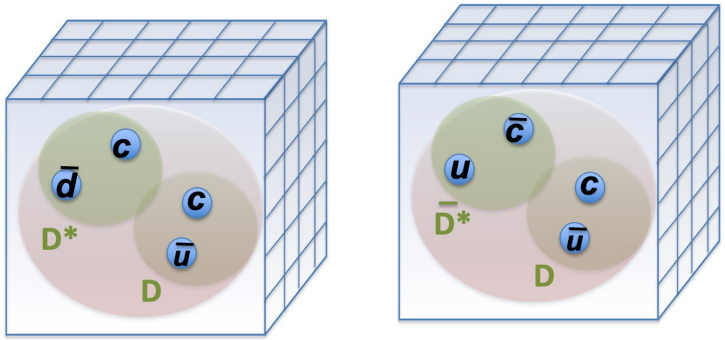
Tetraquarks with two heavy quarks from lattice QCD



Sasa Prelovsek

University of Ljubljana &

Jozef Stefan Institute, Slovenia

Exotic Hadron Spectroscopy
Durham University
April 2023

Tetraquarks with two heavy quarks

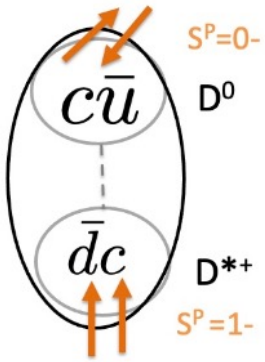
Outline: $QQ\bar{q}\bar{q}'$ $\bar{Q}Q\bar{q}q'$

Q=c,b

q=u,d,s

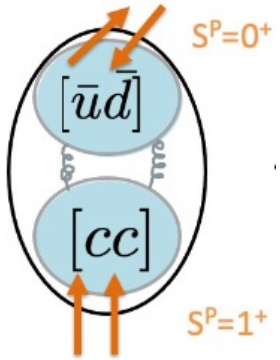
Aim:

- Which states exist? flavor, J^P
- Mass ? Strongly stable ?
- Binding mechanism ?



exchange of light mesons

$\pi, \rho, \pi\pi$?

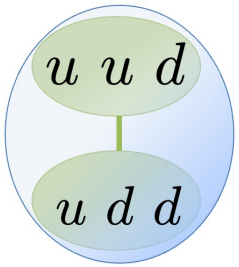


"molecular"

deuterium

$\pi, ..$

$E_b = -2.2$ MeV



p

n

$$QQ\bar{q}\bar{q}'$$

Q=c,b

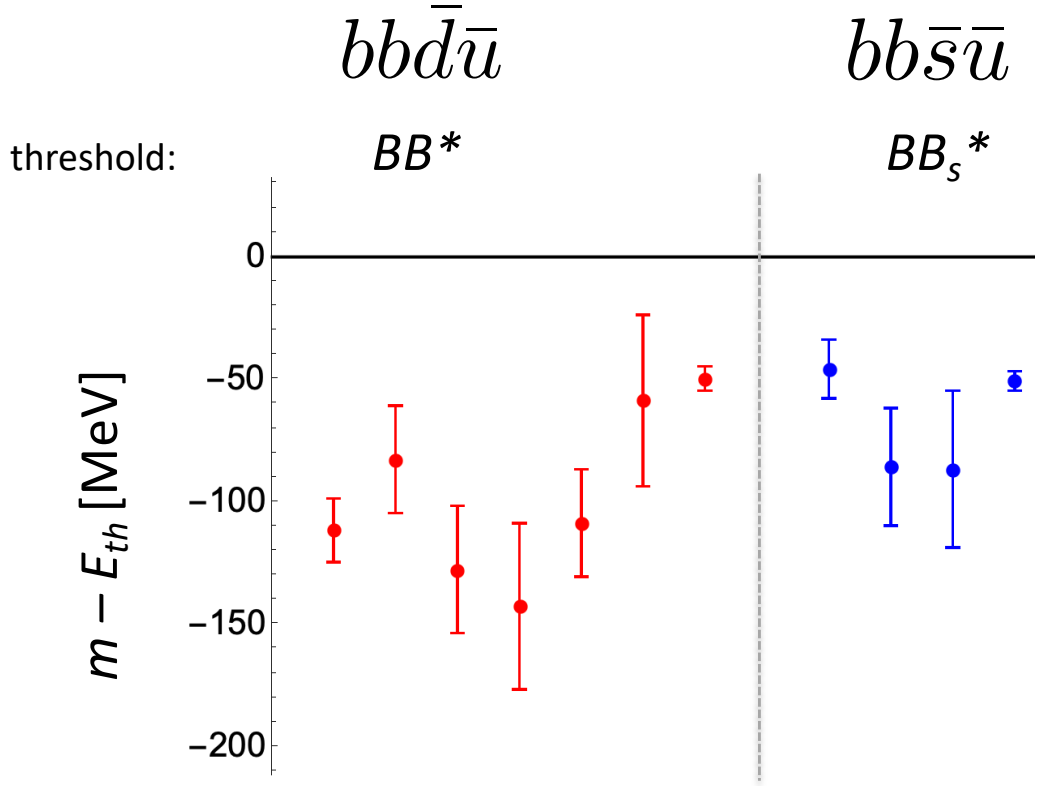
q=u,d,s

Doubly heavy tetraquarks

Doubly bottom tetraquarks

not found in exp, difficult to find

$bb\bar{d}\bar{u}$ $bb\bar{s}\bar{u}$ $I=0,\ J^P=1^+$

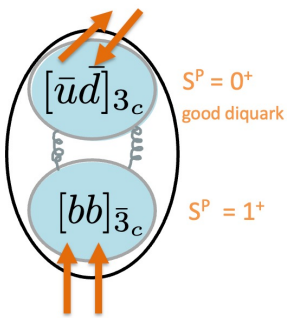


references from left to right (lattice QCD)

Hudspith, Mohler, 2303.17295
HALQCD, 2212.00202
Leskovec, Meinel, Pflaumer, Wagner, 1904.04197
Junnarkar, Mathur, Padmanth, 1810.12285
Frances, Colquhoun, Hudspith, Maltman (2021 PosLat)
Bicudo, Wagner et al. 1612.02758, static potentials
Brown, Orginost, 1210.1953, static potentials

Hudspith, Mohler, 2303.17295
Meinel, Pflaumer, Wagner, 2205.13982
Junnarkar, Mathur, Padmanth 1810.12285
Frances, Colquhoun, Hudspith, Maltman (2021, PosLat)

likely dominant
(B and B* to close
in BB* molecule
with binding ~0.1 GeV)

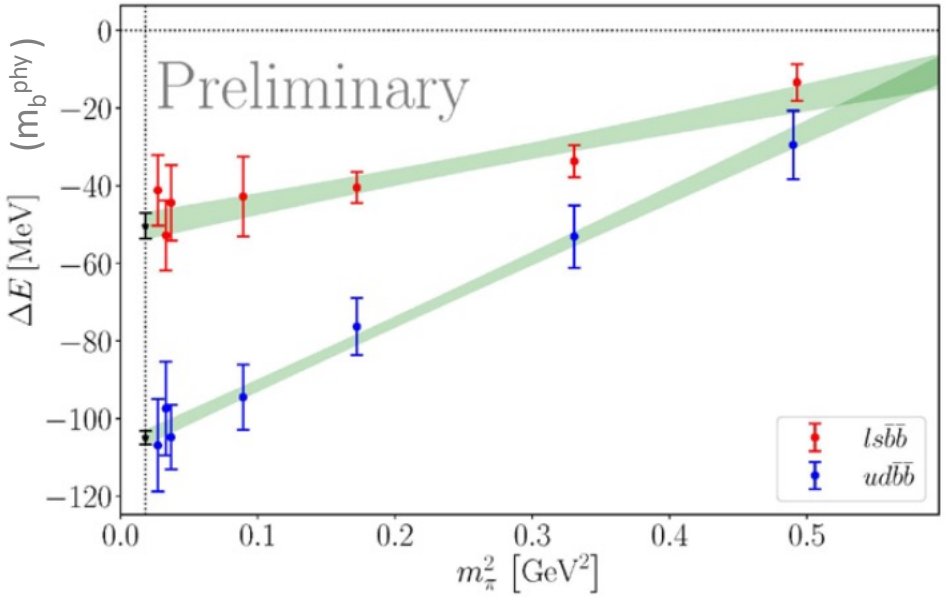


Doubly bottom tetraquarks

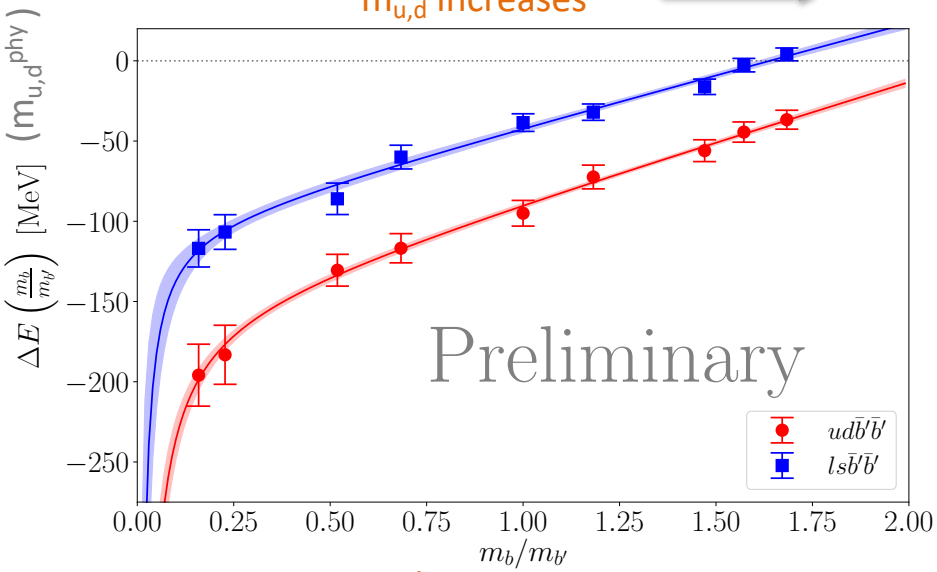
$bb\bar{d}\bar{u}$ $bb\bar{s}\bar{u}$ $I=0,\ J^P=1^+$

lattice: dependence on m_b and $m_{u,d}$

Frances, Colquhoun, Lewis, Maltman (2021)
PoS LATTICE2021 (2022) 144



$m_{u,d}$ increases



$m_{b'}$ decreases

Other $QQ'\bar{q}\bar{q}'$ and J^P

$bc\bar{q}\bar{q}'$, $cc\bar{q}\bar{q}'$ q=u,d,s

↓

talk by M. Pflaumer

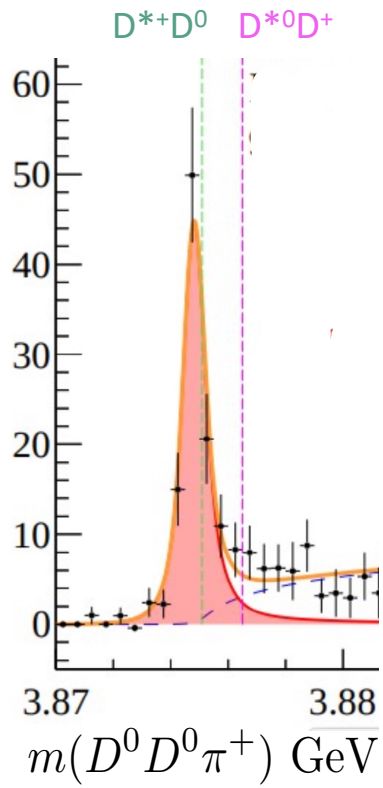
Theoretically expected near or above threshold

States near or above threshold have to be identified as poles in scattering T(E): much more challenging

Experimental discovery of T_{cc} $cc\bar{d}\bar{u}$

The longest lived exotic hadron ever discovered

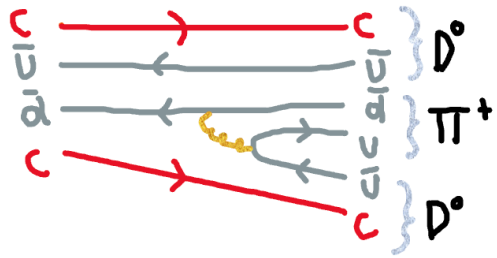
I=0, $J^P=1^+$ (most likely)



$$\delta m = m - (m_{D^{*+}} + m_{D^0})$$

$$\delta m_{pole} = -0.36 \pm 0.04 \text{ MeV}$$

LHCb 2109.01038, 2109.01056, Nature Physics



Omitting $D^* \to D\pi,\ T_{cc} \to DD\pi$

T_{cc} would-be a bound state

T_{cc} from lattice

$cc\bar{d}\bar{u}$ I=0, J^P=1+

- pre-2020 simulations extracted E_n, not T(E): Junnarkar et al 1810.12285, HadSpec 1709.01417
- near-threshold states require extraction of scattering amplitude T(E)
- states correspond to poles in T(E)

Our lattice study of T_{cc} channel

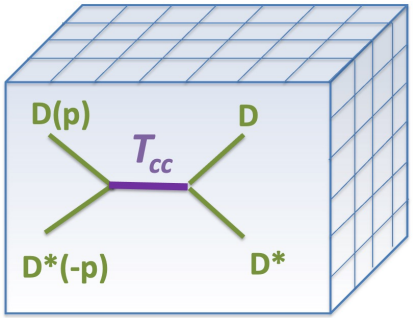
Padmanath, S.P.: 2202.10110, *PRL 2022*

CLS 2+1 ensembles, a≃0.086 fm, L = 2.1 fm, 2.7 fm

m_π=280(3) MeV

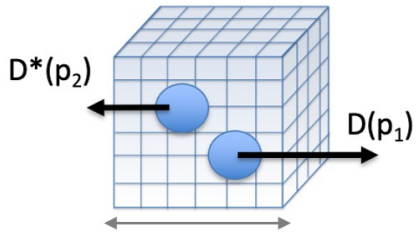
$D^* \not\to D\pi,\ T_{cc} \not\to DD\pi$

