hadspec

hadron spectrum collaboration

hadspec.org

exploring meson resonances using lattice QCD

Jozef Dudek

a lot of progress, can only give you a flavour …

WILLIAM & MARY
CHARTERED 1693

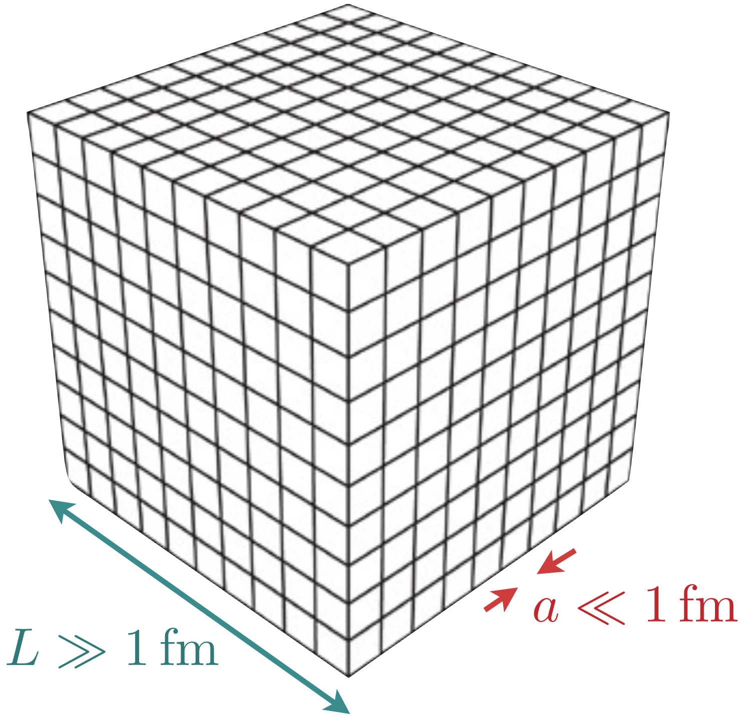
Jefferson Lab
Thomas Jefferson National Accelerator Facility

desire: excited hadrons in first-principles QCD

lattice QCD is controlled QCD

can compute correlation functions

$\rightsquigarrow$ access to spectrum & matrix elements



lattice spacing / discretization choice

$\rightsquigarrow$ relatively unimportant here

finite-volume

$\rightsquigarrow$ tool to access scattering

choice of quark mass

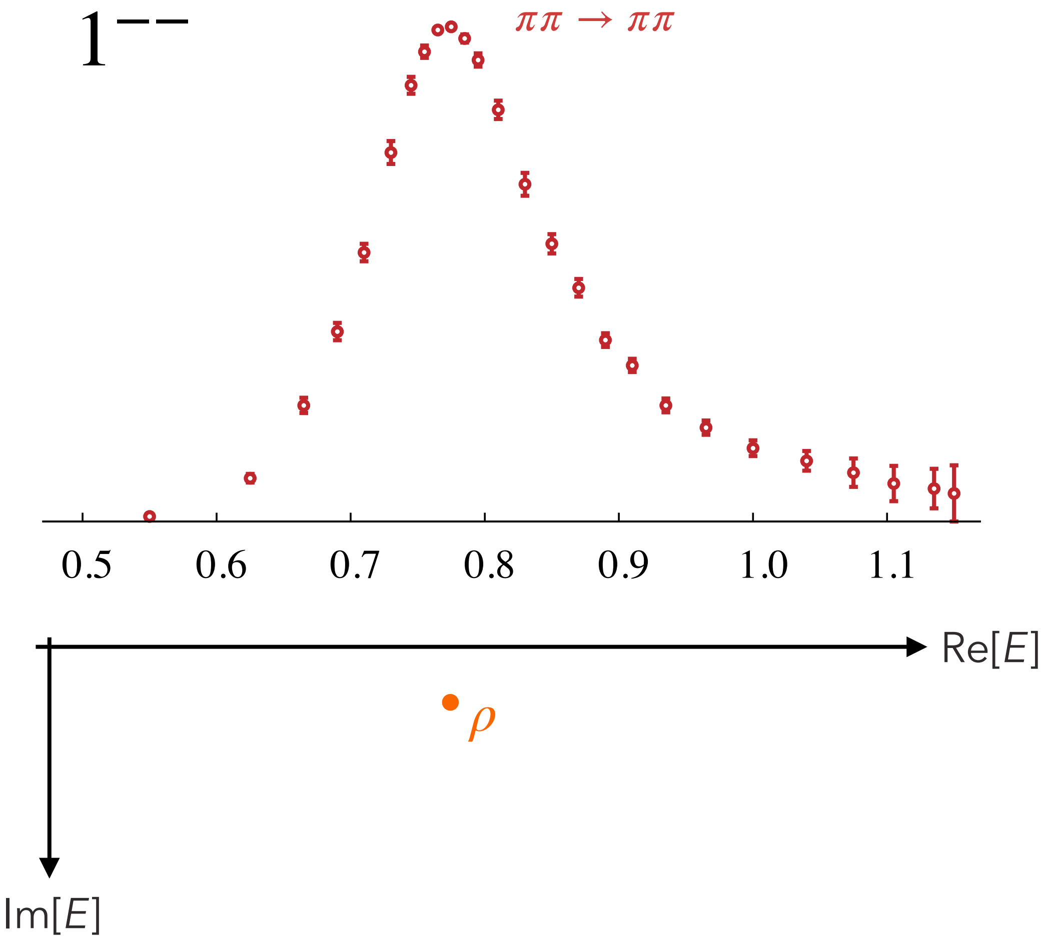
$\rightsquigarrow$ tool to explore QCD dynamics

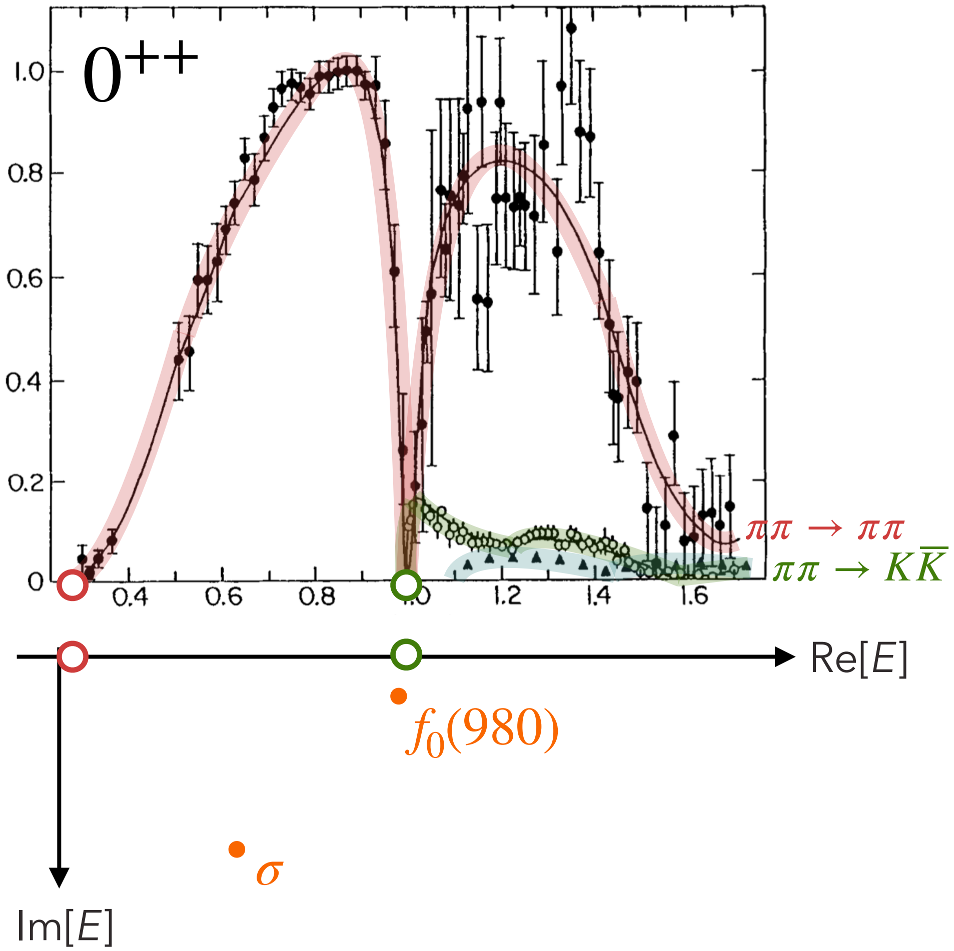
not looking for precision (at first)

but what are we really trying to get at?

excited hadrons are **resonances** $\rightsquigarrow$ scattering amplitudes

‘production’ closely related, see later …

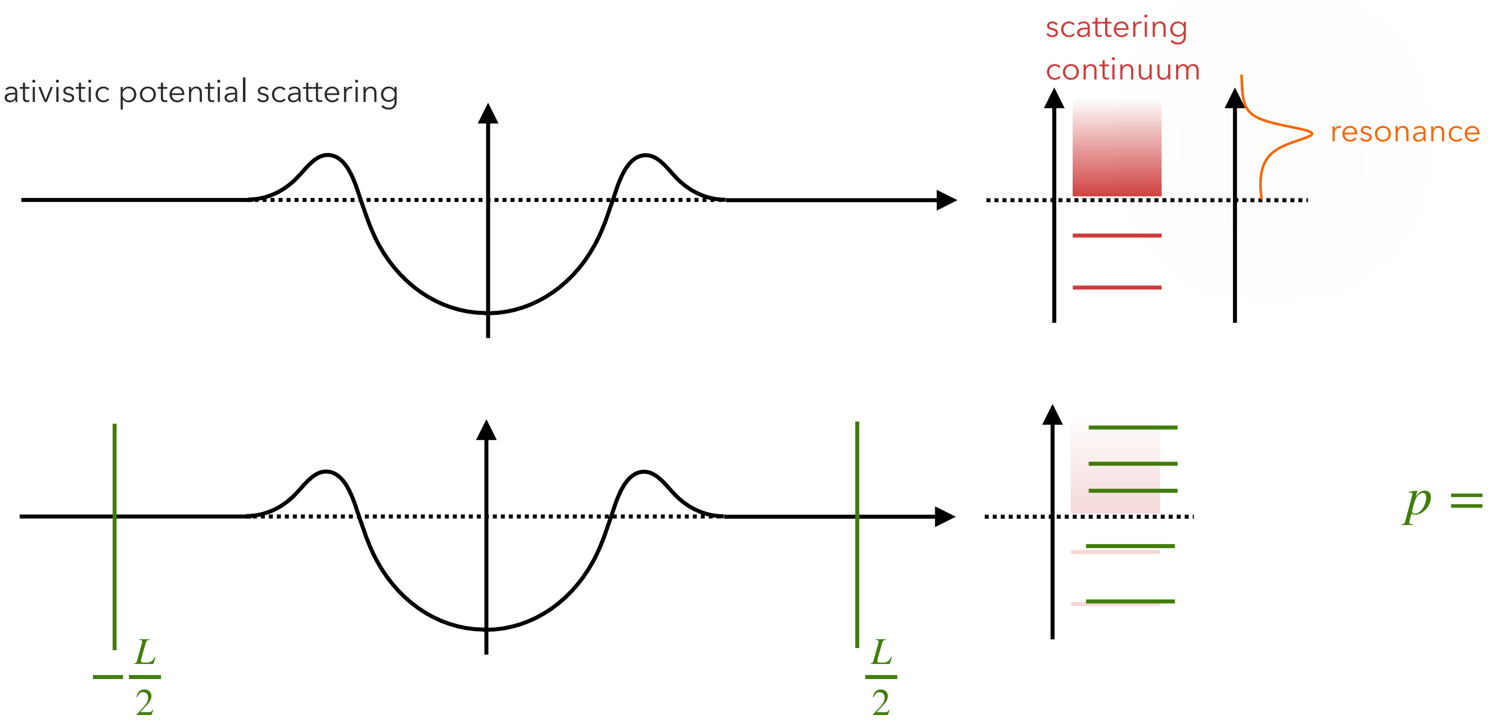




scattering amplitudes in finite-volume

simple concept – **discrete spectrum** in a finite volume controlled by scattering

e.g. non-relativistic potential scattering



$$p = \frac{2\pi}{L} - \frac{2}{L}\delta(p)$$

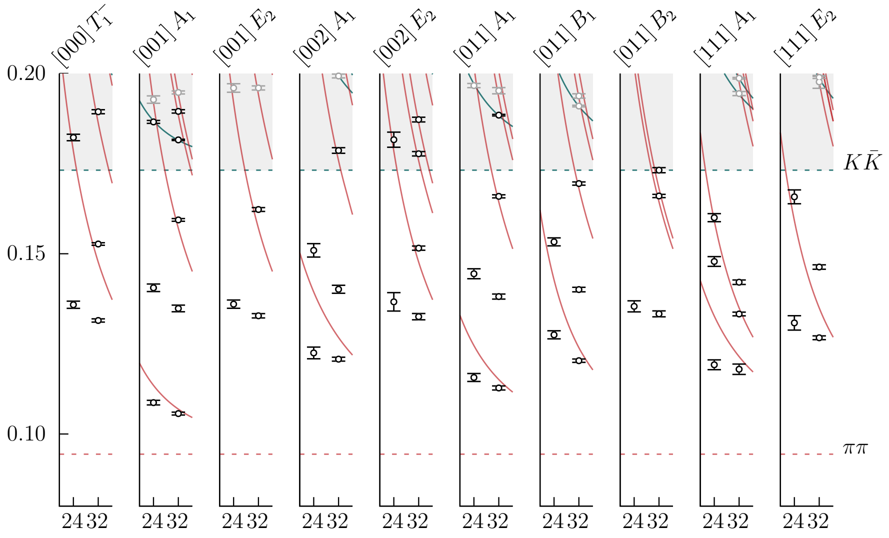
relativistic QFT in periodic cubic volume

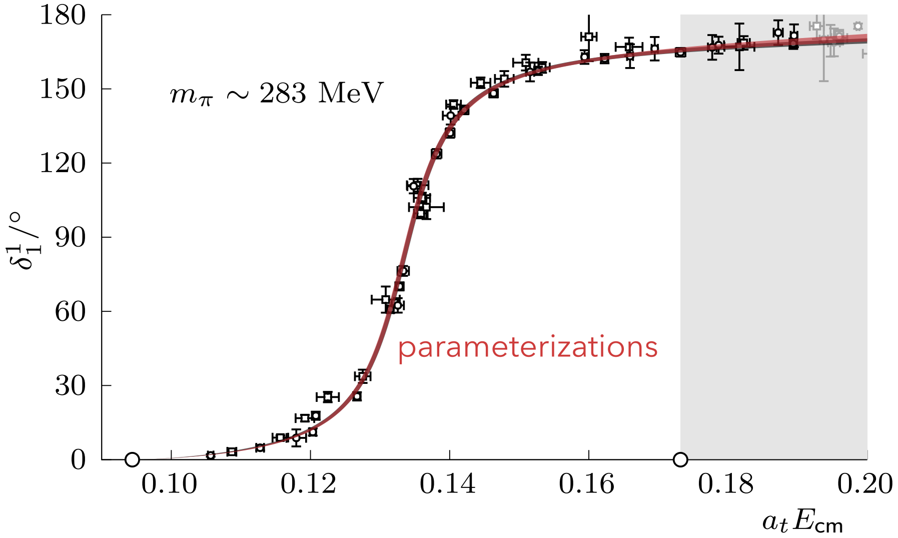
$$0 = \det\Big[1 + \underbrace{i\rho(E)}_{\text{phase space}}\,\underbrace{t(E)}_{\text{scattering } t\text{-matrix}}\big(1 + \underbrace{i\mathcal{M}(E,L)}_{\text{finite-volume functions}}\big)\Big]$$

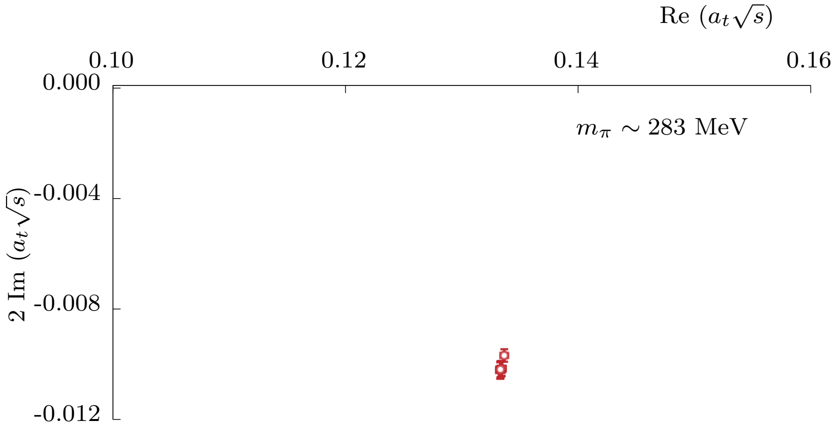
a.k.a the "Lüscher method"

‘simple’ case of elastic scattering

e.g. ρ in $\pi\pi\ I = 1$ scattering

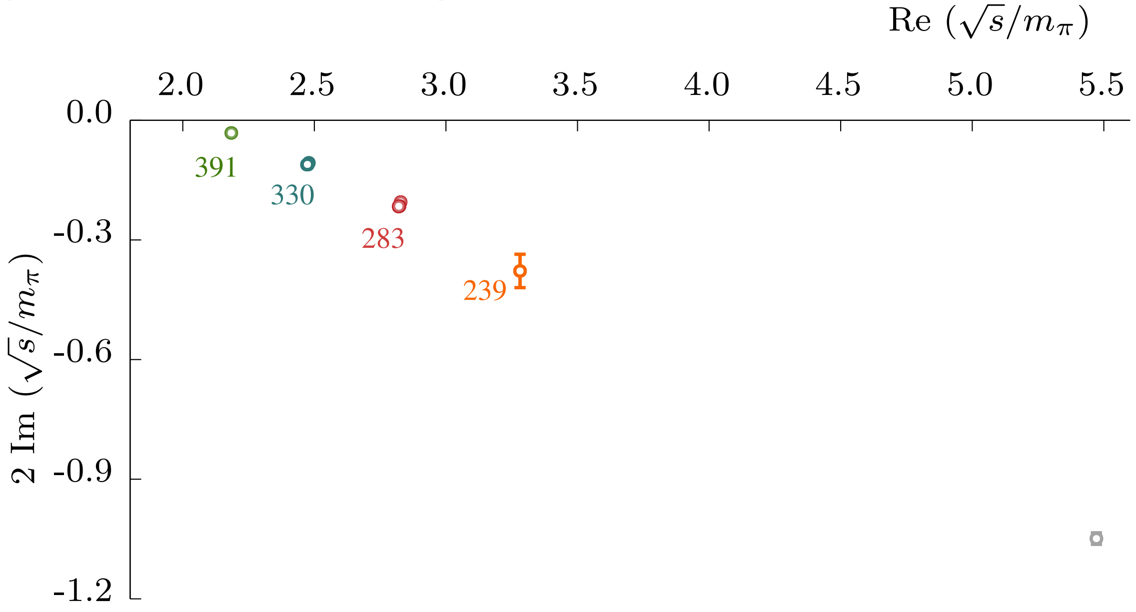






arXiv:2303.10701

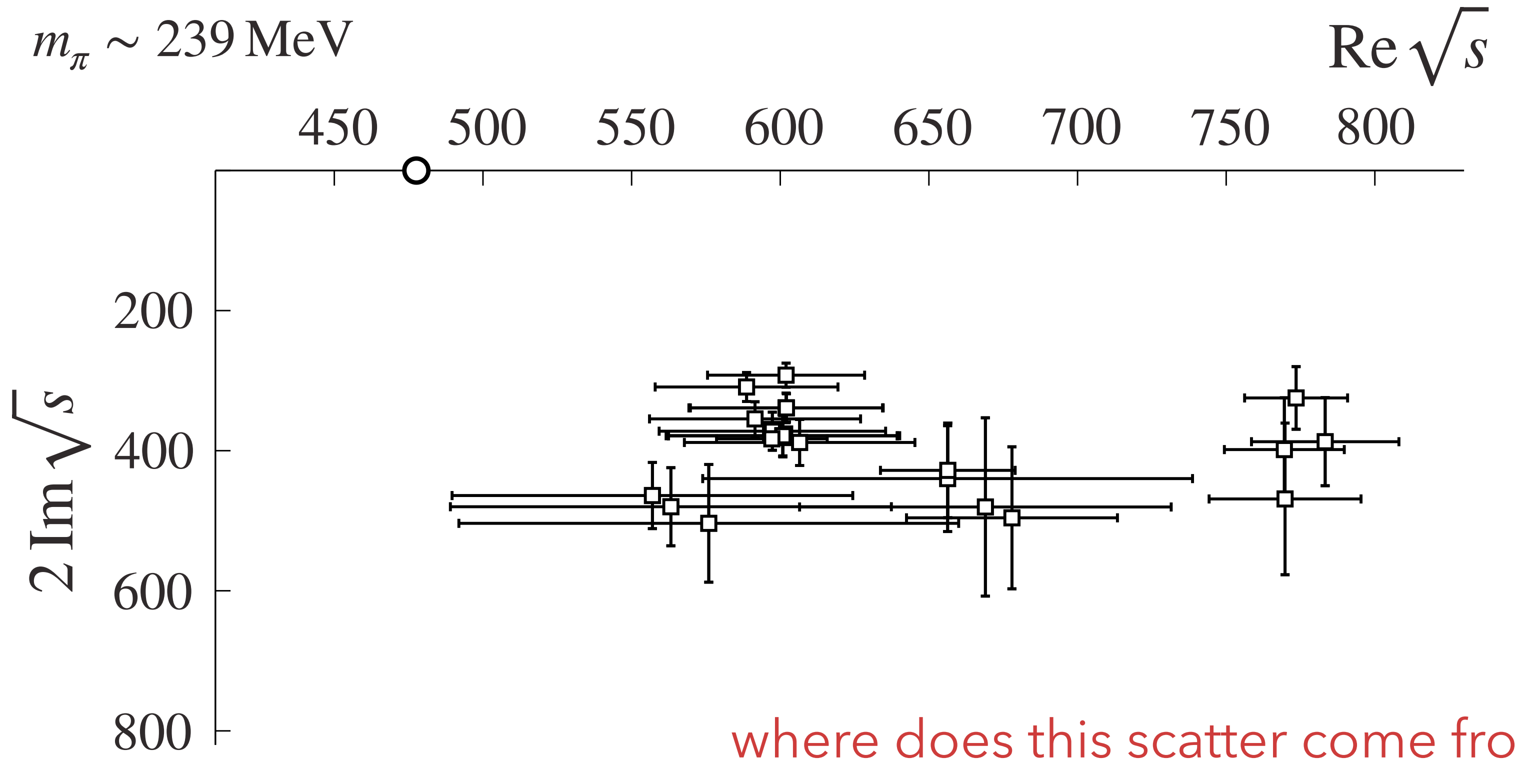
ρ pole moves with quark mass



‘simple’ case of elastic scattering

but … σ in $\pi\pi\ I = 0$ scattering … ?

PRL 118 022002 (2017)



where does this scatter come from?

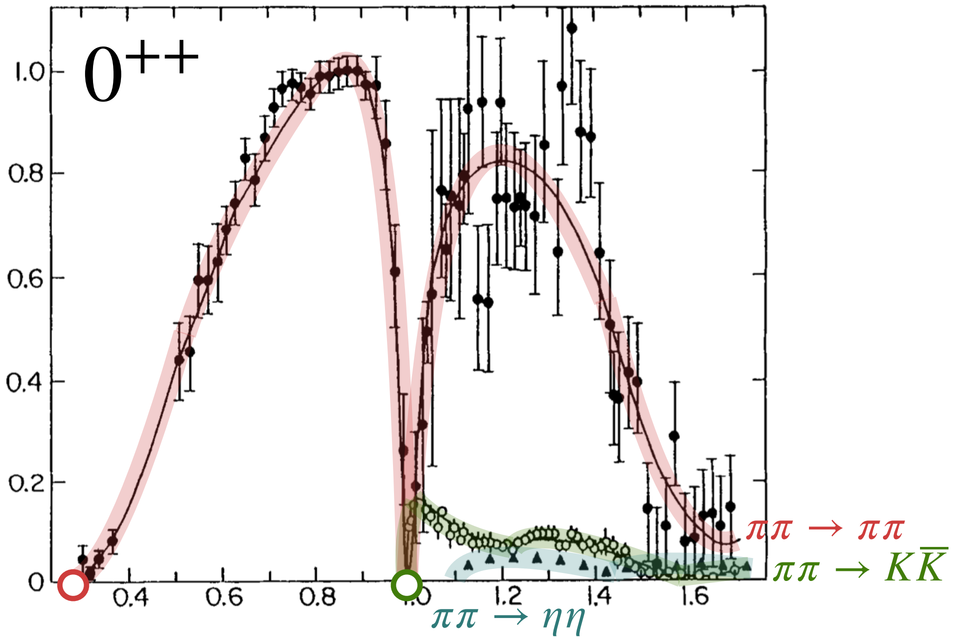
will come back to this

coupled-channel scattering

mandatory in finite-volume to consider all open (and nearly open) channels

$$0 = \det\Big[1 + i\underbrace{\rho(E)}_{\text{phase space}}\,\underbrace{t(E)}_{\text{scattering } t\text{-matrix}}\big(1 + i\underbrace{\mathscr{M}(E,L)}_{\text{finite-volume functions}}\big)\Big]$$

matrix in space of coupled-channels (and partial-waves)



$$\mathbf{t}(E) = \begin{pmatrix} t_{\pi\pi,\pi\pi} & t_{\pi\pi,K\overline{K}} & t_{\pi\pi,\eta\eta} \\ t_{\pi\pi,K\overline{K}} & t_{K\overline{K},K\overline{K}} & t_{K\overline{K},\eta\eta} \\ t_{\pi\pi,\eta\eta} & t_{K\overline{K},\eta\eta} & t_{\eta\eta,\eta\eta} \end{pmatrix}$$

subject to constraint of unitarity $\mathrm{Im}\, t_{ij}(E) = -\rho_i(E)\,\delta_{ij}$

coupled-channel scattering

compute the discrete spectrum of states in one or more volumes (and/or moving frames)

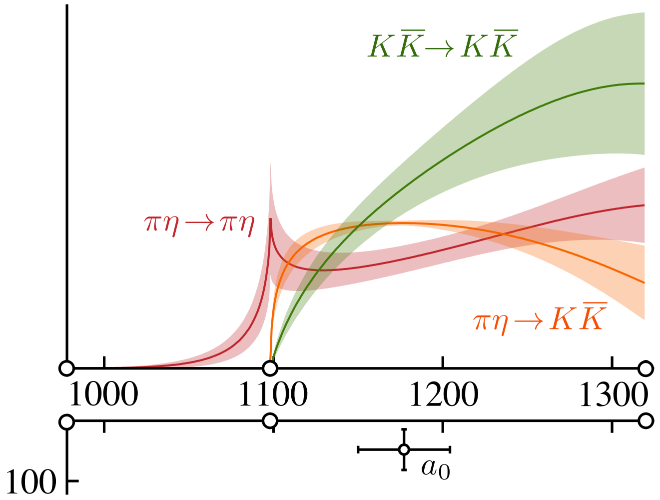
parameterize the *t*-matrix, solve $0 = \det\left[1 + i\rho(E)\, t(E)\big(1 + i\mathcal{M}(E,L)\big)\right]$ for the 'model spectrum'

adjust parameters until the lattice QCD spectrum is described

try several variations of parameterization to check your result is robust

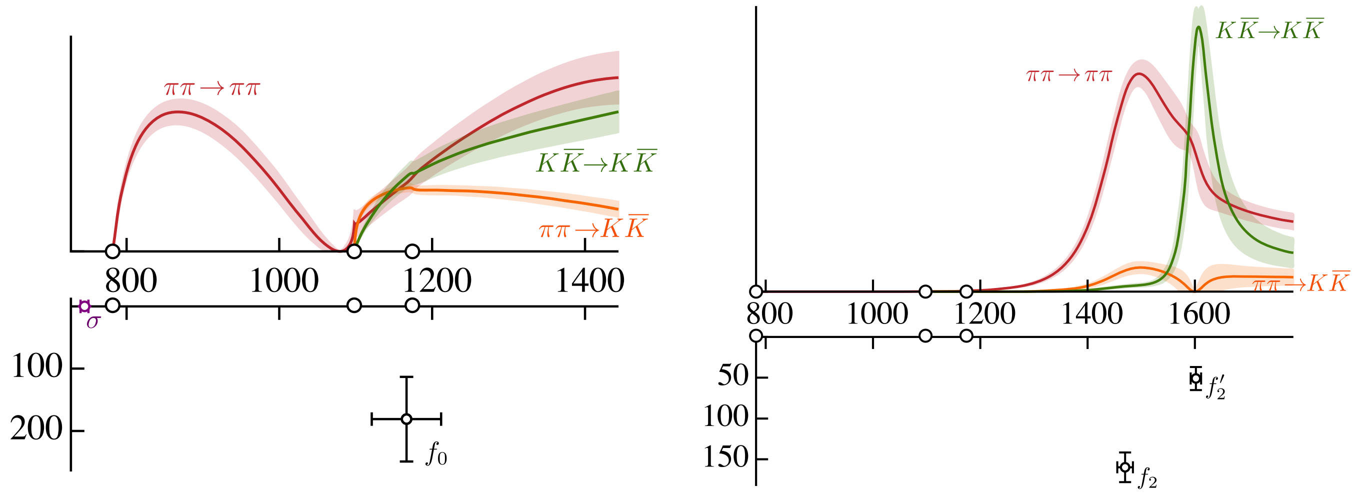
scalars, tensors, axials ...

