Pushing the QCD frontier in $b \to s\ell\ell$

Beyond the flavour anomalies IV – 19/04/2023

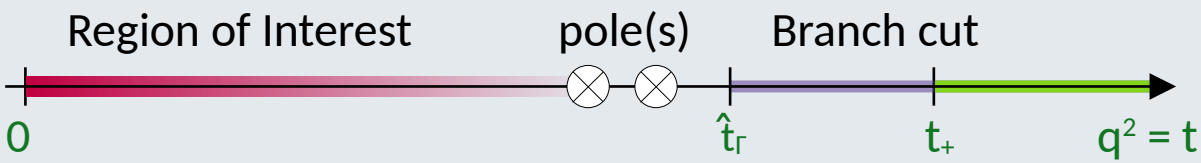
Méril Reboud

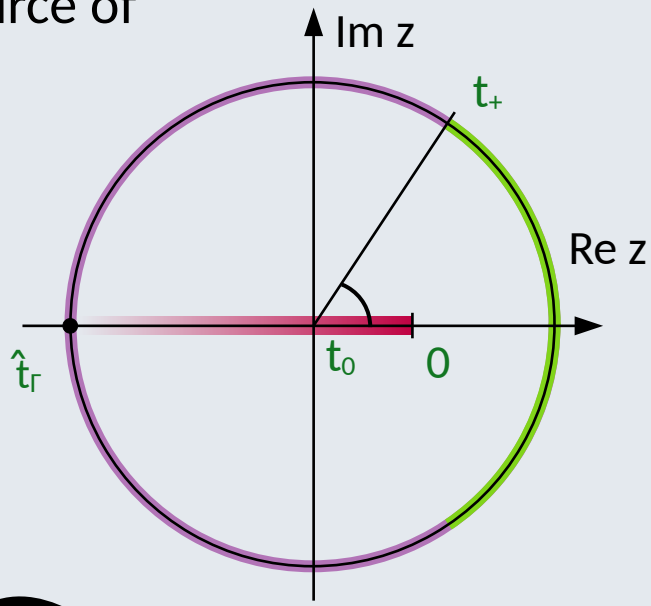
Based on:

- Gubernari, van Dyk, JV 2011.09813
- Gubernari, Reboud, van Dyk, JV 2206.03797
- Ahmis, Bordone, Reboud 2208.08937
- Gubernari, Reboud, van Dyk, JV 2305.XXXX

Introduction and Outline

- Nico's talk:
  - **Local** and **non-local form factors** are the main source of uncertainties in $b \to s\ell\ell$ decays
  - Both follow the same analytic structure:





  - The GRvDV parametrization diagonalizes the dispersive bounds:

$$\hat{z}(q^2) = \frac{\sqrt{\hat{t}_\Gamma - q^2} - \sqrt{\hat{t}_\Gamma}}{\sqrt{\hat{t}_\Gamma - q^2} + \sqrt{\hat{t}_\Gamma}}$$

$$\hat{\mathcal{H}}(\hat{z}) = \sum_{n=0}^{\infty} \beta_n \, p_n(\hat{z})$$

Orthonormal polynomials of the arc of the unit circle

Introduction and Outline

$$\sigma_{\text{incl}}\left(\sim\!\!<^{b}_{\bar{s}}\right) > \sigma_{\text{excl}} \sim \int dPS \left| \sim\!\!\ll^{B}_{K^*} \right|^2$$

$$\sim \sum_{\lambda} \int_{t_{th}}^{\infty} dq^2\, \omega_\lambda(q^2)\, |\mathcal{H}_\lambda(q^2)|^2$$

$$\sim \sum_{\lambda} \int_{\text{arc}(t_+)} dz\, |\hat{\mathcal{H}}_\lambda(q^2)|^2 = \sum_{\lambda, n} |\beta_n^\lambda|^2$$

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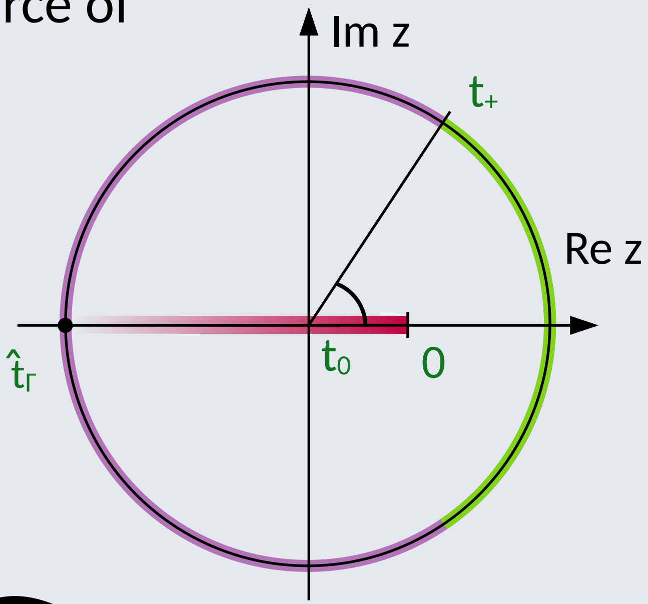
- The GRvDV parametrization diagonalizes the dispersive bounds:

$$\hat{z}(q^2) = \frac{\sqrt{\hat{t}_\Gamma - q^2} - \sqrt{\hat{t}_\Gamma}}{\sqrt{\hat{t}_\Gamma - q^2} + \sqrt{\hat{t}_\Gamma}} \qquad \hat{\mathcal{H}}(\hat{z}) = \sum_{n=0}^{\infty} \beta_n\, p_n(\hat{z})$$



Orthonormal polynomials of the arc of the unit circle

Introduction and Outline

- This talk:

  1) The parametrization in practice with $\Lambda_b \to \Lambda^* \ell\ell$

  2) Combined analysis of $B \to K$, $B \to K^*$ and $B_s \to \phi$ local form factors with improved dispersive bounds

