Accelerating Black Holes in 2+1 and other assorted things

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Overview

- A brief review on accelerating black holes in 3+1 dimensions.
- Black holes in 2+1?
- C-metric in 2+1 dimensions
  - Class I solutions: Accelerating particles
  - Class II solutions: Accelerating BTZ
  - Class Ic: Accelerating Black Hole (not quite like BTZ)
- Concluding remarks

Accelerating black holes in 3+1

- To accelerate a black hole, we need to be able to push or pull it.
- Anything touching the event horizont must fall in unless traveling at the speed of light.
- This means that the physical object that touches the horizon must have energy = tension ($T^0_0 \sim T^1_1$)
- Fortunately, there is a candidate ➡ Cosmic String!

Cosmic string

- Very thin quasilinear object, which is fully characterised by its mass per unit length μ.

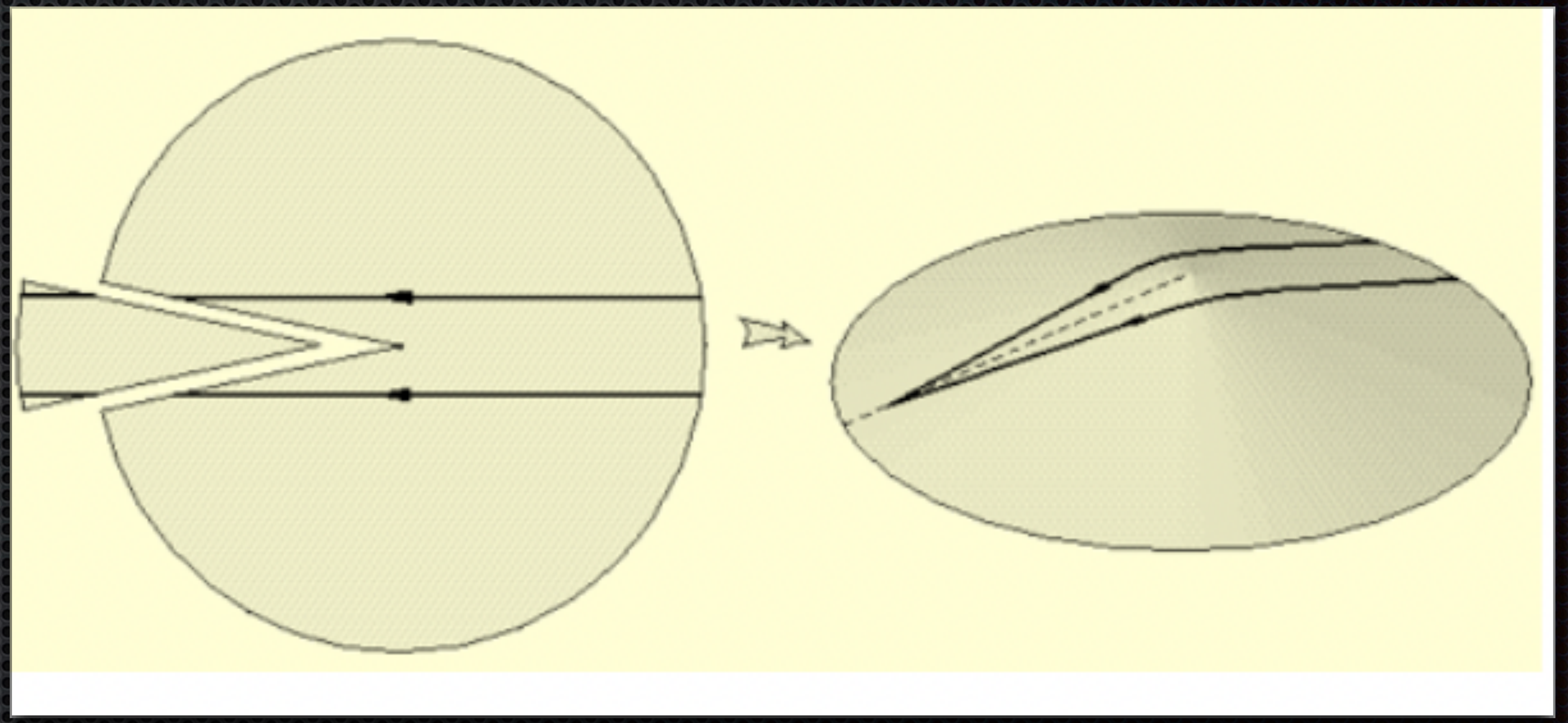
$$T^{\mu}_{\nu} \simeq \delta^{(2)}(r)\mathrm{diag}(\mu, \mu, 0, 0)$$

- The string produces a conical defect

$$\delta = 8\pi G\mu$$

The conical defect is what accelerates the black hole!

- No long range spacetime curvature.