$J^P = 0^+ \; I^G = 1^- \; (\eta\pi, K\bar{K})$



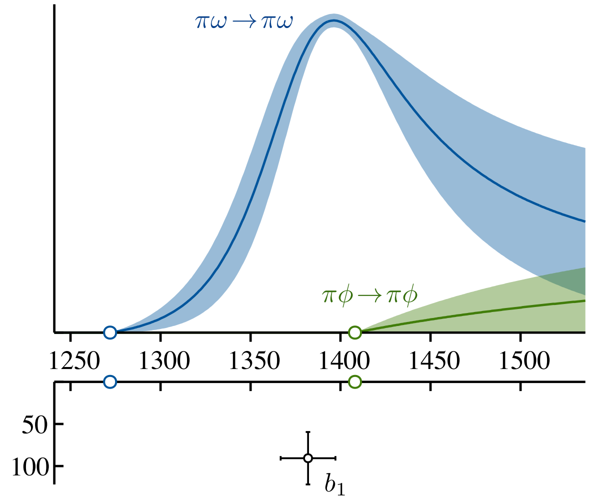
PRD 93 094506 (2016)

$J^P = 0^+ \; I^G = 0^+ \; (\pi\pi, K\bar{K}, \eta\eta)$ $J^P = 2^+ \; I^G = 0^+ \; (\pi\pi, K\bar{K}, \eta\eta)$



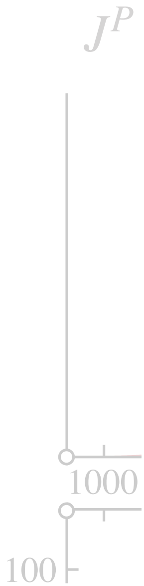
PRD 97 054513 (2018)

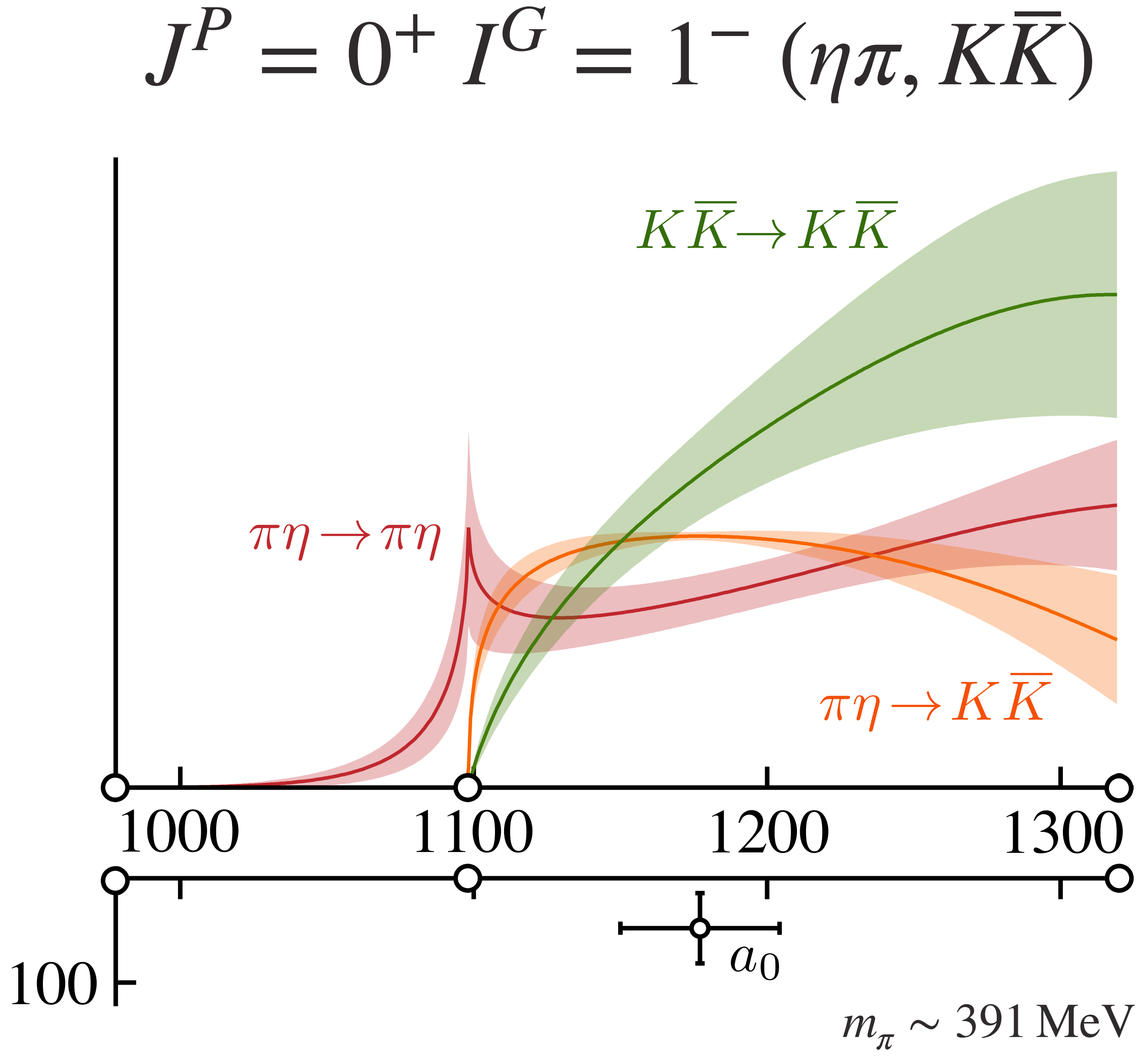
$J^P = 1^+ \; I^G = 1^- \; (\pi\omega, \pi\phi)$

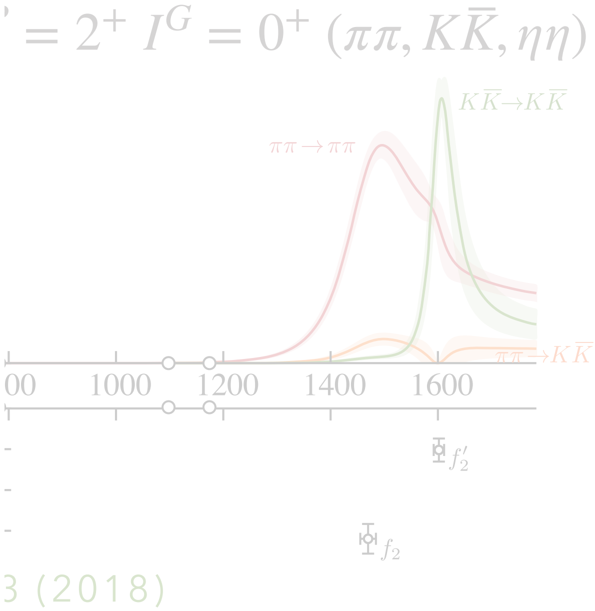


PRD 100 054506 (2019)

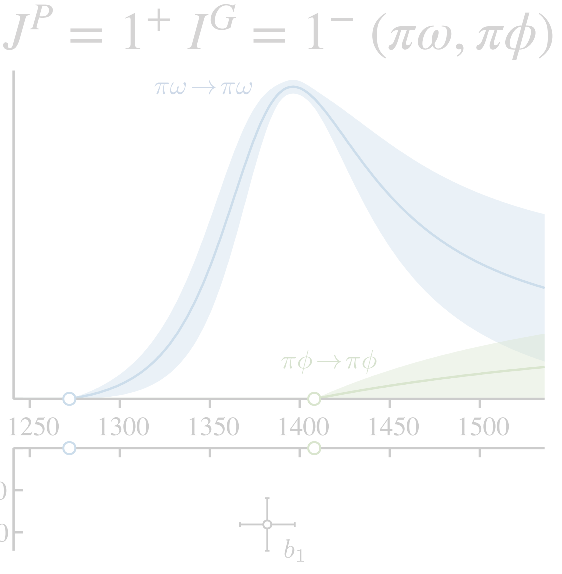
scalars, tensors, axials …







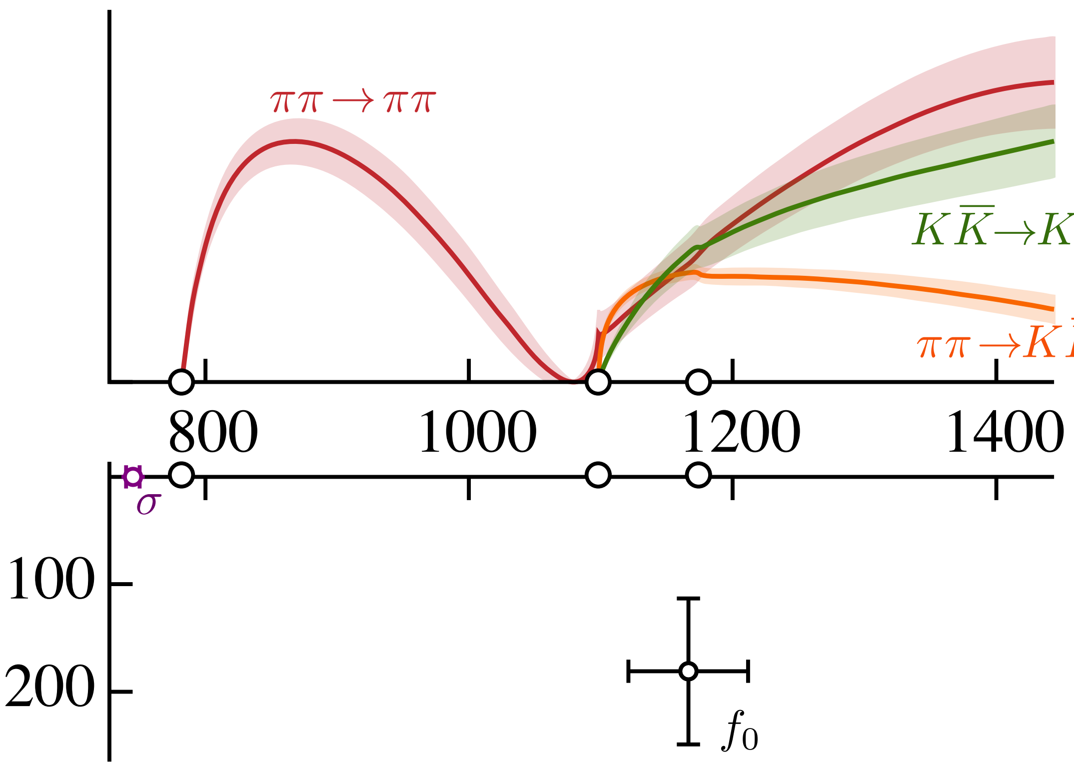
3 (2018)



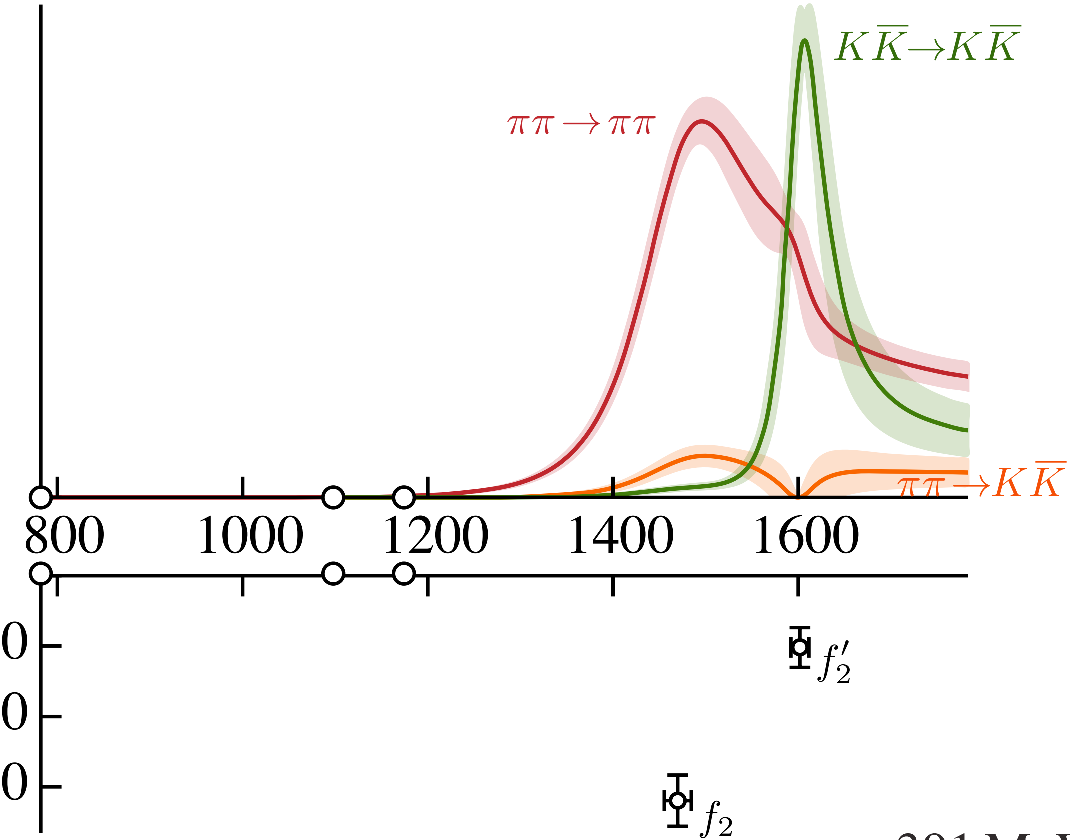
PRD 100 054506 (2019)

scalars, tensors, axials …

$$J^P = 0^+ \; I^G = 0^+ \; (\pi\pi, K\bar{K}, \eta\eta)$$



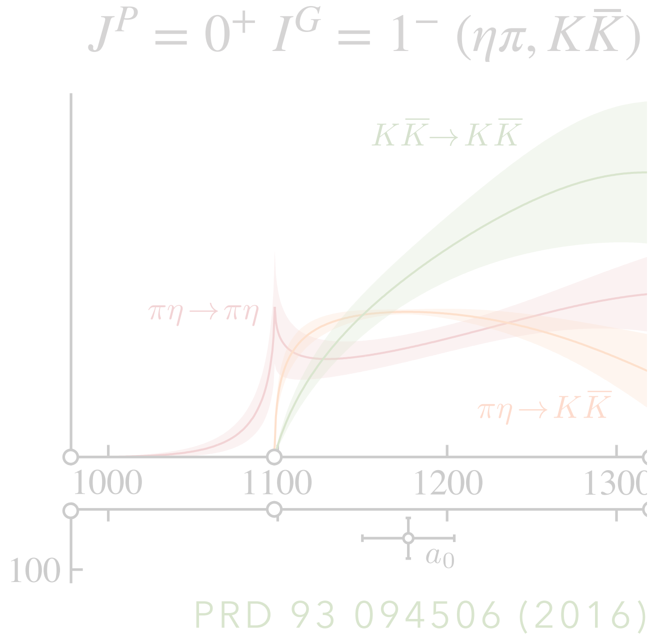
$$J^P = 2^+ \; I^G = 0^+ \; (\pi\pi, K\bar{K}, \eta\eta)$$



$m_\pi \sim 391$ MeV

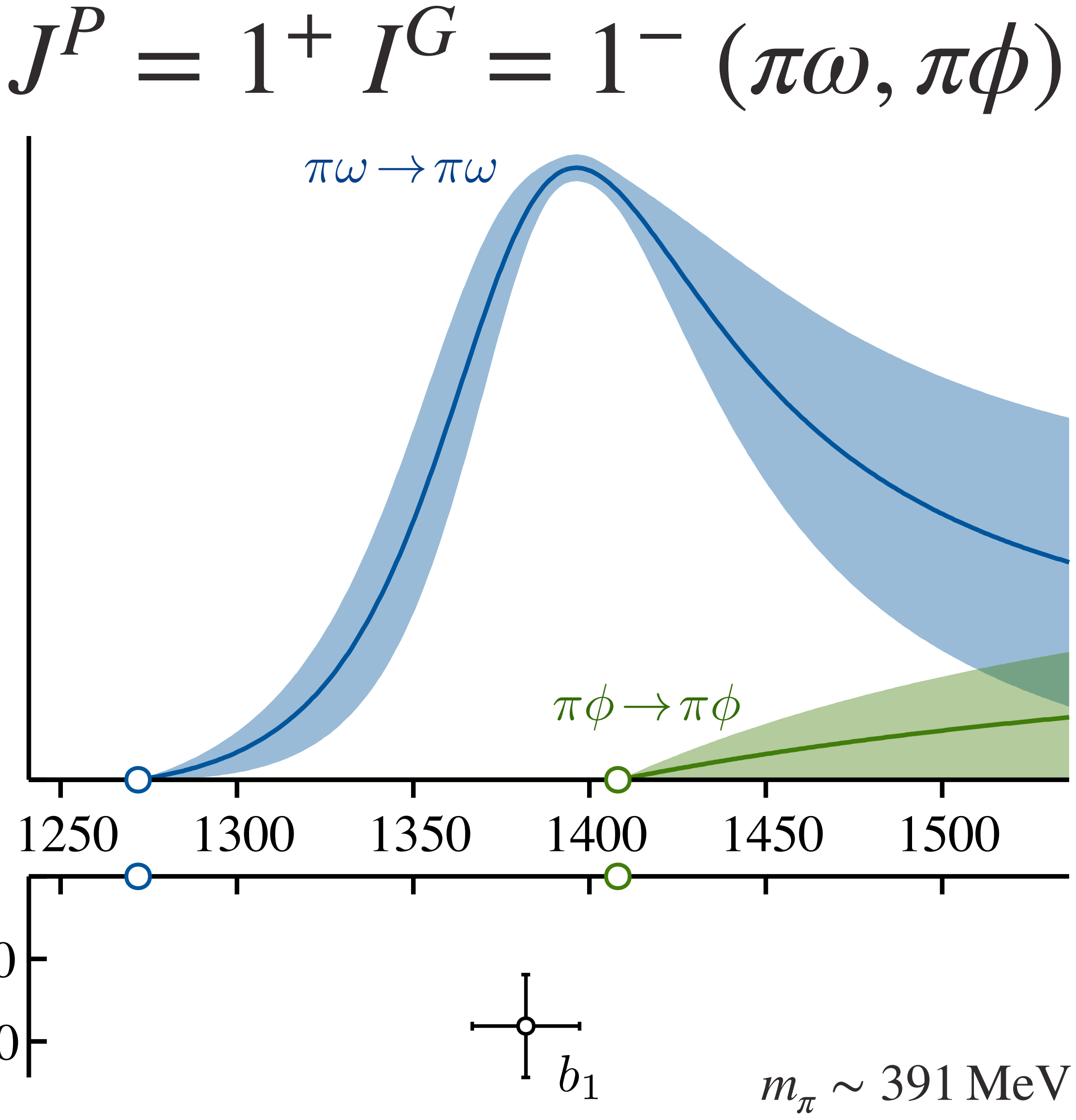
PRD 100 054506 (2019)

scalars, tensors, axials ...

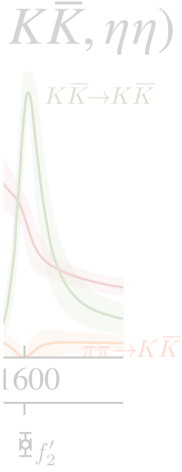


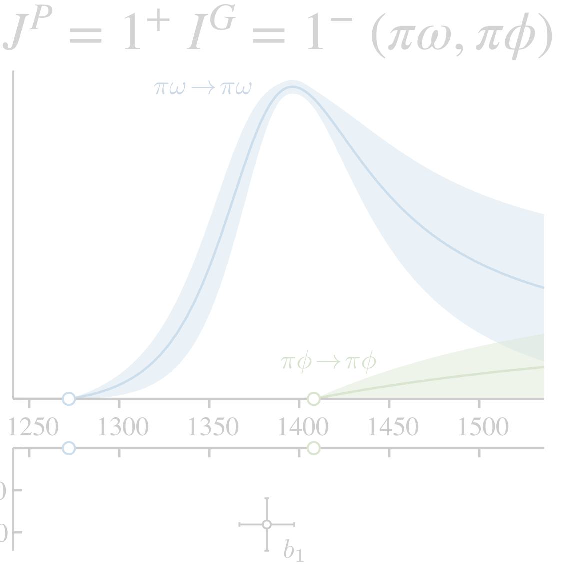
$J^P = 0^+ \; I^G = 1^- \; (\eta\pi, K\bar{K})$

PRD 93 094506 (2016)



$J^P = 1^+ \; I^G = 1^- \; (\pi\omega, \pi\phi)$





$J^P = 1^+ \; I^G = 1^- \; (\pi\omega, \pi\phi)$

PRD 100 054506 (2019)

not shown here, the **D/S ratio** also determined

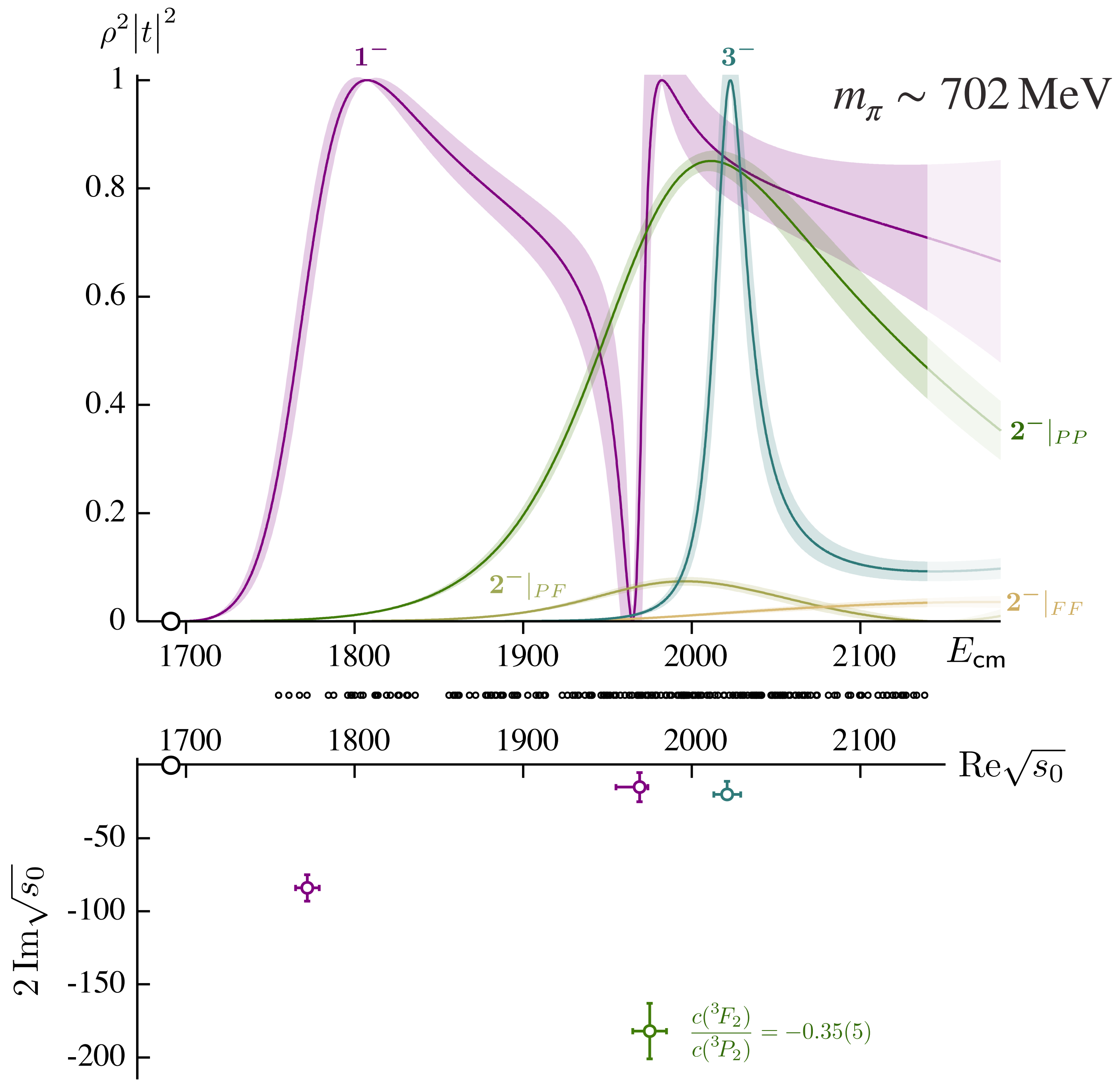
overlapping resonances

QCD in the SU(3) limit, $m_u = m_d = m_s$

$\eta_8\,\omega_8 \to \eta_8\,\omega_8$

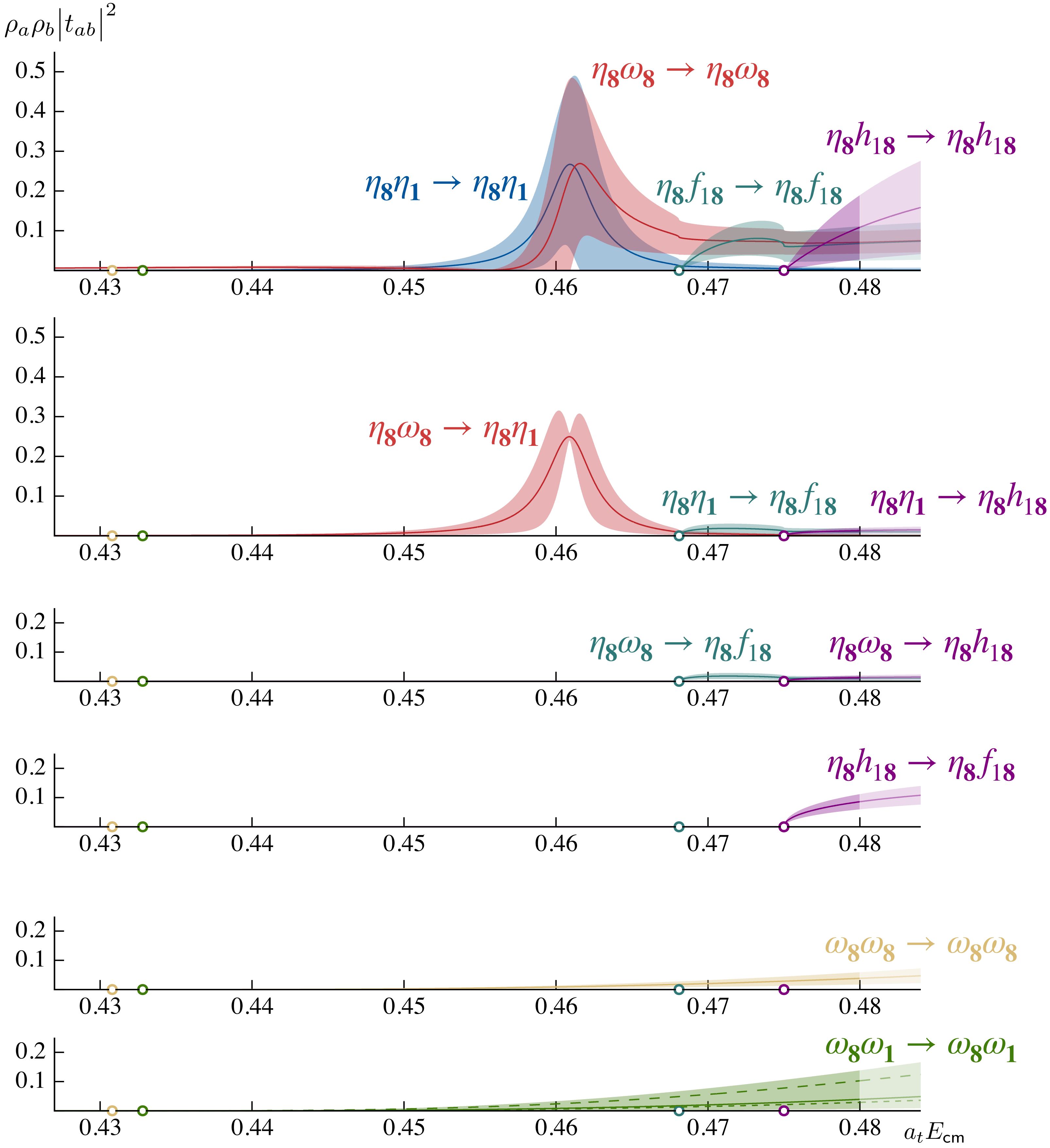
c.f. $\omega'_J \to \pi\rho$

PRD 103 074502 (2021)



decays of an exotic hybrid 1^{-+}

PRD 103 054502 (2021)

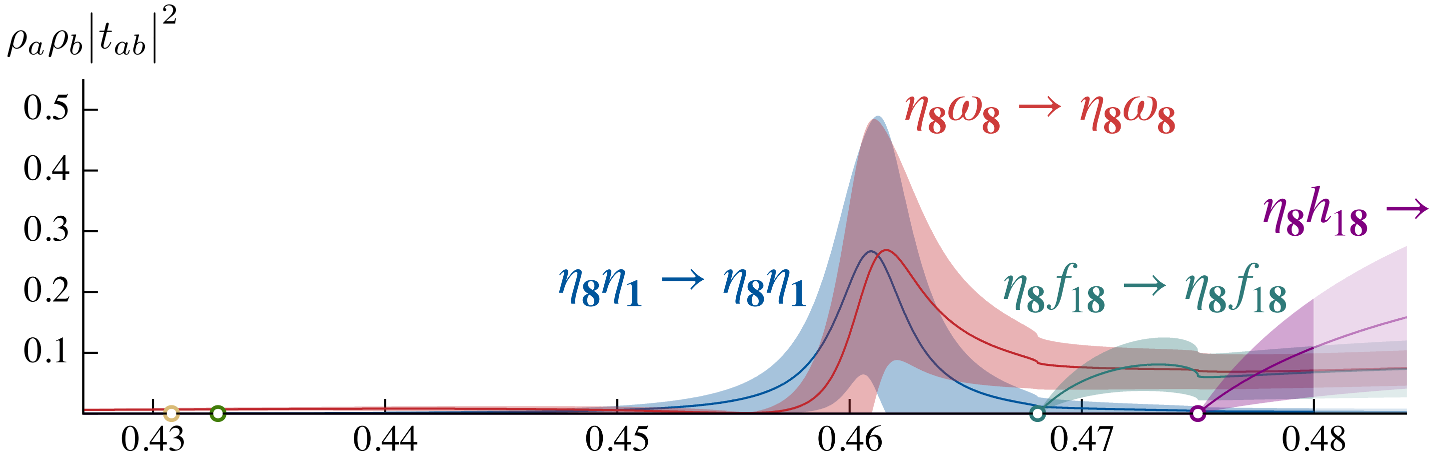


QCD in the SU(3) limit, $m_u = m_d = m_s$

'eight' coupled channels

decays of an exotic hybrid 1^{-+}

PRD 103 054502 (2021)