  3) Analysis of non-local form factors

  4) Benchmark BSM study

I. The method in practice

Example with $\Lambda_b \to \Lambda(1520)\ell\ell$

14 Form Factors → 17 pars (N=1), 31 pars (N=2)
21 LQCD "points" + 9 SCET relations } 30 inputs.

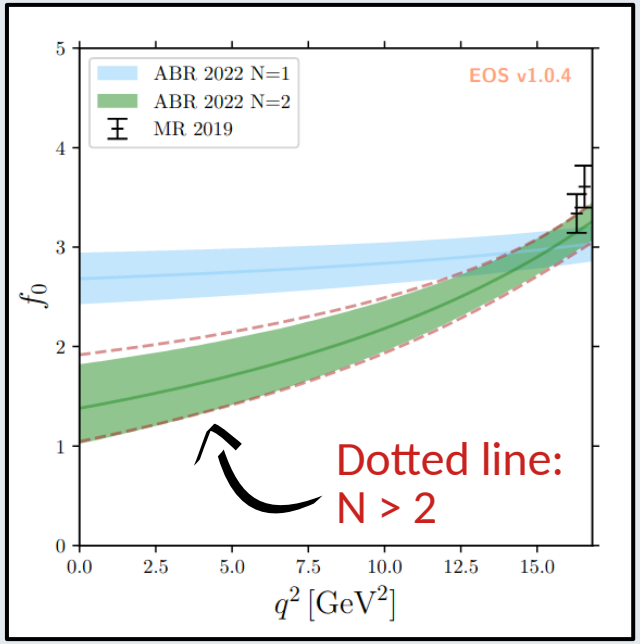
- Inputs:
  - **LQCD** [Meinel, Rendon '21]
  - no LCSR → use (loose) **SCET relations** [Descotes-Genon, M. Novoa-Brunet '19]

$$f_{\perp'}(0) = 0 \pm 0.2\,, \quad g_{\perp'}(0) = 0 \pm 0.2\,, \quad h_{\perp'}(0) = 0 \pm 0.2\,,$$
$$\tilde{h}_{\perp'}(0) = 0 \pm 0.2\,, \quad f_+(0)/f_\perp(0) = 1 \pm 0.2\,, \quad f_\perp(0)/g_0(0) = 1 \pm 0.2\,,$$
$$g_\perp(0)/g_+(0) = 1 \pm 0.2\,, \quad h_+(0)/h_\perp(0) = 1 \pm 0.2\,, \quad f_+(0)/h_+(0) = 1 \pm 0.2\,,$$

$O(\alpha_s/\pi, \Lambda_{QCD}/m_b)$

- Use an **under-constrained fit** (N>1) and allows for saturation of the dispersive bound

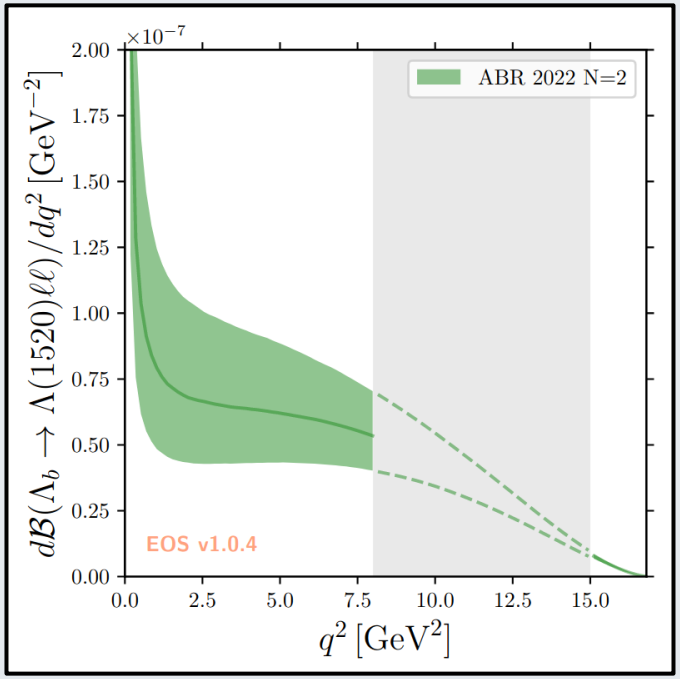
  → The uncertainties are model-independent, increasing the expansion order does not change their size



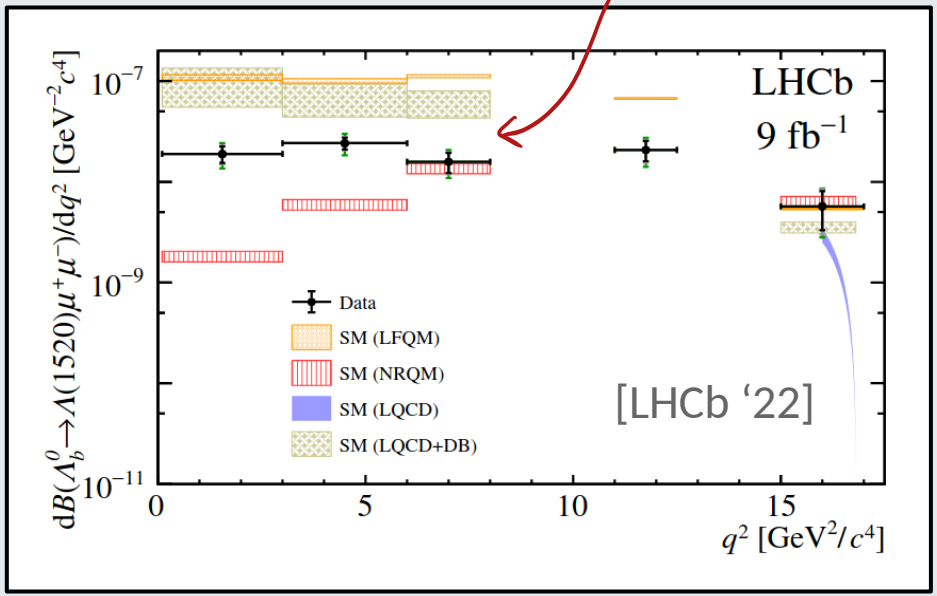
[Ahmis, MR, Bordone '22]

Phenomenology

- Uncertainties are large but **under control** and **systematically improvable**
- LHCb analysis confirmed the usual b → sℓℓ tension at low q^2



bsμμ Pattern.

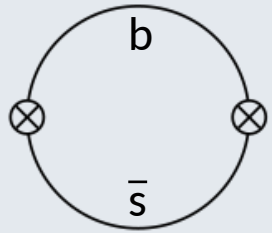


II. Improved dispersive bounds

Correlator and Helicities

- **Main idea:** Compute the inclusive $e^+e^- \to \bar{b}s$ cross-section and relate it to the form factors [Bharucha, Feldmann, Wick '10]

Insertion of a scalar, vector or tensor current



+ other diagrams: loops, quark and gluon condensates...