The C-metric

- Let's consider the Plebanski-Demianski metric with non-zero cosmological constant $\Lambda = -\frac{3}{\ell^2}$ parametrised by two parameters

$$ds^2 = \frac{1}{A^2(x-y)^2}\left(-P(y)dt^2 + \frac{1}{P(y)}dy^2 + \frac{1}{Q(x)}dx^2 + Q(x)dz^2\right)$$

Griffiths and Podolsky 2006'

$$P(y) = \left(\frac{1}{A^2\ell^2} - 1\right) + y^2 - 2mAy^3\,,$$

$$Q(x) = 1 - x^2 - 2mAx^3$$

- The (conformal) boundary $x = y$
- Useful form to see range of the variables that preserves Lorenzian signature
- Not so clear view of black hole structure

- Introduce radial and angular variables $y = \frac{-1}{Ar}$, $x = \cos\theta$, $z = \frac{\phi}{K}$ and rescaling the time $t \to tA$, we get

$$ds^2 = \frac{1}{\Omega}\left[-f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2\left(\frac{d\theta^2}{g(\theta)} + g(\theta)\sin^2\theta\frac{d\phi^2}{K^2}\right)\right]$$

Hong and Teo 2003'

$$f(r) = \left(1 - \frac{2m}{r}\right)(1 - A^2r^2) + \frac{r^2}{\ell^2}, \quad g(\theta) = 1 + 2mA\cos\theta\,, \quad \Omega(r,\theta) = 1 + Ar\cos\theta$$

- The conformal factor blows up at $r = \frac{-1}{A\cos\theta}$
- The horizon(s) structure is determined by an interplay between m, A and ℓ
  - If A is big enough, there is an accelerating horizon (+ the black hole horizon)
  - If $A^2\ell^2 < 1$, we have a single black horizon ($r_h \sim 2m$)
- K determines the conical singularity $\delta = 2\pi\left(1 - \frac{1}{K}\right) = 8\pi\mu$

Aryal, Ford, Vilenkin 1986'
Achucarro, Gregory, Kuijken 1995'

- We can see the effect of A by considering $m = 0$: $f(r) = 1 - A^2r^2 + \frac{r^2}{\ell^2}$, $g(\theta) \to 1$

$$ds^2 = \frac{1}{\Omega}\left[-\left(1 - A^2r^2 + \frac{r^2}{\ell^2}\right)dt^2 + \frac{1}{\left(1 - A^2r^2 + \frac{r^2}{\ell^2}\right)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta\frac{d\phi^2}{K^2}\right)\right]$$

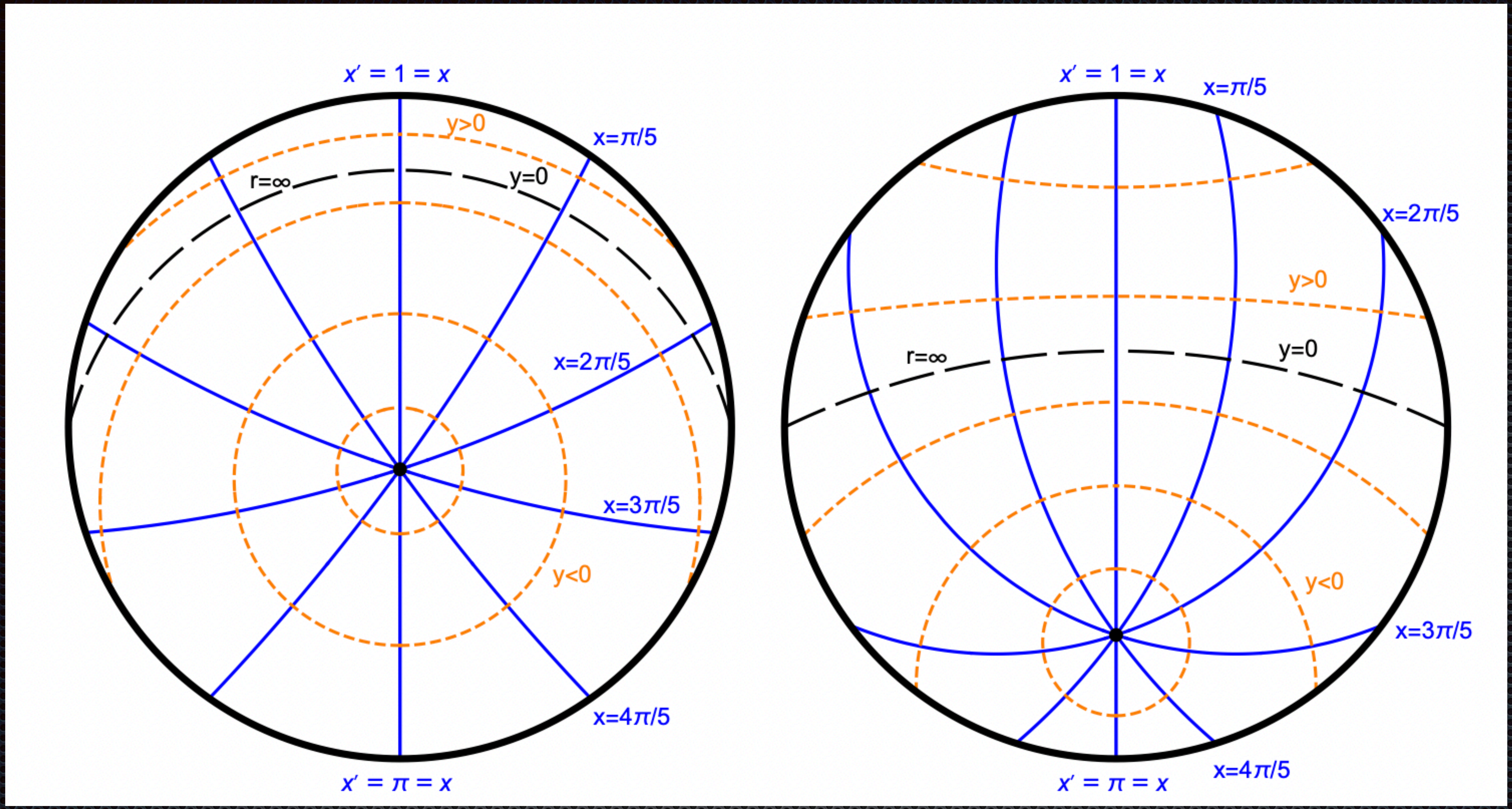
- We restrict ourself to the slowly accelerating limit $A^2\ell^2 < 1$ and taking the following redefinition,

