An unbinned approach to measuring the $B^{\pm} \to \pi^{\pm}\mu^{+}\mu^{-}$ decay rate

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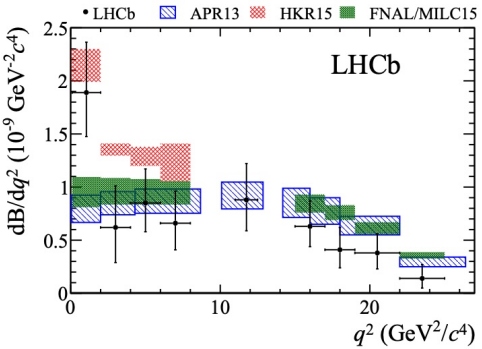
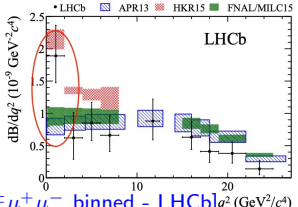
Introduction

- Recent developments from both theory and experiment allow for the possibility of unbinned measurements of channels such as $B^{\pm} \to K^{\pm}\mu^+\mu^-$ and $B^0 \to K^{*0}\mu^+\mu^-$.
- An unbinned approach exploits the full q^2 shape information.
- Fit the q^2 spectra of $B^{\pm} \to \pi^{\pm}\mu^+\mu^-$ extracting C_9 (+ phase) and C_{10}, floating the majority of non-local parameters.
- The lower stats of this channel motivates incorporating a constraint from theory to help pin down hadronic contributions.
- Such an approach maximises the experimental sensitivity to new physics contributions in $B^{\pm} \to \pi^{\pm}\mu^+\mu^-$ decays.

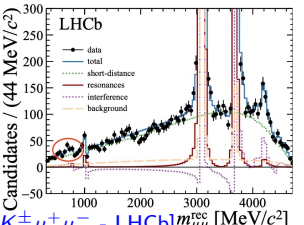
- Previous binned (in q^2) measurement of $\mathcal{B}(B^{\pm} \to \pi^{\pm}\mu^+\mu^-)$ and $\mathcal{A}_{CP}(B^{\pm} \to \pi^{\pm}\mu^+\mu^-)$ [$B^{\pm} \to \pi^{\pm}\mu^+\mu^-$ binned - LHCb]



Differences with respect to $B^{\pm} \to K^{\pm}\mu^{+}\mu^{-}$



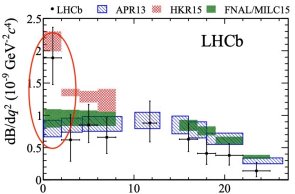
$[B^{\pm} \to \pi^{\pm}\mu^{+}\mu^{-}$ binned - LHCb$]$



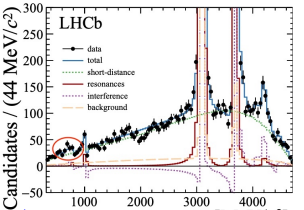
$[B^{\pm} \to K^{\pm}\mu^{+}\mu^{-}$ - LHCb$]$

- $|V_{ts}/V_{td}|^2 \approx 22$ - greatly reduced decay rate across the board.
  - Is this the case for any NP? Is NP minimal flavour violating?
- The ρ and ω resonances are more significant (relative to EW penguin mode) due to the additional *CKM* suppression of EW penguin mode.
- Contributions from weak annihilation and light quark loops (the light quark continuum) cannot be ignored as they are in $B^{\pm} \to K^{\pm}\mu^{+}\mu^{-}$, for the same reason.
- Fitting B^{+} and B^{-} separately is essential due to the potential for large *CP*-asymmetries in $B^{\pm} \to \pi^{\pm}\mu^{+}\mu^{-}$ - even in the SM.

Differences with respect to $B^{\pm} \to K^{\pm}\mu^{+}\mu^{-}$



$[B^{\pm} \to \pi^{\pm}\mu^{+}\mu^{-}$ binned - LHCb$]$



$[B^{\pm} \to K^{\pm}\mu^{+}\mu^{-}$ - LHCb$]$

Experimental considerations:

- Larger combinatorial background (generally more pions) requires a tighter selection $\to$ signal loss.
- Different physics background considerations - for example, significant $B^{\pm} \to K^{\pm}_{\to \pi}\mu^{+}\mu^{-}$.
- Can potentially nicely avoid floating any resolution parameters - by extracting these parameters from the higher statistics $B^{\pm} \to K^{\pm}\mu^{+}\mu^{-}$ results.