$DD\pi$ above analyzed region

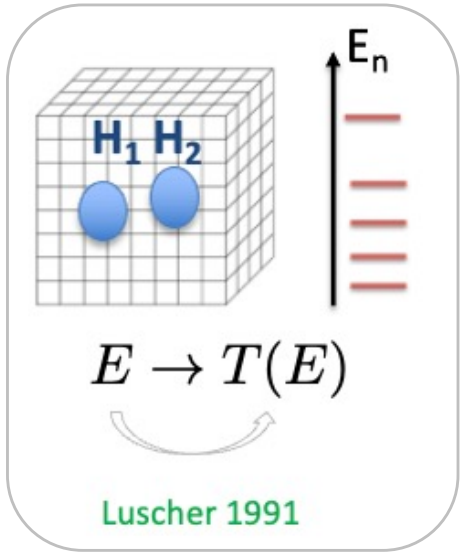


$$T(E) \propto \frac{1}{E^2 - m^2}$$

for E~m



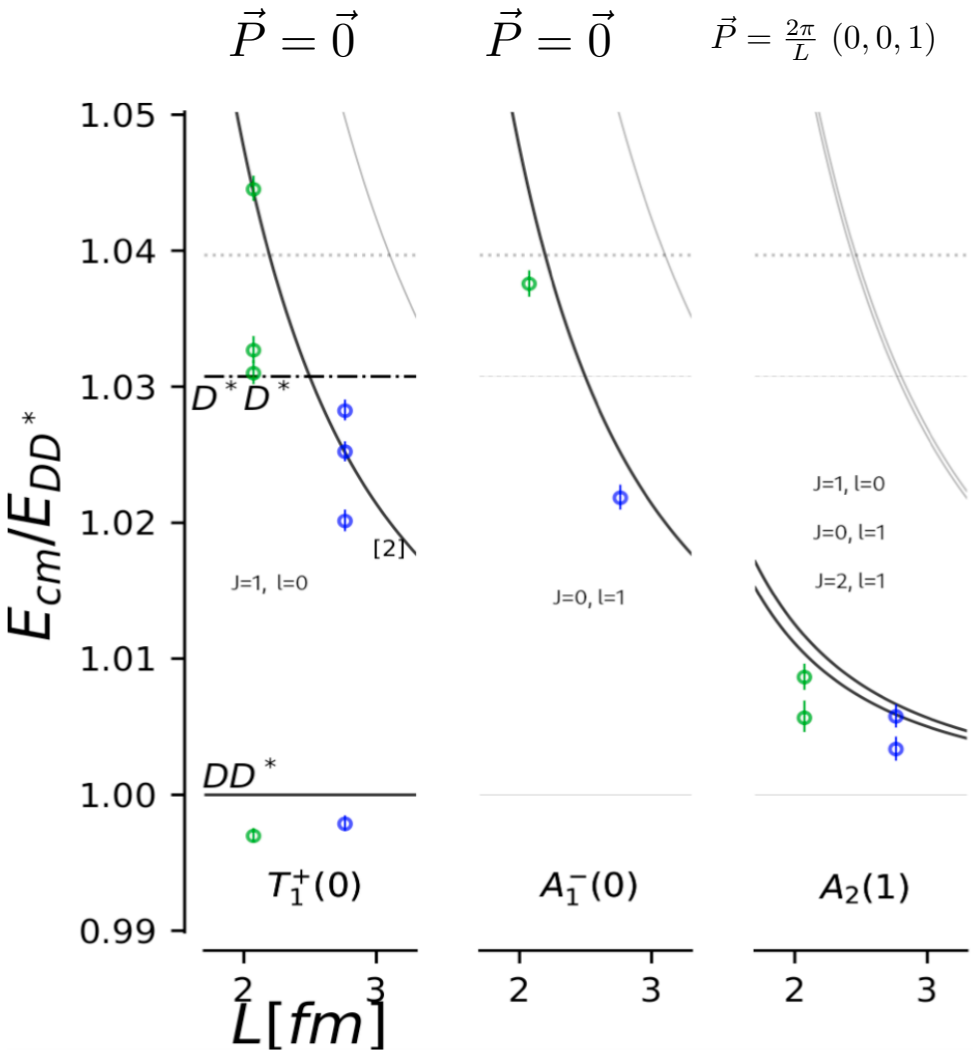
L = 2.1 fm, 2.7 fm



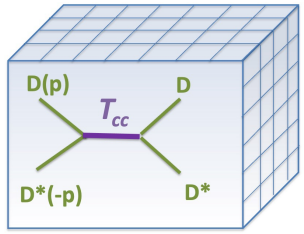
T_{cc} channel: finite-volume energies and scattering amplitude

Padmanath, S.P.: 2202.10110, *PRL 2022*

at $m_\pi \approx 280\ MeV$



$$E_{DD^*} \equiv m_D + m_{D^*}$$

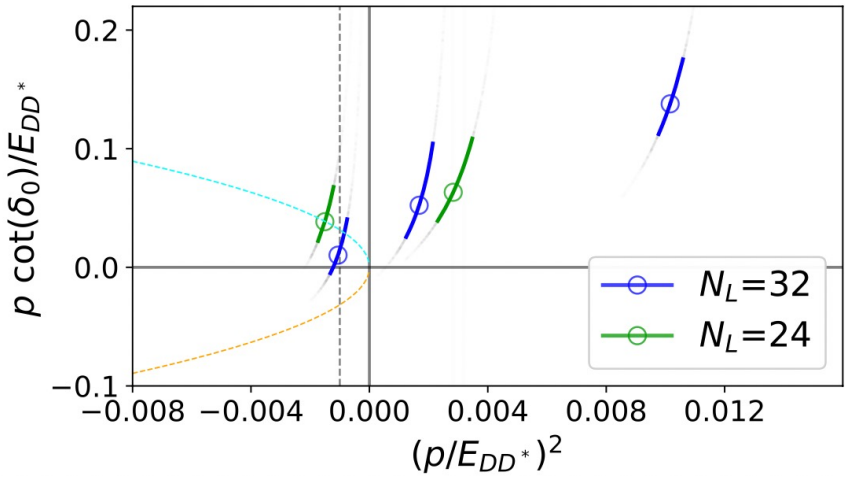


Luscher's relation

E -> T(E), $\delta(E)$

$$T = \frac{E}{2}\frac{1}{p\cot\delta - ip}$$

partial wave l=0



effects of left-hand cut (pion exchange in t-channel) omitted,

effective range approx. employed

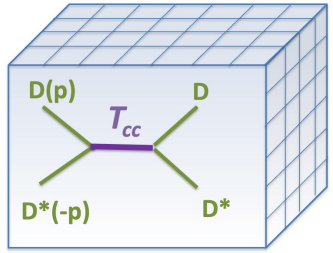
see discussion later on in the talk and

Du, F.K. Guo et al. 2303.09441

T_{cc} channel: scattering amplitude and pole

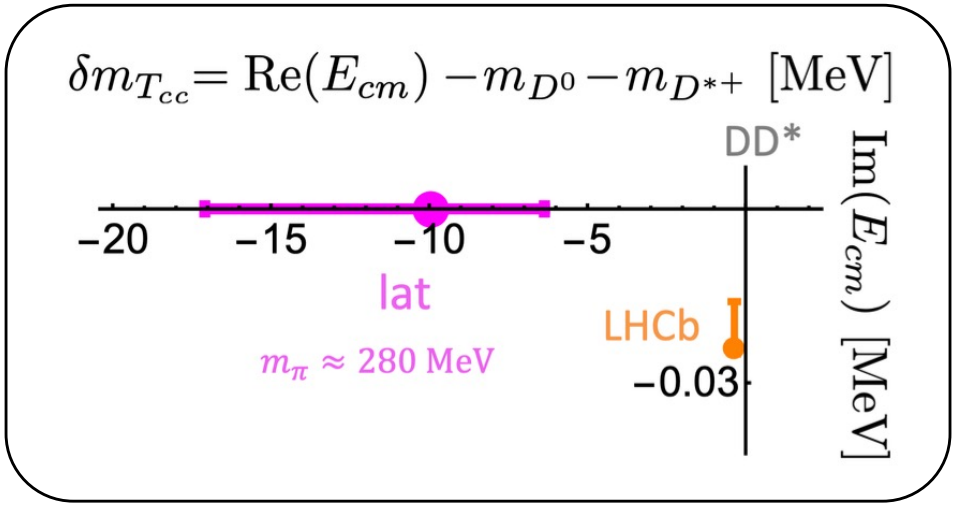
Padmanath, S.P.: 2202.10110, *PRL*

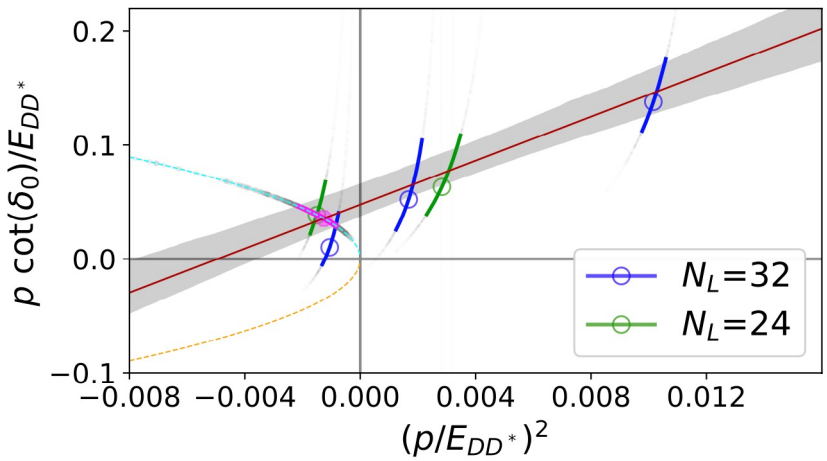
at $m_\pi \approx 280\ MeV$



$$T = \frac{E}{2}\frac{1}{p\cot\delta - ip}$$

partial wave l=0





$$p\cot\delta_0 = \frac{1}{a_0} + \frac{1}{2}\, r_0\, p^2$$

$a_0 = 1.04(0.29)$ fm & $r_0 = 0.96(^{+0.18}_{-0.20})$ fm

Lattice: virtual bound st. pole

Binding energy:

$\delta m_{T_{cc}} = -9.9(^{+3.6}_{-7.2})$ MeV

Nature (LHCb): (would-be) bound st. pole

omitting $D^* \to D\pi,\ T_{cc} \to DD\pi$

T_{cc} channel: dependence on $m_{u/d}$ and m_c

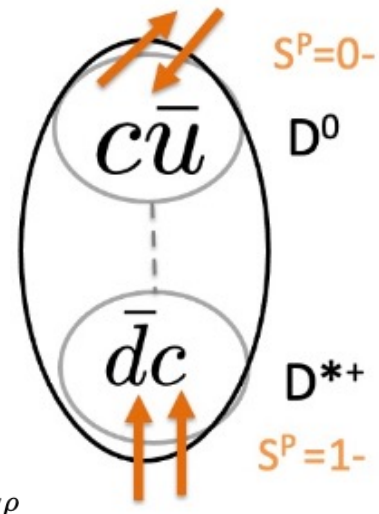
in case of molecular binding mechanism

$c c \bar{d} \bar{u}$

Padmanath, S.P.: 2202.10110, *PRL*
Supplemental material

Simple arguments in QM :

$$\pi, \rho, \pi\pi \ ?$$



$m_{u/d}$

$$V(r) \propto -\frac{e^{-m_{ex} r}}{r}$$

$$m_{ex} : \ m_\pi, m_\rho$$

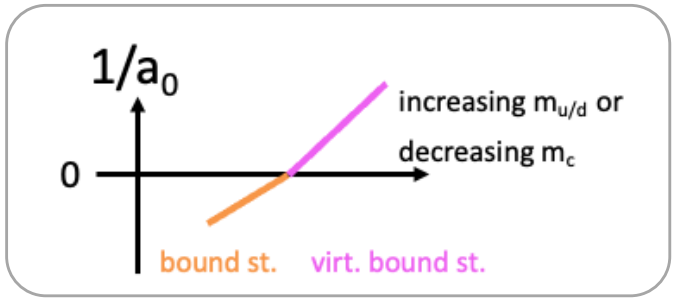
m_c

$$\hat{H}_{kin} = \frac{\hat{p}^2}{2\ m_{red}}$$

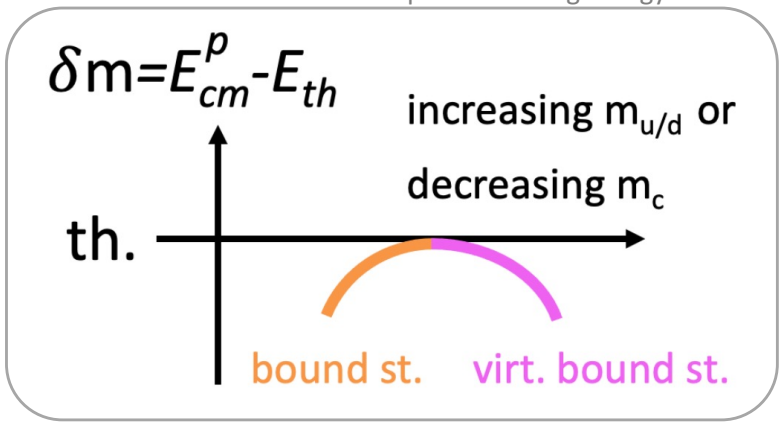
$$m_r \simeq \frac{m_D m_{D^*}}{m_D + m_{D^*}}$$

