Bound States and Resonances in Doubly Heavy Tetraquark States from Lattice QCD

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Why study Heavy-Light Tetraquarks?

Experimental background

- In the last decade: Experiments observed many states which cannot be explained by ordinary quark-antiquark pairs (e.g. $Z_b(10610)^+$)
- Four-quark structure can describe quantum numbers correctly!
- Quite recently: LHCb found a bound tetraquark T_{cc}^+ $(cc\bar{u}\bar{b})$

  R. Aaij *et al.* [LHCb], Nature Commun. **13**, 3351 (2022)

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Theoretical background

- We study heavy-light tetraquarks

$$\bar{Q}\bar{Q}'qq' \quad \text{with:} \qquad Q, Q' \in \{b, c\} \text{ and } q, q' \in \{u, d, s\}$$

- In the limit $m_{Q,Q'} \to \infty$ this tetraquark is shown to be stable

  J. Carlson *et al.*, Phys. Rev. D 37, 744 (1988), A. V. Manohar *et al.*, Nucl. Phys. B 399, 17 (1993), E. J. Eichten *et al.*, Phys. Rev. Lett. 119, no. 20, 202002 (2017)

Physical Background (1)

Four-Quark Systems covered in this talk:

$\bar{b}\bar{b}ud$ $\bar{b}\bar{b}us$ $\bar{b}\bar{c}ud$

Four-Quark System $\bar{b}\bar{b}ud$ with $I(J^P) = 0(1^+)$

- Most promising candidate for bound tetraquark
- Extensively studied in lattice QCD
- All studies predict a bound state
- Binding energy is about $100\,\mathrm{MeV} \ldots 130\,\mathrm{MeV}$ below BB^*

P. Bicudo *et al.*, Phys. Rev. D 92, no. 1, 014507, Z. S. Brown *et al.*, Phys. Rev. D 86, 114506, A. Francis *et al.*, Phys. Rev. Lett. 118, no. 14, 142001, P. Junnarkar *et al.*, Phys. Rev. D 99, no. 3, 034507, L. Leskovec *et al.*, Phys. Rev. D 100, no. 1, 014503, P. Mohanta *et al.*, Phys. Rev. D 102, 094516

Physical Background (2)

Four-Quark System $\bar{b}\bar{b}us$ with $J^P = 1^+$

- Also promising as m_s only slightly larger than m_d
- Lattice QCD results all predict bound state
- Binding energy is expected to be about $30\,\text{MeV} \ldots 80\,\text{MeV}$

Four-Quark System $\bar{b}\bar{c}ud$ with $I(J^P) = 0(0^+)$ and $I(J^P) = 0(1^+)$

- Slightly different structure as $\bar{b} \to \bar{c}$ allows two spin realizations
- Lattice QCD results are inconsistent
- No clear evidence for a bound state

A. Francis *et al.*, Phys. Rev. D 99, no. 5, 054505, S. Meinel *et al.*, Phys. Rev. D 100, no. 1, 014503,
R. J. Hudspith *et al.*, Phys. Rev. D 102, 114506, N. Mathur *et al.* PoS **LATTICE2021**, 443

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⇒ Extend existing studies and investigate finite volume effects

Interpolating Operators (1)

We use two different types of interpolating operators:

- **Local operators**
  - Four quarks at the same space-time position
  - Jointly projected to zero momentum
  - Describe local tetraquark structure
- **Scattering operators**
  - Two mesons separated in space-time position
  - Separately projected to zero momentum
  - Describe meson-meson scattering states

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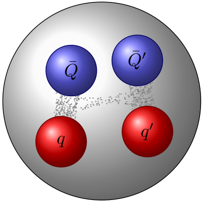
Expectation:

- **Local operators**: good overlap to ground state (stable four-quark)
- **Scattering operators**: sizeable overlap to higher excited states (meson-meson state)

⇨ Allows precise determination of low-lying energy spectrum

Interpolating Operators (2)

