



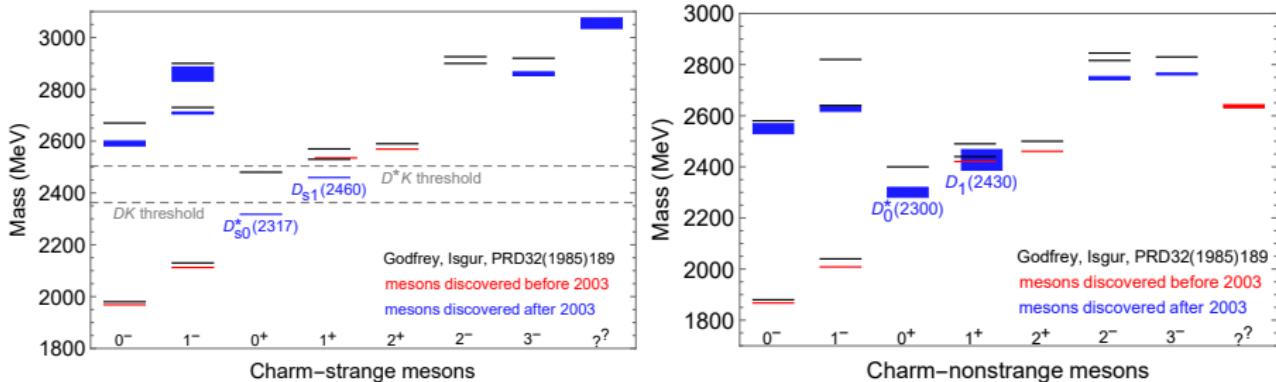
Low-lying exotic mesons in the charm and bottom sectors

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Puzzles of positive-parity charmed mesons



- **Puzzle 1:** Why are $D_{s0}^*(2317)$ and $D_{s1}(2460)$ so light?
- **Puzzle 2:** Why $\underbrace{M_{D_{s1}(2460)} - M_{D_{s0}^*(2317)}}_{(141.8 \pm 0.8) \text{ MeV}} \simeq \underbrace{M_{D^{*\pm}} - M_{D^{\pm}}}_{(140.67 \pm 0.08) \text{ MeV}}$?
- **Puzzle 3:** Why $M_{D_0^*(2300)} \gtrsim M_{D_{s0}^*(2317)}$ and $M_{D_1(2430)} \sim M_{D_{s1}(2460)}$?
 $D_0^*(2300)$ was denoted as $D_0^*(2400)$ up to PDG2018

Solution to the puzzles in the hadronic molecular model

- Solution to Puzzle 1: not quark model $c\bar{s}$ mesons:

$$D_{s0}^*(2317) [\simeq DK(I=0)], D_{s1}(2460) [\simeq D^*K(I=0)]$$

Barnes, Close, Lipkin (2003); van Beveren, Rupp (2003); Kolomeitsev, Lutz (2004); FKG et al. (2006);
FKG, Hanhart, Meißner (2009); ...

- Solution to Puzzle 2: HQSS \Rightarrow similar binding energies

$$M_D + M_K - M_{D_{s0}^*} \simeq 45 \text{ MeV}$$

$$M_{D_{s1}(2460)} - M_{D_{s0}^*(2317)} \simeq M_{D^*} - M_D \text{ is natural}$$

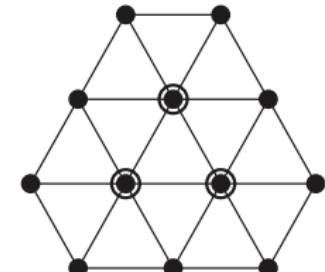
- Solution to Puzzle 3: SU(3) irreps: $\bar{\mathbf{3}} \otimes \mathbf{8} = \bar{\mathbf{15}} \oplus \mathbf{6} \oplus \bar{\mathbf{3}}$

$$S = 2$$

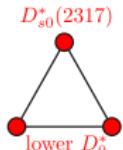
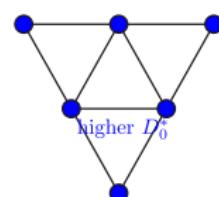
$$S = 1$$

$$S = 0$$

$$S = -1$$



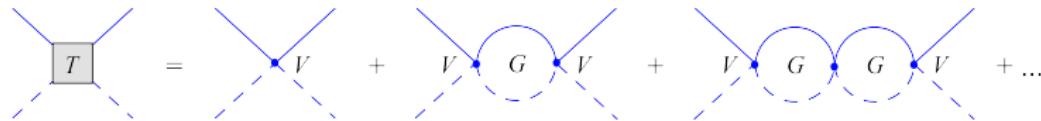
Albaladejo et al., PLB767(2017)465



two D_0^* , the SU(3) partner of $D_{s0}^*(2317)$ is the lower one

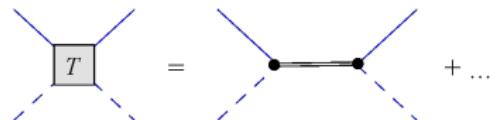
Unitarized chiral perturbation theory

- *S*-wave interactions between (D, D_s) and pseudo-Goldstone bosons (π, K, η)



- not far from the thresholds \Rightarrow chiral EFT for matter fields
- D_{s0}^*/D_0^* should appear as poles in scattering amplitudes:

$$D_{s0}^* : DK, D_s\eta; \quad D_0^* : D\pi, D\eta, D_s\bar{K}$$



\Rightarrow needs a nonperturbative treatment: ChPT + unitarization (**UChPT**)

Truong (1988); Oller, Oset (1997); Oller, Oset, Peláez (1998); Oller, Meiñner (2001); ...

$$T^{-1}(s) = V^{-1}(s) - G(s)$$

$V(s)$: from SU(3) chiral Lagrangian, 6 LECs up to NLO (fixed w/ lattice inputs)

L. Liu, Orginos, FKG, Hanhart, Meiñner, PRD86(2013)014508

LO (universal from χ sym.): **15**: repulsive; **6**: attractive; **3**: most attractive

- Heavy quark spin + flavor symmetry: more predictions, heavy-strange

meson	J^P	prediction (MeV)	PDG2022 (MeV)	lattice (MeV)
D_{s0}^*	0^+	2315^{+18}_{-28}	2317.8 ± 0.5	
D_{s1}	1^+	2456^{+15}_{-21}	2459.5 ± 0.6	
B_{s0}^*	0^+	5720^{+16}_{-23}	—	5711 ± 23 [1]
B_{s1}	1^+	5772^{+15}_{-21}	—	5750 ± 25 [1]

- Heavy-nonstrange, two $I = 1/2$ states ($M, \Gamma/2$):