$$p \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2}\ r_0\ p^2$$

sketch of expected scattering lenght a0

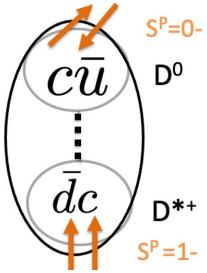


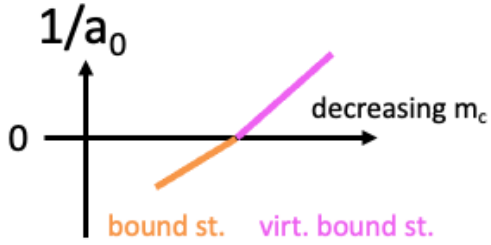
sketch of expected binding energy

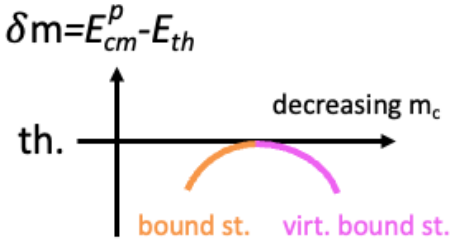


trend already partly verified on lattice

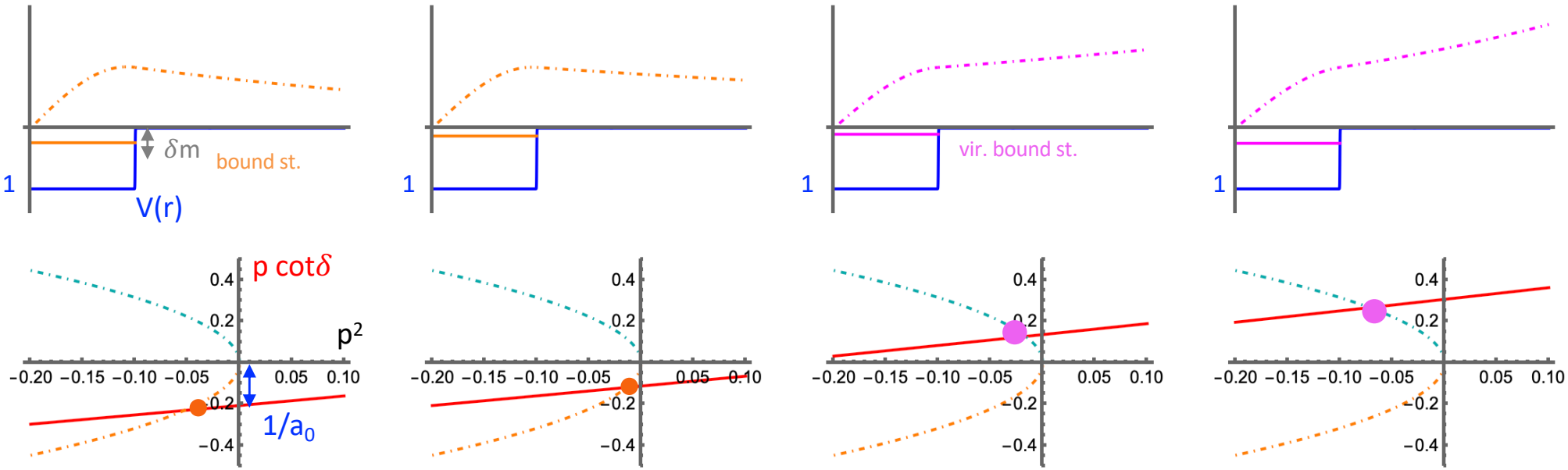
Dependence on m_c





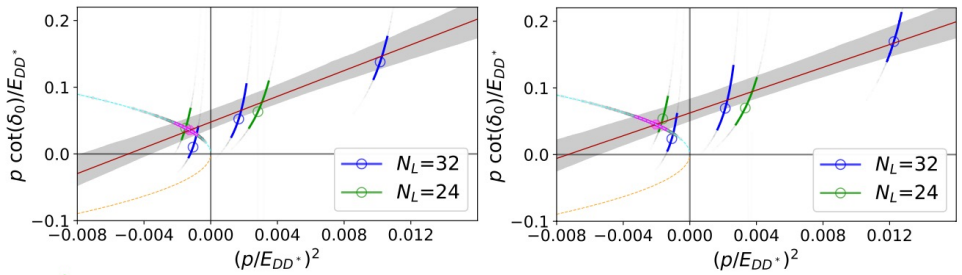


Square well potential (analogous conclusion for other fully attractive shapes), s-wave



decreasing m_c and m_r

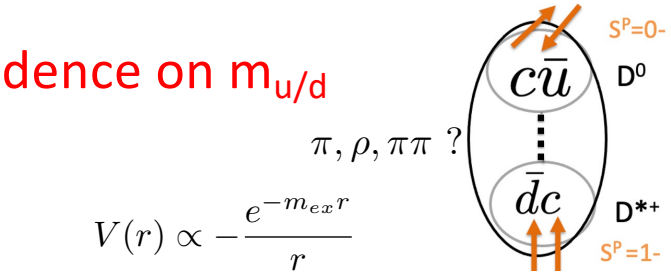
	m_D [MeV]	$a_{l=0}^{(J=1)}$ [fm]	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
$m_c^{(h)}$	1927(1)	1.04(29)	$-9.9^{+3.6}_{-7.2}$	virtual bound st.
$m_c^{(l)}$	1762(1)	0.86(0.22)	$-15.0(^{+4.6}_{-9.3})$	virtual bound st.

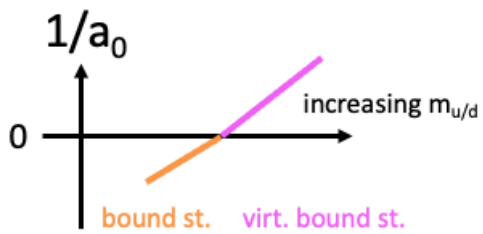


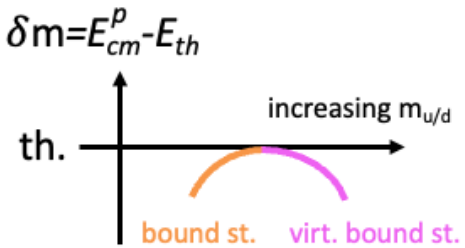
Padmanath, S.P.: 2202.10110, *PRL*

Dependence on $m_{u/d}$

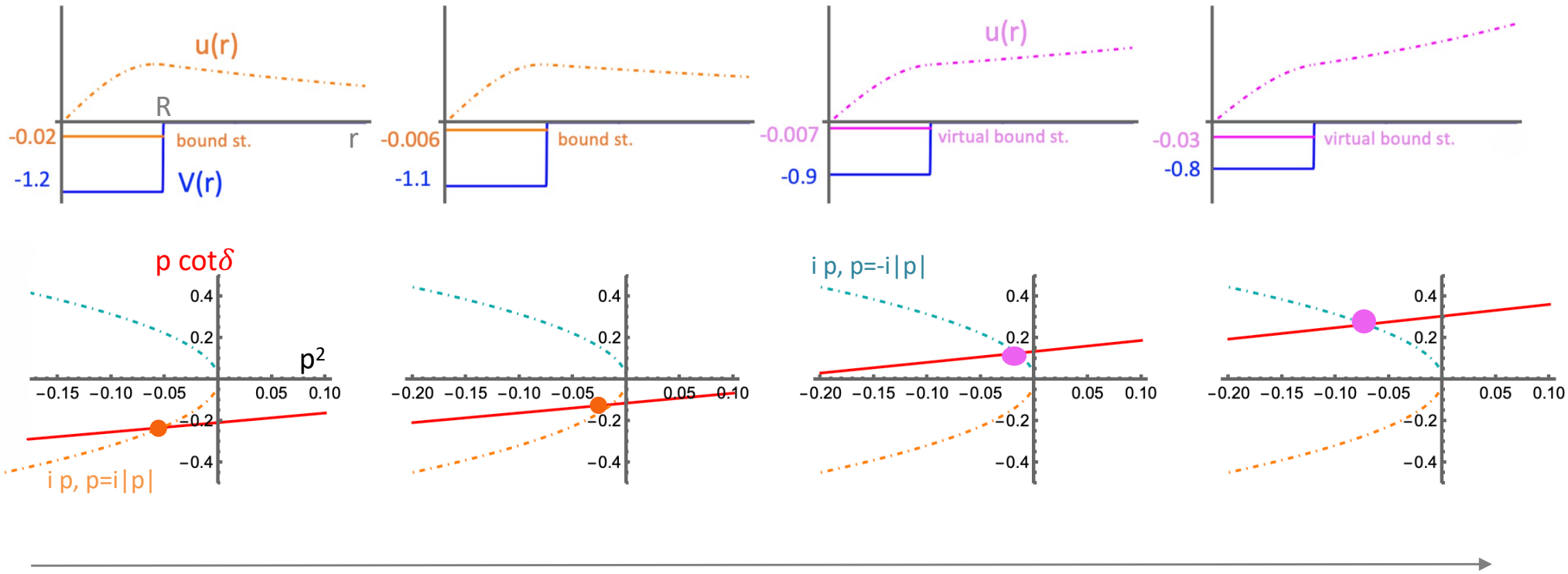
$$V(r) \propto -\frac{e^{-m_{ex} r}}{r}$$







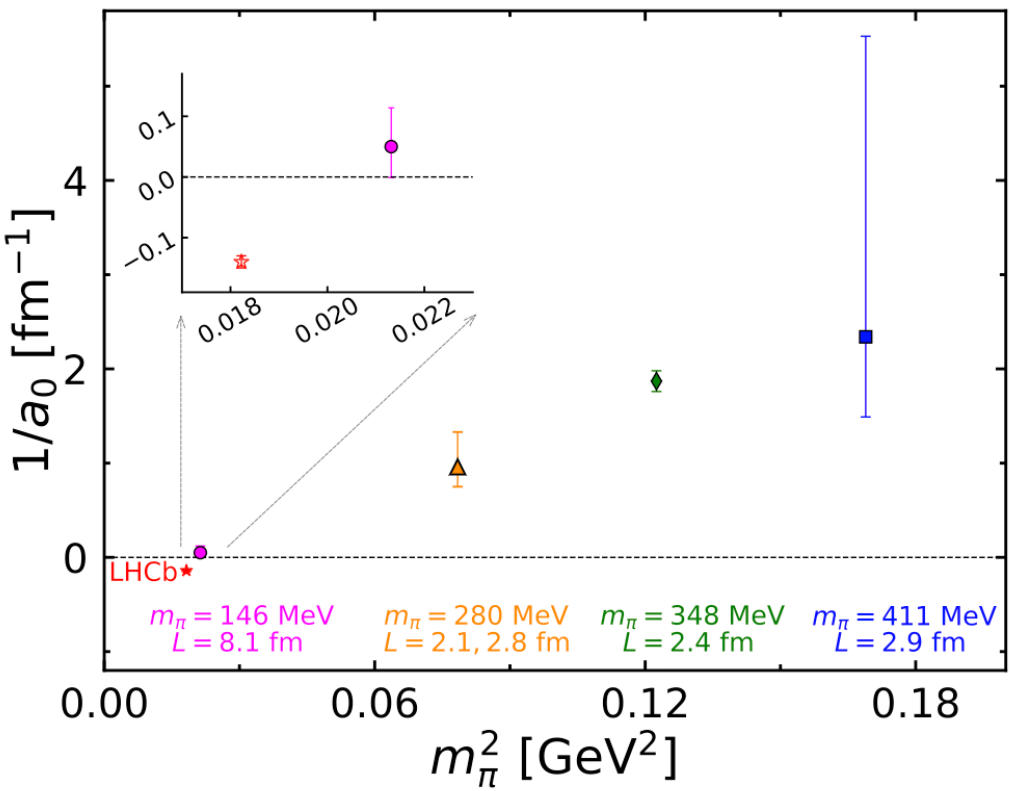
Square well potential (analogous conclusion for other shapes), s-wave



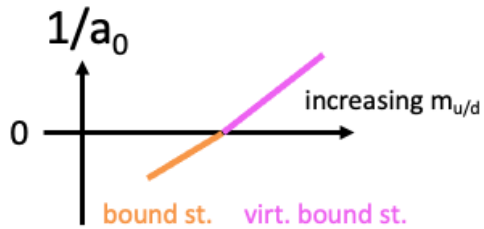
increasing $m_{u/d}$, decreasing attraction

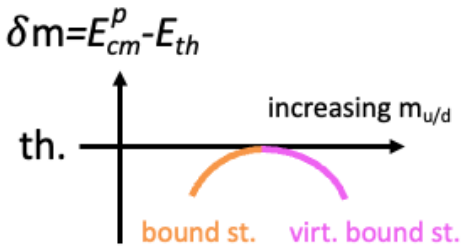
T_{cc} from lattice: dependence on $m_{u/d}$

summary from 2302.04505



$$p \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2}\, r_0\, p^2$$





	LHCb	subsequent lattice sim. HALQCD coll, 2302.04505 HALQCD method	our lattice sim. Luscher's method	subsequent lat. sim. CLQCD 2206.06185 Luscher's method
δm :	- 0.36(4) MeV	- 0.045(77) MeV	$-9.9(^{+3.6}_{-7.2})$ MeV.	

bound st.

virtual bound st.

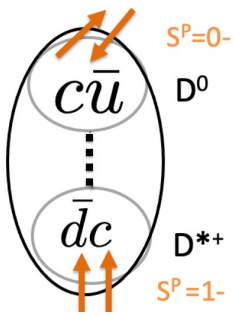
T_{cc} channel: dominant exchanged particles

at $m_\pi \approx 348$ MeV

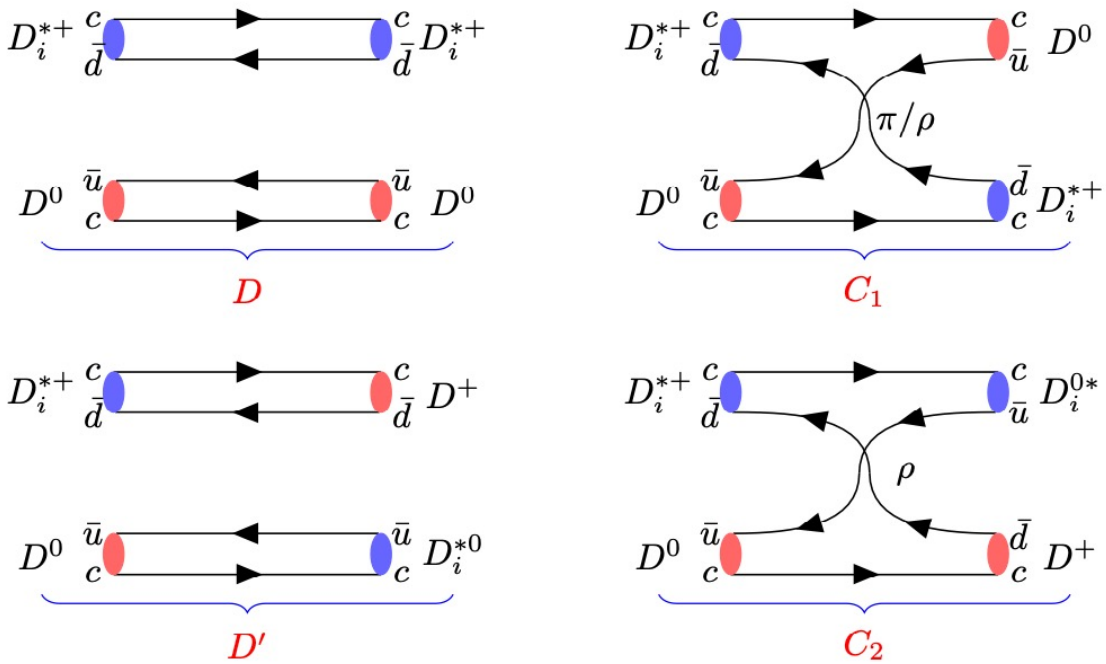
subsequent lattice study via Luscher's method
CLQCD, Chen et al. 2206.06185, PLB