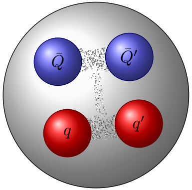
Local meson-meson Operators



N operators corresponding to the lowest thresholds

$\mathcal{O} = \sum_{\boldsymbol{x}} M^1(\boldsymbol{x}) M_j^2(\boldsymbol{x})$

Local diquark-antidiquark operator

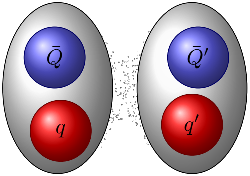


1 operator using "good" diquarks if possible

(Jaffe, Phys. Rept. **409**)

$\mathcal{O} = \sum_{\boldsymbol{x}} D_j^b(\boldsymbol{x}) d^l(\boldsymbol{x})$

Meson-meson scattering operator



N operators corresponding to the lowest thresholds

$\mathcal{O} = \sum_{\boldsymbol{x}} M^1(\boldsymbol{x}) \sum_{\boldsymbol{y}} M_j^2(\boldsymbol{y})$

Interpolating Operators (3)

- Up to now, *only* local operators have been used in studies
- In our previous studies we use *additionally* scattering operators at the sink

  L. Leskovec *et al.*, Phys. Rev. D 100, no. 1, 014503, S. Meinel *et al.*, Phys. Rev. D 100, no. 1, 014503

⇨ We found for all systems that including scattering operators:

- Decreases the lowest energy level significantly
- Allows to determine lowest levels more precisely

⇨ Now: Compute the full matrix, i.e. scattering operators also at source

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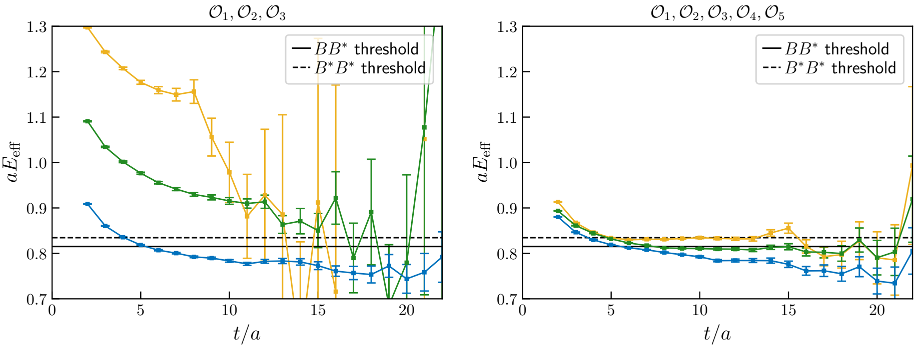
Why is this important?

- Most tetraquarks are close to the relevant threshold or even resonances
  ⇒ Finite volume effects might be significant
  ⇒ Scattering analysis must be done which requires scattering operators
- We study finite volume effects in $\bar{b}\bar{b}ud$ with $I(J^P) = 0(1^+)$ and $\bar{b}\bar{b}us$ with $J^P = 1^+$

Investigation of $\bar{b}\bar{b}ud$ with $I(J^P) = 0(1^+)$

- 3 local and 2 scattering operators ➡ 5×5 matrix
- Meson-meson operators resemble BB^* and B^*B^* structures
- Lowest *finite volume* energy levels are extracted from a GEVP



3 lowest energy eigenstates for **left**: only local operators and **right:** local and scattering operators.