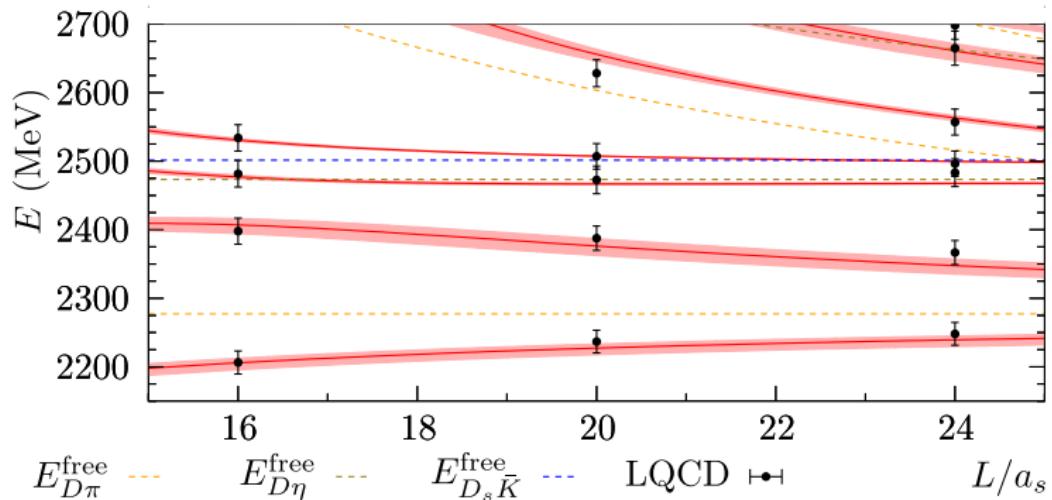
	Lower (MeV)	Higher (MeV)	PDG2022 (MeV)
D_0^*	$(2105^{+6}_{-8}, 102^{+10}_{-11})$	$(2451^{+36}_{-26}, 134^{+7}_{-8})$	$(2343 \pm 10, 115 \pm 8)$
D_1	$(2247^{+5}_{-6}, 107^{+11}_{-10})$	$(2555^{+47}_{-30}, 203^{+8}_{-9})$	$(2412 \pm 9, 157 \pm 15)$
B_0^*	$(5535^{+9}_{-11}, 113^{+15}_{-17})$	$(5852^{+16}_{-19}, 36 \pm 5)$	—
B_1	$(5584^{+9}_{-11}, 119^{+14}_{-17})$	$(5912^{+15}_{-18}, 42^{+5})$	—

[1] Lang, Mohler, Prelovsek, Woloshyn, PLB750(2015)17

Agreements with lattice results (1)

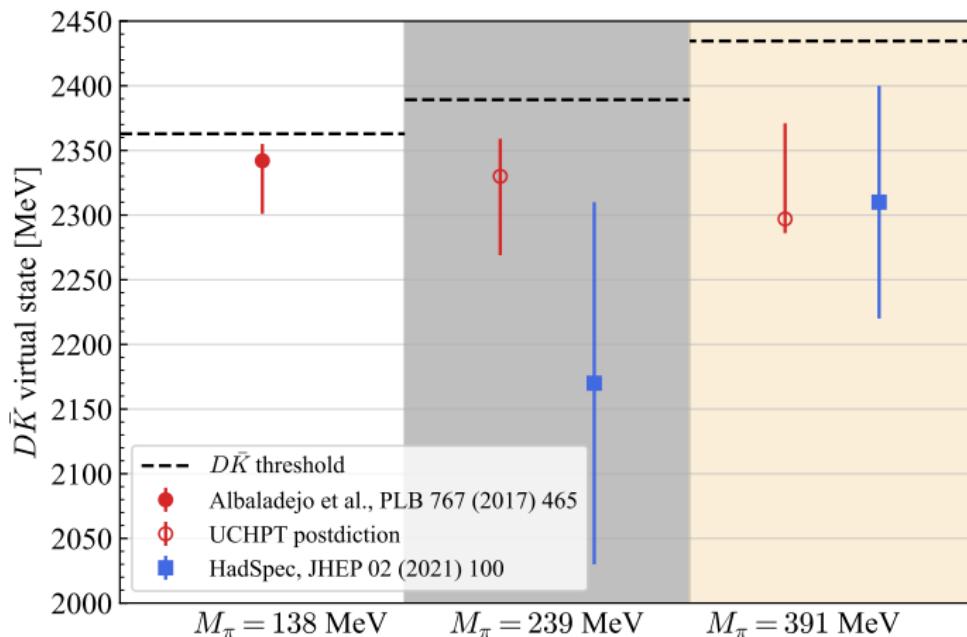
- Postdicted $I = 1/2 D\pi, D\eta, D_s \bar{K}$ finite volume energy levels in rest frame v.s. lattice QCD results (more later) in [G. Moir *et al.* [HadSpec], JHEP1610(2016)011]
NOT a fit!

M. Albaladejo, P. Fernandez-Soler, FKG, J. Nieves, PLB767(2017)465



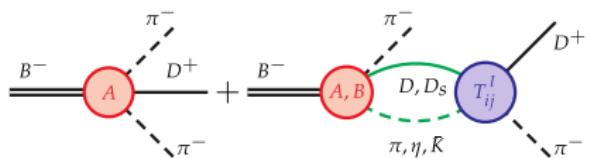
Agreements with lattice results (2)

- Predicted $(S, I) = (-1, 0)$ $D\bar{K}$ virtual state got support from lattice



Agreements with exp. (1)

M.-L. Du et al., PRD98(2018)094018



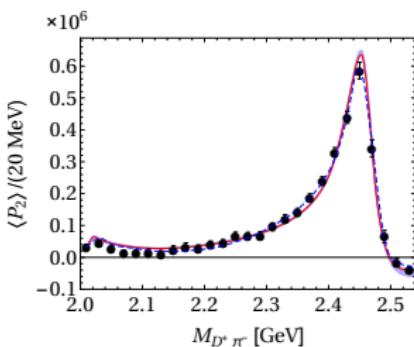
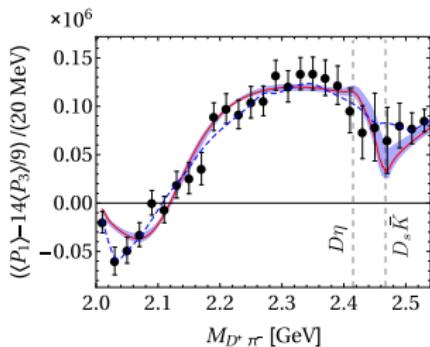
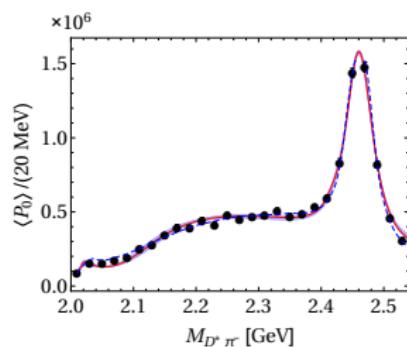
- SU(3) symmetry Savage, Wise (1989)
- *S*-wave: FSI, two new parameters
- *P, D*-wave: BWs from the LHCb fit