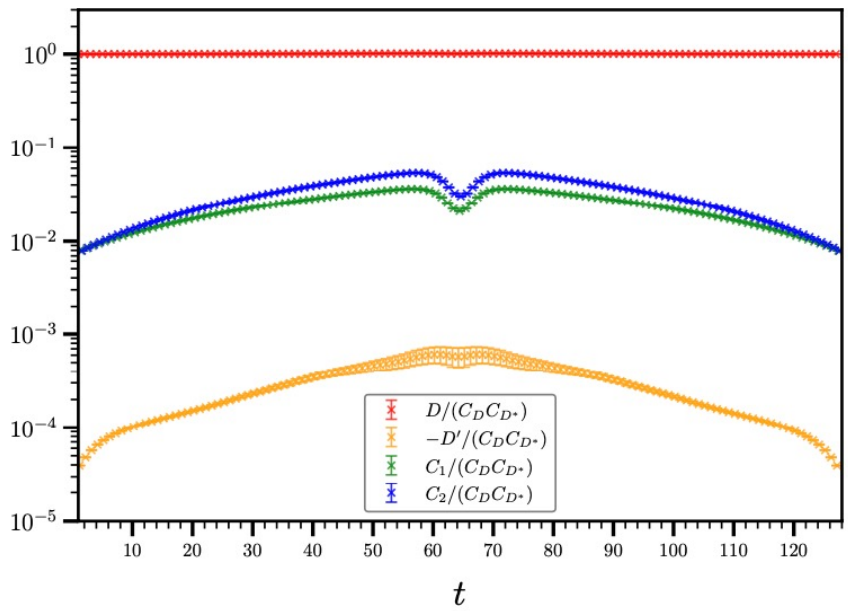
comparison of I=0,1 :
attraction in I=0 channel arises
mainly from ϱ exchange

$\pi, \rho, \pi\pi$?



$$C^{(I)}(p,t) = D - C_1(\pi/\rho) + (-)^{I+1}\,(D' - C_2(\rho))$$





at $m_\pi \approx 146$ MeV, $L \approx 8.1$ fm !!

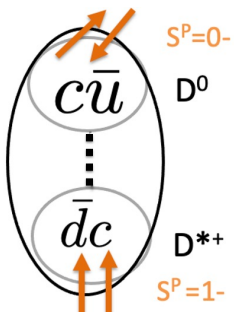
T_{cc} channel: dominant exchanged particles

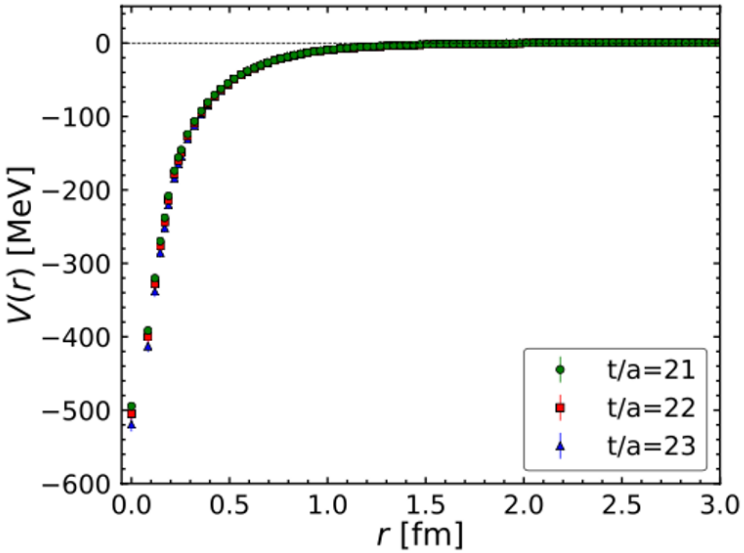
subsequent lattice sim.

HALQCD coll, 2302.04505

HALQCD method

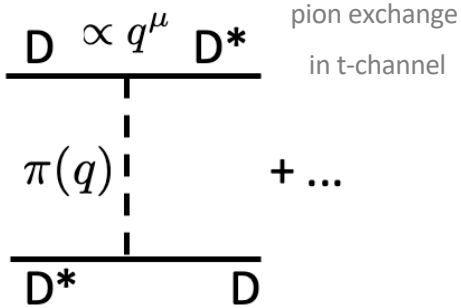
$\pi, \rho, \pi\pi$?





$$V(r) \sim -\frac{e^{-2m_\pi r}}{r^2} \quad r > 1 \text{ fm}$$

T_{cc} channel: pion exchange and left-hand cut



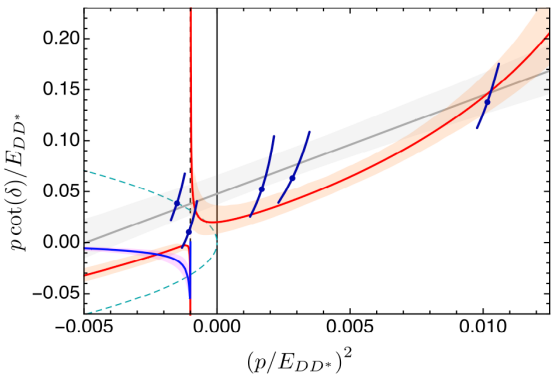
pion exchange in t-channel

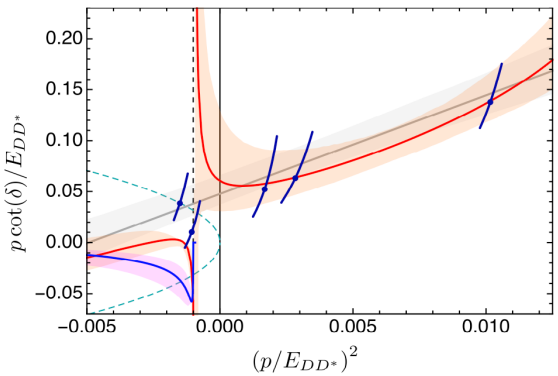
- possible effects from left-hand cut : requires further work
- pion exchange: suppressed near threshold due to derivative coupling
- pheno studies: one-pion exchange not dominant
- CLQCD, HALQCD lattice studes: one-pion exchanges not dominant
- generalization of Luscher's relation on left-hand cut: 2301.03981, Raposo& Hansen @ lat22

- reanalysis our Tcc data incorporating left-hand cut

Du, F.K. Guo et al. 2303.09441

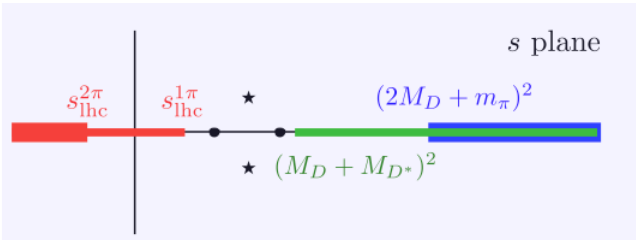
".. The appearance of a pair of virtual states is indeed natural near the point where they are about to turn to a narrow resonance ... "



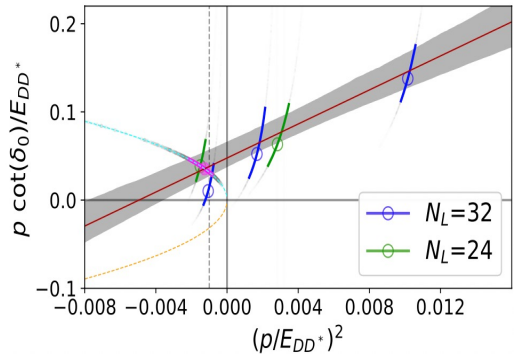


● two virtual bound states

★ two conjugate poles of one narrow resonance



Padmanath, S.P.: 2202.10110, *PRL*



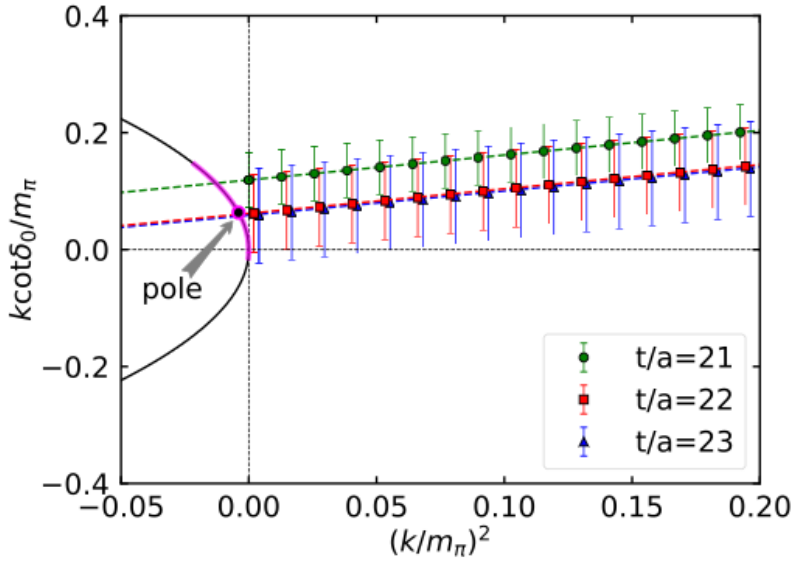
one virtual bound state

both conclusions support the presence of significant attraction and poles, likely due to Tcc

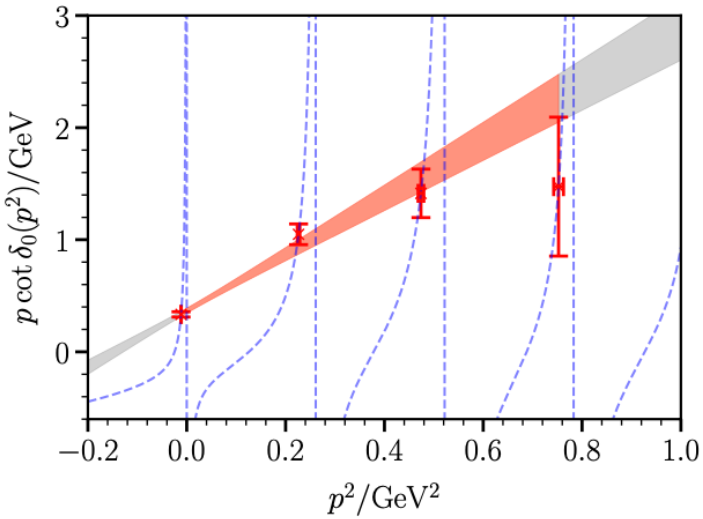
Intermezzo: p cot δ_0 in Tcc chanel from available lattice simulations (other two simulations will be detailed later in the talk)

eff. range approx.: $p \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2}\, r_0\, p^2$

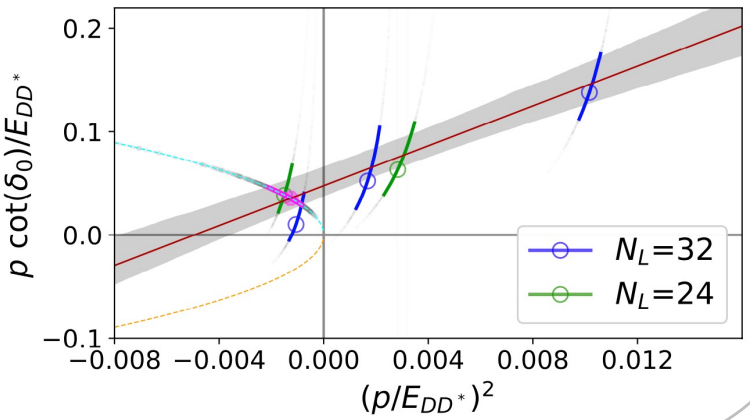
HALQCD, 2302.04505, $m_\pi \approx 146$ MeV



CLQCD 2206.06185, $m_\pi \approx 348$ MeV, PLB



Padmanath, S.P.: 2202.10110, $m_\pi \approx 280$ MeV *PRL*



$$\bar{Q}Q\bar{q}q'$$

q=u,d,s

charmonium-like and bottomonium-like states

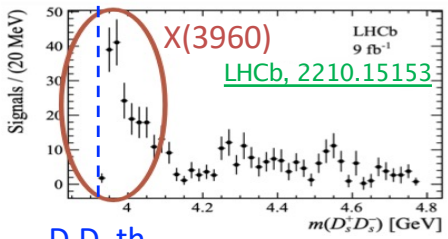
Charmonium(like) resonances and bound states

$\bar{c}c$, $\bar{c}q\bar{q}c$ q=u,d,s $I=0$

$$\frac{|c^2_{D\bar{D}}|}{|c^2_{D_s\bar{D}_s}|} = 0.02\ ^{+0.02}_{-0.01}$$

$$T_{ij}(E_{cm}) \sim \frac{c_i\ c_j}{E^2_{cm} - m^2}$$

$\bar{D}_s D_s$ J^P=0$^+$ state



LHCb, 2210.15153

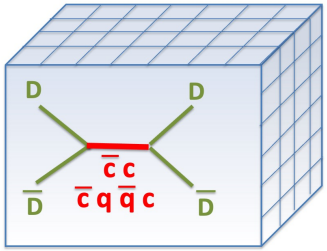
D_sD_s th.

