# Unbinned analyses of $b \to s\ell\ell$

## Beyond the Flavour Anomalies IV


Matthew Birch[1], Andrea Mauri[1], Rui Wang[2]

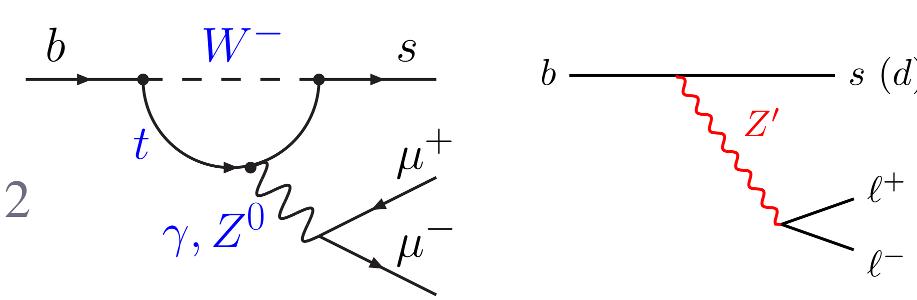
[1] *Imperial College London*, [2] *University of Bristol*


April 19, 2023

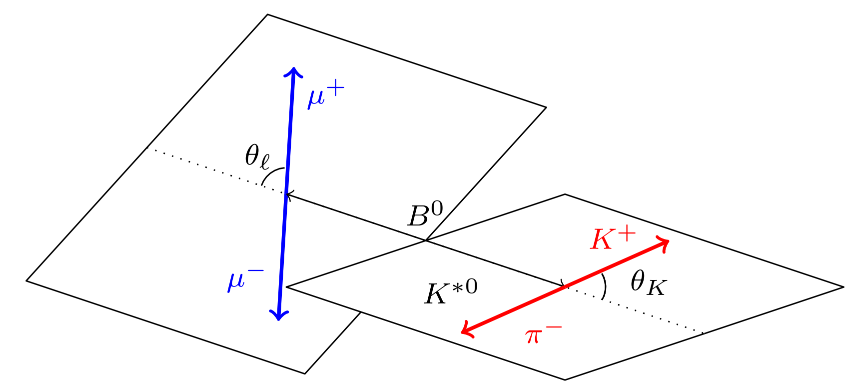
# Motivation

- $B^0 \to K^{*0}\mu^+\mu^-$ decays are mediated by FCNC
  - Suppressed in the SM $\longrightarrow$ sensitive to NP
  - Decay fully described by $\overrightarrow{\Omega} = (\cos\theta_L, \cos\theta_K, \phi), q^2$

SM vs NP



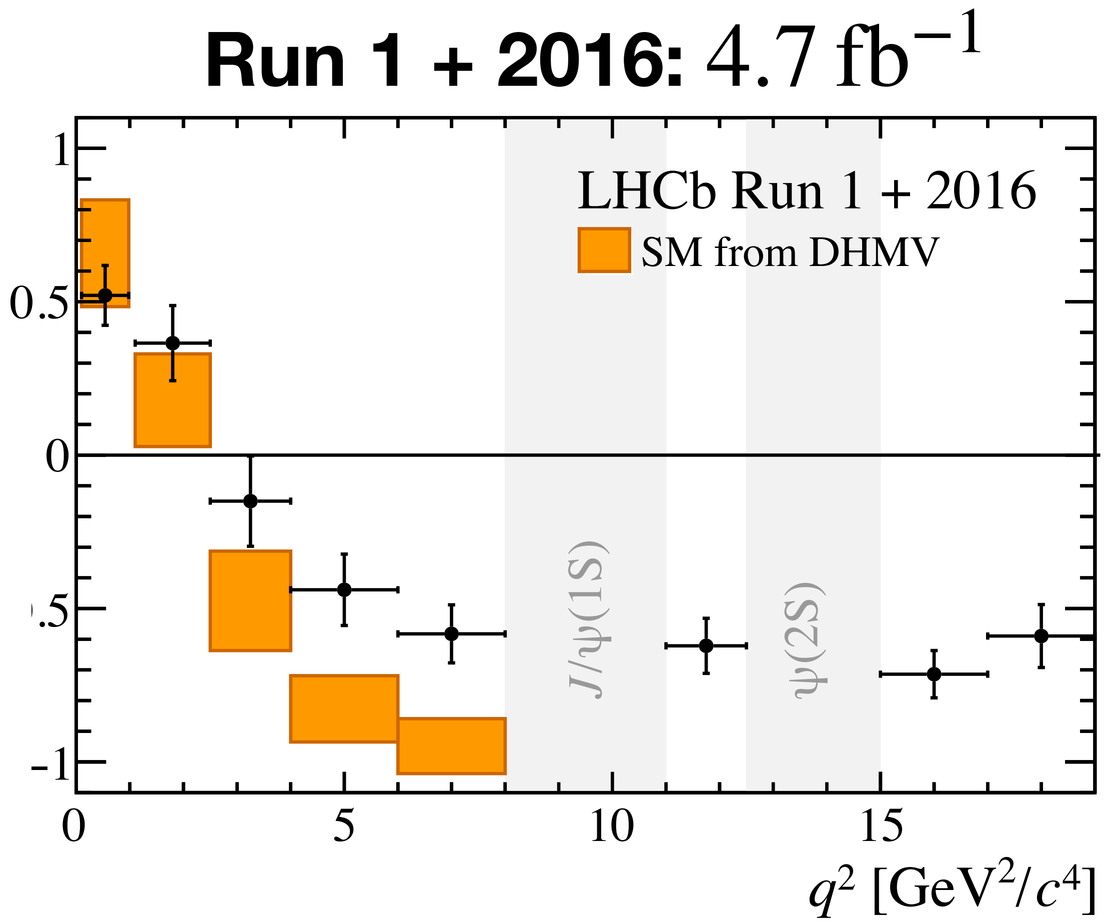

$$\frac{\mathrm{d}^4\Gamma[\bar{B}^0 \to \bar{K}^{*0}\mu^+\mu^-]}{\mathrm{d}q^2\,\mathrm{d}\vec{\Omega}} = \frac{9}{32\pi}\sum_i \underbrace{I_i(q^2)}_{\text{angular coefficients}}\underbrace{f_i(\vec{\Omega})}_{\text{angular functions}}$$

