A dynamical formulation of ghost-free massive gravity

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Based on work with Toby Wiseman, Claudia de Rham and Andrew Tolley

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Overview

1. What is massive gravity and why you might care about it
2. The Gauge is Gone: dynamical massive gravity
3. The numerics are weird, but we think we know why
4. What the future might hold

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Massive gravity 101

(see 1401.4173)

$$\mathcal{L}_{GR} = -\frac{1}{4} h^T_{\mu\nu} \mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta}$$

Unique stable mass term

Breaks gauge invariance!

$$\mathcal{L}_{MG} = \mathcal{L}_{GR} - \frac{1}{8} m^2 (h^2_{\mu\nu} - h^2)$$

Main changes: weaker on large scales, 5 dofs

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Problem I

$$-\frac{1}{2}(\nabla^2 - m^2)h_{\mu\nu} + \partial_{(\mu}\chi_{\nu)} = T_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu}T$$

vs.

vDVZ discontinuity!

$$-\frac{1}{2}\nabla^2 h_{\mu\nu} + \partial_{(\mu}\chi_{\nu)} = T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T$$

Where $\chi_\mu = \partial^\rho h_{\rho\mu} - \frac{1}{2}\partial_\mu h$

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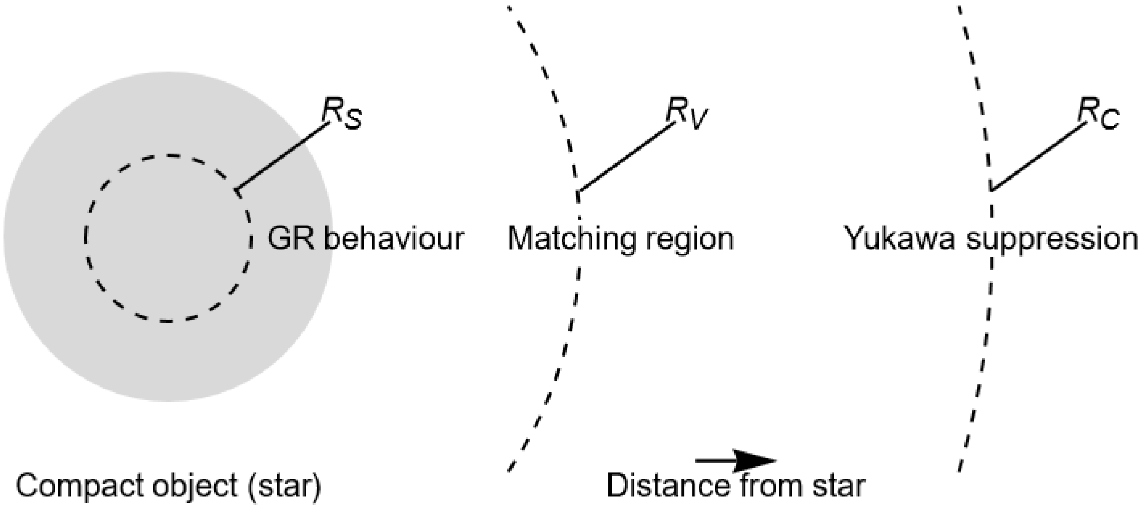
Solution I

$$h = -\frac{2}{3m^2} T$$

This blows up in the small mass limit. Hence, we need to account for non-linearities!



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Problem II

Suppose non-linearly we have

$$G_{\mu\nu} + m^2 M_{\mu\nu} = T_{\mu\nu}$$

This gives us 4 constraints

$$\nabla^{\mu} M_{\mu\nu} = 0$$

For a total of 6 dofs. But we're only meant to have 5! The 6th is the **Boulware-Deser ghost**

Solution II

To avoid the ghost, we must use the **dRGT mass term**:

$$\mathcal{L}_{dRGT} = \frac{m^2}{4}\sqrt{-g}\sum_{n=0}^{4}\beta_n \mathcal{L}_n[E]$$

$$E := \sqrt{g^{-1}f}$$

Fixed reference metric
(you need to contract g with *something*)

Symmetric polynomials in the eigenvalues.
Includes a cosmological constant and
a non-dynamical term for $n = 4$

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EOM

For the purposes of this talk, the equations of motion are

$$G_{\mu\nu} = T_{\mu\nu} - m_1^2 M_{\mu\nu}^{(1)} - m_2^2 M_{\mu\nu}^{(2)}$$

$$M_{\mu\nu}^{(1)} = -E_{\mu\nu} + [E]g_{\mu\nu} - 3g_{\mu\nu}$$

$$M_{\mu\nu}^{(2)} = \frac{1}{2} E_\mu{}^\alpha E_{\alpha\nu} - \frac{1}{2}[E]E_{\mu\nu} - \frac{1}{4}\left([E^2] - [E]^2\right) g_{\mu\nu} - \frac{3}{2} g_{\mu\nu}$$

$$m^2 = m_1^2 + m_2^2$$

Where the square brackets denote a trace.