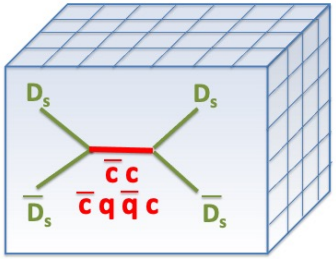
$\bar{D}D$ J^P=0$^+$ state

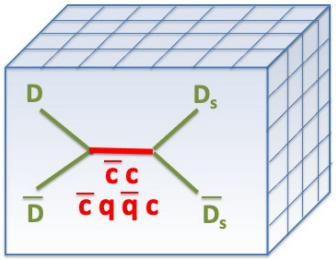
predicted in models [Oset et al, 0612179 PRD, Guo et al 2101.01021]

seen in re-analysis of exp. [Danilkin et al 2111.15033, Ji, F.K. Guo et al., 2212.00613]

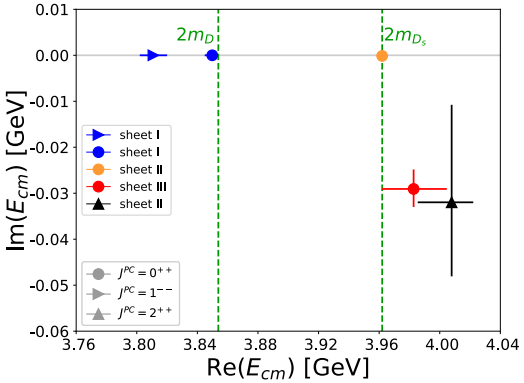
+ expected conventional charmonia



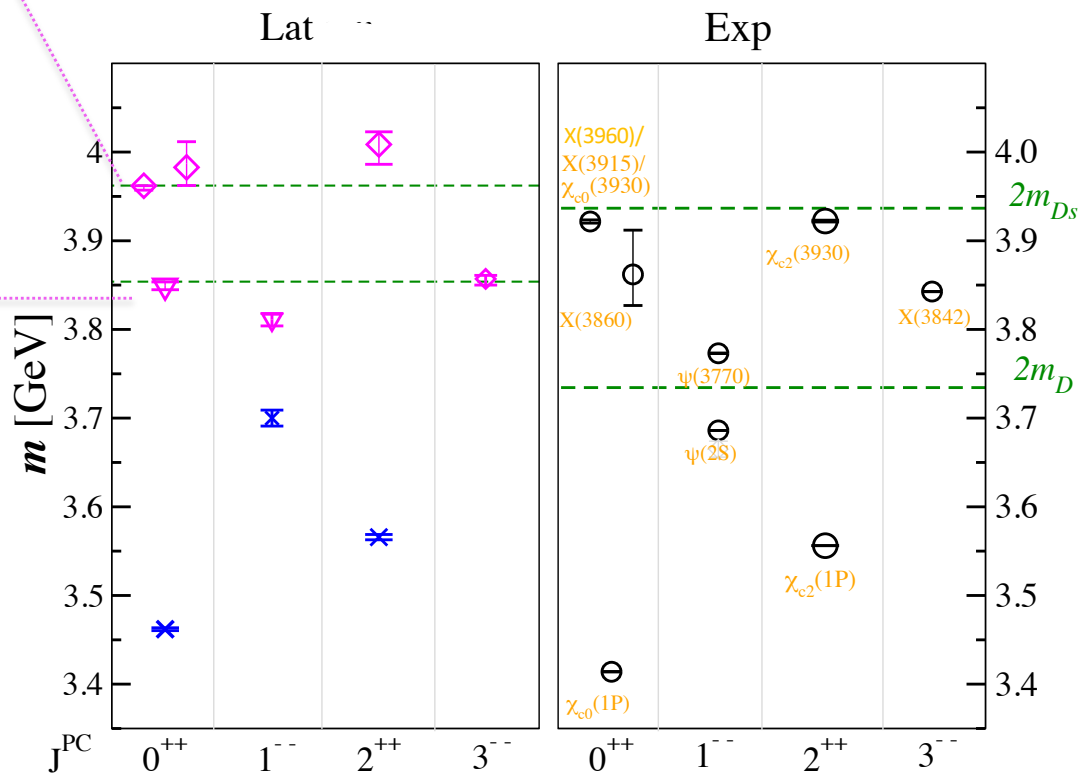




$\bar{D}D - \bar{D}_sD_s$



$m_\pi \simeq 280$ MeV



S.P. , Collins, Padmanath, Mohler, Piemonte
2011.02541 JHEP,
1905.03506 PRD

CLS ensembles

puzzling !

$$Z_b = \bar{b}b\bar{d}u$$

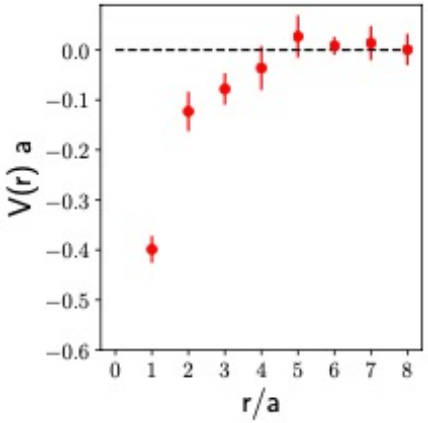
$$Z_c(3900) = \bar{c}c\bar{d}u$$

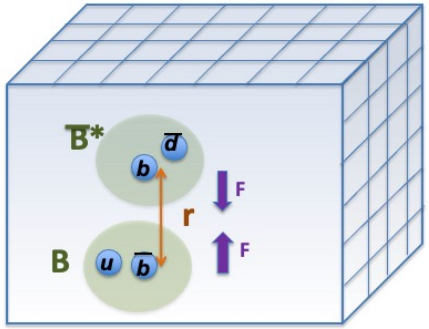
$\bar{b}b\bar{d}u \to B\bar{B}^*,\ \Upsilon\pi$

static b quarks &
Born-Oppenheimer approach

Peter, Wagner, Bicudo

SP, Bahtiyar, Petkovic, Sadl 2019, 2020,





attraction between B and B* likely responsible for Zb

$\bar{c}c\bar{d}u \to D\bar{D}^*,\ J/\psi\pi$

non-static c quarks

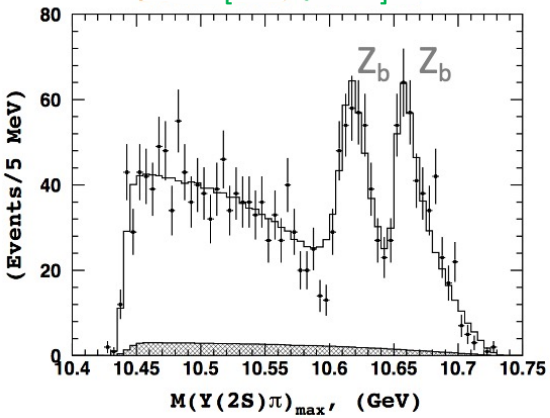
several lattice studies find $E_n \simeq E_{H1} + E_{H2}$

[Leskovec Mohler Lang SP: 1308.2097,1405.7623
HadSpec 1709.01417
Liuming Liu et al. 1907.03371, 1911.08560] $|\vec{P}| = 0;$ single volume

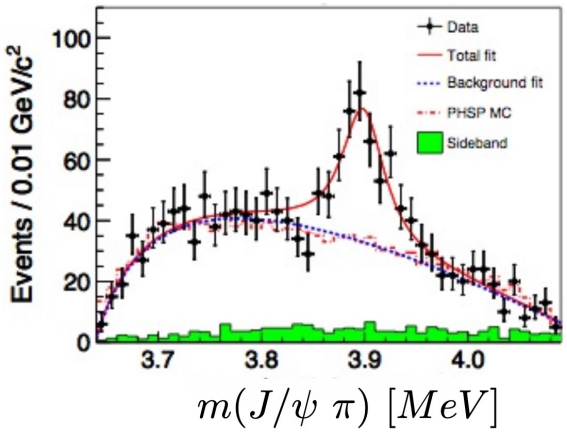
Sadl, SP, Padmanath, Collins 2212.04835: preliminary] $|\vec{P}| = 0, 1;$ two volumes

already constrains interpretations of Zc

[Belle 2011]



[BessIII, Belle 2013]



Conclusions

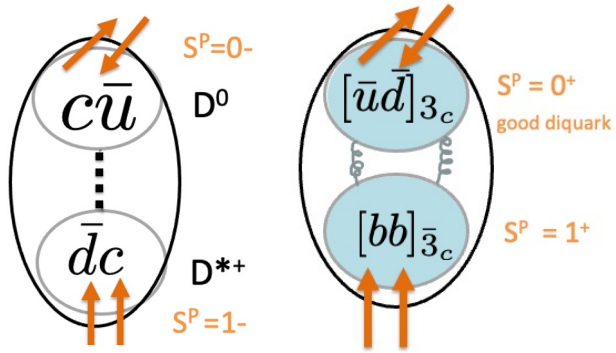
$QQ\bar{q}\bar{q}'$

- Tcc=ccud is the longest-lived exotic hadron ever discovered
- doubly heavy tetraquarks are good probes for binding mechanisms
- valuable theoretical probe: explore states as a function of quark masses
- excited to see whether more states get discovered in exp or theory

$$cc\bar{u}\bar{d},\ bb\bar{u}\bar{d},\ bc\bar{u}\bar{d},\ cc\bar{u}\bar{s},\ldots$$

: talk by M. Pflaumer

likely dominant



In general:

lots of progress, a number of challenges remain

Backup

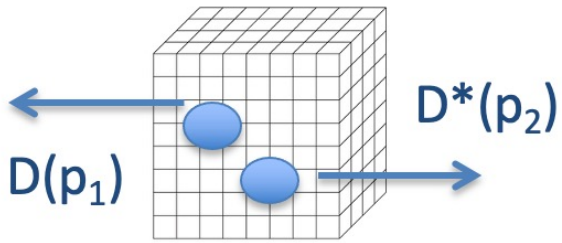
Interpolators for Tcc

Example: P=0

$J^P=1^+$ -> cubic irrep T_1^+

P=D, V=D*

$$
\begin{aligned}
O^{l=0} =& P(\{0,0,0\})V_z(\{0,0,0\}) \\
O^{l=0} =& P(\{1,0,0\})V_z(\{-1,0,0\}) + P(\{-1,0,0\})V_z(\{1,0,0\}) \\
&+P(\{0,1,0\})V_z(\{0,-1,0\}) + P(\{0,-1,0\})V_z(\{0,1,0\}) \\
&+P(\{0,0,1\})V_z(\{0,0,-1\}) + P(\{0,0,-1\})V_z(\{0,0,1\})] \\
O^{l=2} =& P(\{1,0,0\})V_z(\{-1,0,0\}) + P(\{-1,0,0\})V_z(\{1,0,0\}) \\
&+P(\{0,1,0\})V_z(\{0,-1,0\}) + P(\{0,-1,0\})V_z(\{0,1,0\}) \\
&-2[P(\{0,0,1\})V_z(\{0,0,-1\}) + P(\{0,0,-1\})V_z(\{0,0,1\})] \\
O^{l=0} =& V_{1x}[0,0,0]V_{2_y}[0,0,0] - V_{1y}[0,0,0]V_{2x}[0,0,0]
\end{aligned}
$$



$$\chi^2(\{a\}) = \sum_L \sum_{\vec{P}\Lambda n} \sum_{\vec{P}'\Lambda' n'} dE_{cm}(L, \vec{P}\Lambda n; \{a\}) \qquad (1)$$
$$\mathcal{C}^{-1}(L; \vec{P}\Lambda n; \vec{P}'\Lambda' n') dE_{cm}(L, \vec{P}'\Lambda' n'; \{a\}) \ .$$

Here

$$dE_{cm}(L, \vec{P}\Lambda n; \{a\}) = E_{cm}(L, \vec{P}\Lambda n) - E_{cm}^{an.}(L, \vec{P}\Lambda n; \{a\})$$

$$(t_l^{(J)})^{-1} = \frac{2(\tilde{K}_l^{(J)})^{-1}}{E_{cm} p^{2l}} - i\frac{2p}{E_{cm}}, \quad (\tilde{K}_l^{(J)})^{-1} = p^{2l+1} \cot \delta_l^{(J)} \qquad (5)$$

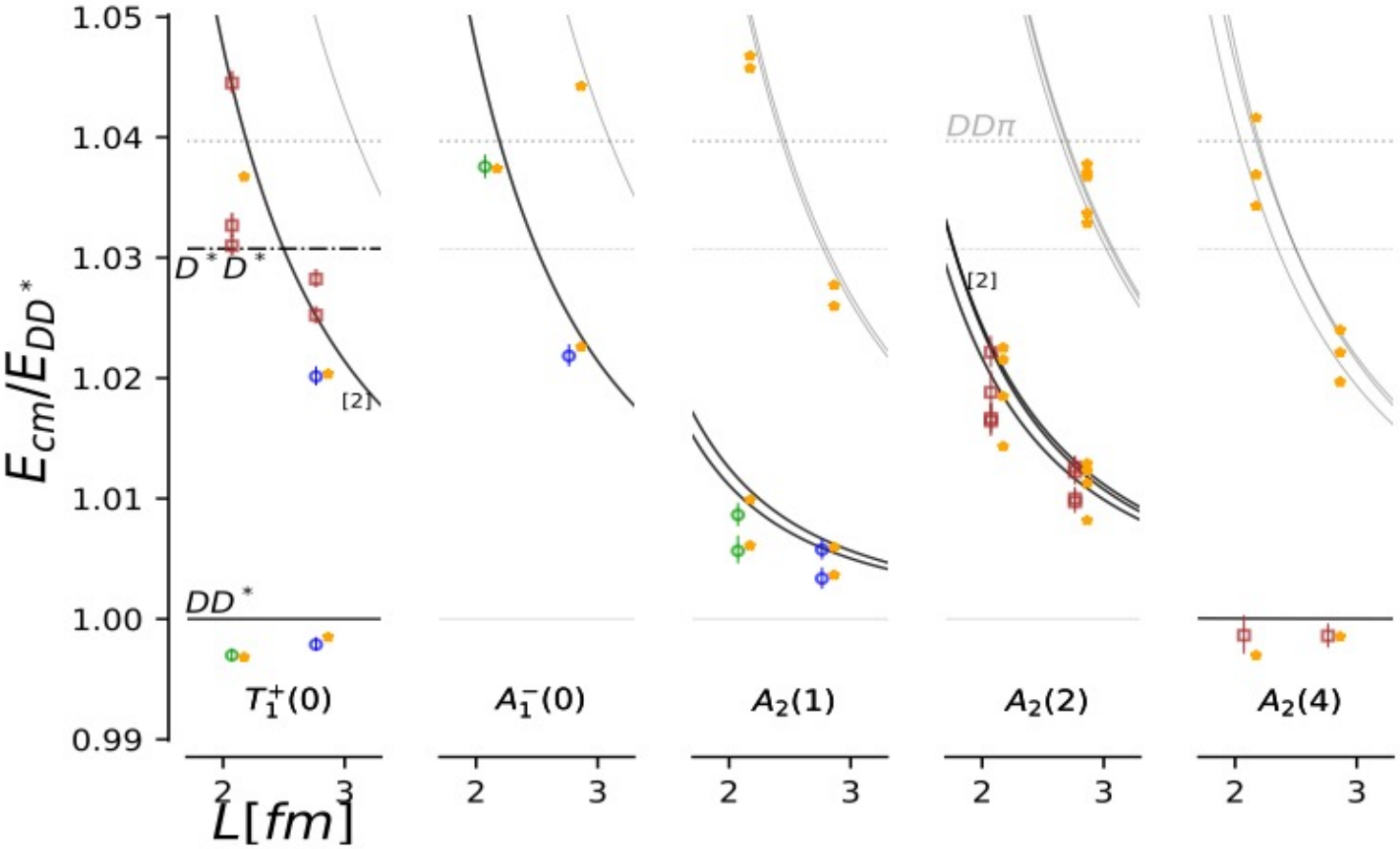
We parametrize it with the effective range expansion

$$\tilde{K}^{-1} = \begin{bmatrix} \frac{1}{a_0^{(1)}} + \frac{r_0^{(1)} p^2}{2} & 0 & 0 \\ 0 & \frac{1}{a_1^{(0)}} + \frac{r_1^{(0)} p^2}{2} & 0 \\ 0 & 0 & \frac{1}{a_1^{(2)}} \end{bmatrix} . \qquad (6)$$