Angular moments:

$$\langle P_0 \rangle \propto |\mathcal{A}_0|^2 + |\mathcal{A}_1|^2 + |\mathcal{A}_2|^2, \quad \langle P_2 \rangle \propto \frac{2}{5}|\mathcal{A}_1|^2 + \frac{2}{7}|\mathcal{A}_2|^2 + \frac{2}{\sqrt{5}}|\mathcal{A}_0||\mathcal{A}_2| \cos(\delta_2 - \delta_0),$$

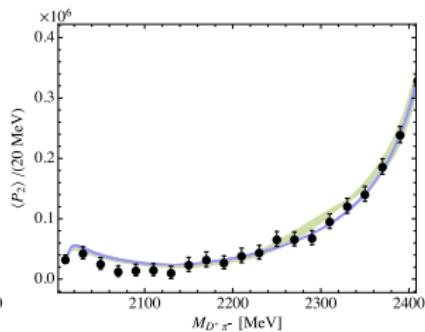
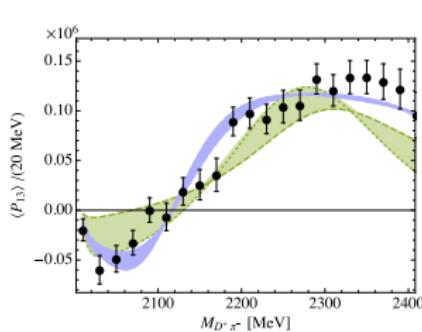
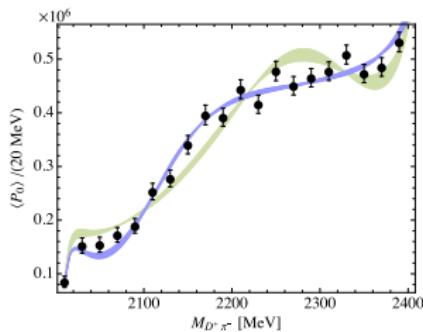
$$\langle P_{13} \rangle \equiv \langle P_1 \rangle - \frac{14}{9} \langle P_3 \rangle \propto \frac{2}{\sqrt{3}} |\mathcal{A}_0| |\mathcal{A}_1| \cos(\delta_1 - \delta_0)$$

Data: LHCb, PRD94(2016)072001



- Fast variation in [2.4, 2.5] GeV in $\langle P_{13} \rangle$: cusps at $D\eta$ and $D_s\bar{K}$ thresholds

- Fits with the Khuri-Treiman equation taking into account three-body unitarity:
 - ★ using S -wave $D\pi$ scattering phase from UCHPT ($\chi^2/\text{d.o.f.} = 1.2$)
 - ★ from Breit-Wigner for $D_0^*(2300)$ w/ PDG parameters ($\chi^2/\text{d.o.f.} = 2.0$)

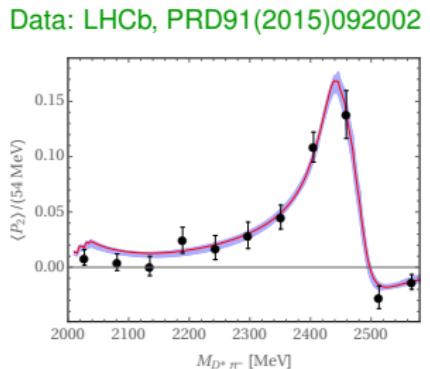
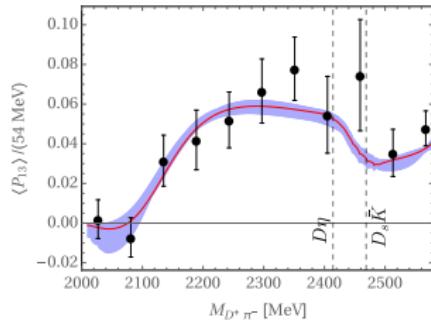
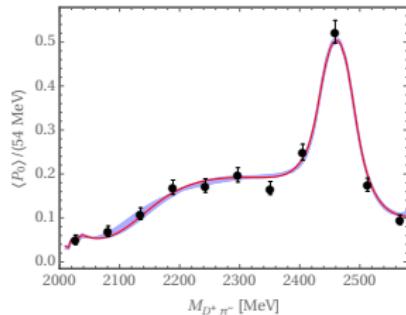


- The LHCb data are well described with UCHPT amplitude with two D_0^* states; the lower one has a mass about 2.1 GeV

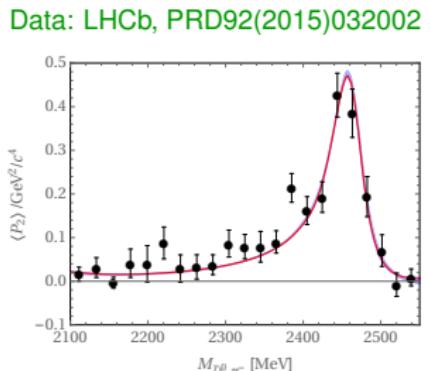
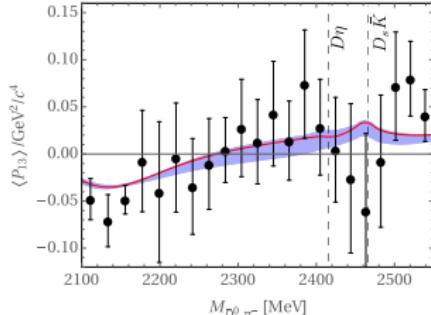
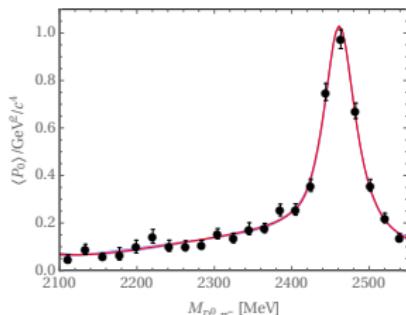
Agreements with exp. (3)

M.-L. Du, FKG, U.-G. Meißner, PRD99(2019)114002

More B decays: $B^- \rightarrow D^+ \pi^- K^-$



$B^0 \rightarrow \bar{D}^0 \pi^- \pi^+$



and also $B^0 \rightarrow \bar{D}^0 \pi^- K^+$, $B^- \rightarrow D^+ \pi^- K^-$, $B_s^0 \rightarrow \bar{D}^0 K^- \pi^+$

