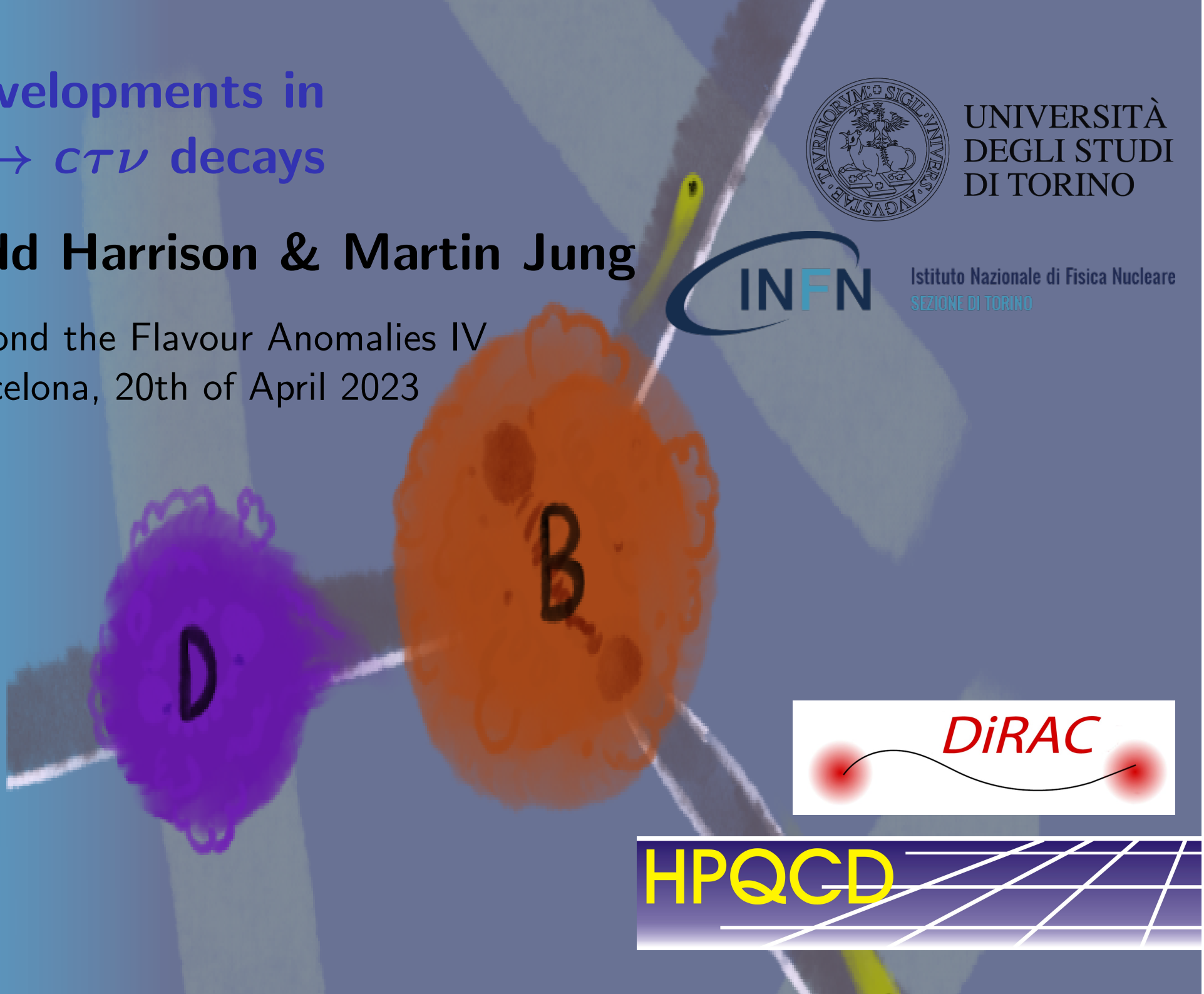
Developments in $b \to c\tau\nu$ decays

Judd Harrison & Martin Jung

UNIVERSITÀ DEGLI STUDI DI TORINO

INFN Istituto Nazionale di Fisica Nucleare SEZIONE DI TORINO

Beyond the Flavour Anomalies IV
Barcelona, 20th of April 2023



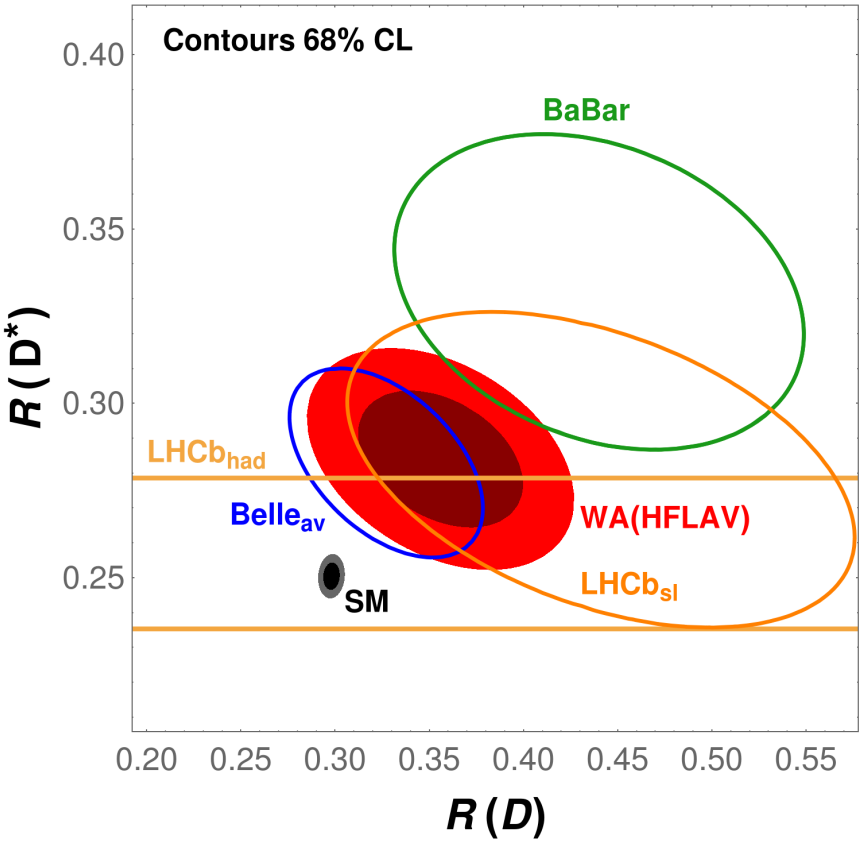
DiRAC

HPQCD

Lepton-non-Universality in $b \to c\tau\nu$

$$R(X) \equiv \frac{\mathrm{Br}(B \to X\tau\nu)}{\mathrm{Br}(B \to X\ell\nu)}$$

- Partial cancellation of uncertainties
  - ➥ Precise predictions (and measurements)



- $R(D^{(*)})$: BaBar, Belle, LHCb
  - ➥ average $\sim 3 - 4\sigma$ from SM

More flavour $b \to c\tau\nu$ observables:

- τ-polarization ($\tau \to$ had) [1608.06391]
- $B_c \to J/\psi\tau\nu$ [1711.05623] : huge
- Differential rates from Belle, BaBar
- Total width of B_c
- $b \to X_c\tau\nu$ by LEP
- D^* polarization (Belle)
- $R(\Lambda_c) \to$ below SM

NP interpretation:

[talks by B. Stefanek and F. Wilsch]

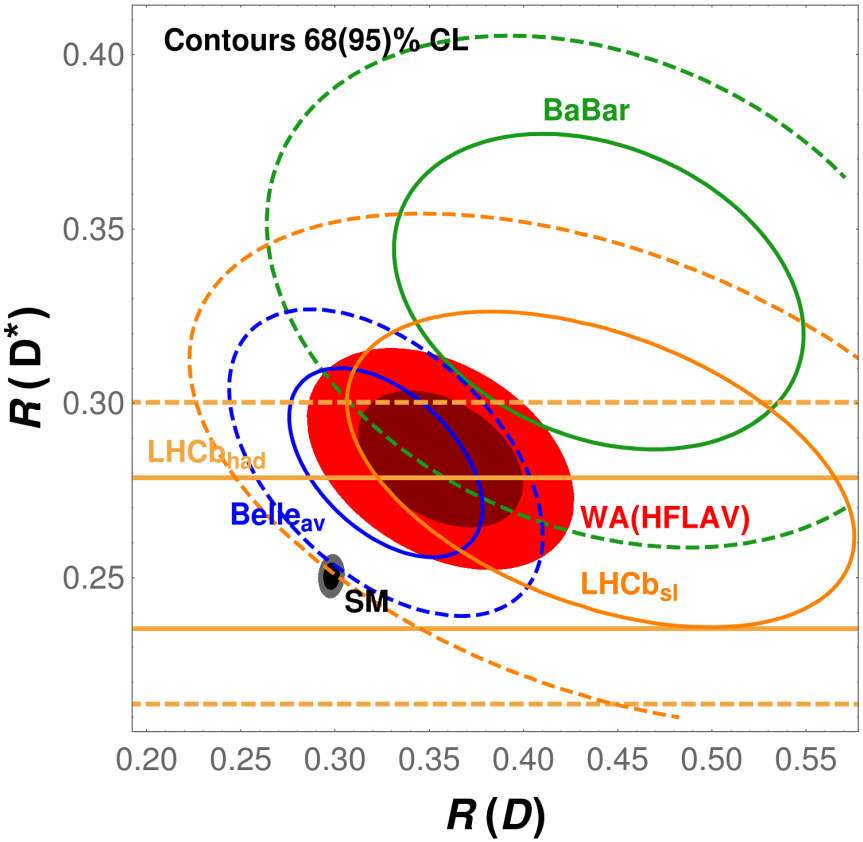
Note: only 1 result $\geq 3\sigma$ from SM

In the following: discuss SM + NP predictions

Lepton-non-Universality in $b \to c\tau\nu$

$$R(X) \equiv \frac{\mathrm{Br}(B \to X\tau\nu)}{\mathrm{Br}(B \to X\ell\nu)}$$

- Partial cancellation of uncertainties
  - ➥ Precise predictions (and measurements)



- $R(D^{(*)})$: BaBar, Belle, LHCb
  - ➥ average $\sim 3 - 4\sigma$ from SM

More flavour $b \to c\tau\nu$ observables:

- τ-polarization ($\tau \to$ had) [1608.06391]
- $B_c \to J/\psi\tau\nu$ [1711.05623] : huge
- Differential rates from Belle, BaBar
- Total width of B_c
- $b \to X_c\tau\nu$ by LEP
- D^* polarization (Belle)
- $R(\Lambda_c) \to$ below SM

NP interpretation:

[talks by B. Stefanek and F. Wilsch]

Note: only 1 result $\geq 3\sigma$ from SM

In the following: discuss SM + NP predictions

Exclusive decays: Form factors

In exclusive decays, hadronic information encoded in Form Factors
They parametrize fundamental mismatch:

Theory (e.g. SM) for partons (quarks)
vs.
Experiment with hadrons

$$\langle D_q(p')|\bar{c}\gamma^\mu b|\bar{B}_q(p)\rangle = (p+p')^\mu f_+^q(q^2) + (p-p')^\mu f_-^q(q^2)\,,\; q^2 = (p-p')^2$$

Most general matrix element parametrization, given symmetries:
Lorentz symmetry plus P- and T-symmetry of QCD
$f_\pm(q^2)$: real, scalar functions of one kinematic variable

How to obtain these functions?

- Calculable w/ non-perturbative methods (Lattice, LCSR,...)
  Precision?
- Measurable e.g. in semileptonic transitions
  Normalization? Suppressed FFs? NP?

q^2 dependence

- q^2 range can be large, e.g. $q^2 \in [0, 12]\ \mathrm{GeV}^2$ in $B \to D$
- Calculations give usually one or few points

➥ Knowledge of functional dependence on q^2 crucial

- This is where discussions start...

Give as much information as possible independently of this choice!

q^2 dependence

- q^2 range can be large, e.g. $q^2 \in [0, 12]\ \mathrm{GeV}^2$ in $B \to D$
- Calculations give usually one or few points

➥ Knowledge of functional dependence on q^2 crucial

- This is where discussions start...

Give as much information as possible independently of this choice!

In the following: discuss BGL and HQE ($\to$ CLN) parametrizations

q^2 dependence usually rewritten via conformal transformation:

$$z\left(t = q^2, t_0\right) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

$t_+ = (M_{B_q} + M_{D_q^{(*)}})^2$: pair-production threshold

$t_0 < t_+$: free parameter for which $z(t_0, t_0) = 0$

Usually $|z| \ll 1$, e.g. $|z| \leq 0.06$ for semileptonic $B \to D$ decays

➥ Good expansion parameter

The BGL parametrization [Boyd/Grinstein/Lebed, 90's]

FFs are parametrized by a few coefficients the following way:

1. Consider analytical structure, make poles and cuts explicit
2. Without poles or cuts, the rest can be Taylor-expanded in z
3. Apply QCD symmetries (unitarity, crossing)
   ➥ dispersion relation
4. Calculate partonic part (mostly) perturbatively

The BGL parametrization [Boyd/Grinstein/Lebed, 90's]

FFs are parametrized by a few coefficients the following way:

1. Consider analytical structure, make poles and cuts explicit
2. Without poles or cuts, the rest can be Taylor-expanded in z
3. Apply QCD symmetries (unitarity, crossing)
   ➥ dispersion relation
4. Calculate partonic part (mostly) perturbatively

Result: Model-independent parametrization

$$F(t) = \frac{1}{P(t)\phi(t)} \sum_{n=0}^{\infty} a_n [z(t, t_0)]^n \,.$$

- a_n: real coefficients, the only unknowns
- $P(t)$: Blaschke factor(s), information on poles below t_+
- $\phi(t)$: Outer function, chosen such that $\sum_{n=0}^{\infty} a_n^2 \leq 1$

Series in z with bounded coefficients (each $|a_n| \leq 1$)!
➥ Uncertainty related to truncation is calculable!

$B \to D\ell\nu$

$B \to D\ell\nu$, aka "The teacher's pet":

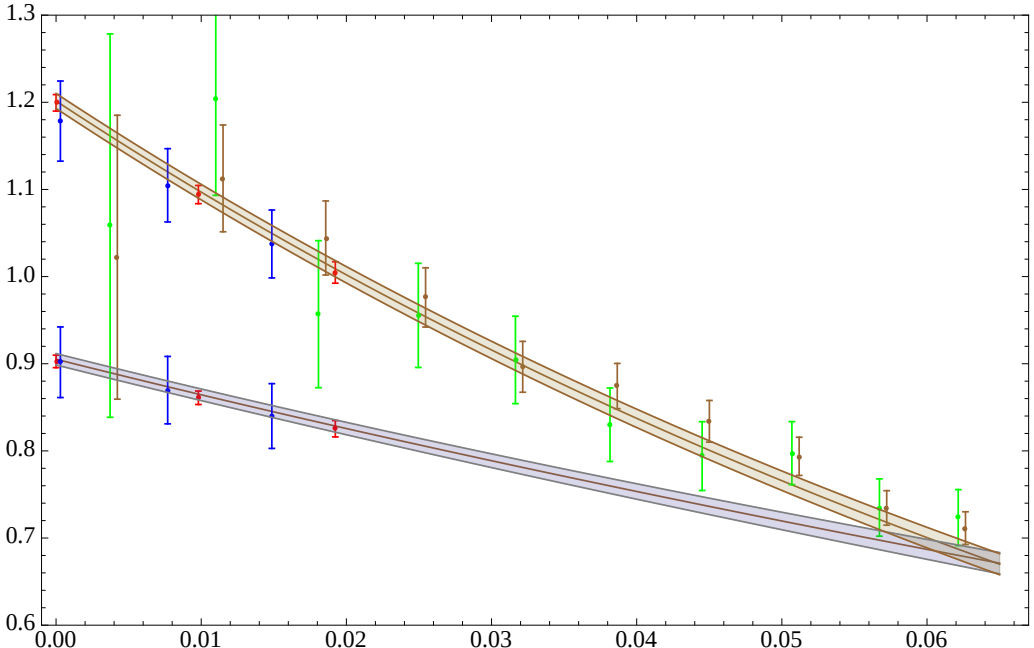
- Excellent agreement between experiments [BaBar'09,Belle'16]
- Excellent agreement between two lattice determinations [FNAL/MILC'15,HPQCD'16]

➥ Lattice data inconsistent with CLN parametrization! (but consistent w/ HQE@1/m, discussed later)

- BGL fit [Bigi/Gambino'16] :

$$R(D) = 0.299(3)\,.$$

See also [Jaiswal+,Berlochner+'17,MJ/Straub'18,Bordone/MJ/vanDyk'19]



$f_{+,0}(z)$, inputs:

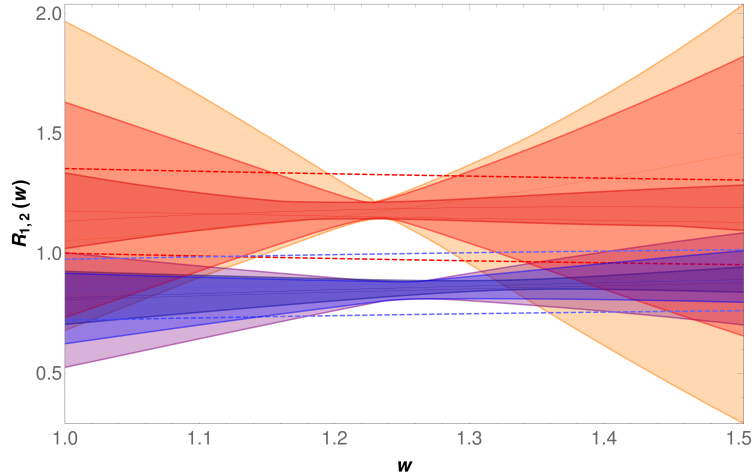
- FNAL/MILC'15
- HPQCD'16
- BaBar'09
- Belle'16

$V_{cb} + R(D^*)$ w/ data + lattice + unitarity [Gambino/MJ/Schacht'19]

Belle'18(+'17) provide FF-independent data for 4 single-differential rates

BGL analysis:

- Datasets compatible
- d'Agostini bias + syst. important
- Expand FFs to z^2
  ➥ 50% increased uncertainties



- Belle'18: no parametrization dependence
- Belle'17 never published → replace w/ Belle'23, not available yet
- Tension w/ inclusive reduced, but not removed

$$R(D^*) = 0.253^{+0.007}_{-0.006} \quad \text{(including LCSR point)}$$

HQE parametrization

Heavy-Quark Expansion (HQE) employs additional information:

- $m_{b,c} \to \infty$: all $B \to D^{(*)}$ FFs given by 1 Isgur-Wise function
- Systematic expansion in $1/m_{b,c}$ and α_s
- Higher orders in $1/m_{b,c}$: FFs remain related
  - ➥ Parameter reduction, necessary for NP analyses!