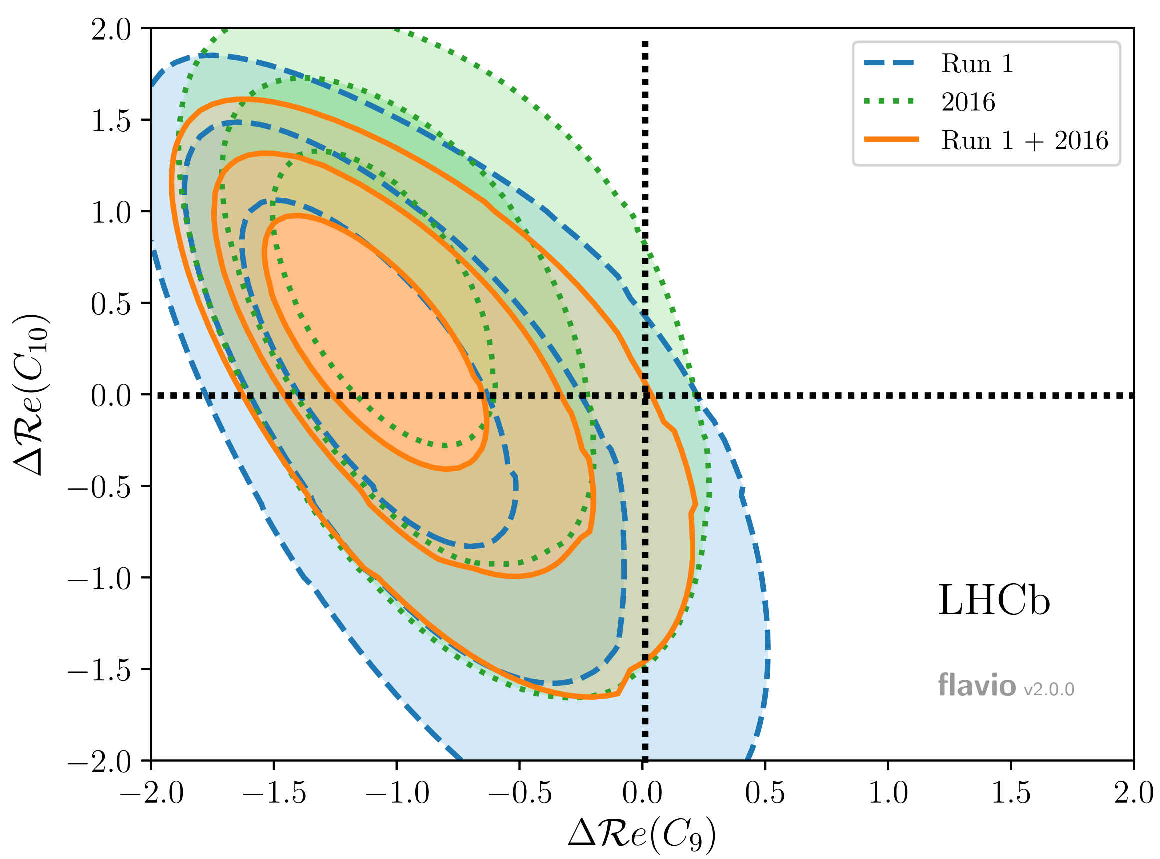



- NP particles can affect branching ratio and angular distributions

# Current status of the anomalies

- Angular analysis in bins of $q^2$
- Tension w.r.t. the SM




PRL 125 (2020) 011802

# How to move on

More data

More information

full Run1 + Run2

improved selection

binned ang. obs. + BR + A_CP

see next talk

Unbinned measurements

this talk

# Three K*mumu unbinned measurements

- Different choices of $q^2$ treatment

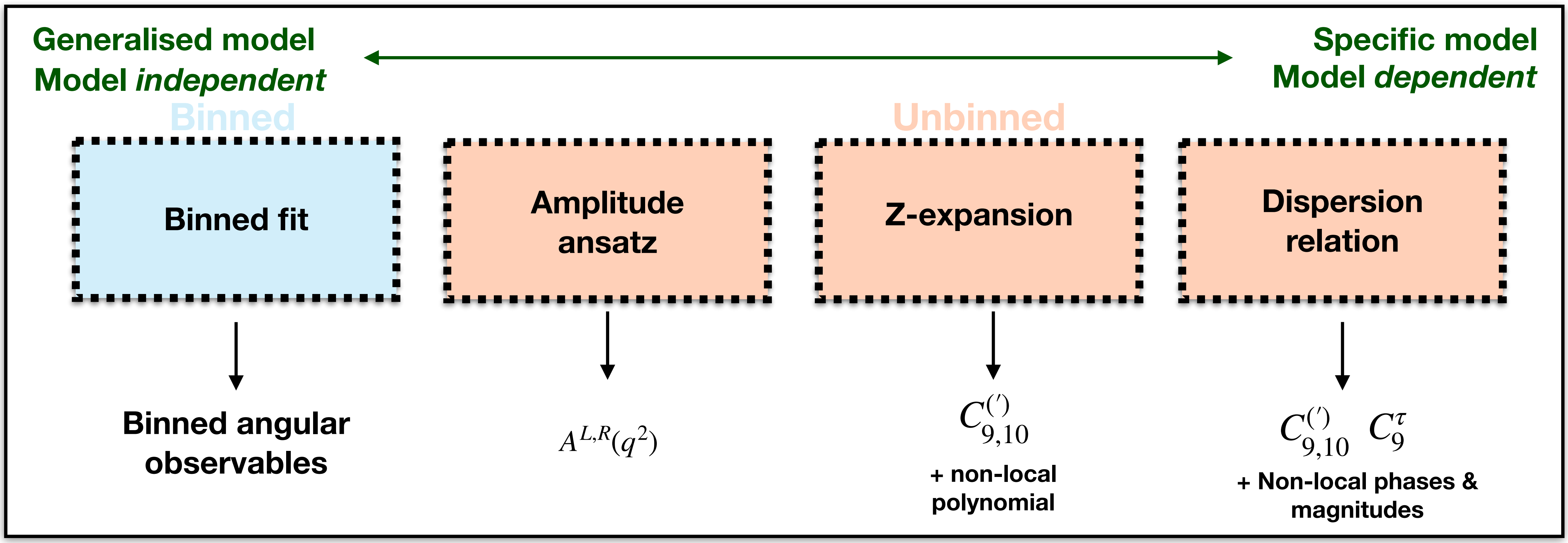
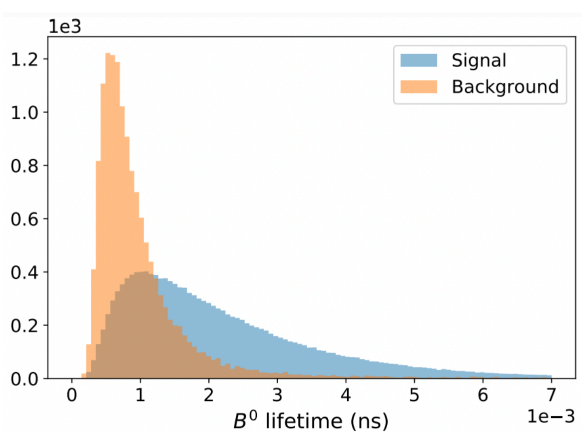
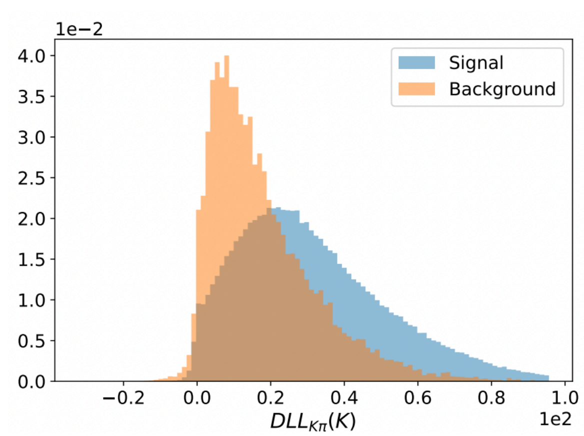


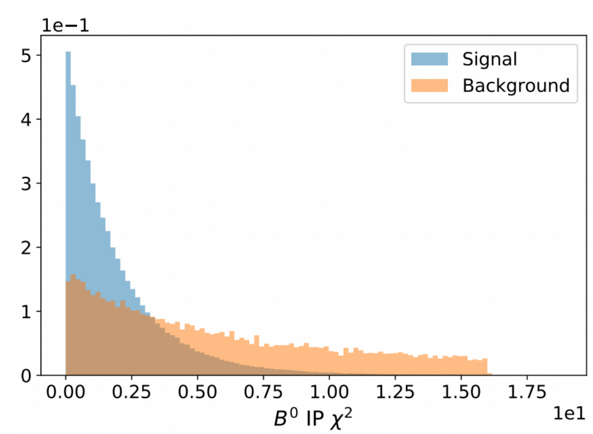

diagram by T. Hadavizadeh

# Commonalities

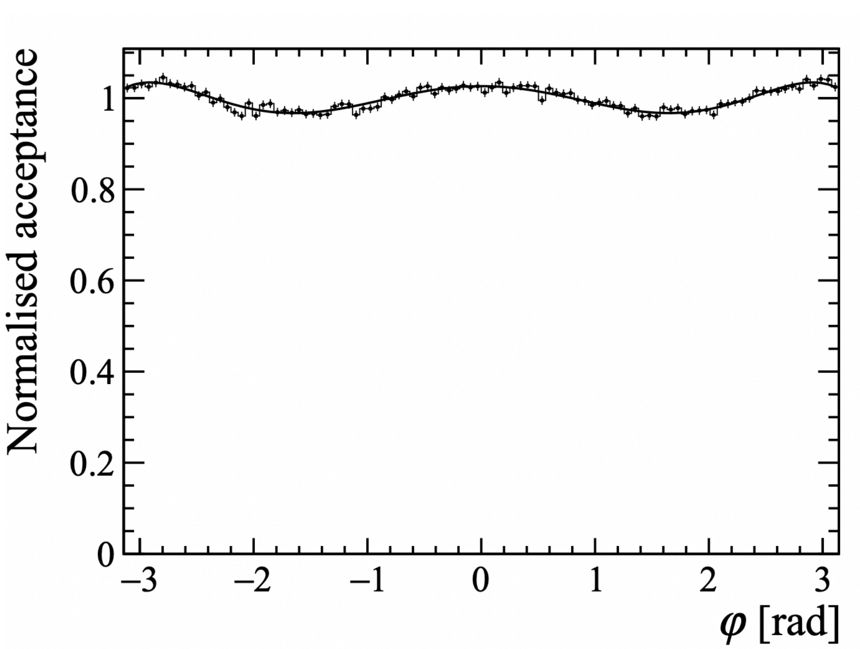
- Signal selection aligned where possible
- Background rejection
  - veto peaking backgrounds
  - remove combinatorial