- In practice, the correlator $\Pi_\Gamma^{\mu\nu}(q) \equiv i \int d^4x\, e^{iq\cdot x} \langle 0|\mathcal{T}\left\{J_\Gamma^\mu(x) J_\Gamma^{\dagger,\nu}(0)\right\}|0\rangle$

can be decomposed on an helicity basis:

$$\Pi_\Gamma^{\mu\nu}(q) \equiv \sum_{\lambda,\lambda'} \epsilon^\mu(\lambda)\epsilon^{\nu*}(\lambda')\, \Pi_\Gamma^{(\lambda,\lambda')}(q^2)$$

Polarization vectors

Correlator and Helicities

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Compared to the usual:

$$\Pi_\Gamma^{\mu\nu} = \underbrace{\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)}_{\sum_{\lambda=0,\pm,\parallel} \epsilon^\mu(\lambda)\,\epsilon^{\nu *}(\lambda)} \Pi_\Gamma^{(J=1)} + \frac{q^\mu q^\nu}{q^2} \Pi_\Gamma^{(J=0)}$$

$\epsilon^\mu(t)\,\epsilon^{\nu *}(t)$

- In practice, the correlator $\Pi_\Gamma^{\mu\nu}(q) \equiv i \int d^4x\, e^{iq\cdot x} \langle 0|\mathcal{T}\left\{J_\Gamma^\mu(x) J_\Gamma^{\dagger,\nu}(0)\right\}|0\rangle$

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Polarization vectors

Correlator and Helicities

- **Main advantage:**
  - The OPE calculation is independent of the helicities:

$$\Pi_\Gamma^{(J=1)}\big|_{\mathrm{OPE}} = \Pi_\Gamma^{(0)}\big|_{\mathrm{OPE}} = \Pi_\Gamma^{(\|)}\big|_{\mathrm{OPE}} = \Pi_\Gamma^{(\perp)}\big|_{\mathrm{OPE}}$$

    → The calculation of Ref. [Bharucha, Feldmann, Wick '10] still applies!

  - Remove spurious correlations between form factors:
    - e.g. A_1 and A_{12} now fulfill different bounds
    - decorrelate completely $B \to K$ from ($B \to K^*$, $B_s \to \phi$)

Correlator and Helicities

- **In equations:**
  - This is the bound used in the literature:

$$\chi_A^{(J=1)}\Big|_{BK^*} = \frac{\eta^{B\to K^*}}{24\pi^2}\int_{(M_B+M_{K^*})^2}^{\infty} ds \frac{\lambda_{\rm kin}^{1/2}}{s^2(s-Q^2)^3}\left[s\,(M_B+M_{K^*})^2 \boxed{|A_1^{B\to K^*}|^2} + 32\,M_B^2 M_{K^*}^2 \boxed{|A_{12}^{B\to K^*}|^2}\right]$$

  - And this is what we propose:

$$\chi_A^{(0)}\Big|_{\bar{B}K^*} = \frac{\eta^{B\to K^*}}{\pi^2}\int_{(M_B+M_{K^*})^2}^{\infty} ds \frac{\lambda_{\rm kin}^{1/2}}{s^2(s-Q^2)^3} 4\,M_B^2 M_{K^*}^2 \boxed{|A_{12}^{B\to K^*}|^2},$$

$$\chi_A^{(\|)}\Big|_{\bar{B}K^*} = \frac{\eta^{B\to K^*}}{8\pi^2}\int_{(M_B+M_{K^*})^2}^{\infty} ds \frac{\lambda_{\rm kin}^{1/2}}{s^2(s-Q^2)^3} s\,(M_B+M_{K^*})^2 \boxed{|A_1^{B\to K^*}|^2},$$

Local form factors fit

- With this framework we perform a **combined fit** of $B \to K$, $B \to K^*$ and $B_s \to \phi$ LCSR and lattice QCD inputs:
  - $B \to K$:
    - [HPQCD '13 and '22; FNAL/MILC '17]
    - ([Khodjamiriam, Rusov '17]) → large uncertainties, not used in the fit
  - $B \to K^*$:
    - [Horgan, Liu, Meinel, Wingate '15]
    - [Gubernari, Kokulu, van Dyk '18]
  - $B_s \to \varphi$:
    - [Horgan, Liu, Meinel, Wingate '15]
    - [Gubernari, van Dyk, Virto '20]

(we have LQCD @ low q^2)

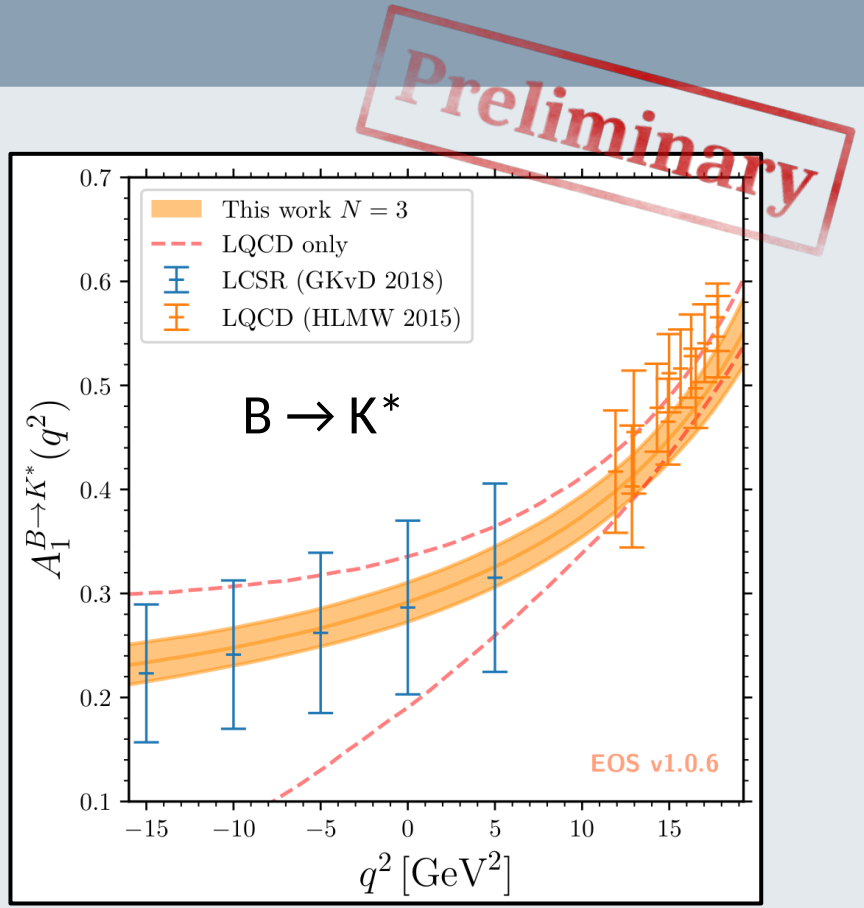
+ 10% width effect in observable.