$$1 + \frac{R^2}{\ell^2} = \frac{1 + (1 - A^2\ell^2)\frac{r^2}{\ell^2}}{\left(1 - A^2\ell^2\right)\Omega^2} \quad , \quad R\sin\Theta = \frac{r\sin\theta}{\Omega}$$

We get

$$ds^2 = -\left(1 + \frac{R^2}{\ell^2}\right)\beta dt^2 + \frac{1}{\left(1 + \frac{R^2}{\ell^2}\right)}dR^2 + R^2\left(d\Theta^2 + \sin^2\Theta\frac{d\phi^2}{K^2}\right)$$

$$\beta = \sqrt{1 - A^2\ell^2}$$



$A\ell = 0.3$ $A\ell = 0.9$

A. Scoins, 2022

- We have an off-centre global AdS perspective. As we increase A, the point mass is "pulled" closer to the AdS boundary.
- The system is "accelerating" and yet, remains static suspended due to the string tension $\mu_s = mA/K$ $(\leftrightarrow F = MA)$

Anabalon, Appels, Gregory, Kubiznak, and Mann, 2018

Black holes in 2+1?

- Gravity in 2+1 dimensions is topological.
- Nevertheless, there are black hole solutions (for negative cosmological constant) which resembles some properties of Schwarzschild black holes.

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\theta^2$$

Bañados, Teitelboim, and Zanelli 1992
Bañados, Henneaux, Teitelboim, and Zanelli 1993

$$f(r) = -M + \frac{r^2}{\ell^2}$$

- Most important feature: No curvature singularity. Instead, there is a conical singularity.

C-metric in 2+1 dimensions

- We start with an ansatz similar to Plebanski-Demianski metric.

$$ds^2 = \frac{1}{A^2(x-y)^2}\left(-P(y)dt^2 + \frac{1}{P(y)}dy^2 + \frac{1}{Q(x)}dx^2\right)$$

- We eliminate all the gauge freedom of the metric functions getting quadratic eqs for $P(y)$ and $Q(x)$.
- We restrict the coordinates for ranges where Lorenzian signature holds.
- We get 3 classes of solutions depending on the range of x

Class	$Q(x)$	$P(y)$	Maximal range of x
I	$1-x^2$	$\frac{1}{A^2\ell^2}+(y^2-1)$	$\lvert x\rvert<1$
II	x^2-1	$\frac{1}{A^2\ell^2}+(1-y^2)$	$x>1$ or $x<-1$
III	$1+x^2$	$\frac{1}{A^2\ell^2}-(1+y^2)$	$\mathbb{R}$

- Finally, we insert a (single) domain wall by cutting and gluing the spacetime at $x = \mathbf{const}$. Israel's condition give the tension of the wall $\mu = \pm \frac{A}{4\pi}\sqrt{Q(x)}$.

