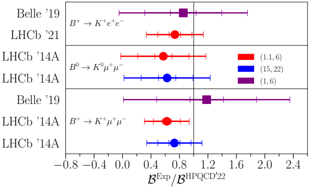
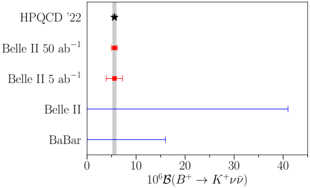
The search for new physics in $B \to K\ell^+\ell^-$ and $B \to K\nu\bar{\nu}$ using precise lattice QCD form factors

William Parrott
parrott@yorku.ca

York University

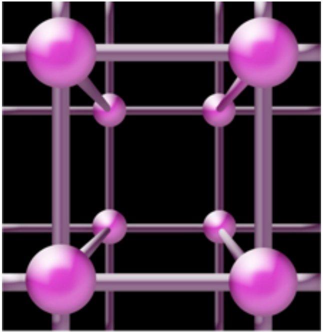




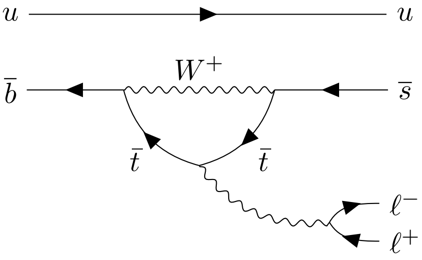
C. Bouchard, C.T.H. Davies

Overview

- Based on *2207.12468*, *2207.13371*
- $B \to K$ motivation
- Calculation of hadronic form factors on the lattice
- Studying $B \to K$ using heavy-HISQ
- Results:
  $B \to K$ form factors and phenomenology
- Subsequent work



$B \to K$ motivation

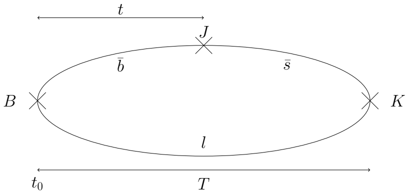


- Rare flavour changing neutral currents require loops
- Highly suppressed in the SM
- A good place to look for new physics
- We need very precise theoretical and experimental determinations to test SM.
- Theory requires precise form factors for the hadronic part of the decay, which we calculate on the lattice

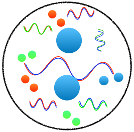
Form factors on the lattice

$$\frac{d\Gamma^{D\to K}}{dq^2} = \frac{G_F^2|V_{cs}|^2}{24\pi^3}|\vec{p}_K|^3 \; |f_+(q^2)|^2$$

- Parameterise the ‘QCD bit’ in a differential decay rate
- Interested in $f_0(q^2)$, $f_+(q^2)$ and $f_T(q^2)$ form factors for $B \to K$.
- Encode meson structure and describe the shape in $q^2 = (p_{\mathrm{mother}} - p_{\mathrm{daughter}})^2$ space.
- Form factors are constructed from three-point functions calculated on the lattice.



Form factors on the lattice



- Problem: QCD non-perturbative at low energies.
- Need to solve path integral. Discretise Euclidean action on a 4D lattice of space-time points with spacing a.
- Many discretisations of the action exist. We use a highly improved (HISQ) action for all quarks. This removes effects associated with the naive discretisation process through order a^2.
- Calculate correlation functions by inserting operators and performing Monte Carlo integration. For QCD we must integrate over quark and gluon fields.
- Repeat for many a values and extrapolate to continuum ($a = 0$).

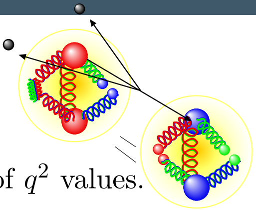
$$\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}Ae^{-S[\bar{\psi},\psi,A]}$$

Form factors on the lattice

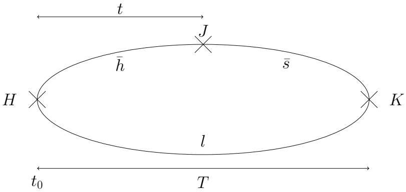
We can calculate two and three point functions to study meson masses and decays from first principles, but with some limitations:

- The computational cost of calculations is very large. This grows quickly with lighter masses and finer lattices.
- For $B \to K$ we need to include ensembles with physical light quark masses.
- We also need fine lattices for $am_b \leq 0.8$. Not achievable on most ensembles.