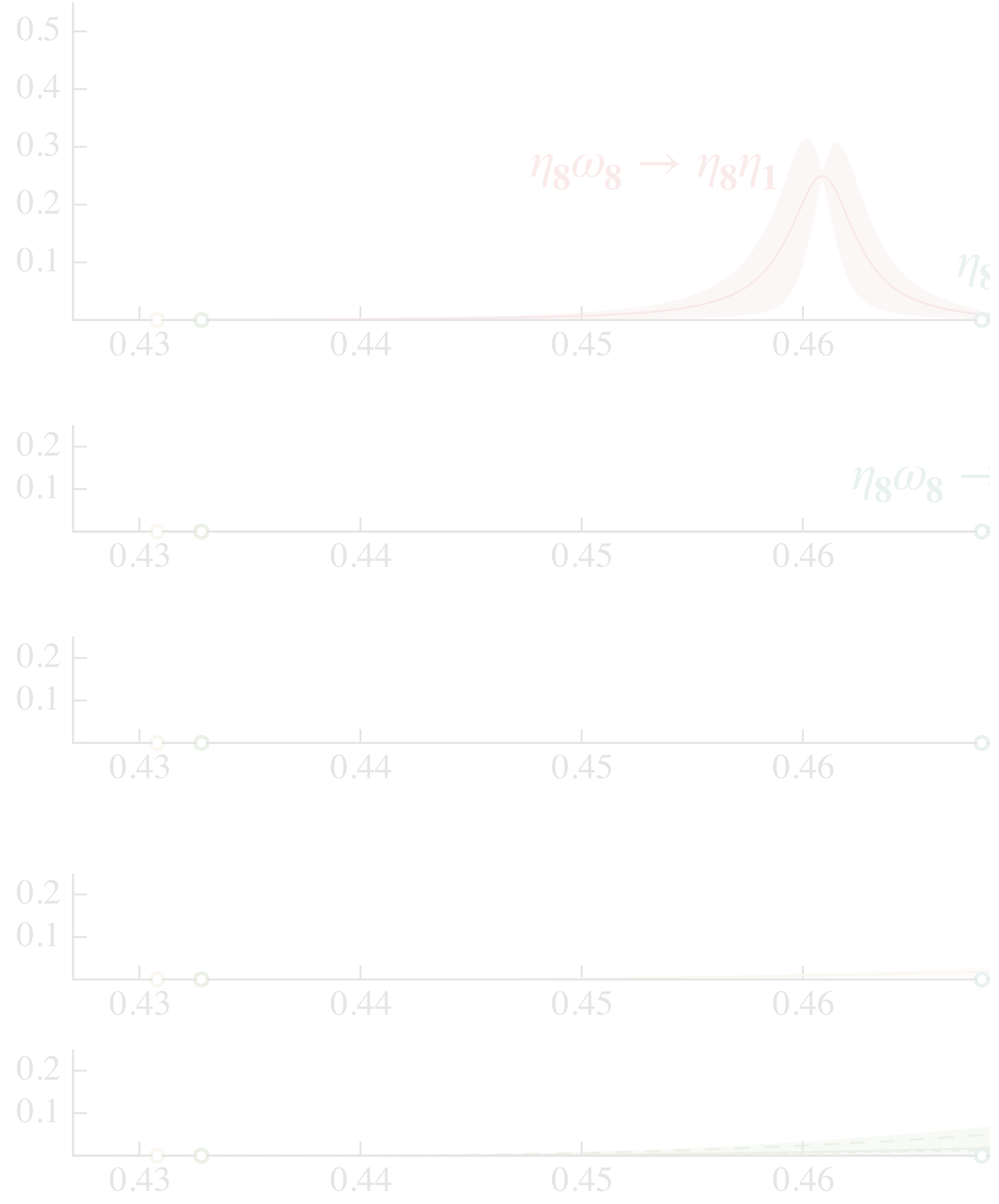
QCD in the SU(3) limit, $m_u = m_d = m_s$

$\eta_8\eta_1 \sim \pi\eta'$

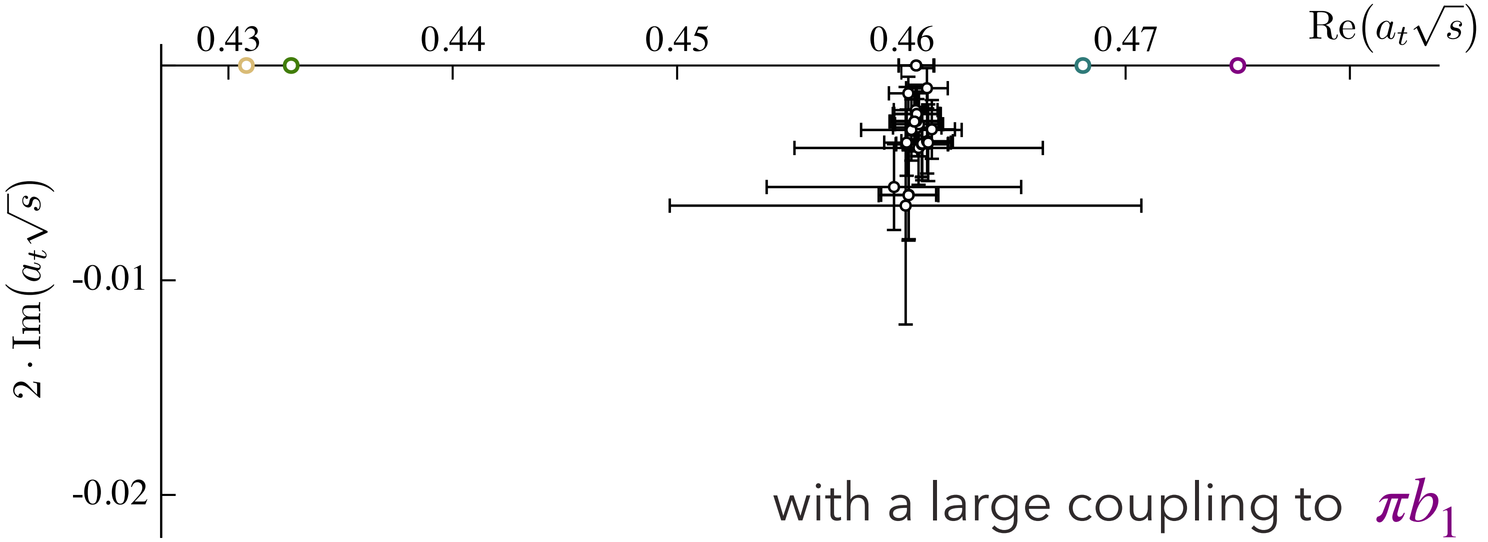
$\eta_8\omega_8 \sim \pi\rho$

$\eta_8 f_{18} \sim \pi f_1, \eta a_1$

$\eta_8 h_{18} \sim \pi b_1$



a single narrow resonance



with a large coupling to πb_1

(speculative) extrapolation to physical point gives broad resonance with dominant πb_1 decay

production amplitudes

can also couple to an external current, e.g. electroweak production

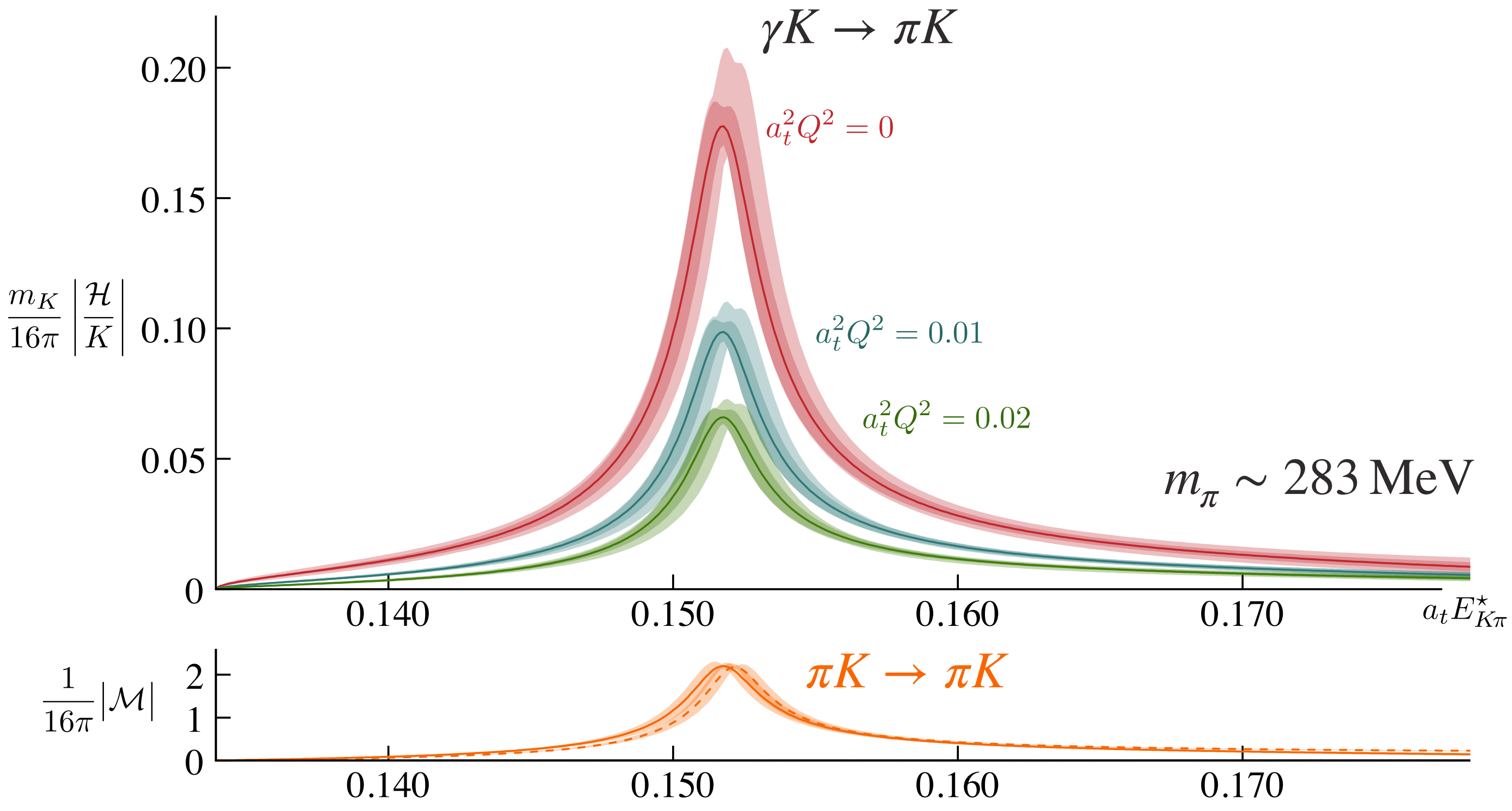
photoproduction, *B*-decays, radiative charmonium decays …

production amplitudes

can also couple to an external current, e.g. electroweak production

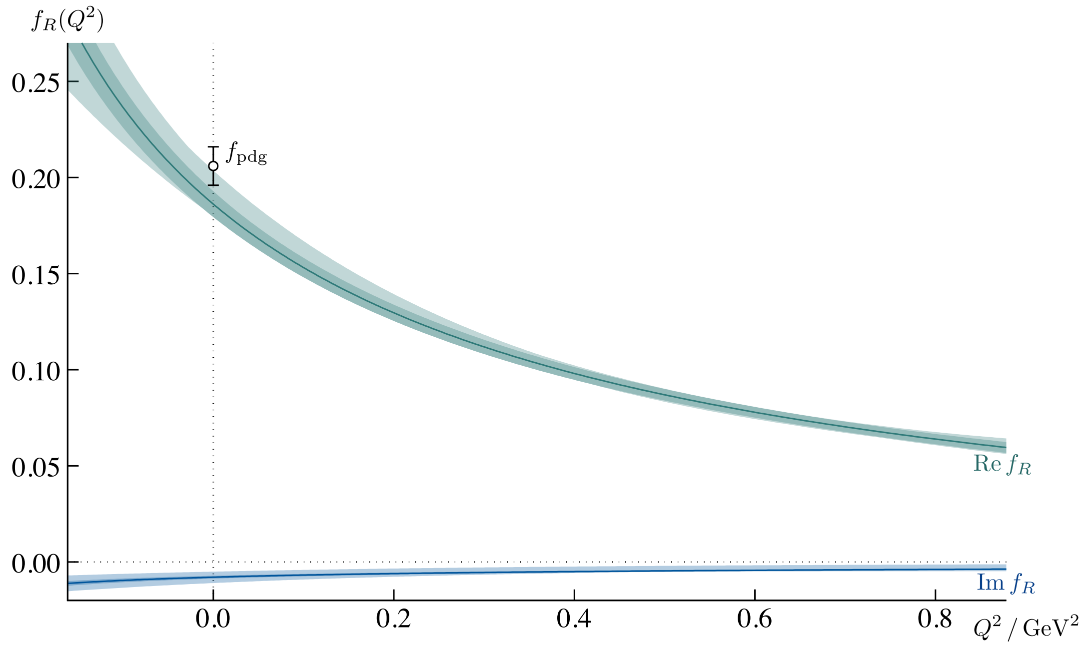
compute three-point functions with a current insertion
correct for the finite-volume

a recent example of the approach …



PRD 106 114513 (2022)

resonance transition form-factor $K^* \to K\gamma$

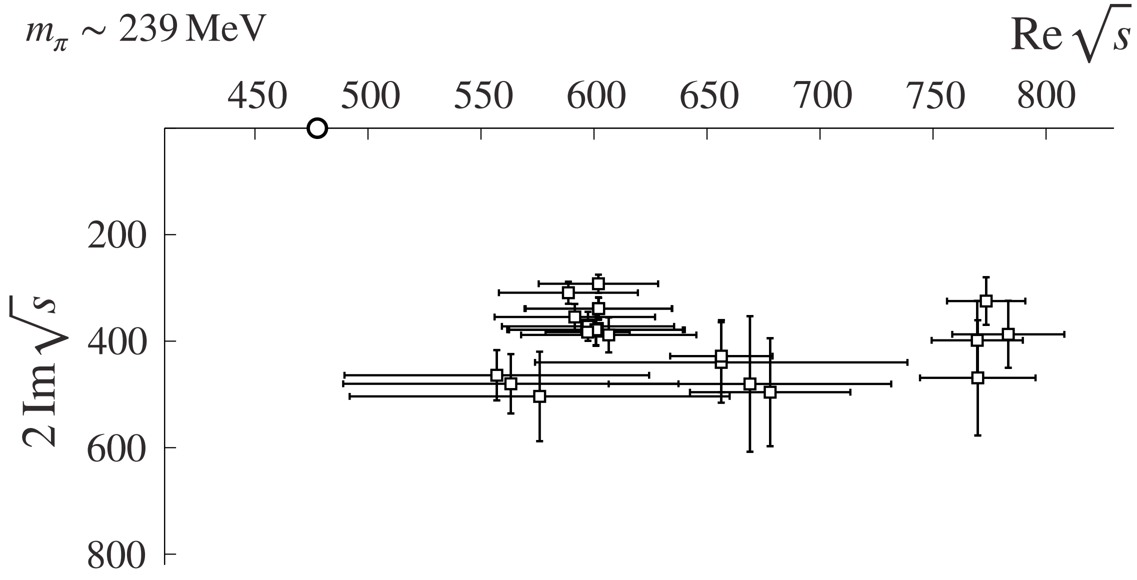


parameterizing amplitudes – always reliable?

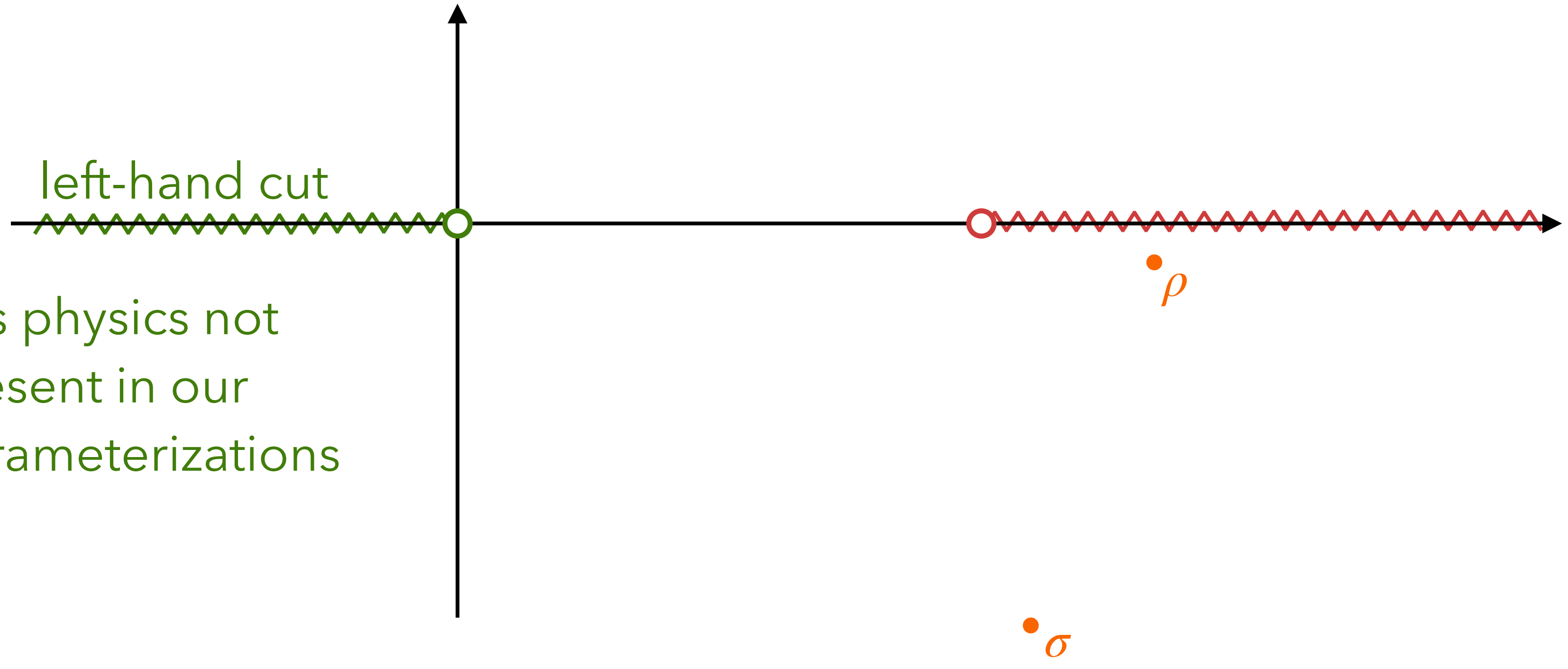
‘narrow’ resonances tend to be robust, but some hadrons are broad …

parameterizing amplitudes – always reliable?

‘narrow’ resonances tend to be robust, but some hadrons are broad …



different parameterizations analytically continued far into the complex energy plane

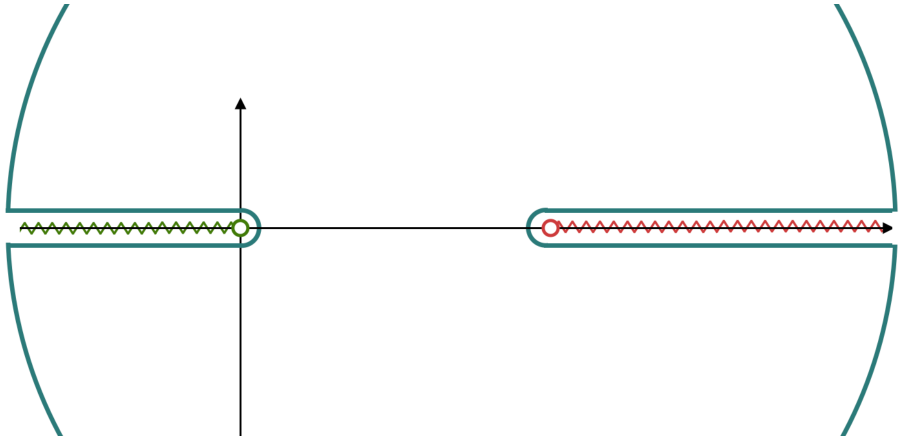


this physics not present in our parameterizations

parameterizing amplitudes – always reliable?

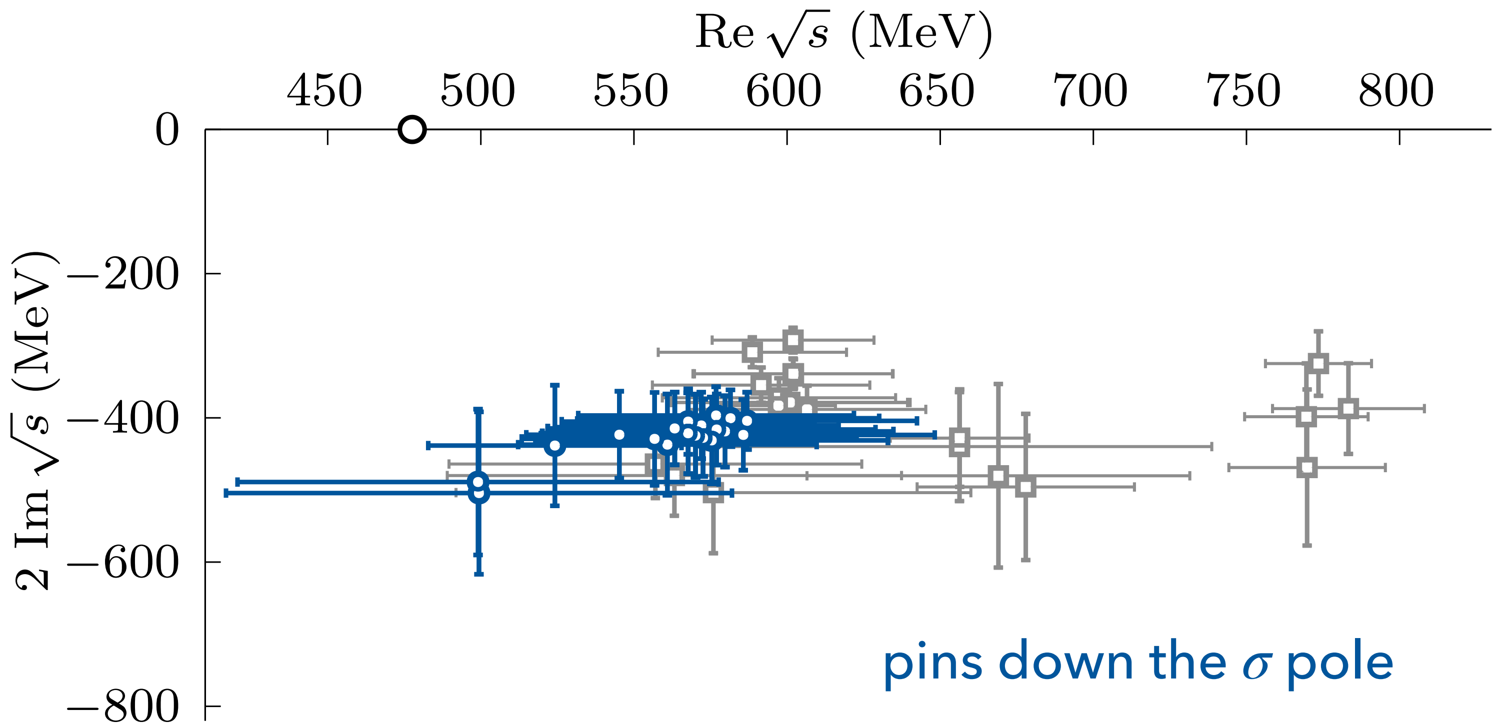
additional constraints of crossing symmetry/analyticity implemented through dispersion relations

$$\tilde{t}^I_\ell(s) = \tau^I_\ell(s) + \sum_{I',\ell'} \int_{s_{\text{thr.}}}^{\infty} ds'\, K^{II'}_{\ell\ell'}(s',s)\, \text{Im}\, t^{I'}_{\ell'}(s')$$



lattice QCD input: partial-wave amplitudes in $I = 0,1,2$ selects those combinations of parameterizations compatible with crossing

arXiv:2304.03762

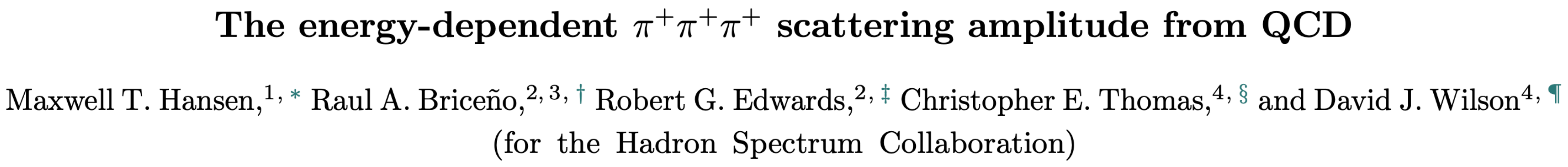


pins down the σ pole

the three-body frontier

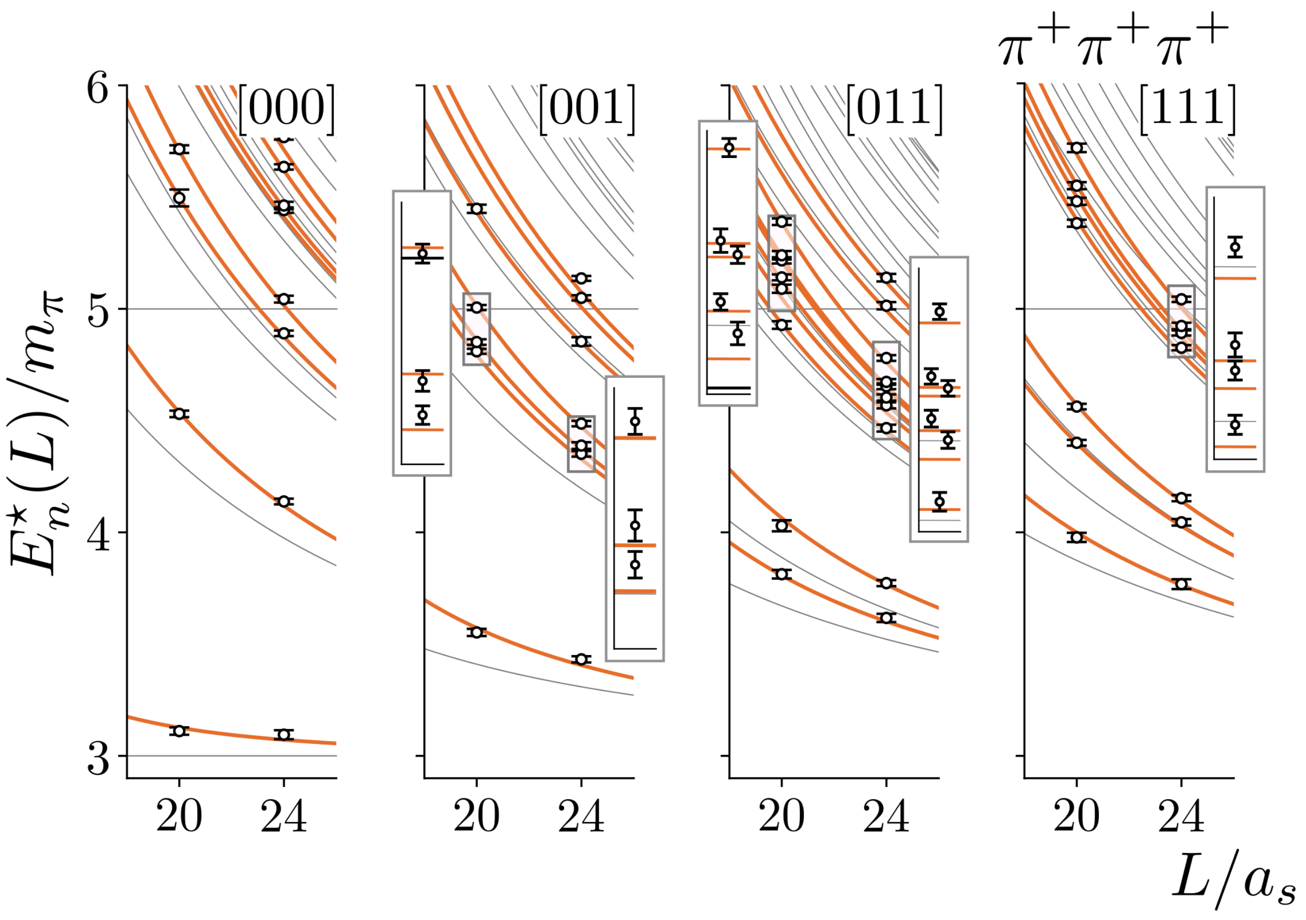
many resonances decay to three mesons

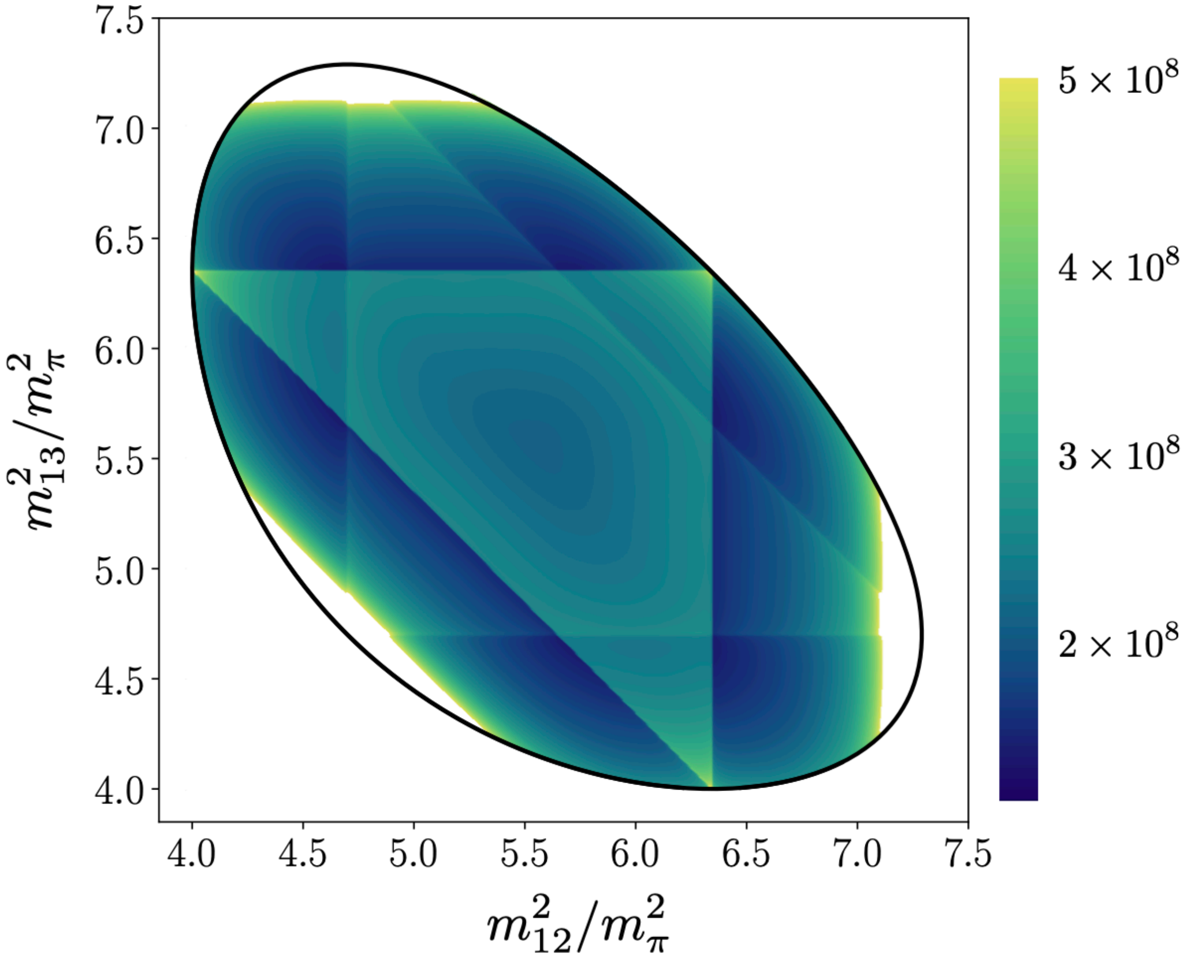
formalism to handle three-body scattering amplitudes (in infinite or finite volume) much more complicated