Details on Tcc

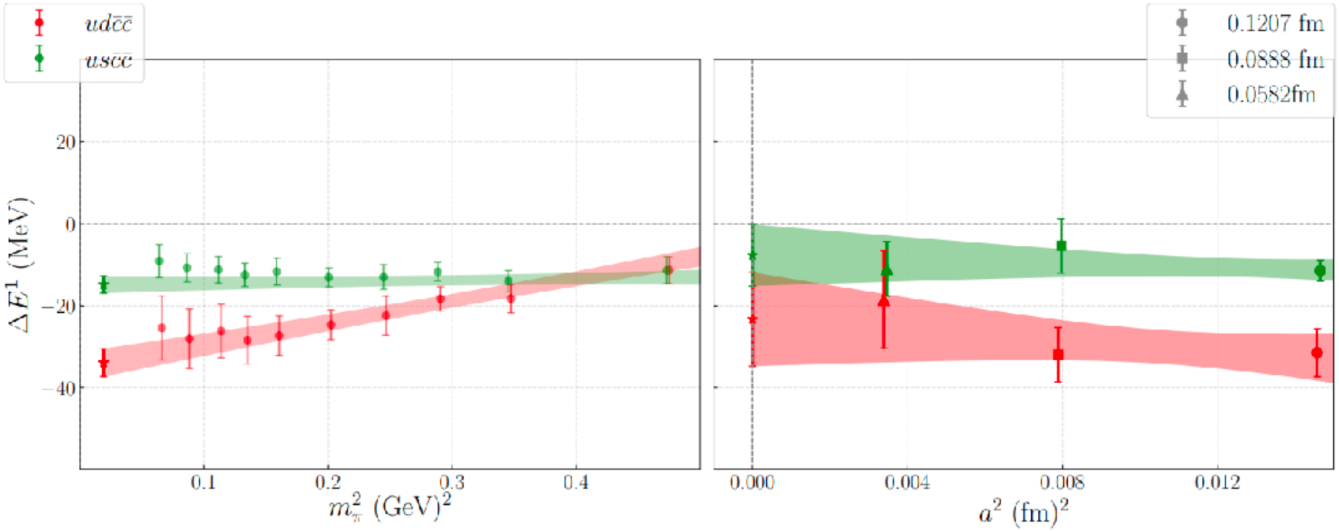
$\vec{P}$	LG	Λ^P	J^P	l	interpolators: $M_1(\vec{p}_1{}^2)M_2(\vec{p}_2{}^2)$
$(0,0,0)$	O_h	T_1^+	1^+	$0,2$	$D(0)D^*(0)$, $D(1)D^*(1)$ [2], $D^*(0)D^*(0)$
$(0,0,0)$	O_h	A_1^-	0^-	1	$D(1)D^*(1)$
$(0,0,1)\frac{2\pi}{L}$	Dic_4	A_2	$0^-,1^+,2^-$	$0,1,2$	$D(0)D^*(1)$, $D(1)D^*(0)$
$(1,1,0)\frac{2\pi}{L}$	Dic_2	A_2	$0^-,1^+,2^-,2^+$	$0,1,2$	$D(0)D^*(2)$, $D(1)D^*(1)$ [2], $D(2)D^*(1)$
$(0,0,2)\frac{2\pi}{L}$	Dic_4	A_2	$0^-,1^+,2^-$	$0,1,2$	$D(1)D^*(1)$

	m_D [MeV]	m_{D^*} [MeV]	M_{av} [MeV]	$a_{l=0}^{(J=1)}$ [fm]	$r_{l=0}^{(J=1)}$ [fm]	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(h)}$)	1927(1)	2049(2)	3103(3)	1.04(29)	$0.96(^{+0.18}_{-0.20})$	$-9.9^{+3.6}_{-7.2}$	virtual bound st.
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(l)}$)	1762(1)	1898(2)	2820(3)	0.86(0.22)	$0.92(^{+0.17}_{-0.19})$	$-15.0(^{+4.6}_{-9.3})$	virtual bound st.
exp. [2, 37]	1864.85(5)	2010.26(5)	3068.6(1)	-7.15(51)	[-11.9(16.9),0]	$-0.36(4)$	bound st.



Previous lattice QCD study of T_{cc} channel

Junnarkar, Mathur, Padmanath, PRD 99, 034507 (2019), 1810.12285



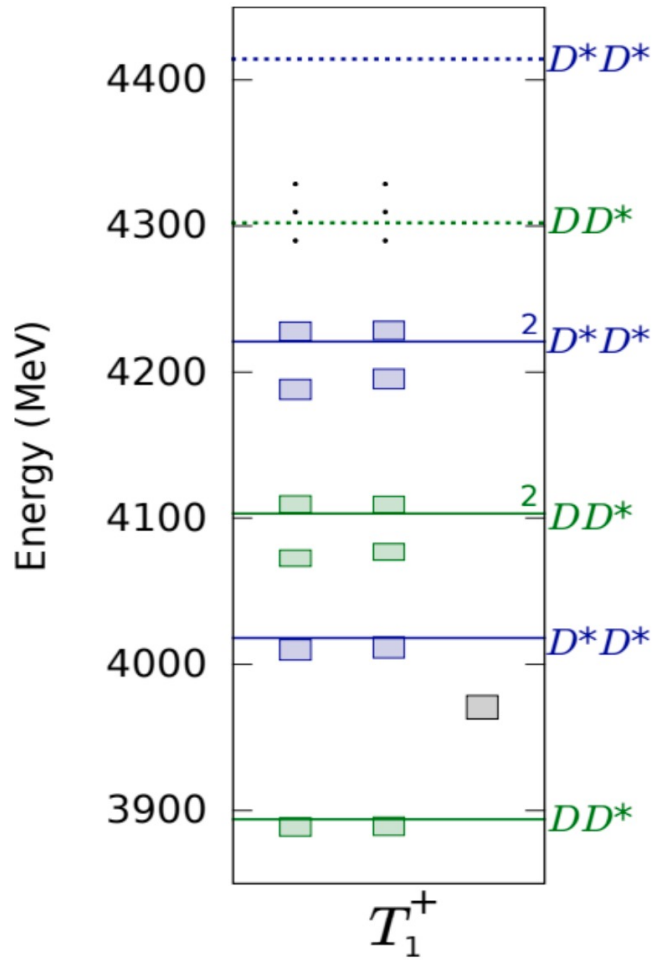
lowest finite-volume eigen-energy for P=0, $J^P=1^+$, I=0

- Study performed on LQCD ensembles with different lattice spacings.
  Single volume and only rest frame finite-volume irreps considered.
- Including a meson-meson and diquark-antidiquark interpolator.
  Diquark-antidiquark interpolators do not influence the low energy spectrum.
- The ground state energy subjected to chiral and continuum extrapolations.
- A finite-volume energy level 23(11) MeV below DD^* threshold.
  No rigorous scattering analysis and no pole structure determined.

Previous lattice QCD study of T_{cc} channel

Hadron Spectrum, JHEP 11, 033 (2017), 1709.01417

finite-volume
eigen-energies for
P=0, $J^P=1^+$, I=0

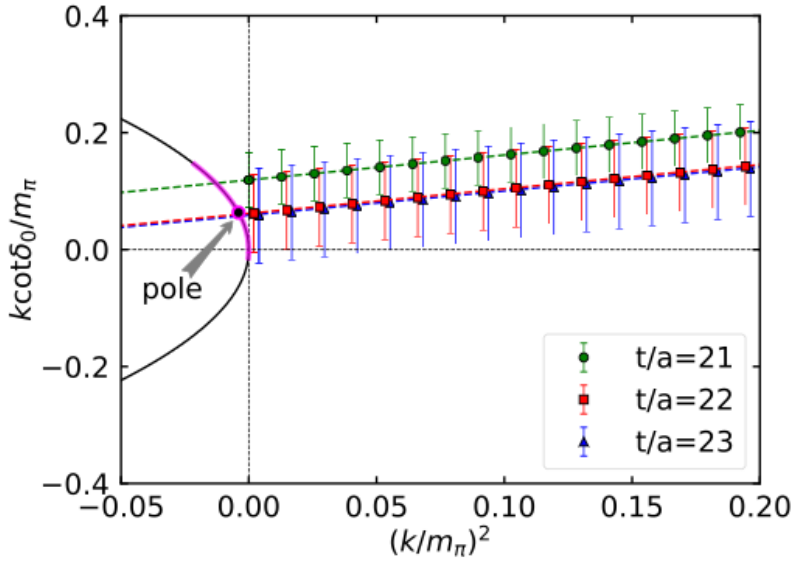


- Single volume rest frame study on a relatively coarse lattice ($a_s \sim 0.12$ fm).
- Large basis of meson-meson and diquark-antidiquark interpolators.
- Diquark-antidiquark interpolators do not influence the low energy spectrum.
- No statistically significant energy shifts observed near DD^* threshold.
  $\Rightarrow$ No scattering amplitude extraction.

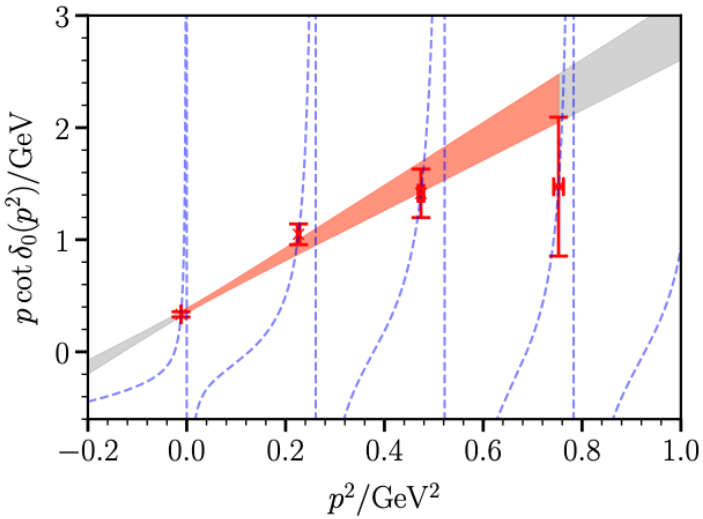
Intermezzo: p cot δ_0 in Tcc chanel from available lattice simulations (other two simulations will be detailed later in the talk)

eff. range approx.: $p \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2}\, r_0\, p^2$

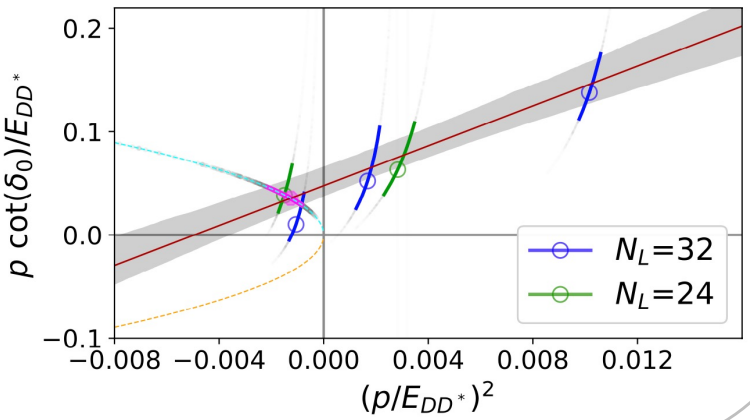
HALQCD, 2302.04505, $m_\pi \approx 146$ MeV



CLQCD 2206.06185, $m_\pi \approx 348$ MeV, PLB

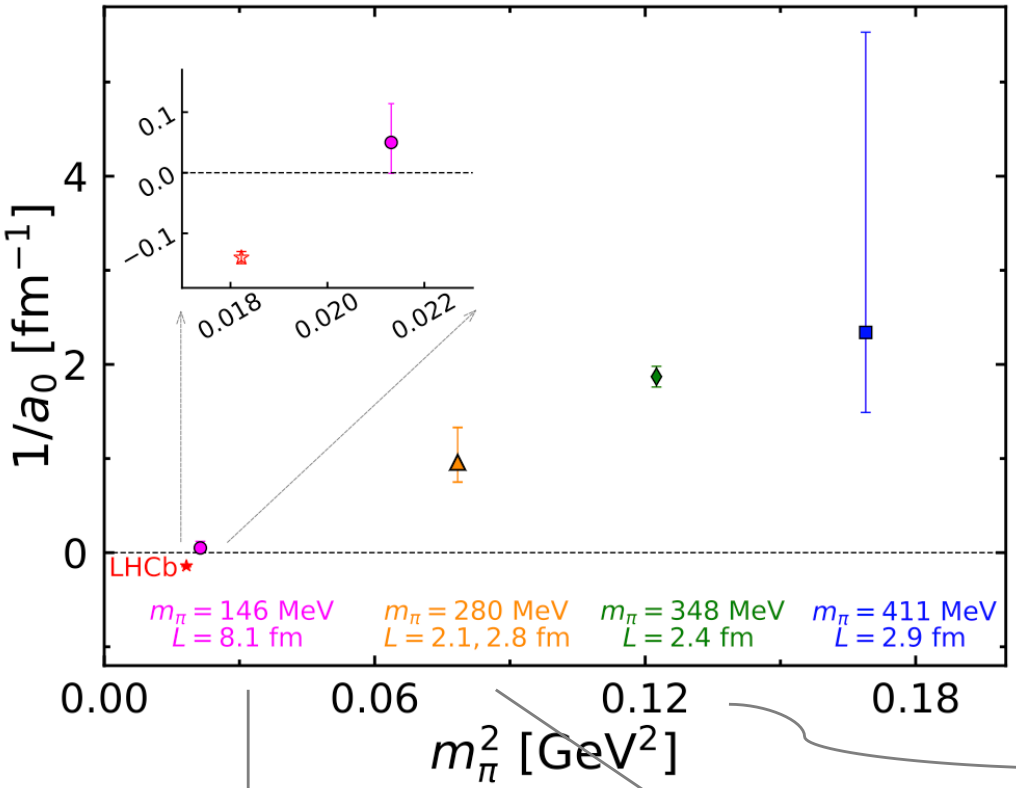


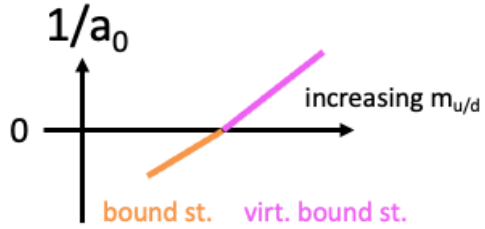
Padmanath, S.P.: 2202.10110, $m_\pi \approx 280$ MeV *PRL*



