Higher Order Corrections in Pure Spinor SYM

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Sections

1. Introduction
2. Non-Linear Super Yang-Mills
3. Ansatz Building
4. α'^3 Corrections

Non Linear Fields

- Pure Spinor Superstring OPEs inspired a recursive approach [1012.398]

  λ^α where $\lambda\gamma^m\lambda = 0$.

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- Inspired Non-linear fields that generate Multiparticle fields

$$\mathbb{A}_\alpha(x,\theta)\,,\ \mathbb{A}^m(x,\theta)\,,\ \mathbb{F}^{mn}(x,\theta)\,,\ \mathbb{W}^\alpha(x,\theta)$$

Non Linear Fields

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$$\mathbb{A}_\alpha(x,\theta)\,,\ \mathbb{A}^m(x,\theta)\,,\ \mathbb{F}^{mn}(x,\theta)\,,\ \mathbb{W}^\alpha(x,\theta)$$

- Capture the n-point expansion of the field and KineMatic factors!

$$\boxed{\text{Tree-Level} \rightarrow \tfrac{1}{3}\mathrm{Tr}\langle \mathbb{V}\,\mathbb{V}\,\mathbb{V}\rangle} \longleftarrow [1510.08843]$$

$$\langle \lambda^3\theta^5\rangle = 2880\,,\quad \mathbb{V} = \lambda^\alpha \mathbb{A}_\alpha$$

Higher Order Corrections & Methods

- Non-Abelion α'^4 Corrections found using Superspace Methods
  + Cohomology
  [1004.3466]

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- BRST operator in PS = Simple!

$$Q = \lambda^\alpha D_\alpha \qquad \text{[hep-th/0001035].}$$

Spinor Derivative

$$D_\alpha \equiv \partial_\alpha + \tfrac{1}{2} (\gamma^m \theta)_\alpha \partial_m$$

Higher Order Corrections & Methods

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- BRST operator in PS = Simple!

$$Q = \lambda^\alpha D_\alpha$$ [hep-th/0001035].

- Find BRST Closed (NOT EXACT) expression at Correct Mass dimension

↳ Obtain the Perturbiner expansion

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Non-Linear Equation of Motion

- Non-Linear SYM is built from the Spinor and Vector Covariant derivatives,

$$\nabla_\alpha = D_\alpha - A_\alpha \,, \quad \nabla_m = \partial_m - A_m$$

- We impose the following Constraint,

$$\{\nabla_\alpha, \nabla_\beta\} = \gamma^m_{\alpha\beta} \nabla_m$$

[Witten '86]

Non-Linear Equation of Motion

- Using the Bianchi identity and the above Constraint yields the equations of Motion:

$$
\begin{aligned}
\{D_{(\alpha}, \mathbb{A}_{\beta)}\} &= \gamma^m_{\alpha\beta}\mathbb{A}_m + \{\mathbb{A}_\alpha, \mathbb{A}_\beta\}, \\
\{D_\alpha, \mathbb{W}^\beta\} &= \frac{1}{4}(\gamma^{mn})_\alpha{}^\beta \mathbb{F}_{mn} + \{\mathbb{A}_\alpha, \mathbb{W}^\beta\}, \\
[D_\alpha, \mathbb{A}_m] &= [\partial_m, \mathbb{A}_\alpha] + (\gamma_m \mathbb{W})_\alpha + [\mathbb{A}_\alpha, \mathbb{A}_m], \\
[D_\alpha, \mathbb{F}^{mn}] &= (\mathbb{W}^{[m}\gamma^{n]})_\alpha + [\mathbb{A}_\alpha, \mathbb{F}^{mn}],
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[D_\alpha, \mathbb{F}^{mn}] &= (\mathbb{W}^{[m}\gamma^{n]})_\alpha + [\mathbb{A}_\alpha, \mathbb{F}^{mn}],
\end{aligned}
$$

We Can Use these for BRST Variations $\longrightarrow$ $[Q, \mathbb{A}_m] = [\partial_m, \lambda \mathbb{A}] + (\lambda \gamma_m \mathbb{W}) + [\lambda \mathbb{A}, \mathbb{A}_m]$.