$H \to K$ form factors

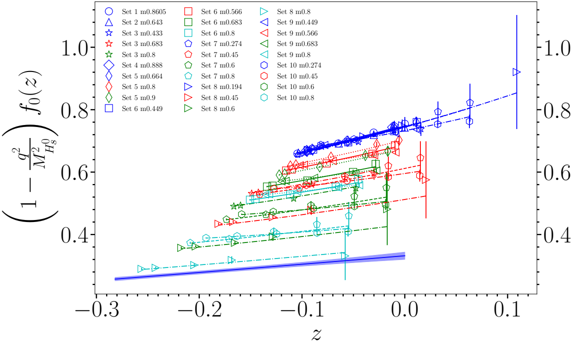


- Want meson form factors over the full range of q^2 values.
- f_0, f_+ and f_T form factors use matrix elements from 3-point correlation functions with scalar, vector and tensor current insertions.
- We fit the time dependence to extract matrix elements from correlators.



Moving to $B \to K$

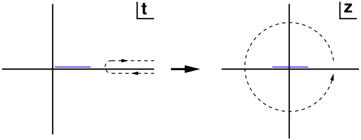
Heavy-HISQ:



- Proceed for $H \to K$ using 'heavy' mass m_h.
- $am_c \leq am_h \leq 0.8$ on each ensemble.
- f_0 & f_+ non-pert. normalised. For f_T, use normalisation from [2008.02024].
- Use 5 lattice spacings with 3 physical m_l ensembles.
- First fully relativistic calculation - all HISQ quarks.

Moving to $B \to K$

Convert to z space and extrapolate in heavy mass too:



$$f_0(q^2) = \frac{1+L}{1-\frac{q^2}{M^2_{H^*_{s0}}}} \sum_{n=0}^{N-1} a_n^0 z^n,$$

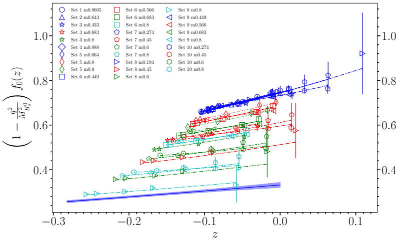
$$N = 3$$

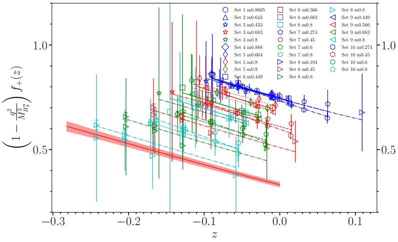
$$x_\pi = \frac{M_\pi^2}{(4\pi f_\pi)^2}$$

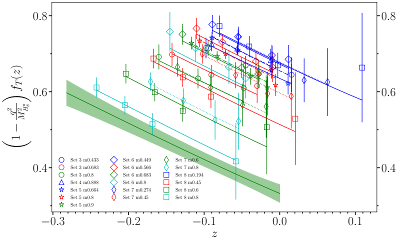
$$f_{+,T}(q^2) = \frac{1+L}{1-\frac{q^2}{M^2_{H^*_s}}} \sum_{n=0}^{N-1} a_n^{+,T}\Big(z^n - \frac{n}{N}(-1)^{n-N} z^N\Big).$$

$$a_n^{0,+,T} = \Big(\frac{M_D}{M_H}\Big)^{\zeta_n}\Big(1+\rho_n^{0,+,T}\log\Big(\frac{M_H}{M_D}\Big)\Big)(1+\mathcal{N}_n^{0,+,T})\times$$
$$\sum_{i,j,k,l=0}^{N_{ijkl}-1} d_{ijkln}^{0,+T}\Big(\frac{\Lambda_{\mathrm{QCD}}}{M_H}\Big)^i\Big(\frac{am_h^{\mathrm{val}}}{\pi}\Big)^{2j}\Big(\frac{a\Lambda_{\mathrm{QCD}}}{\pi}\Big)^{2k}(x_\pi - x_\pi^{\mathrm{phys}})^l.$$