[Descotes-Genon, Khodjamirian, Virto 2019]

- Baryonic decays should be added, but there are currently only few constraints

Results

- Bayesian analysis using EOS
- Truncate the series expansion to N = 2, 3, 4
  - Uncertainties stable for N > 2
  - Provide machine-readable numerical results
- **Main conclusions:**
  - With the current inputs, BSZ still performs well (including uncertainties) for $q^2 > 0$
  - Our approach is essential for $q^2 < 0$



(No under constrained fit)
↳ stability criterion.

eos.github.io

[Gubernari, Reboud, van Dyk, JV 2305.XXXX]

III. Parametrization of non-local form factors

Parametrization of the charm loop

- Still focusing on $B \to K$, $B \to K^*$ and $B_s \to \varphi$

  Inputs:

  - 4 theory point at negative q^2 from the light cone OPE
  - Experimental results at the J/ψ

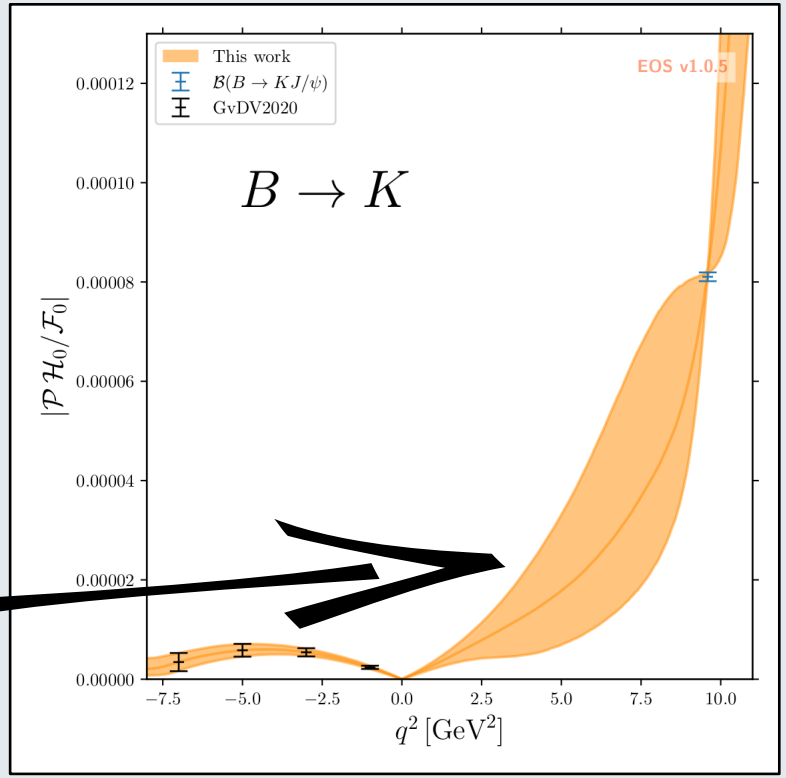
- Use again an under-constrained fit (N = 5) and allows for saturation of the dispersive bound

  → The uncertainties are **model-independent**, increasing the expansion order does not change their size

  → All p-values are larger than 11%

trunc. order indep.

[Gubernari, MR, van Dyk, Virto '22]



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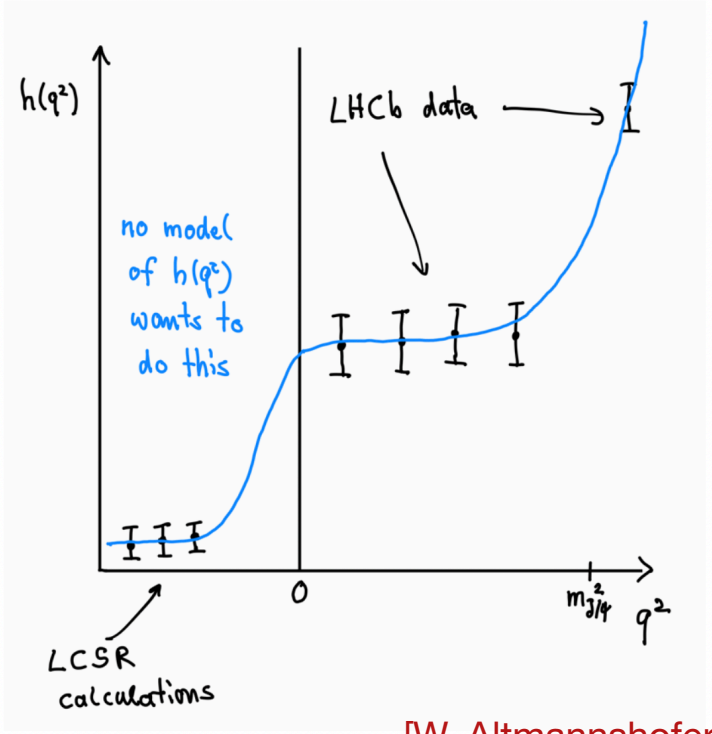
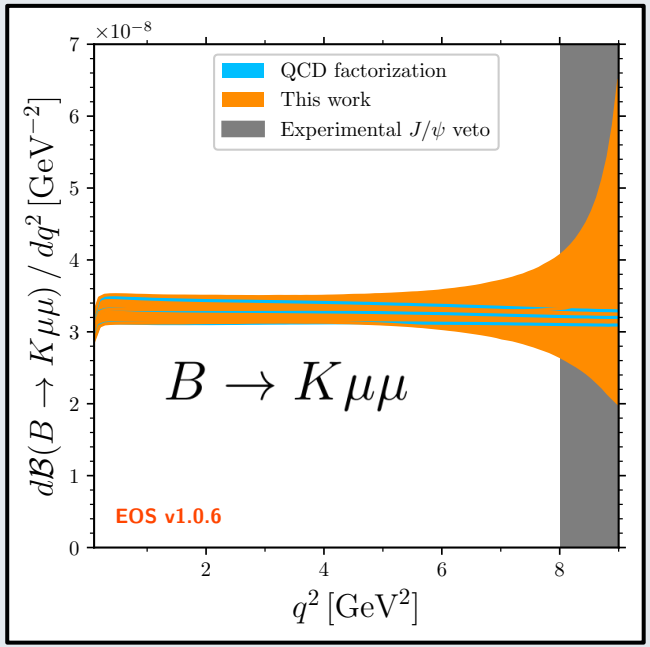


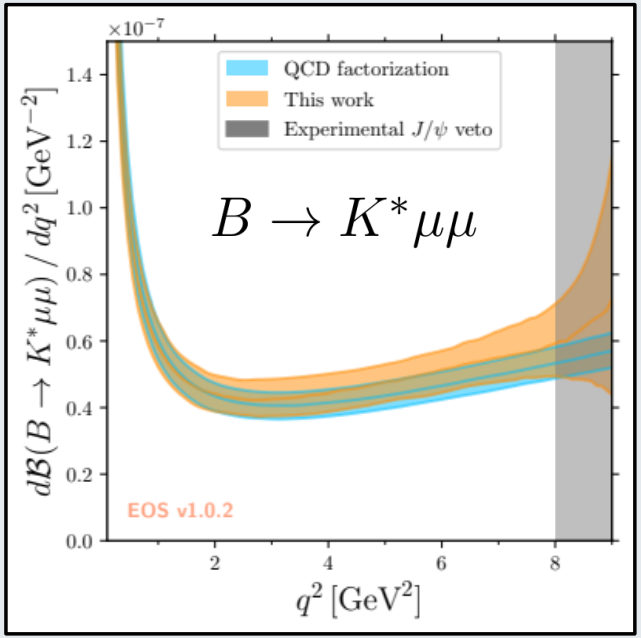
[W. Altmannshofer]