Describing the decay rate

- The $B^{\pm} \to \pi^{\pm}\mu^{+}\mu^{-}$ decay can be fully described with two variables, q^2 and $cos(\theta_\ell)$.
- For now we integrate over $cos(\theta_\ell)$.

The decay rate is then as follows[1]:

$$\begin{aligned}\frac{d\Gamma(B^{\pm} \to \pi^{\pm}\mu^{+}\mu^{-})}{dq^2} = & \frac{G_F^2\alpha^2|V_{tb}V_{td}^*|^2}{2^7\pi^5}|\boldsymbol{k}|\Bigg\{\frac{2}{3}|\boldsymbol{k}|^2\beta_+^2|C_{10}f_+(q^2)|^2 \\ & + \frac{m_\ell^2(M_B^2 - M_\pi^2)^2}{q^2 M_B^2}|C_{10}f_0(q^2)|^2 \\ & + |\boldsymbol{k}|^2\left[1 - \frac{1}{3}\beta_+^2\right]\left|C_9^{eff,B^{\pm}}f_+(q^2) + 2C_7^{eff}\frac{m_b + m_d}{M_B + M_\pi}f_T(q^2)\right|^2\Bigg\},\end{aligned}$$

where non-local components ($Y(q^2)$) are baked into C_9^{eff},

$$C_9^{eff}(q^2) = C_9 + Y(q^2).$$

[1] This requires an assumption of no (pseudo-)scalar and (pseudo-)tensor new physics.

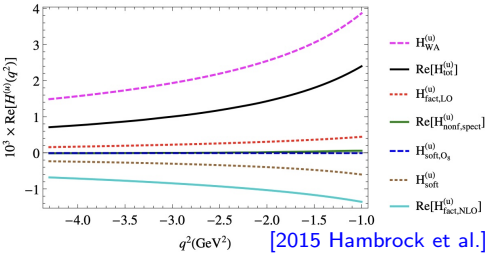
Building the $q^2 < 0$ constraint

- Non-local contributions to $B^\pm \to \pi^\pm \mu^+ \mu^-$ can be computed in the $q^2 < 0$ region as in [2015 Hambrock et al.].
  - Employing the operator-product expansion, QCD factorization and light-cone sum rule techniques to compute the $\mathcal{H}^{(q)}$.
- The sum of all the relevant contributions can be related to $\Delta C_9(q^2)$ at various points in the $q^2 < 0$ region:

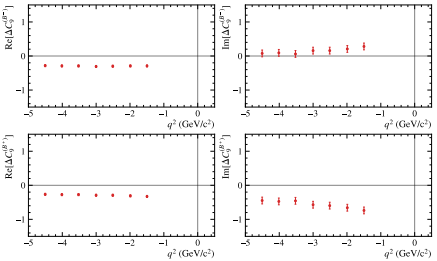
$$\Delta C_9(q^2) = -16\pi^2 \frac{(\lambda_u \mathcal{H}^{(u)}(q^2) + \lambda_c \mathcal{H}^{(c)}(q^2))}{\lambda_t f^+(q^2)}.$$

- Build the constraint using the following dispersion relation:

$$\Delta C_9(q^2) - \Delta C_9(q_0^2) = (q^2 - q_0^2)\Big[Y_{\rho,\omega}(q^2) + Y^{2P}_{c\bar{c}}(q^2) + Y_{\textit{light quark continuum}}(q^2) + Y_{J/\psi,\psi(2S),\ldots}(q^2)\Big].$$



[2015 Hambrock et al.]



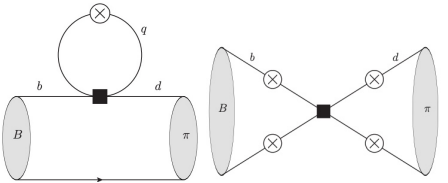
Describing the hadronic contributions - $Y_{\textit{light quark continuum}}(q^2)$

$$C_9^{eff}(q^2) = C_9 + Y(q^2)$$

- Non-local components are handled by combining two dispersion relations ($\mathcal{H}^{(u)}$ and $\mathcal{H}^{(c)}$) into one:

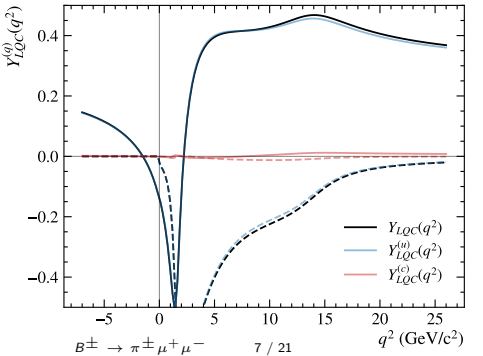
$$\Delta C_9(q^2) - \Delta C_9(q_0^2) = (q^2 - q_0^2)\Big[Y_{\rho,\omega}(q^2) + Y^{2P}_{c\bar{c}}(q^2) + Y_{\textit{light quark continuum}}(q^2) + Y_{J/\psi,\psi(2S),\ldots}(q^2)\Big].$$