$D_0^*(2300)$

$$I(J^P) = 1/2(0^+)$$

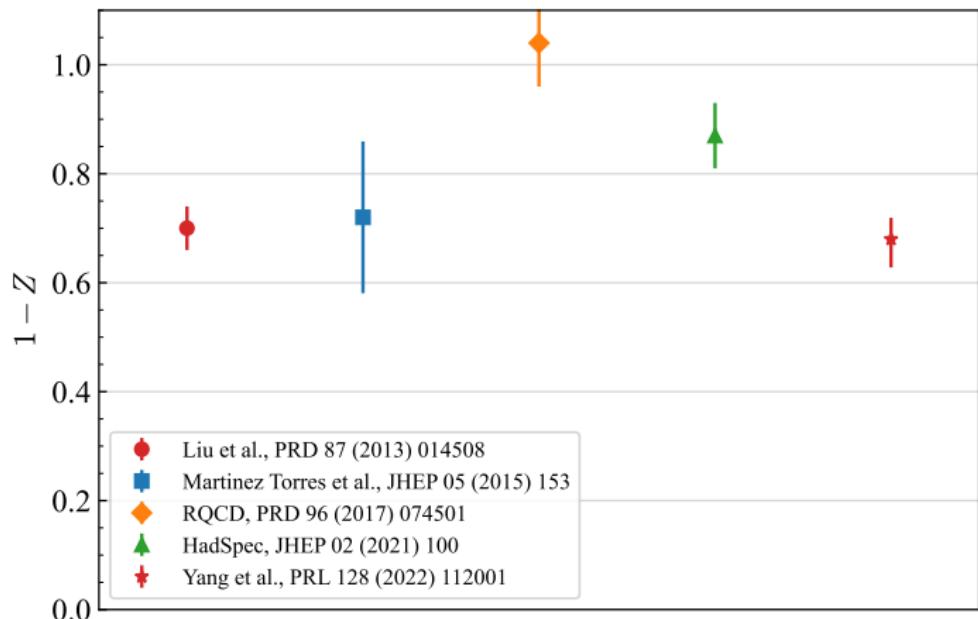
was $D_0^*(2400)$

There is a strong evidence that recent data on $B \rightarrow D\pi\pi$ ([AAIJ 2015Y](#), [AAIJ 2016AH](#)) and $B \rightarrow D\pi K$ ([AAIJ 2014BH](#), [AAIJ 2015V](#), [AAIJ 2015X](#)) call for two poles in the scalar $I = 1/2 \pi D$ amplitude in this mass range. The data are consistent with a lower pole at $(2105^{+6}_{-8}) - i(102^{+10}_{-11})$ MeV and a higher pole at $(2451^{+35}_{-26}) - i(134^{+7}_{-8})$ MeV ([DU 2018A](#), [DU 2019](#), [DU 2021](#)). For details see review on "Heavy Non- $q\bar{q}$ Mesons."

Independent analyses of data from other groups are called for!!

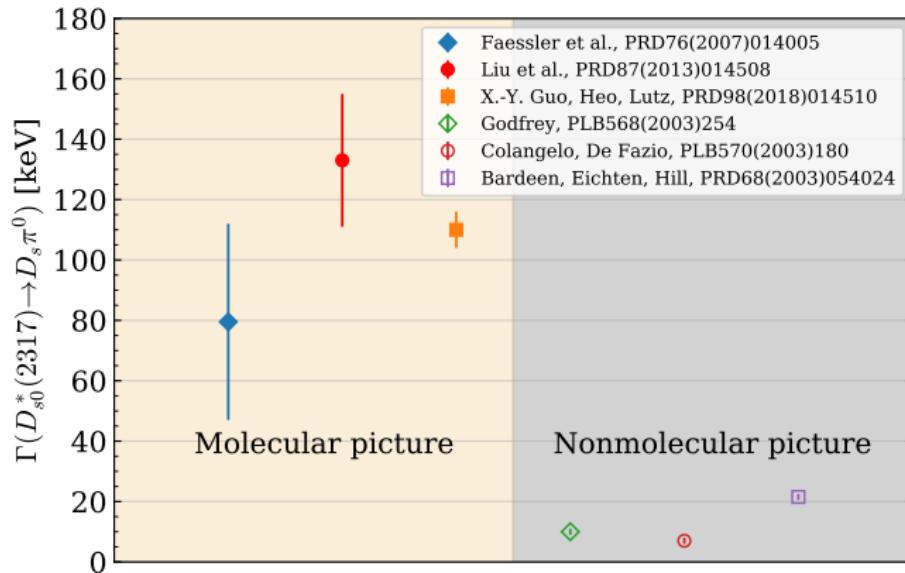
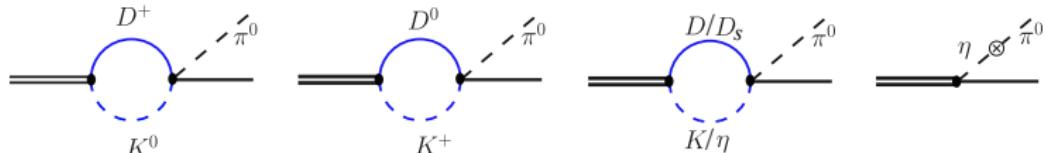
D_{s0}^* as DK molecule from lattice QCD

- $1 - Z$: compositeness (DK component) for $D_{s0}^*(2317)$ from (in)direct lattice QCD calculations: DK as the main component, \Rightarrow hadronic molecule
 - reviews of compositeness: Hyodo, IJMPA28(2013)1330045; FKG, Hanhart, Mei β nner, Wang, Zhao, Zou, RMP90(2018)015004



Decays of $D_{s0}^*(2317)$

- Dominant channel: $D_{s0}^*(2317) \rightarrow D_s\pi^0$
- Measurement of total width planned at PANDA



Hadronic decays of $D_{s1}(2460)$

- Experimental measurement:

$$\frac{\Gamma(D_{s1}(2460)^+ \rightarrow D_s^+ \pi^+ \pi^-)}{\Gamma(D_{s1}(2460)^+ \rightarrow D_s^{*+} \pi^0)} = \begin{cases} 0.14 \pm 0.04 \pm 0.02 & \text{Belle, PRL 92(2004)012002} \\ 0.09 \pm 0.02 & \text{PDG fit} \end{cases}$$

- Isospin breaking $D_{s1} \rightarrow D_s^* \pi^0$: (111 ± 15) keV H.-L. Fu et al., EPJA58(2022)70
- How about $D_{s1}(2460)^+ \rightarrow D_s^+ \pi^+ \pi^-$?
 - CHPT result from S. Fajfer, A. Prapotnik Brdnik, PRD92(2015)074047

$$\Gamma(D_{s1}(2460)^+ \rightarrow D_s^+ \pi^+ \pi^-) = 0.25(4)(7) \left(\begin{array}{c} +2 \\ -4 \end{array}\right) \text{ keV}$$