$B \to K$ form factors

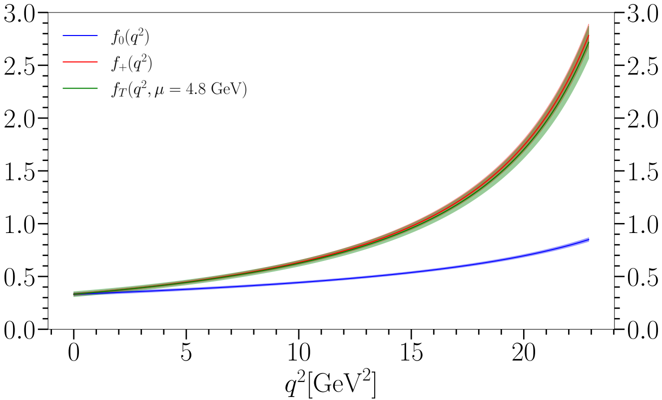






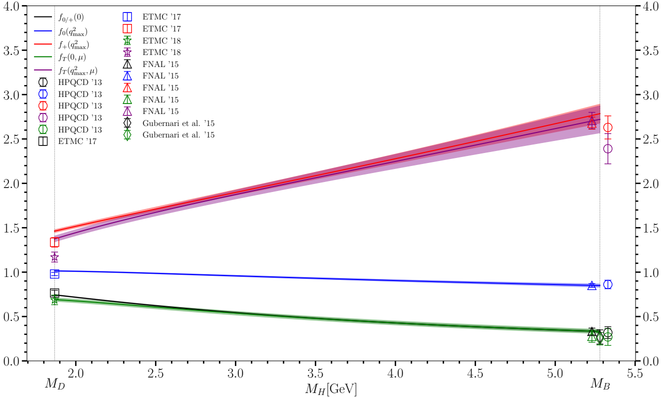
The z expansion is well behaved in all cases.

$B \to K$ form factors



Evaluate at the continuum, physical point and B mass to give precise form factors across whole q^2 range.

$B \to K$ form factors



Heavy-HISQ fits behaviour in M_H at fixed q^2. Improvements in precision, particularly at low q^2.

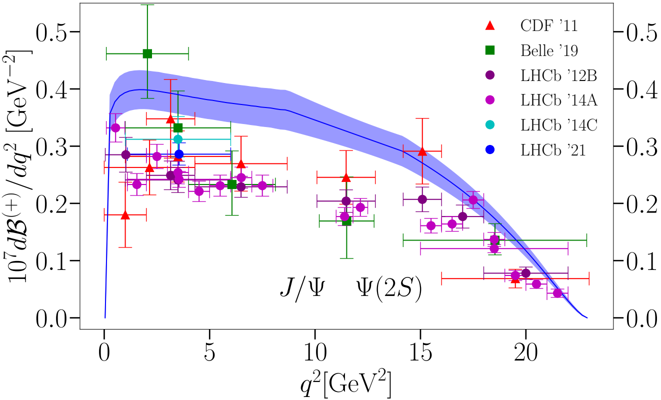
$B \to K\ell^+\ell^-$ phenomenology

As with $D \to K$ (above) we can use the form factors to get at the differential decay rate for $B \to K\ell^+\ell^-$:

$$\frac{d\Gamma^{B\to K\ell^+\ell^-}}{dq^2} = \mathcal{F}(q^2, f_0, f_+, f_T, W_i)$$

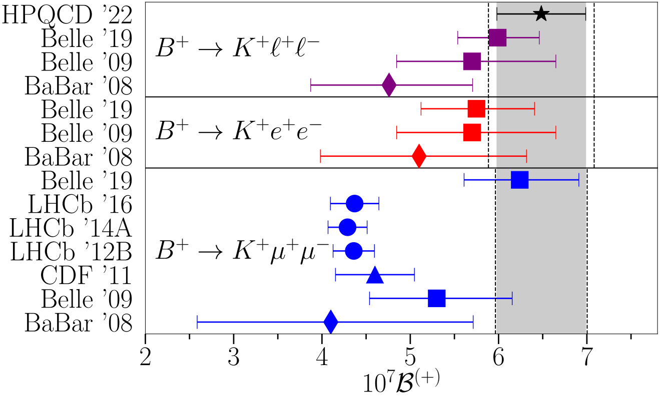
where W_i are Wilson coefficients and $\mathcal{F}$ is a complicated function (see e.g. arXiv 1510.02349). Does not account for $c\bar{c}$ resonances. We can compare this with experiment, in differential form and integrate to get $\mathcal{B} = \Gamma\tau_B$.

$B \to K\ell^+\ell^-$ phenomenology



We can compare the differential branching fraction with binned experimental data.

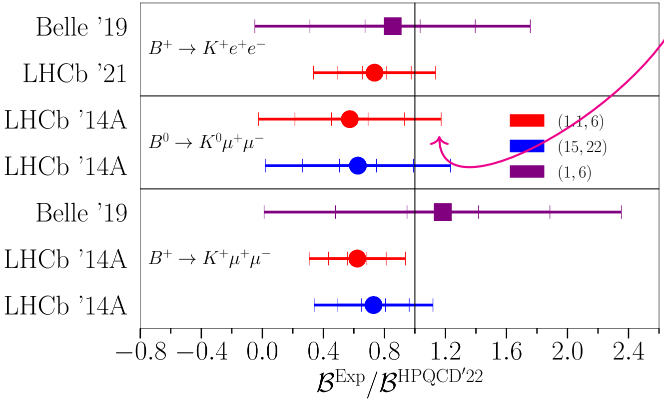
$B \to K\ell^+\ell^-$ phenomenology



Can also integrate across the whole q^2 range to get the branching fraction. Vetoed region contributes $\approx 15\%$.