$\bar{b}\bar{b}ud$ - Scattering Analysis

- Determine *finite* volume energy spectrum
- *Finite* volume energy levels can be related to *infinite* volume scattering amplitude T via quantization condition

$$\det[\mathbf{1} + i\boldsymbol{\rho}(s) \cdot \mathbf{T}(s)(\mathbf{1} + i\mathbf{M}(q^2, L))] = 0$$

$$\rho_{ij}(s) = 2k_i/\sqrt{s}\delta_{ij}; \qquad \mathbf{M}(q^2, L)) \text{ analytic function}$$

- Use reasonable parametrization for T-Matrix

$$[T^{-1}(s)]_{ij} = [K^{-1}(s)]_{ij} + I(s)_{ij}, \quad I(s) \text{ known function of C.o.M. energy } s$$

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Parametrizations

single channel

$$K^{-1}(s) = c^{(0)} + c^{(1)}s$$

(effective range expansion (ERE))

coupled channel

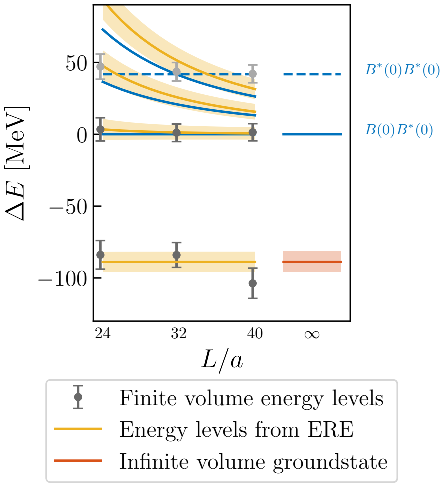
$$K^{-1}(s) = \begin{pmatrix} c_{00}^{(0)} & c_{01}^{(0)} \\ c_{10}^{(0)} & c_{11}^{(0)} \end{pmatrix}$$

Preliminary Results $\bar{b}\bar{b}ud$, $I(J^P) = 0(1^+)$ (1)

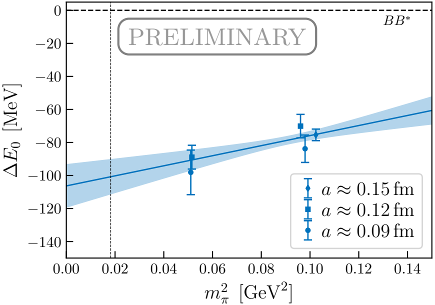
- Perform single channel scattering analysis
- Parameters are determined such that energy levels from parametrization match *finite* volume energy spectrum
- Search for pole of scattering amplitude T in complex energy plane
- Found pole on the negative real axis
- This corresponds to a bound state in the *infinite* volume

single channel

$a \approx 0.12\,\text{fm}$, $m_\pi \approx 220\,\text{MeV}$



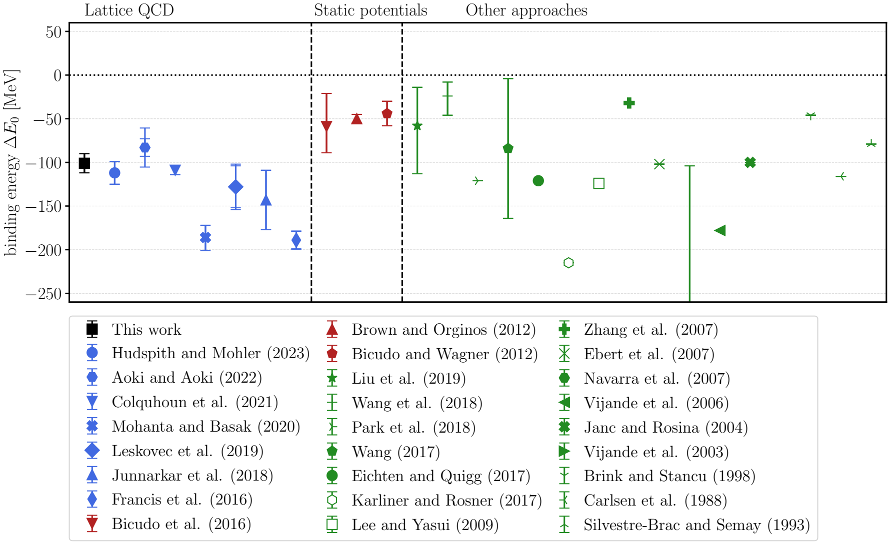
Preliminary Results $\bar{b}\bar{b}ud$, $I(J^P) = 0(1^+)$ (2)