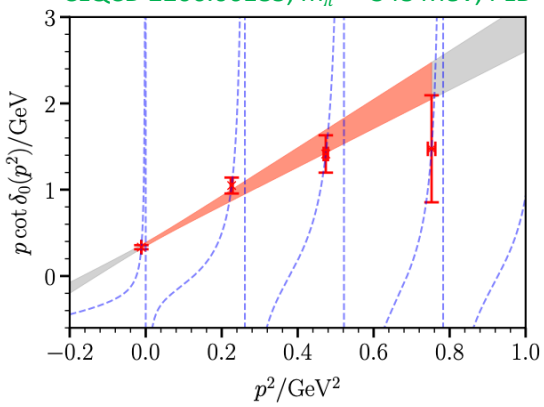
Tcc from lattice: dependence of 1/a0 on $m_{u/d}$

summary from 2302.04505

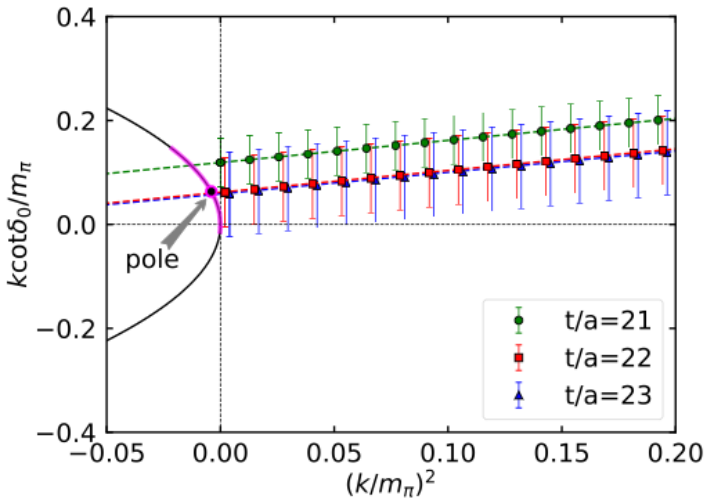




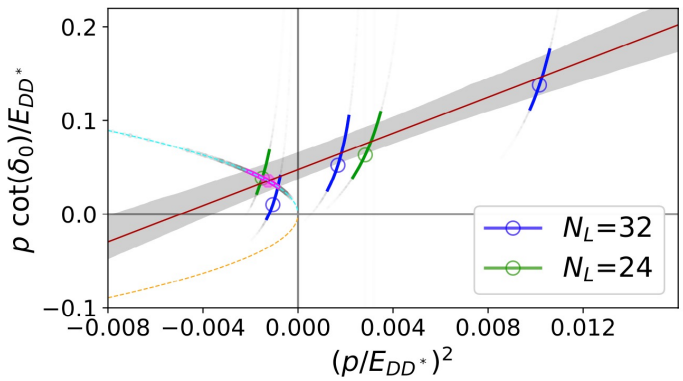
CLQCD 2206.06185, $m_\pi \approx$ 348 MeV, PLB



HALQCD, 2302.04505, $m_\pi \approx$146 MeV



Padmanath, S.P.: 2202.10110, $m_\pi \approx$ 280 MeV *PRL*



HALQCD study of Tcc

Lyu, Aoki et al, 2302.04505

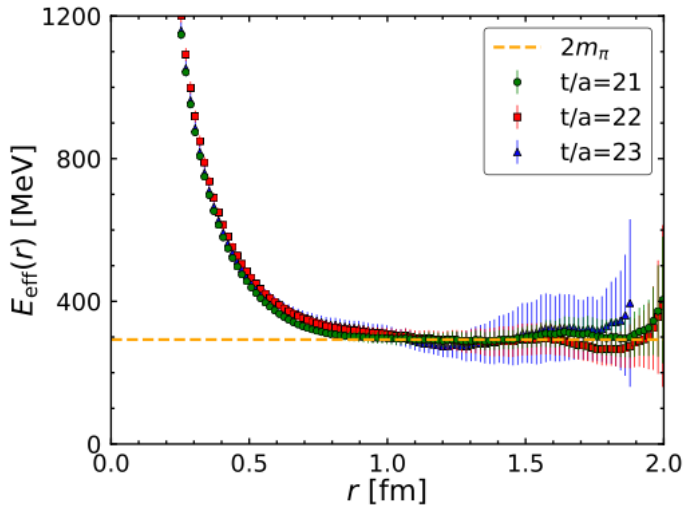
$$R(\boldsymbol{r},t) = \sum_{\boldsymbol{x}} \langle 0|D^*(\boldsymbol{x}+\boldsymbol{r},t)D(\boldsymbol{x},t)\overline{\mathcal{J}}(0)|0\rangle / e^{-(m_{D^*}+m_D)t}$$

$$\left[\frac{1+3\delta^2}{8\mu}\partial_t^2 - \partial_t - H_0 + O(\delta^2\partial_t^3)\right] R(\boldsymbol{r},t) = \int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')R(\boldsymbol{r}',t).$$

$$V(r) = R^{-1}(\boldsymbol{r},t)\left[\frac{1+3\delta^2}{8\mu}\partial_t^2 - \partial_t - H_0\right] R(\boldsymbol{r},t)$$

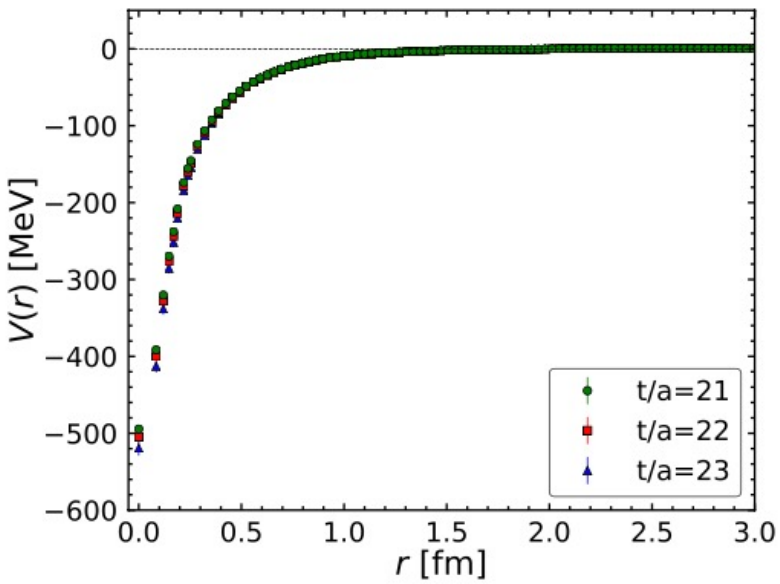
V(r) $\sim \frac{e^{-2m_\pi r}}{r^2}$ r > 1 fm

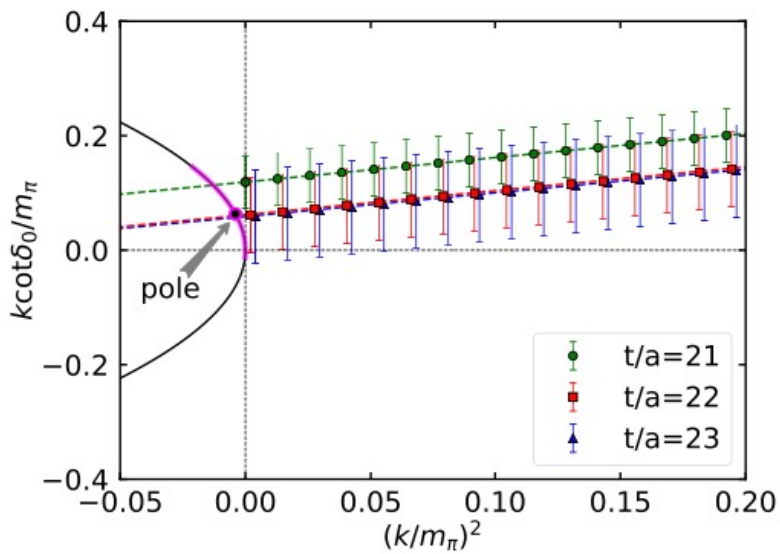
$$E_{\rm eff}(r) = -\frac{\ln[V(r)r^2/a_3]}{r}$$



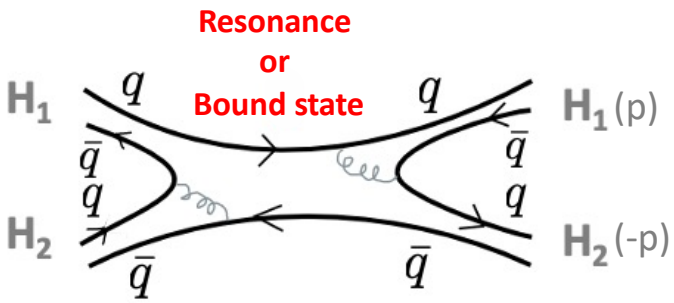
$$V^B_{\rm fit}(r;m_\pi) = \sum_i a_i e^{-(r/b_i)^2} + a_3(1-e^{-(r/b_3)^2})^n V_\pi^n$$

parameter set, $(a_1, a_2) = (-284(36), -201(60))$ in MeV, $a_3 = -45(12)$ MeV $\cdot$ fm^2, and $(b_1, b_2, b_3) = (0.15(2), 0.32(12), 0.49(24))$ in fm. Also, we find that

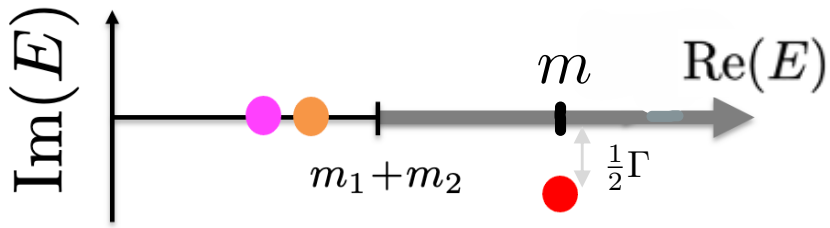




Extract resonances and (virtual) bound states from H_1 H_2 scattering



scattering matrix T(E)

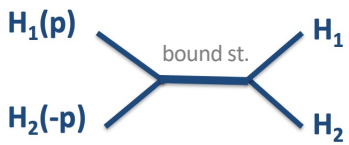


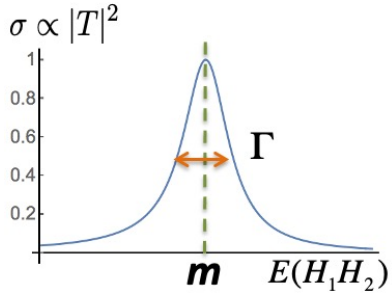
Virtual bound st. | Bound st. | Resonance

p = - i |p| | p = i |p|

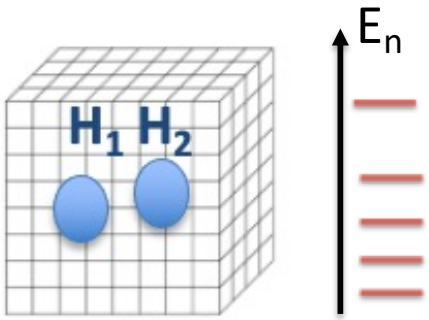
$$T(E) \propto \frac{1}{E^2 - m^2}$$

$$T(E) \propto \frac{1}{E^2 - m^2 + iE\Gamma}$$





Scattering matrix T(E) from lattice QCD



periodic b.c.