The energy-dependent $\pi^+\pi^+\pi^+$ scattering amplitude from QCD

Maxwell T. Hansen,[1,*] Raul A. Briceño,[2,3,†] Robert G. Edwards,[2,‡] Christopher E. Thomas,[4,§] and David J. Wilson[4,¶]
(for the Hadron Spectrum Collaboration)





summary

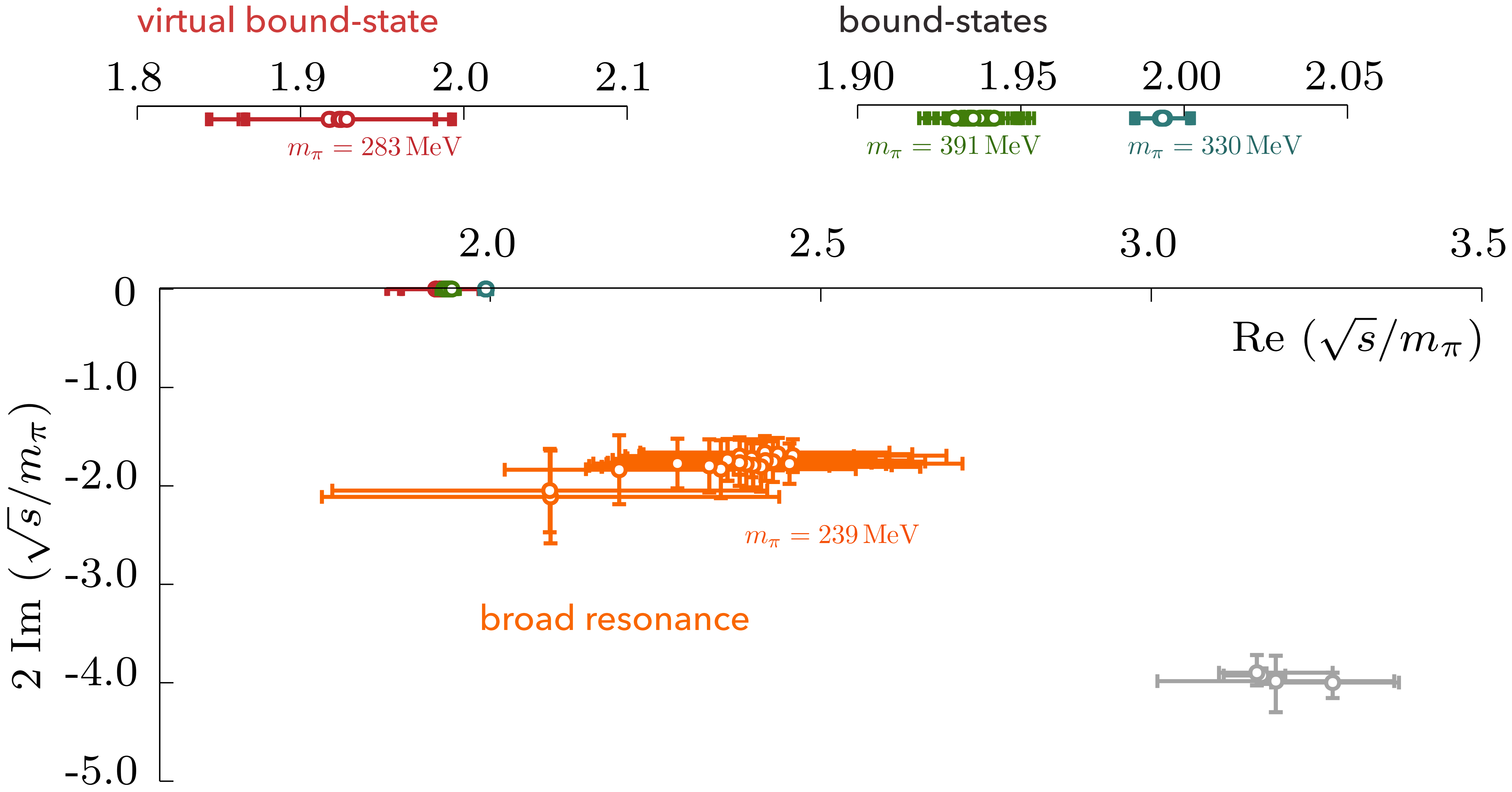
lattice QCD as a tool to investigate hadron spectroscopy within QCD has progressed rapidly

many tools from amplitude analysis have found application here

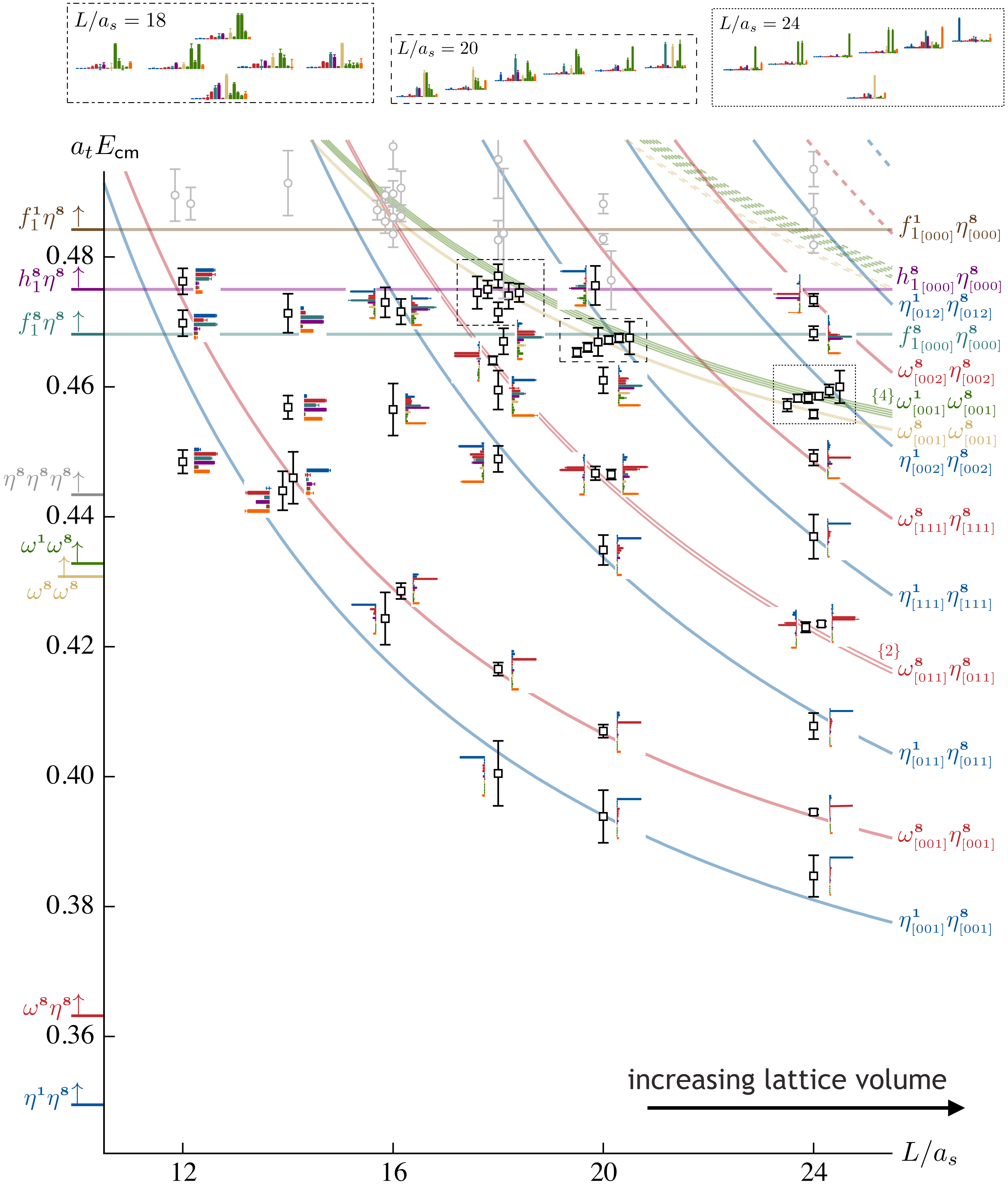
mostly calculations at unphysical quark masses – exploration of sensitivity of QCD

haven’t shown the impressive progress with charm quarks
interrogate the Cambridge crew (Christopher, Dave, Travis, Daniel)

σ pole quark mass evolution



lattice QCD spectrum computed in 6 volumes



$\eta^8 \rightsquigarrow \pi, K, \eta$

$\eta^1 \rightsquigarrow \eta'$

$\omega^8, \omega^1 \rightsquigarrow \rho, K^*, (\omega, \varphi)$

$h_1{}^8, h_1{}^1 \rightsquigarrow b_1, K_1, (h_1, h_1')$

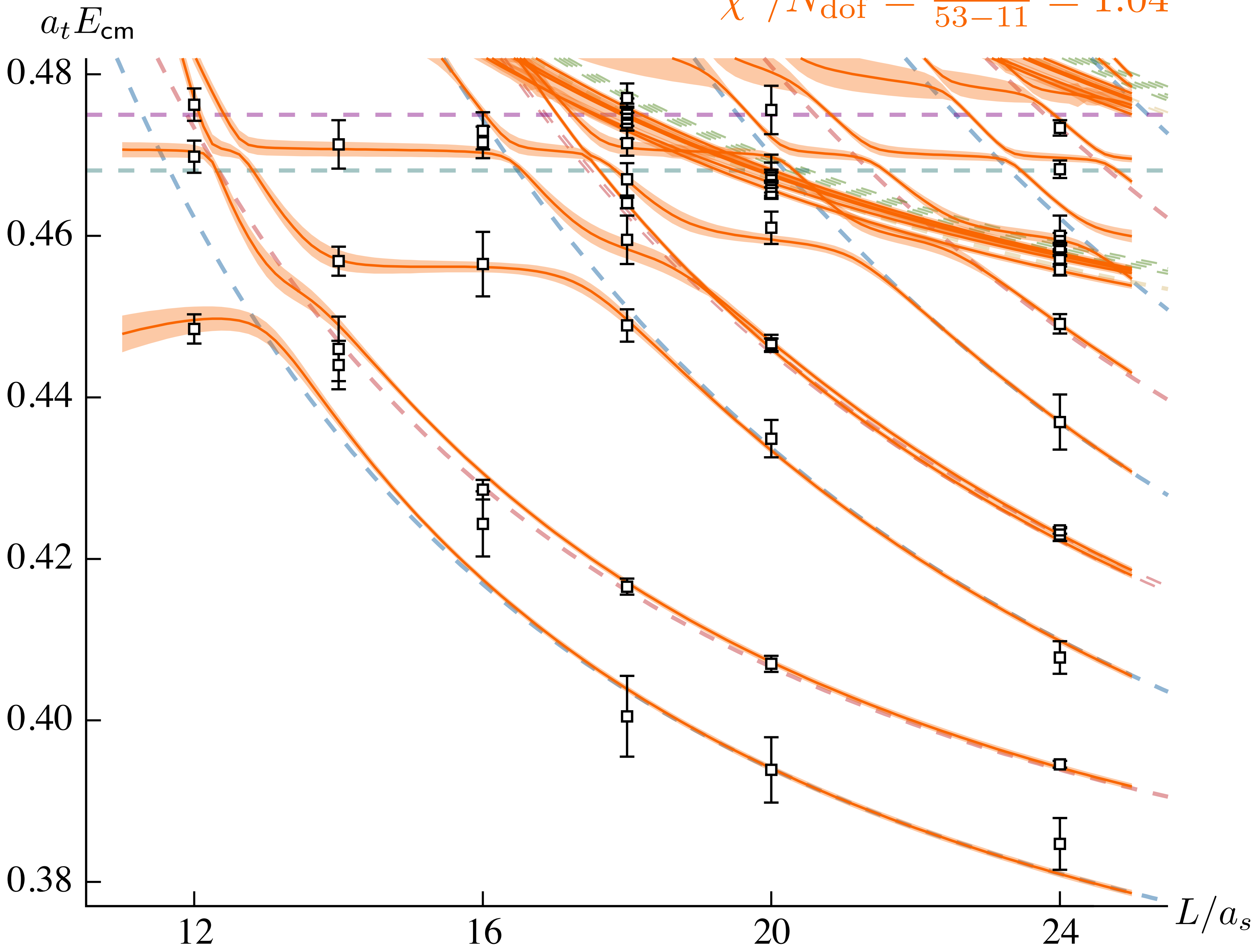
$f_1{}^8, f_1{}^1 \rightsquigarrow a_1, K_1, (f_1, f_1')$

53 energy levels to constrain
'eight' channel scattering

an ‘eight’ channel scattering amplitude

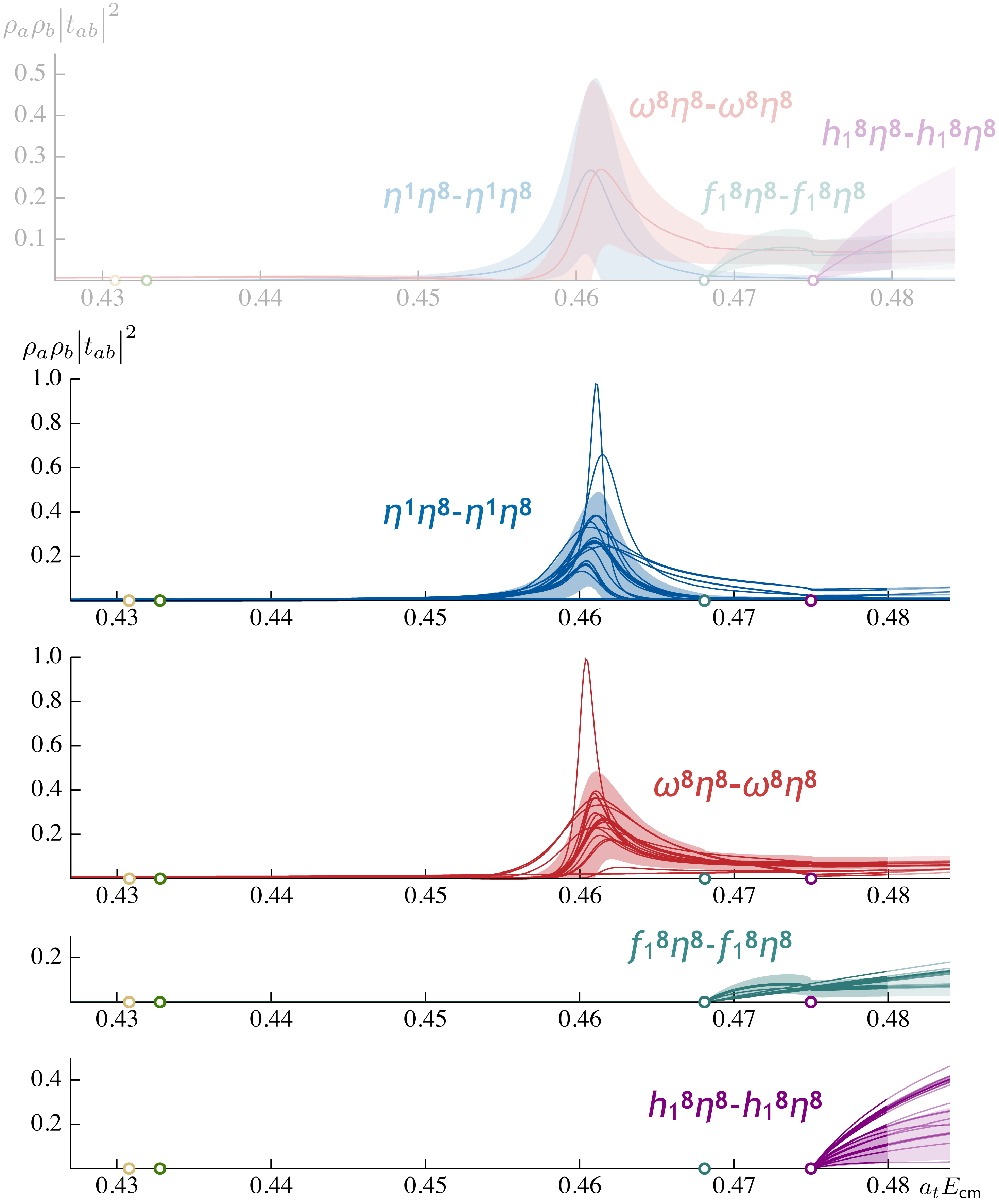
describe scattering by a unitarity-preserving *K*-matrix featuring a pole
(11 free parameters)

$\chi^2/N_{\mathrm{dof}} = \frac{43.6}{53-11} = 1.04$

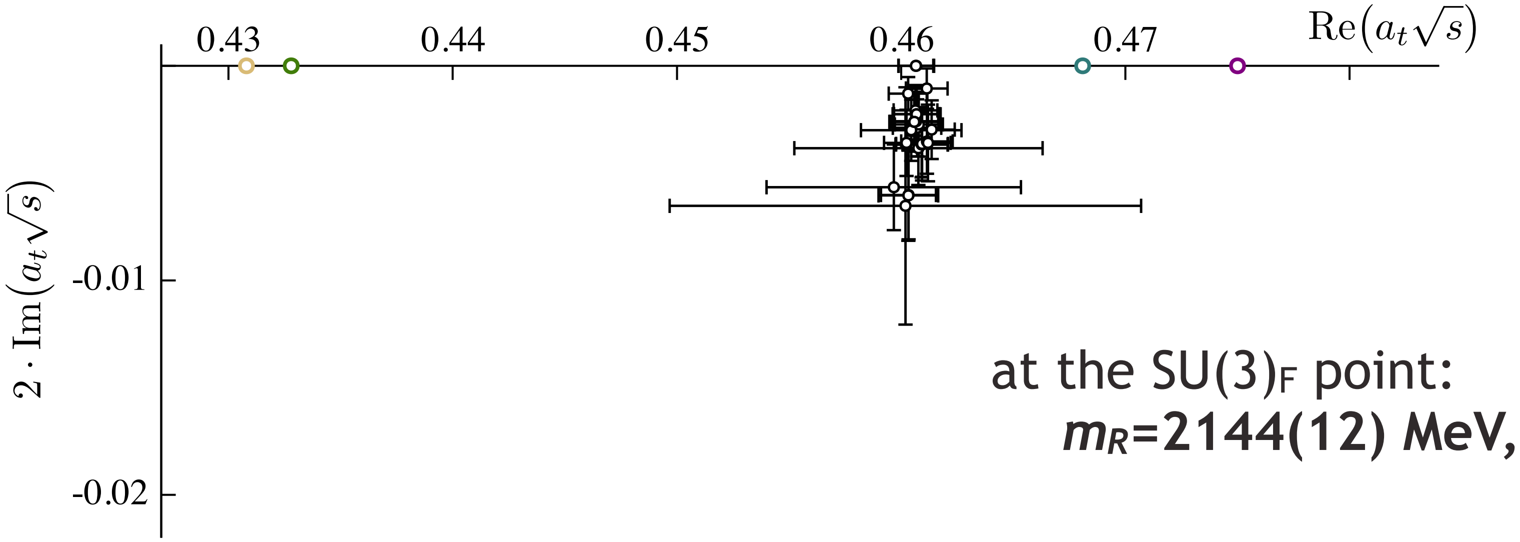


a good description of the spectrum …

‘eight’ channel scattering amplitudes - varying parameterization



octet 1^{-+} resonance pole & couplings

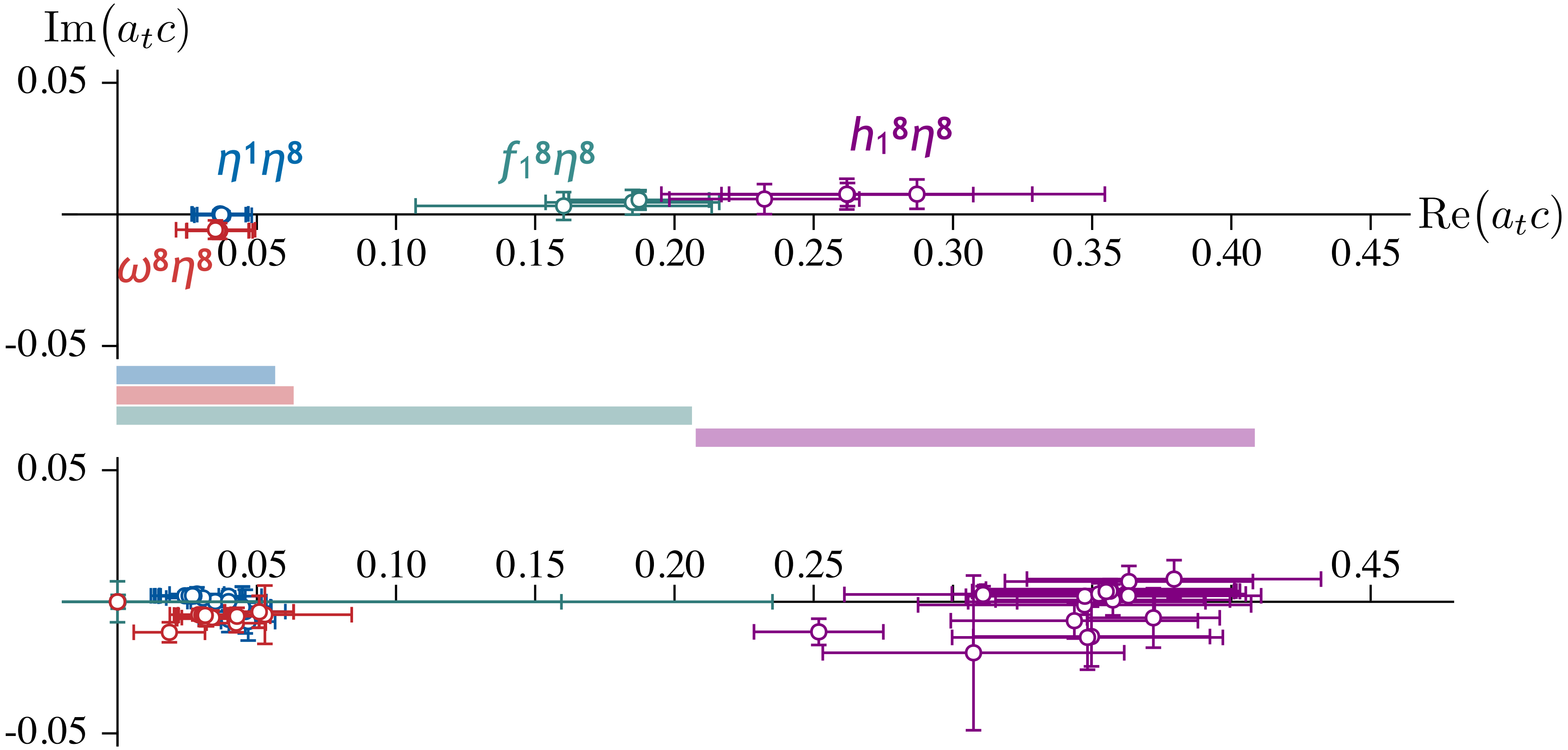


$$t_{ab}(s) \sim \frac{c_a c_b}{s_0 - s}$$

$$\sqrt{s_0} = m_R - i\tfrac{1}{2}\Gamma_R$$

at the SU(3)$_F$ point:

m_R=2144(12) MeV, Γ_R=21(21) MeV (a narrow resonance)



resonance <u>below</u> $h_1{}^8\eta^8$ threshold, but with a large coupling

extrapolation

$$|c|^{\text{phys}} = \left|\frac{k^{\text{phys}}(m_R^{\text{phys}})}{k(m_R)}\right|^{\ell} |c|.$$

$$\Gamma(R \to i) = \frac{|c_i^{\text{phys}}|^2}{m_R^{\text{phys}}} \cdot \rho_i(m_R^{\text{phys}}).$$

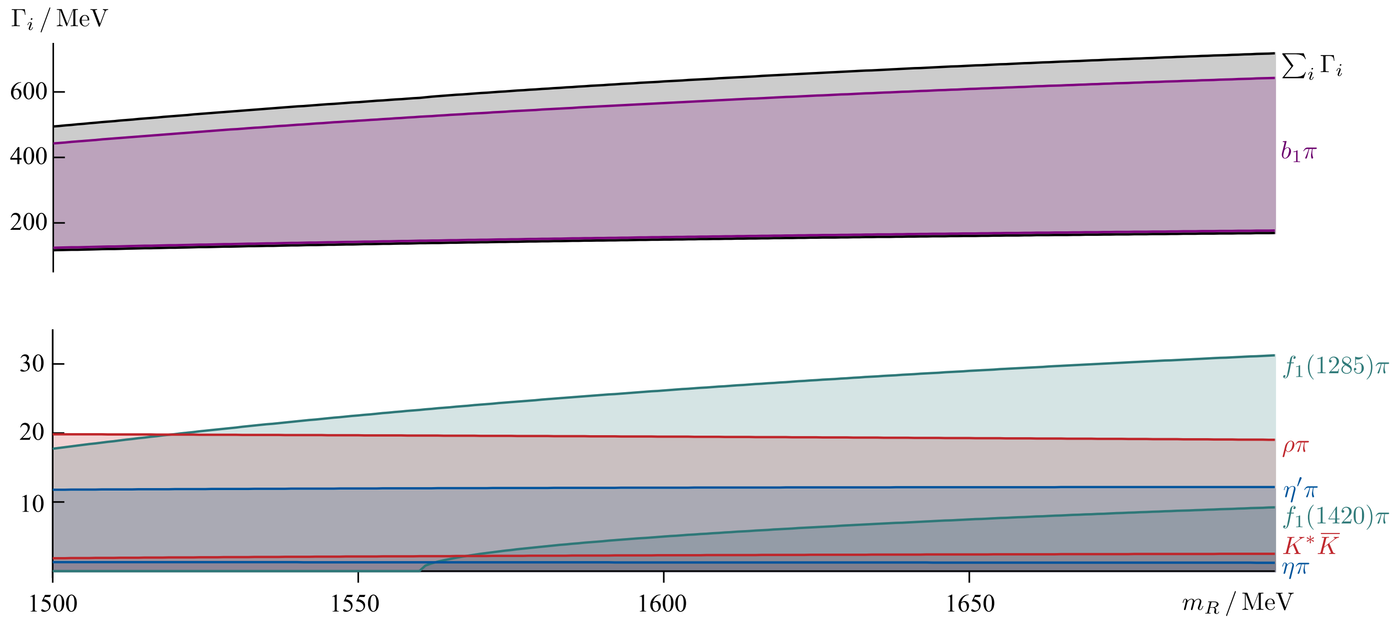
example ‘success’ – f_2,f_2' calculated at m_π~400 MeV

	Scaled	PDG
$\lvert c(f_2 \to \pi\pi)\rvert$	488(28)	453^{+9}_{-4},
$\lvert c(f_2 \to K\bar{K})\rvert$	139(27)	132(7),
$\lvert c(f_2' \to \pi\pi)\rvert$	103(32)	33(4),
$\lvert c(f_2' \to K\bar{K})\rvert$	321(50)	389(12),

$$\frac{1}{\sqrt{3}}(\pi^+\rho^0 - \pi^0\rho^+) + \frac{1}{\sqrt{6}}(K^+\bar{K}^{*0} - \bar{K}^0 K^{*+}),$$

$$-\sqrt{\frac{3}{10}}(K_{1A}^+\bar{K}^0 + \bar{K}_{1A}^0 K^+) + \frac{1}{\sqrt{5}}(a_1^+\eta_8 + (f_1)_8\pi^+),$$

$$\frac{1}{\sqrt{6}}(K_{1B}^+\bar{K}^0 - \bar{K}_{1B}^0 K^+) + \frac{1}{\sqrt{3}}(b_1^+\pi^0 - b_1^0\pi^+),$$



crude extrapolation to physical point

core assumption: couplings scale only with the relevant barrier factor $\boldsymbol{k^{\ell}}$

use PDG masses & COMPASS/JPAC π_1 mass

generates for a π_1 at 1564 MeV:

Γ_{TOT} ~ 140-600 MeV

$\Gamma(\pi\eta) \lesssim$ 1 MeV

$\Gamma(\pi\eta') \lesssim$ 20 MeV

$\Gamma(\pi\rho) \lesssim$ 12 MeV

$\Gamma(\pi b_1)$ ~ 140-530 MeV

if extrapolation correct,
suggests prior observations in $\pi\eta$, $\pi\eta'$, $\pi\rho$
are in heavily suppressed decay channels

JPAC/COMPASS candidate:

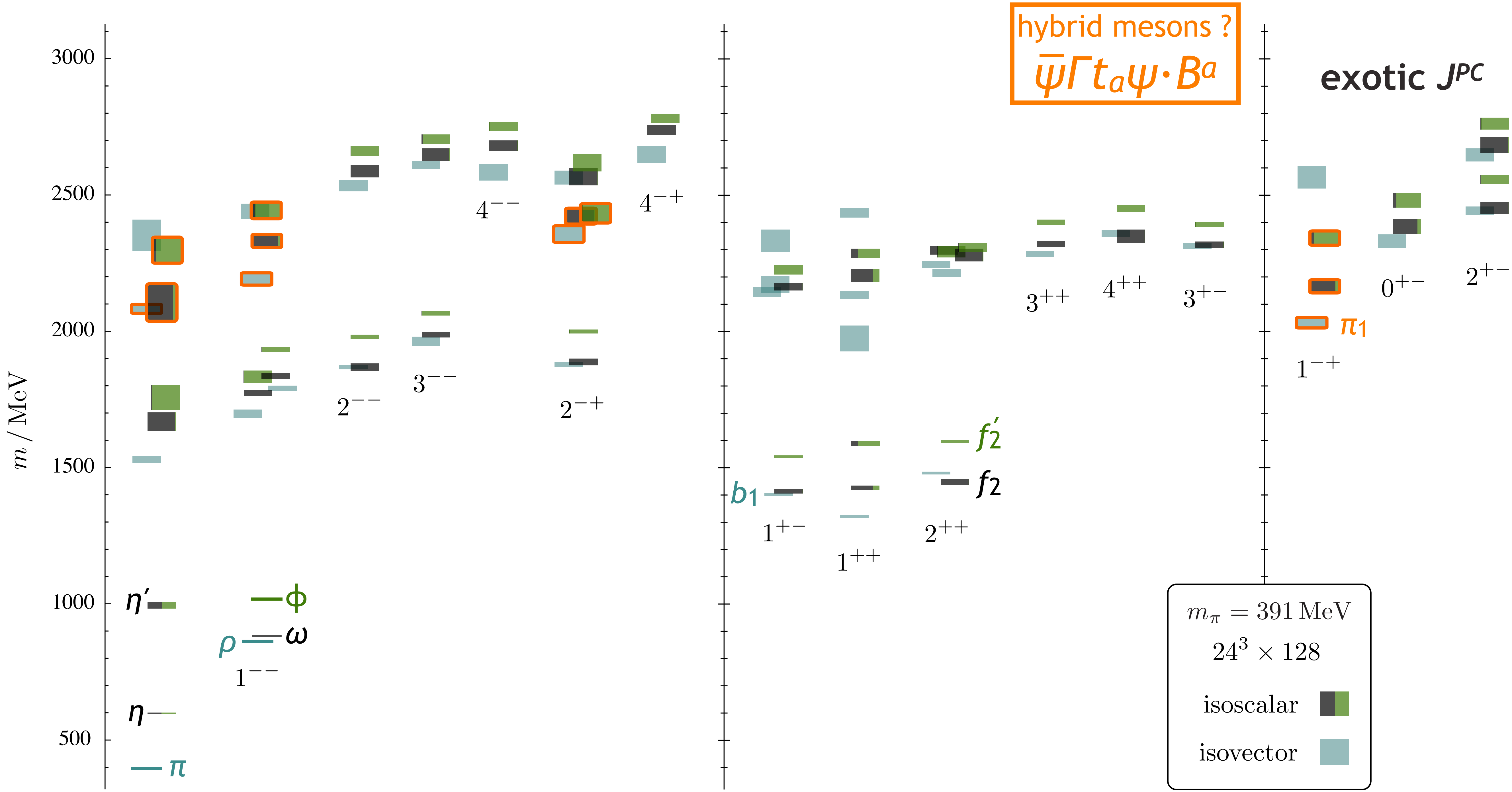
Γ_{TOT} ~ 492(115) MeV

Kopf et al analysis:

Γ_{TOT} ~ 455(170) MeV

$\Gamma(\pi\eta')$ / $\Gamma(\pi\eta)$ ~ 5.5(20)

(incomplete) lattice QCD spectrum of mesons



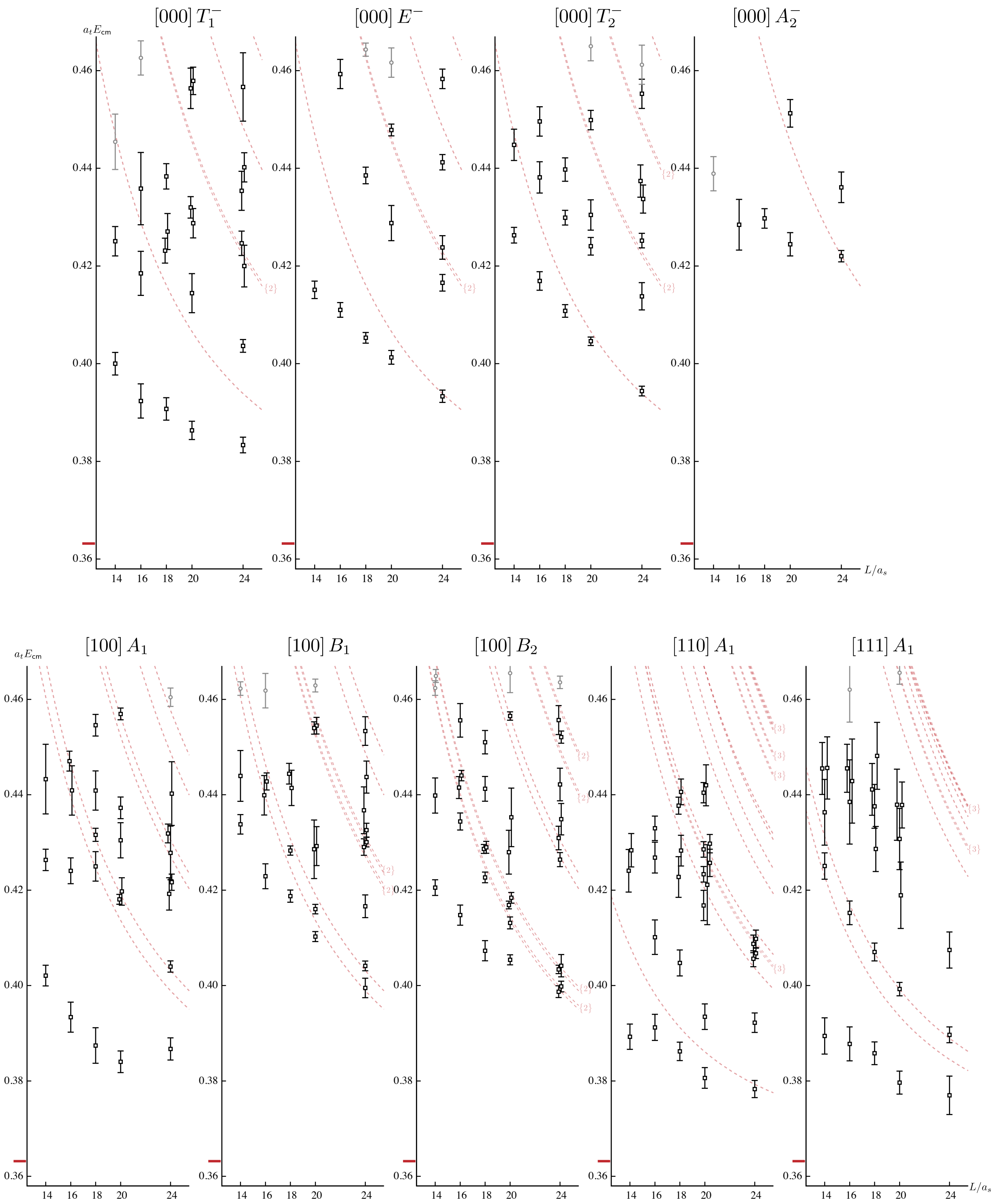
hybrid mesons ?
$\bar{\psi}\Gamma t_a \psi \cdot B^a$

exotic J^{PC}

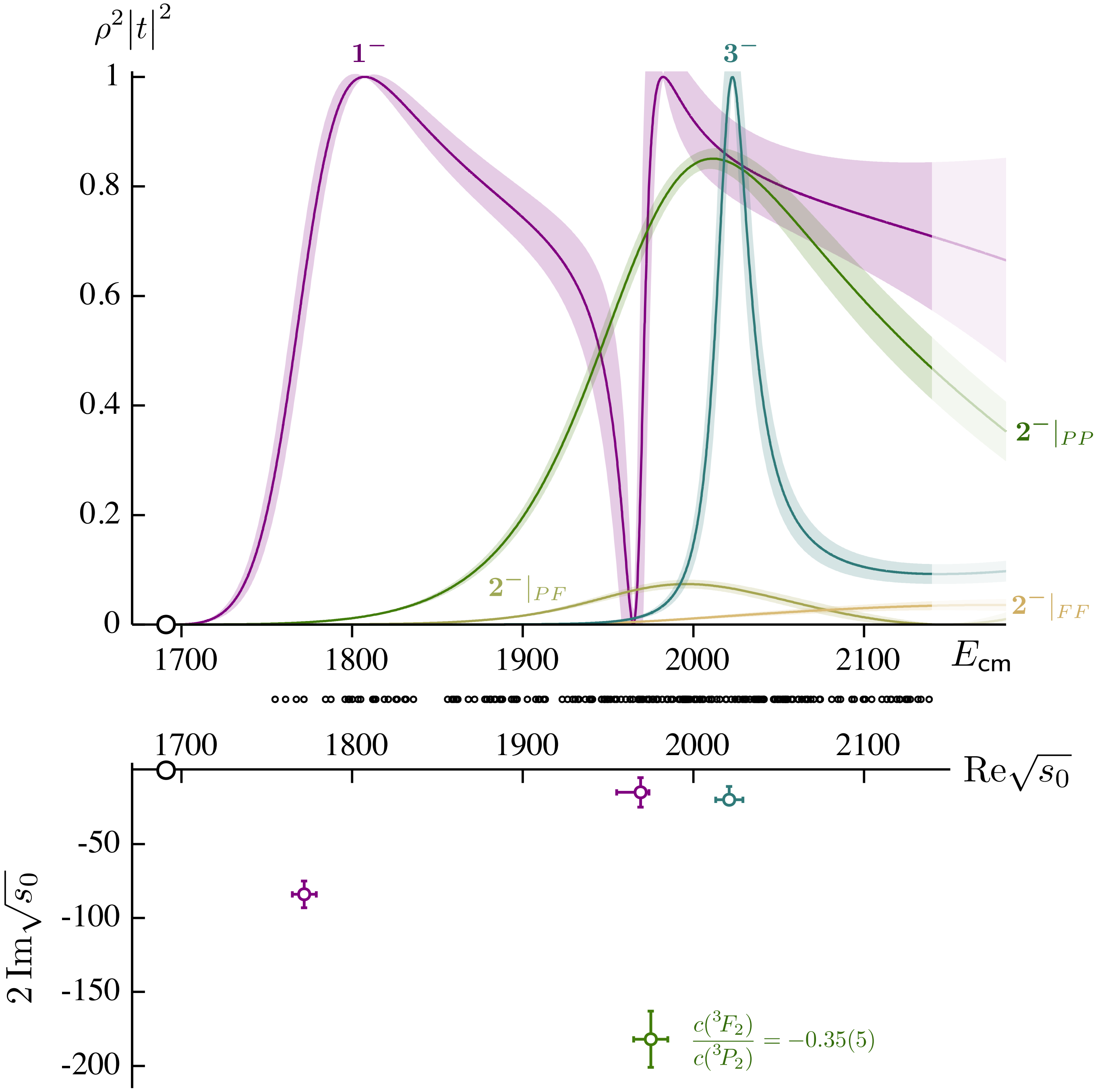
PRD 88 094505 (2013)

excited J^{--} meson resonances – m_π~700 MeV

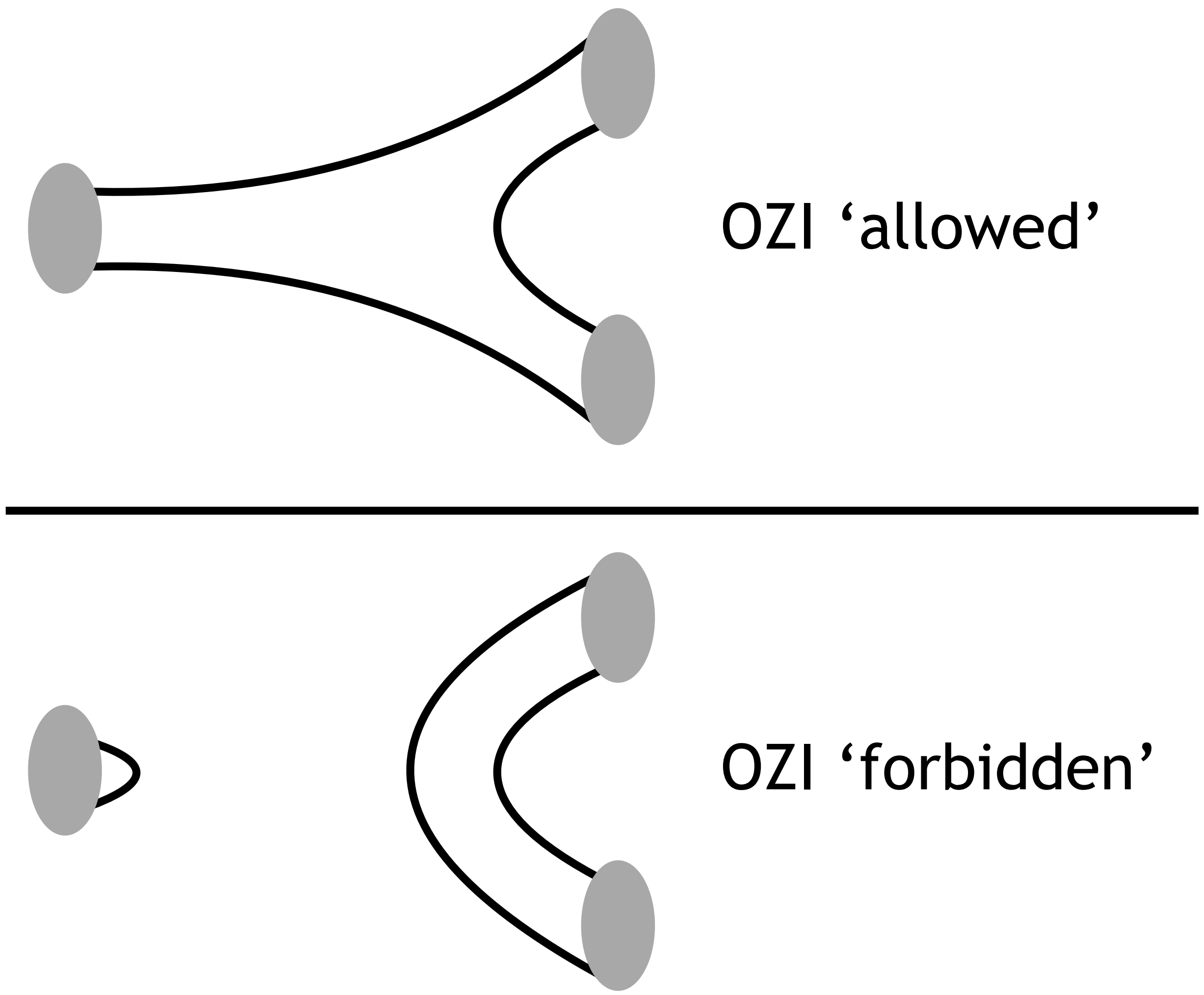
exact SU(3) flavor symmetry $\omega_J^{\mathbf{1}} \to \eta^{\mathbf{8}}\omega^{\mathbf{8}}$



unprecedented number of energy levels

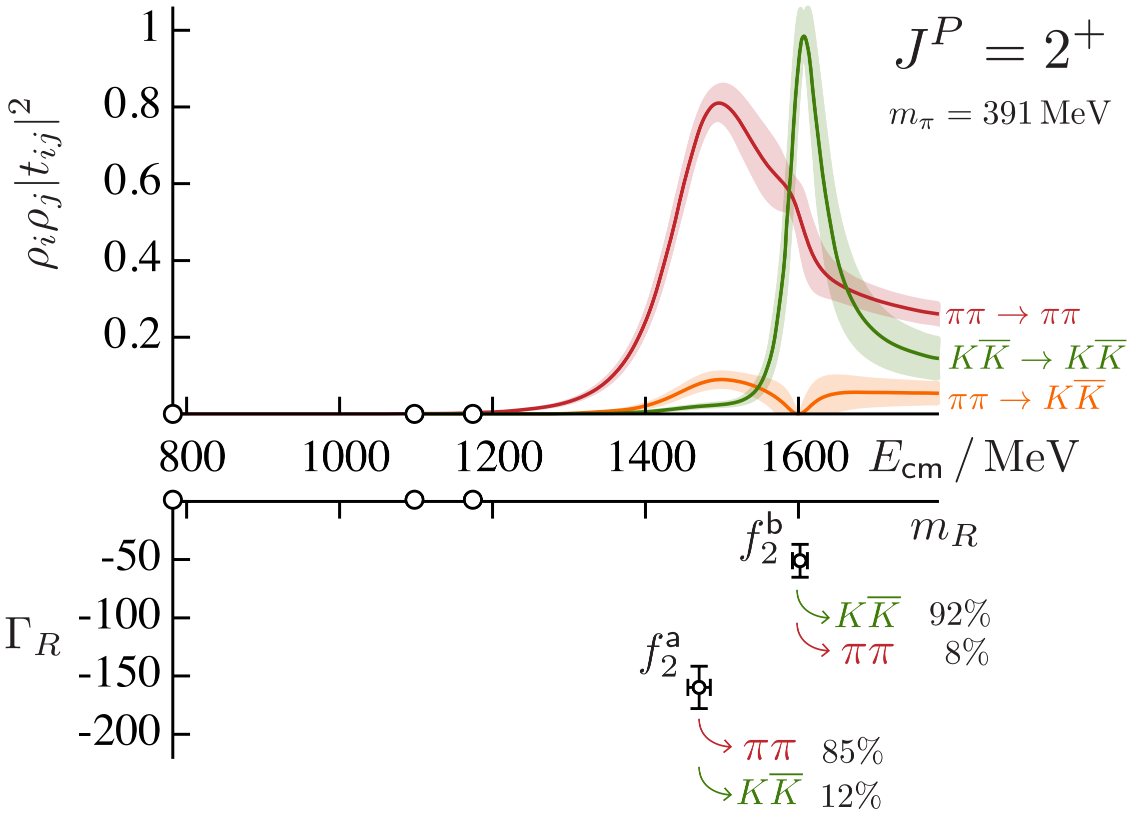


f_2 resonances – decay couplings & OZI



OZI ‘allowed’

OZI ‘forbidden’



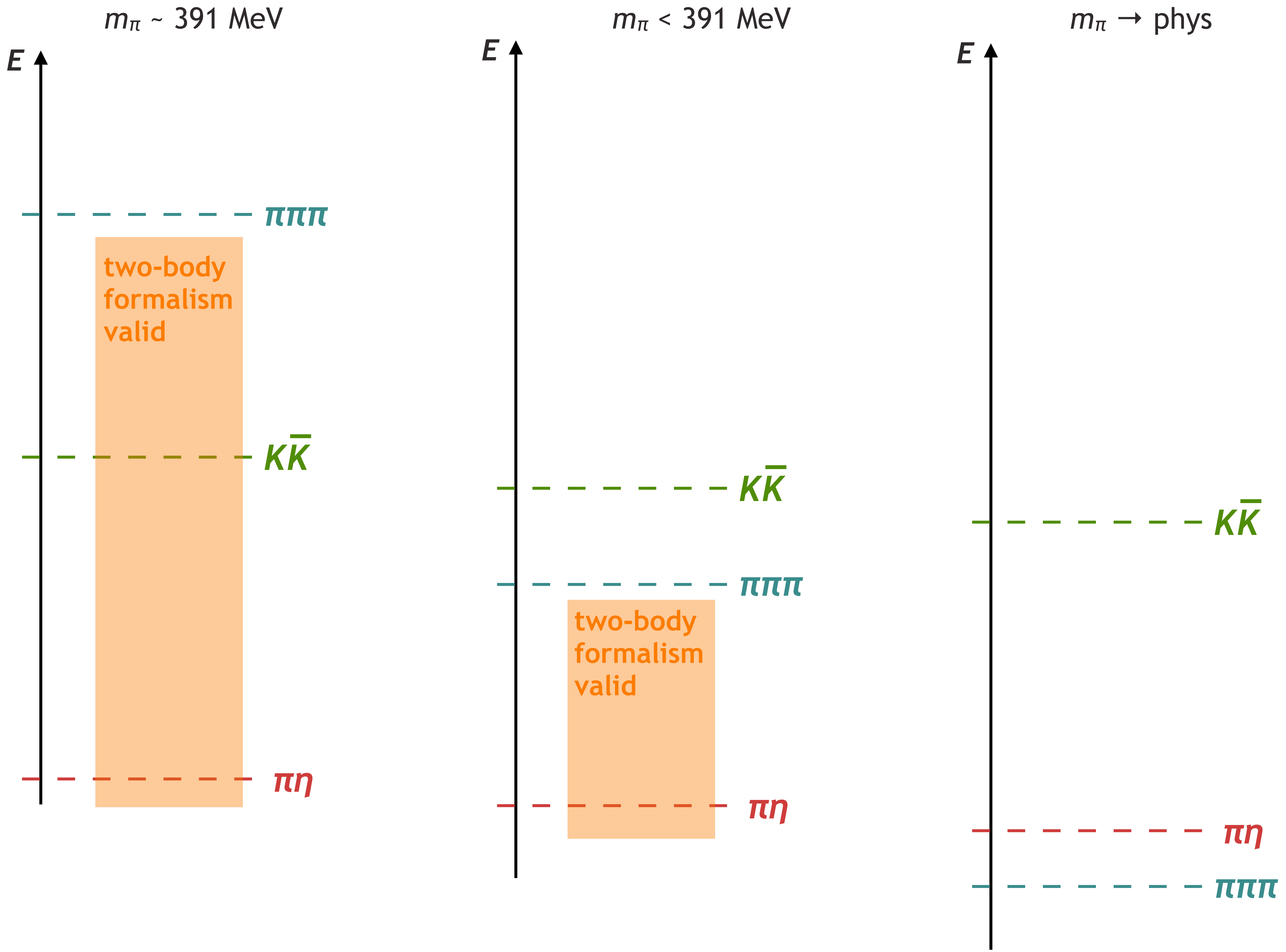
$$f_2^{\mathsf{a}} \sim u\bar{u} + d\bar{d} \qquad f_2^{\mathsf{b}} \sim s\bar{s}$$

couplings from pole residue

	$\frac{a_t\lvert c_{\pi\pi}\rvert}{(a_t k_{\pi\pi})^2}$	$\frac{a_t\lvert c_{K\bar{K}}\rvert}{(a_t k_{K\bar{K}})^2}$
f_2^{a}	$7.1(4)$	$4.8(9)$
f_2^{b}	$1.0(3)$	$5.5(8)$

zero in ‘OZI’ limit
— requires $s\bar{s}$ annihilation

physical pion masses = low-lying multipion channels



what do you calculate

calculate correlation functions

e.g. $\langle 0 | \mathcal{O}_i(t) \mathcal{O}_j(0) | 0 \rangle$

where the operators are constructed from quark and gluon fields
and have the quantum numbers of the hadronic system you want to study

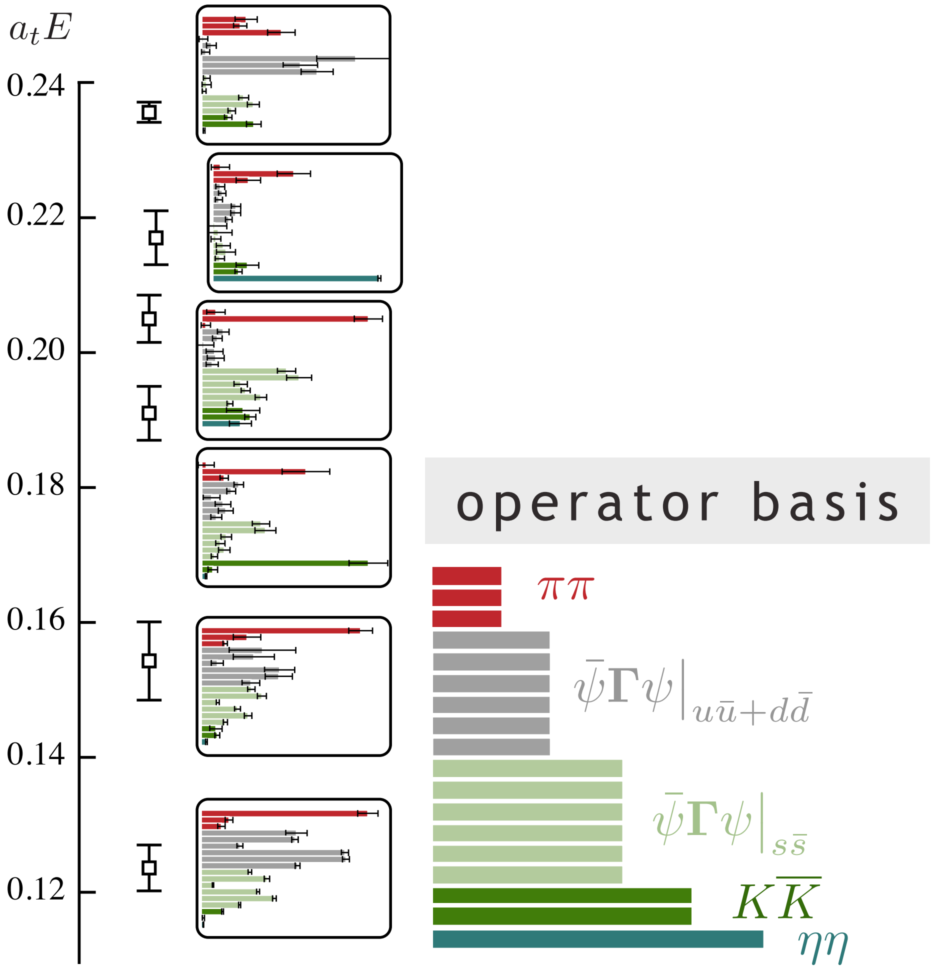
$$\langle 0 | \mathcal{O}_i(t) \mathcal{O}_j(0) | 0 \rangle = \sum_n \langle 0 | \mathcal{O}_i | n \rangle \langle n | \mathcal{O}_j | 0 \rangle \, e^{-E_n t}$$

a superposition of the (finite-volume) eigenstates of QCD

powerful approach:

- use a large basis of operators*
- form a matrix of correlation functions
- diagonalize this matrix

e.g. [000] A_1^+ 24^3



* could give a whole interesting talk on the construction of these operators

operator basis – $I=0$ $\pi\pi$, $K\bar{K}$, $\eta\eta$

operator basis: ‘single-meson’ + ‘meson-meson’

$$\bar{\psi}\boldsymbol{\Gamma}\psi$$

(& if you like,
tetraquark & …)

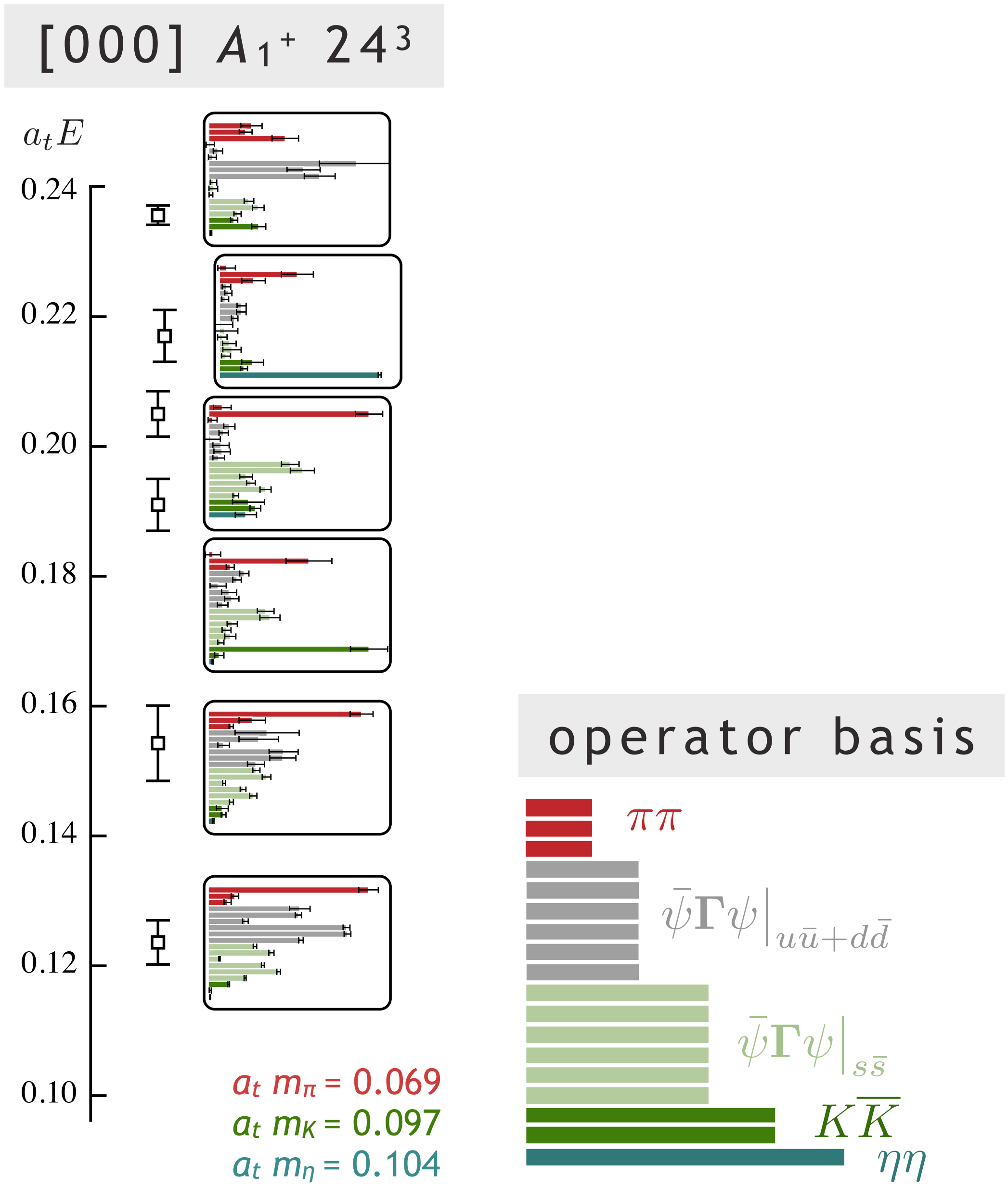
$$\sum_{\hat{\mathbf{p}}_1,\hat{\mathbf{p}}_2} C(\mathbf{p}_1,\mathbf{p}_2;\mathbf{p})\, M_1(\mathbf{p}_1)\, M_2(\mathbf{p}_2)$$

$$\mathbf{p} = \frac{2\pi}{L}[n_x, n_y, n_z]$$

maximum momentum
guided by non-interacting
energies

$$\sqrt{m_1^2+\mathbf{p}_1^2}+\sqrt{m_2^2+\mathbf{p}_2^2}$$

solutions of the det equation
when **t** = **0**



operator basis

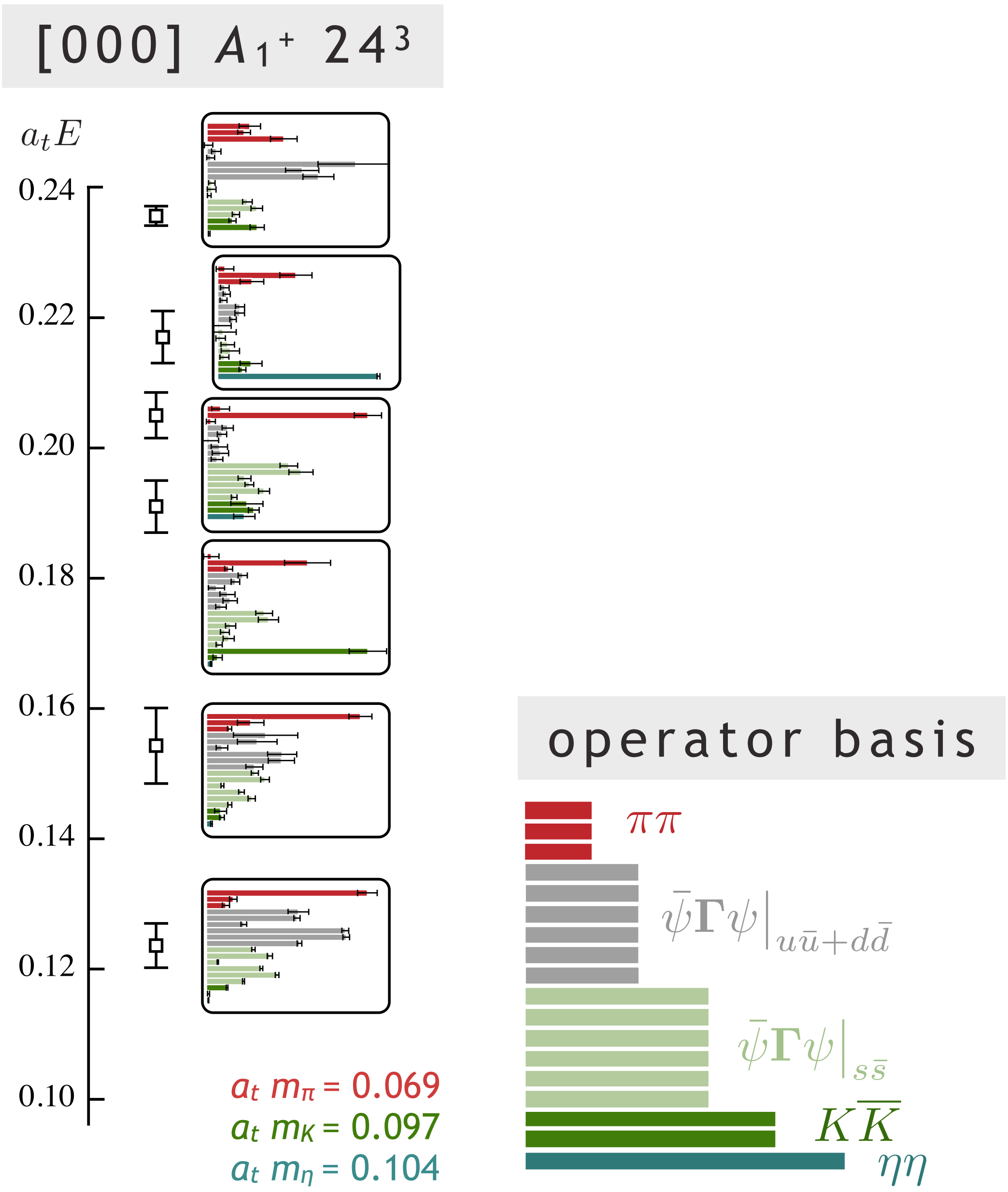
operator basis: ‘single-meson’ + ‘meson-meson’