- This yields to

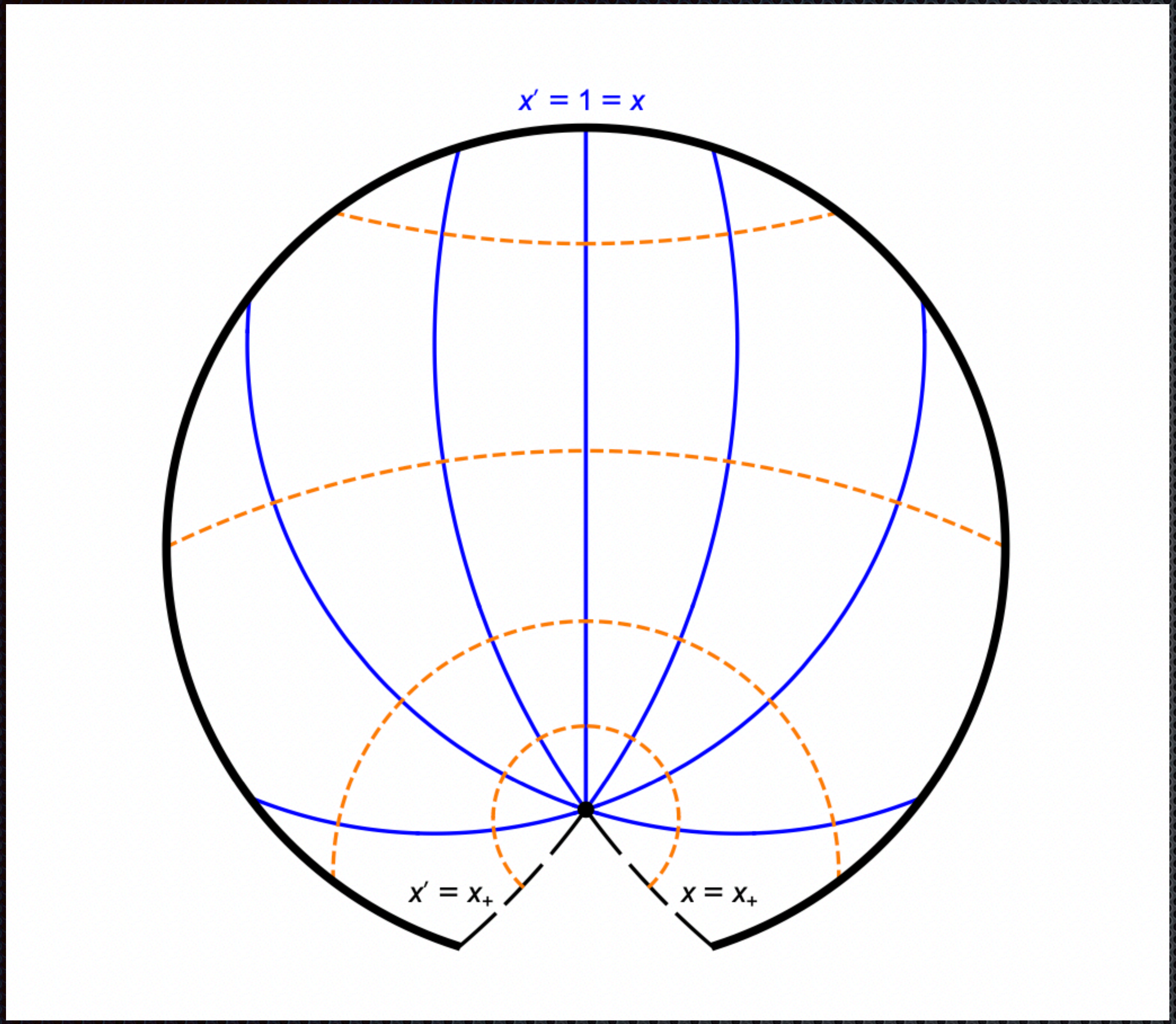
Class I → Accelerating particles: pulled by a wall or pushed by a strut