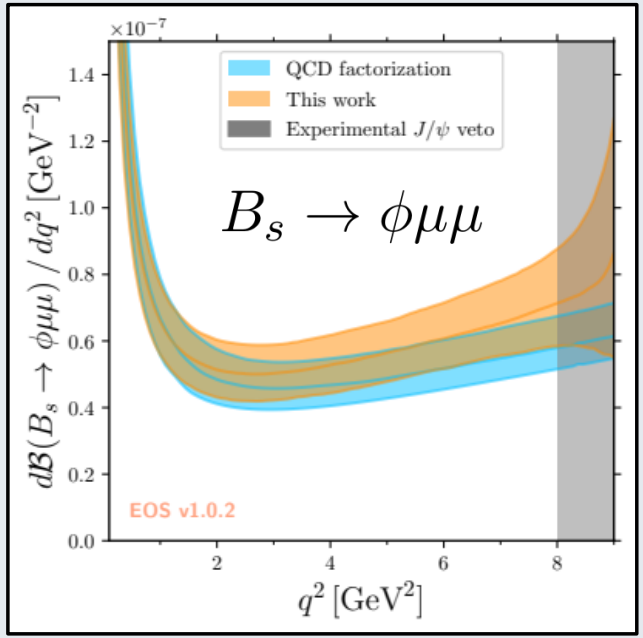
SM predictions

- Good overall agreement with previous theoretical approaches [Beneke, Feldman, Seidel '01 & '04]
  - Small deviation in the slope of $B_s \to \phi\mu\mu$
- Larger but controlled uncertainties especially near the J/ψ

  → The approach is **systematically improvable** (new channels, ψ(2S) data…)



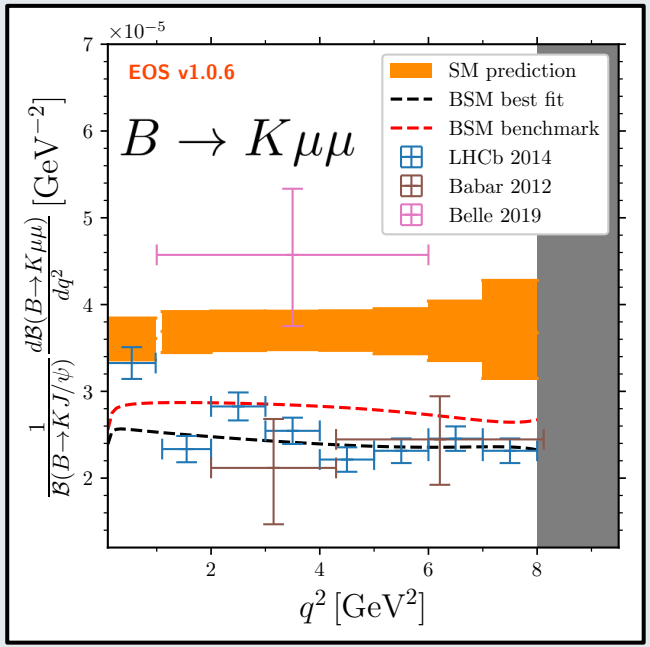


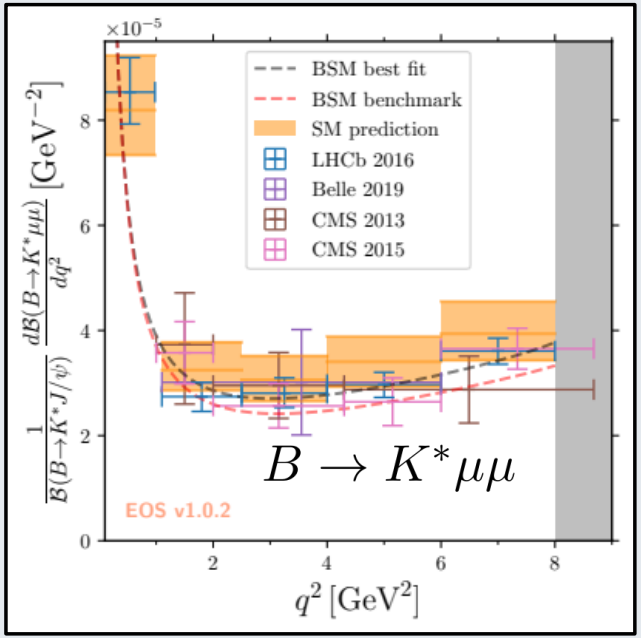


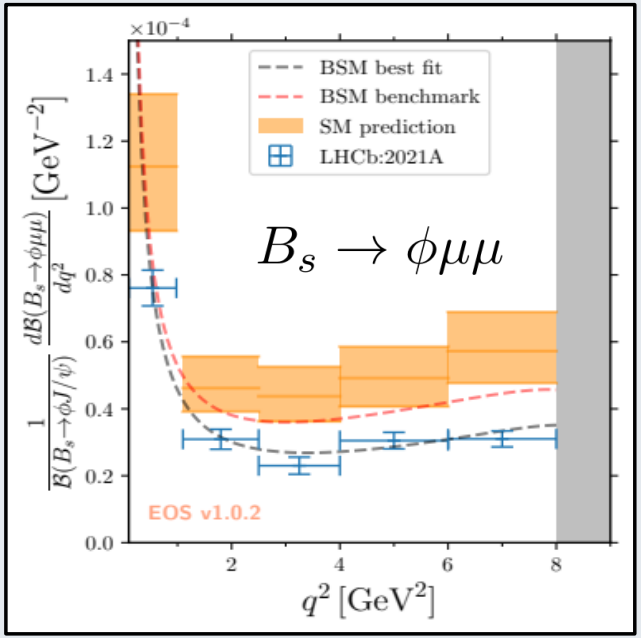
Confrontation with data

- Conservatively accounting for the non-local form factors does not solve the b → sμμ anomalies
- The largest source of theoretical uncertainty still comes from local form factors

Experimental results:
[Babar: 1204.3933; Belle: 1908.01848, 1904.02440; ATLAS: 1805.04000, CMS: 1308.3409, 1507.08126, 2010.13968, LHCb: 1403.8044, 2012.13241, 2003.04831, 1606.04731, 2107.13428]