- We recognize m_π dependence of ground state energy
- Perform chiral extrapolation of *infinite* volume levels to the physical point
- Discretization effects due to finite a are negligible
- Final binding energy for $\bar{b}\bar{b}ud$ in $I(J^P) = 0(1^+)$ channel:

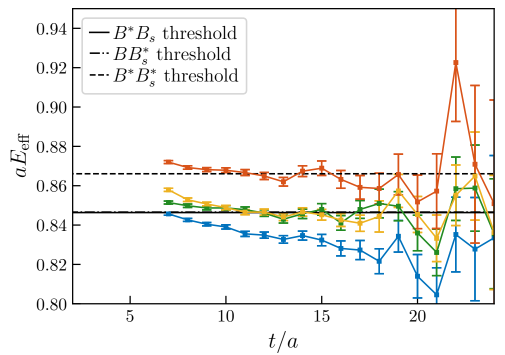
$$E_{\text{bind}} = (-101 \pm 11)\,\text{MeV}$$

Comparison of Different Results for $\bar{b}\bar{b}ud$



Investigation of $\bar{b}\bar{b}us$ with $J^P = 1^+$

- 4 local and 3 scattering operators ➡ 7×7 matrix
- Meson-meson operators resemble B^*B_s, BB_s^* and $B^*B_s^*$ structures
- First and second excited state are almost degenerated
  ➡ Requires coupled channel analysis



4 lowest energy eigenstates for full 7×7 matrix with local and scattering operators.

Preliminary Results $\bar{b}\bar{b}us$, $J^P = 1^+$ (1)

- Determine the 3 lowest energy levels
- Perform coupled channel scattering analysis
- Parameters are determined such that energy levels from parametrization match *finite* volume energy spectrum
- Search for pole of scattering amplitude T in complex energy plane
- Found pole on the negative real axis
- This corresponds to a bound state in the *infinite* volume

coupled channel

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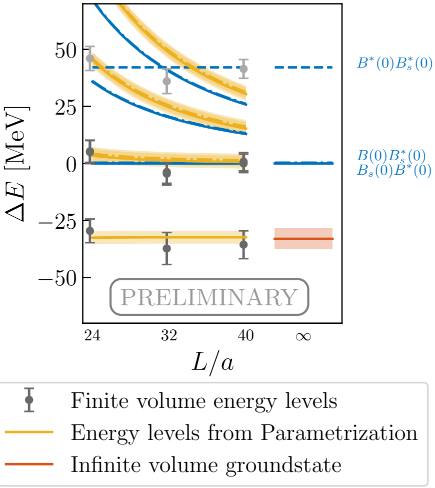
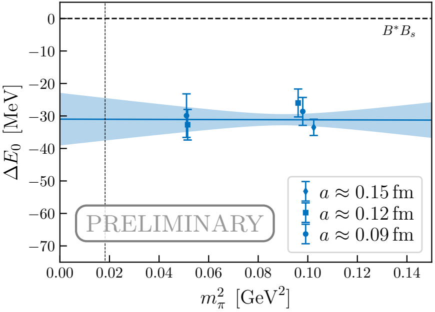