Class I → Type I_C: An accelerating black hole pushed by a strut

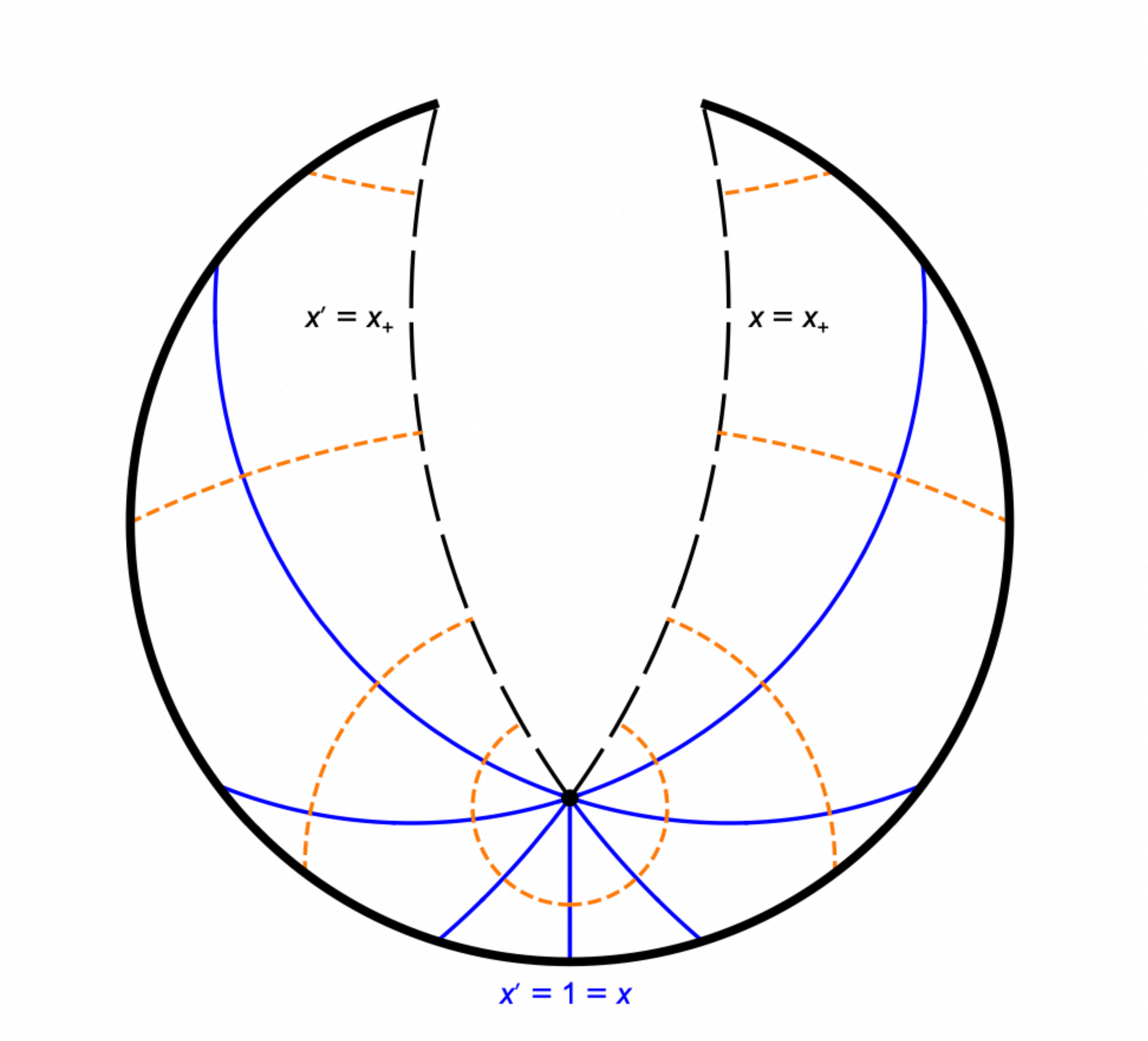
Class II → Accelerating BTZ: pulled by a wall or pushed by a strut

Class III → No single wall solutions. Braneworld-type of geometries allowed.