HQE parametrization

Heavy-Quark Expansion (HQE) employs additional information:

- $m_{b,c} \to \infty$: all $B \to D^{(*)}$ FFs given by 1 Isgur-Wise function
- Systematic expansion in $1/m_{b,c}$ and α_s
- Higher orders in $1/m_{b,c}$: FFs remain related
  - ➥ Parameter reduction, necessary for NP analyses!

CLN parametrization [Caprini+'97] :
HQE to order $1/m_{b,c}, \alpha_s$ plus (approx.) constraints from unitarity
[Bernlochner/Ligeti/Papucci/Robinson'17] : identical approach, updated and consistent treatment of correlations

Problem: Contradicts Lattice QCD (both in $B \to D$ and $B \to D^*$)
Dealt with by varying calculable (@$1/m_{b,c}$) parameters, e.g. $h_{A_1}(1)$
➥ Not a systematic expansion in $1/m_{b,c}$ anymore!
➥ Related uncertainty remains $\mathcal{O}[\Lambda^2/(2m_c)^2] \sim 5\%$, insufficient

HQE parametrization

Heavy-Quark Expansion (HQE) employs additional information:

- $m_{b,c} \to \infty$: all $B \to D^{(*)}$ FFs given by 1 Isgur-Wise function
- Systematic expansion in $1/m_{b,c}$ and α_s
- Higher orders in $1/m_{b,c}$: FFs remain related
  - ➥ Parameter reduction, necessary for NP analyses!

CLN parametrization [Caprini+'97] :
HQE to order $1/m_{b,c}, \alpha_s$ plus (approx.) constraints from unitarity
[Bernlochner/Ligeti/Papucci/Robinson'17] : identical approach, updated and consistent treatment of correlations

Problem: Contradicts Lattice QCD (both in $B \to D$ and $B \to D^*$)
Dealt with by varying calculable (@$1/m_{b,c}$) parameters, e.g. $h_{A_1}(1)$
➥ Not a systematic expansion in $1/m_{b,c}$ anymore!
➥ Related uncertainty remains $\mathcal{O}[\Lambda^2/(2m_c)^2] \sim 5\%$, insufficient

Solution: Include systematically $1/m_c^2$ corrections
[Bordone/MJ/vDyk'19,Bordone/Gubernari/MJ/vDyk'20] ,using [Falk/Neubert'92]
[Bernlochner+'22] : model for $1/m_c^2$ corrections $\to$ fewer parameters

Theory determination of $b \to c$ Form Factors

[Bordone/MJ/vanDyk'19,Bordone/Gubernari/MJ/vanDyk'20]

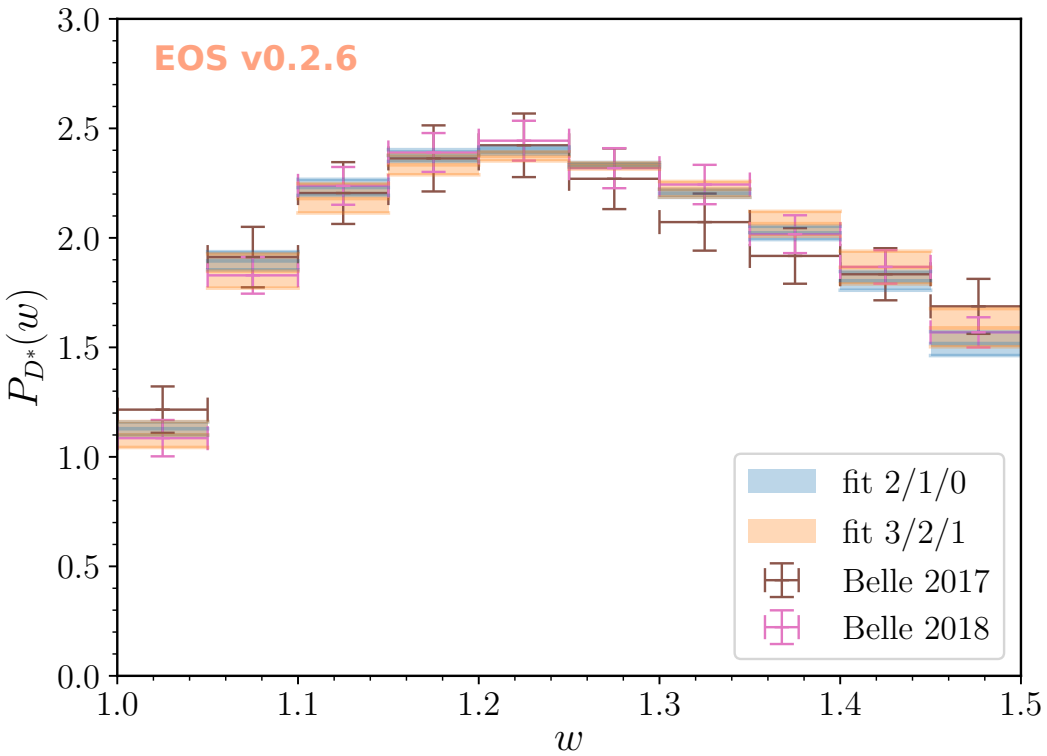
For general NP analysis, FF shapes needed from theory!

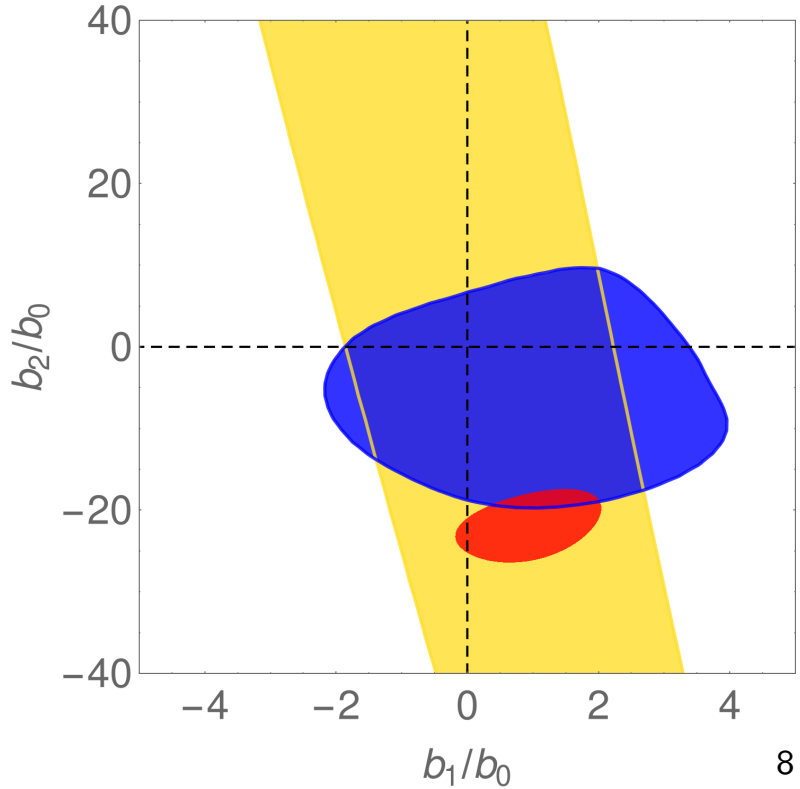
Fit to all $B \to D^{(*)}$ FFs, using lattice, LCSR, QCDSR and unitarity

[CLN,BGL,HPQCD'15'17,FNAL/MILC'14'15,Gubernari+'18,Ligeti+'92'93]

k/l/m: order in z for leading/subleading/subsubleading IW functions

- 2/1/0 works, but only 3/2/1 captures uncertainties
- Consistent V_{cb} value from Belle'17+'18
- Predictions for diff. rates, perfectly confirmed by data





Form-factor truncation

Key question: Where do we truncate our expansions?

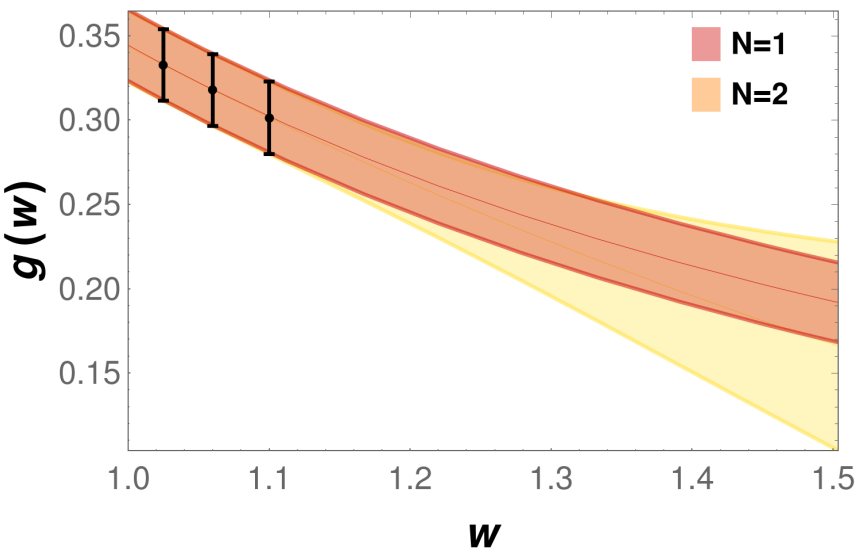
- A [Bernlochner+'19] : include parameter only if χ^2 decreases significantly
- B (GJS, BGJvD): include one "unnecessary" order

Comments:

- Large difference, $\sim 50\%$ difference in uncertainty
- Motivation for A: convergence, avoid overfitting
- Motivation for B: avoid underestimating uncertainties
- Different perspectives: only describing data, A is ok.
  However: we extrapolate to regions where we lack sensitivity

Example: $g(w)$ from FNAL/MILC

- perfect description at $\mathcal{O}(z)$
- large impact from $\mathcal{O}(z^2)$
- Nevertheless: $\mathcal{O}(z^2) \leq 6\% \times \mathcal{O}(z)$
  - overfitting limited



Just because you're not sensitive, doesn't mean it's not there!

Priors and potential biases

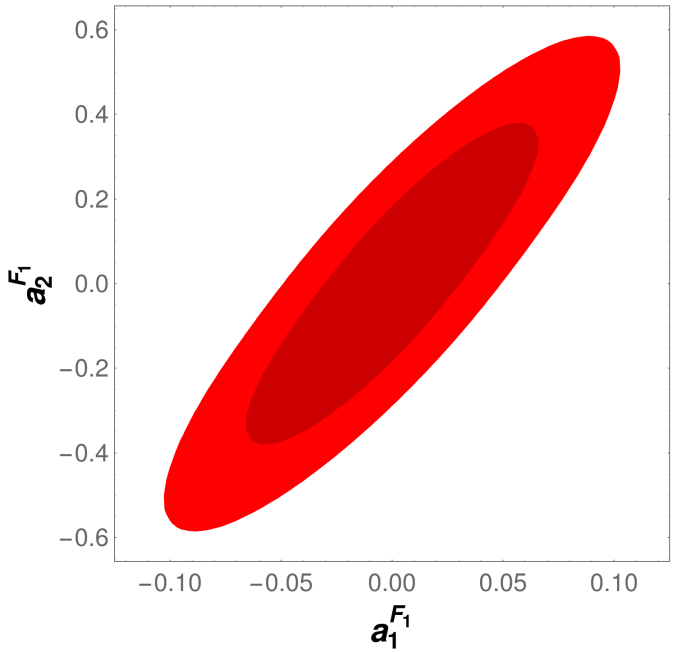
Typical error estimate:

$$\delta X \sim \mathcal{O}(1) \times \text{ known factor}$$

- What's "$\mathcal{O}(1)$"?
  - Answer seems to be community-dependent
  - Often in lattice analyses: gaussian around 0, width 1
  - BGJvD HQET FFs: flat range $[-20, 20]$
  - potentially large differences
  - needs to be checked and communicated

Similarly: treatment of BGL coefficients

- FNAL/MILC and HPQCD: series in $(w-1)^n$
- Priors can be strong in BGL space
- Plot: prior information, only (HPQCD)
- Order of final result



Form Factors

The effective Hamiltonian relevant for semileptonic $b \to c$ decays can be written, assuming left-handed neutrinos,

$$\begin{aligned}\mathcal{H}_{\text{eff}} = \sqrt{2} G_F V_{cb} \Big[& g_V \bar{c}\gamma_\mu b \bar{\ell}_L \gamma^\mu \nu_L + g_A \bar{c}\gamma_\mu\gamma_5 b \bar{\ell}_L \gamma^\mu \nu_L \\ & + g_S \bar{c} b \bar{\ell}_R \nu_L \\ & + g_P \bar{c}\gamma_5 b \bar{\ell}_R \nu_L \\ & + g_T \bar{c}\sigma_{\mu\nu} b \bar{\ell}_R \sigma^{\mu\nu} \nu_L + \text{h.c.} \Big]\end{aligned}$$

matrix elements are parameterised in terms of form factors (FFs)

$$\begin{aligned}\langle D^* | \bar{c}\gamma^\mu b | \overline{B} \rangle &= i\sqrt{M_B M_{D^*}}\, \varepsilon^{\mu\nu\alpha\beta} \epsilon^*_\nu v'_\alpha v_\beta h_V \\ \langle D^* | \bar{c}\gamma^\mu\gamma^5 b | \overline{B} \rangle &= \sqrt{M_B M_{D^*}} \big[h_{A_1}(w+1)\epsilon^{*\mu} \\ &\quad - h_{A_2}(\epsilon^* \cdot v) v^\mu - h_{A_3}(\epsilon^* \cdot v) v'^\mu \big] \\ \langle D^* | \bar{c}\sigma^{\mu\nu} b | \overline{B} \rangle &= -\sqrt{M_B M_{D^*}}\, \varepsilon^{\mu\nu\alpha\beta} \big[h_{T_1} \epsilon^*_\alpha (v+v')_\beta \\ &\quad + h_{T_2} \epsilon^*_\alpha (v-v')_\beta + h_{T_3} (\epsilon^* \cdot v) v_\alpha v'_\beta \big],\end{aligned}$$

$B \to D^*$ vector, axial-vector and tensor form factors for the full q^2 range from lattice QCD [2304.03137]

Use "Heavy-HISQ" approach:

- Compute FFs on multiple lattices using heavy quark h with mass m_h, using **H**ighly **I**mproved **S**taggered **Q**uark action for all quarks
  - $\to$ Small discretisation effects - no tree level a^2 effects, greatly reduced $(am)^4$ effects
  - $\to$ High numerical efficiency enables good statistics
  - $\to$ Fully relativistic
  - $\to$ Nonperturbative renormalisation of currents
- Fit lattice data including discretisation effects, dependence on heavy-quark mass and chiral effects.