Both D_s and D_{s1} were treated statically, so P -wave happens between π^+ and $\pi^- \Rightarrow I(\pi^+ \pi^-) = 1$, isospin breaking

- But isospin is conserved (!!) for P -wave between D_s and isoscalar $\pi^+ \pi^-$

$D_{s1}(2460)^+ \rightarrow D_s^+ \pi^+ \pi^-$ (1)

M.-N. Tang, Y.-H. Lin, FKG, C. Hanhart, U.-G. Meißner, arXiv:2303.18225 (Commun. Theor. Phys., in print)

- Hadronic molecule v.s. other (compact) components

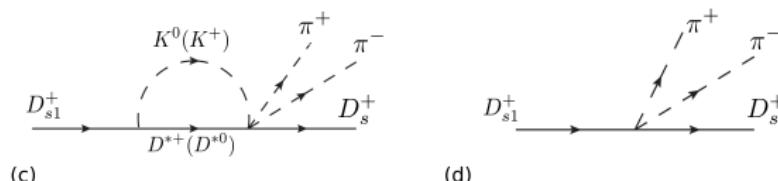
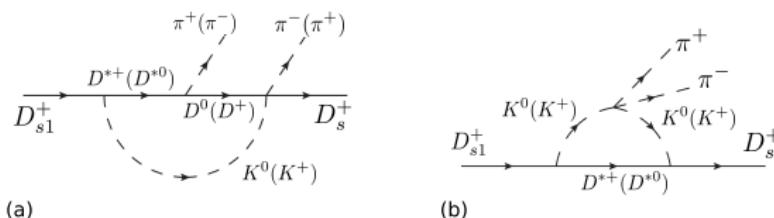
Effective coupling contains crucial information

S. Weinberg, PR137(1965)B672

$$g^2 \propto (1 - Z) \sqrt{2\mu E_B}$$

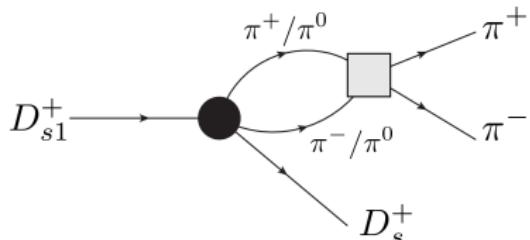
is maximized for a pure molecular state

- Diagrams for $D_{s1}(2460)^+ \rightarrow D_s^+ \pi^+ \pi^-$: (a,b,c): leading for molecular state



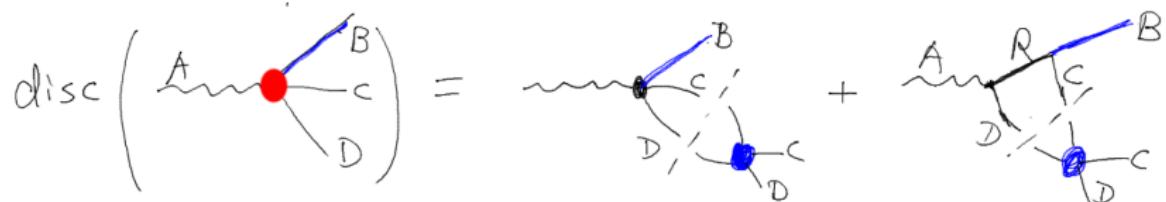
$$D_{s1}(2460)^+ \rightarrow D_s^+ \pi^+ \pi^-$$

- Phase space: $m_{\pi^+ \pi^-} \in [2M_{\pi^\pm}, 0.49 \text{ GeV}]$
- $\pi\pi$ final state interaction (FSI) is important in S -wave: $f_0(500)$ (σ) meson



Black filled circle: all the one-loop diagrams in the previous page, $\hat{\mathcal{A}}_L(s)$

- Unitarity \Rightarrow dispersion relation



$$\frac{1}{2i} \text{disc} \left[\mathcal{A}_L(s) + \underbrace{\hat{\mathcal{A}}_L(s)}_{\text{no right-hand cut}} \right] = \mathcal{A}_L(s) \rho(s) T_L^*(s) + \hat{\mathcal{A}}_L(s) \rho(s) \underbrace{T_L^*(s)}_{\pi\pi \text{ scattering}}$$

$$D_{s1}(2460)^+ \rightarrow D_s^+ \pi^+ \pi^- (\mathbf{3})$$

Solution to the dispersive equation, δ_L L-wave phase shift:

$$\frac{1}{2i} \operatorname{disc} \mathcal{A}_L(s) = [\mathcal{A}_L(s) + \hat{\mathcal{A}}_L(s)] \sin \delta_L(s) e^{-i\delta_L(s)}$$

- Rewrite the unitarity relation using $\operatorname{disc} \mathcal{A}_L(s) = \mathcal{A}_L(s + i\epsilon) - \mathcal{A}_L(s - i\epsilon)$:

$$\frac{1}{2i} [\mathcal{A}_L(s + i\epsilon) e^{-i\delta_L(s)} - \mathcal{A}_L(s - i\epsilon) e^{i\delta_L(s)}] = \hat{\mathcal{A}}_L(s) \sin \delta_L(s)$$

- Using properties of the Omnès function:

$$|\Omega_L(s)| = \Omega_L(s + i\epsilon) e^{-i\delta_L(s)} = \Omega_L(s - i\epsilon) e^{i\delta_L(s)},$$

$$\frac{1}{2i} \left[\frac{\mathcal{A}_L(s + i\epsilon)}{\Omega_L(s + i\epsilon)} - \frac{\mathcal{A}_L(s - i\epsilon)}{\Omega_L(s - i\epsilon)} \right] = \frac{\hat{\mathcal{A}}_L(s) \sin \delta_L(s)}{|\Omega_L(s)|}$$