Non-Linear Equation of Motion

- Dirac and Yang-Mills:

$$\gamma^m_{\alpha\beta}[\nabla_m, \mathbb{W}^\beta] = 0, \quad \partial_m \mathbb{F}^{mn} = \gamma^n_{\alpha\beta}\{\mathbb{W}^\alpha, \mathbb{W}^\beta\}$$

Non-Linear Equation of Motion

- Dirac and Yang-Mills :

$$\gamma^m_{\alpha\beta}[\nabla_m, W^\beta] = 0 , \quad \partial_m \mathbb{F}^{mn} = \gamma^n_{\alpha\beta}\{W^\alpha, W^\beta\}$$

- We can treat higher mass fields like $[\nabla_m, W^\alpha]$ as 'fundamental',

$$\mathbb{K}^{m_1 \dots m_k | N} = [\nabla^{m_1}, \mathbb{K}^{m_2 \dots m_k | N}]$$

$\longmapsto$ They also obey 'Dirac + Yang-Mills' eqns. [2210.14241].

Contains Deconcatenation.

PS Review

- Non-linear fields can be expanded as a Lie algebra valued generating series with BG coefficients.

[1501.05562]. $\mathbb{K} = \sum_{p=1}^{\infty} \sum_{i_1 \dots i_p} \mathrm{K}_{\underbrace{i_1 \dots i_p}} t^{i_1} \dots t^{i_p}$

BG Currents

Multiparticle Labels.

Lie algebra generators

- Non-linear fields can be expanded as a Lie algebra valued generating series with BG coefficients.

$$\mathbb{K} = \sum_{p=1}^{\infty} \sum_{i_1 \dots i_p} K_{i_1 \dots i_p} t^{i_1} \dots t^{i_p}$$

- Multiparticle BG currents have correct pole structure,

$$K_P = \frac{1}{s_P} \sum_{XY=P} \underbrace{K_{[X,Y]}}_{\text{From BG EoM}}$$

$P = i_1 i_2 \dots i_p$

[1510.08843].

Sections

Dynkin Labels of $\mathfrak{so}(10)$

- Consider all terms of correct mass dimension
  - At α'^3 this is dimension 7.5.

$$\nabla_m \leftrightarrow 1$$

$$A_\alpha \leftrightarrow 0.5$$

$$A_m \leftrightarrow 1$$

$$F_{mn} \leftrightarrow 2$$

$$W^\alpha \leftrightarrow 1.5$$

Dynkin Labels of $\mathfrak{so}(10)$

- Consider all terms of correct mass dimension
  At α'^3 this is dimension 7.5.

- Use Dynkin labels to determine if in SO(10) group structure.

Also check constraints
+ scalars

Dynkin Labels of $\mathfrak{so}(10)$

- Consider all terms of correct mass dimension
  At α'^3 this is dimension 7.5.
- Use Dynkin labels to determine if in SO(10) group structure.
- Find all allowed permutations + contractions
  → This gives the ansatz.

Dynkin Labels of $\mathfrak{so}(10)$

- Basis labels :

$$\begin{aligned}
\text{Scalar } A &= (0,0,0,0,0),\\
\text{Vector } A^m &= (1,0,0,0,0),\\
\text{2-Form } A^{[mn]} &= (0,1,0,0,0),\\
\text{3-Form } A^{[mnp]} &= (0,0,1,0,0),\\
\text{Anti-Weyl } \xi_\alpha &= (0,0,0,1,0),\\
\text{Weyl } \xi^\alpha &= (0,0,0,0,1),
\end{aligned}$$

[hep-th\0205165]

Ansatz Terms

- Example : $\mathbb{V}(\lambda\gamma W)(\lambda\gamma W)D^2\mathbb{F}$
  - Decomposition $\Rightarrow 1(0,0,0,0,0)+\cdots$
  - Only a Single Scalar :

    $$\mathbb{V}(\lambda\gamma^{m}W)(\lambda\gamma^{p}W)\mathbb{F}^{mnlpn}$$

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- Example : $\mathbb{V}(\lambda\gamma\mathbb{W})(\lambda\gamma\mathbb{W})D^2\mathbb{F}$
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- Repeating this for all terms we can think of gives 162 terms $\longrightarrow$ After PS + EoM.