We include data for $B_s \to D_s^*$ FFs, which we include in our fit using HMχPT.
Use second generation MILC $n_f = 2 + 1 + 1$ HISQ gluon field configurations, with $a \approx 0.09\mathrm{fm}, 0.06\mathrm{fm}, 0.044\mathrm{fm}$ and pion masses from $300\mathrm{MeV}$ down to the physical value.

Our fit function for each FF takes the form

$$F^{Y^{(s)}}(w) = \sum_{n=0}^{3} a_n^Y (w-1)^n \mathcal{N}_n^Y + \frac{g_{D^*D\pi}^2}{16\pi^2 f_\pi^2}\left(\mathrm{logs}_{SU(3)}^{Y^{(s)}} - \mathrm{logs}_{SU(3)\mathrm{phys}}^{Y}\right)$$
$$+ \tilde{a}^Y \left(\left(\frac{M_\pi^{\mathrm{phys}}}{\lambda_\chi}\right)^2 - \left(\frac{M_{\pi(K)}}{\lambda_\chi}\right)^2\right),$$

where $Y_{(s)}$ labels the form factor, $\mathcal{N}_n^Y \approx 1$ encodes sea and valence quark mass mistuning effects, and the coefficients, a_n^Y, for each form factor take the form

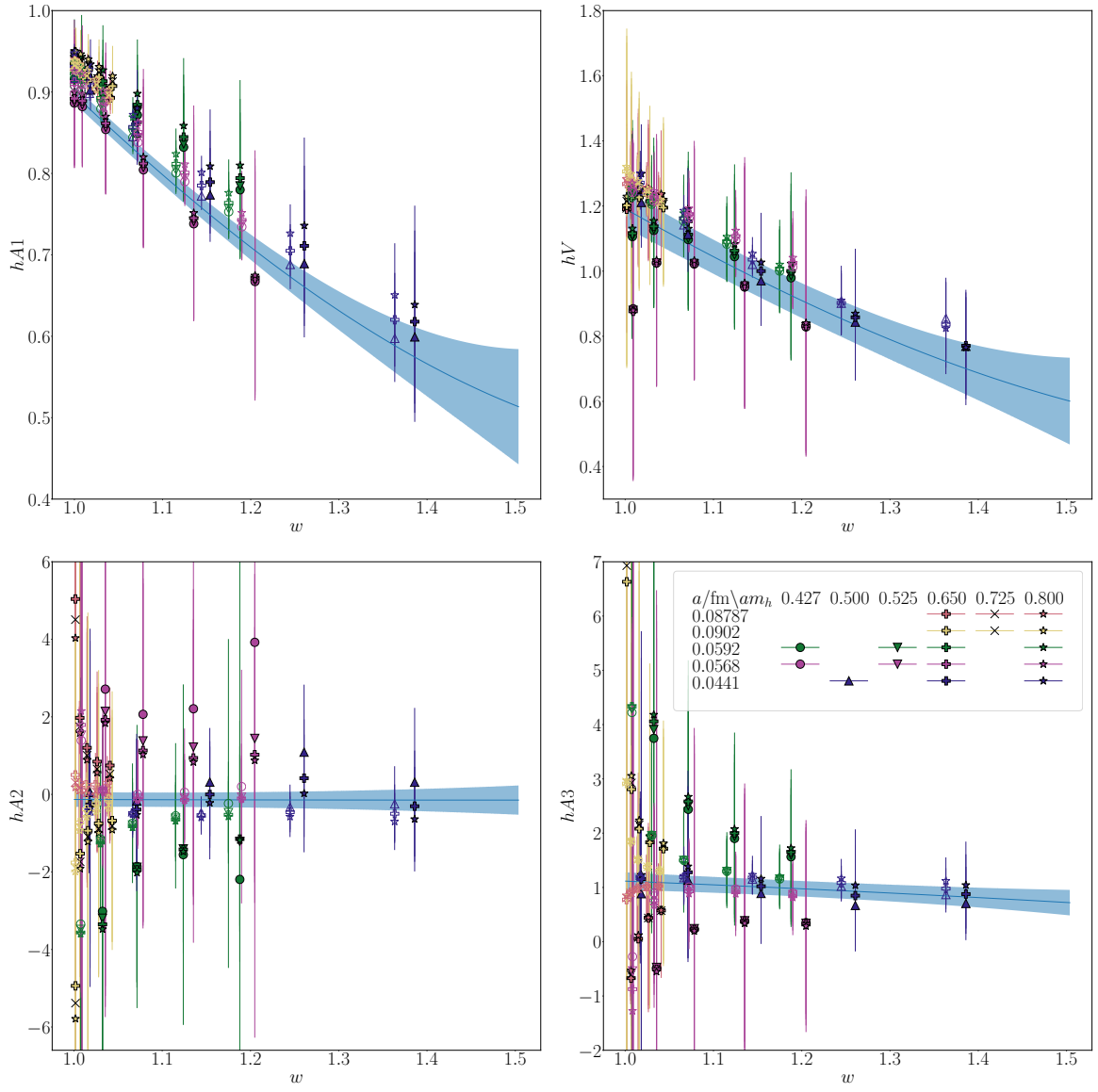
$$a_n^Y = \sum_{j,k,l=0}^{3} b_n^{Y,jkl} \Delta_h^{(j)} \left(\frac{am_c^{\mathrm{val}}}{\pi}\right)^{2k} \left(\frac{am_h^{\mathrm{val}}}{\pi}\right)^{2l},$$

where

$$\Delta_h^{(0)} = 1, \qquad \Delta_h^{(j\neq 0)} = \left(\frac{\Lambda}{M_{H_s}}\right)^j - \left(\frac{\Lambda}{M_{B_s}^{\mathrm{phys}}}\right)^j.$$

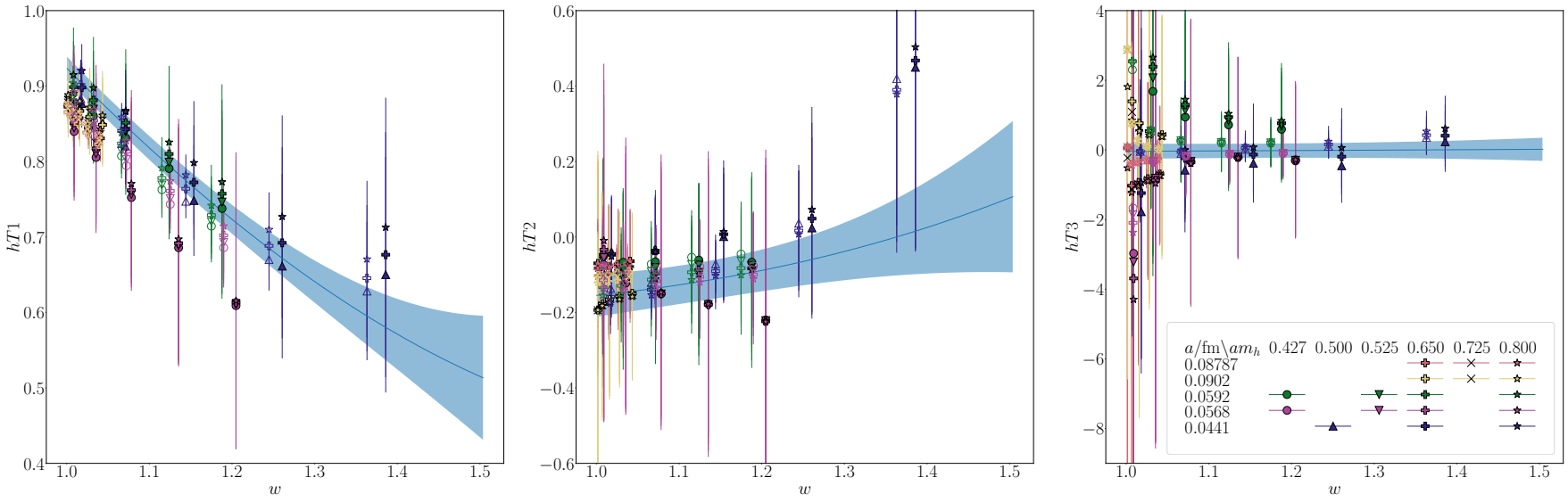
Take $\mathcal{N}_n^Y \to 1$, $am_c \to 0$, $am_h \to 0$, $M_{H_s} \to M_{B_s}^{\mathrm{phys}}$ and $M_\pi \to M_\pi^{\mathrm{phys}}$ to recover physical form factors at the b mass.

SM Form Factors



We compute the SM FFs with good coverage of the full q^2 range.

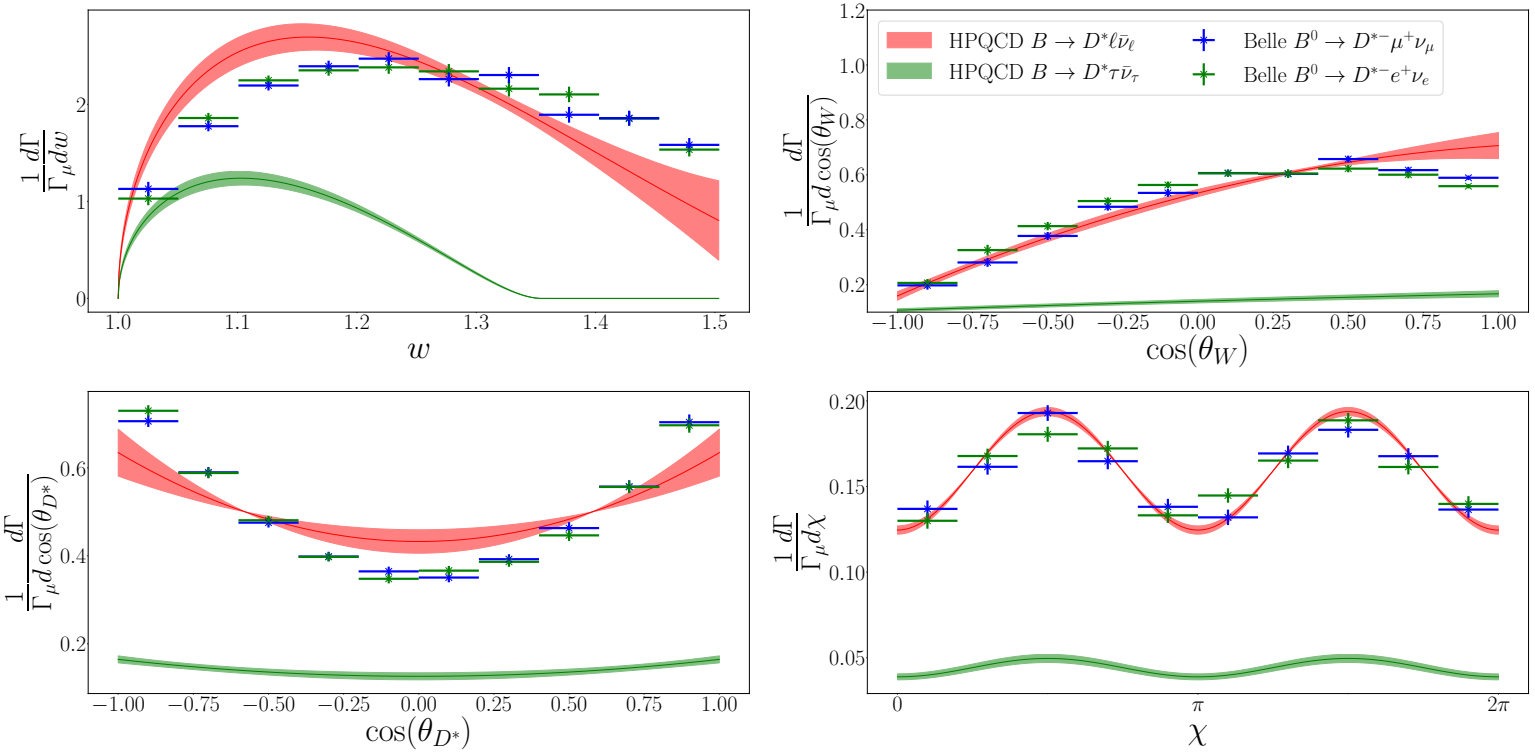
Tensor Form Factors



We also compute the tensor FFs.

$R(D^*)$

Comparison to Belle data [1809.03290].

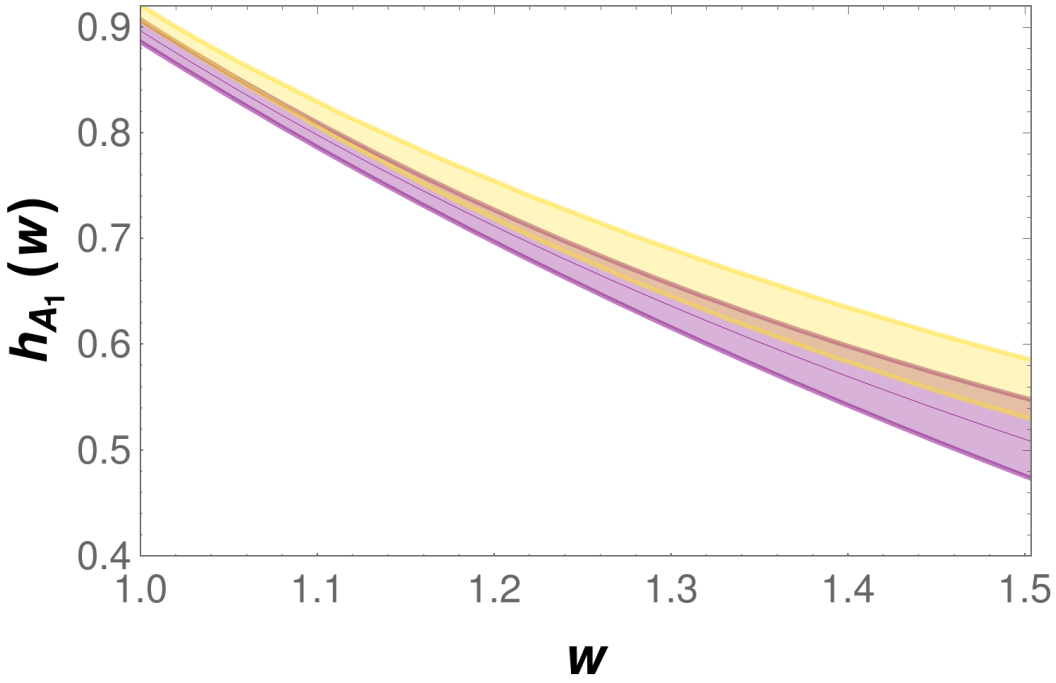


Looks terrible! However, including correlations only $\approx 1.5\sigma$ tension. We find that when fitting FFs to the Belle data $R(D^*)$ is shifted downwards significantly.