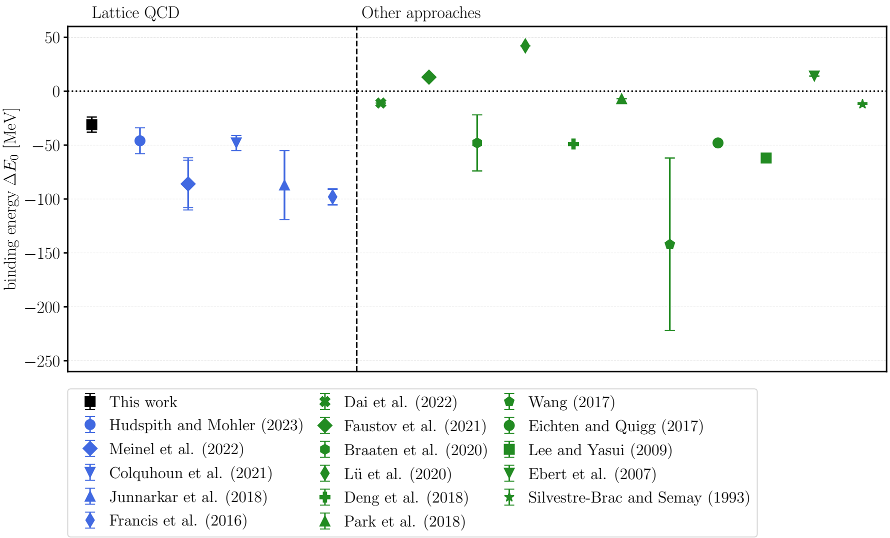
Preliminary Results $\bar{b}\bar{b}us$, $J^P = 1^+$ (2)



- We recognize no m_π dependence of ground state energy
- Perform chiral extrapolation of *infinite* volume levels to the physical point
- Discretization effects due to finite a are negligible
- Final binding energy for $\bar{b}\bar{b}us$ in $J^P = 1^+$ channel:

$$E_{\text{bind}} = (-31 \pm 7)\,\text{MeV}$$

Comparison of Different Results for $\bar{b}\bar{b}us$



Search for Bound State or Resonances in $\bar{b}\bar{c}ud$ Four Quark Systems

Why is $\bar{b}\bar{c}ud$ of interest?

- $\bar{b}\bar{b}ud$ is consistently predicted to be deeply bound ($\sim 100\,\text{MeV}$) by lattice studies
- Lattice study found $\bar{c}\bar{c}ud$ virtual bound state (M. Padmanath, S. Prelovsek, Phys. Rev. Lett. **129**)
- Weakly bound $\bar{c}\bar{c}ud$ has been found by LHCb (R. Aaij *et al.* [LHCb], Nature Commun. **13**)

⇒ Naively, one would expect $\bar{b}\bar{c}ud$ somewhere in between

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Current Status in Lattice Studies

- Previous studies do not show evidence for bound state in $\bar{b}\bar{c}ud$ sector
- in e.g. S. Meinel *et al.* we found level close to lowest threshold

⇒ Finite volume effects might be large and bound state or resonance cannot be ruled out

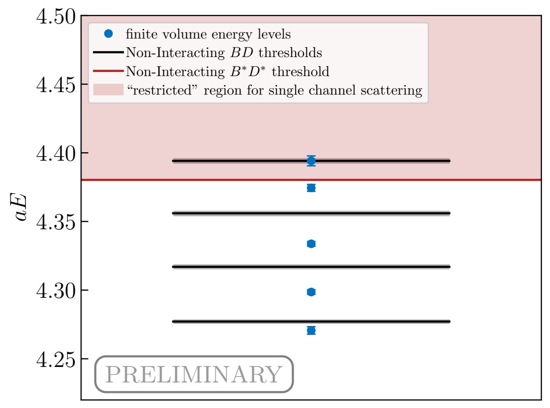
Interpolating Operators for $\bar{b}\bar{c}ud$

- Two different channels: $I(J^P) = 0(0^+)$ and $I(J^P) = 0(1^+)$
- **Local** operators
  - N meson-meson operators corresponding to the N lowest 2-meson thresholds
  - 1 diquark-antidiquark operator
- **Scattering** operators
  - Meson-meson scattering operators for the lowest 2-meson threshold
  - Include all relevant back-to-back momenta $p^2 = n(2\pi/L)^2$

$I(J^P) = 0(0^+)$	$I(J^P) = 0(1^+)$
3 local operators	4 local operators
4 scattering operators:	3 scattering operators:
BD with $n = 0 \ldots 3$	B^*D with $n = 0 \ldots 2$
7×7 matrix	7×7 matrix