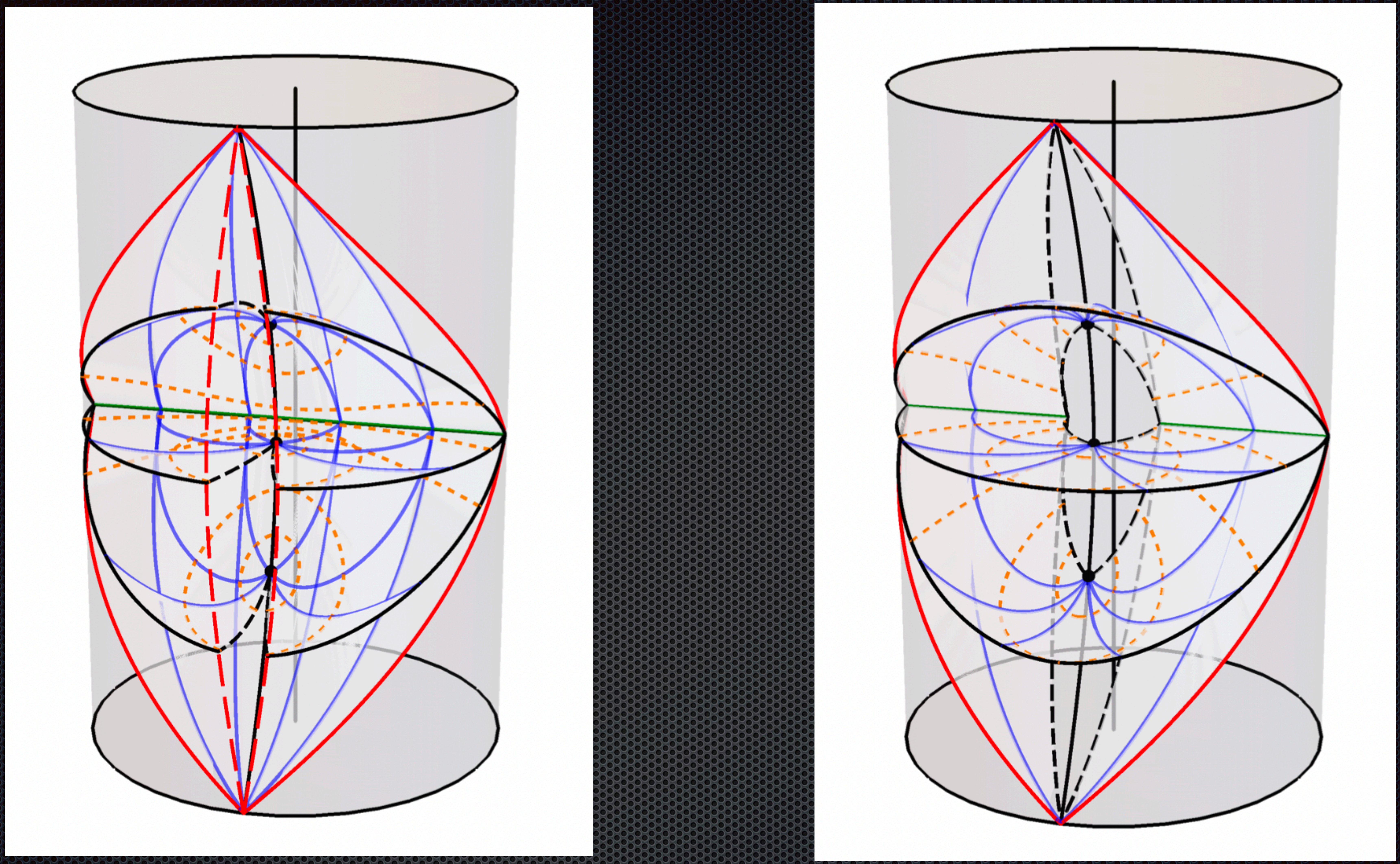
Class I: Accelerating particles



Slowly accelerating conical defect
Pulled by a wall $A = 0.9\ell$



Slowly accelerating conical defect
pushed by a strut $A = 0.9\ell$



Embedding within global AdS_3: The particle worldline is shown in solid black. Several surfaces of constant t are plotted. The event horizons are demonstrated by the surfaces at early and late t. The bifurcation surface is shown as a green line. The boundary of the classically accessible subset of the global boundary is shown in red. Lines of constant x are shown in blue, with lines of constant y in dashed orange.

Class II: Accelerating BTZ

M. Astorino, 2011

$$ds^2 = \frac{1}{\Omega}\left[-F(r)dt^2 + \frac{1}{F(r)}dr^2 + r^2 d\psi^2\right]$$

$$F(r) = -m^2\left(1 - A^2r^2\right) + \frac{r^2}{\ell^2} \quad , \quad \Omega(r,\psi) = 1 \mp Ar\cosh\psi$$