	'lattice-only'	'lattice+experiment'
$R(D^*)$	0.279(13)	0.2471(19)
$R(D_s^*)$	0.265(9)	0.2464(27)

Comparison with new lattice calculations

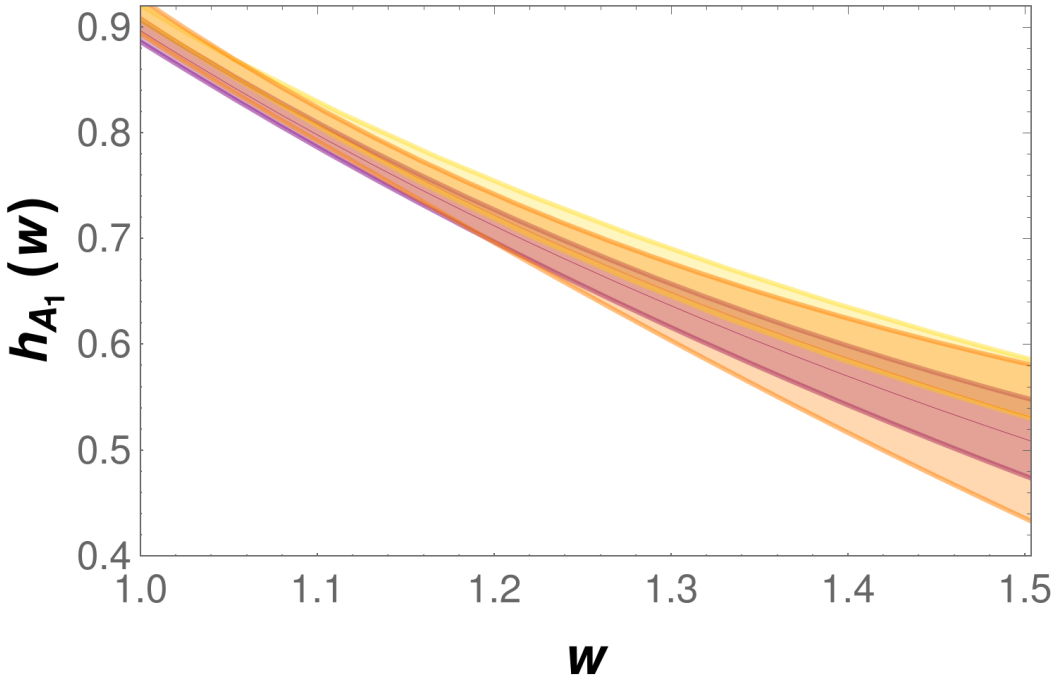
Major improvement: $B_{(s)} \to D^*_{(s)}$ FFs@$w > 1$! (B_s: [Harrison+'22])



- FNAL/MILC'21
- HQE@$1/m_c^2$
- Exp (BGL)
- JLQCD prel
- HPQCD'23

Comparison with new lattice calculations

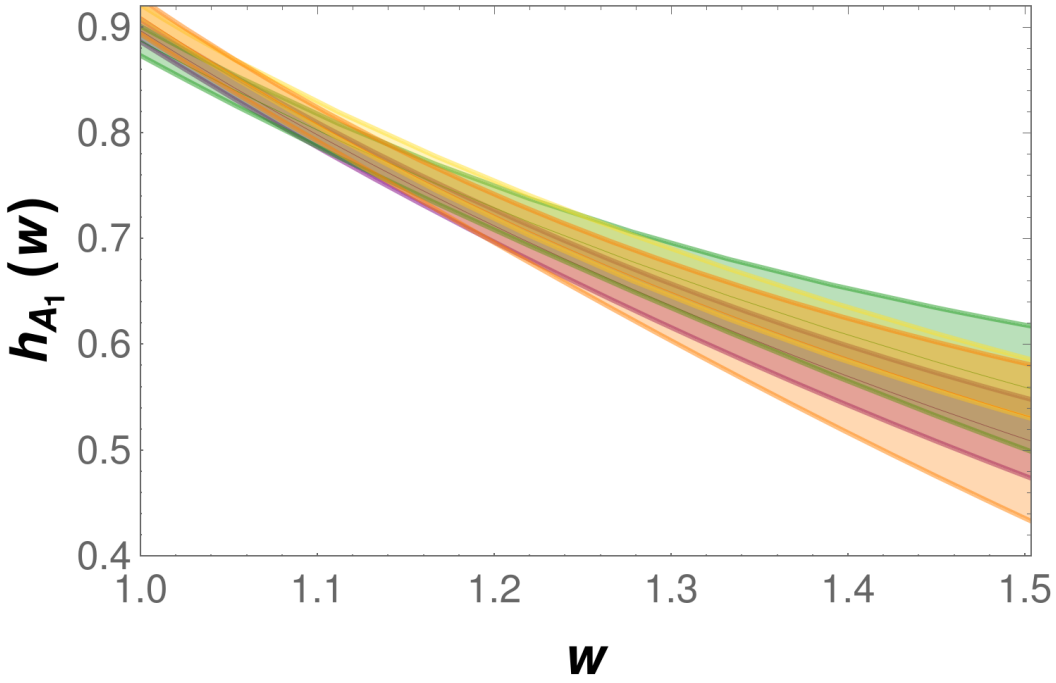
Major improvement: $B_{(s)} \to D^*_{(s)}$ FFs@$w > 1$! (B_s: [Harrison+'22])



- FNAL/MILC'21
- HQE@$1/m_c^2$
- Exp (BGL)
- JLQCD prel
- HPQCD'23

Comparison with new lattice calculations

Major improvement: $B_{(s)} \to D^*_{(s)}$ FFs@$w > 1$! (B_s: [Harrison+'22])

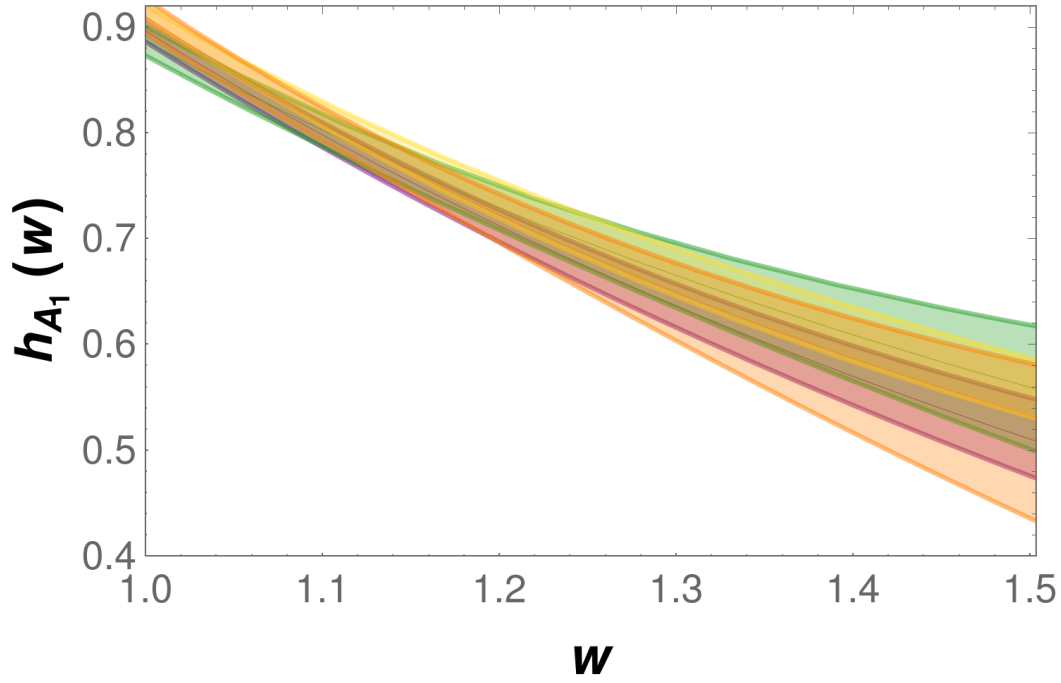


- FNAL/MILC'21
- HQE@$1/m_c^2$
- Exp (BGL)
- JLQCD prel
- HPQCD'23

➥ Compatible

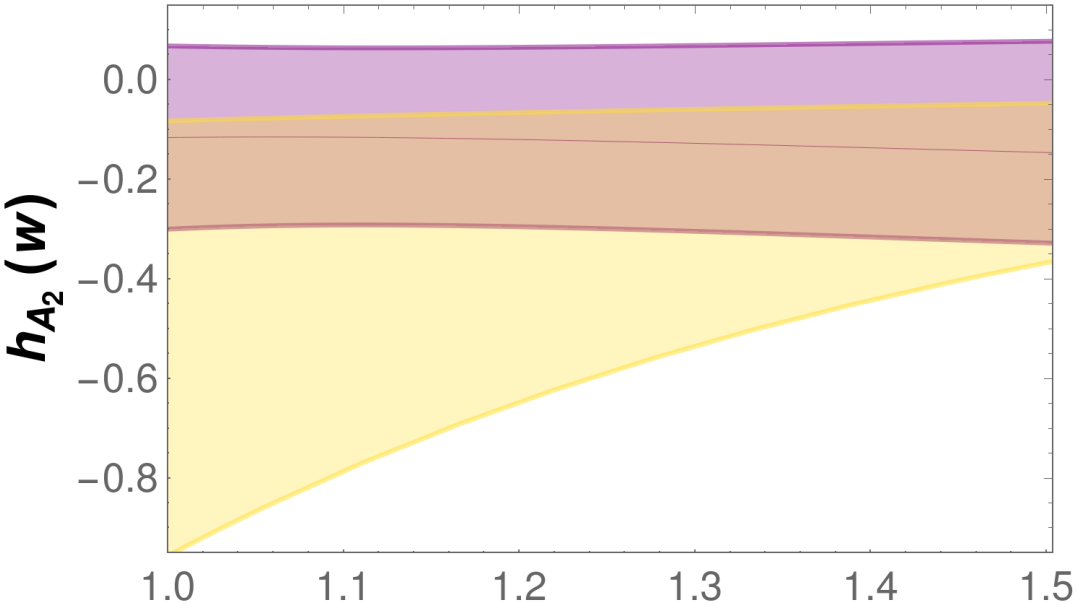
Comparison with new lattice calculations

Major improvement: $B_{(s)} \to D^*_{(s)}$ FFs@$w > 1$! (B_s: [Harrison+'22])



- FNAL/MILC'21
- HQE@$1/m_c^2$
- Exp (BGL)
- JLQCD prel
- HPQCD'23

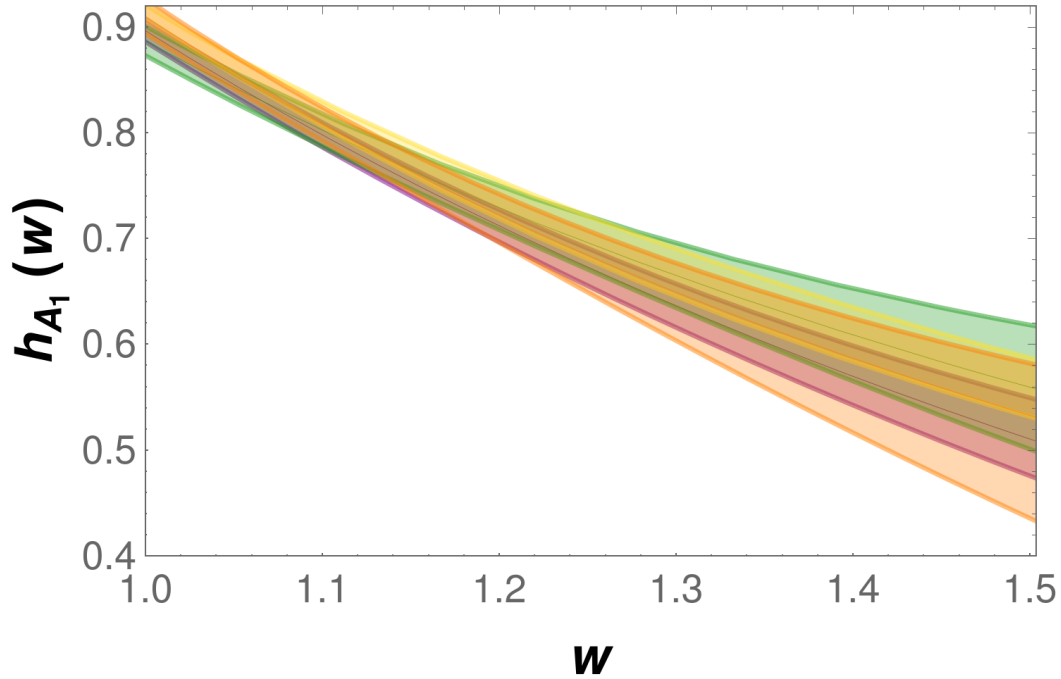
➥ Compatible



- HPQCD and BGJvD compatible

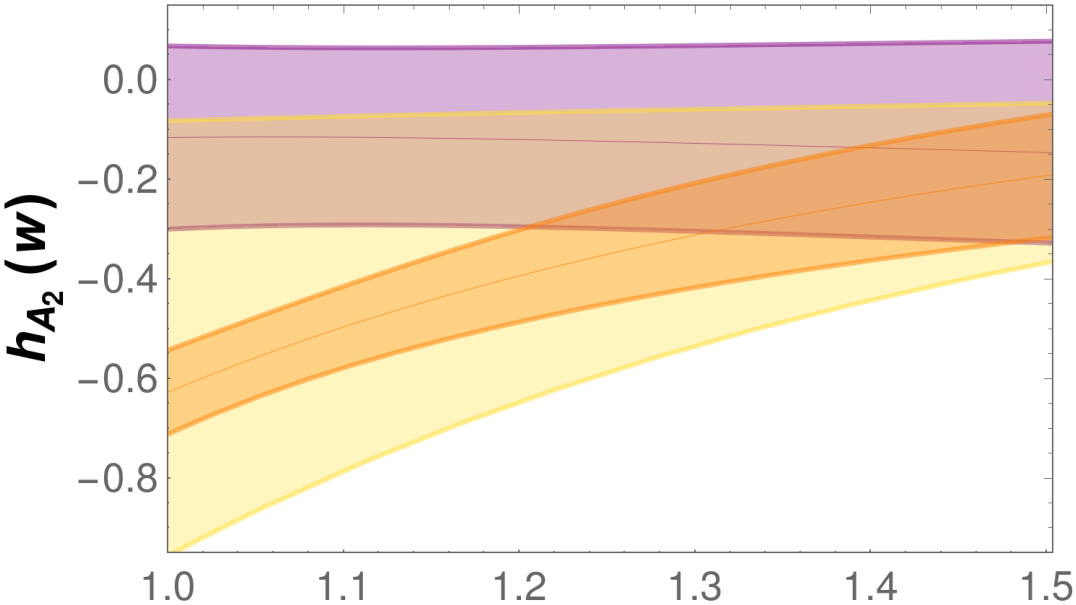
Comparison with new lattice calculations

Major improvement: $B_{(s)} \to D^*_{(s)}$ FFs@$w > 1$! (B_s: [Harrison+'22])



- FNAL/MILC'21
- HQE@$1/m_c^2$
- Exp (BGL)
- JLQCD prel
- HPQCD'23

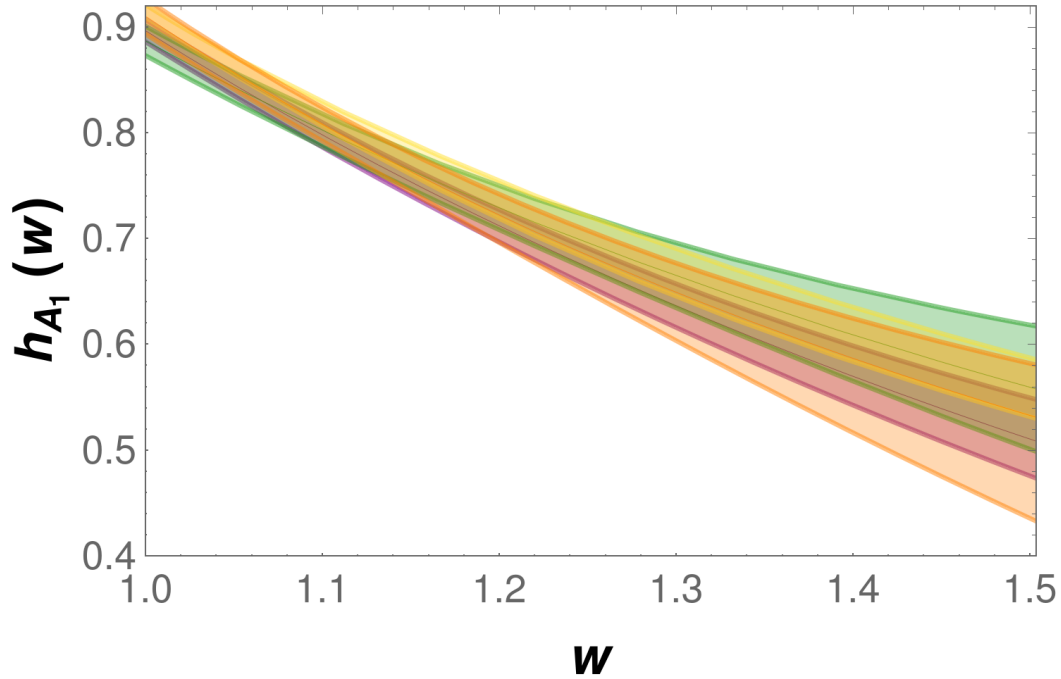
➥ Compatible



- HPQCD and BGJvD compatible
- Slope HPQCD-FNAL/MILC?