Preliminary Finite Volume $\bar{b}\bar{c}ud$ Energy Levels

$I(J^P) = 0(0^+)$



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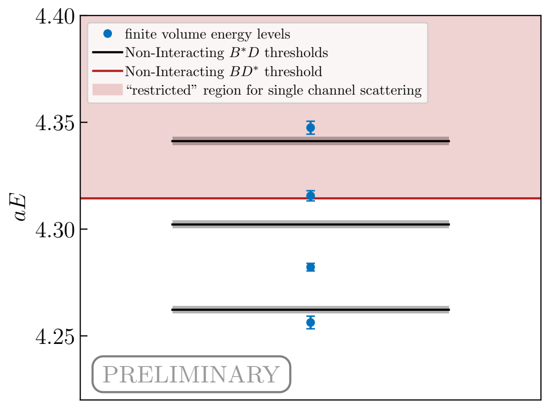

- One additional energy level compared to the non-interacting case!
- Something interesting seems to happen!

Towards the Infinite Volume ...

- Perform a scattering analysis via Lüscher's method
  (M. Lüscher, Nucl. Phys. **B354**, 531)
- Find parametrization of the scattering amplitude T
- Reasonable ansatz might by quadratic parametrization of phase shift $\delta(s)$:

$$\frac{k}{\sqrt{s}} \cot(\delta(s)) = as^2 + bs + s$$

- Using this parametrization
  ⇨ Search for T-matrix poles in the complex energy plane

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Preliminary Results for Pole Search

- We find a pole on the negative real axis at around $\approx -40 \cdots - 50\,\text{MeV}$ in both channels!
- Might be indication for a bound state in $\bar{b}\bar{c}ud$

Summary

- Study bound states in doubly heavy tetraquarks
- Consider local and additionally scattering interpolating operators
- Investigate finite volume effects performing a scattering analysis

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- Confirm bound tetraquarks for $\bar{b}\bar{b}ud$ with $0(1^+)$ and $\bar{b}\bar{b}us$ with 1^+
- Observed no significant finite volume effects
- Determined infinite volume binding energies as

$$E_{\mathrm{bind},\bar{b}\bar{b}ud} = (-101 \pm 11)\,\mathrm{MeV}, \qquad E_{\mathrm{bind},\bar{b}\bar{b}us} = (-31 \pm 7)\,\mathrm{MeV}$$

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- Found some indication for bound tetraquarks in these channels
- Still more work needs to be done here

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Thank You for Your Attention!

Back-Up

Previous work on heavy-light tetraquarks

Born-Oppenheimer study of $\bar{b}\bar{b}ud$ tetraquark (with static $\bar{b}$-quarks):

- bound tetraquark with $I(J^P) = 0(1^+)$ and $E_{\text{bind}} \approx -90\,\text{MeV}$
- resonance with $I(J^P) = 0(1^-)$ and $E_{\text{res}} \approx +20\,\text{MeV}$, $\Gamma \approx 100\,\text{MeV}$

[Z. S. Brown and K. Orginos, Phys. Rev. D **86**, 114506 (2012)]
[P. Bicudo *et al.* [ETMC], Phys. Rev. D **87**, no. 11, 114511 (2013)]
[P. Bicudo, K. Cichy, A. Peters, B. Wagenbach, M. Wagner, Phys. Rev. D **92**, no. 1, 014507 (2015)]
[P. Bicudo, J. Scheunert and M. Wagner, Phys. Rev. D **95**, no. 3, 034502 (2017)]
[P. Bicudo, M. Cardoso, A. Peters, M.P. and M. Wagner, Phys. Rev. D **96**, no. 5, 054510 (2017)]

Non-Relativistic QCD

- Search for doubly-heavy tetraquark bound states in $\bar{b}\bar{b}ud$, $\bar{b}\bar{b}us$, $\bar{b}\bar{c}ud$
- bound tetraquark for $\bar{b}\bar{b}ud$ with $I(J^P) = 0(1^+)$ and $E_{\text{bind}} \approx -120\,\text{MeV}$
- bound tetraquark for $\bar{b}\bar{b}us$ with $J^P = 1^+$ and $E_{\text{bind}} \approx -80\,\text{MeV}$