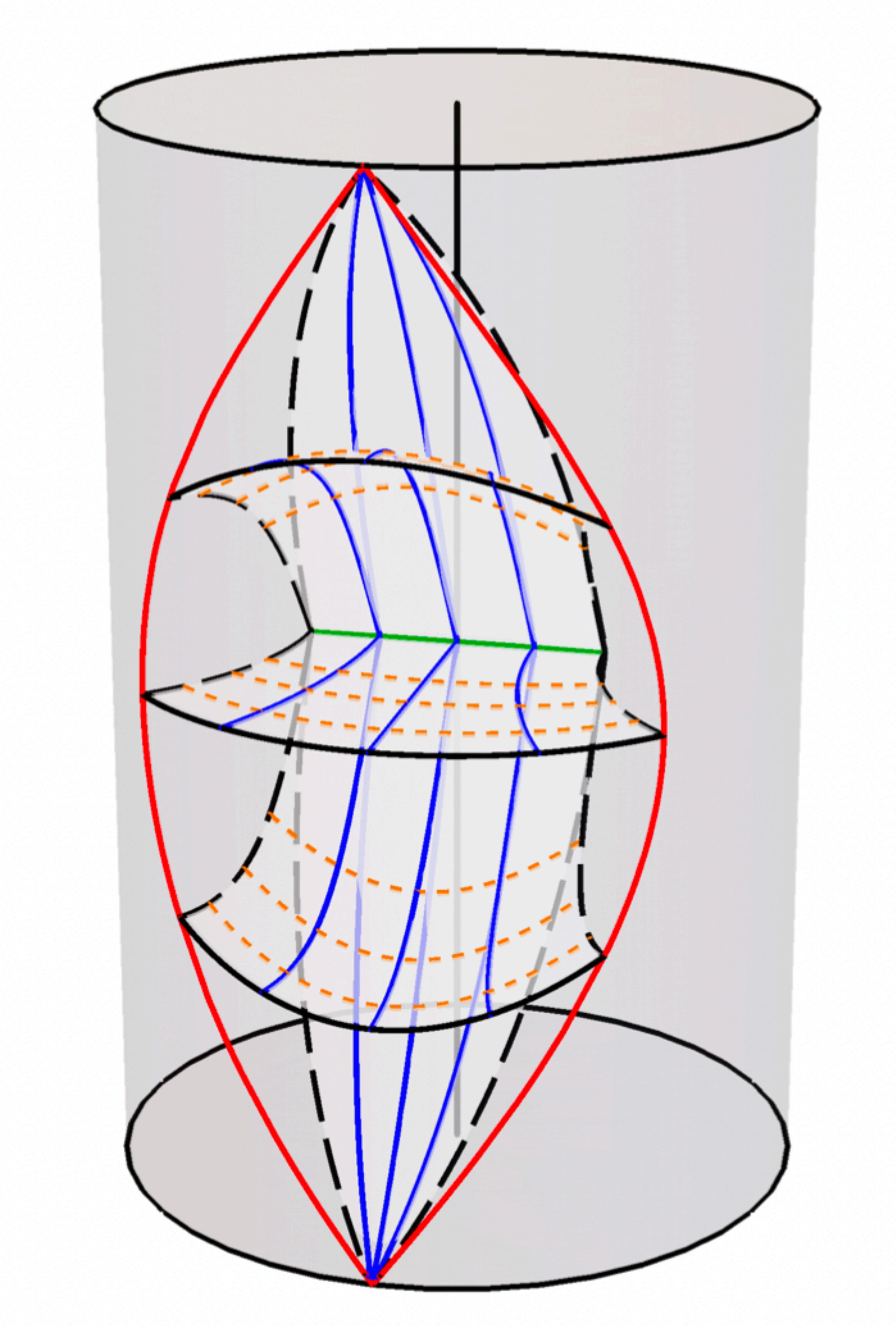
+ Pulled by a wall

- Pushed by a strut

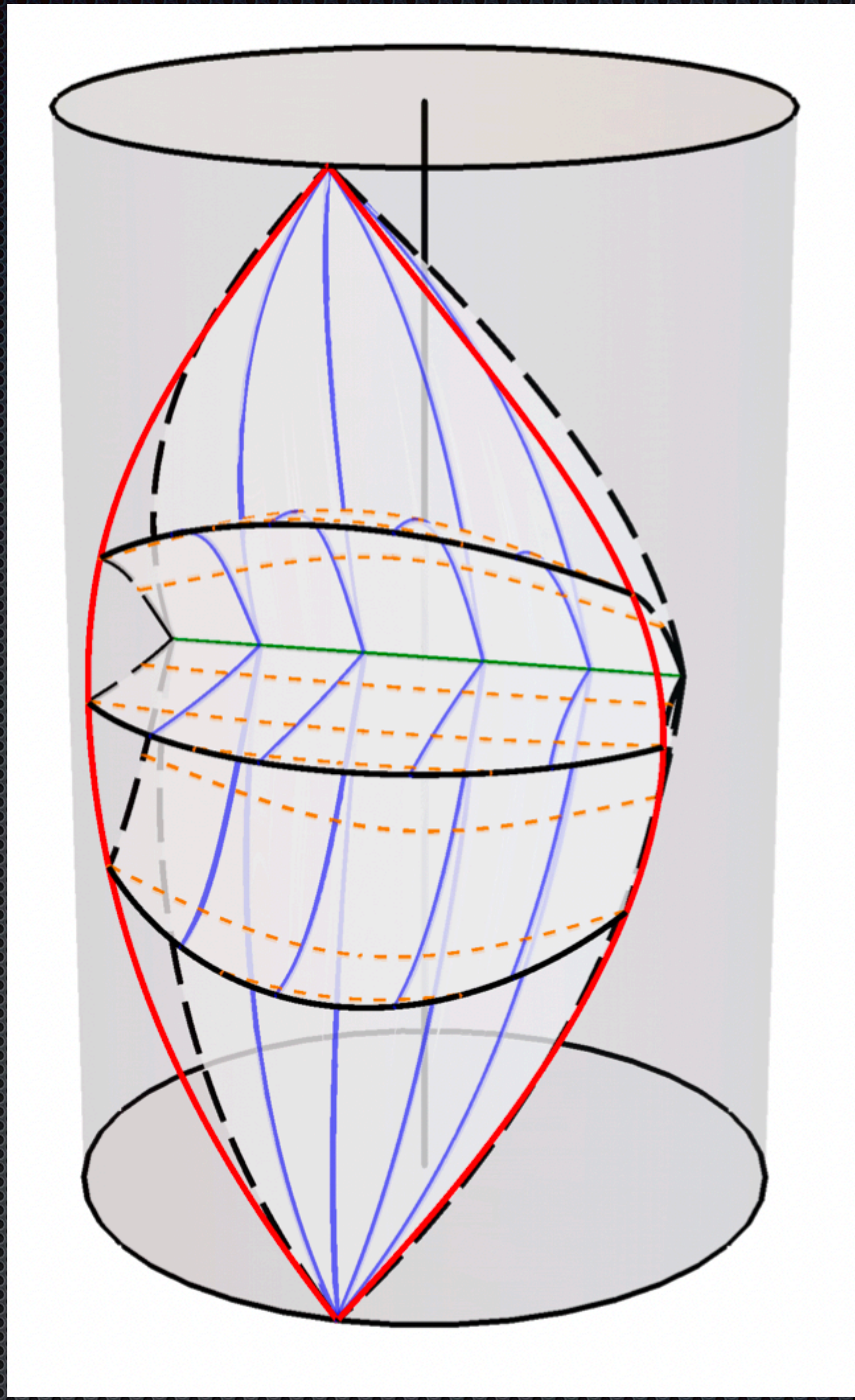
Class I_C: A black hole pulled by a wall

$$ds^2 = \frac{1}{\Omega}\left[-F(r)dt^2 + \frac{1}{F(r)}dr^2 + r^2 d\theta^2\right]$$