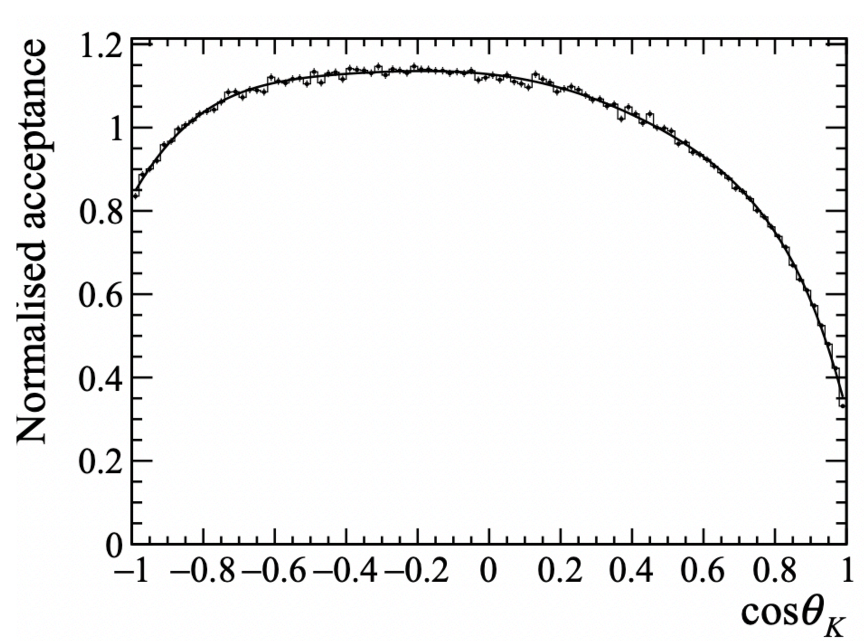


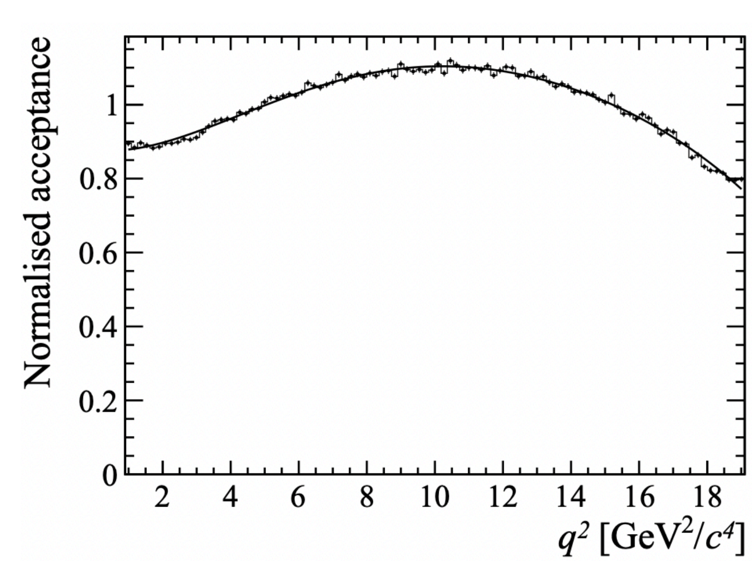




- Acceptance

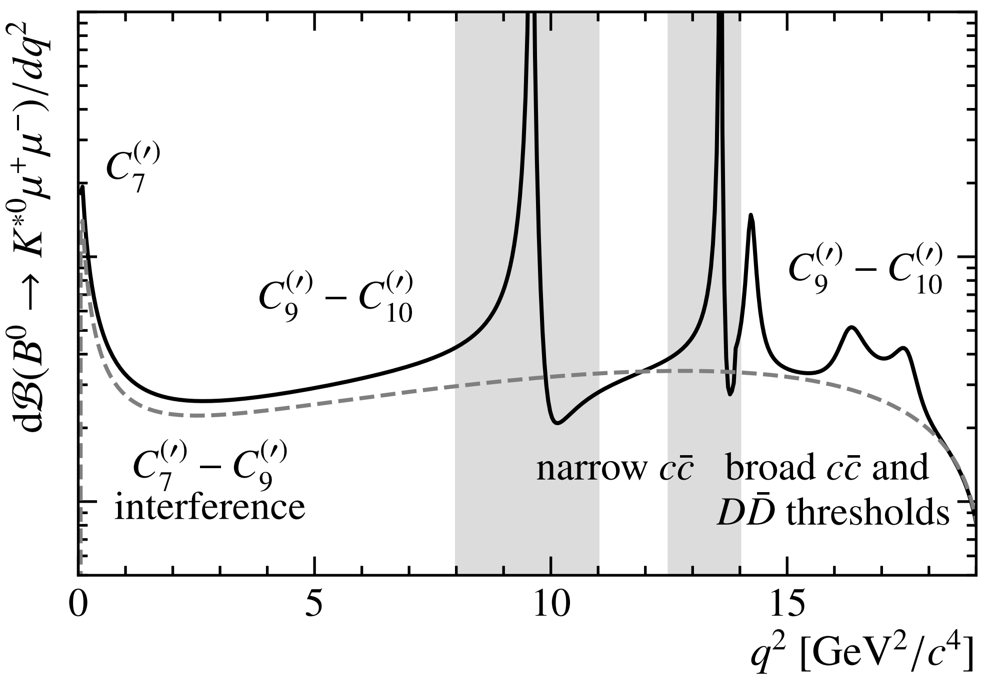


# Differences

## $q^2$ range




1.25 8.0 11.0 12.5

ansatz (Run1+2)

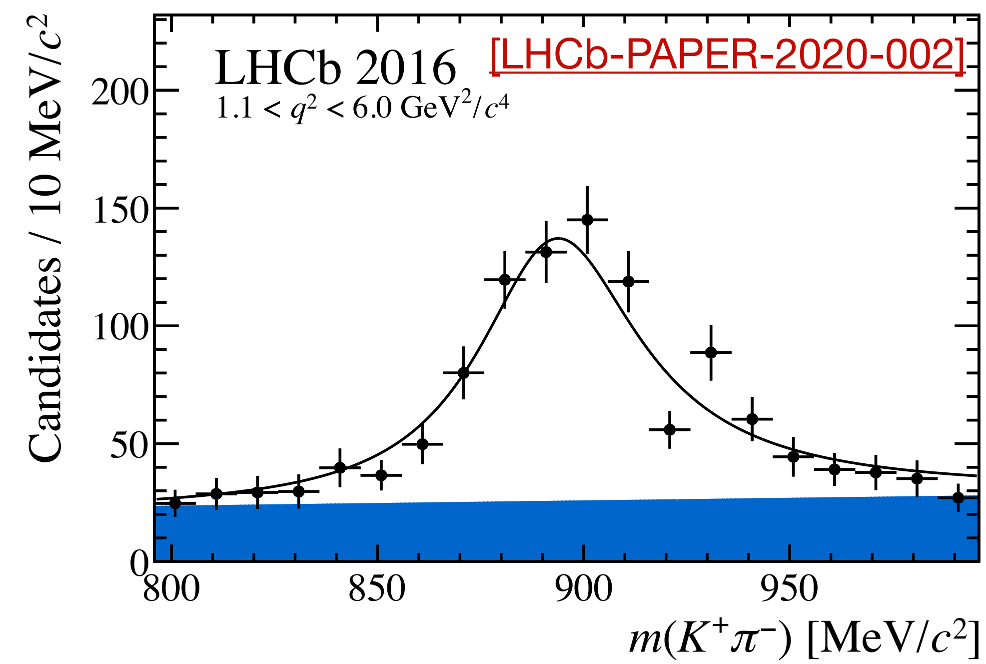
1.1 8.0 11.0 12.5

z-expansion (Run1+2016)

0.1 19.0

dispersion (Run1+2)

## $m_{K\pi}$ treatment




ansatz : integrated over

z-expansion : parametrized

dispersion : integrated over

# Ansatz

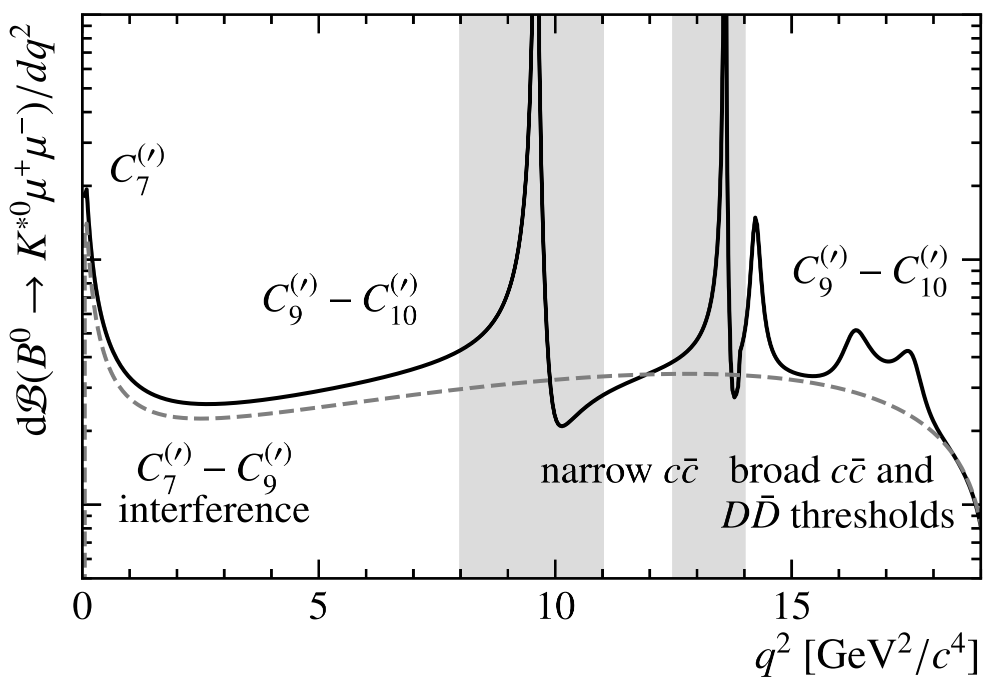
- Perform a measurement of the $q^2$ dependent amplitudes which as model independent as possible

*J. High Energ. Phys.* **2015**, 84 (2015)

- Apply the ansatz

$$A = \Sigma_i \alpha_i L_i(q^2)$$

to the amplitudes, where $L_i$ are Legendre polynomials of order $i$.