- In $B^\pm \to \pi^\pm \mu^+\mu^-$, both the rare mode ($V_{tb}V_{td}^*$) and these light quark diagrams ($V_{ub}V_{ud}^*$) go as $\sim \lambda^3$.
  - In contrast in $B^\pm \to K^\pm \mu^+\mu^-$ the rare mode ($V_{tb}V_{ts}^*$) goes as $\sim \lambda^2$.



[2015 Hambrock et al.]

$Y_{\textit{light quark continuum}}(q^2)$



Describing the hadronic contributions - $Y^{2P}_{c\bar{c}}(q^2)$

$$C_9^{eff}(q^2) = C_9 + Y(q^2)$$

- Non-local components are handled by combining two dispersion relations ($\mathcal{H}^{(u)}$ and $\mathcal{H}^{(c)}$) into one:

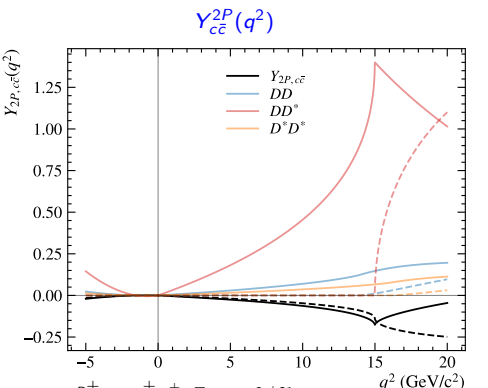
$$\Delta C_9(q^2) - \Delta C_9(q_0^2) = (q^2 - q_0^2)\Big[Y_{\rho,\omega}(q^2) + Y^{2P}_{c\bar{c}}(q^2) + Y_{light\ quark\ continuum}(q^2) + Y_{J/\psi,\psi(2S),\ldots}(q^2)\Big].$$

- 2P charmonium contribution $Y^{2P}_{c\bar{c}}(q^2)$ is the following rescattering:

$$B^{\pm} \to \pi^{\pm} MM' \to \pi^{\pm}\mu^+\mu^-$$

$$MM' = \{DD, DD^*, D^*D^*\}$$

- Following the recipe in [2020 Cornella et al.] that models the two particle spectral density as 2-body phasespace accounting for angular momentum.



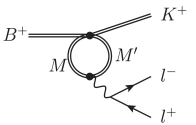
Describing the hadronic contributions - $Y^{2P}_{c\bar{c}}(q^2)$

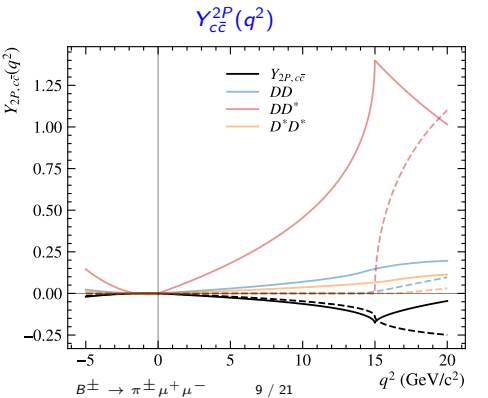
$$C_9^{eff}(q^2) = C_9 + Y(q^2)$$

- Non-local components are handled by combining two dispersion relations ($\mathcal{H}^{(u)}$ and $\mathcal{H}^{(c)}$) into one:

$$\Delta C_9(q^2) - \Delta C_9(q_0^2) = (q^2 - q_0^2)\Big[Y_{\rho,\omega}(q^2) + Y^{2P}_{c\bar{c}}(q^2) + Y_{light\ quark\ continuum}(q^2) + Y_{J/\psi,\psi(2S),\ldots}(q^2)\Big].$$

- Approximate the sum of DD, D^*D^* and DD^* contributions as a single component with a single magnitude and phase.





