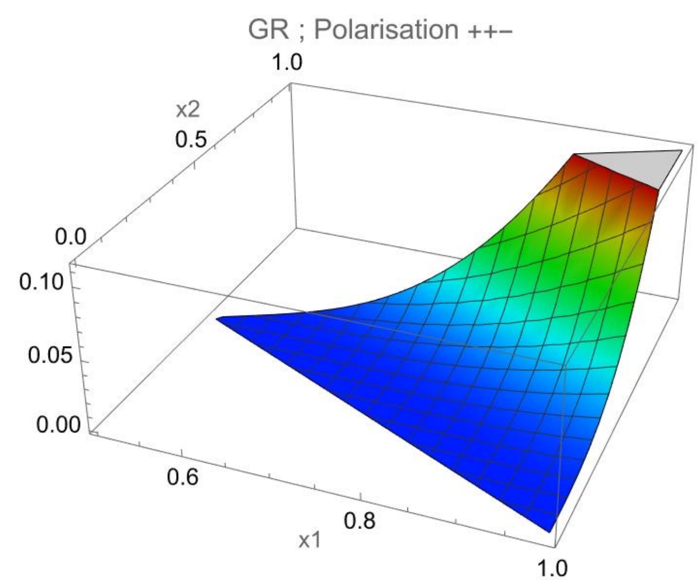
e.g. $B^{+++}_{3BB,\alpha=0} = \dfrac{e_3^2\, SH_{+++}}{e_3^3 k_T^p}\ \text{Poly}_{p-3}(k_T, e_2, e_3)$

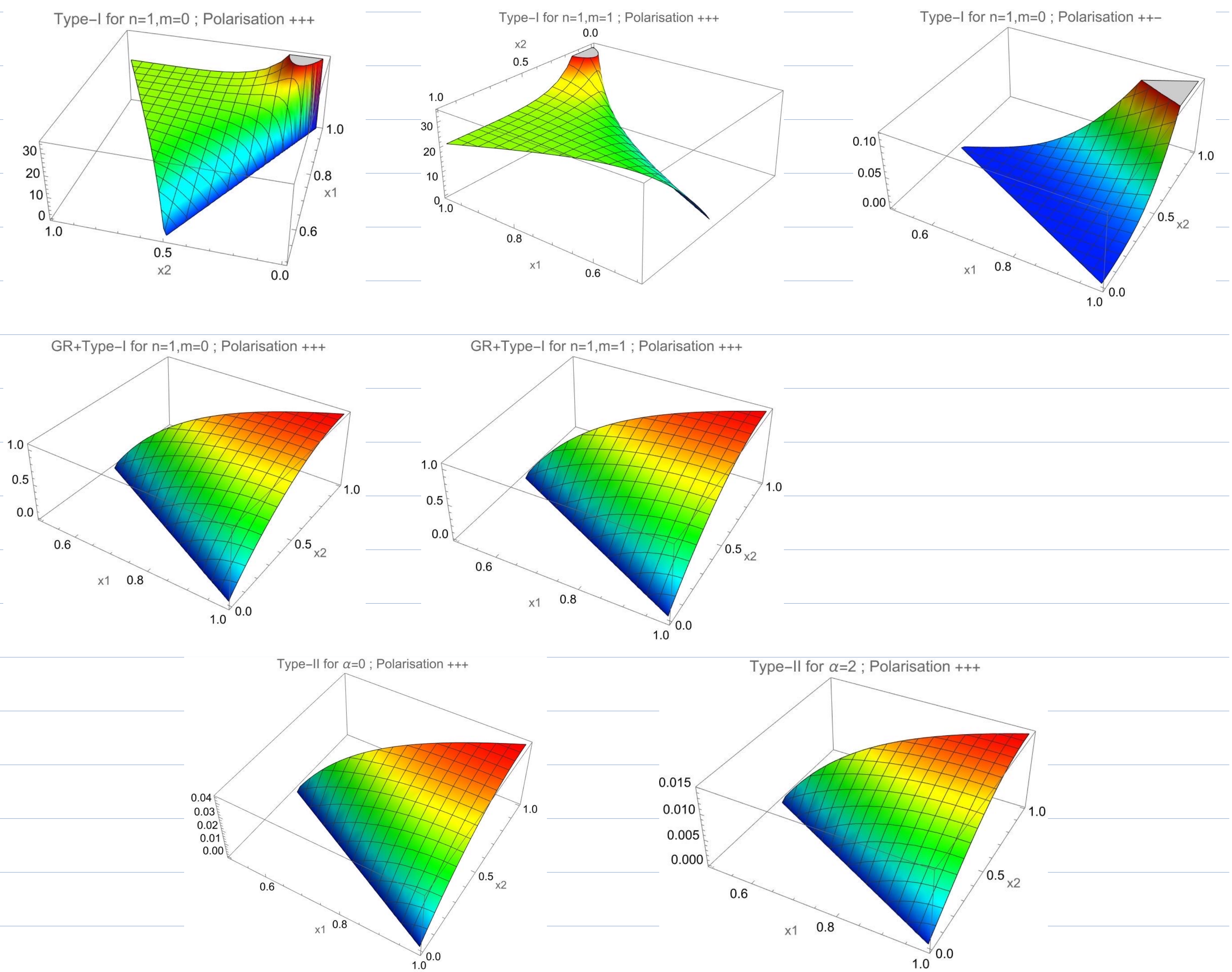
Conclusions

We derived all tree-level graviton 3-pt functions in the EFToI to all orders in derivatives:

- Showed for the first time that they can be constructed out of the extrinsic curvature only.
- Classified all graviton bispectra for the most general class of single-field inflationary models.
- Gives observers an idea of what to look for in the (very far) future, i.e. bispectra shapes to use as templates for B-mode searches.
- To do this we used a combination of bulk computations and bootstrap methods.
- Future work involves reproducing these results in a fully “bootstrap” way using consistency relations, i.e. soft theorems to sub-leading order.
- Construct mixed correlators, specifically those we hope to measure before $\langle \gamma\gamma\gamma \rangle$, i.e. $\langle \zeta\zeta\gamma \rangle$, $\langle \zeta\gamma\gamma \rangle$, by developing bootstrap tools to efficiently determine the form of these mixed correlators.
- Constrain bispectra by demanding consistency of higher-point functions, such as the trispectrum.







Type−I for n=1,m=0 ; Polarisation +++
Type−I for n=1,m=1 ; Polarisation +++
Type−I for n=1,m=0 ; Polarisation ++−
GR+Type−I for n=1,m=0 ; Polarisation +++
GR+Type−I for n=1,m=1 ; Polarisation +++
Type−II for α=0 ; Polarisation +++
Type−II for α=2 ; Polarisation +++
x1
x2

Born rule - WFU coefficients to Cosmological Correlators

$$\Psi[\eta_0; \varphi(\underline{k})]$$ $$\varphi^*(\underline{k}) = \varphi(-\underline{k})$$

$$= \exp\left[-\sum_{n=2}^{\infty} \frac{1}{n!} \int_{\underline{k}_1,\dots,\underline{k}_n} \psi_n(\{k\};\{\underline{k}\})\, \varphi(\underline{k}_1)\dots\varphi(\underline{k}_n) \right]$$

$$-\log(\Psi\Psi^*) = \sum_{n=2}^{\infty} \frac{1}{n!} \int_{\underline{k}_1,\dots,\underline{k}_n} \psi_n(\{k\};\{\underline{k}\})\varphi(\underline{k}_1)\dots\varphi(\underline{k}_n)$$

$$+ \sum_{n=2}^{\infty} \frac{1}{n!} \int_{\underline{k}_1,\dots,\underline{k}_n} \psi_n^*(\{k\};\{\underline{k}\})\varphi(-\underline{k}_1)\dots\varphi(-\underline{k}_n)$$

$$= \sum_{n=2}^{\infty} \frac{1}{n!} \int_{\underline{k}_1,\dots,\underline{k}_n} \psi_n(\{k\};\{\underline{k}\})\varphi(\underline{k}_1)\dots\varphi(\underline{k}_n)$$

$\{\underline{k}\} \to \{-\underline{k}\}$
for second integral

$$+ \sum_{n=2}^{\infty} \frac{1}{n!} \int_{\underline{k}_1,\dots,\underline{k}_n} \psi_n^*(\{k\};\{-\underline{k}\})\varphi(\underline{k}_1)\dots\varphi(\underline{k}_n)$$

$$\Rightarrow \Psi\Psi^* = \exp\left[-\sum_{n=2}^{\infty} \frac{1}{n!} \int_{\underline{k}_1,\dots,\underline{k}_n} \left[\psi_n(\{k\};\{\underline{k}\}) + \psi_n^*(\{k\};\{-\underline{k}\})\right]\varphi(\underline{k}_1)\dots\varphi(\underline{k}_n) \right]$$

Note that the COT states $\psi_n(\{k\};\{\underline{k}\}) + \psi_n^*(\{-k\};\{-\underline{k}\})$

$$B_n^{\text{contact}} = \langle \varphi(\underline{k}_1) \dots \varphi(\underline{k}_n) \rangle' = \frac{\int D\varphi\, \Psi \Psi^* \varphi(\underline{k}_1) \dots \varphi(\underline{k}_n)}{\int D\varphi\, \Psi \Psi^*}$$

becomes Gaussian integral

$\because$ of φ^2 for each $\underline{k}_a$

$\Rightarrow \frac{1}{2\,\mathrm{Re}\,\psi_2'(k_a)}$

$$= \frac{\psi_n'(\{k\};\{\underline{k}\}) + \psi_n'^*(\{k\};\{-\underline{k}\})}{\prod_{a=1}^{n} 2\,\mathrm{Re}\,\psi_2'(k_a)}$$

Extra Slides