1.25 8.0 11.0 12.5

**P-wave** **4 parameters per amplitude** **2 parameters per amplitude**

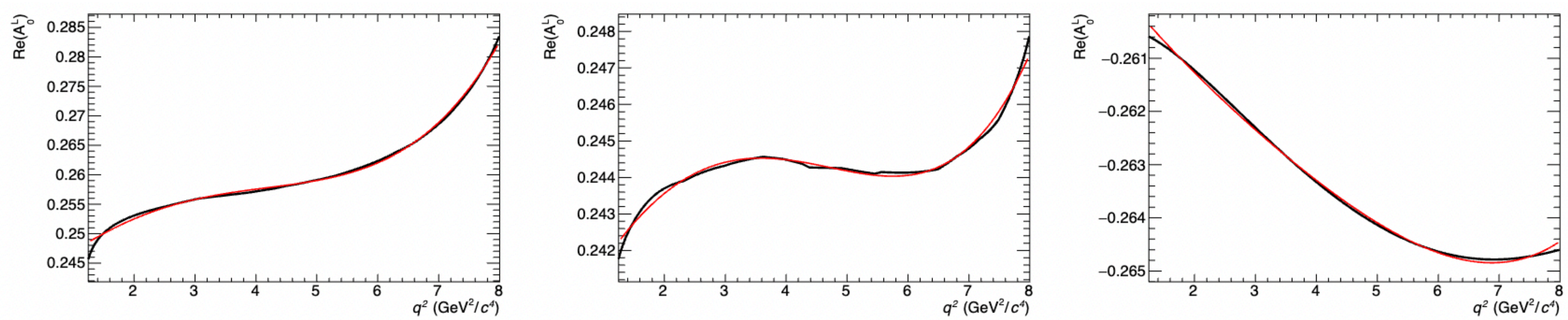
- Treat the S–wave to be flat in $q^2$ (i.e. nuisance parameters)

# Ansatz

- Due to symmetries in the PDF, need to define which amplitude basis to work in
- Work in the basis where

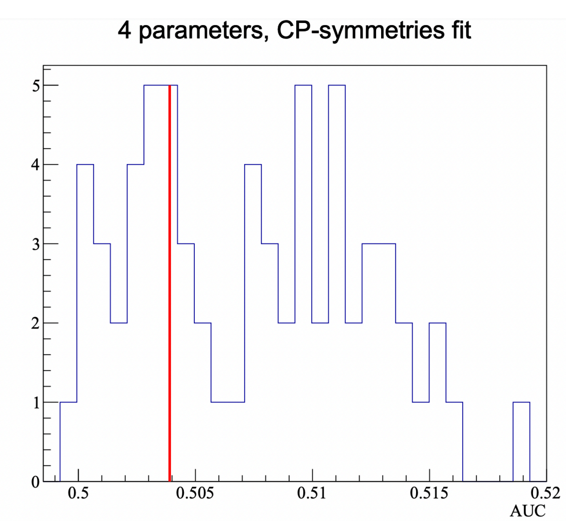
$$Im(A_\perp^R) = Im(A_0^L) = Re(A_0^R) = Im(A_0^R) = 0$$

- Our amplitude ansatz can describe a variety of models




**SM**, $J/\psi$ and $\phi$ phases $= 0$ **SM**, phases $= \frac{\pi}{2}$ **NP** with $\Delta C_9 = -1$

- and validated by goodness_of_fits to the data



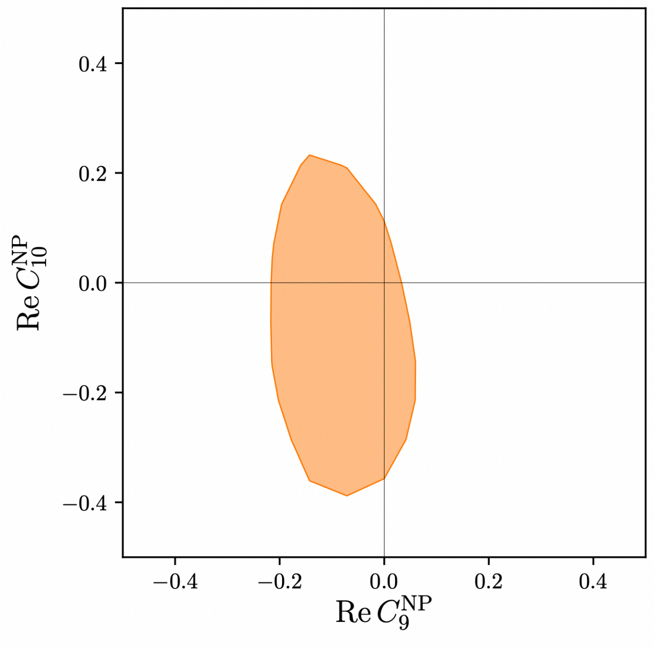

p-value = 70%

# Ansatz

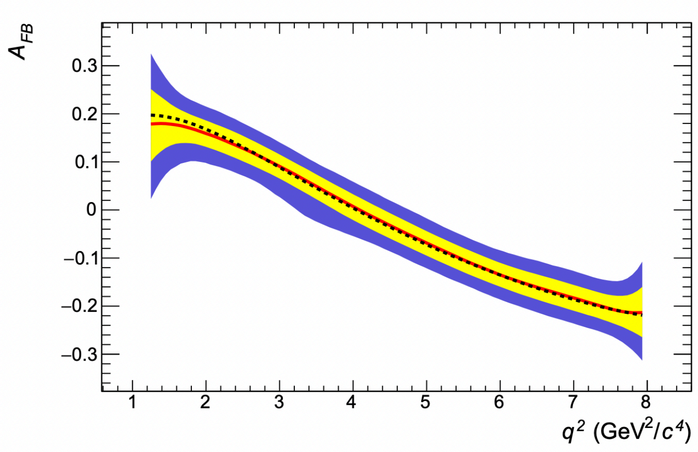
- Publish the P-wave amplitude coefficients with covariance matrix.

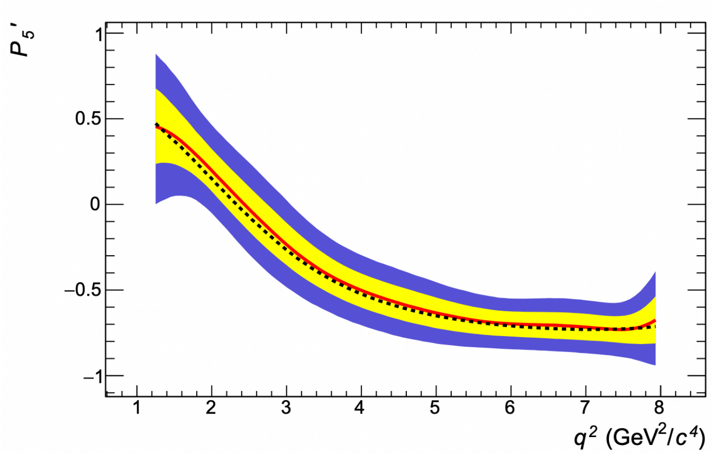
  - **A model-independent* parameterisation of the LHCb dataset which can be used to generate synthetic datasets and fit back with any choice of model!**

- Example use:




- Can also compute the amplitudes and thus the observables directly from the fit results






* model-independent up to the choice of polynomial order

# z-expansion

- SM description of local amplitudes
  - Wilson coefficients
  - form factors
- Parametric form (polynomials) for non-local contributions

$$\mathcal{A}_\lambda^{L,R} = \mathcal{N}_\lambda \left\{ \left[ (\mathcal{C}_9 \pm \mathcal{C}_9') \mp (\mathcal{C}_{10} \pm \mathcal{C}_{10}') \right] \mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2} \left[ (\mathcal{C}_7 \pm \mathcal{C}_7') \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

z-expansion for non-local contributions (H)

$$\mathcal{H}_\lambda(z) = \frac{1 - z z_{J/\psi}^*}{z - z_{J/\psi}} \frac{1 - z z_{\psi(2S)}^*}{z - z_{\psi(2S)}} \times \; \ldots \; \times \sum_n \alpha_{\lambda,n} p_n(z)$$

# z-expansion : the non-local contribution

Combine theory & experimental information

- Prior knowledge can be used to constrain polynomial parameters
- enters as constraints on the value of $\mathcal{H}(q^2)$ at ψ poles and negative $q^2$

**External experimental inputs**

PRD 76 031102(R) (2007)