[A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, Phys. Rev. Lett. **118**, no. 14, 142001 (2017)]
[P. Junnarkar, N. Mathur and M. Padmanath, Phys. Rev. D **99**, no. 3, 034507 (2019)]
[A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, Phys. Rev. D **99**, no. 5, 054505 (2019)]
[L. Leskovec, S. Meinel, M.P. and M. Wagner, Phys. Rev. D **100**, no.1, 014503 (2019)]
[R. J. Hudspith, B. Colquhoun, A. Francis, R. Lewis and K. Maltman, Phys. Rev. D **102**, 114506 (2020)]
[M.P., L. Leskovec, S. Meinel and M. Wagner, arXiv:2009.10538 [hep-lat]].

More details of previous publications

- Previous studies by Francis et. al., Junnarkar et al. and Leskovec et al. predict bound states in $\bar{b}\bar{b}ud$ and $\bar{b}\bar{b}us$
- For $\bar{b}\bar{c}ud$, the predictions are not as clear
  $\rightarrow$ Might be weakly bound or no binding
  $\rightarrow$ Replacing $\bar{b}$ with $\bar{c}$ opens additional channel
  (flavour can now be also anti-symmetric)

For all systems, there have been used:

- Local operators; basically used in all previous studies
- Scattering operators; only in Leskovec et al. at the sink

Lattice Setup

- Gauge link configurations from the **MILC collaboration**

  A. Bazavov *et al.* [MILC], Phys. Rev. D **87**, 054505 (2013)

- Highly improved staggered quark (HISQ) action for sea quarks
- Wilson-Clover action for the valence quarks

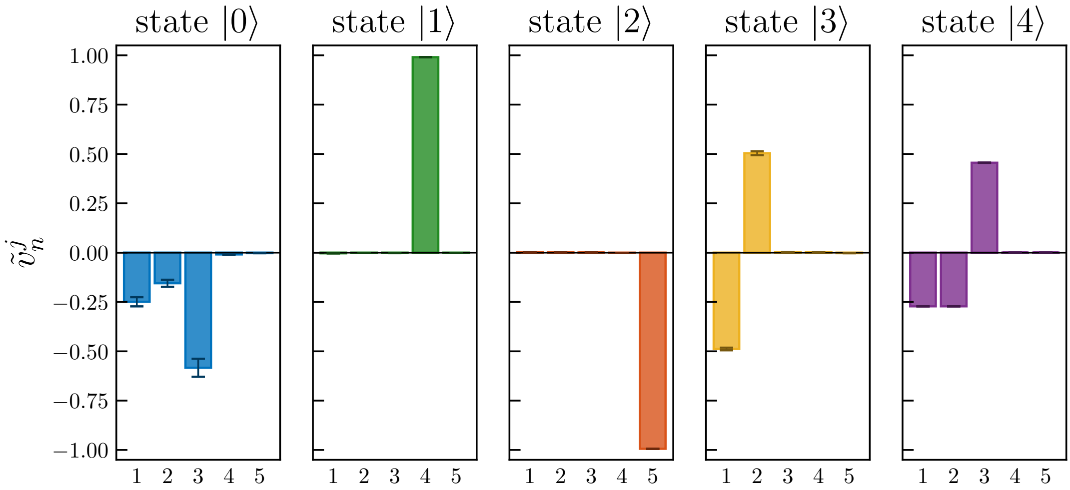
  T. Bhattacharya *et al.* [PNDME], Phys. Rev. D **92**, 094511 (2015)
  R. Gupta *et al.* [PNDME], Phys. Rev. D **98**, 034503 (2018)