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EOM

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$$G_{\mu\nu} = T_{\mu\nu} - m_1^2 M_{\mu\nu}^{(1)} - m_2^2 M_{\mu\nu}^{(2)}$$

Now what?

$$M_{\mu\nu}^{(1)} = -E_{\mu\nu} + [E]g_{\mu\nu} - 3g_{\mu\nu}$$

$$M_{\mu\nu}^{(2)} = \frac{1}{2}E_{\mu}{}^{\alpha}E_{\alpha\nu} - \frac{1}{2}[E]E_{\mu\nu} - \frac{1}{4}([E^2] - [E]^2)\, g_{\mu\nu} - \frac{3}{2}g_{\mu\nu}$$

$$m^2 = m_1^2 + m_2^2$$

Where the square brackets denote a trace.

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MG trouble

Two metrics: g and f, but only enough gauge freedom for one!

For us, f is Minkowski, and we work in Cartesians, so

$$f_{\mu\nu} = \mathrm{diag}(-1, +1, +1, +1)$$

We will work with the components of the $E_{\mu\nu}$ vierbein directly

We also decompose its spatial part as

$$E_{ij} = \tilde{E}_{ij} + \tilde{E}\delta_{ij}$$

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Constraints galore

Recall the MG equation of motion:

$$\mathcal{E}_{\mu\nu} := G_{\mu\nu} + m_1^2 M^{(1)}_{\mu\nu} + m_2^2 M^{(2)}_{\mu\nu} - T_{\mu\nu} = 0$$

A priori, this propagates 10 dofs in E: E_{tt}, E_{ti}, $\tilde{E}$ and $\tilde{E}_{ij}$

Take divergence of EOM:

$$0 = V_\mu = \nabla^\nu \left(m_1^2 M^{(1)}_{\mu\nu} + m_2^2 M^{(2)}_{\mu\nu} \right)$$

This **vector constraint** can be used to solve the second order dynamics of E_{ti} and $\tilde{E}$

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Constraints galore

Define

$$\xi_\mu := E_{\mu\alpha} f^{\alpha\beta} V_\beta$$

Then, somewhat magically

$$0 = \frac{1}{2}(m_1^2 g^{\mu\nu} + m_2^2 E^{\mu\nu})\mathcal{E}_{\mu\nu} + \nabla \cdot \xi$$

Gives a **scalar constraint** which can determine E_{tt} **algebraically**

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Constraints galore

Finally, we still have the usual **Hamiltonian** and **Momentum constraints**

$$\mathcal{E}^t{}_t, \mathcal{E}^t{}_i$$

Which fix E_{ti} and $\tilde{E}$ on the initial surface.

In the end, we only have 5 propagating dofs: they are the $\tilde{E}_{ij}$

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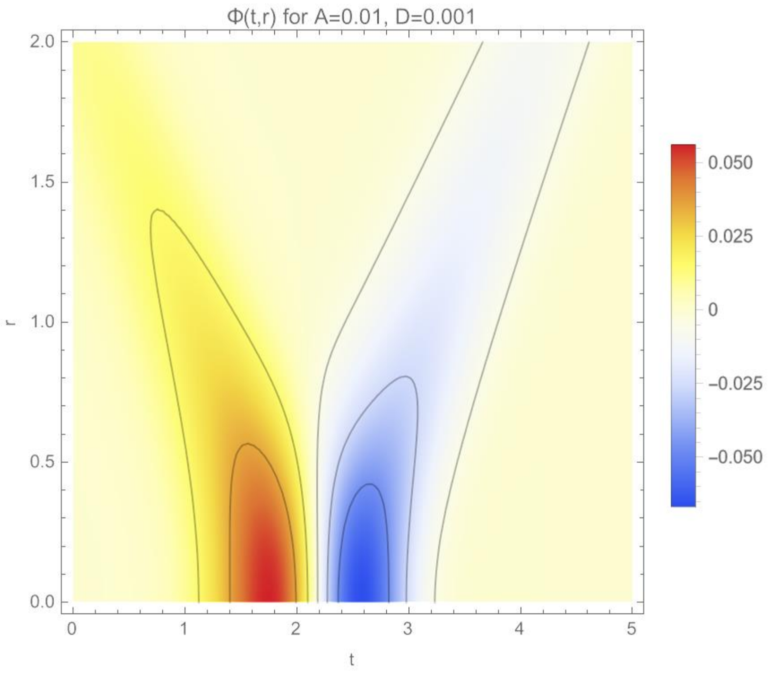
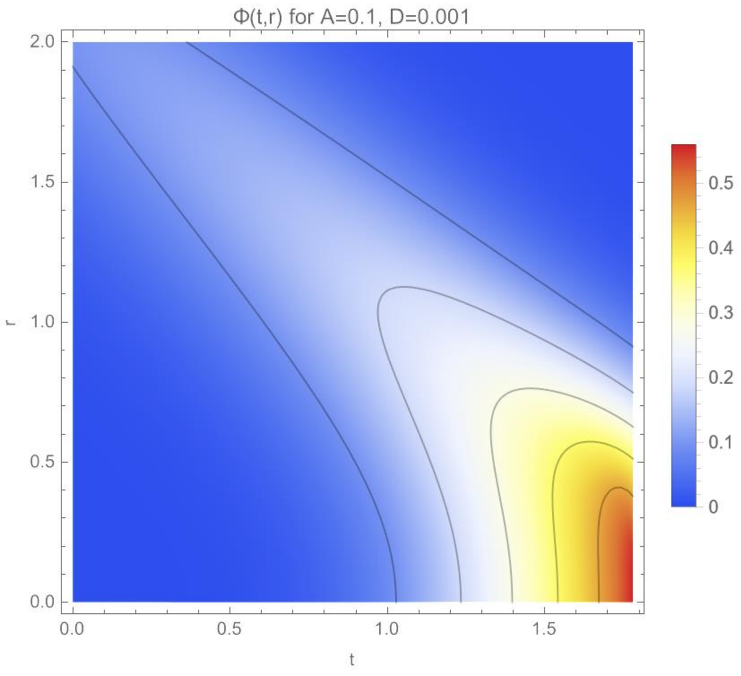
A sliver of results