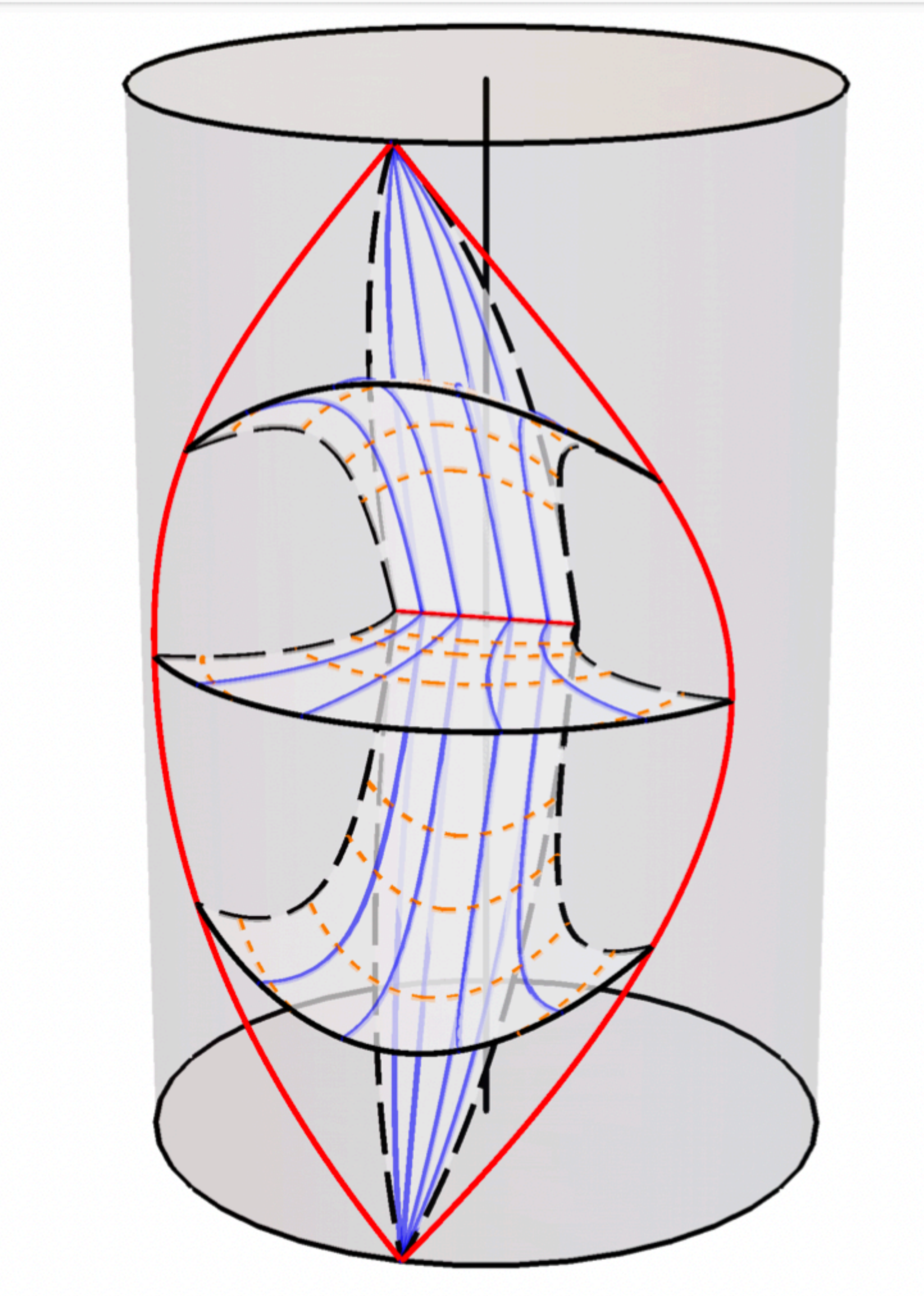
$$F(r) = -m^2\left(A^2r^2 - 1\right) + \frac{r^2}{\ell^2} \quad , \quad \Omega(r,\theta) = Ar\cos m\theta - 1 \quad , \quad \frac{1}{m} \leq A\ell < \frac{1}{m\sin(m\pi)}$$



Standard BTZ



BTZ pulled by a wall



BTZ pushed by a strut

Concluding remarks

- We have constructed a broad family of solutions in 2+1 dimensions resembling the four-dimensional C-metric, showing that the set of possible geometries is much richer than previously acknowledged in the literature.
- Class I_C has no analogous object in higher-dimensions. Deeper understanding is needed.
- Class III have not been studied so far.
- Next step in understanding the three-dimensional C-metric is to establish a consistent thermodynamic description of the system.
- Holographic implications.

Questions?

Thanks!