- Six ensembles with

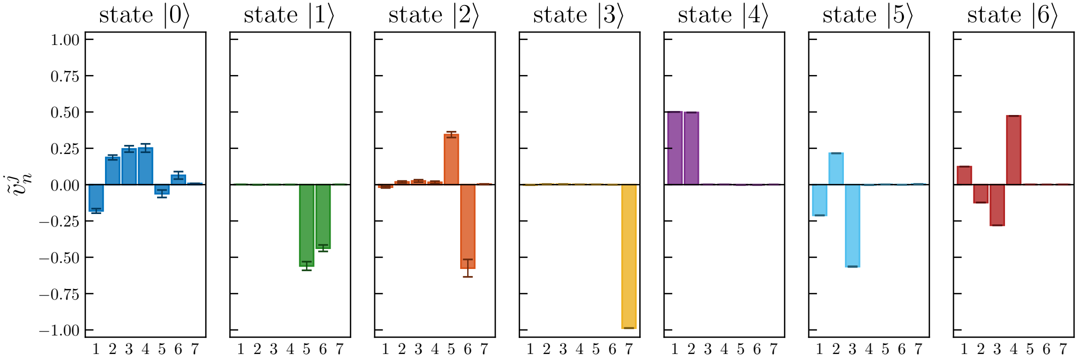
$$\begin{array}{lll} \text{lattice size} & L & \approx 2.84\,\text{fm} \ldots 4.76\,\text{fm}, \\ \text{pion mass} & m_\pi & \approx 220\,\text{MeV},\ 310\,\text{MeV}, \\ \text{lattice spacing} & a & \approx 0.09\,\text{fm},\ 0.12\,\text{fm} \end{array}$$

- NRQCD for bottom quarks
- for light quarks smeared ...
  - ... point-to-all propagators if local operator at source
  - ... stochastic timeslice-to-all propagators if scattering operator at source

Eingevector components for $\bar{b}\bar{b}ud$



Eingevector components for $\bar{b}\bar{b}us$



Lattice Setup of first Calculations

- Use gauge link configuration generated by RBC and UKQCD collaboration
  [Y. Aoki *et al.* [RBC and UKQCD Collaborations], Phys. Rev. D **83**, 074508 (2011)]
  [T. Blum *et al.* [RBC and UKQCD Collaborations],Phys. Rev. D **93**, no. 7, 074505 (2016)]
- $2+1$ flavours domain-wall fermions and Iwasaki gauge action
- Five different ensembles which differ in

$$\begin{array}{lll} \text{lattice spacing} & a & \approx 0.083\,\text{fm}\ldots 0.114\,\text{fm}, \\ \text{lattice size} & L & \approx 2.65\,\text{fm}\ldots 5.48\,\text{fm}, \\ \text{pion mass} & m_\pi & \approx 139\,\text{MeV}\ldots 431\,\text{MeV} \end{array}$$

- Here: only results for one ensemble presented
- explored also dependence on L, m_π via scattering analysis and chiral extrapolation
- Smeared point-to-all propagators for the up and down quarks

Energy Spectrum for the $\bar{Q}\bar{Q}'qq'$ system

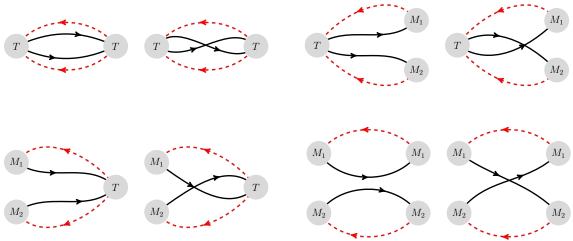
- Apply multi-exponential matrix fitting: employable also for non-symmetric matrices

$$C_{jk}(t) \approx \sum_{n=0}^{N-1} Z_j^n Z_k^n \mathrm{e}^{-E_n t},$$

E_n : n-th energy eigenvalue

$Z_j^n = \langle \Omega | \mathcal{O}_j | n \rangle$: overlap factor

- or GEP: beneficial for symmetric matrices



Schematic representation of Wick contractions for different correlation matrix elements