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Effective Field Theory Wavefunction Coefficients in de Sitter

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Abstract

The Cosmological Scattering Equations lead to a natural formulation of EFT wavefunction coefficients in de Sitter written in terms of conformal generators in the future boundary. The corresponding integrands can be assembled from simple building blocks (including mass deformations and curvature corrections) leading to a double copy prescription. We can also analyse the operator form of the wavefunction coefficients in the soft limit, letting us link EFT soft theorems with Lagrangian symmetries in curved spacetimes [1, 2].

Background

We focus on objects living on the future boundary of dS_{d+1}. These "coefficients of the wavefunction of the universe" can be computed via a Wick rotation of Witten diagrams in AdS. We present an alternative description obtained using the Cosmological Scattering equations (CSE), which leads naturally to expressions written in terms of CFT operators acting on the future boundary (conformal time $\eta \to 0$). These objects are linked to observables in inflationary cosmology and also provide a way to probe concepts like colour-kinematics, the double copy and soft theorems beyond flat space.

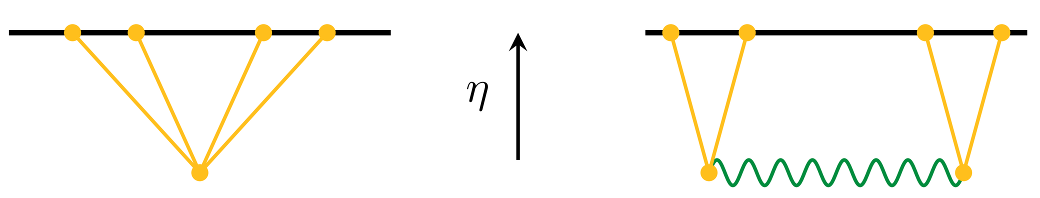

Figure 1: Particles interact in the bulk of dS with external states living on the future boundary.

1 Introduction

We work with scalars obeying the massive Klein-Gordon equation in dS

$$\left[\eta^2\partial_\eta^2 + (1-d)\eta\partial_\eta + \eta^2 k^2 + m^2\right]\phi = 0, \tag{1}$$

with $m^2 = \Delta(d-\Delta)$ for a scalar of conformal weight Δ. The general solutions are Hankel functions and for $k^2 > 0$ are known as bulk-to-boundary propagators

$$\mathcal{K}_\nu(k,\eta) = \mathcal{N} k^\nu \eta^{d/2} H_\nu(-k\eta).,\tag{2}$$

with $\nu = \Delta - d/2$. We can express the same equation of motion using the Casimir operators built of conformal generators

$$\mathcal{D}_a\cdot\mathcal{D}_b = \frac{1}{2}(P_a^i K_{bi} + K_{ai}P_b^i - M_{a,ij}M_b^{ij}) + D_a D_b,\tag{3}$$

where a, b are particle labels and i, j run over the spatial directions. The wavefunctions then obey $(\mathcal{D}^2 + m^2)\mathcal{K}_\nu(k,\eta) = 0$. We will use the operators to build scalar EFT wavefunction ceofficients taking the general form

$$\mathcal{A}_n = f(\{\mathcal{D}_a\cdot\mathcal{D}_b\})\mathcal{C}_n,\tag{4}$$

where $\mathcal{C}_n = \int \frac{d\eta}{\eta^{d+1}}\prod_{a=1}^n \mathcal{K}_\nu(k_a,\eta)$ is the n-point contact diagram. The operators $\mathcal{D}_a\cdot\mathcal{D}_b$ can appear in the numerator or as propagators and in general do not commute with each other.

