Electroweak Input Schemes in the SMEFT

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$$\mathcal{L}(g_1, g_2, v_T)$$
$$\downarrow$$
$$\mathcal{L}(\text{inputs})$$
$$\downarrow$$
Finite Predictions

The choice of Electroweak input scheme is how we eliminate the SU(2) and hypercharge gauge couplings g_1 and g_2 as well as the Higgs doublet field vacuum expectation value v_T in favour of three physical input parameters.

We have chosen to consider three input schemes:

- The α_μ scheme, which uses the inputs $\{G_F, M_W, M_Z\}$
- The α scheme, which uses the inputs $\{\alpha, M_W, M_Z\}$
- The LEP scheme, which uses the inputs $\{\alpha, G_F, M_Z\}$

The Z boson mass is usually chosen as an input due to the precision of its measurement.

To implement the α_μ scheme, we first rewrite the tree level Lagrangian in terms of v_T, M_W and M_Z using

$$\bar{g}_1 = \frac{2M_W s_w}{c_w v_T}\left[1 + c_w s_w\left(C_{HWB} + \frac{c_w}{4s_w}C_{HD}\right)v_T^2\right]$$
$$\bar{g}_2 = \frac{2M_W}{v_T},$$

The renormalised Lagrangian is obtained by treating tree level parameters as bare ones and replacing them with the renormalised ones plus a counter term. The boson masses are renormalised through 2-pt functions, and the bare vev is renormalised through the following, where the corrections ensure the tree level muon decay is exact to all orders.

$$\frac{1}{v_{T,0}^2} = \frac{1}{v_\mu^2}\left[1 - v_\mu^2 \Delta v_\mu^{(6,0,\mu)} - \frac{1}{v_\mu^2}\Delta v_\mu^{(4,1,\mu)} - \Delta v_\mu^{(6,1,\mu)}\right]$$

To use the α scheme, we define

$$v_\alpha = \frac{M_W^2 s_w^2}{\pi\alpha}$$

At tree level the relation between v_T and v_α is given by

$$\frac{1}{v_T^2} = \frac{1}{v_\alpha^2}\left(1 + 2v_\alpha^2 \frac{c_w}{s_w}\left[C_{HWB} + \frac{c_w}{4s_w}C_{HD}\right]\right)$$

Again treating as a bare equation and replacing tree level parameters with renormalised ones plus counter terms defined through two point functions, we have:

$$\frac{1}{v_{T,0}^2} = \frac{1}{v_\alpha^2}\left[1 - v_\alpha^2 \Delta v_\alpha^{(6,0,\alpha)} - \frac{1}{v_\alpha^2}\Delta v_\alpha^{(4,1,\alpha)} - \Delta v_\alpha^{(6,1,\alpha)}\right]$$

If we have a result in the α_μ scheme, we can obtain the result in the α scheme by using the following equation, derived by equating the two expansions for v_T

$$\frac{v_\alpha^2}{v_\mu^2} \equiv 1 + \Delta r = 1 + v_\alpha^2 \Delta r^{(6,0)} + \frac{1}{v_\alpha^2}\Delta r^{(4,1)} + \Delta r^{(6,1)}$$

The LEP doesn't use the W boson mass as an input and so is slightly more complicate to use. We can use the definition of v_α to find a relation for the on shell W boson mass in terms of Δr.

Using this equation iteratively on itself to re-express the on shell W boson mass within the Δr , gives an expression for M_W in the LEP. This is then substituted into any expression in the α_μ scheme. Once we have expand to desired order in loop and dimension we obtain the expression in the LEP scheme.

$$M_W^2 = \hat{M_W}^2\left[1 - \frac{\hat{s}_w^2}{\hat{c}_{2w}}\Delta r - \frac{\hat{c}_w^2 \hat{s}_w^4}{\hat{c}_{2w}^3}\Delta r^2\right] + \mathcal{O}\left(\Delta r^3\right)$$

$$\hat{M_W}^2 = \frac{M_Z^2}{2}\left(1 + \sqrt{1 - \frac{4\pi\alpha v_\mu^2}{M_Z^2}}\right)$$

$$M_W = \hat{M_W}\left[1 + v_\mu^2 \hat{\Delta}_W^{(6,0,\mu)} + \frac{1}{v_\mu^2}\hat{\Delta}_W^{(4,1,\mu)} + \hat{\Delta}_W^{(6,1,\mu)}\right]$$

The perturbative convergence of any weak process in the different schemes is easily understood. The differences between the schemes is characterised by the $\Delta r^{(4,1)}$ and numerically these evaluate to:

$$\frac{\Delta r^{(4,1)}}{v_\alpha^2} = -4\% \qquad \frac{-\hat{s}_w^2}{2\hat{c}_w^2 - 1}\frac{\hat{\Delta} r^{(4,1)}}{v_\mu^2} = 1.5\%$$

	Electroweak SM Corrections		
Scheme	$W \to \tau\nu_\tau$	$Z \to \tau^+\tau^-$	$H \to b\bar{b}$
α	-4.2%	-4.0%	-4.8%
α_μ	-0.3%	<0.1%	-0.8%
LEP	1.2%	0.1%	-0.7%

The LEP and α_μ only differ when M_W appears in the tree level matrix element. For full SMEFT results read the paper [arXiv:2212.TBC]

Number of Wilson Coefficients introduced

Scheme		M_W	M_Z	v_T	Total unique
α	LO	0	0	2	2
	NLO	12	29	29	29
α_μ	LO	0	0	3	3
	NLO	13	30	12	33
LEP	LO	5	0	3	5
	NLO	33	30	12	33