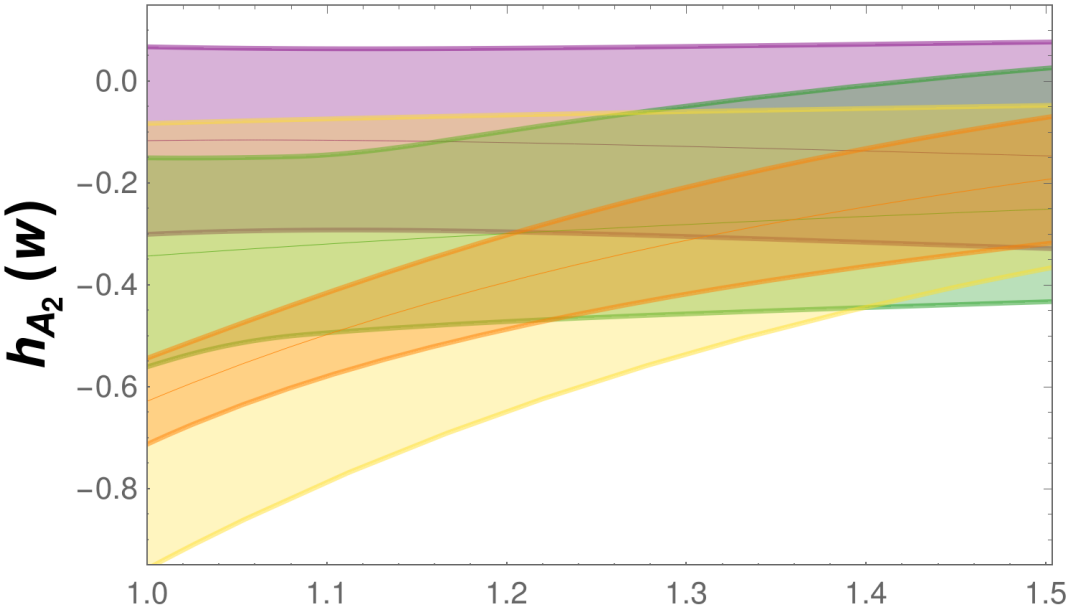
Comparison with new lattice calculations

Major improvement: $B_{(s)} \to D^*_{(s)}$ FFs@$w > 1$! (B_s: [Harrison+'22])



- FNAL/MILC'21
- HQE@$1/m_c^2$
- Exp (BGL)
- JLQCD prel
- HPQCD'23

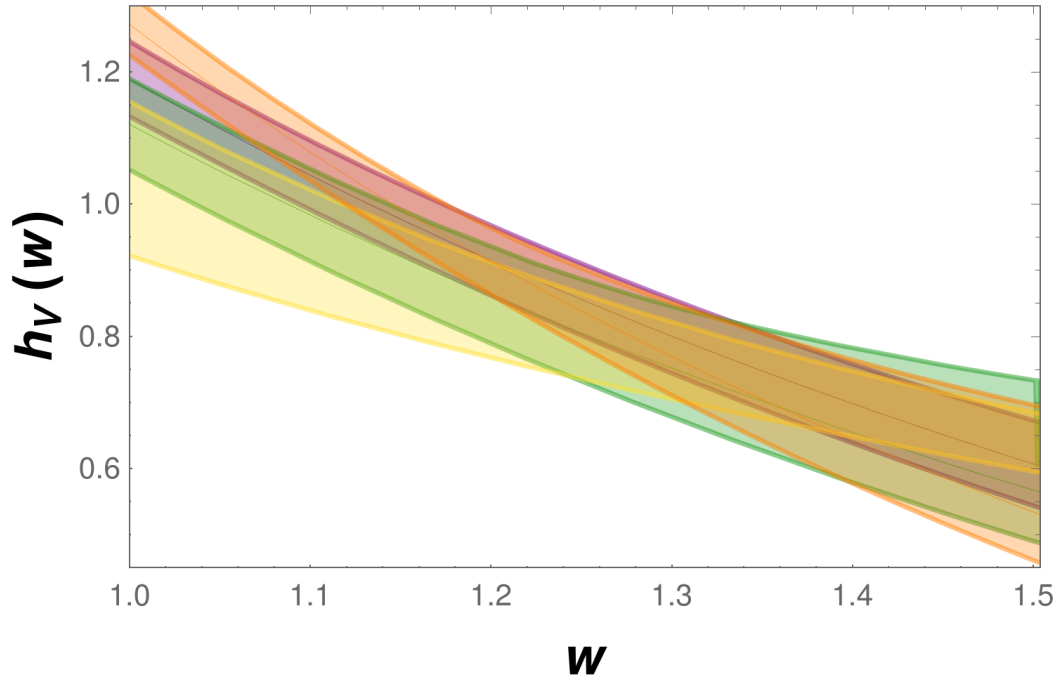
➥ Compatible



- HPQCD and BGJvD compatible
- Slope HPQCD-FNAL/MILC?
- JLQCD "diplomatic"

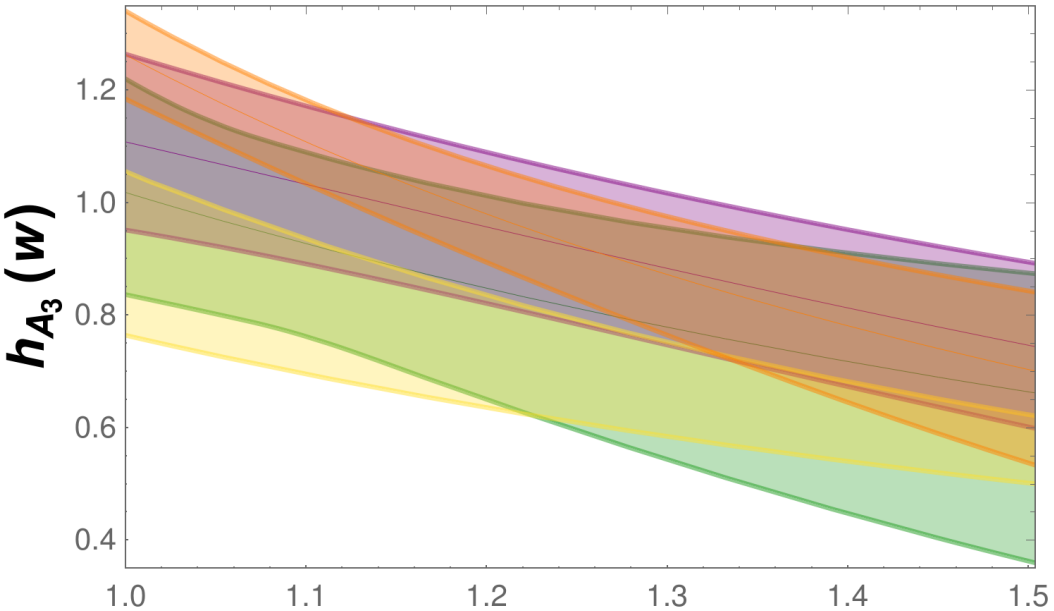
Comparison with new lattice calculations

Major improvement: $B_{(s)} \to D^*_{(s)}$ FFs@$w > 1$! (B_s: [Harrison+'22])



- FNAL/MILC'21
- HQE@$1/m_c^2$
- Exp (BGL)
- JLQCD prel
- HPQCD'23

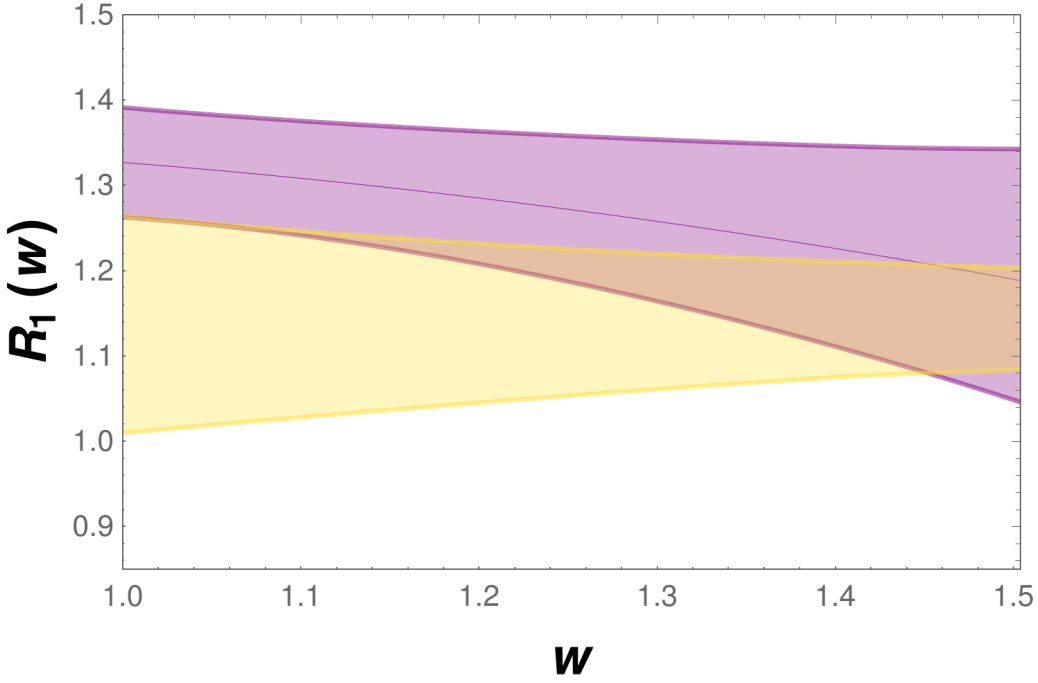
➥ Compatible



- Similar pattern in h_V and h_{A_3}
- Tension between BGJvD and FNAL/MILC in h_{A_2}

Comparison with new lattice calculations

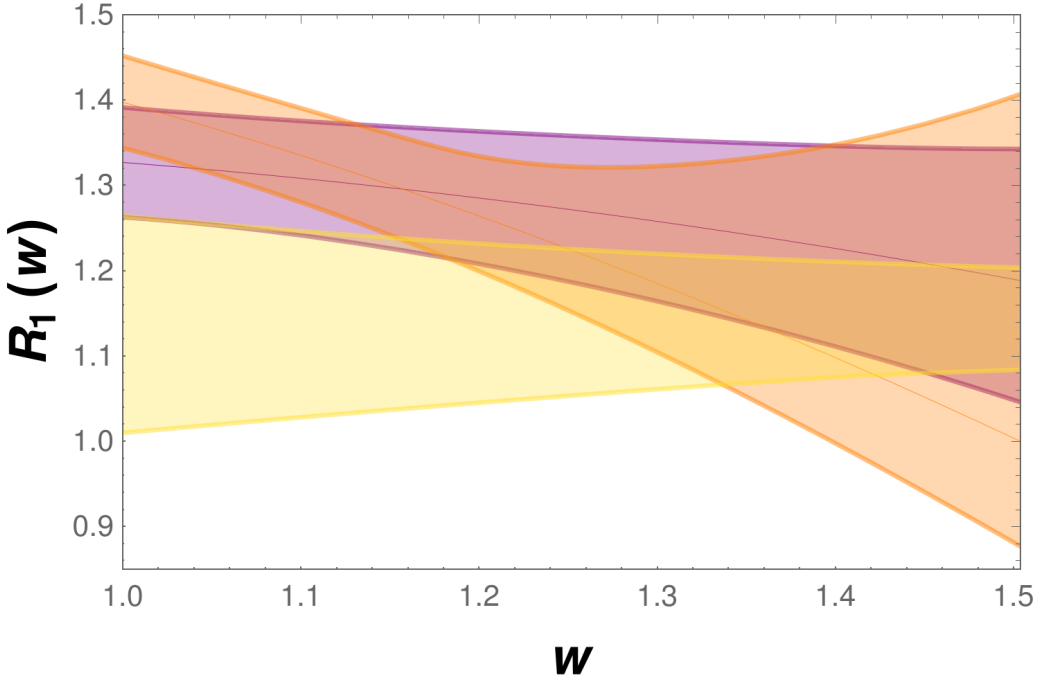
Major improvement: $B_{(s)} \to D^*_{(s)}$ FFs@$w > 1$! (B_s: [Harrison+'22])



- FNAL/MILC'21
- HQE@$1/m_c^2$
- Exp (BGL)
- JLQCD prel
- HPQCD'23

Comparison with new lattice calculations

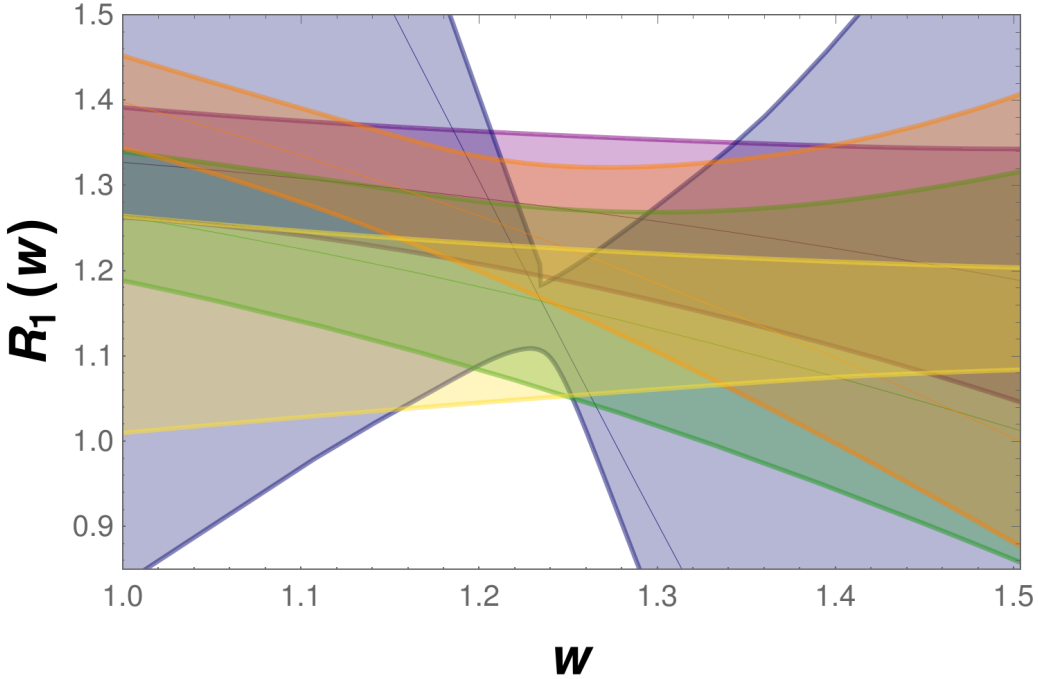
Major improvement: $B_{(s)} \to D^*_{(s)}$ FFs@$w > 1$! (B_s: [Harrison+'22])



- FNAL/MILC'21
- HQE@$1/m_c^2$
- Exp (BGL)
- JLQCD prel
- HPQCD'23

Comparison with new lattice calculations

Major improvement: $B_{(s)} \to D^*_{(s)}$ FFs@$w > 1$! (B_s: [Harrison+'22])

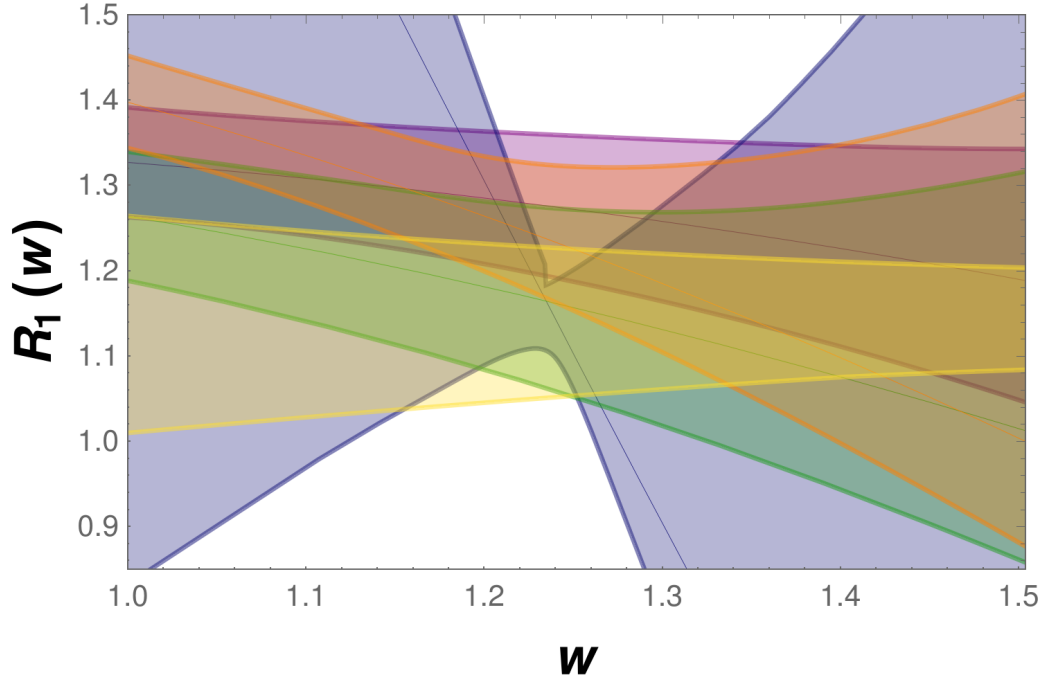


- FNAL/MILC'21
- HQE@$1/m_c^2$
- Exp (BGL)
- JLQCD prel
- HPQCD'23

➥ Compatible. Slope?