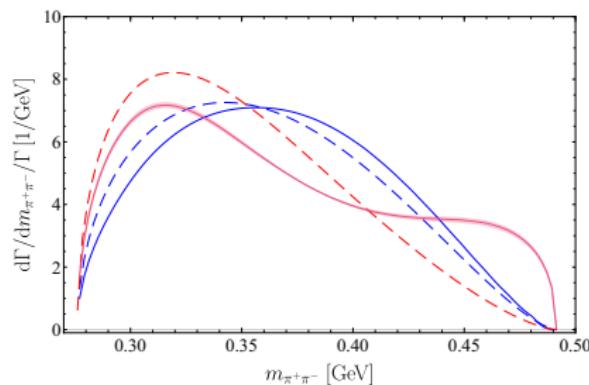
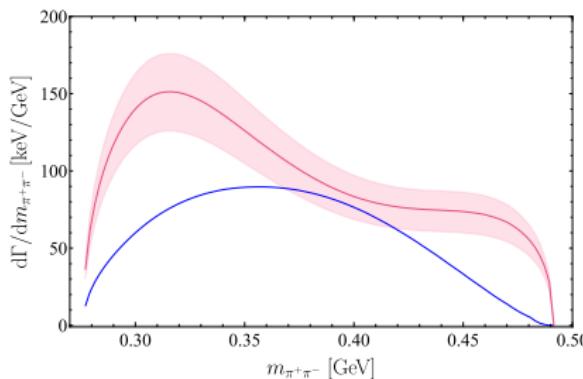
- Dispersion relation for $\mathcal{A}_L(s)/\Omega_L(s) \Rightarrow$ the total decay amplitude:

$$\mathcal{A}_{\text{tot}}(s, \cos \theta) = \sum_L [\hat{\mathcal{A}}_L(s) + \mathcal{A}_L(s)] P_L(\cos \theta)$$

$$\mathcal{A}_L(s) = \Omega_L(s) \left[P_{n-1}(s) + \frac{s^n}{\pi} \int_{s_{\text{th}}}^{\infty} dz \frac{\hat{\mathcal{A}}_L(z) \sin \delta_L(z)}{z^n (z - s) |\Omega_L(z)|} \right]$$

$D_{s1}(2460)^+ \rightarrow D_s^+ \pi^+ \pi^-$ (4)

- Double bump structure in the $\pi\pi$ invariant mass distribution as a feature of the hadronic molecular picture
 - ★ Red: molecular, assuming the $D_{s1}D_s\pi\pi$ contact term to vanish
 - ★ Blue: compact, without D^*K loops



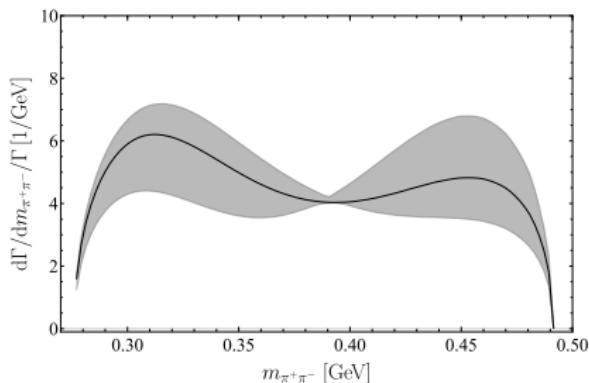
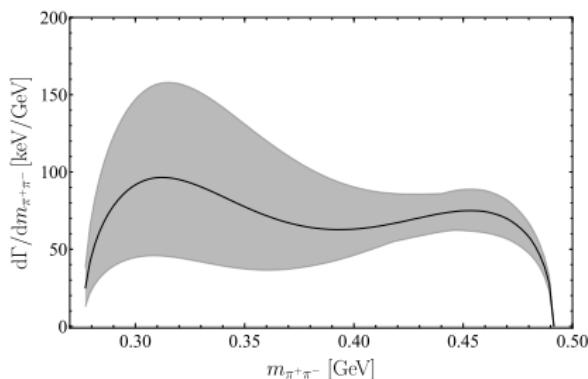
Dashed: without $\pi\pi$ FSI

- agree with Belle measurement $0.14 \pm 0.04 \pm 0.02$:

$$\frac{\Gamma(D_{s1}(2460)^+ \rightarrow D_s^+ \pi^+ \pi^-)}{\Gamma(D_{s1}(2460)^+ \rightarrow D_s^{*+} \pi^0)} \Big|_{\text{mol.}} = 0.19^{+0.07}_{-0.05}$$

$D_{s1}(2460)^+ \rightarrow D_s^+ \pi^+ \pi^-$ (5)

- With the contact term fixed from reproducing the Belle measurement (consistent with zero)



- Prediction for the bottom analogue:

$$\Gamma(B_{s1}^0 \rightarrow B_s^0 \pi^+ \pi^-) = (3 \pm 1) \text{ keV}$$

Summary

We are close to understanding the nature of the low-lying 0^+ and 1^+ charmed mesons.

Missing building blocks: more measurements for $D_{s0}^*(2317)$ and $D_{s1}(2460)$:

- Full widths
- $\pi^+\pi^-$ invariant mass distribution of $D_{s1}(2460)^+ \rightarrow D_s^+\pi^+\pi^-$

Looking forward to more experimental observations!

Thank you for your attention !

