

The importance of $|V_{cb}|$

Another CKM unitarity test is the Unitarity Triangle (UT) formed by

$$1 + \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0$$

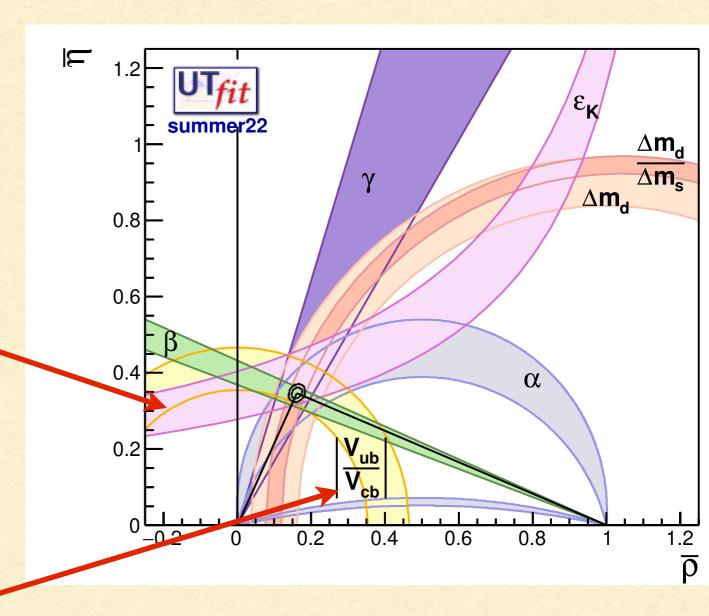
V_{cb} plays an important role in UT

$$\varepsilon_K \approx x |V_{cb}|^4 + \dots$$

and in the prediction of FCNC:

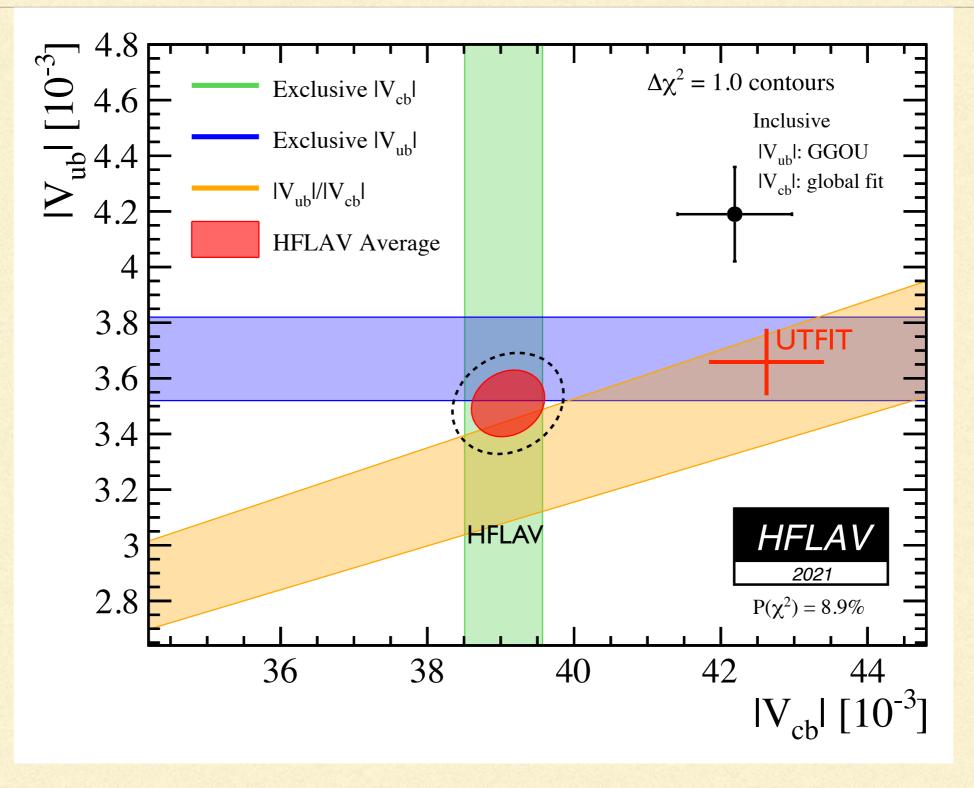
$$\propto |V_{tb}V_{ts}|^2 \simeq |V_{cb}|^2 \left[1 + O(\lambda^2)\right]$$

where it often dominates the theoretical uncertainty. V_{ub}/V_{cb} constrains directly the UT



Our ability to determine precisely V_{cb} is crucial for indirect NP searches

Since several years the inclusive and exclusive determinations of |V_{cb}| diverge