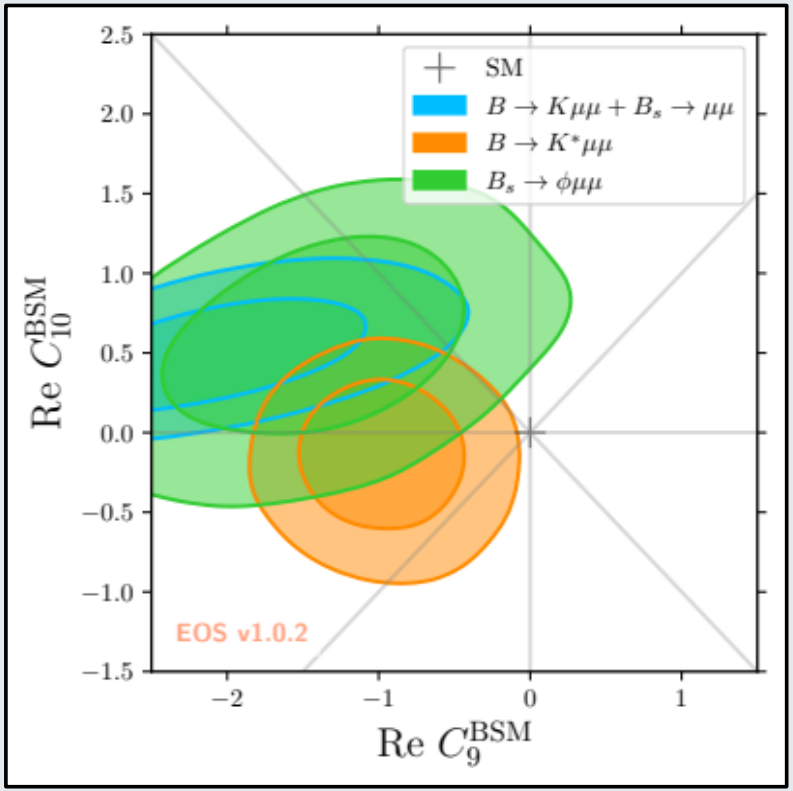
Additional plots can be found in the paper: 2206.03797

IV. BSM analysis: proof of concept

BSM 'proof-of-concept' analysis

Dispersive bound.

- A combined BSM analysis would be **very CPU expensive** (130 correlated, non-Gaussian, nuisance parameters!)
- Fit C_9 and C_{10} **separately** for the three channels:
  - $B \to K\mu^+\mu^- + B_s \to \mu^+\mu^-$ (*)
  - $B \to K^*\mu^+\mu^-$
  - $B_s \to \varphi\mu^+\mu^-$



(*) Need to be updated with CMS' $B_s \to \mu^+\mu^-$ measurement [2212.10311] and HPQCD '22 $B \to K$ form factors

Conclusion

Discussing BSM models requires a solid understanding of the hadronic physics:

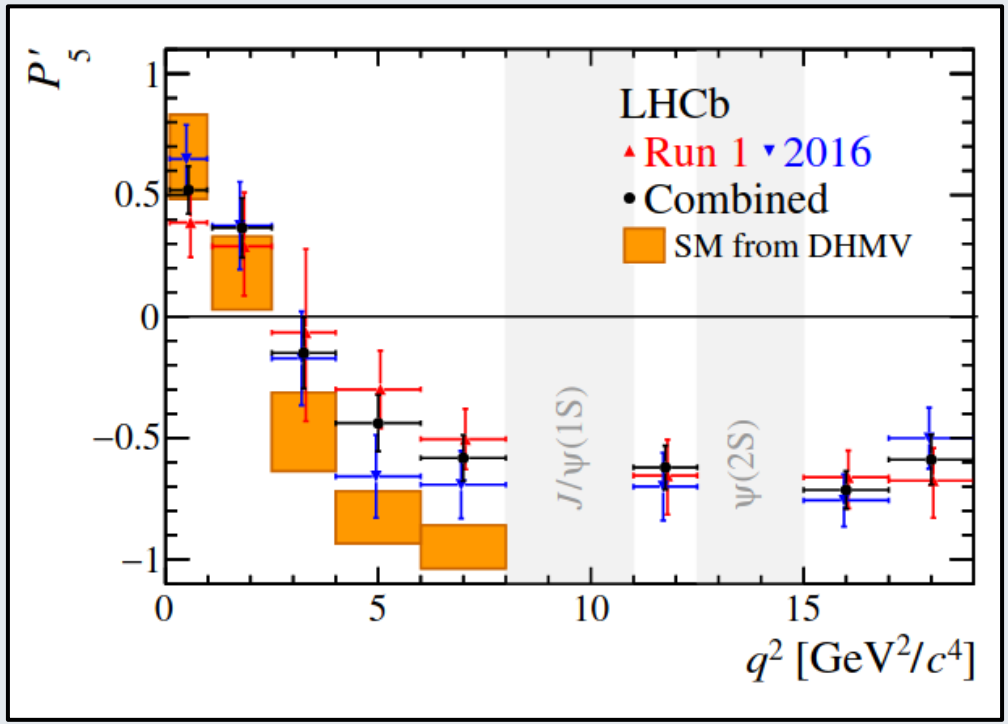
- **Local form factors** uncertainties can be controlled and reduced by using improved dispersive bound and a *appropriate* parametrization
- **Non-local form factors** can also be constrained by theory calculation and experimental measurements

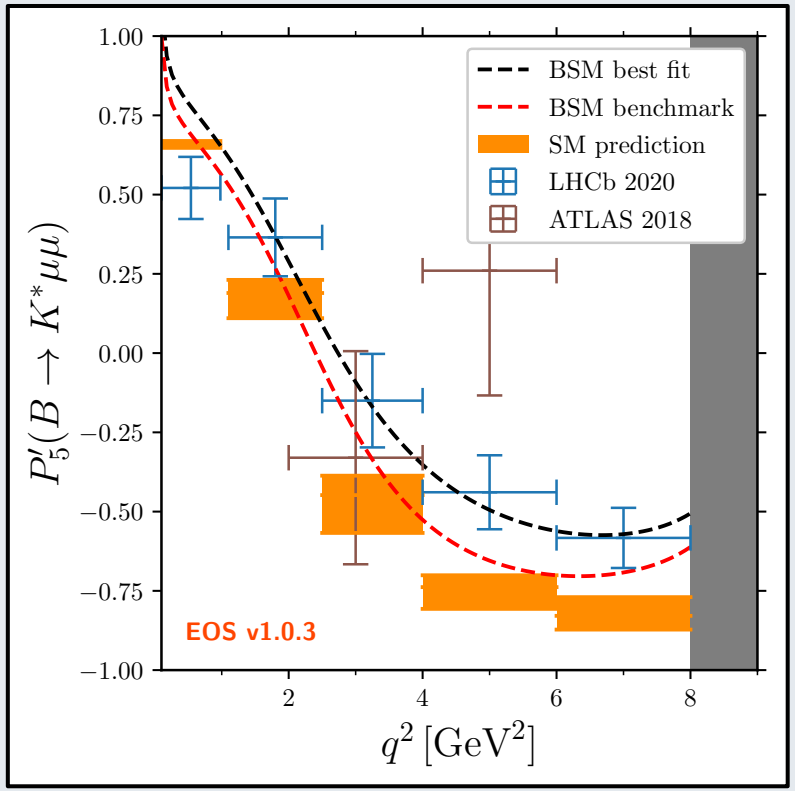
→ In both cases:

- Uncertainties are still large, but controlled by dispersive bounds
- Our approach is systematically improvable

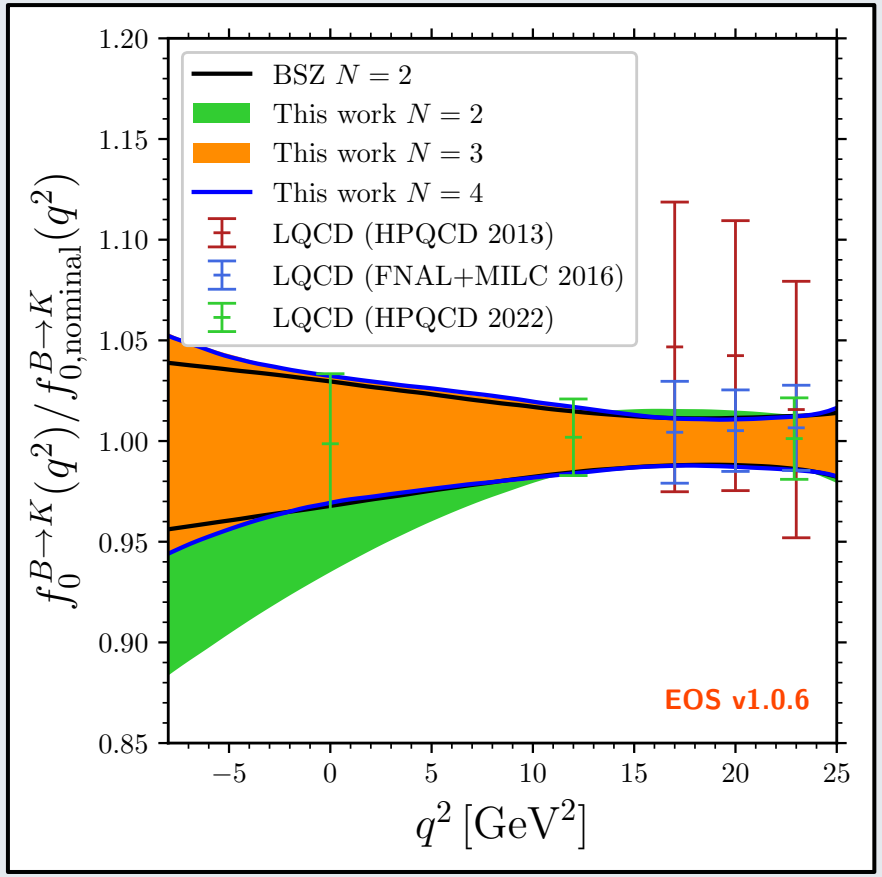
Back-up

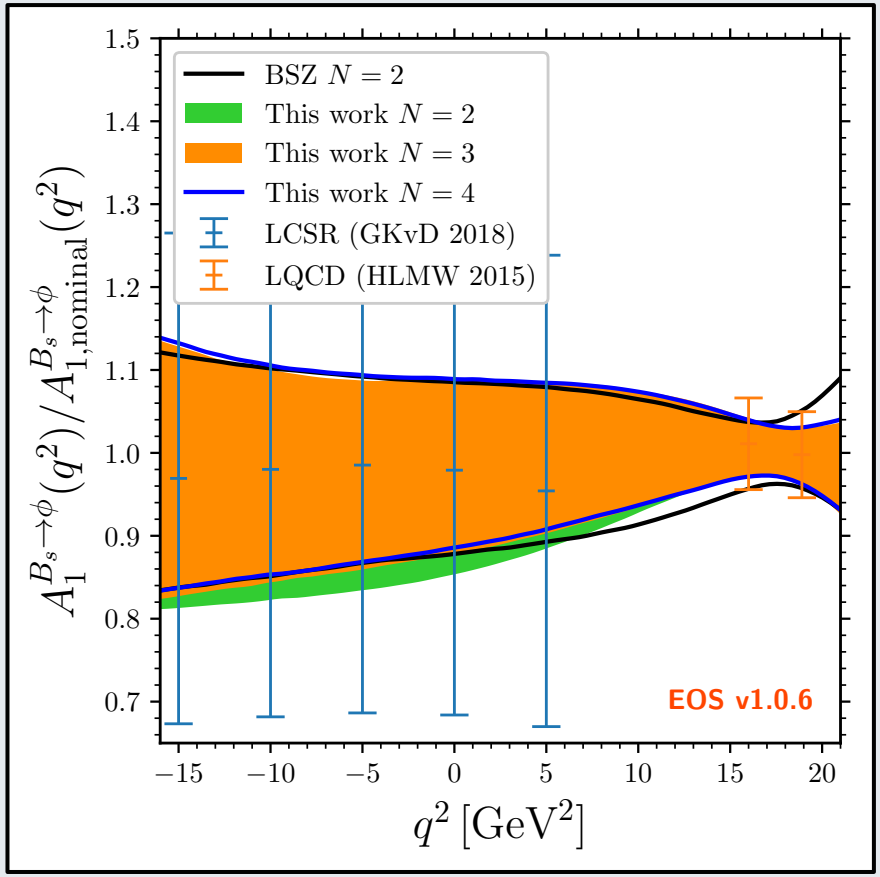
$B \to K^* P'_5$


P'_5
LHCb
Run 1
2016
Combined
SM from DHMV
$J/\psi(1S)$
$\psi(2S)$
q^2 [GeV^2/c^4]


BSM best fit
BSM benchmark
SM prediction
LHCb 2020
ATLAS 2018
EOS v1.0.3
$P'_5(B \to K^* \mu\mu)$
q^2 [GeV^2]

Model comparisons





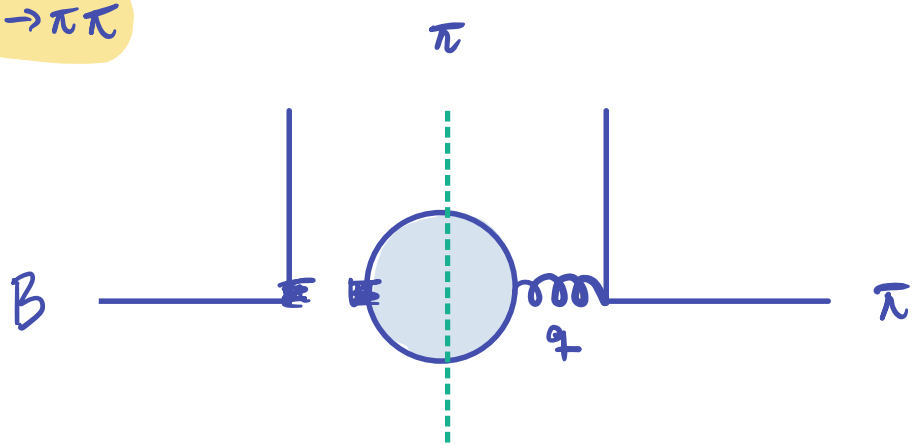
Non-local form factors: QCDF

What is QCD Factorization doing for us?