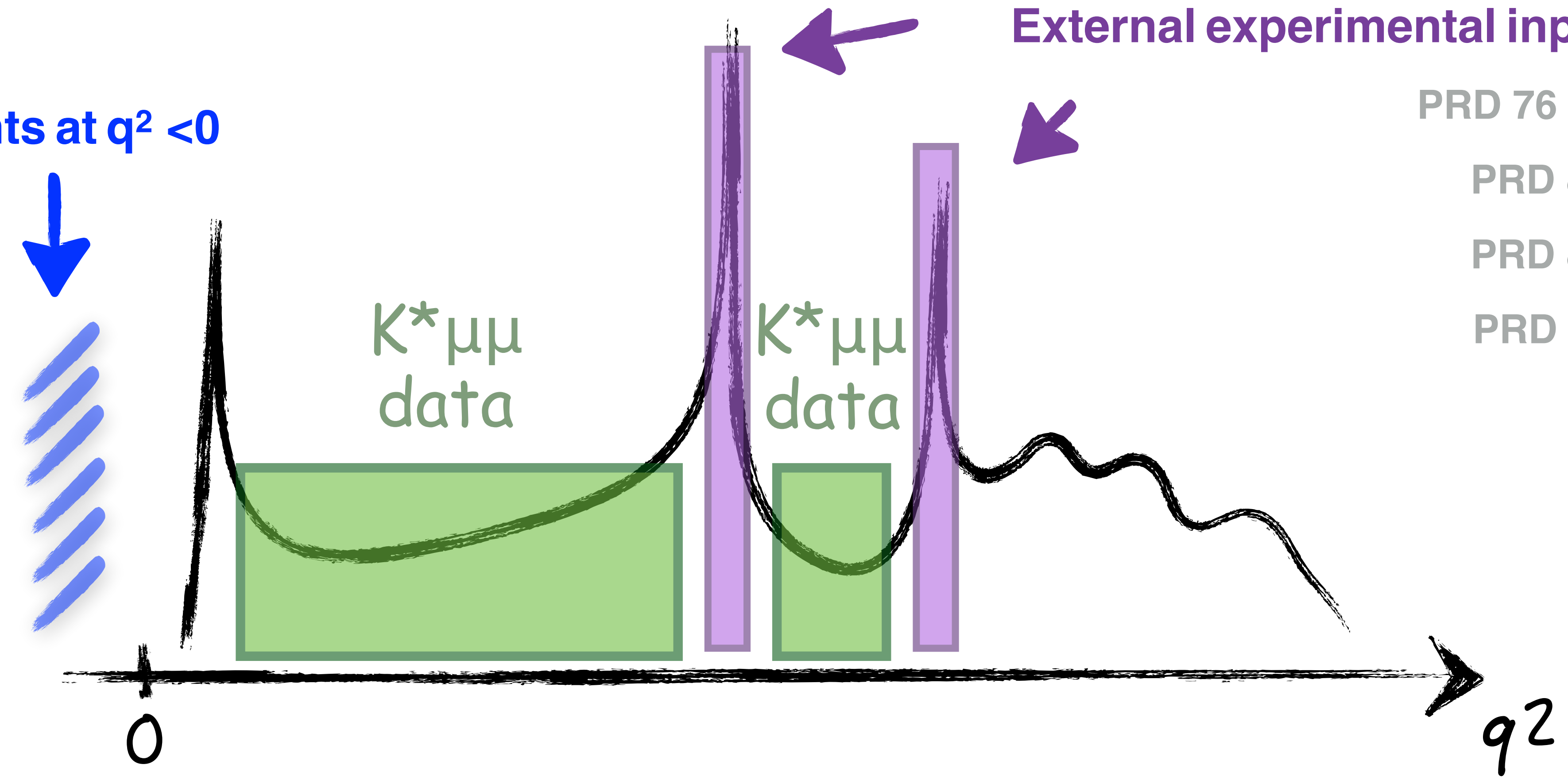
PRD 88 052002 (2013)

PRD 88 074026 (2013)

PRD 90 112009 (2014)

**Theory points at q² <0**

*JHEP* 09 (2022) 133




Fit provided in two configurations:

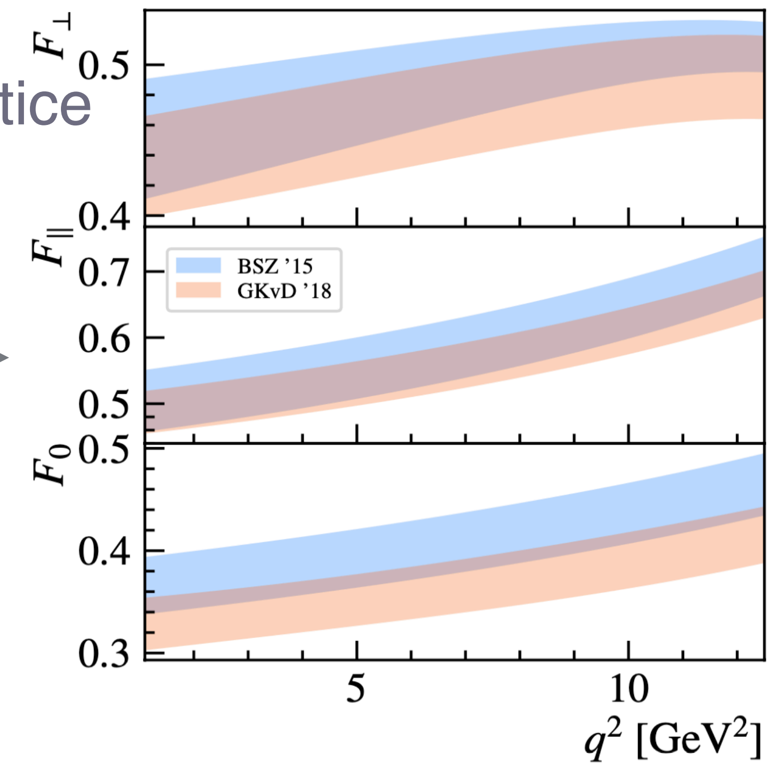
- *with theory:* include theory points @ q2<0
- *without theory:* exclude theory points @ q2<0

# Challenge #1: The form factors

Should not forget the importance of the FF

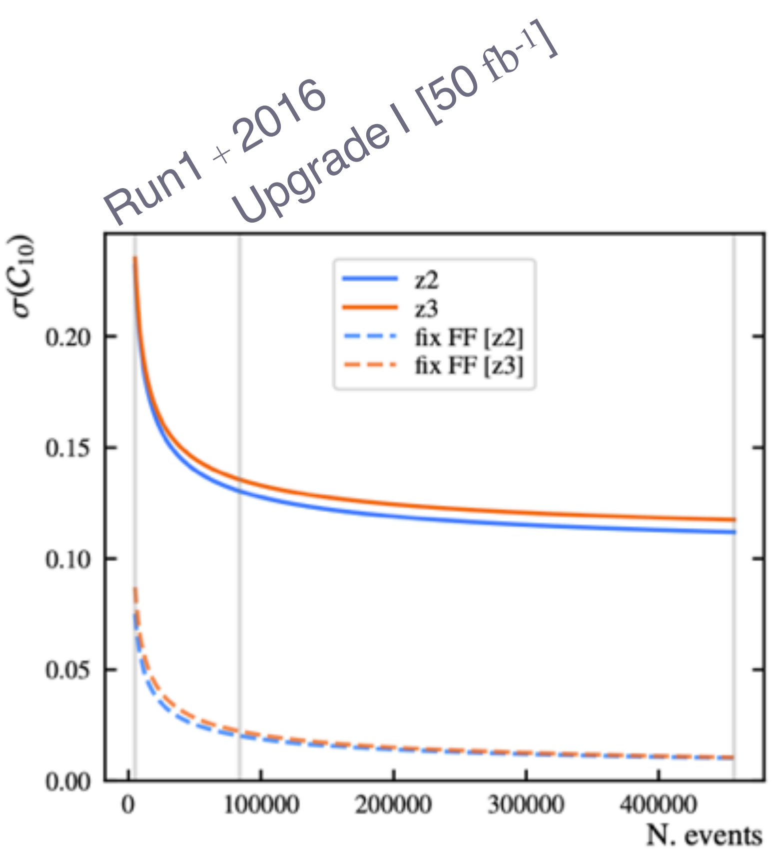
- P-wave FF constrained to theory predictions from LCSR + Lattice
- Several sets of values available in the literature
  - BSZ '15 arXiv:1503.05534
  - GKvD '18 arXiv:1811.00983
  - Virto et al. arXiv:1908.02267 → suggest 9% increase due to K* width

home-made comparison →



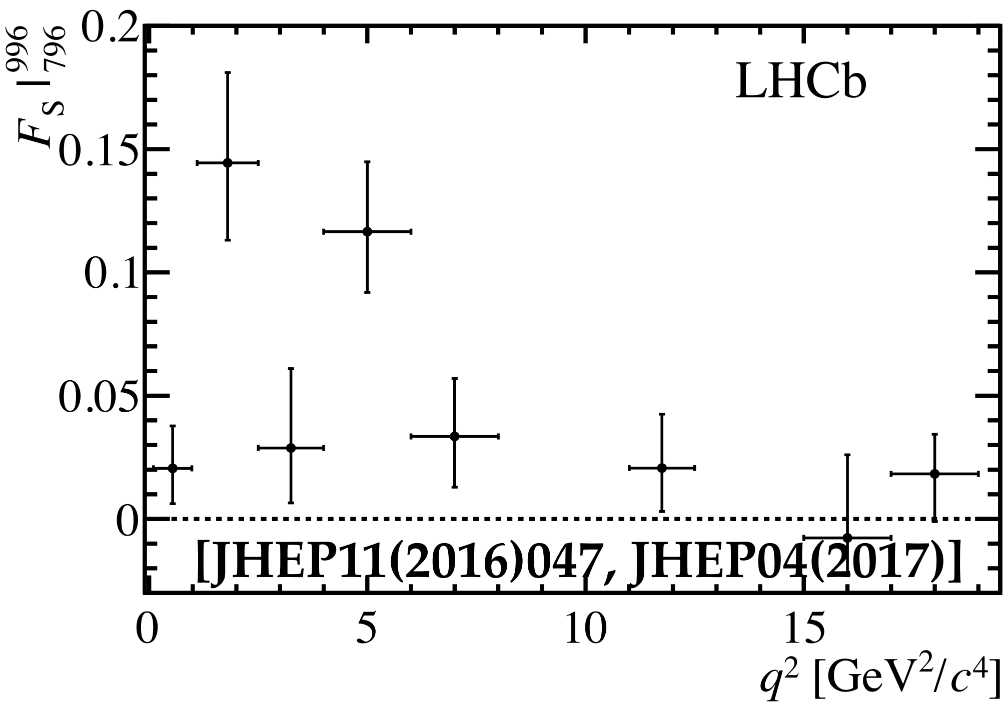

→ Direct impact on the branching ratio…

They will soon become a limiting factor to the determination of the Wilson coeff.