$$\overline{\psi}\boldsymbol{\Gamma}\psi$$

$$\sum_{\hat{\mathbf{p}}_1,\hat{\mathbf{p}}_2} C(\mathbf{p}_1,\mathbf{p}_2;\mathbf{p})\,\boxed{M_1(\mathbf{p}_1)\,M_2(\mathbf{p}_2)}$$

$$\boxed{\sum_{\mathbf{x}} e^{i\mathbf{p}_1\cdot\mathbf{x}}\,\bar{\psi}_{\mathbf{x}}\boldsymbol{\Gamma}\psi_{\mathbf{x}}\ \sum_{\mathbf{y}} e^{i\mathbf{p}_2\cdot\mathbf{y}}\,\bar{\psi}_{\mathbf{y}}\boldsymbol{\Gamma}'\psi_{\mathbf{y}}}$$

sampling the whole lattice volume



prefer to use
optimized single-meson operators …

‘dropping’ ops in $\rho \to \pi\pi$

exclude the $K\bar{K}$ operator

exclude all meson-meson

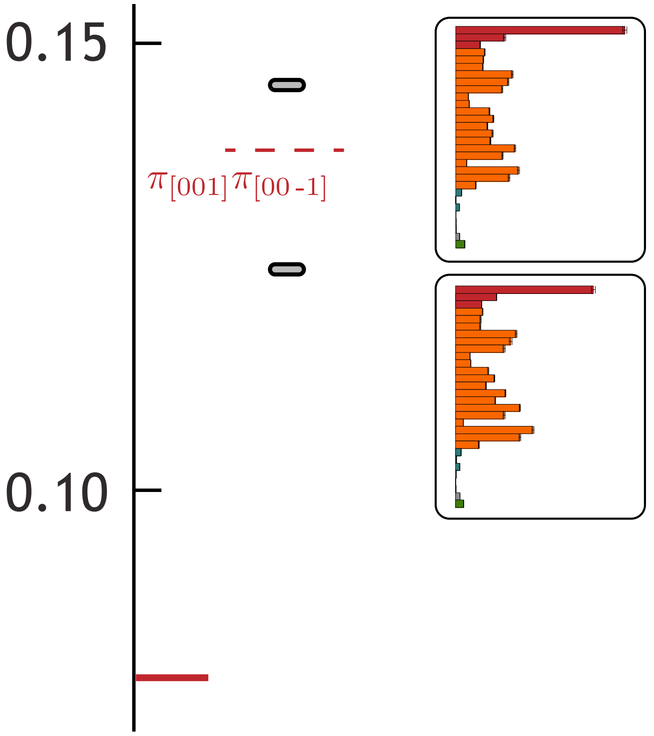
exclude all $\bar{\psi}\Gamma\psi$

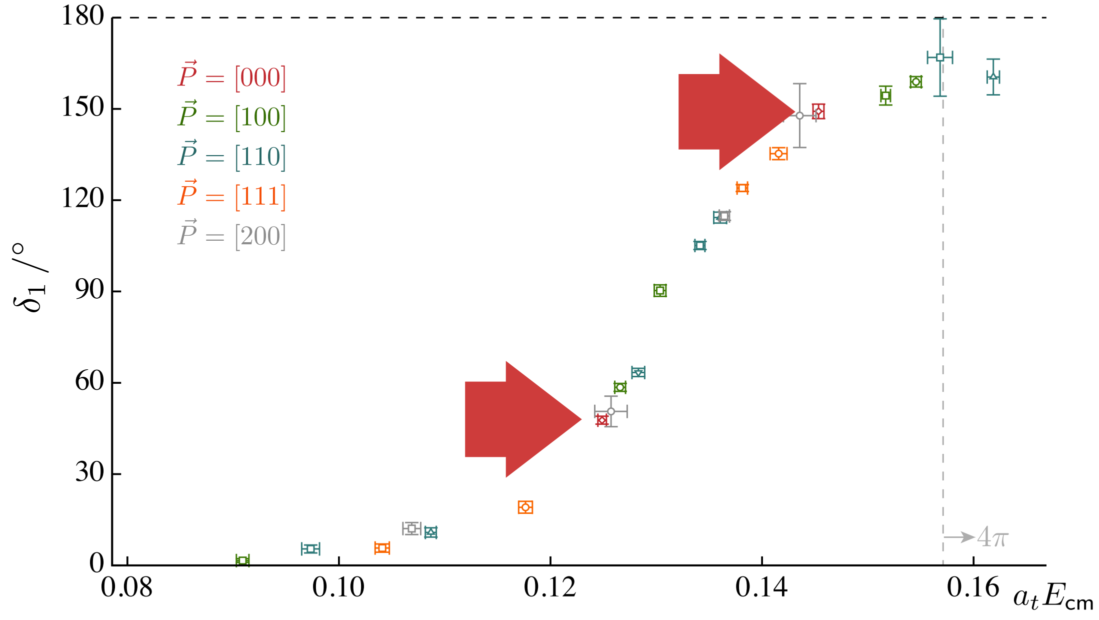
$m_\pi = 0.039$
$m_K = 0.083$

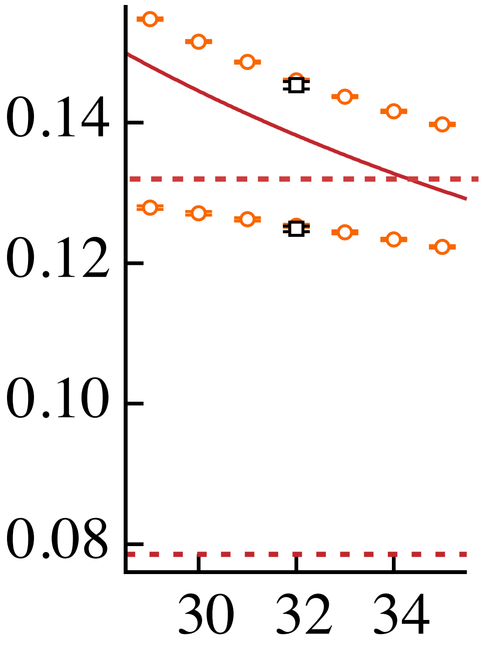
L ~ 3.8 fm

what’s happening here ?

focus on the lowest two states

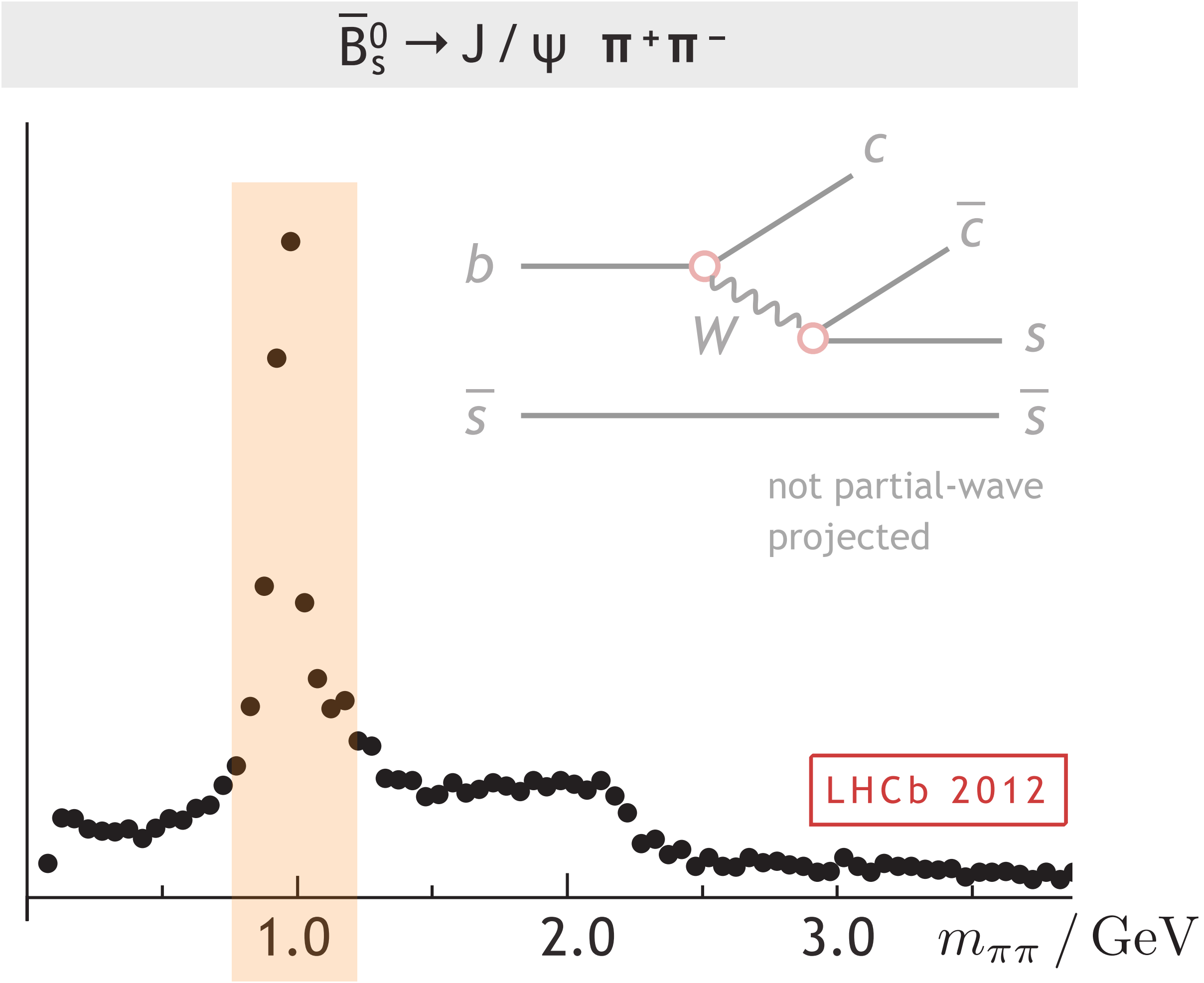




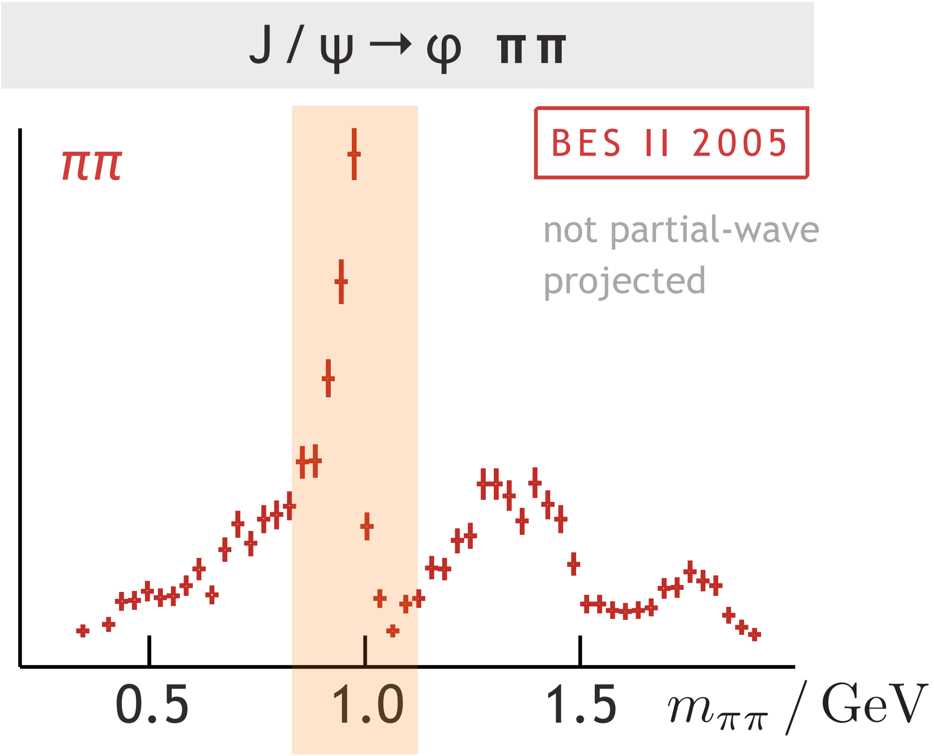


an avoided level crossing

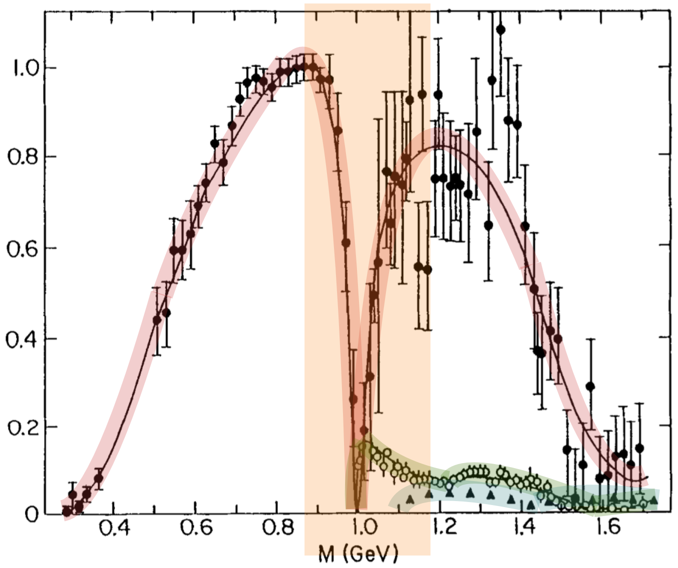
“production” of *ππ* (as opposed to scattering)



$\bar{B}^0_s \to J/\psi\ \pi^+\pi^-$

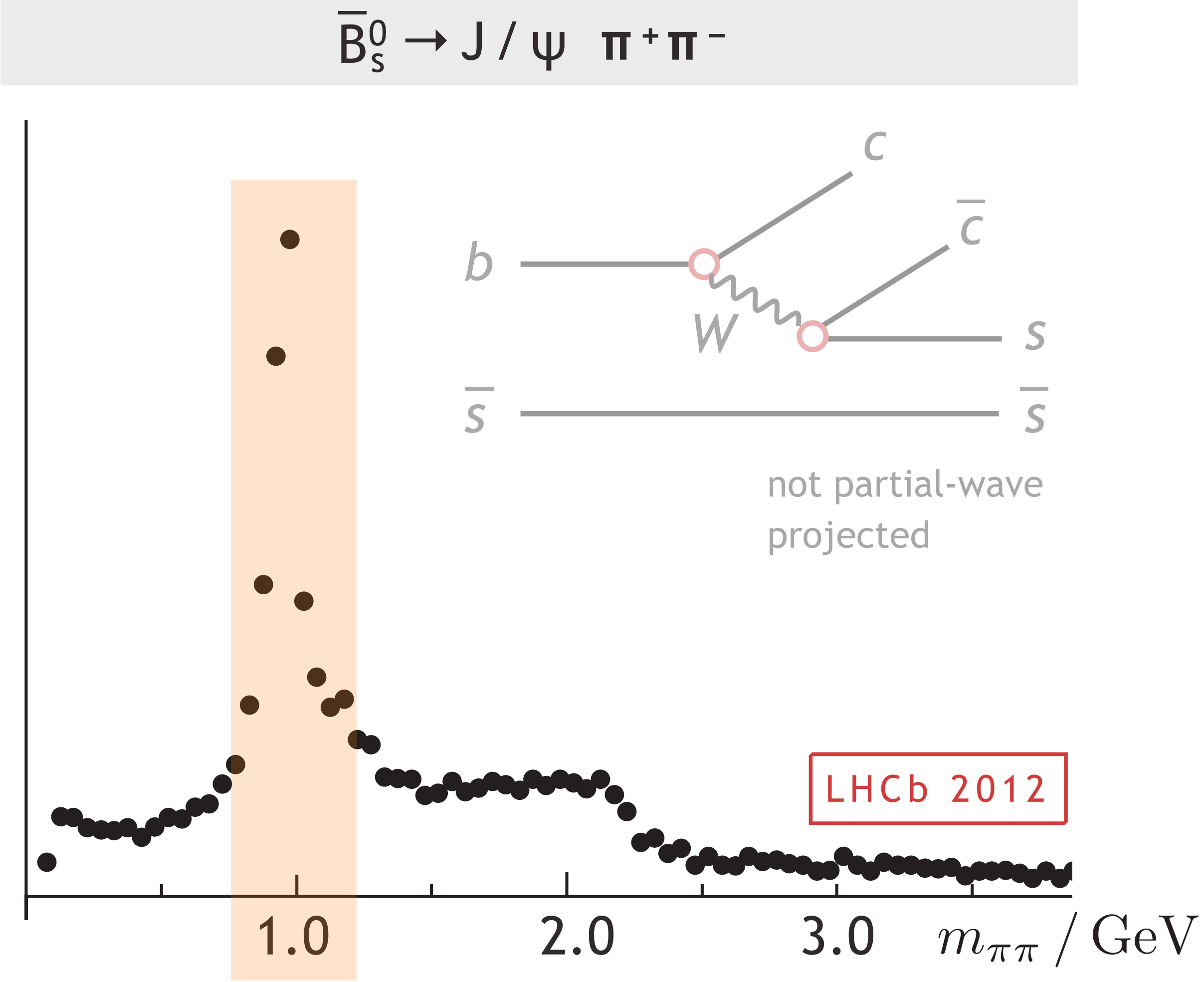


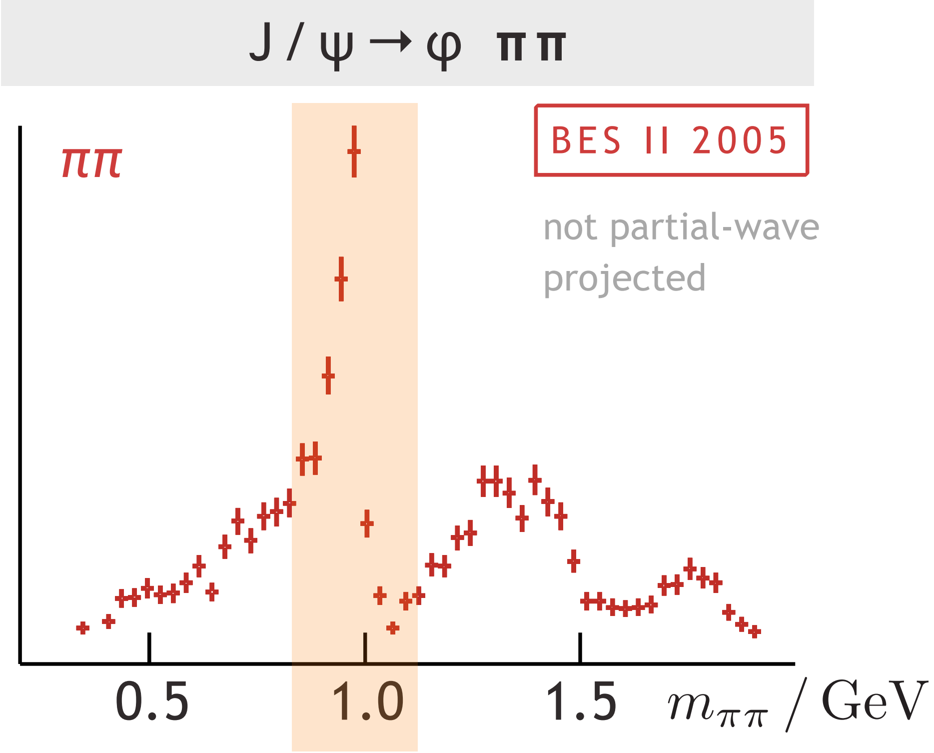
$J/\psi \to \varphi\ \pi\pi$

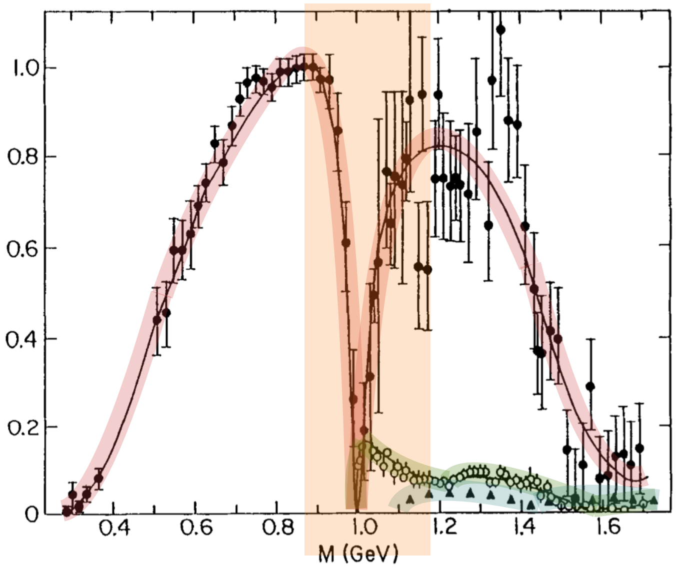


can ‘look’ drastically different to scattering !

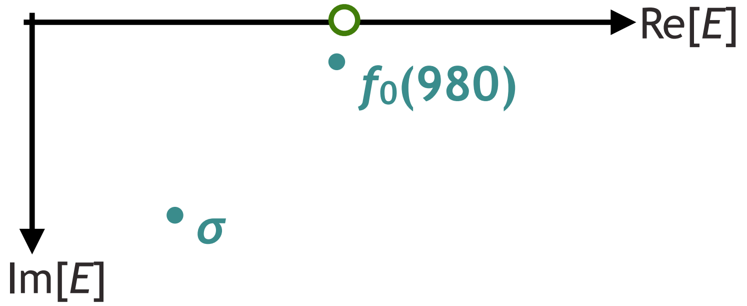
"production" of ππ (as opposed to scattering)





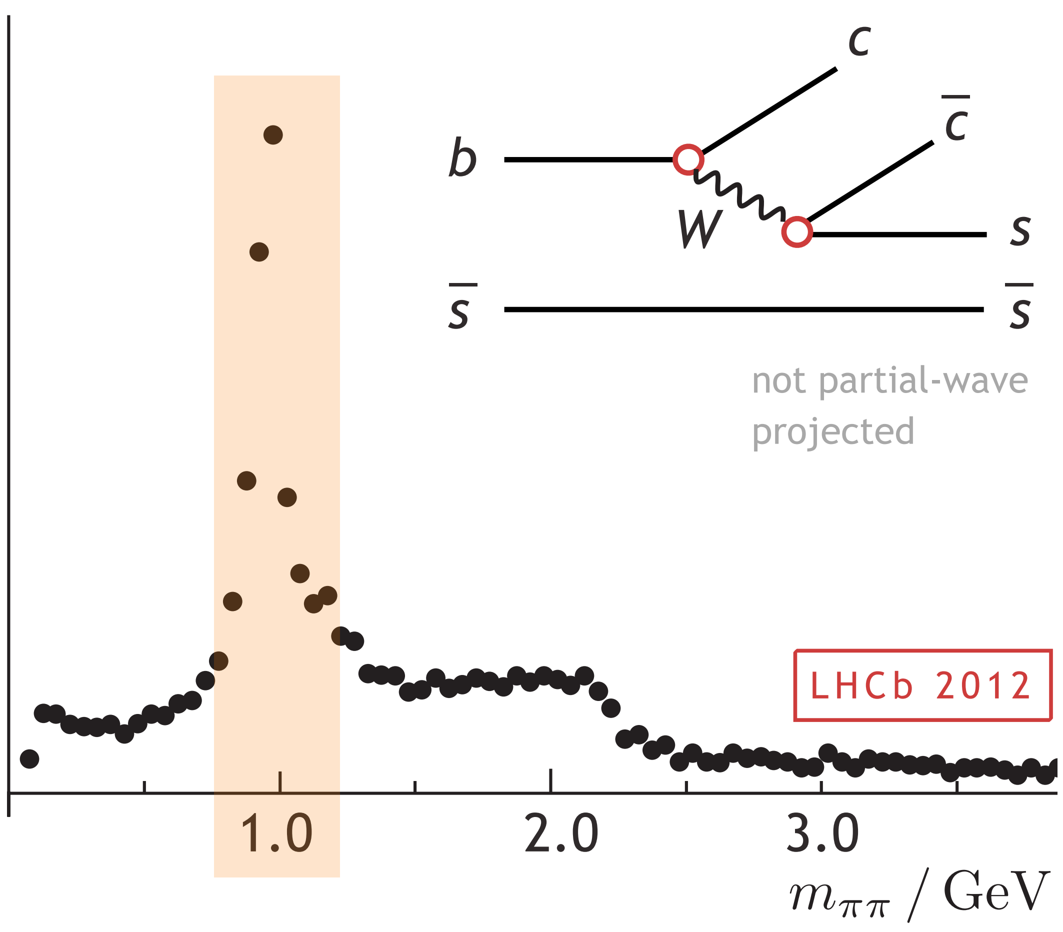


... **same poles** (σ, $f_0(980)$) — different couplings ...



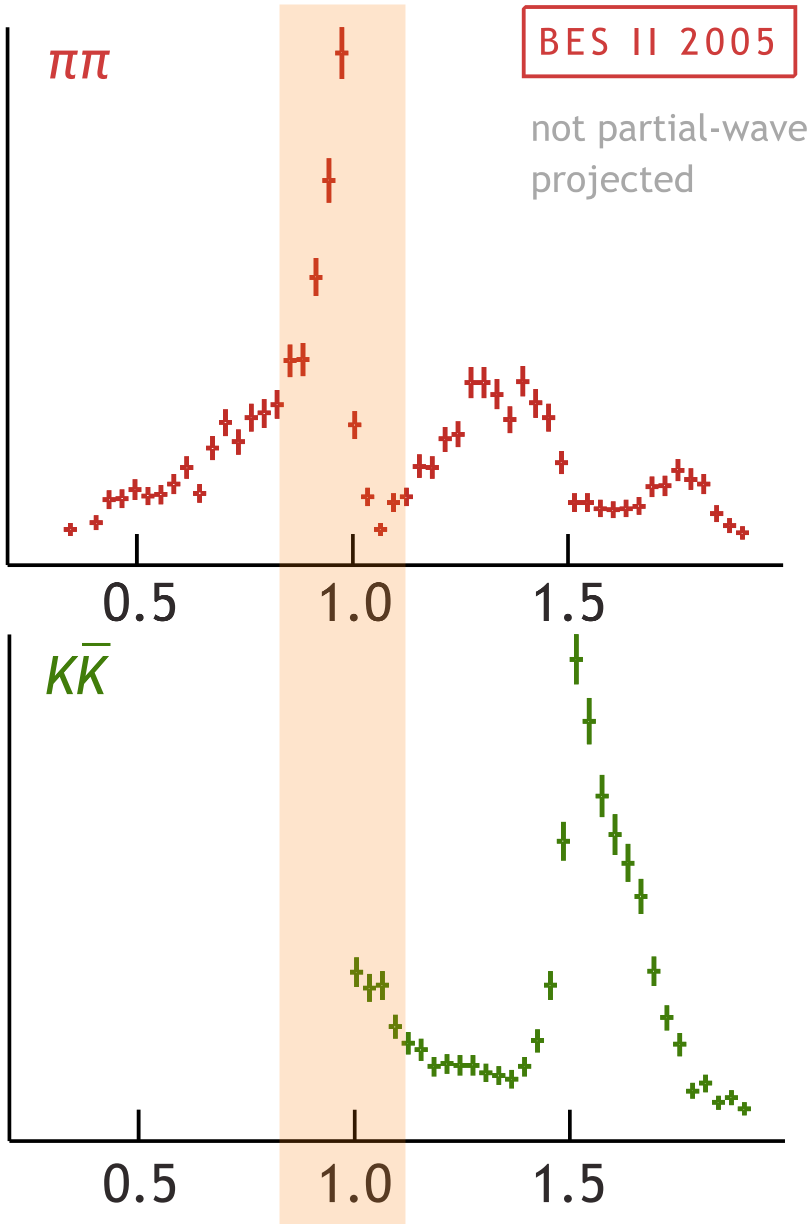
$f_0(980)$ as a peak in "$s\bar{s}$" production