2 Cosmological Scattering Equations

To construct EFT wavefunction coefficients, we use a CHY formalism adapted to dS [4, 6]. We replace the Mandelstam variables $k_a\cdot k_b$ in the original formalism with Casimir operators plus a mass deformation to obtain

$$S_a = \sum_{b\neq a}\frac{2\left(\mathcal{D}_a\cdot\mathcal{D}_b\right) + \mu_{ab}}{\sigma_{ab}} \equiv \sum_{b\neq a}\frac{\alpha_{ab}}{\sigma_{ab}}.\tag{5}$$

The wavefunction coefficients are then given by the worldsheet integral

$$\Psi_n = \delta^d(\vec{k}_T)\int_\gamma \prod_{a\neq b,c,d}^n \mathrm{d}\sigma_a\, S_a^{-1}(\sigma_{bc}\sigma_{cd}\sigma_{db})^2\,\mathcal{I}_n\mathcal{C}_n^\Delta.\tag{6}$$

Both the integrand $\mathcal{I}$ (which contains the theory information) and the scattering equations are now operators. As solving the scattering equations for the σ is therefore not well understood, the amplitude is instead mapped to a sum over Witten diagrams using the global residue theorem [7, 5].

3 EFT Integrands/ Generalised Double Copy

CHY integrands for dS are built from Parke-Taylor factors and Pfaffians just as for flat space EFTs. They depend on the worldsheet coordinates σ in the same way but the Mandelstams are uplifted to the operators α_{ab}. For NLSM we can write

$$\mathcal{I}_4^{NLSM} = \lambda^2\mathrm{PT}\left(\mathrm{Pf}'A\right)^2 + c\mathrm{PT}\,\mathrm{Pf}X|_{\mathrm{conn}}\,\mathrm{Pf}'A,\tag{7}$$

with c an unfixed coefficient for a curvature correction – equivalent to adding a ϕ^4 term in the Lagrangian. We find that with a replacement

$$\lambda^2\mathrm{PT} \to a\mathrm{Pf}'A\left(\mathrm{Pf}'A + m^2\,\mathrm{Pf}X|_{\mathrm{conn}}\right) + b\left(\mathrm{Pf}'A\mathrm{Pf}X + m^2\mathrm{PT}\right),\tag{8}$$

in the NLSM integrand, we can reproduce any amplitude comprised of DBI, sGal and ϕ^4 contributions. This "generalised double copy" mirrors the mapping between YM and gravity or NLSM and DBI/ sGal integrands in flat space.

The results are the same as those obtained by uplifting flat space amplitudes via the $k_a\cdot k_b \to \alpha_{ab}$ replacement although above 4pt the ordering of operators needs to be considered carefully.

4 Soft Limits

The form of the wavefunction coefficients in (4) naturally leads to integrals over η with an integrand that is a function of the 3d boundary momenta. The explicit momentum dependence makes it easy to take soft limits. Flat space EFTs with hidden symmetries have enhanced soft limits, it is therefore natural to probe the conditions (values of Δ, d and any curvature corrections) that reproduce this behavior in AdS.

Consider NLSM as a simple example

$$\begin{aligned}\mathcal{A}_4|_{\vec{k}_1\to 0} &= \mathcal{D}_1\cdot\mathcal{D}_3\,\mathcal{C}_4|_{\vec{k}_1\to 0},\\ &\sim (\Delta - d)D_3\tilde{\mathcal{C}}_3.\end{aligned}\tag{9}$$

We can express the 4pt soft limit in terms of conformal generators acting on a 3pt contact and see that it will only vanish for $\Delta = d$ and with any possible curvature corrections set to zero. DBI and sGal have a richer behavior with subleading limits fixing non-zero curvature corrections. This lets us uniquely determine the a, b, c in the generalised double copy.

5 Outlook

- CSE give a natural way to uplift scalar amplitudes in flat space to dS wavefunction coefficients written in terms of Casimirs acting on a contact diagram
- We see double copy structure for EFT integrands as in flat space CHY
- This description lets us explore amplitude properties outside flat space and is particularly suited to taking soft limits
- Imposing enhanced soft limits uniquely fixes the conformal weight and form of the Lagrangian – recovering cases with shift symmetry [3, 2].

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