Describing the hadronic contributions - Resonances

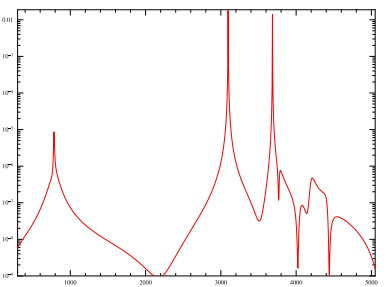
$$C_9^{eff}(q^2) = C_9 + Y(q^2)$$

- Non-local components are handled by combining two dispersion relations ($\mathcal{H}^{(u)}$ and $\mathcal{H}^{(c)}$) into one:

$$\Delta C_9(q^2) - \Delta C_9(q_0^2) = (q^2 - q_0^2)\Big[Y_{\rho,\omega}(q^2) + Y_{c\bar{c}}^{2P}(q^2) + Y_{\textit{light quark continuum}}(q^2) + Y_{J/\psi,\psi(2S),\ldots}(q^2)\Big].$$

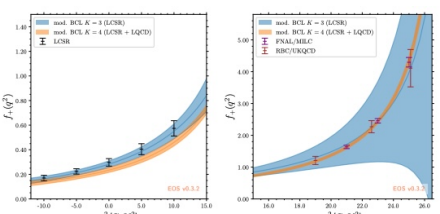
- Resonances are described with relativistic Breit–Wigner distributions.
- Each resonance has a unique phase (δ_V) and a unique magnitude (η_V) for the B^+ and the B^- PDF.
  - This enables us to model any CP-violation.
- We introduce constraints on resonance branching fractions using existing measurements ($BF \propto \eta_V^2$).

$|Y_{\rho,\omega}(q^2) + Y_{J/\psi,\psi(2S),\ldots}(q^2)|^2$



$B \to \pi$ local form factors

- Taken from [2021 Leljak et al.].
- Nominal is the $K = 4$ LCSR+LQCD option.
  - K is the maximal order of the z-expansion.
- In our fit the form factor parameters are fixed.
- We will assess an uncertainty on the Wilson coefficients as a systematic using the covariance matrix provided in [2021 Leljak et al.].



The $\bar{B} \to \pi$ form factors from QCD and their impact on $|V_{ub}|$

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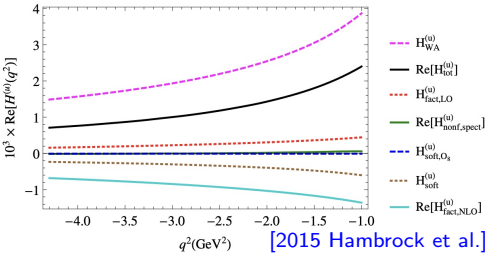
scenario / param.	LCSR+LQCD $K = 3$	LCSR+LQCD $K = 4$	LCSR $K = 3$
$f_+(0)$	$0.237^{+0.017}_{-0.017}$	$0.235^{+0.019}_{-0.019}$	$0.283^{+0.027}_{-0.027}$
b_1^+	$-2.38^{+0.33}_{-0.38}$	$-2.45^{+0.49}_{-0.54}$	$-1.0^{+3.5}_{-3.6}$
b_2^+	$-0.82^{+0.76}_{-0.81}$	$-0.2^{+1.1}_{-1.2}$	$-2.8^{+4.9}_{-4.7}$
b_3^+	—	$-0.9^{+4.2}_{-4.0}$	—

Correlations in the constraint

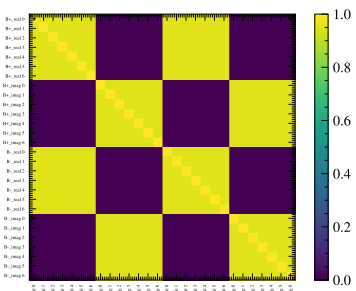
We do not have access to the correlations between the individual pieces of the $q^2 < 0$ information, so in our fits we make the assumption of no correlations (a conservative choice).

If one did wish to make a different assumption:

- Some contributions are small and can be ignored.
- Others are known to be dominated by local form factor uncertainties $\rightarrow$ cancel in $\Delta C_9(q^2)$.
- The uncertainty from $\mathcal{H}^{(p)}_{\mathrm{WA}}$ dominates the real components - assume correlated between B^+ and B^-.
- The uncertainty from $\mathcal{H}^{(p)}_{\mathrm{nonf,spect}}$ dominates the imaginary components - assume correlated between B^+ and B^-.
- Assume points at different q^2 points in the same component are correlated.



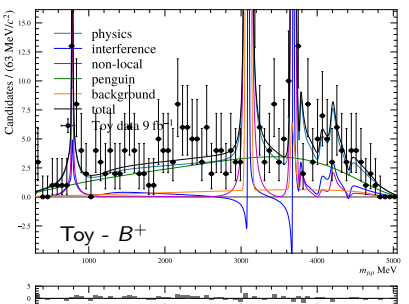
[2015 Hambrock et al.]