Experiments

Lattice

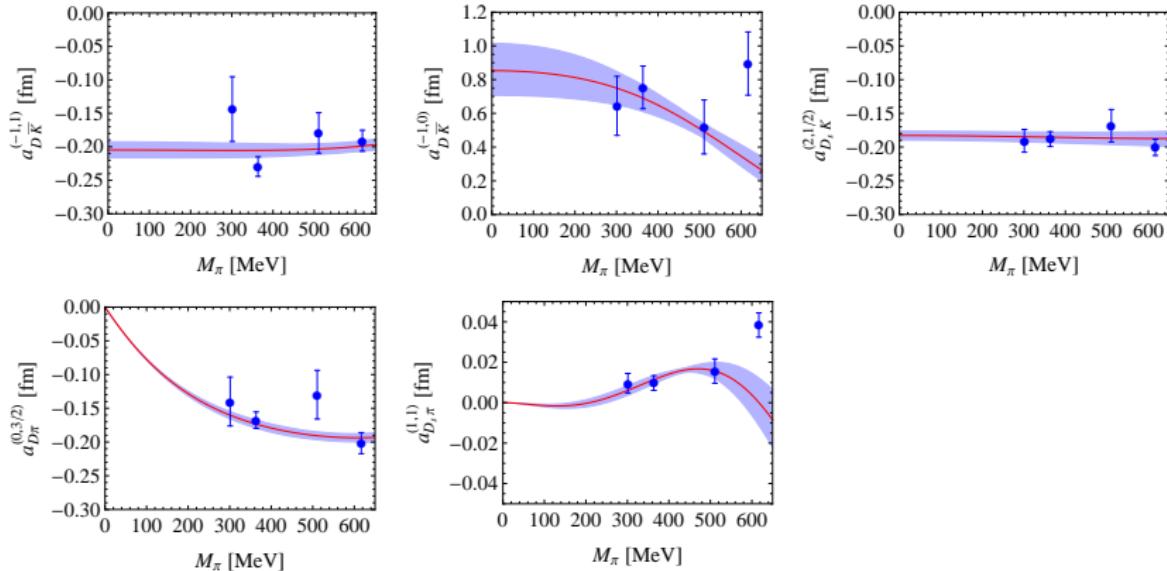
Backup slides

EFT, models

Fit to lattice data

L. Liu, Orginos, FKG, Hanhart, Meißner, PRD86(2013)014508

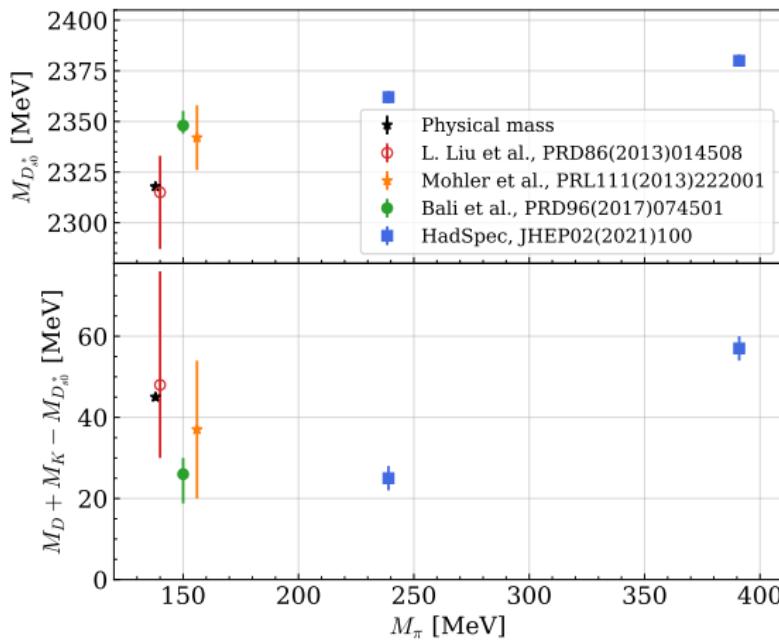
- Fit to lattice data on scattering lengths in 5 **relatively simple channels**:
 $D\bar{K}(I=1, I=0)$, $D_s K$, $D\pi(I=3/2)$, $D_s\pi$: no disconnected contribution
 5 parameters: h_2, h_3, h_4, h_5 and $a(\mu)$



- N_c counting fulfilled: $\underbrace{h_2 \simeq 0.2, h_4 M_D^2 \simeq -0.3}_{\mathcal{O}(N_c^{-1})}$, $\underbrace{h_3 \simeq 2.1, h_5 M_D^2 \simeq -1.9}_{\mathcal{O}(N_c^0)}$

Lattice studies of the charmed scalar mesons: strange

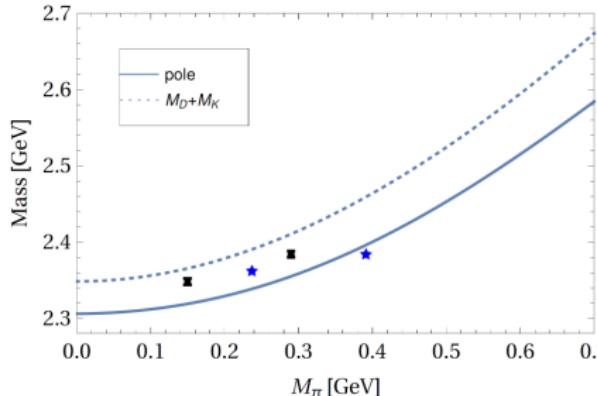
- $SU(3) \Rightarrow M_{D_{s0}^*} = 2315^{+18}_{-28}$ MeV Liu, Orginos, FKG, Hanhart, Mei β nner, PRD86(2013)014508
- Lattice results with $c\bar{s} + DK$ interpolators: \sim right mass



- Early studies using only $c\bar{s}$ -type interpolators typically give mass sizeably larger than that of $D_{s0}^*(2317)$ Bali (2003); UKQCD (2003); HadSpec (2013); ...

D_{s0}^* as DK molecule from lattice QCD (2)

- Lattice results in G. Bali et al., PRD96(2017)074501



M_π [MeV]	150	290
$M_{D_{s0}^*(2317)}$ [MeV]	2348 ± 4	2384 ± 3
M_{D_s} [MeV]	1977 ± 1	1980 ± 1

strong M_π dependence!

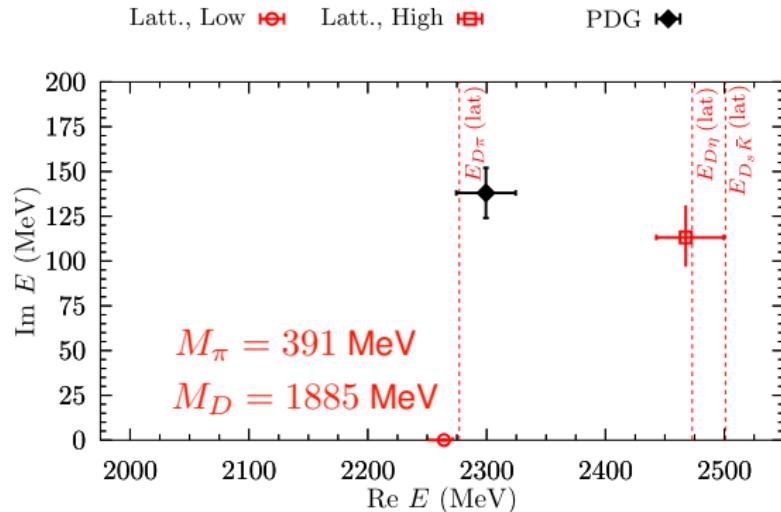
curves: prediction in Du et al., EPJC77(2017)728

- Lattice results in HadSpec, JHEP 02 (2021) 100

M_π [MeV]	239	391
$M_{D_{s0}^*(2317)}$ [MeV]	2362 ± 3	2380 ± 3

Pion mass dependence

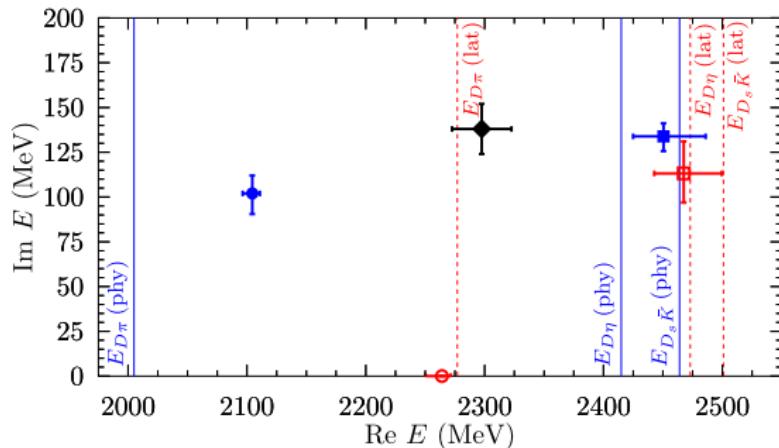
Masses	M (MeV)	$\Gamma/2$ (MeV)	RS	$ g_{D\pi} $	$ g_{D\eta} $	$ g_{D_s\bar{K}} $
lattice	2264^{+8}_{-14}	0	(+++)	$7.7^{+1.2}_{-1.1}$	$0.3^{+0.5}_{-0.3}$	$4.2^{+1.1}_{-1.0}$
	2468^{+32}_{-25}	113^{+18}_{-16}	(--+)	$5.2^{+0.6}_{-0.4}$	$6.7^{+0.6}_{-0.4}$	$13.2^{+0.6}_{-0.5}$