$\bar{B}_s^0 \to J/\psi\ \pi^+\pi^-$



not partial-wave projected

$J/\psi \to \varphi(\pi\pi, K\bar{K})$

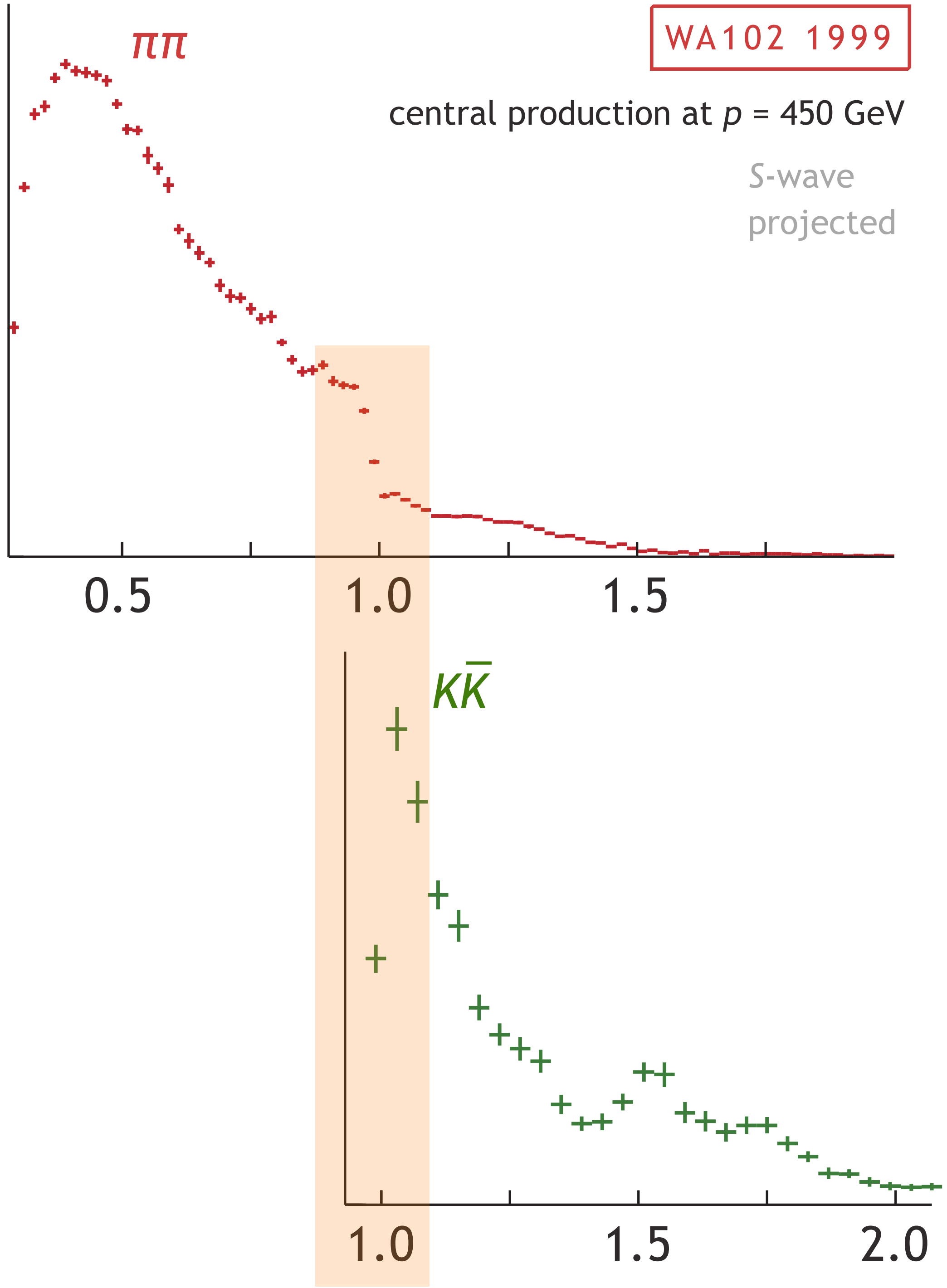


not partial-wave projected

note the rapid turn-on of $K\bar{K}$ at threshold

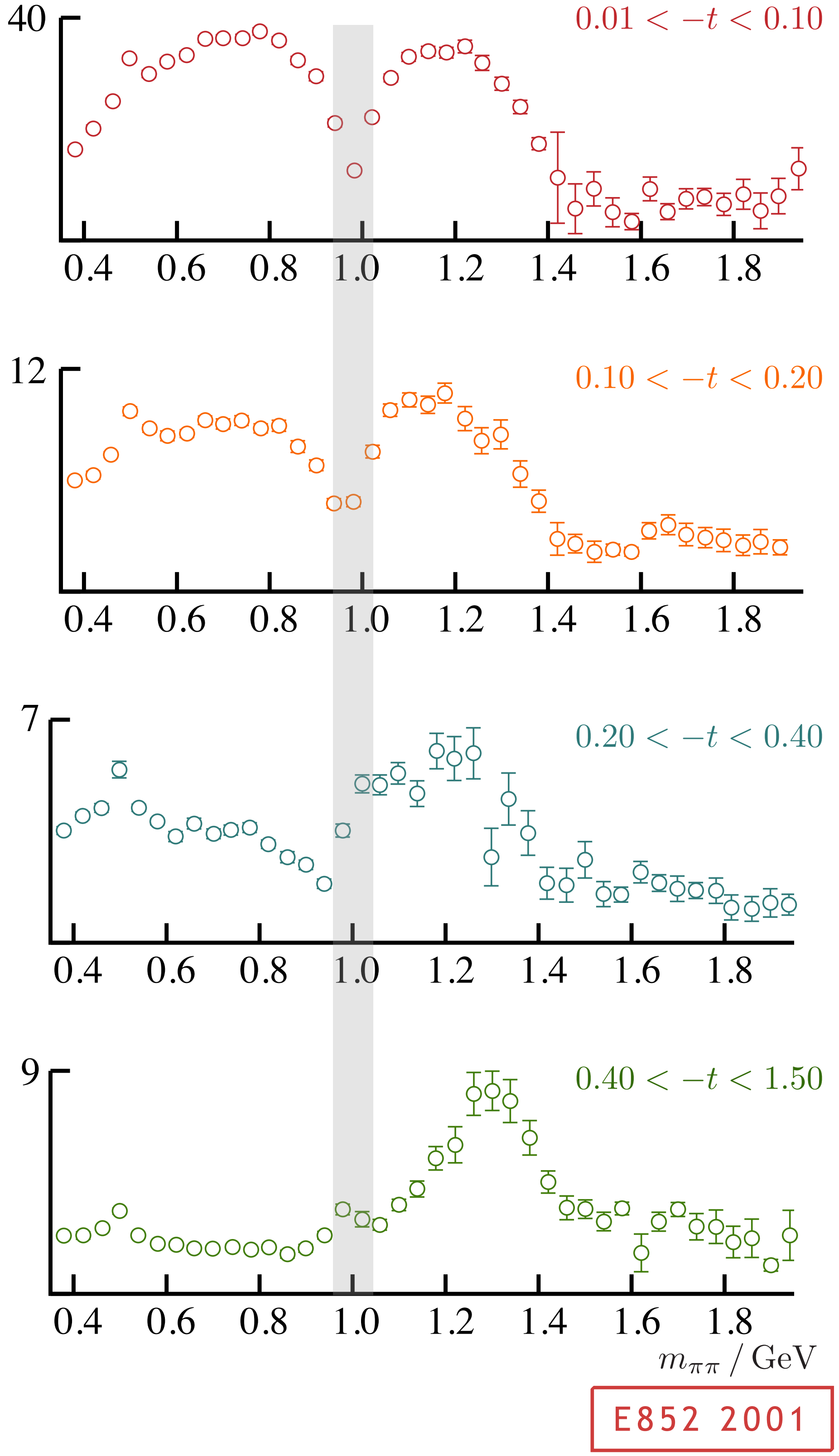
f_0(980) as ?

$pp \to p(\pi\pi, K\bar{K})p$

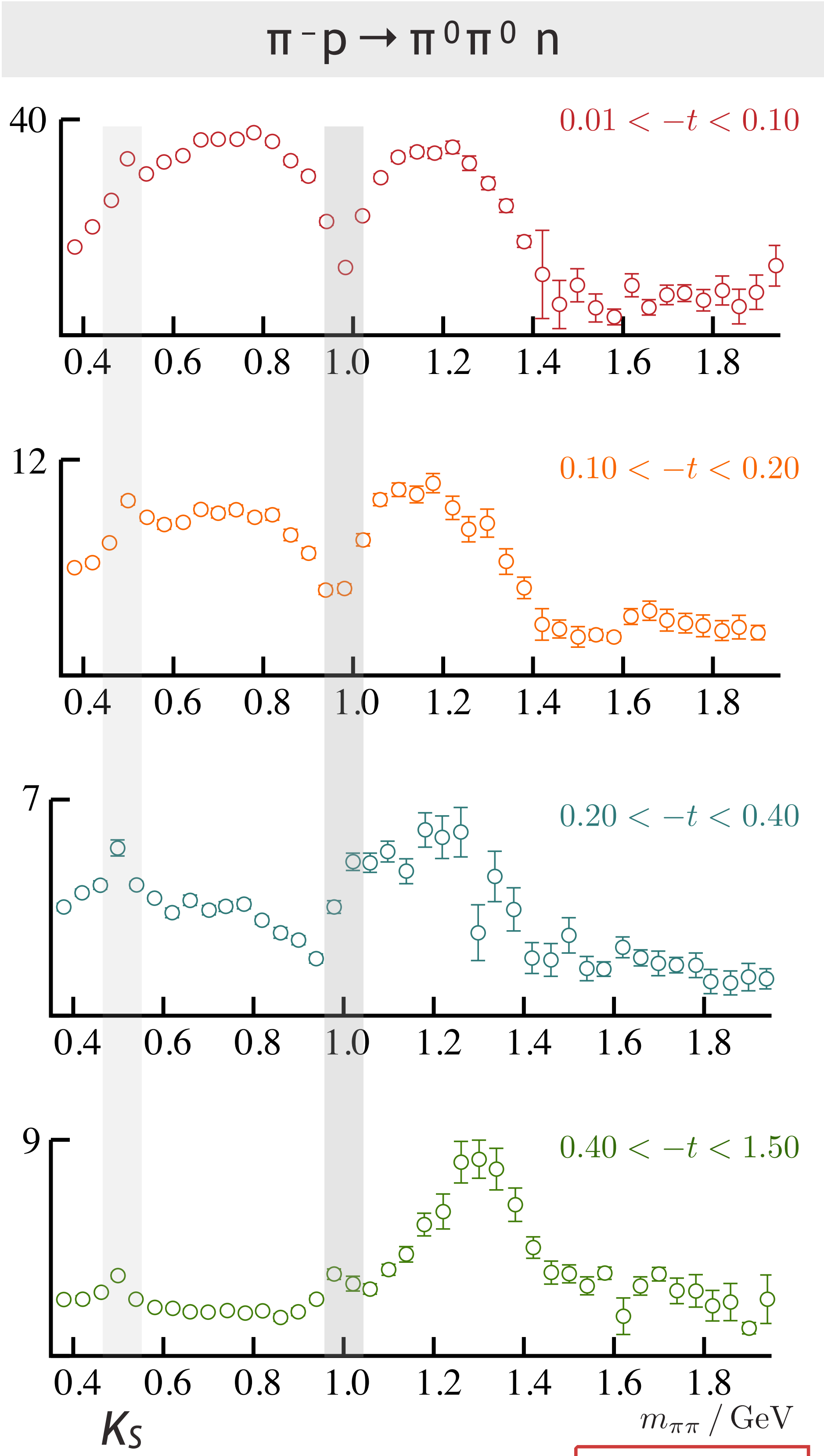


f_0(980) as a shoulder on
a large σ 'background'

$\pi^- p \to \pi^0\pi^0\, n$

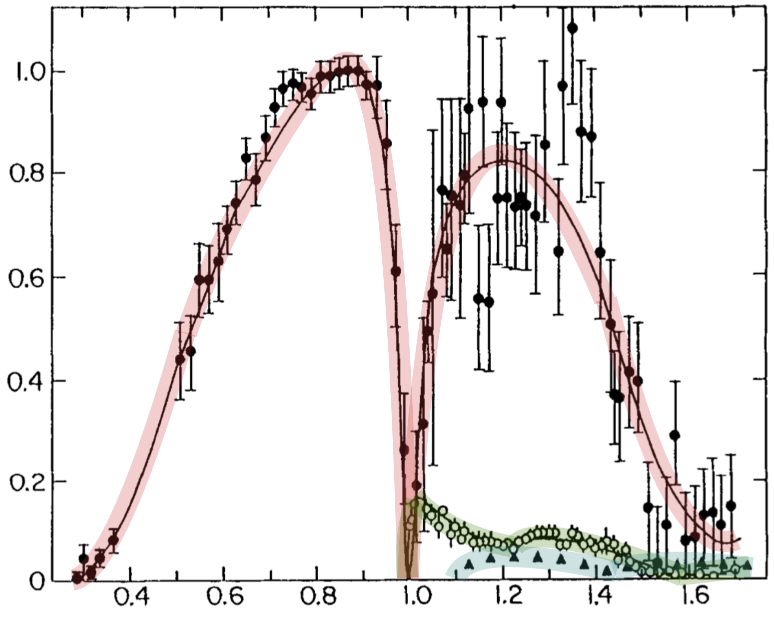


S–wave *ππ* production



E852 2001

dominated by *π* exchange
— looks like the the 1970s
elastic phase-shift data →



other (non-π) exchanges
becoming significant,
$f_0(980)$ dip less pronounced

σ no longer large,
$f_0(980)$ starting to be a peak ?

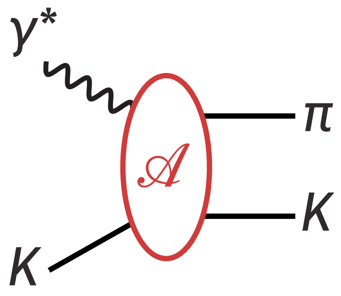
resonance transition form-factors

the process of interest is

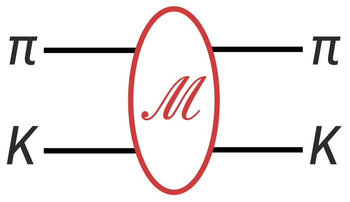
current + stable hadron → resonance → hadron–hadron pair

e.g. $\gamma K \to \pi K$ in a *P*-wave

after the current produces $K\pi$ …



… $K\pi$ strongly rescatters



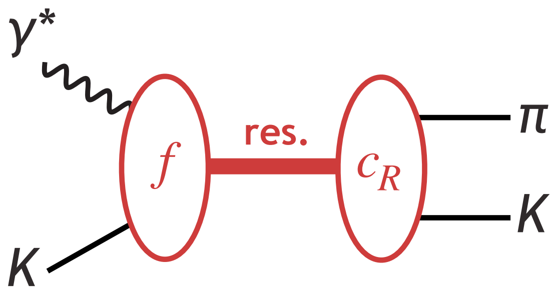
$$\mathcal{H}(Q^2, E^\star_{K\pi}) \equiv \langle K \,|\, j \,|\, K\pi; E^\star_{K\pi} \rangle$$

$$= \mathscr{A}\left(Q^2, E^\star_{K\pi}\right) \cdot \frac{1}{k^\star_{K\pi}} \cdot \mathscr{M}^{\ell=1}(E^\star_{K\pi})$$

strong scattering amplitude, $\mathscr{M}$, can have resonance poles $\mathscr{M}^{\ell=1}(s) \sim \dfrac{c_R^2}{s_0 - s}$ $\sqrt{s_0} = m_R - i\frac{1}{2}\Gamma_R$

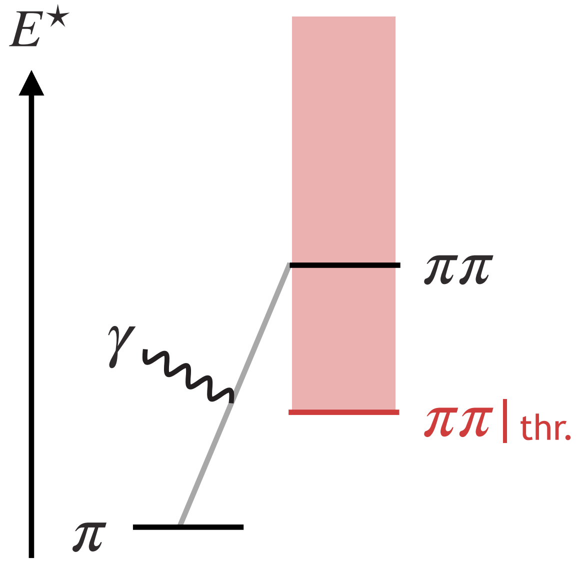
hence $\mathcal{H}(Q^2, s) \sim \dfrac{c_R\, f(Q^2)}{s_0 - s}$

residue at the complex pole



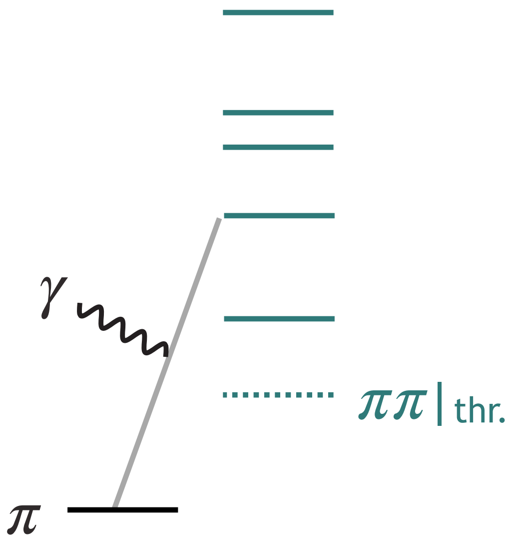
current matrix-elements in a finite-volume – cartoon

infinite volume



can transition
to any energy
in the $\pi\pi$ continuum

finite volume



can only transition
to one of the discrete
f.v. eigenstates

finite-volume matrix element

$${}_L\langle \pi | j | \pi\pi; E_n^\star \rangle_L$$

single hadron state

$$|\pi\rangle_L \sim |\pi\rangle_\infty + \mathcal{O}(e^{-m_\pi L})$$

hadron-hadron state

$$|\pi\pi; E_n^\star\rangle_L \sim \underline{\sqrt{\tilde{R}_n}}\, |\pi\pi; E_{\pi\pi}^\star = E_n^\star\rangle_\infty$$

effective f.v.
normalization

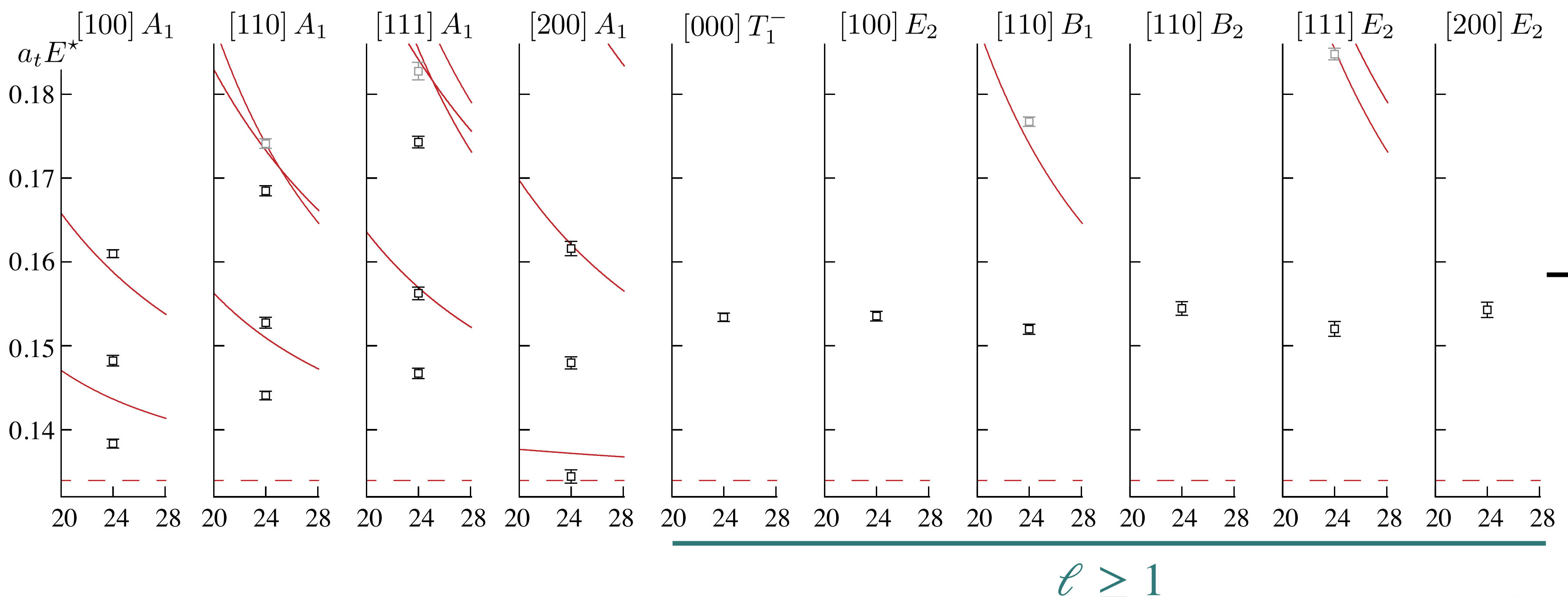
c.f. "Lellouch-Lüscher" factor

$$\tilde{R}_n(L) \equiv 2E_n \cdot \lim_{E\to E_n} \left(E - E_n\right)\left(F^{-1}(E^\star; L) + \underline{\mathcal{M}(E^\star)}\right)^{-1}$$

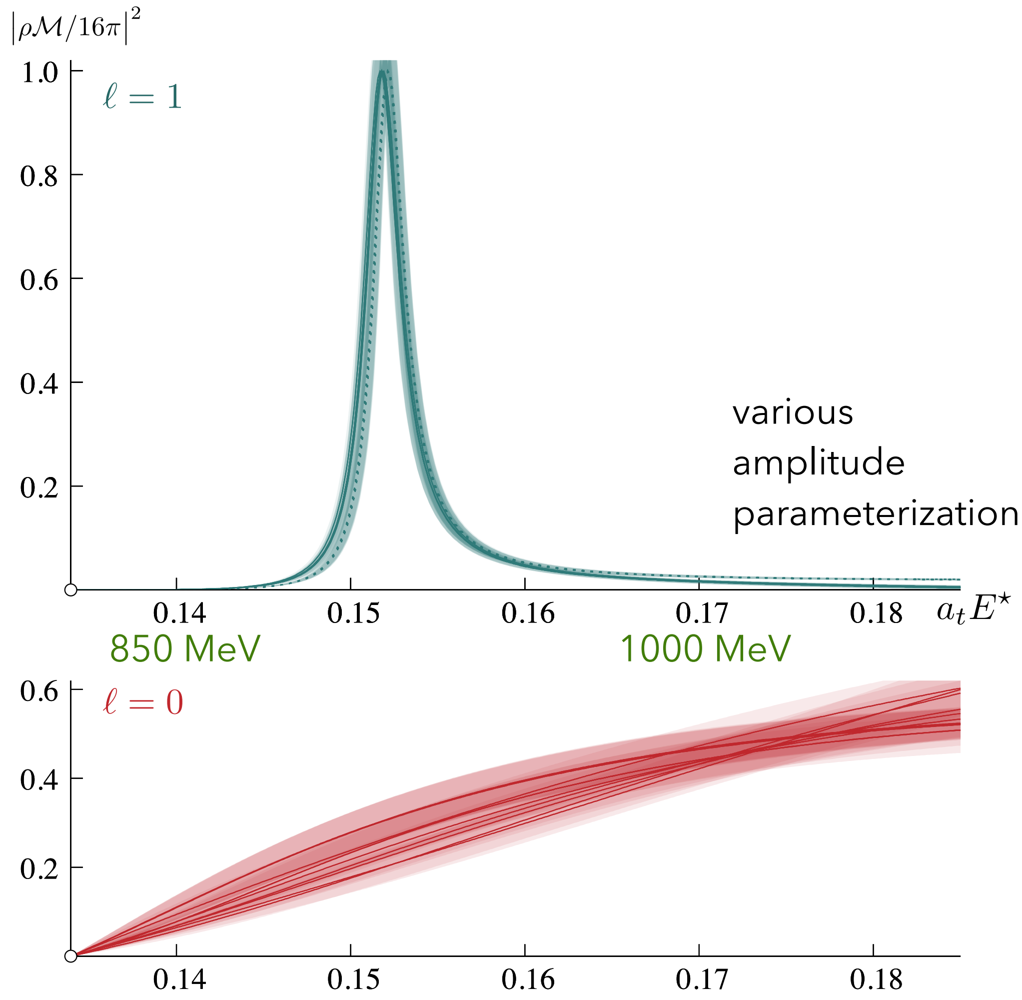
effective f.v. normalization
depends on the scattering
amplitude

finite-volume spectrum → *S*,*P*-wave amplitudes

$m_\pi \sim 284\,\mathrm{MeV}$



$$\longrightarrow \det\left[F^{-1}(E^\star;L) + \mathcal{M}(E^\star)\right] = 0$$



various
amplitude
parameterizations

$\ell = 2$ found to be negligible in this energy region

what's different in $\gamma K \to \pi K$?

relation between finite-volume matrix element, and infinite-volume matrix element, $\mathscr{H}$

$$\left|{}_L\langle K\,|\,j\,|\,K\pi\rangle_L\right| \propto \left(\mathscr{H}\cdot\tilde{R}_n\cdot\mathscr{H}\right)^{1/2}$$

where the residue of the finite-volume hadron-hadron propagator appears

$$\tilde{R}_n(L) \equiv 2E_n \cdot \lim_{E\to E_n}\left(E-E_n\right)\left(\underbrace{F^{-1}(E^\star;L)}_{\text{matrix in } \ell\,=\,0,1} + \underbrace{\mathscr{M}(E^\star)}_{\text{diagonal matrix in } \ell\,=\,0,1}\right)^{-1}$$

using an eigen-decomposition $F + \mathscr{M}^{-1} = \sum_i \mu_i\,\mathbf{w}_i\mathbf{w}_i^\intercal$

$$\mathbf{w}_i = \begin{pmatrix}\mathbf{w}_i^{\ell=0}\\ \mathbf{w}_i^{\ell=1}\end{pmatrix}$$

the residue factorizes $\tilde{R}_n = \left(-\frac{2E_n^\star}{\underbrace{\mu_0^{\star\prime}}_{\text{slope of zero crossing eigenvalue}}}\right)\mathscr{M}^{-1}\mathbf{w}_0\,\underbrace{\mathbf{w}_0^\intercal}_{\text{zero crossing eigenvector}}\mathscr{M}^{-1}$

and the net finite-volume correction is $F(Q^2, E^\star_{K\pi}\!=\!E^\star_n) = \dfrac{1}{\tilde{r}_n(L)}F_L(Q^2, E^\star_n)$

remember,
no $\gamma K \to (K\pi)_{\ell=0}$
amplitude

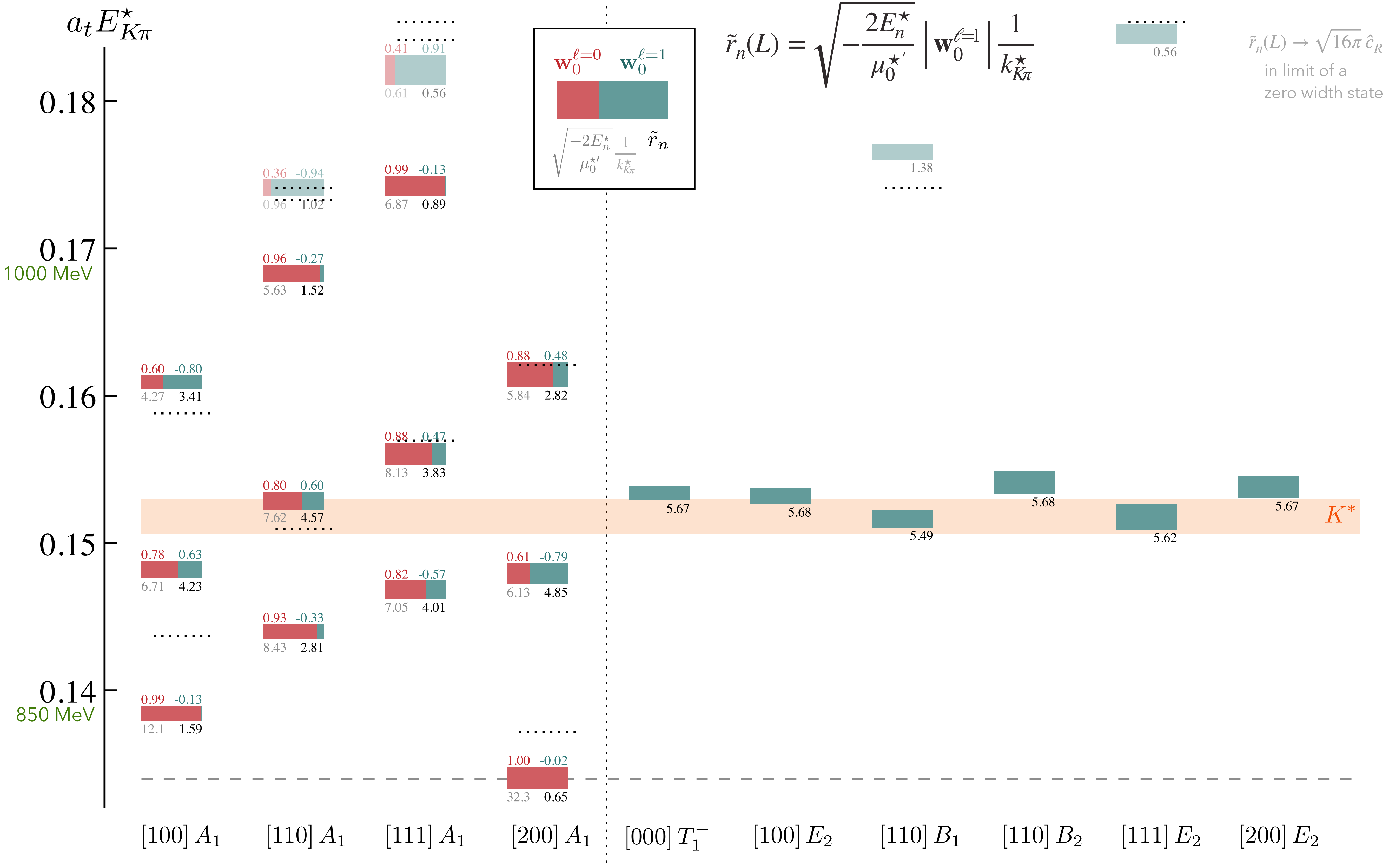
$$\mathscr{H} = \mathscr{A}\cdot\frac{1}{k^\star_{K\pi}}\cdot\mathscr{M}^{\ell=1}$$

$$\mathscr{A} = \underbrace{K}_{\text{kinematic factor}}\cdot\underbrace{F}_{\text{form-factor}}$$

where $\tilde{r}_n(L) = \sqrt{-\dfrac{2E_n^\star}{\mu_0^{\star\prime}}}\;\left|\mathbf{w}_0^{\ell=1}\right|\dfrac{1}{k^\star_{K\pi}}$

finite-volume correction factors

$$F(Q^2, E^\star_{K\pi} = E^\star_n) = \frac{1}{\tilde{r}_n(L)} F_L(Q^2, E^\star_n)$$