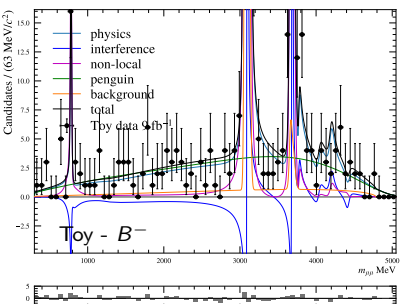


Toy studies

Use toys to study fit stability and to estimate expected precision.

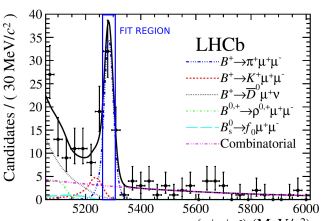
- We run toys at the SM, using hadronic parameters obtained from fits to negative q^2 points ([See slide 16]), these are compatible with [2015 Hambrock et al.].
- Fit B^+ and B^- simultaneously sharing C_{10}, C_9 and the phase of C_9 (flipping sign under CP).
- Fix the light quark continuum contribution ($Y_{light\ quark\ continuum}(q^2)$).
- Float both the phase and magnitude the $Y^{2P}_{c\bar{c}}(q^2)$ component, and separately for both B^+ and B^-.
- Avoid local minima by fitting each generated toy multiple times from random start points and pick lowest $\mathcal{NLL}$.



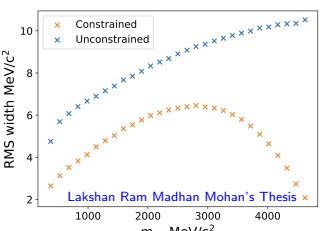


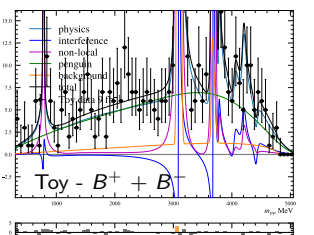
Toy studies - fit technicalities

- Select only events ± 40 MeV around B mass, fit B mass constrained q^2.
- The detector resolution (assume at J/ψ) is convolved with the fit model using a Fast Fourier Transform, and the same resolution function as used in [$B^\pm \to K^\pm \mu^+ \mu^-$ - LHCb] is employed.
- A combinatorial background shape is included.
- We employ simple q^2-dependent efficiency function based on that of [$B^\pm \to K^\pm \mu^+ \mu^-$ - LHCb].
- Constraints are employed on the BFs of resonances from existing measurements (based on BF and A_{CP} measurements).



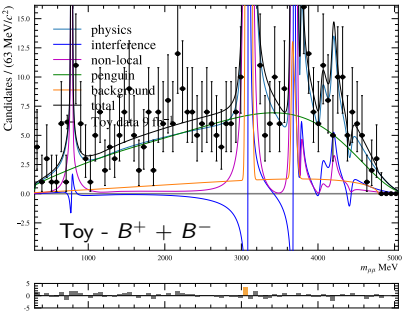
[$B^\pm \to \pi^\pm \mu^+ \mu^-$ binned - LHCb]





Choosing a q^2 region to fit

- With the statistics available from LHCb run $1 + 2$ it is no surprise that we cannot float the parameters of the open charm resonances.
- We, therefore, suggest cutting out the open charm region.
  - We cut q^2 just below the $\psi(3770)$ resonance.
- This avoids model dependence related to fixing these parameters.



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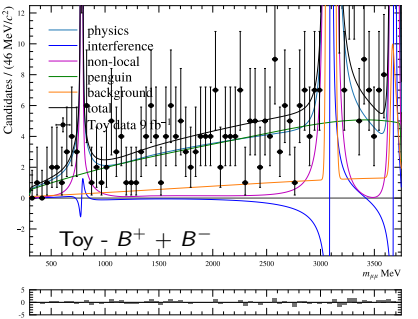


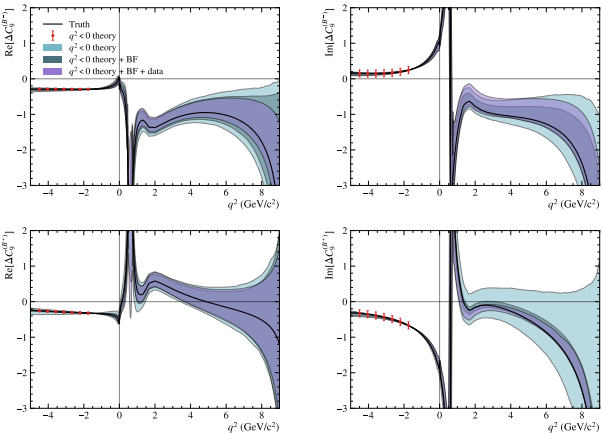
Hadronic contributions with q^2 - 9 fb^{-1}

- Pick starting hadronic parameters such that $\Delta C_9(q^2)$ distributions are compatible with [2015 Hambrock et al.].