$B \to K\ell^+\ell^-$ phenomenology

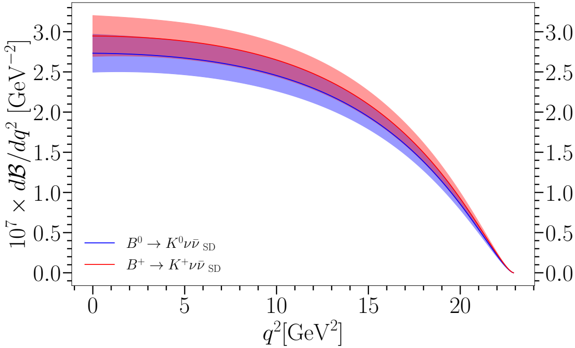
Caps @ 1, 3, 5 σ



We find large tensions in the theoretically clean regions of q^2: 1.1-6 GeV^2 and 15-22 GeV^2.

YORK UNIVERSITÉ UNIVERSITY

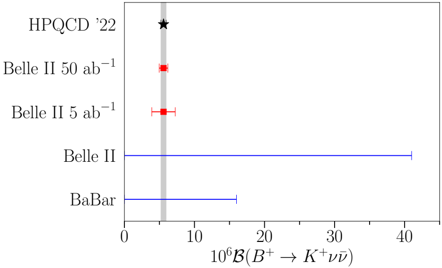
$B \to K\nu\bar{\nu}$ phenomenology

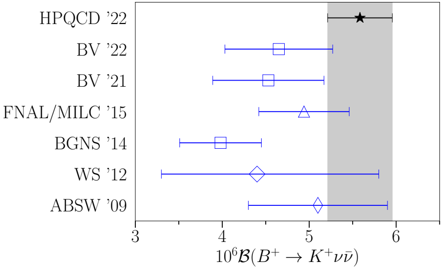


$$\frac{d\mathcal{B}(B \to K\nu\bar{\nu})_{\mathrm{SD}}}{dq^2} = \frac{G_F^2 \alpha_{\mathrm{EW}}^2 X_t^2}{32\pi^5 \sin^4\theta_W} \tau_B |V_{tb}V_{ts}^*|^2 |\vec{p}_K|^3 f_+^2(q^2)$$

YORK UNIVERSITÉ UNIVERSITY

$B \to K\nu\bar{\nu}$ phenomenology





Subsequent work on $B \to K\ell^+\ell^-$

Looking at the tension in $\mathcal{B}(B^+ \to K^+\mu^+\mu^-)_{[1.1,6]\ \mathrm{GeV}^2}$
Subsequent work [A. Buras *2209.03968*] uses NP clean ratios of $\Delta F = 2$ observables and our form factors - tension persists at a level of 5.1σ.
Other work [R. Fleischer et al. *2212.09575*] looked at the effect of inclusive vs exclusive $|V_{ts}V_{tb}|$ values, using our ffs. Found 3.5σ tension for hybrid inclusive (ex. $|V_{ub}|$ and inc. $|V_{cb}|$) and 2.4σ for the exclusive. We used a value close to their hybrid inclusive one, from B mixing [R.J. Dowdall et al. *1907.01025*]

Subsequent work on $B \to K\nu\bar{\nu}$

The work in [D. Becirevic et al. *2301.06990*] uses an average of form factors, including ours, to find $\mathcal{B}(B^+ \to K^+\nu\bar{\nu}) = 5.06(31) \times 10^{-6}$, slightly lower than the $5.58(37) \times 10^{-6}$ that we obtain.

[A. Issadykov et al. *2211.10683*] use a convarient confined quark model to obtain form factors, from which they obtain $\mathcal{B}(B^+ \to K^+\nu\bar{\nu}) = 4.96(74) \times 10^{-6}$, in agreement with our result.

Conclusions

- First fully relativistic calculation of $B \to K$ form factors
- Reduced uncertainty, particularly at low q^2
- $B \to K$ branching fractions show $3 - 5\sigma$ tension with LHCb in clean regions
- Branching fractions for $B \to K\nu\bar{\nu}$ now with $< 10\%$ error
- Belle II promised similar uncertainty at 50 ab^{-1} [*2101.11573*]
- Subsequent work using our form factors finds similar tensions

Thanks for listening. Any questions?