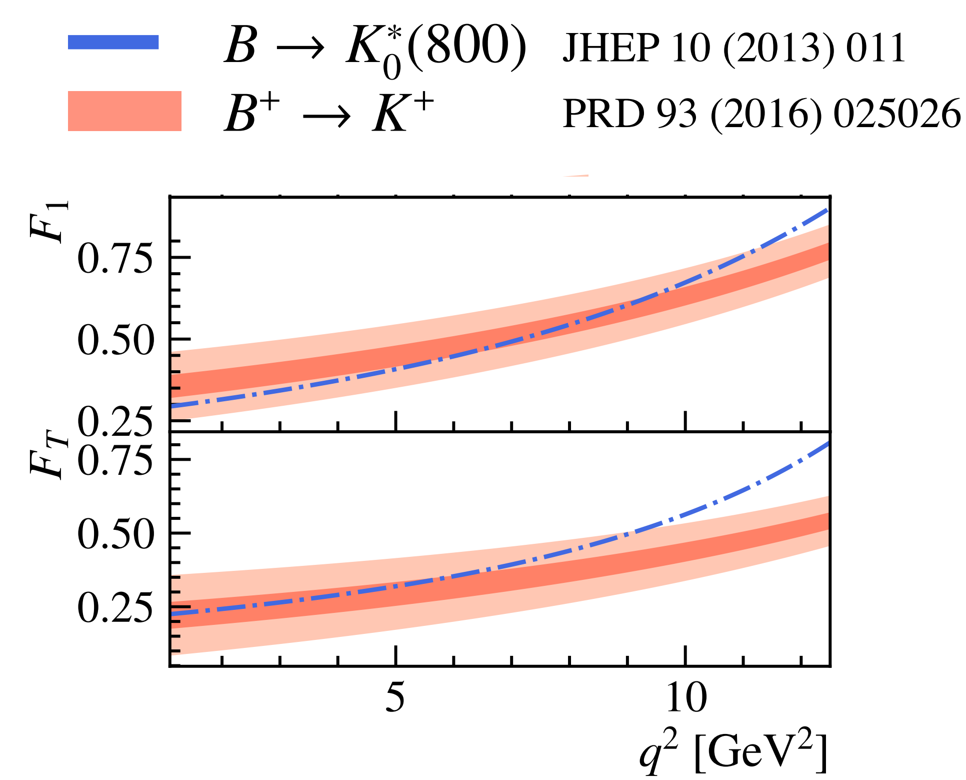
# Challenge #2: The S wave

- Non-negligible S wave component in $796 < m_{K\pi} < 996$ MeV
- Independently if you model $m_{K\pi}$ or not, scalar $q^2$ amplitude must be included




$$A_{00}^{L,R}(q^2) \propto \sqrt{\beta_\ell}\lambda_{K_0^*}\left[(C_9 \mp C_{10})f_+(q^2) + C_7 2m_b \frac{f_T(q^2)}{(m_B + m_{K_0^*})}\right]$$

- S wave form factor poorly known
  - different possible choices (all non ideal)
  - requires systematic uncertainty



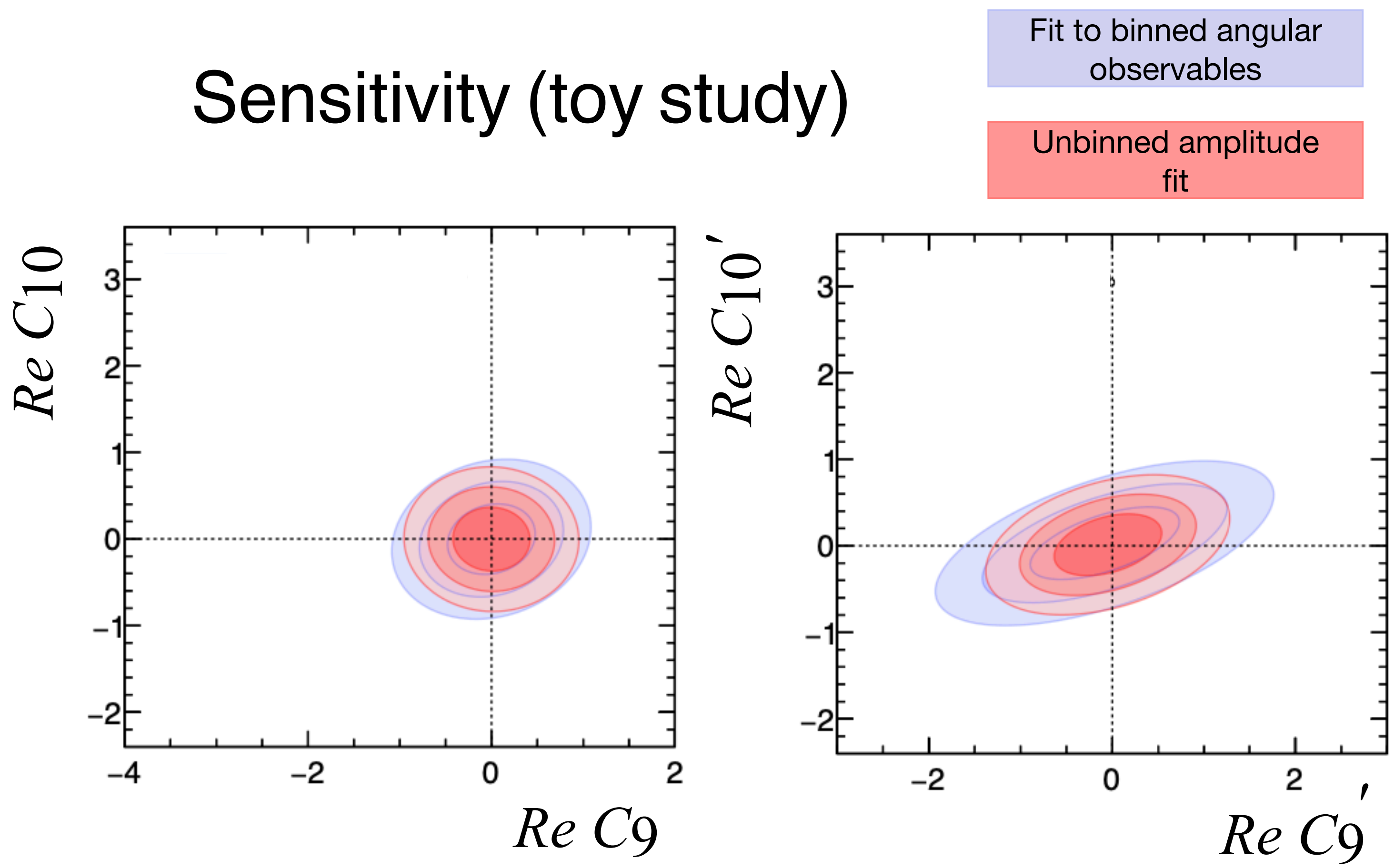
# z-expansion : overview

Aiming to publish:

- Wilson coefficients C9, C10, C9’, C10’
  - 1D and 2D likelihood profile
- Form factors
- Non-local hadronic parameters (polynomial coeff.)

Sensitivity (toy study)

Improved sensitivity w.r.t. binned approach



# Dispersion relations

- Amplitude described in terms of effective Wilson coefficients

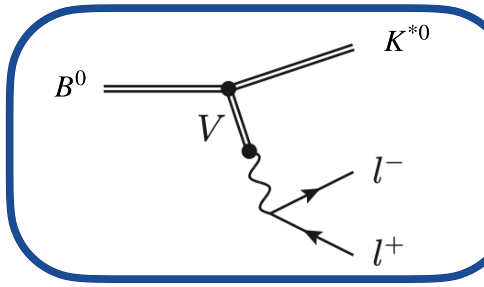
$$C_7^{\mathrm{eff},\lambda}(q^2) = C_7^{\mu} + \epsilon^{\lambda} e^{i\omega^0}$$

$$C_9^{\mathrm{eff},\lambda}(q^2) = C_9^{\mu} + \boxed{Y_{c\bar{c}}^{(0),\lambda}} + \boxed{Y_{c\bar{c}}^{1P,\lambda}(q^2) + Y_{\mathrm{light}}^{1P,\lambda}(q^2)} + \boxed{Y_{c\bar{c}}^{2P,\lambda}(q^2)} + \boxed{Y_{\tau\bar{\tau}}(q^2)}$$

This is determined theoretically at negative $q^2$ values

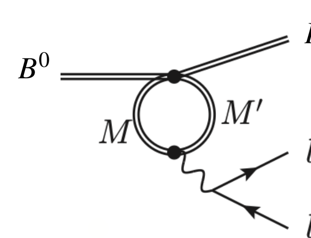
**Constant offset**

Asatrian, Greub, Virto [JHEP 04 (2020) 012]