Compare uncertainty of $\Delta C_9(q^2)$ for three scenarios:

- Using just the $q^2 < 0$ information...
- ... then adding BF constraints.
- ... then adding LHCb run 1 + 2 pseudo-data.

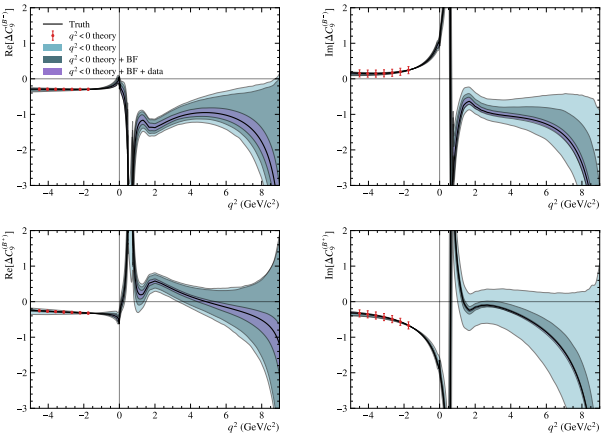
The improvement in sensitivity to non-local contributions from adding 9 fb^{-1} LHCb data is small.



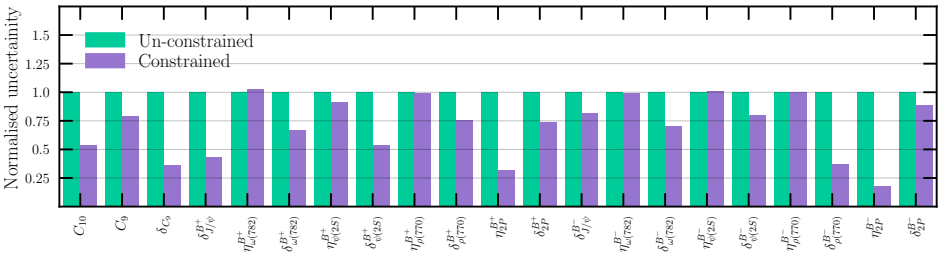
Hadronic contributions as a function of q^2 - 300 fb^{-1}

- We can repeat the same exercise using 300 fb^{-1} of LHCb data ($\sim$ 2035).
- This brings the $B^\pm \to \pi^\pm \mu^+ \mu^-$ event yields to similar to LHCb run 1 + 2 $B^\pm \to K^\pm \mu^+ \mu^-$ yields.

At this point the LHCb data is providing a clear improvement in sensitivity to non-local contributions.



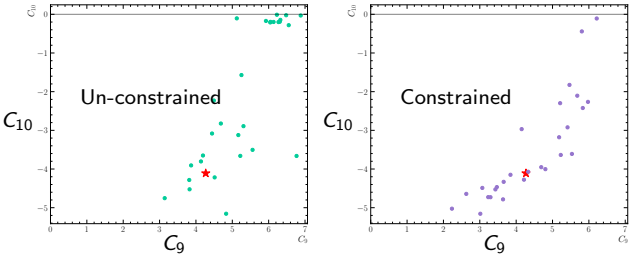
How much does the $q^2 < 0$ information add?



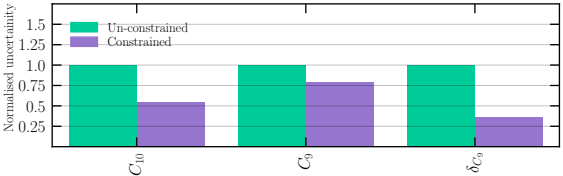
- We run fits to generated pseudo-datasets representative of 9 fb^{-1} of LHCb data.
- Fit each dataset both with and without the $q^2 < 0$ constraint, as such any differences are more significant.
- Report uncertainties from Hesse matrix and combine any bias into the overall uncertainty.
- Largest improvements are in the phases of the resonances, and both the phases and the magnitudes of the $Y^{2P}_{c\bar{c}}(q^2)$.
- **This increase in sensitivity to non-local parameters translates into better precision on the Wilson coefficients describing the short-distance physics.**

How much does the $q^2 < 0$ information add?