Pion mass dependence

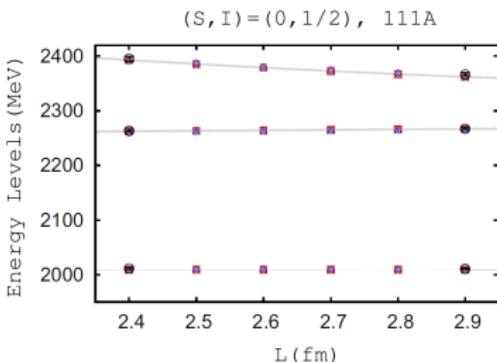
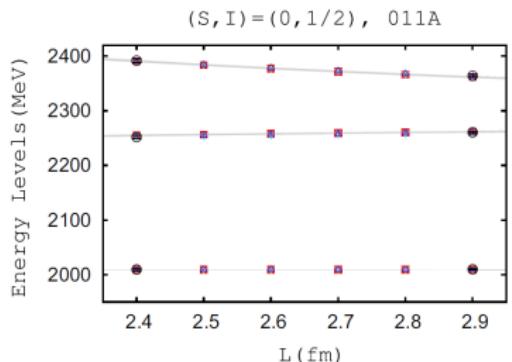
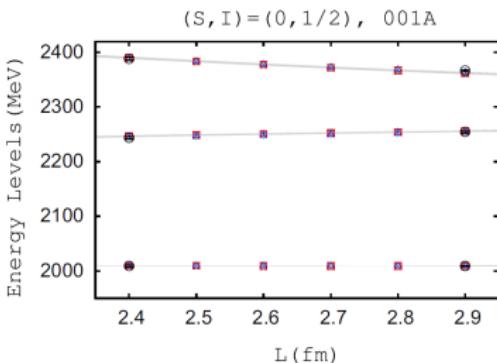
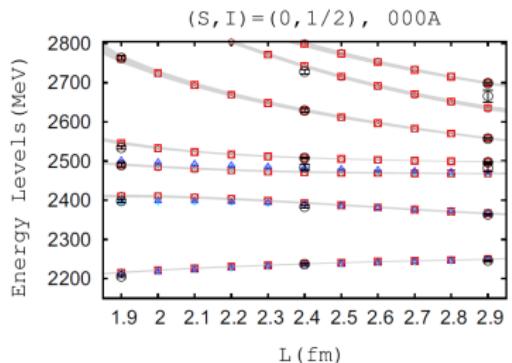
Masses	M (MeV)	$\Gamma/2$ (MeV)	RS	$ g_{D\pi} $	$ g_{D\eta} $	$ g_{D_s\bar{K}} $
lattice	2264^{+8}_{-14}	0	(+++)	$7.7^{+1.2}_{-1.1}$	$0.3^{+0.5}_{-0.3}$	$4.2^{+1.1}_{-1.0}$
	2468^{+32}_{-25}	113^{+18}_{-16}	(--+)	$5.2^{+0.6}_{-0.4}$	$6.7^{+0.6}_{-0.4}$	$13.2^{+0.6}_{-0.5}$
physical	2105^{+6}_{-8}	102^{+10}_{-11}	(-++)	$9.4^{+0.2}_{-0.2}$	$1.8^{+0.7}_{-0.7}$	$4.4^{+0.5}_{-0.5}$
	2451^{+36}_{-26}	134^{+7}_{-8}	(--)	$5.0^{+0.7}_{-0.4}$	$6.3^{+0.8}_{-0.5}$	$12.8^{+0.8}_{-0.6}$

Latt., Low Latt., High PDG
 Phys., Low Phys., High



A more recent fit to lattice data including moving frame ones

Z.-H. Guo et al., EPJC79(2019)13



Determined parameters (from Fit IIB) are similar

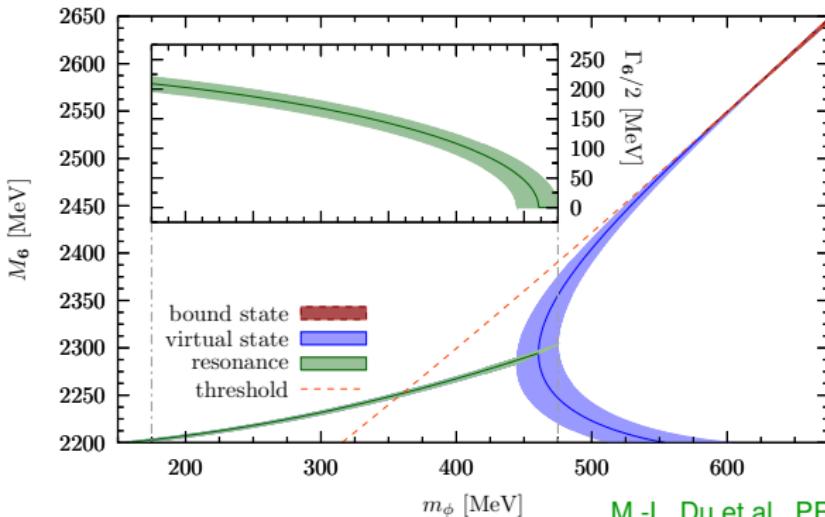
Lattice data: Moir et al., JHEP1610,011

Searching for the higher nonstrange state: lattice

- Tuning interaction strength by varying quark masses:

Expectation: WT term $\propto E_\pi$, increasing M_π leads to stronger interaction
increasing S -wave interaction strength \Rightarrow resonance \rightarrow below-th. resonance \rightarrow
virtual state \rightarrow bound state, then easier for lattice to get a signal

- $SU(3)$ symmetric, then the sextet decouples from the triplet;
prediction (qualitative for large m_q), to check with large m_q on lattice:



M.-L. Du et al., PRD98(2018)094018

Lattice results

E. B. Gregory, FKG, C. Hanhart, S. Krieg, T. Luu, arXiv.2106.15391

Lattice results of the energy shift at SU(3) symmetric point with $M_\pi = 612(90)$ MeV

- Evidence for a bound state in the **sextet**
- The $\bar{15}$ has a repulsive interaction (in a diquark-antidiquark tetraquark model, $\bar{15}$ would also exist)