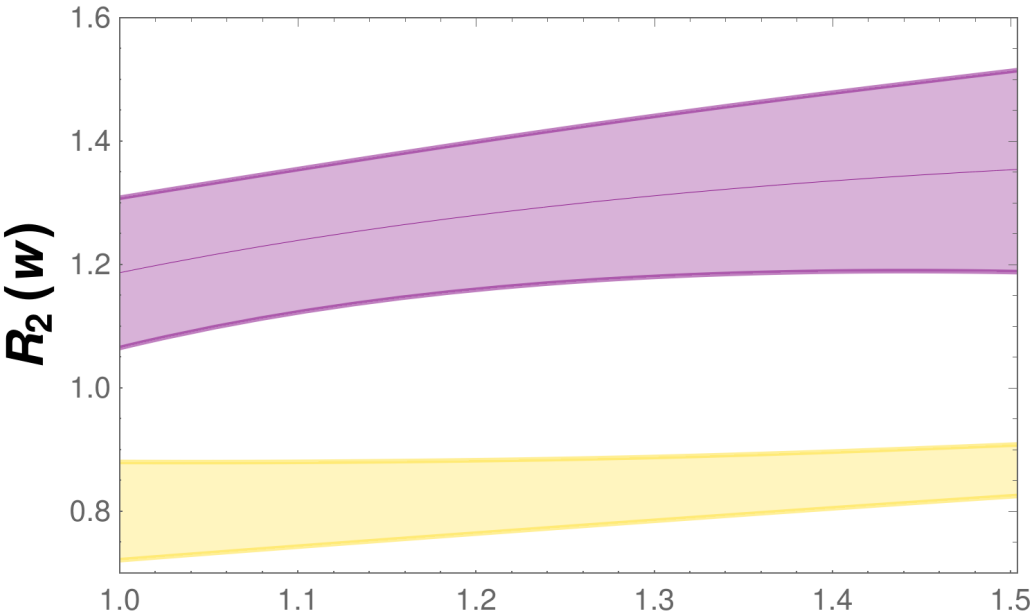
Comparison with new lattice calculations

Major improvement: $B_{(s)} \to D^*_{(s)}$ FFs@$w > 1$! (B_s: [Harrison+'22])



- FNAL/MILC'21
- HQE@$1/m_c^2$
- Exp (BGL)
- JLQCD prel
- HPQCD'23

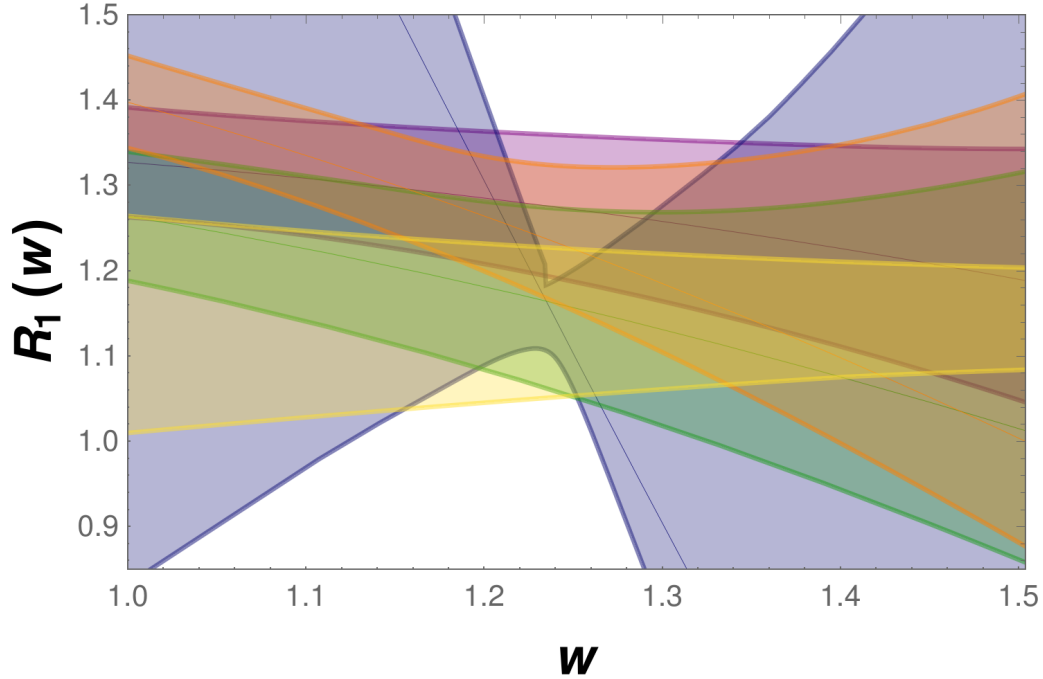
➥ Compatible. Slope?



- Deviation HPQCD-BGJvD

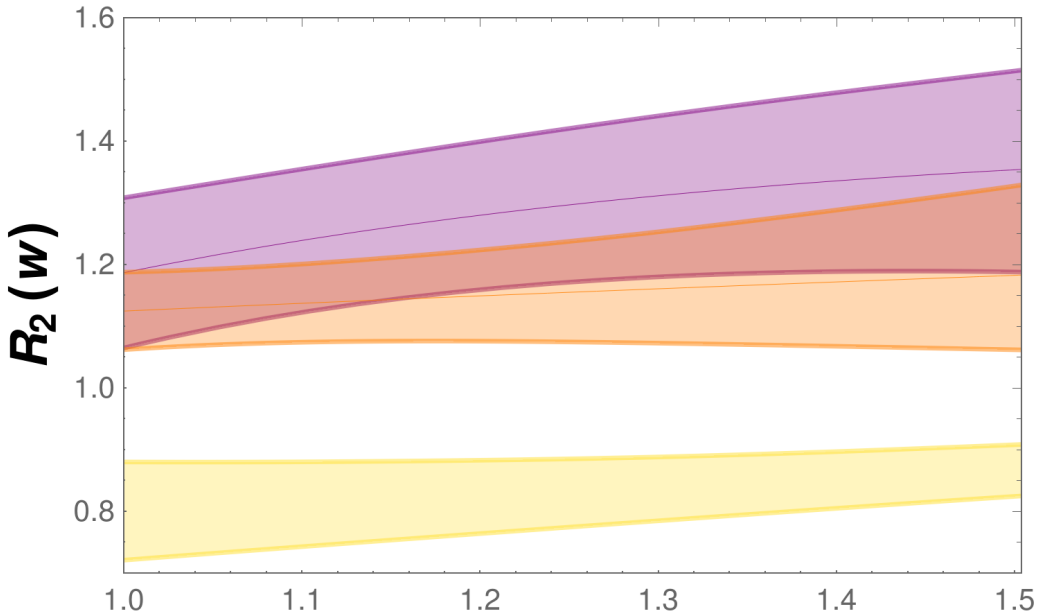
Comparison with new lattice calculations

Major improvement: $B_{(s)} \to D^*_{(s)}$ FFs@$w > 1$! (B_s: [Harrison+'22])



- FNAL/MILC'21
- HQE@$1/m_c^2$
- Exp (BGL)
- JLQCD prel
- HPQCD'23

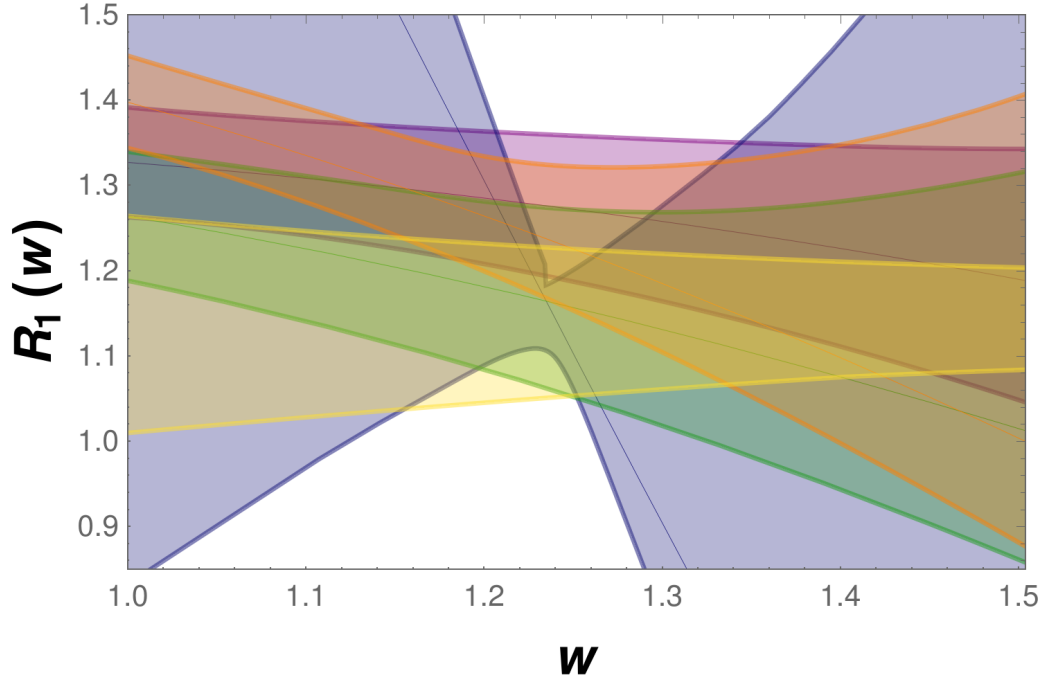
➥ Compatible. Slope?



- Deviation HPQCD-BGJvD
- FNAL/MILC close to HPQCD

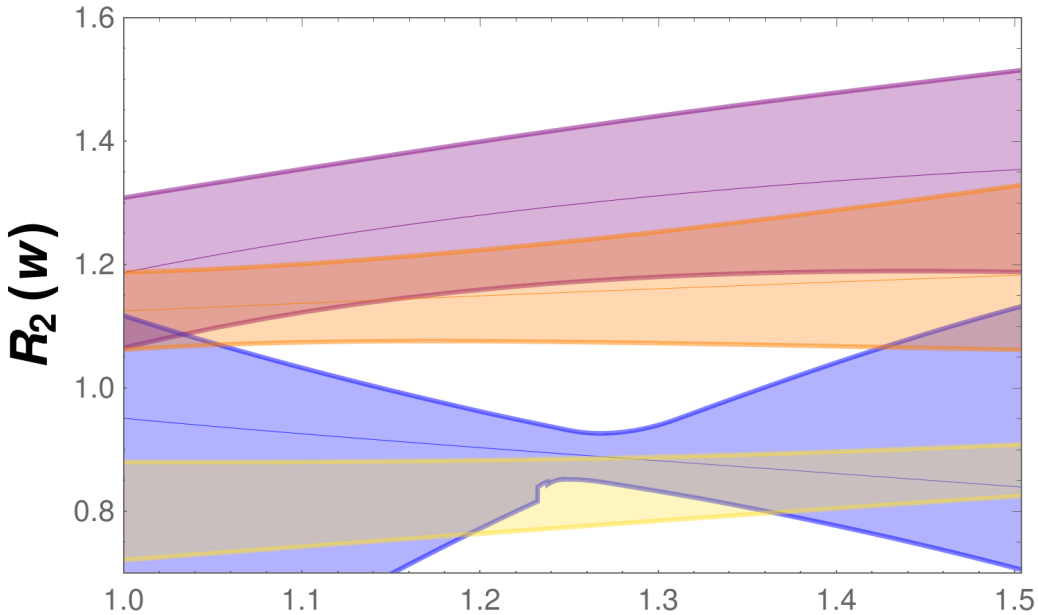
Comparison with new lattice calculations

Major improvement: $B_{(s)} \to D^*_{(s)}$ FFs@$w > 1$! (B_s: [Harrison+'22])



- FNAL/MILC'21
- HQE@$1/m_c^2$
- Exp (BGL)
- JLQCD prel
- HPQCD'23

➥ Compatible. Slope?

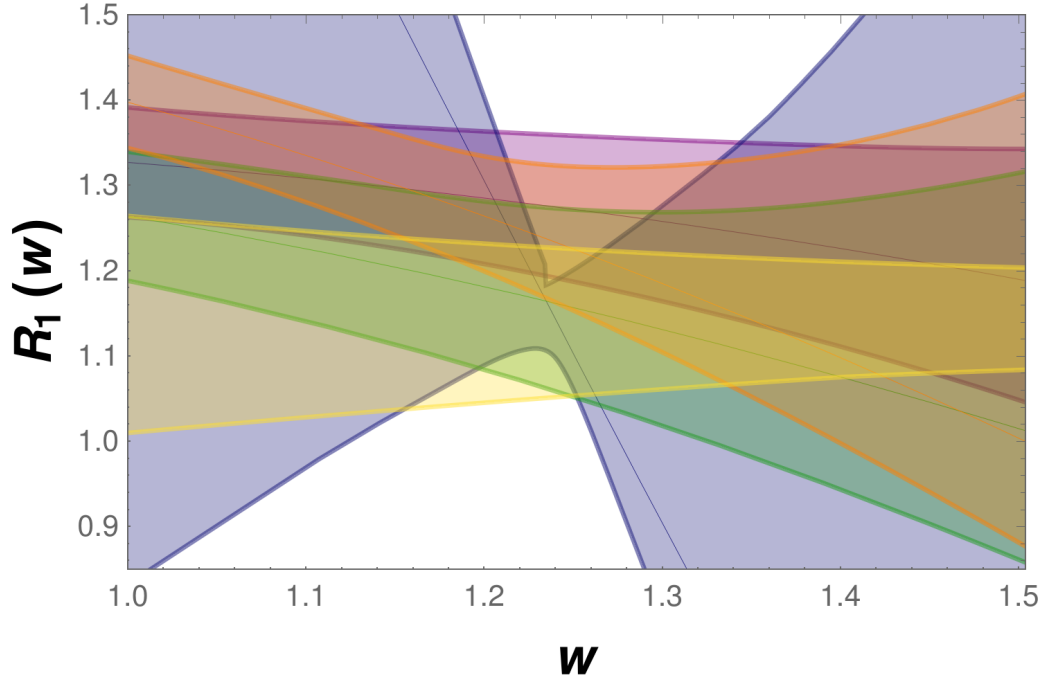


- Deviation HPQCD-BGJvD
- FNAL/MILC close to HPQCD
- Deviation wrt experiment

➥ Requires further investigation!

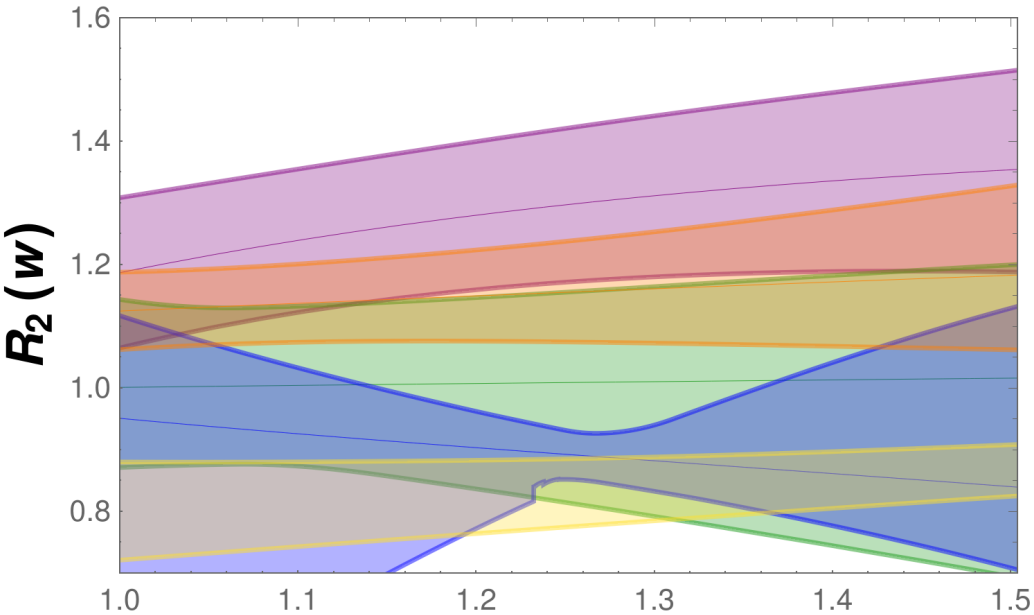
Comparison with new lattice calculations

Major improvement: $B_{(s)} \to D^*_{(s)}$ FFs@$w > 1$! (B_s: [Harrison+'22])



- FNAL/MILC'21
- HQE@$1/m_c^2$
- Exp (BGL)
- JLQCD prel
- HPQCD'23

➥ Compatible. Slope?