real E → for real E → for complex E

$$E \rightarrow T(E) \rightarrow T(E^c)$$

analytic relation: Luscher 1991

analytic contin. to complex E

simple argument: next slide

Simplest Example: scattering in square-well potential in QM

$\delta = \arctan[\tan(qR)\frac{p}{q}] - pR$

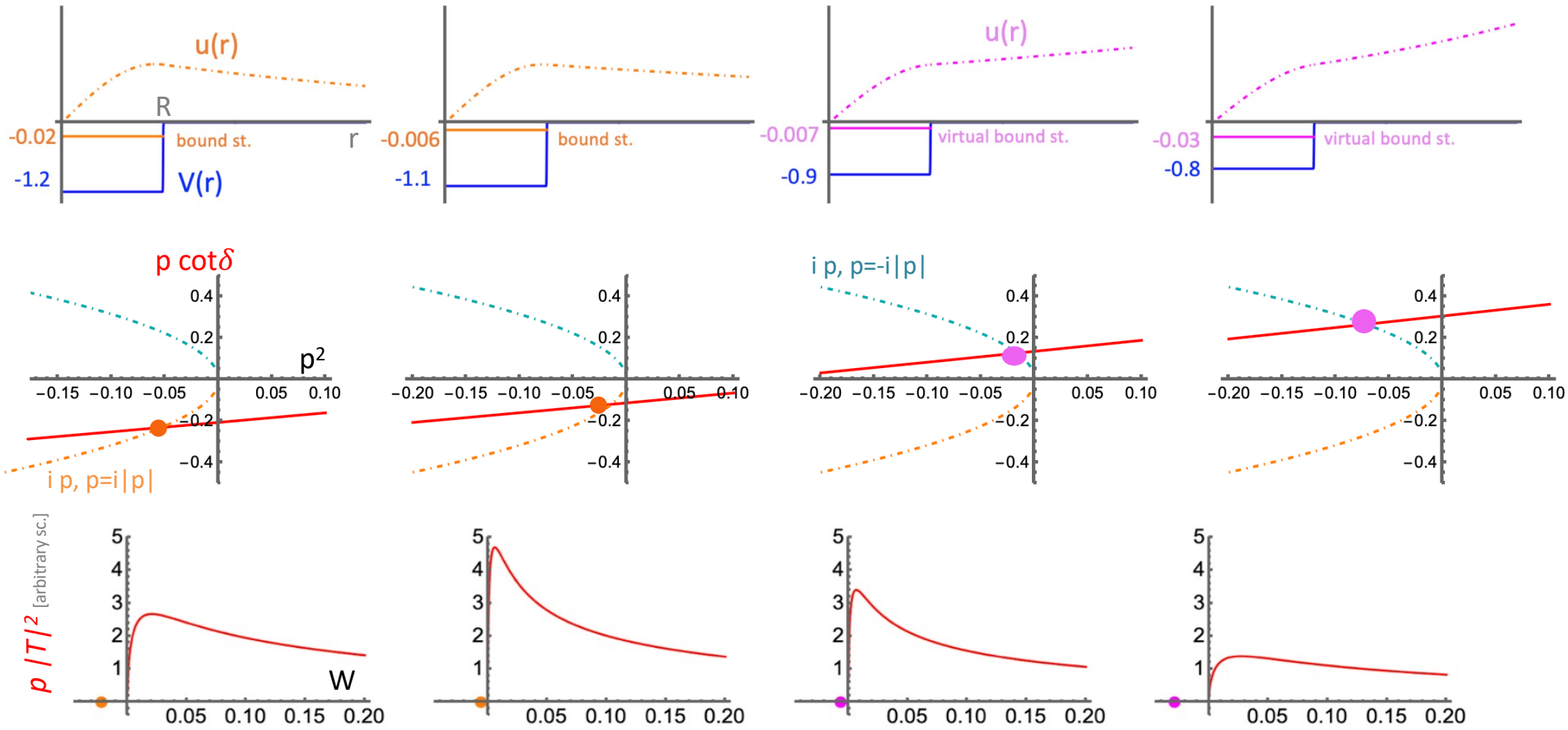
partial wave l=0

$T \propto (p\cot\delta - ip)^{-1}$

$u(r) = A\sin(qr)$ $\quad u(r) = B\sin(pr+\delta)$

p=i|p| $\quad e^{ipr} = e^{-|p|r}$

p=-i|p| $\quad e^{ipr} = e^{|p|r}$



increasing $m_{u/d}$, decreasing attraction V_0 (or decreasing R)

Charmonium(like) resonances and bound states

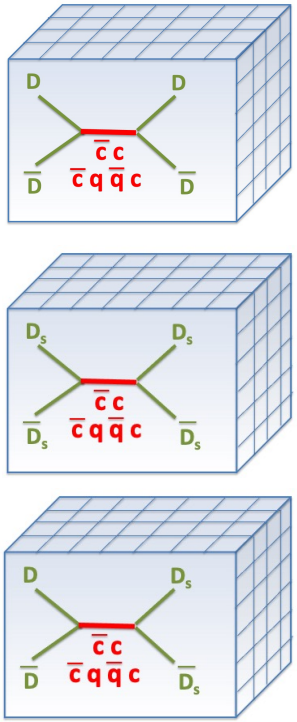
$\bar{c}c$, $\bar{c}q\bar{q}c$ q=u,d,s $I=0$

$$T_{ij}(E_{cm}) \sim \frac{c_i\ c_j}{E_{cm}^2 - m^2}$$

near the pole

lat: $\frac{|c^2_{D\bar{D}}|}{|c^2_{D_s\bar{D}_s}|} = 0.02\ ^{+0.02}_{-0.01}$

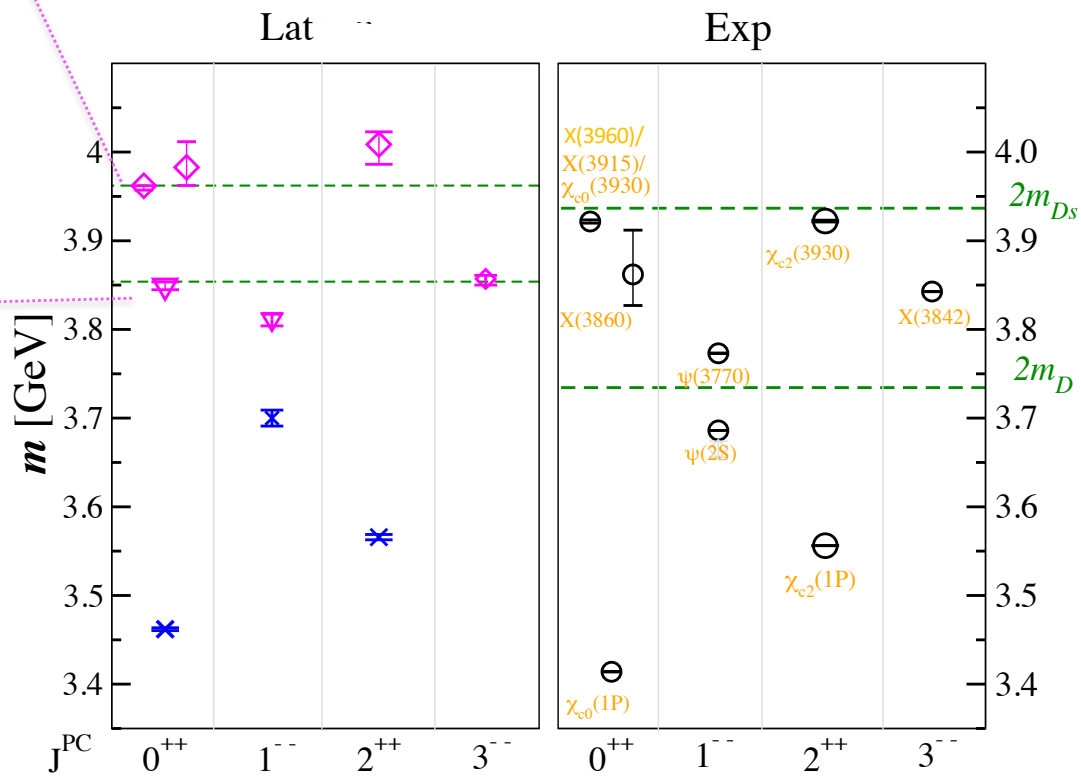
$\bar{c}s\bar{s}c$



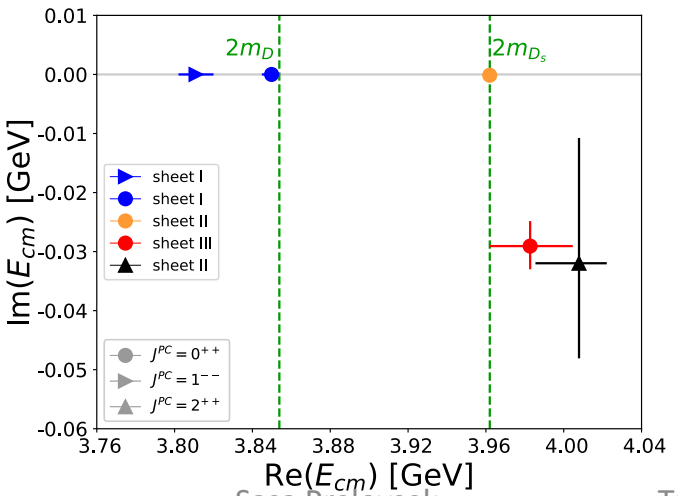
S.P. , Collins, Padmanath, Mohler, Piemonte
2011.02541 JHEP,
1905.03506 PRD
2111.02934

q=u,d $\bar{c}q\bar{q}c$

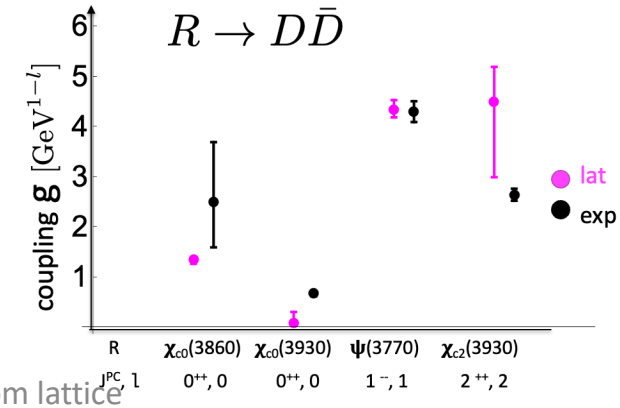
$m_\pi \simeq 280$ MeV



+ expected conventional charmonia



$$\Gamma \equiv g^2\ \frac{p_D^{2l+1}}{m^2}$$



$Z_c(3900)$: puzzling on the lattice

$\bar{c}c\bar{d}u$

challenging: $\bar{c}c\bar{d}u \quad \rightarrow \quad (\bar{c}u)\ (\bar{d}c) = \bar{D}^* D$

$\rightarrow \quad (\bar{c}c)\ (\bar{u}d) = J/\psi\ \pi, \ldots$

several lattice studies find

almost non-interacting finite-volume energies

symbols lines

$$E_n \simeq E_{H1} + E_{H2}$$

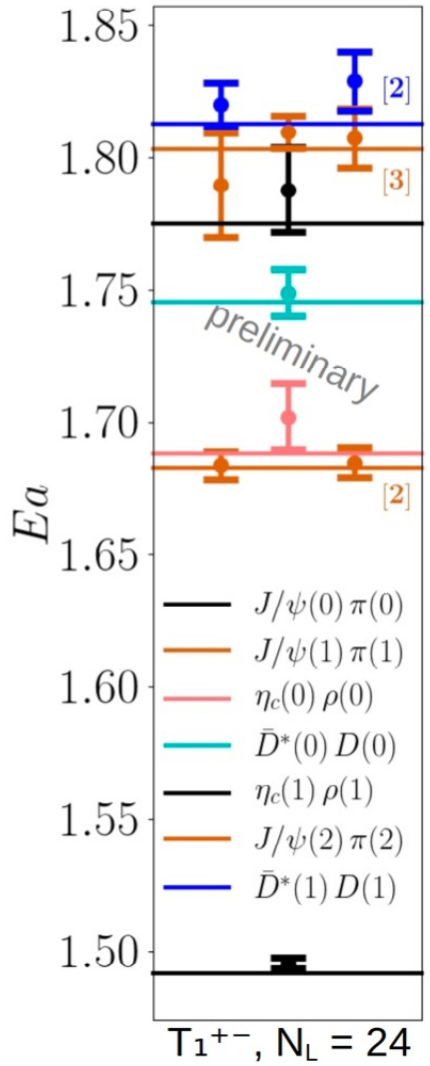
- Leskovec Mohler Lang SP: 1308.2097,1405.7623
- HadSpec 1709.01417
- Liuming Liu et al. 1907.03371, 1911.08560

$|\vec{P}| = 0;$ single volume

- Sadl, SP, Padmanath, Collins 2212.04835: preliminary

$|\vec{P}| = 0, 1;$ two volumes

Sadl, SP et al

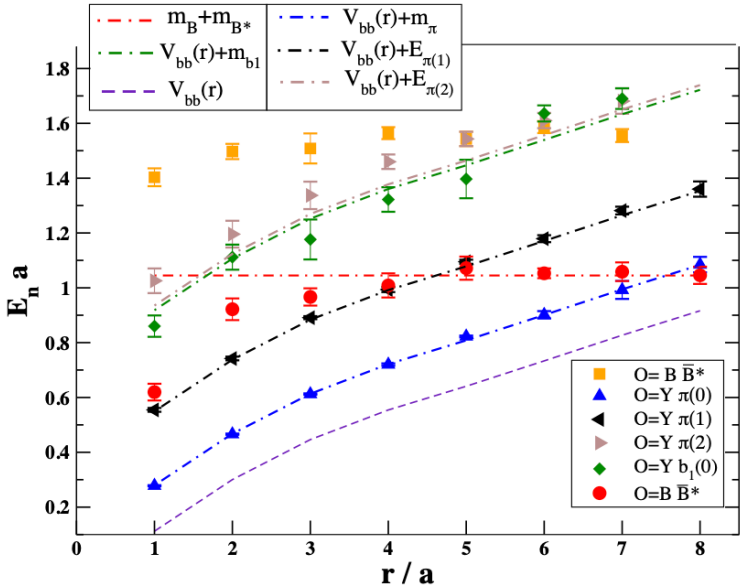


This may already be constaining interpretations of Zc(3900)

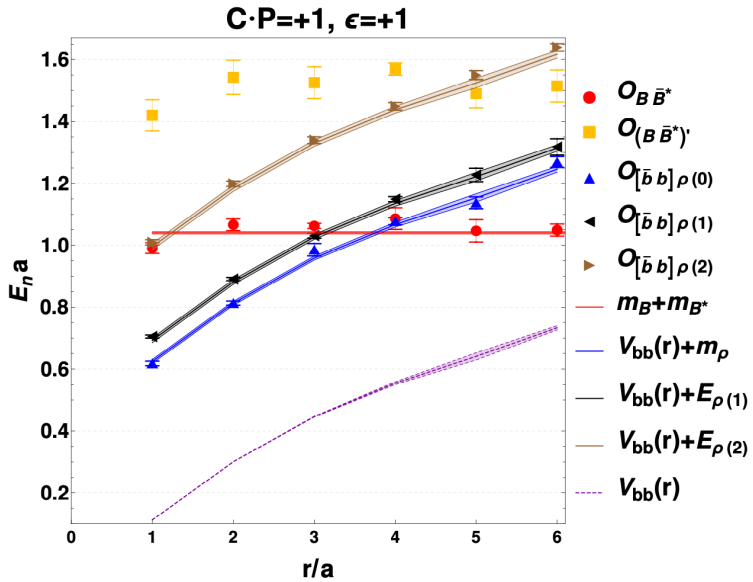
Is it suggesting significant coupled-channel effect?

Supported by HALQCD method Ikeda et al. PRL (2016)

Zb channel in the Born Oppenheimer approach



SP, Bahtiyar, Petkovic, 1912.02656



M. Sadl and SP, 2109.08560