Higher Mass Identities

- Higher order terms can often be re-written as lower order

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- Higher order terms can often be re-written as lower order
- E.g. $D^N \mathbb{F}$ and $D^N \mathbb{W}$, $N \geq 2$ can be decomposed into sym. Traceless part + other fields.

$$\mathbb{F}^{ab|mn} = \hat{\mathbb{F}}^{(ab)|mn} + \frac{\delta^{ab}}{5}([\mathbb{F}^{mp}, \mathbb{F}^{pn}] + \{(\mathbb{W}^{[m}\gamma^{n]}), \mathbb{W}\}) + \frac{1}{2}[\mathbb{F}^{mn}, \mathbb{F}^{ab}]$$

Sym. + Traceless in a,b.

Sections

α'^3 Corrections

- After applying BRST Closure → Fix last unknown with 4-point amp.

BRST fixes the Correction up to a Mandelstam invariant
→ We fix this

α'^3 Corrections

- After applying BRST Closure → Fix last unknown with 4-point amp.
- The Final result gives

'Stringy' SYM Corrections

$$\begin{aligned}\mathbb{V}\Big(&\mathbb{F}^{mn}(\lambda\gamma^n\mathbb{W}^p)(\lambda\gamma^p\mathbb{W}^m)+(\lambda\gamma^m\mathbb{W}^n)\mathbb{F}^{np}(\lambda\gamma^p\mathbb{W}^m)\\
&+\frac{1}{2}[\mathbb{F}^{mn}\mathbb{F}^{np},\{(\lambda\gamma^m\mathbb{W}),(\lambda\gamma^p\mathbb{W})\}]\\
&+\frac{1}{4}\mathbb{F}^{mn}\mathbb{F}^{pq}\{(\lambda\gamma^m\mathbb{W}),(\lambda\gamma^{npq}\mathbb{W})\}\\
&+\frac{1}{4}\mathbb{F}^{mn}(\lambda\gamma^{npq}\mathbb{W})[(\lambda\gamma^m\mathbb{W}),\mathbb{F}^{pq}]\\
&-\frac{1}{4}\mathbb{F}^{pq}(\lambda\gamma^m\mathbb{W})[(\lambda\gamma^{npq}\mathbb{W}),\mathbb{F}^{mn}]\\
&-\frac{1}{4}(\lambda\gamma^{npq}\mathbb{W})(\lambda\gamma^m\mathbb{W})[\mathbb{F}^{mn},\mathbb{F}^{pq}]\Big)\end{aligned}$$

2-Loop 4-point Kin. Factor.

Outlook

- Connection between $\mathbb{V}$ and No-$\mathbb{V}$ Corrections,

$$\mathbb{F}^{mlpq}(\lambda\gamma^p \mathbb{W})(\lambda\gamma^q \mathbb{W})(\lambda\gamma^m \mathbb{W}).$$

Find ghost nº 2 expression

+ Match in Superspace.

Outlook

- Connection between $\mathbb{V}$ and No-$\mathbb{V}$ Corrections,

$$\mathbb{F}^{mlpq}(\lambda\gamma^{p}\mathbb{W})(\lambda\gamma^{q}\mathbb{W})(\lambda\gamma^{m}\mathbb{W}).$$

- $(\alpha')^4$ Corrections in Pure Spinor Superspace
  - → Ansatz Over 12,000 terms
  - → Canonicalization difficult.

↳ Would yield Perturbiner Expansion!

n-point.

Thank You