We work in spherical symmetry. Small initial data works fine, larger data breaks.





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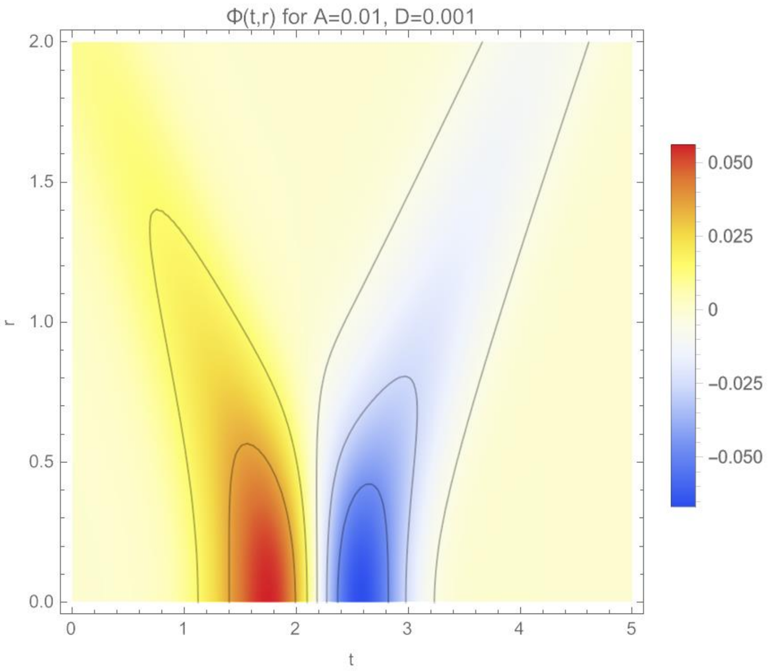

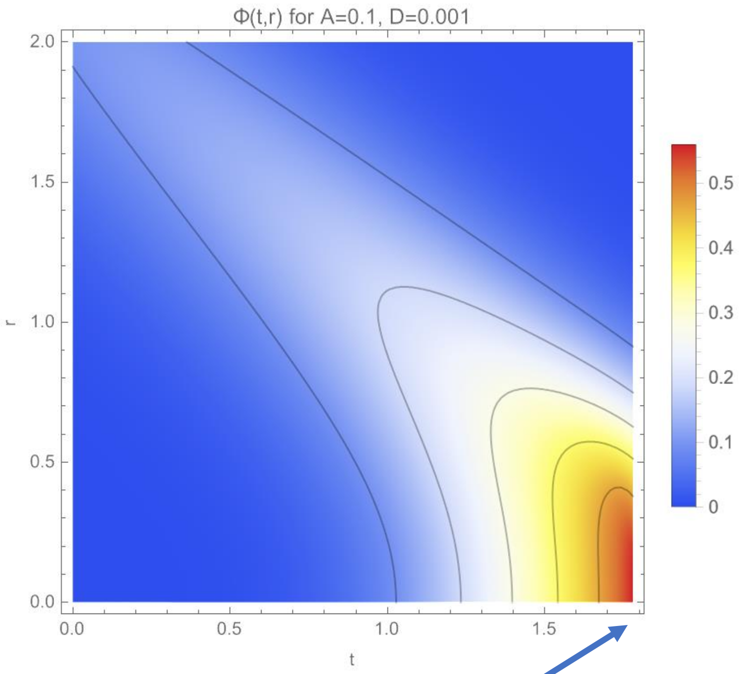

Scalar constraint breaks

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What's next?

- Go to higher mass terms
- Check whether Vainshtein works
- Relax spherical symmetry
- Maybe a merger?
- Try other reference metrics

Thank you!