- Large correlation between C_9 and C_{10}:
  - This is expected. C_9 and C_{10} can swap out so long as $\mathcal{B}(B^\pm \to \pi^\pm \mu^+ \mu^-)_{EWP}$ remains satisfied.
  - This is especially true in the case of small interference with non-local contributions.
- Unconstrained fits are unfeasible. The build-up of results at $C_{10} \approx 0$ makes up a significant fraction of toys.

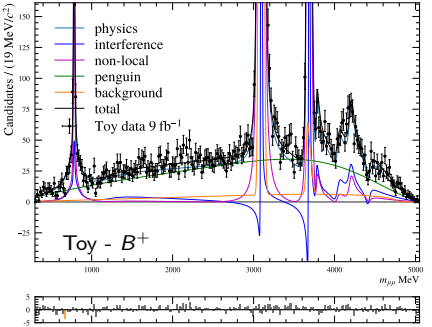


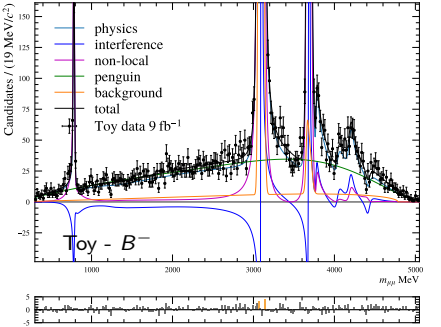
Parameter	$\sigma_{residuals}$ un-constrained	$\sigma_{residuals}$ constrained
C_{10}	2.54	1.41
C_9	1.39	1.12
δ_{C_9}	0.96	0.34



How does the picture change with more data?

- With 300 fb^{-1} the expected $B^{\pm} \to \pi^{\pm}\mu^{+}\mu^{-}$ event yields are similar to those of LHCb run 1 + 2 $B^{\pm} \to K^{\pm}\mu^{+}\mu^{-}$ yields.
- We can now float the open charm resonance parameters.
- Yet to run more than a few toys here, however, we expect the constraint should become less essential but still relevant.

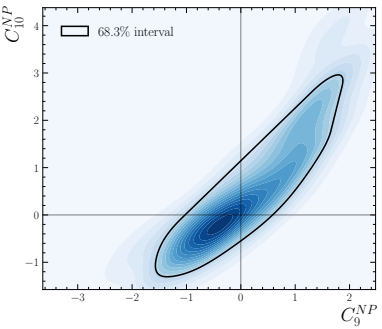


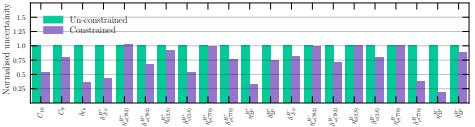


Conclusion

- We have a procedure that demonstrates it possible to fit $B^{\pm} \to \pi^{\pm}\mu^{+}\mu^{-}$ at LHCb in an unbinned way fully accounting for CPV, the largest non-local contributions and all interference effects.
- The $q^2 < 0$ information and the BF constraints do the heavy lifting on pinning down the hadronic components of $B^{\pm} \to \pi^{\pm}\mu^{+}\mu^{-}$, the current LHCb dataset is not large enough to independently control these components.
- We find that employing $q^2 < 0$ information from QCD factorization and light-cone sum rule techniques as a constraint in the likelihood of fits to LHCb data is essential for fits to current and near-future data sets.
- We are working to publish these studies in arXiv:2306.XXXX.

68% interval in C_9 vs C_{10}

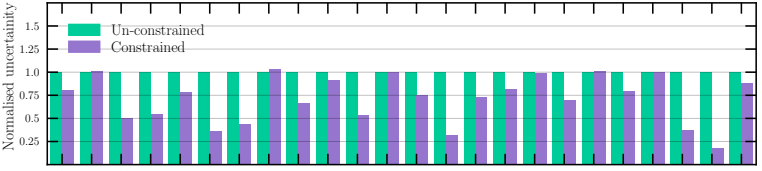




Thanks for listening

BACKUP SLIDES

How much does the $q^2 < 0$ information add? Validation fits



Parameter	$\sigma_{residuals}$ un-constrained	$\sigma_{residuals}$ constrained
A_{CP}	9.5×10^{-4}	7.7×10^{-4}
$\mathcal{B}(B^\pm \to \pi^\pm \mu^+ \mu^-)$	3.0×10^{-9}	3.1×10^{-9}
$\mathcal{B}(B^\pm \to \pi^\pm \mu^+ \mu^-)_{EWP}$	1.5×10^{-9}	7.3×10^{-10}
C_{10}	2.67	1.44
C_9	1.44	1.14
δ_{C_9}	0.93	0.34
$\delta^{B^+}_{J/\psi}$	1.46	0.63
$\eta^{B^+}_{\omega(782)}$	2.96	3.04
$\delta^{B^+}_{\omega(782)}$	1.42	0.94
$\eta^{B^+}_{\psi(2S)}$	17.60	16.10
$\delta^{B^+}_{\psi(2S)}$	1.32	0.71
$\eta^{B^+}_{\rho(770)}$	0.60	0.60
$\delta^{B^+}_{\rho(770)}$	1.12	0.84

