Wilson loops for 5d and 3d conformal linear quivers

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Based on [2209.07536] AF, Carlos Nunez

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Overview

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Introduction

- The classification of Type II or M-theory backgrounds with AdS_{d+1} factors is of importance
- These backgrounds being holographic duals to SCFTs in d dimensions with different amounts of SUSY.
- $\Rightarrow$ Conformal and supersymmetric linear quiver field theories in 3d and 5d preserving eight Poincare supercharges
- SUSY gauge theories $\rightarrow$ the Wilson loop can be computed exactly

Background

- $\mathcal{N} = 1$ SUSY in five dimensions and $\mathcal{N} = 4$ in three dimensions

In three dimensions, the Wilson loop in $\mathcal{N} = 4$ supersymmetric field theories is labelled by a representation $\mathbb{R}$ of a given gauge group,

$$W_{\mathbb{R}} = \mathrm{Tr}_{\mathbb{R}}\, \mathcal{P} e^{i \oint \left(A_\mu \dot{x}^\mu + \sigma_3 \sqrt{-\dot{x}^2} \right) d\tau}.$$

Where σ_3 is one of the three scalars in the vector multiplet.

3d $\mathcal{N} = 4$ linear quiver gauge theories

$$
\begin{array}{ccccccc}
U(N_1) & - & U(N_2) & - \dots - & U(N_{L-1}) & - & U(N_L) \\
| & & | & & | & & | \\
[k_1] & & [k_2] & & [k_{L-1}] & & [k_L]
\end{array}
$$

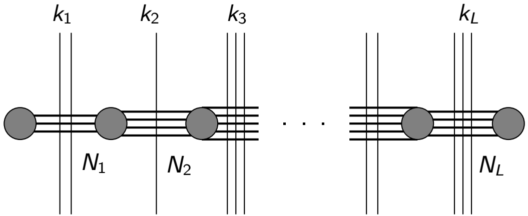
$U(\cdot)$ nodes are nodes connected by bifundamental hypermultiplets. The theory also has $SU(2) \times SU(2)$ global R-symmetry.
These theories can be engineered by configurations of D3-branes which are ending on and suspended between NS5 and D5 branes,

[Hanany, Witten '96, Gaiotto Witten '08].

	0	1	2	3	4	5	6	7	8	9
D3	–	–	–	–						
D5	–	–	–		–	–	–			
NS5	–	–	–					–	–	–
F1	–									–
D5′	–				–	–	–	–	–	

[Coccia, Uhlemann '21]

Each $U(N_t)$ gauge node is represented by N_t D3-branes suspended between NS5 branes, while D5-branes intersecting the D3-branes represent fundamental matter.

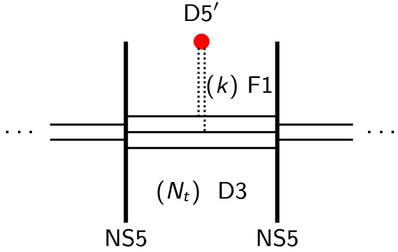


Half-BPS Wilson loops

- preserves $U(1)_C \times SU(2)_H$ subgroup of the $SU(2)_C \times SU(2)_H$ R-symmetry

[Assel, Gomis '15]

Wilson loops in the fundamental representation of the each gauge node are built by a string ending on the stack of D3-branes associated with it.
For the Wilson loops in the antisymmetric representation of rank k of the $U(N_t)$ gauge group, k fundamental strings stretching between D5′ branes and the D3-branes associated with $U(N_t)$, is needed.



Type IIB supergravity solution

The type IIB background in string frame is: *[Akhond, Legramandi, Nunez '21]*

$$ds^2_{10,st} = f_1(\sigma,\eta)\Big[ds^2(\mathrm{AdS}_4) + f_2(\sigma,\eta)ds^2(S^2_1) + f_3(\sigma,\eta)ds^2(S^2_2) + f_4(\sigma,\eta)(d\sigma^2 + d\eta^2)\Big], \quad e^{-2\Phi} = f_5(\sigma,\eta),$$

$$B_2 = f_6(\sigma,\eta)\mathrm{Vol}(S^2_1), \quad C_2 = f_7(\sigma,\eta)\mathrm{Vol}(S^2_2), \quad \tilde{C}_4 = f_8(\sigma,\eta)\mathrm{Vol}(\mathrm{AdS}_4),$$

$$f_1 = f_1(V_3(\sigma,\eta)), \quad f_2 = f_2(V_3(\sigma,\eta)), \quad \cdots$$

Where the fluxes are defined from the potentials as follows,

$$F_1 = 0, \quad H_3 = dB_2 \quad F_3 = dC_2, \quad F_5 = d\tilde{C}_4 + *d\tilde{C}_4.$$