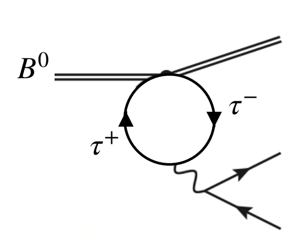

**1-particle contributions**

**Includes:**
$\omega(782)$, $\rho(770)$, $\phi(1020)$, $J/\psi$, $\psi(2S)$, $\psi(3770)$, $\psi(4040)$, $\psi(4160)$

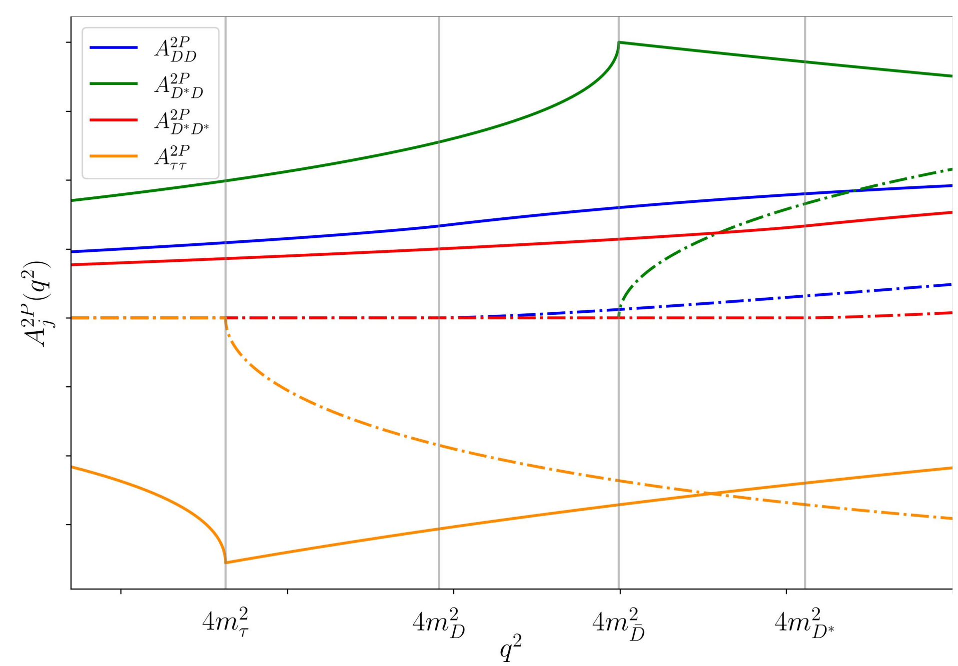



**2-particle contributions**

**Includes:**
$D\overline{D}$, $D^*\overline{D}$, $D^*\overline{D}^*$



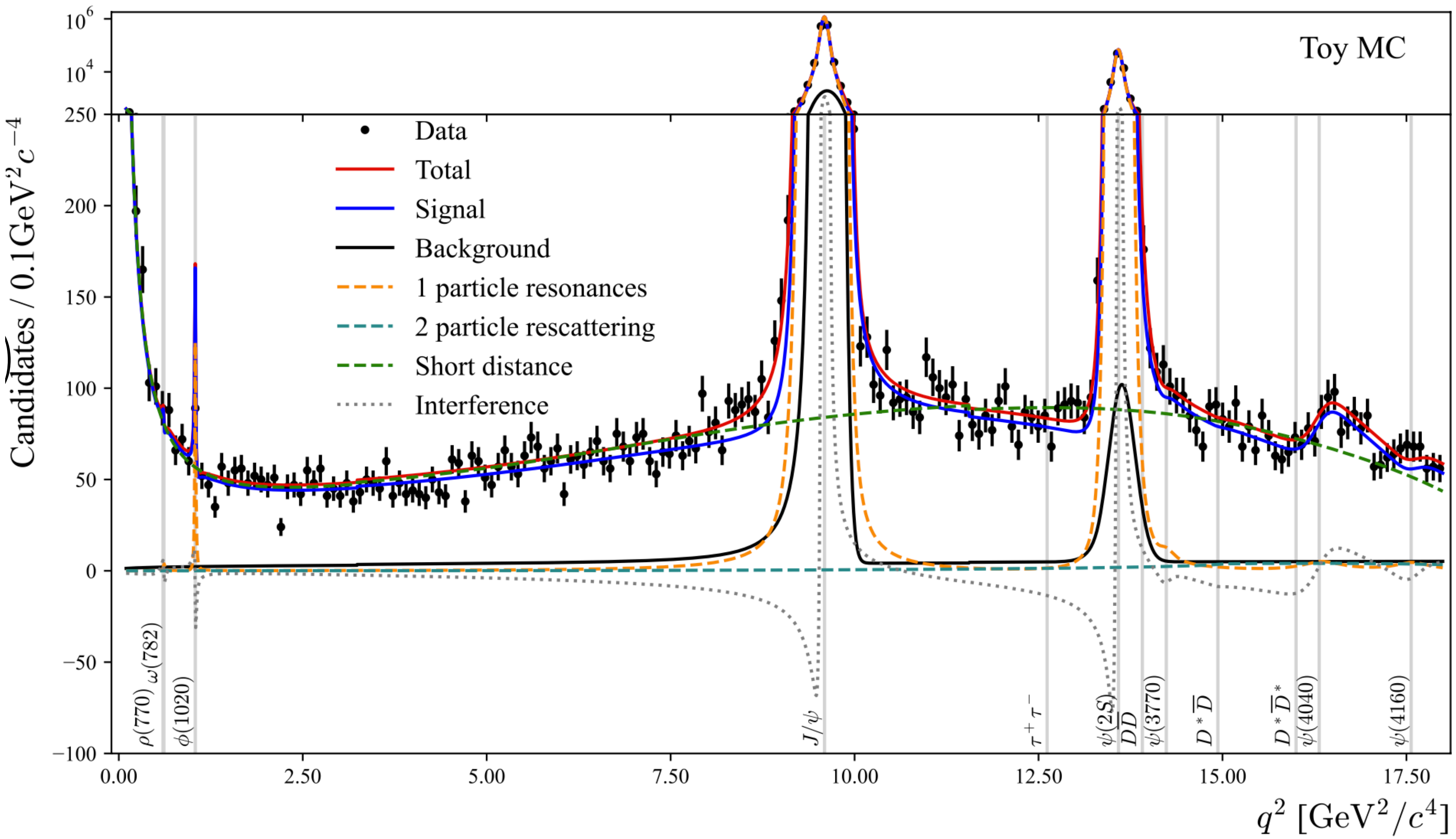

**Tau loop contribution**




C. Cornella, G. Isidori, M. König, S. Liechti, P. Owen, N. Serra [Eur.Phys.J.C 80 (2020) 12, 1095]

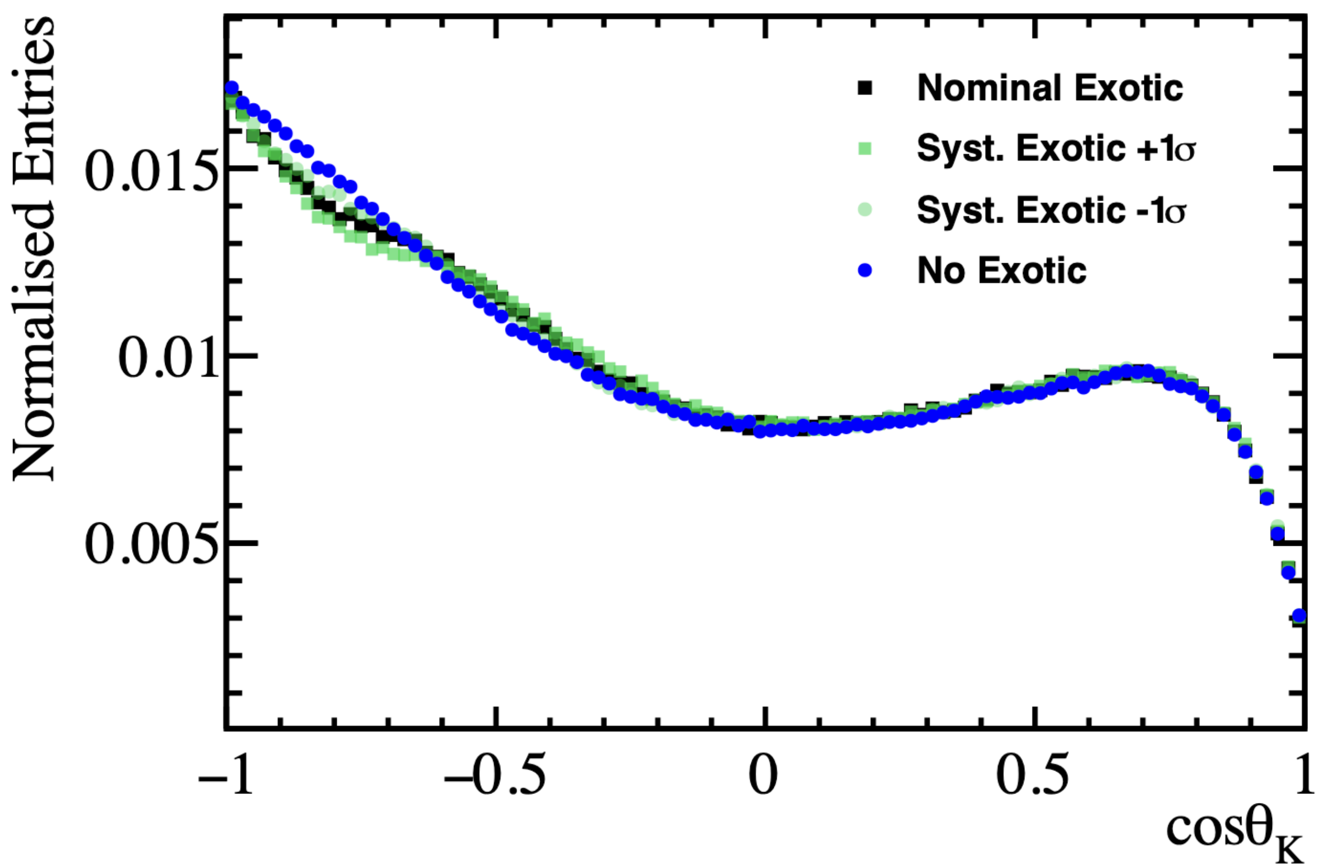
# Challenge #3: resolution

- Fit full $q^2$
- Measure everything w.r.t. $B^0 \to J/\psi K^*$ longitudinal amplitude
- q2 resolution much larger than natural widths of $J/\psi$ and $\psi(2S)$
  - Requires convolution in the pdf



# Challenge #4: Exotic contribution to $B^0 \to J/\psi K^*$

- $B^0 \to J/\psi K^*$ polluted by exotic $B^0 \to ZK^+$ with
  - $Z(4200)^- \to J/\psi\pi^-$
  - $Z(4430)^- \to J/\psi\pi^-$
  - $Z(4430)^- \to \psi(2S)\pi^-$
- Influence observed yield and distorts $\cos\,\theta_\mathrm{K}$



# Dispersion relations: overview

Aiming to publish:

- Wilson coefficients $|C_9^\mu|, \ |C_{10}^\mu|, \ \mathrm{Re}(C_9'), \ \mathrm{Re}(C_{10}'), \ \mathrm{Re}(C_9^\tau)$
- Relative sign between $C_9$ and $C_{10}$
- Form factors
- Magnitude and phase of 1P resonances
- Real and Imaginary $D^{(*)}\bar{D}^{(*)}$
  - unable to float all $D\bar{D}, D^*\bar{D}, D^*\bar{D}^*$ components
  - constrain one to an other
- $\Delta C_7$

| Blinded parameters | $\|C_9\|$ | $\|C_{10}\|$ | $\mathscr{R}(C_9')$ | $\mathscr{R}(C_{10}')$ | $\mathscr{R}(C_9^\tau)$ |
|---|---|---|---|---|---|
| Absolute sensitivity | 0.24 | 0.14 | 0.34 | 0.17 | 232 |

Sensitivity to $C_9^\tau$ better than current limit