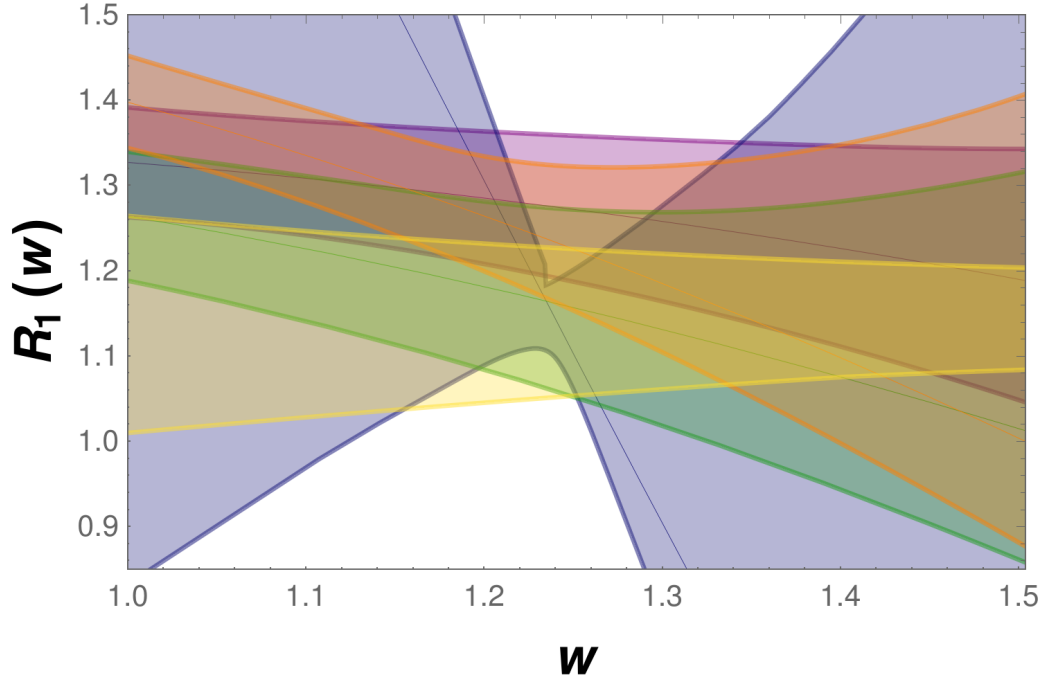
- Deviation HPQCD-BGJvD
- FNAL/MILC close to HPQCD
- Deviation wrt experiment

➥ Requires further investigation!

- JLQCD "diplomatic"

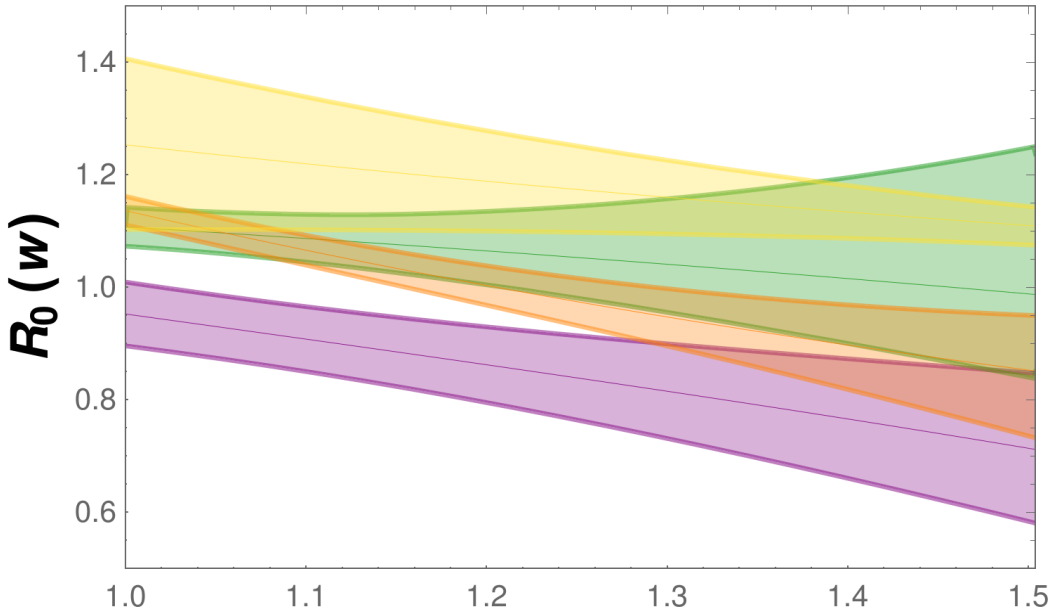
Comparison with new lattice calculations

Major improvement: $B_{(s)} \to D^*_{(s)}$ FFs@$w > 1$! (B_s: [Harrison+'22])



- FNAL/MILC'21
- HQE@$1/m_c^2$
- Exp (BGL)
- JLQCD prel
- HPQCD'23

➥ Compatible. Slope?

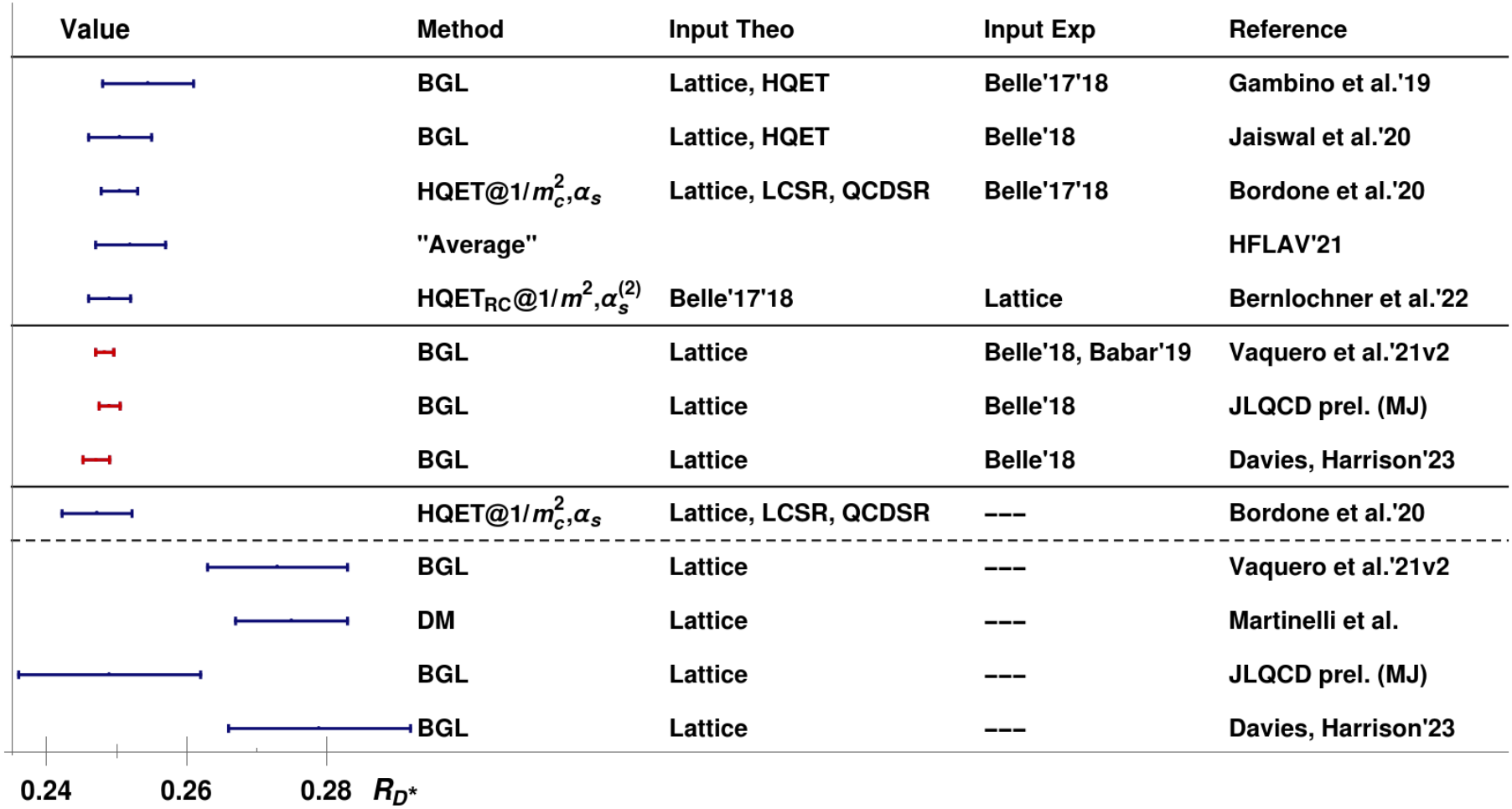


- Also in R_0 deviation wrt BGJvD
- JLQCD again "diplomatic"

➥ Requires further investigation!

➥ Correlations?

Overview over predictions for $R(D^*)$



Lattice $B \to D^*$: $h_{A_1}(w=1)$ [FNAL/MILC'14,HPQCD'17] , [FNAL/MILC'21]
Other lattice: $f_{+,0}^{B\to D}(q^2)$ [FNAL/MILC,HPQCD'15]
QCDSR: [Ligeti/Neubert/Nir'93,'94] , LCSR: [Gubernari/Kokulu/vDyk'18]

Overall consistent SM predictions!
"Explaining" $R(D^*)$ by FM/HPQCD $\to$ NP in $B \to D^*(e,\mu)\nu$!

Conclusions

Semileptonic $b \to c$ transitions remain exciting!

1. q^2 dependence of FFs critical
   - Need parametrization-independent data
2. Inclusion of higher-order (theory) uncertainties essential
3. HQE: systematic expansion in $1/m, \alpha_s$, relates FFs
   - $\mathcal{O}(1/m_c)$ ($\to$ CLN) not sufficient anymore
4. Important first LQCD analyses in $B_{(s)} \to D^*_{(s)}$ @ finite recoil
   - HPQCD: First 2+1+1 results, full q^2 range!
   - Tensions in ratios – correlations?
5. Despite complications: $R(D^{(*)})$ SM prediction robust!

> Central lesson:
> Experiment and theory (lattice + pheno) need to work closely together!

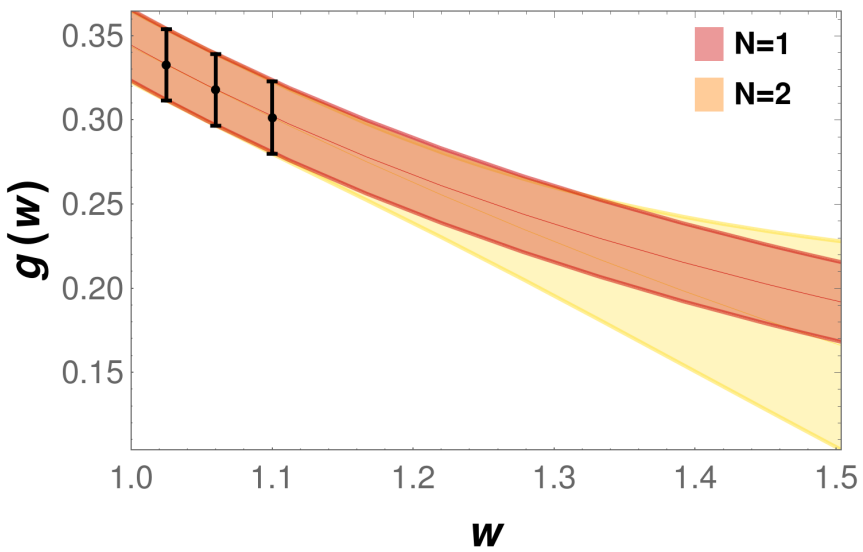
Comparison to Bernlochner+'22

Bernlochner et al. also perform HQE analysis @$1/m_c^2$. Differences:

- Postulate different counting within HQET
  - ➥ Highly constraining model for higher-order corrections
- Avoid use of LCSR (and mostly QCDSR) results
- Include partial α_s^2 corrections
- Include FNAL/MILC results partially
- Expansion in z: 2/1/0 (justified in [Bernlochner+'19])

Observations:

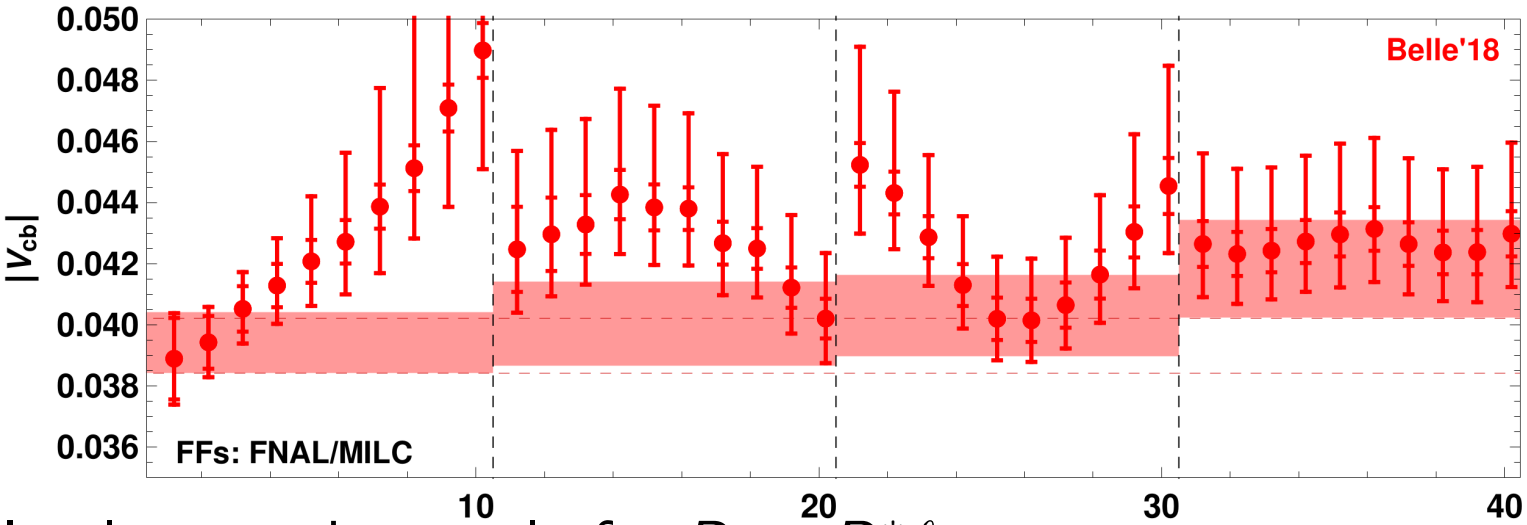
- $1/m_c^2$ corrections necessary
- Overall small uncertainties
- $V_{cb} = (38.7 \pm 0.6) \times 10^{-3}$
  - ➥ smaller due to larger $\mathcal{F}(1)$
- $R(D^*)$: agreement w/ BGJvD
- $R(D) \sim 3\sigma$ from GJS + BGJvD
  - ➥ In my opinion due to model



The Dispersive Matrix (DM) Method

Alternative implementation of unitarity [Bourrely+'81,Lellouch'95] :