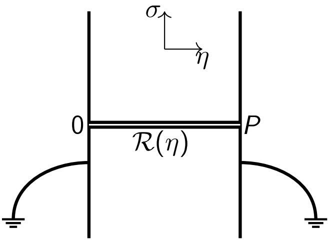
The configuration is solution to the Type IIB equations of motion, if the function $V(\sigma,\eta)$ satisfies,

$$\partial_\sigma\left(\sigma^2\partial_\sigma V_3\right) + \sigma^2\partial^2_\eta V_3 = 0.$$

We define and $\sigma V_3(\sigma,\eta)=\partial_\eta \hat{W}(\sigma,\eta)$ with the coordinates to range in $0\leq\eta\leq P$ and $-\infty<\sigma<\infty$.

$$\partial_\sigma^2 \hat{W}(\sigma,\eta)+\partial_\eta^2 \hat{W}(\sigma,\eta)=0, \qquad \text{(almost everywhere)}$$
$$\hat{W}(\sigma,\eta=0)=0, \quad \hat{W}(\sigma,\eta=P)=0,$$
$$\partial_\sigma \hat{W}(\sigma=0^+,\eta)-\partial_\sigma \hat{W}(\sigma=0^-,\eta)=-\mathcal{R}(\eta).$$

The function $\mathcal{R}(\eta)$ is the input determined by the dual quiver field theory.

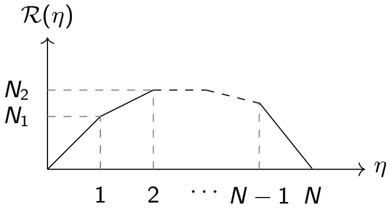


We Fourier decomposition the rank function $\mathcal{R}(\eta)$

$$\mathcal{R}(\eta) = \sum_{k=1}^{\infty} \mathcal{R}_k \sin\left(\frac{k\pi}{P}\eta\right), \quad \mathcal{R}_k = \frac{2}{P}\int_0^P R(\eta) \sin\left(\frac{k\pi\eta}{P}\right) d\eta.$$

Imposing the quantisation of the conserved Page charges in the background it is found that the function $\mathcal{R}(\eta)$ must be a convex piecewise linear function.

$$\mathcal{R}(\eta) = \begin{cases} N_1\eta & 0 \leq \eta \leq 1 \\ N_l + (N_{l+1} - N_l)(\eta - l) & l \leq \eta \leq l+1, \quad l := 1, ..., P-2 \\ N_{P-1}(P-\eta) & (P-1) \leq \eta \leq P. \end{cases}$$

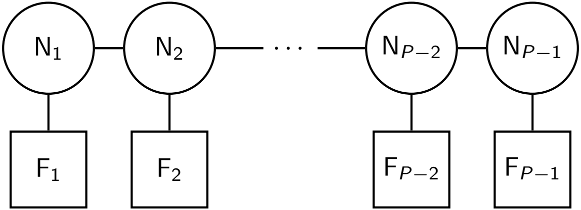


The number of D3 (colour) branes and D5 (flavour) branes in the interval $[k, k+1]$ and the total number of branes are given as,

[Akhond, Legramandi, Nunez '21]

$$N_{D3}[k,k+1]=N_k, \quad N_{D5}[k,k+1]=2N_k-N_{k+1}-N_{k-1}, \quad N_{NS5}^{\text{total}}=P.$$

For the generic rank function $\mathcal{R}(\eta)$, the supergravity background is proposed to be dual to the strongly coupled, IR-fixed point of the quiver in the figure below for which $F_i=2N_i-N_{i+1}-N_{i-1}$. In other words, the quiver is balanced.