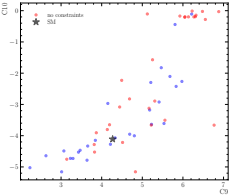
Parameter	$\sigma_{residuals}$ un-constrained	$\sigma_{residuals}$ constrained
$\eta^{B^+}_{2P}$	3.02	0.95
$\delta^{B^+}_{2P}$	1.61	1.18
$\delta^{B^-}_{J/\psi}$	1.31	1.08
$\eta^{B^-}_{\omega(782)}$	4.32	4.30
$\delta^{B^-}_{\omega(782)}$	1.86	1.31
$\eta^{B^-}_{\psi(2S)}$	21.69	21.96
$\delta^{B^-}_{\psi(2S)}$	1.43	1.14
$\eta^{B^-}_{\rho(770)}$	0.62	0.62
$\delta^{B^-}_{\rho(770)}$	1.57	0.58
$\eta^{B^-}_{2P}$	4.97	0.90
$\delta^{B^-}_{2P}$	1.51	1.33

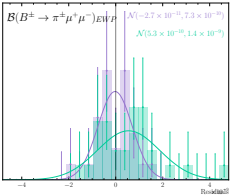
Build up at $C_{10} \approx 0$

- Most likely these are toys stuck at a local minimum, with the correct start point these would converge properly.
  - Current investigating this - looks promising
- C_{10} always appears as $|C_{10}|^2$ and so the PDF symmetrical around $C_{10} \approx 0$

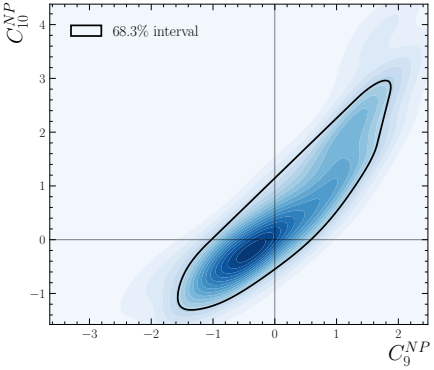
$$\frac{d\Gamma(B^\pm \to \pi^\pm \mu^+ \mu^-)}{dq^2} = \frac{G_F^2 \alpha^2 |V_{tb} V_{td}^*|^2}{2^7 \pi^5} |\boldsymbol{k}| \Bigg\{ \frac{2}{3} |\boldsymbol{k}|^2 \beta_+^2 |C_{10} f_+(q^2)|^2 + \frac{m_\ell^2 (M_B^2 - M_\pi^2)^2}{q^2 M_B^2} |C_{10} f_0(q^2)|^2 + |\boldsymbol{k}|^2 \left[1 - \frac{1}{3}\beta_+^2\right] \left| C_9^{eff,B^\pm} f_+(q^2) + 2 C_7^{eff} \frac{m_b + m_d}{M_B + M_\pi} f_T(q^2) \right|^2 \Bigg\},$$

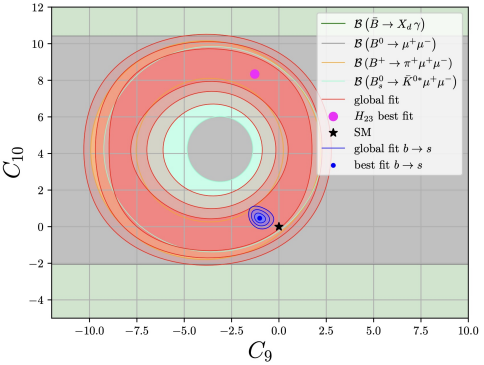
- C_{10} and C_9 are hard to separate and are somewhat interchangeable - so long as the $\mathcal{B}(B^\pm \to \pi^\pm \mu^+ \mu^-)_{EWP}$ remains satisfied.
- We could use $B_s \to \mu^+ \mu^-$ results $(C_{10} - C'_{10})$ to constrain C_{10} however this would require us to assume $C'_{10} = 0$ as our C_{10} is really $C_{10} + C'_{10}$.





Comparison to current limits





- Reduce limit to one corner of the circle
- Current limit assumes real C_9