Beneke, Feldmann, Seidel 2001

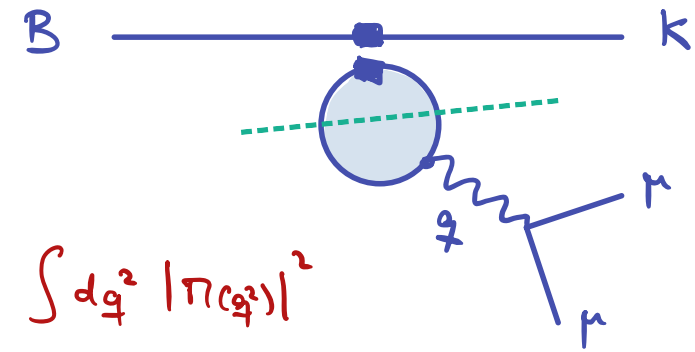
Beneke, Buchalla, Neubert, Sachrajda 2009

$B \to \pi\pi$



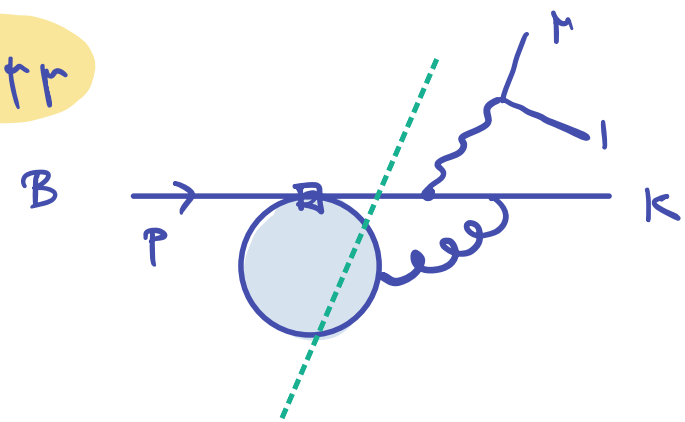
$$\left| \int dq^2 \, \Pi(q^2) \right|^2$$

$B \to K \mu\mu$



$$\int dq^2 \, |\Pi(q^2)|^2$$

$B \to K \mu\mu$



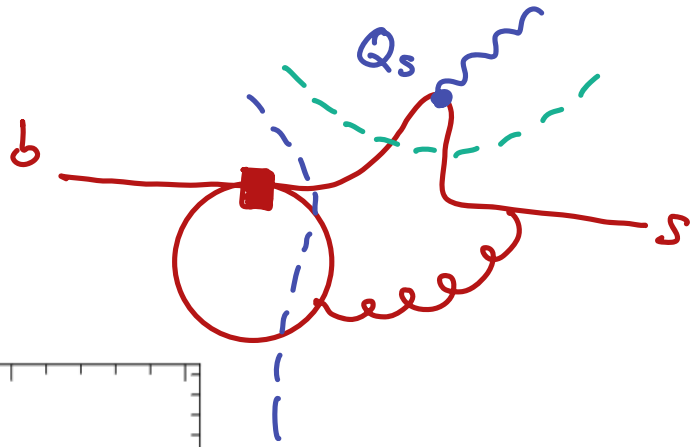
This imaginary part is like the one in $B \to \pi\pi$.

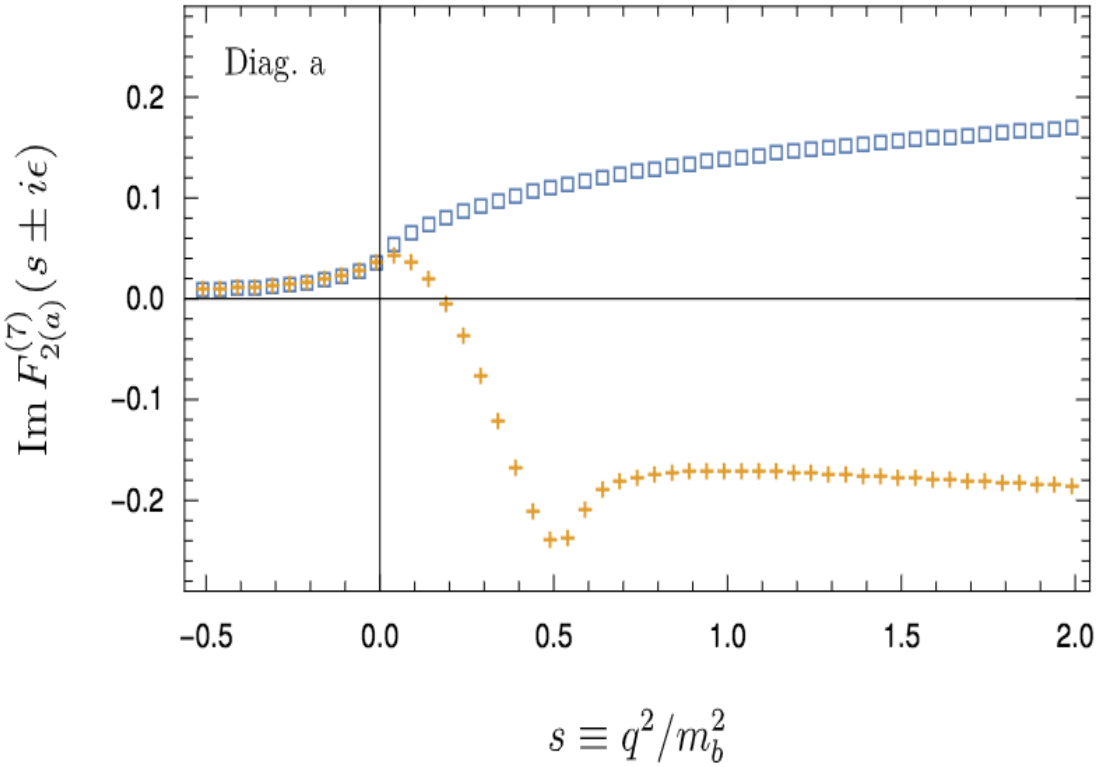
(See also: Beylich, Buchalla, Feldmann.)

Matching calculation at NLO

Asatrian, Greub, Virto 2019

Checking analytic structure of $\mathcal{H}(q^2)$





[Analytic structure in q^2 checked explicitly numerically through a disp. relation]