For Fourier transform of $\hat{W}$ we have

$$\hat{W}(\sigma,\eta)=\sum_{k=1}^{\infty} b_k\left(\frac{P}{k\pi}\right)\sin\left(\frac{k\pi\eta}{P}\right)e^{-\frac{k\pi|\sigma|}{P}}. \tag{1}$$

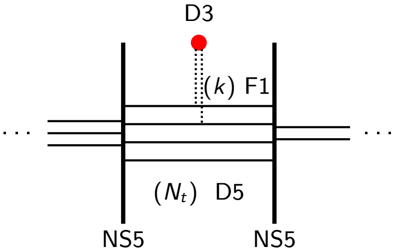
5d $\mathcal{N}=1$ linear quiver gauge theories

$$
\begin{array}{ccccccc}
SU(N_1) & - & SU(N_2) & - \ldots - & SU(N_{L-1}) & - & SU(N_L) \\
| & & | & & | & & | \\
[k_1] & & [k_2] & & [k_{L-1}] & & [k_L]
\end{array}
$$

$SU(\cdot)$ nodes are nodes connected by bifundamental hypermultiplets.
These theories can be engineered by configurations of D5-branes which are ending on and suspended between NS5 and D7 branes.

	0	1	2	3	4	5	6	7	8	9
D5	–	–	–	–	–	–				
NS5	–	–	–	–	–		–			
D7	–	–	–	–	–			–	–	–

For the Wilson loops in the antisymmetric representation of rank k we have



Type IIB supergravity solution

The type IIB background in string frame is [*Legramandi, Nunez '21]*

$$ds^2_{10,st} = f_1(\sigma,\eta)\Big[ds^2(AdS_6) + f_2(\sigma,\eta)ds^2(S^2) + f_3(\sigma,\eta)(d\sigma^2 + d\eta^2)\Big],$$

$$e^{-2\Phi} = f_6(\sigma,\eta), B_2 = f_4(\sigma,\eta)Vol(S^2), \quad C_2 = f_5(\sigma,\eta)Vol(S^2), \quad C_0 = f_7(\sigma,\eta),$$

$$f_1 = f_1(V_5(\sigma,\eta)), \quad f_2 = f_2(V_5(\sigma,\eta)), \quad \cdots$$

The function $V_5(\sigma,\eta)$ solves

$$\partial_\sigma\left(\sigma^2\partial_\sigma V_5\right) + \sigma^2\partial_\eta^2 V_5 = 0.$$

$$\hat{V}_5(\sigma,\eta) = \sum_{k=1}^{\infty} a_k \sin\left(\frac{k\pi}{P}\eta\right) e^{-\frac{k\pi}{P}|\sigma|}, \qquad a_k = \frac{P}{2\pi k}\mathcal{R}_k.$$

Results

- For 3d SCFT Wilson loop

$$\ln\langle W_\wedge\rangle = \pi(-\hat{W}(\sigma,\eta) + \sigma\partial_\sigma \hat{W}(\sigma,\eta))\Big|_{(\sigma*,\eta*)}, \tag{2}$$

- For 5d Wilson loop

$$\ln\langle W_\wedge\rangle = -3\pi(-\hat{V}_5(\sigma,\eta) + \sigma\partial_\sigma \hat{V}_5(\sigma,\eta))\Big|_{(\sigma*,\eta*)}. \tag{3}$$

- $$\mu^{-1}\ln\langle W_\wedge\rangle = \pi\sum_{k=1}^{\infty}\frac{\mathcal{R}_k}{2}\left(\frac{P}{k\pi}\right)\sin\left(\frac{k\pi}{P}\eta^*\right)e^{-\frac{k\pi}{P}|\sigma^*|}\left(\frac{k\pi}{P}|\sigma^*|+1\right),$$

  $\mu_{3d} = \pi, \qquad \mu_{5d} = 3\pi.$

- For triangular quivers, under mirror symmetry the Wilson loop transforms as $N_f^{el}\ln\langle W_\wedge^{el}\rangle = N_f^{mag}\ln\langle W_\wedge^{mag}\rangle$.

Summary

- We summarised the electrostatic description of an infinite family of Type IIB backgrounds dual to SCFTs in five and three spacetime dimensions, preserving eight Poincare supercharges.
- the result for the VEV of Wilson loops for a given gauge group in a given antisymmetric representation.
- The action of mirror symmetry on three dimensional quiver field theories are considered and also how the holographic description of balanced quivers with one flavour node realises this symmetry.
- Generalisation to 4d and 6d SCFTs are expected.

Thank you