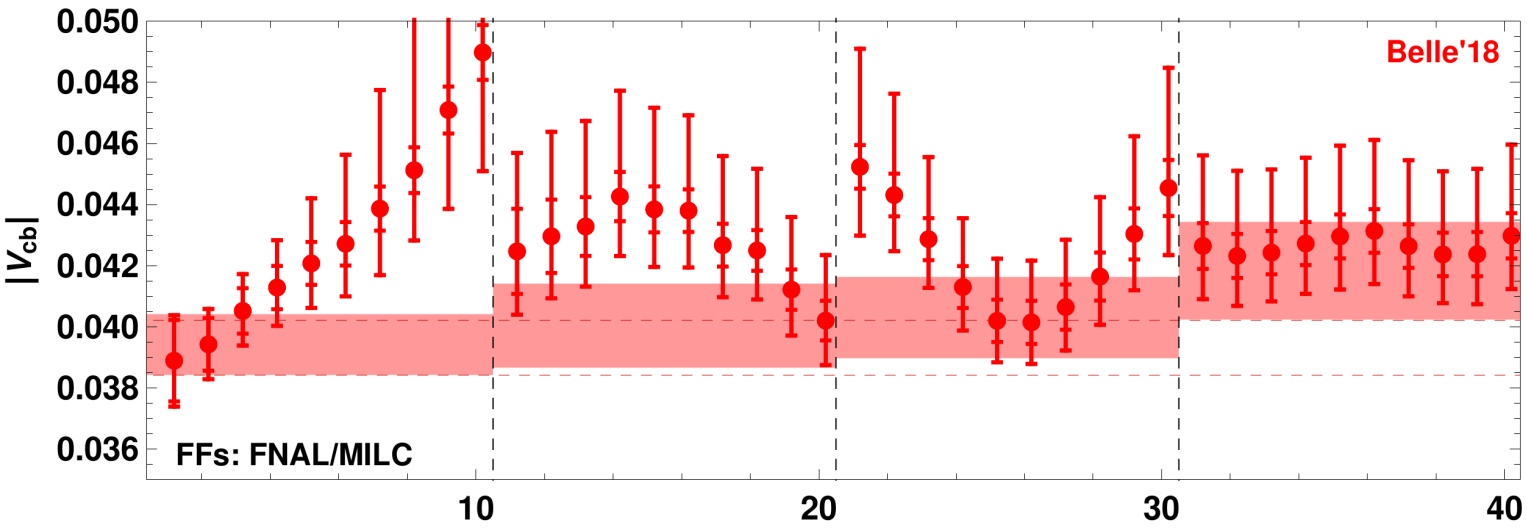
- Identical starting point as BGL: dispersion relation
- Known information in a matrix with positive determinant
  - ➥ Form-factor bounds
- Enables parametrization-free analysis



Implemented recently for $B \to D^* \ell \nu$ [DiCarlo+'21,Martinelli+'21,22] :

- Use DM w/ new FNAL/MILC data to obtain FF bands
- Calculate V_{cb} bin-wise, combine $d\Gamma/dx$ bins ($x = q^2, \cos\theta, \ldots$) (including experimental and theoretical correlations)
  - ➥ 2×4 V_{cb} values. Claim: 0.5σ to $V_{cb}^{\rm incl}$, 1.3σ to $R(D^*)$

The Dispersive Matrix (DM) Method



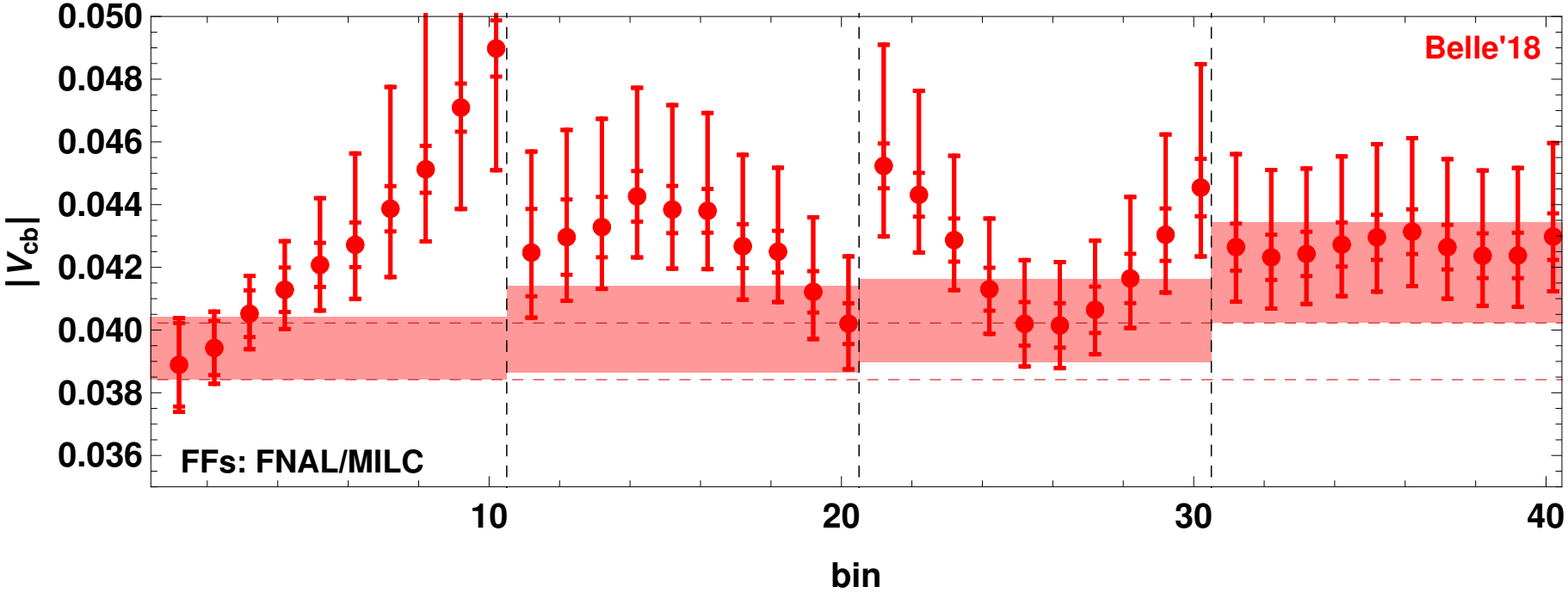
Differences between DM and GJS [Gambino/MJ/Schacht'19] :

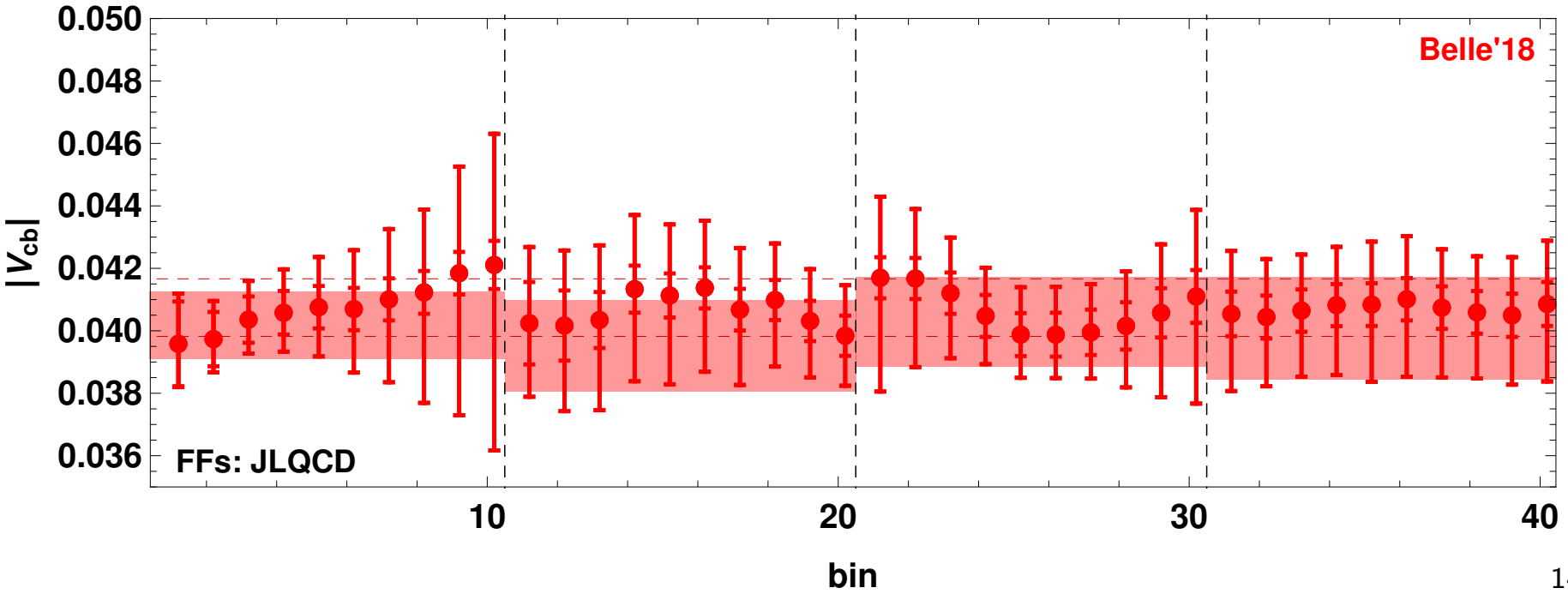
- GJS: Combined fit of lattice and experiment, imposing unitarity
- DM: Unweighted, uncorrelated average of the 4 V_{cb} values:

$$\mu = \frac{1}{N}\sum_{k=1}^{N} x_k\,, \quad \sigma_x^2 = \frac{1}{N}\sum_{k=1}^{N}\sigma_k^2 + \frac{1}{N}\sum_{k=1}^{N}(x_k-\mu_x)^2$$

➥ $V_{cb}^{\rm GJS} = \left(39.2^{+1.4}_{-1.2}\right)\times 10^{-3}, \qquad V_{cb}^{\rm DM} = (40.8\pm 1.7)\times 10^{-3}$

Binned V_{cb} from Belle'18 data: FNAL/MILC vs JLQCD

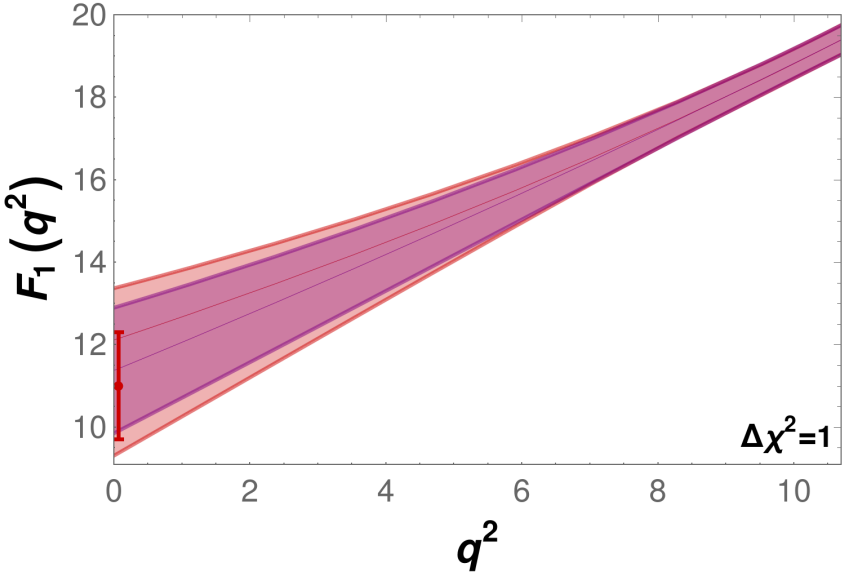


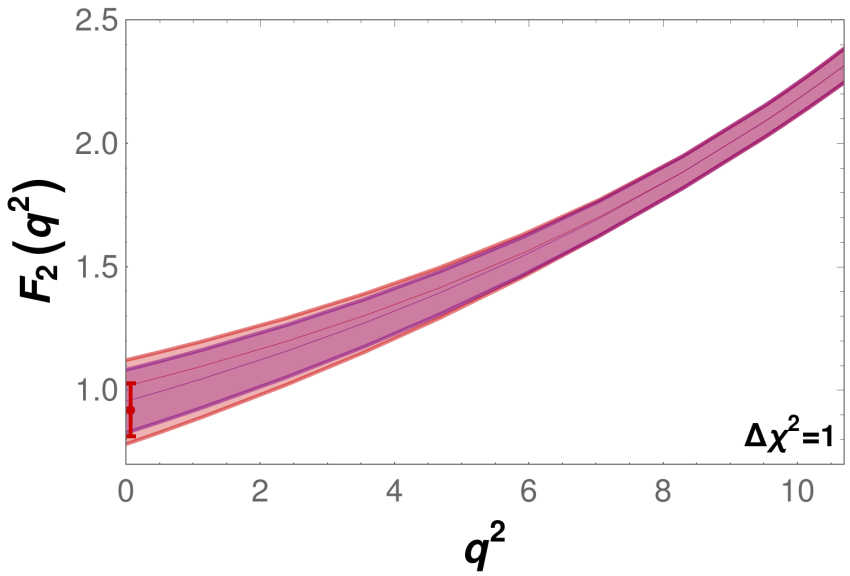


Priors and potential biases

Different conclusions starting from identical information

Example: $R(D^*)$ extraction from FNAL/MILC data





$R(D^*)$ including kinematical identities and weak unitarity

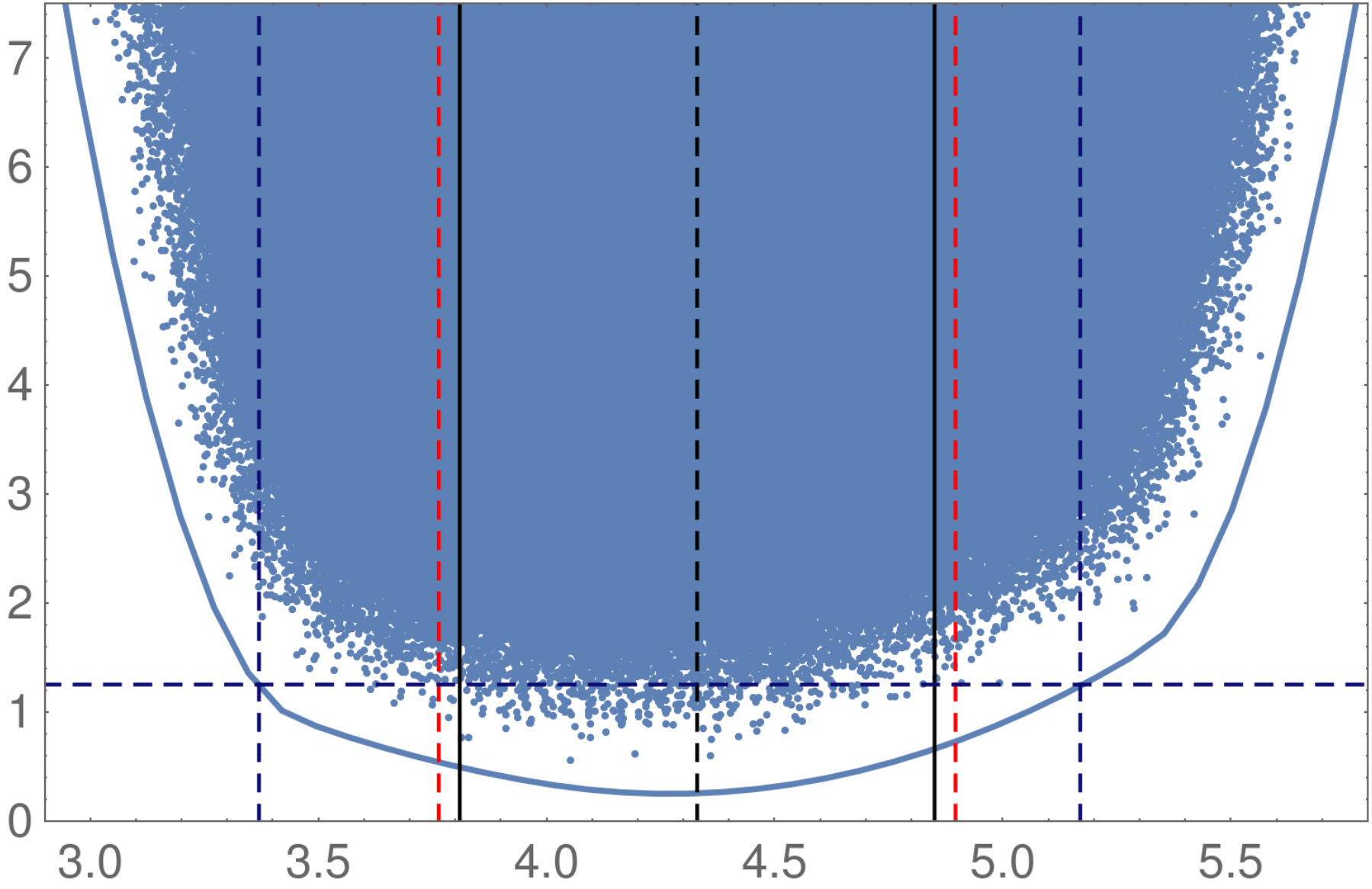
$$R(D^*) \overset{\text{WU}}{=} 0.269^{+\mathbf{0.020}}_{-0.008} \quad \overset{\text{FM}}{=} 0.274 \pm 0.010 \quad \overset{\text{Rome}}{=} 0.275 \pm 0.008\,.$$

Difference WU-FM: FM apply prior on BGL coefficients

Difference WU-Rome (educated guess): iterated "unitarity filter" + different error estimate

Applying data: $R(D^*) = 0.249 \pm 0.001(!)$ **universally**.

Uncertainty determination



MC points together with χ^2 profile (minimizing for each FF value)
Vertical: CV MC, "1σ" MC, symmetric 68.3% interval MC, $\Delta\chi^2 = 1$