Current limit on $\mathscr{B}(B^0 \to K^{*0}\tau^+\tau^-)$ by Belle (2021) : $2.00 \times 10^{-3}$ at 90% CL

$\rightarrow C_9^\tau < 450$

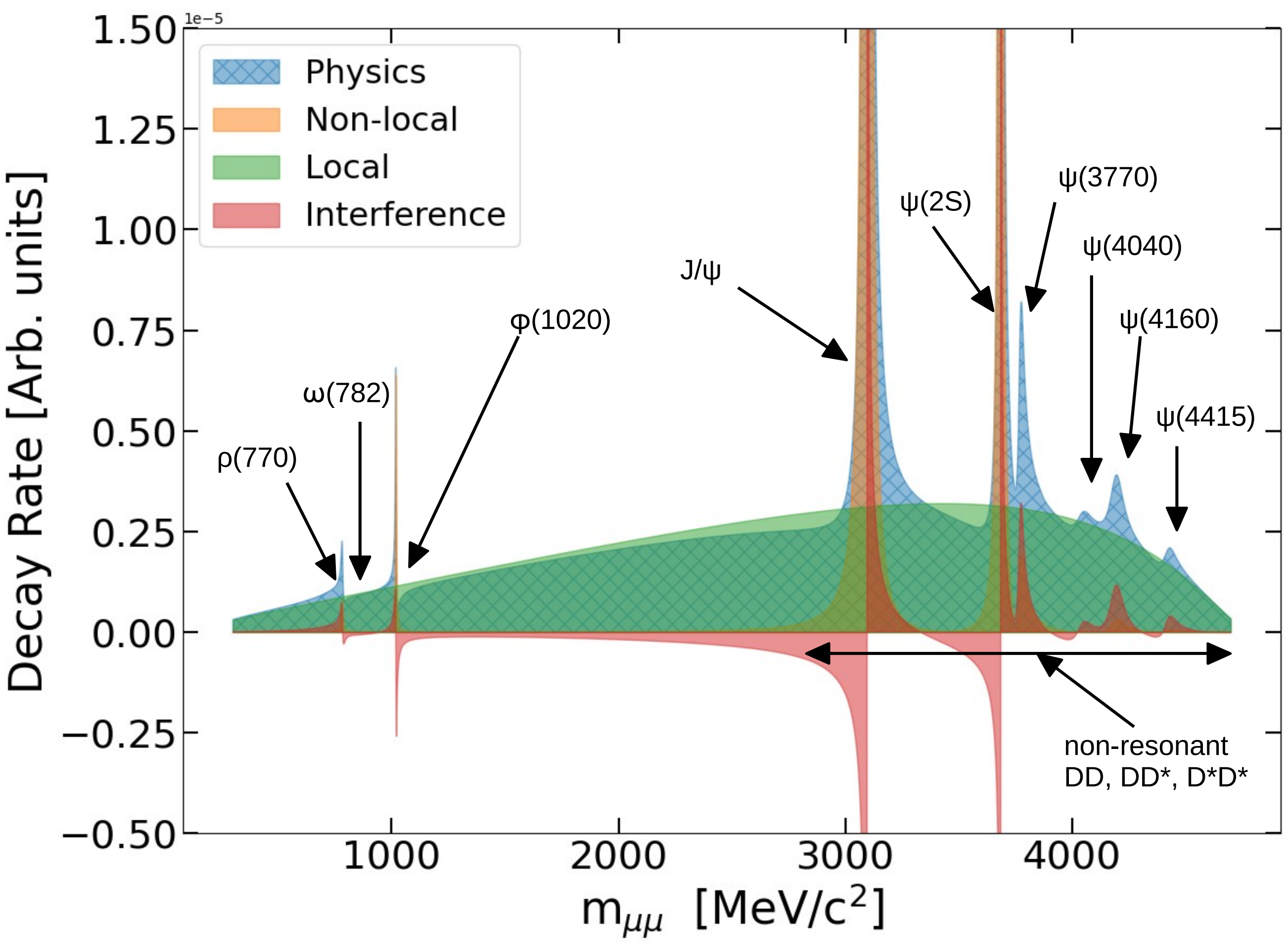
- Fit for the complex Wilson Coefficients is also in progress

# Dispersion relations: $B^+ \to K^+\mu^+\mu^-$

- Similar strategy but simpler angular structure
- Differential decay rate

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2\alpha^2|V_{tb}V_{ts}^*|^2}{2^7\pi^5}|k|\beta\left\{\frac{2}{3}|k|^2\beta^2\left|C_{10}f_+(q^2)\right|^2 + \frac{m_\mu^2(m_B^2-m_K^2)^2}{q^2m_B^2}\left|C_{10}f_0(q^2)\right|^2 + |k|^2\left[1-\frac{1}{3}\beta^2\right]\left|C_9^{eff}f_+(q^2)+2C_7^{eff}\frac{m_b+m_s}{m_B+m_K}f_T(q^2)\right|^2\right\},$$

- New HPQCD July 2022 form factors
  - fixed due to fit instabilities (→ syst.)
- Measure:
  - Wilson coeff.
  - 1P and 2P mag. and phase
  - $C_9^\tau$
  - 4-fold degeneracy observed in $\psi$'s phases



# Summary and conclusions

**Generalised model**
**Model *independent***

**Specific model**
**Model *dependent***

Binned | Unbinned

| **Binned fit** | **Amplitude ansatz** | **Z-expansion** | **Dispersion relation** |
|---|---|---|---|
| **Binned angular observables** | $A^{L,R}(q^2)$ | $C^{(\prime)}_{9,10}$ **+ non-local polynomial** | $C^{(\prime)}_{9,10}$ $C^{\tau}_{9}$ **+ Non-local phases & magnitudes** |

Ansatz: Quasi-model-independent measurement of the $q^2$ dependence of the transversity amplitudes

z-expansion: Measurement of $C^{(\prime)}_9, C^{(\prime)}_{10}$, constrain form-factors, quasi-model-independent measurement of the non-local contributions

Dispersion: Measurement of $C^{(\prime)}_9, C^{(\prime)}_{10}, C_\tau$, non-local phases and magnitudes relative to the $J/\psi K^{*0}$ longitudinal amplitude

**Status:** Pre-WG review | Collaboration review | WG review

Exciting times ahead!