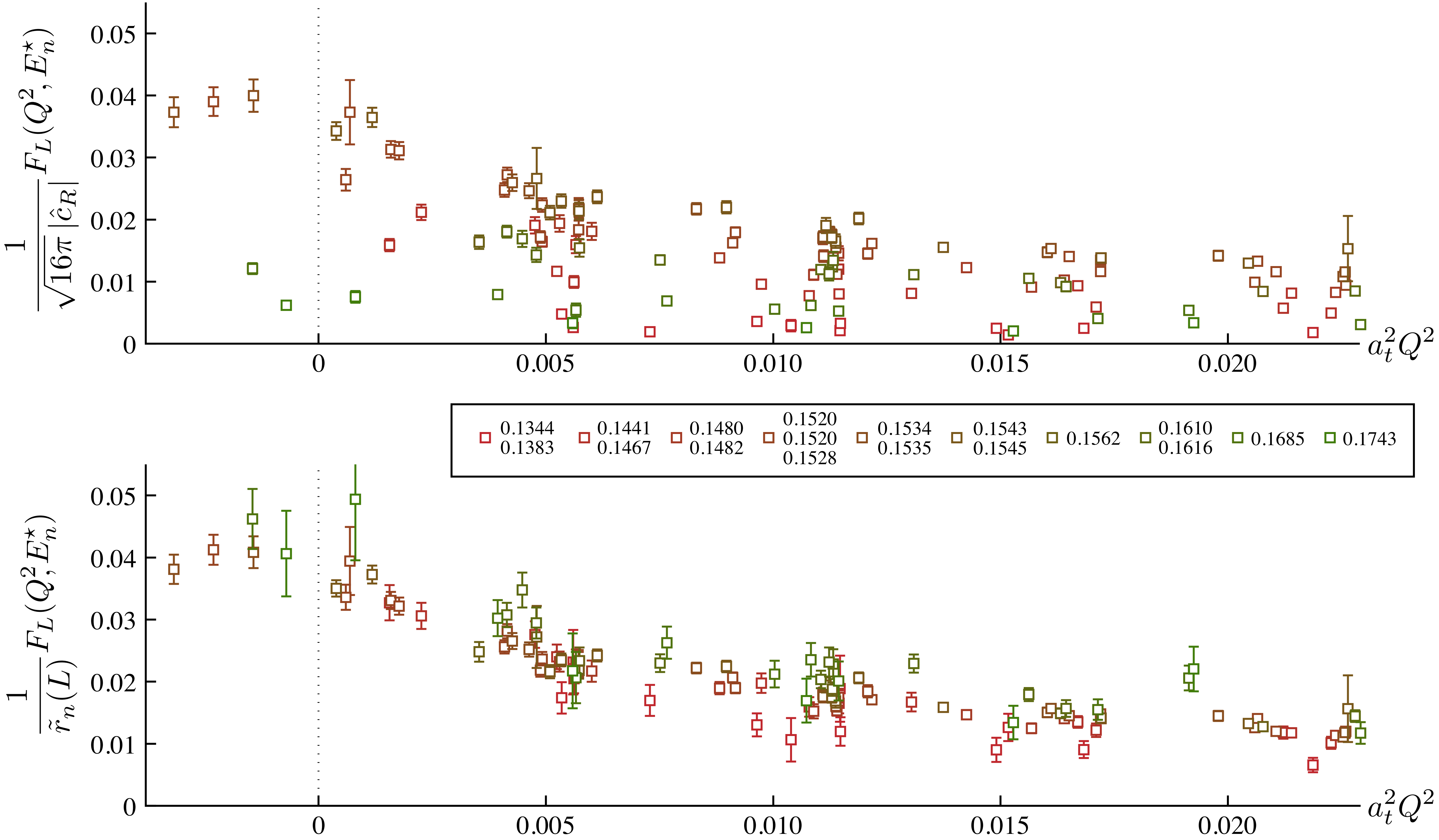
$$\tilde{r}_n(L) = \sqrt{-\frac{2E^\star_n}{\mu_0^{\star\prime}}} \left|\mathbf{w}_0^{\ell=1}\right| \frac{1}{k^\star_{K\pi}}$$

$\tilde{r}_n(L) \to \sqrt{16\pi}\,\hat{c}_R$

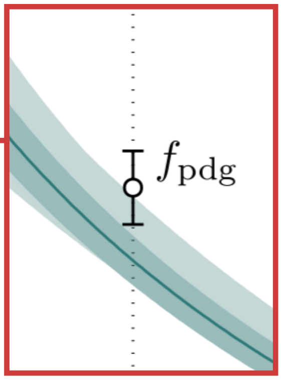
in limit of a

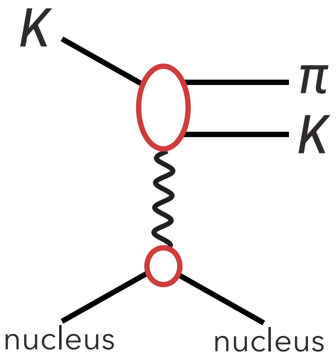
zero width state

infinite-volume form-factor



experimental determination



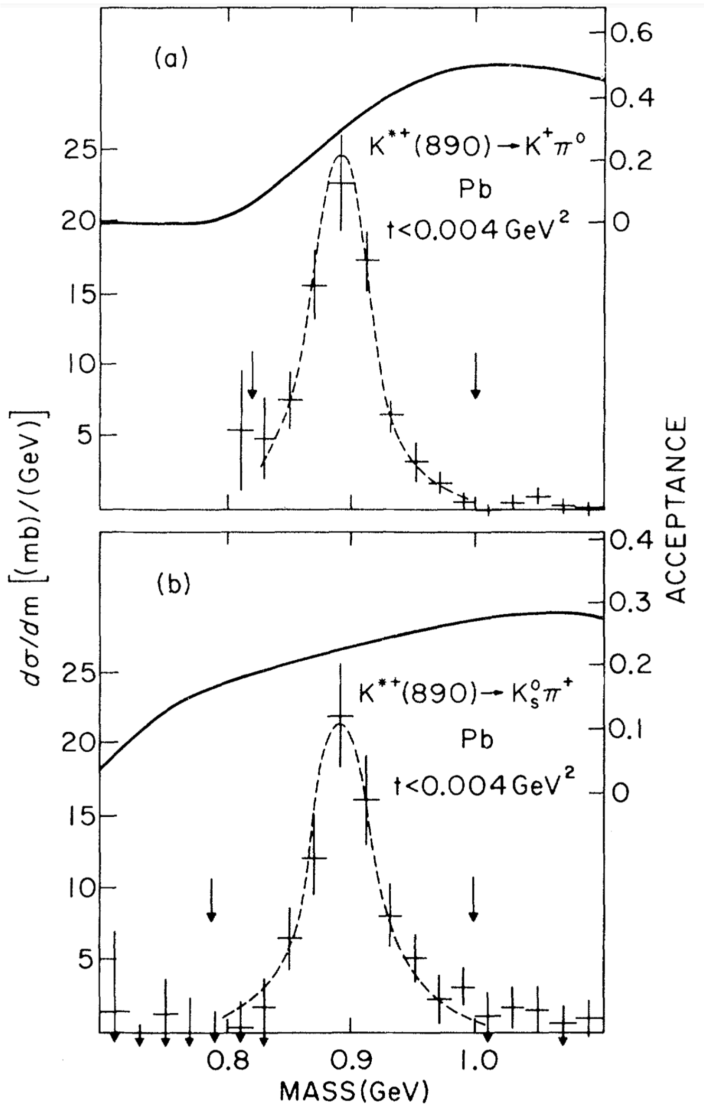


handful of Primakoff
experiments in the 70s, 80s

(very forward production of πK
using $K^{\pm}, K_L^0$ beams on nuclear targets)

pdg average of a couple of experiments $\Gamma(K^{*\pm} \to K^{\pm}\gamma) = 50(5)\,\mathrm{keV}$

$\Gamma(K^{*0} \to K^{0}\gamma) = 116(10)\,\mathrm{keV}$



very simplistic analysis scheme

$$\frac{d\sigma}{dt\,dm} = 3\pi\alpha Z^2 \frac{\Gamma_o}{k_o^3} \frac{t-t_{min,o}}{t^2} \,|\, f_{C_o} |^2 BW(m);$$

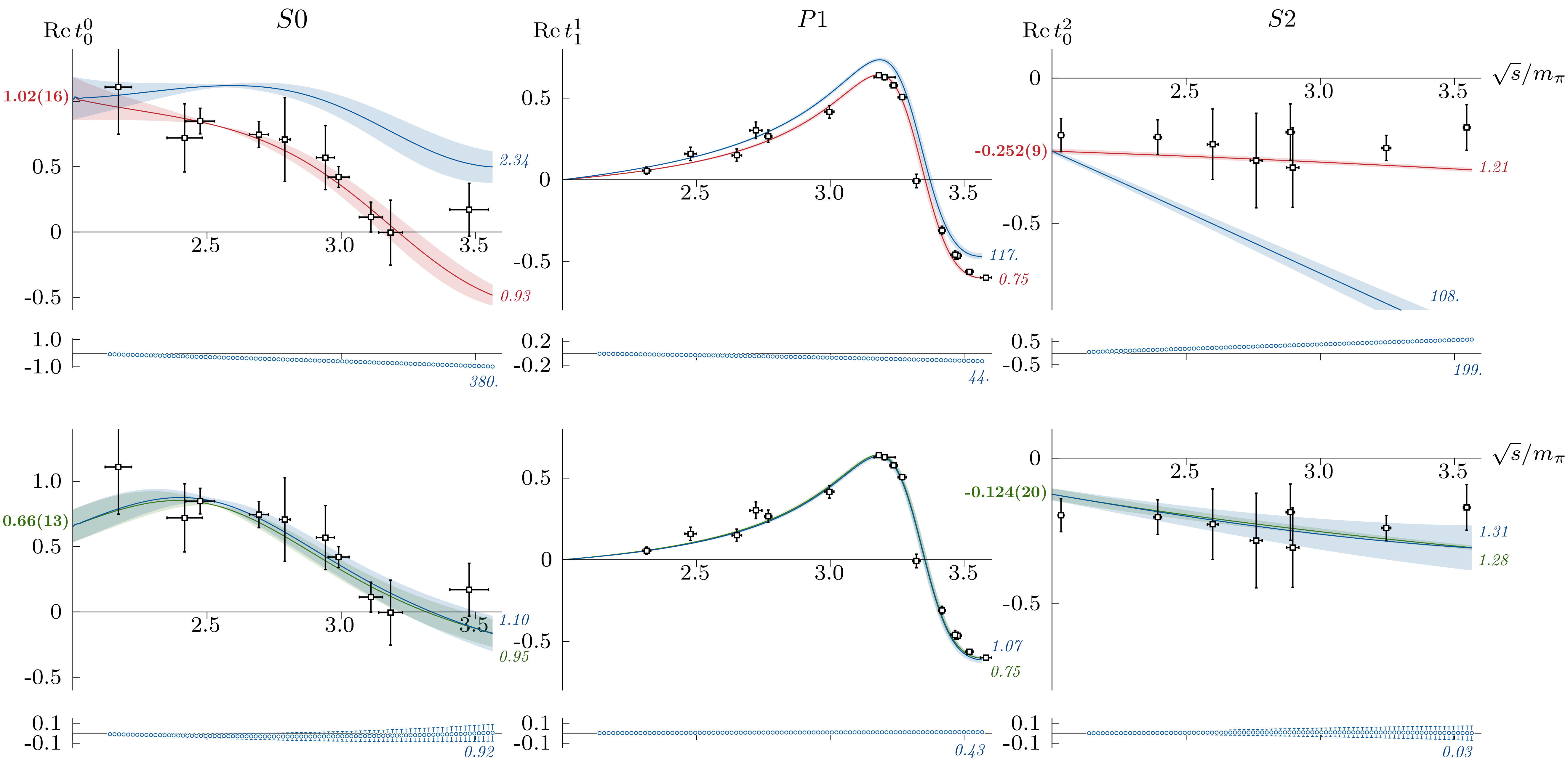
$$BW(m) = \frac{1}{\pi} \frac{m^2\Gamma^{tot}}{[m^2-m_o^2]+[m_o\Gamma^{tot}]^2} \left| \frac{g(k)}{g(k_o)} \right|^2$$

$$\Gamma(K^{*+} \to K^+\gamma) = \frac{4}{3}\alpha \frac{k_{K\gamma}^{\star 3}}{m_K^2} |f|^2$$

loss of rigor here
this is not the pole residue

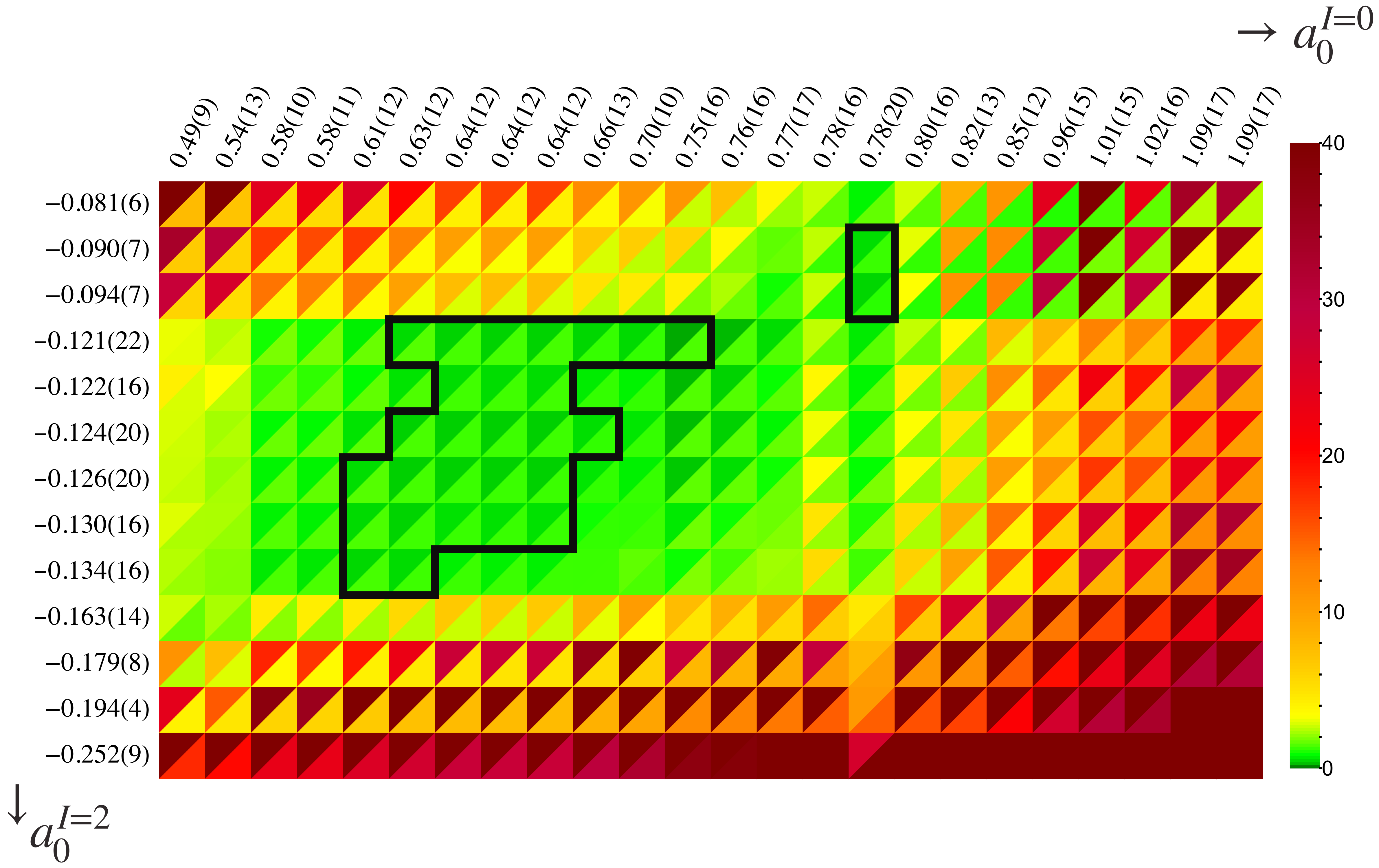
$$|f_{\rm pdg}| = 0.206(10)$$

dispersive amplitudes check

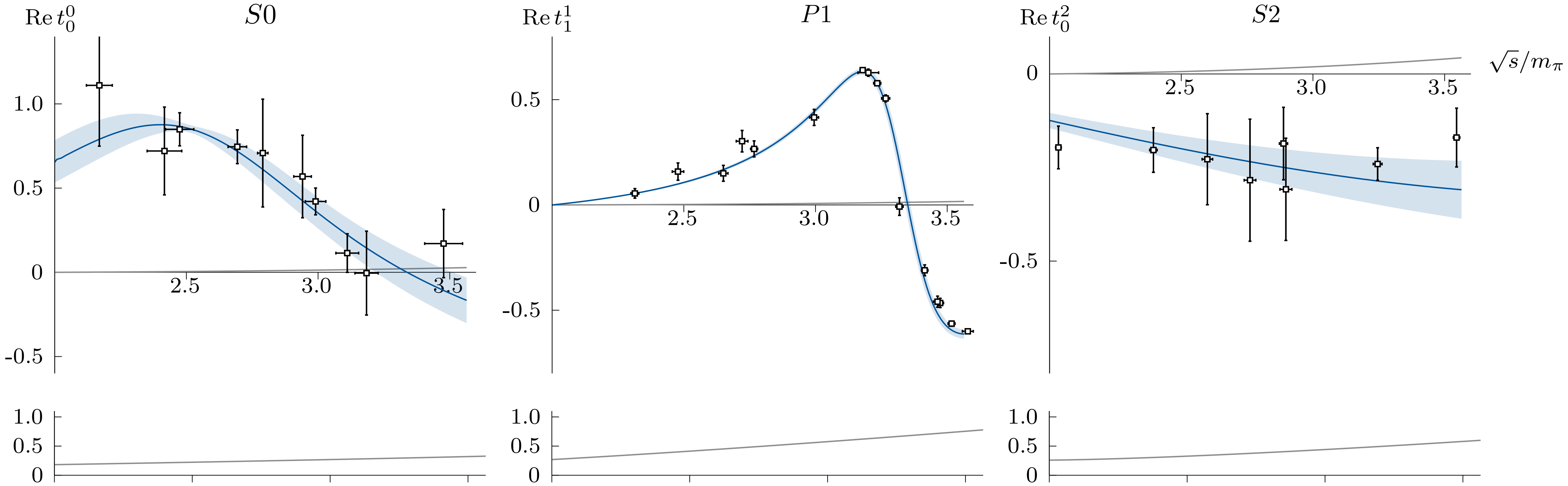


$$\tilde{t}^I_\ell(s) = \tau^I_\ell(s) + \sum_{I',\ell'} \int_{s_{\text{thr.}}}^{\infty} ds'\, K^{II'}_{\ell\ell'}(s',s)\, \mathrm{Im}\, t^{I'}_{\ell'}(s')$$

dispersive amplitudes check



LQCD dispersive $\pi\pi$ analysis

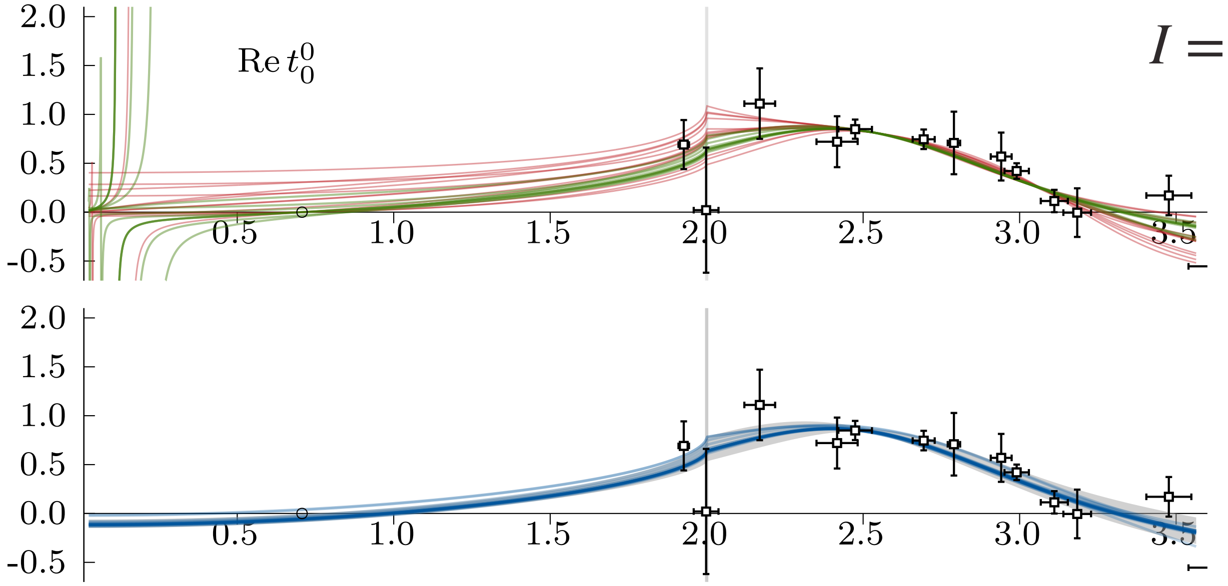


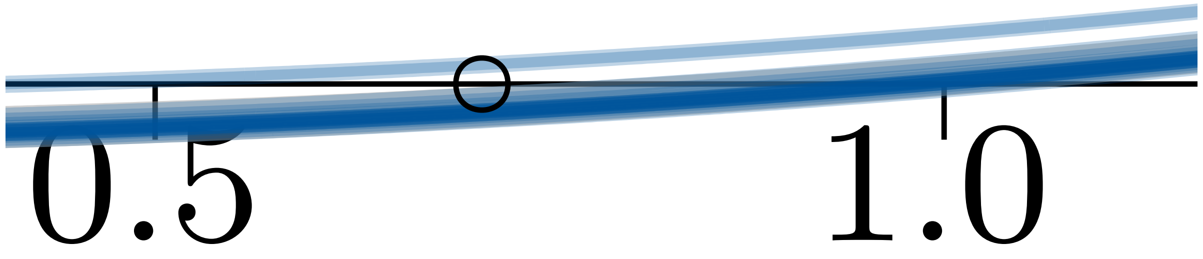
High-energy Regge parameterization details are unimportant

$$\tilde{t}^I_\ell(s) = \tau^I_\ell(s) + \sum_{I',\ell'} \int_{s_{\text{thr.}}}^{\infty} ds'\, K^{II'}_{\ell\ell'}(s', s)\, \mathrm{Im}\, t^{I'}_{\ell'}(s')$$

subthreshold zeroes at $m_\pi = 239\,\mathrm{MeV}$

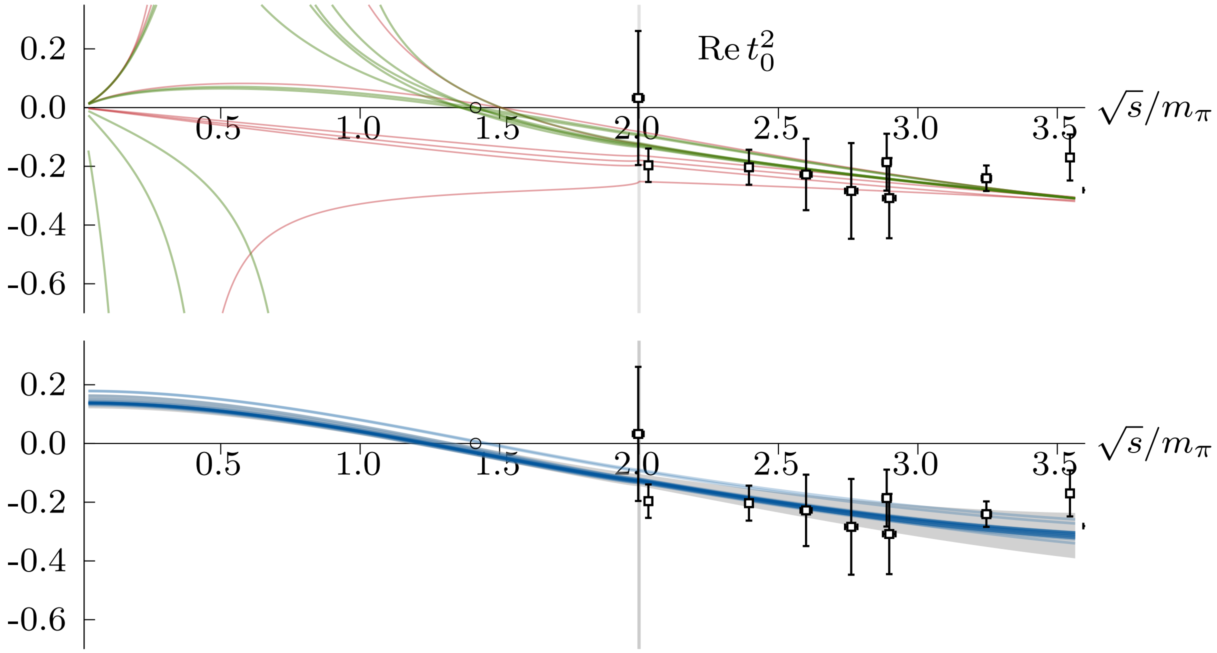
$I = 0$

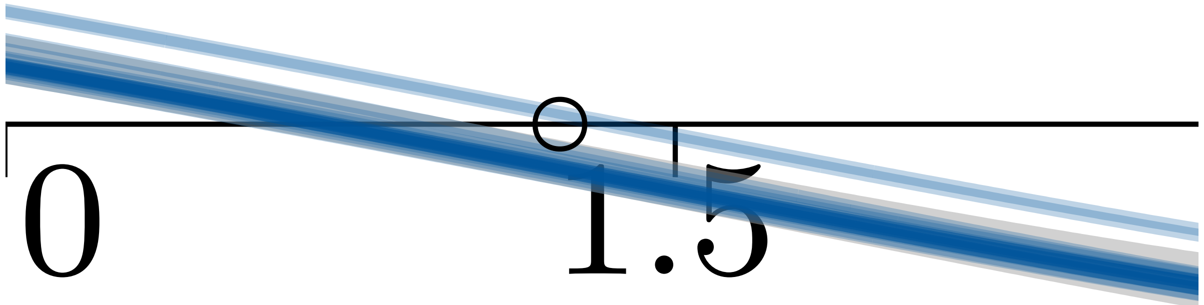




closer to threshold

$I = 2$

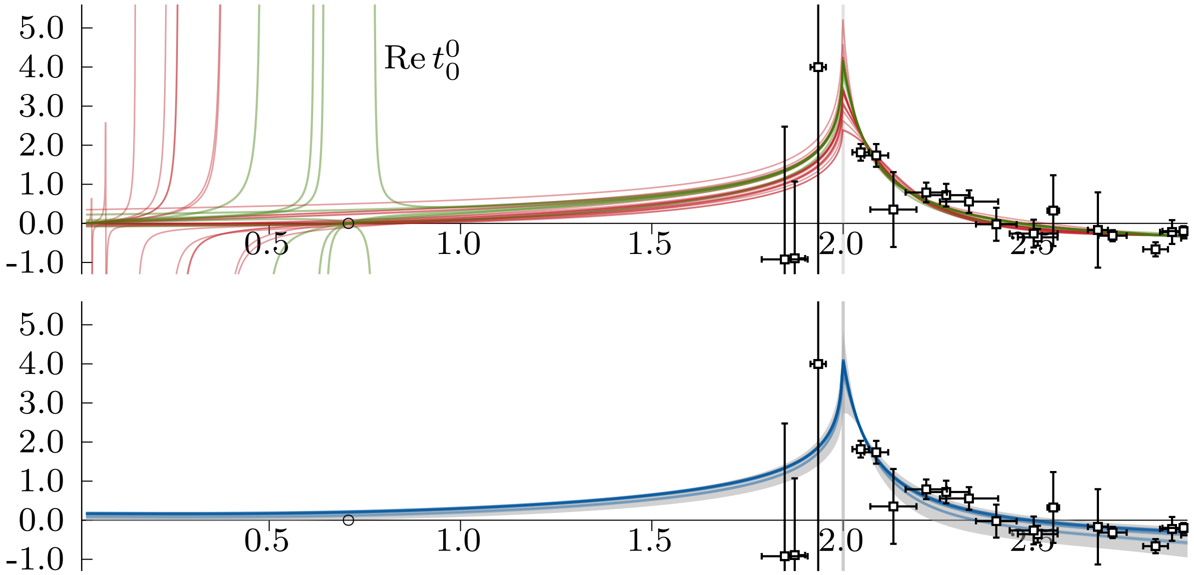




further away from threshold

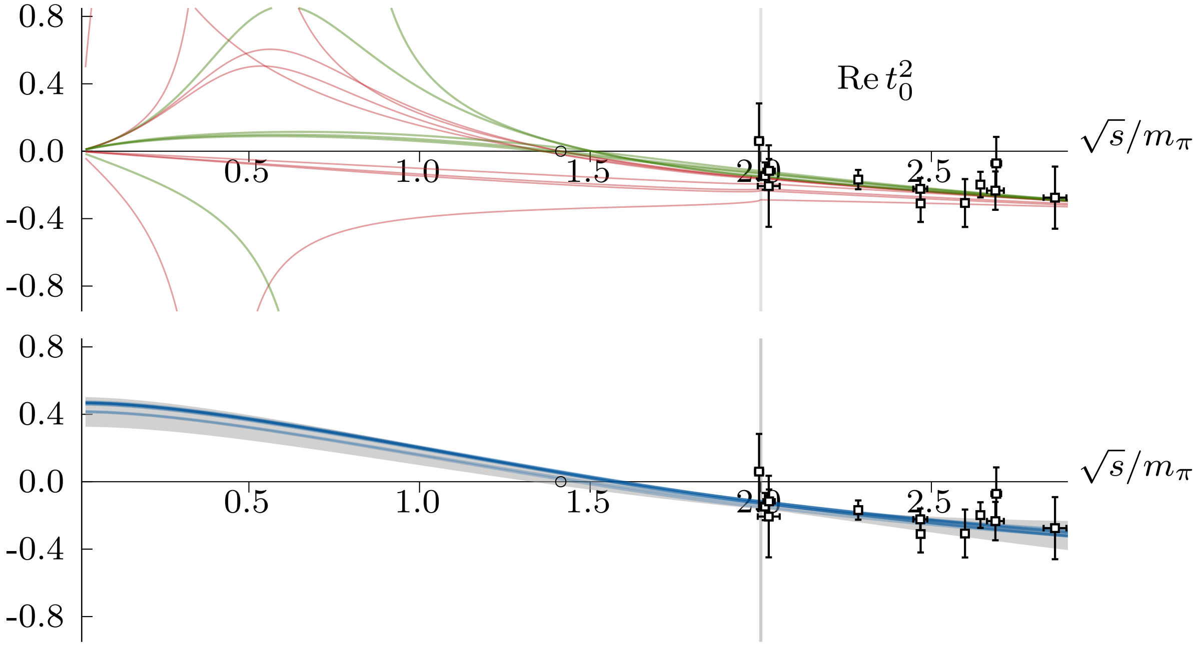
subthreshold zeroes at $m_\pi = 283\,\mathrm{MeV}$

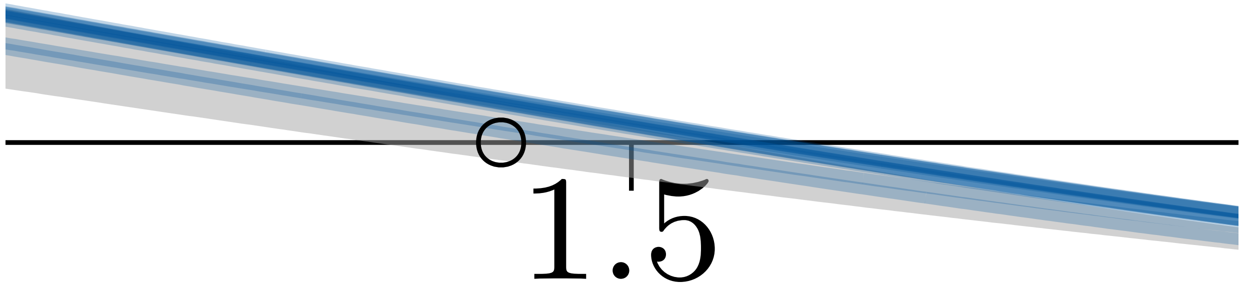
$I = 0$



no zero observed – "further" ?

$I = 2$





closer to threshold