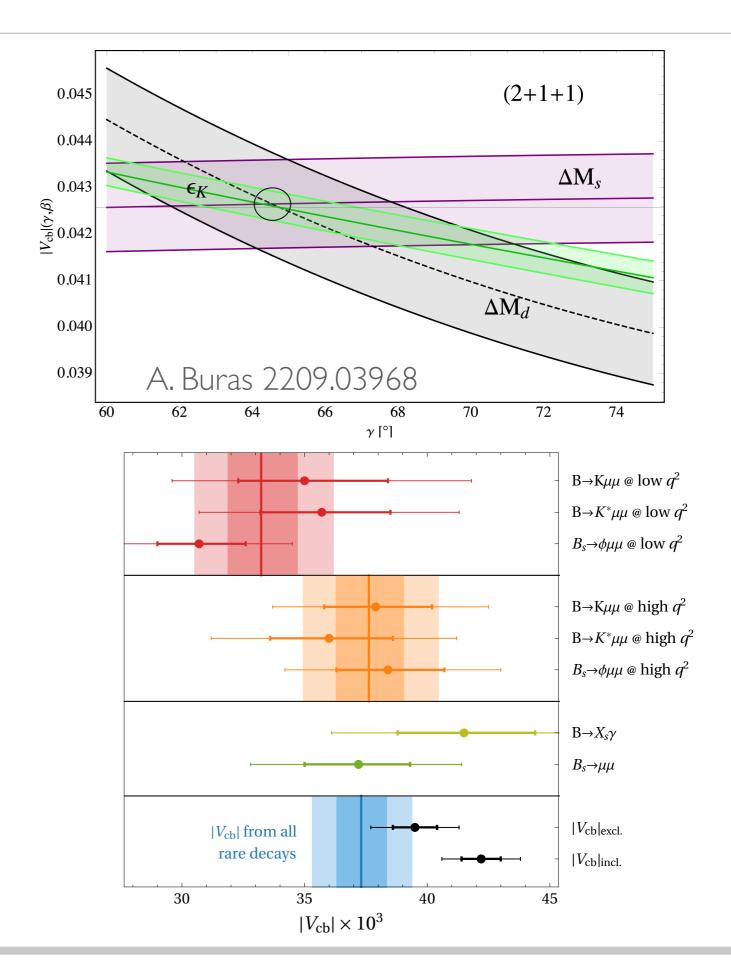


Recently: new calculations of FFs by several lattice collaborations and with light-cone sum rules, new perturbative calculations, all facing the challenges of a precision measurements... and several new measurements as well!

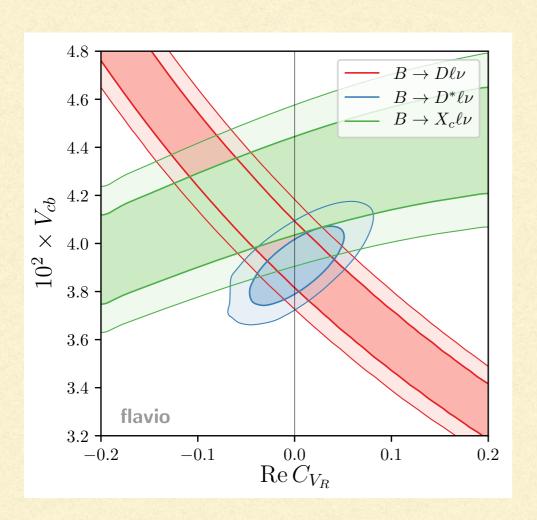
Indirect determinations

of $|V_{cb}|$ from loop induced $\Delta F=2$ processes assuming the SM. They tend to prefer a high $|V_{cb}|$ but sensitive to lattice calculations for mixing

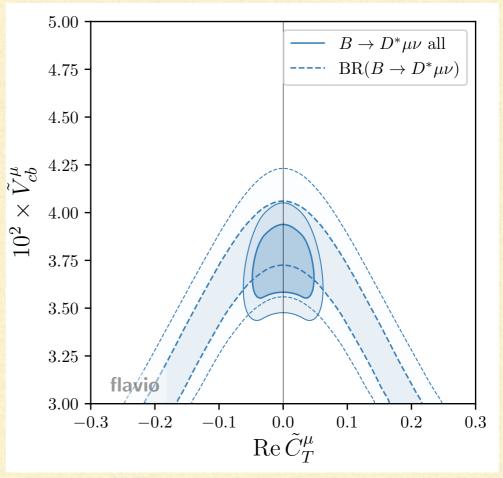
see also
W. Altmannshofer's talk
2112.03437



NEW PHYSICS?

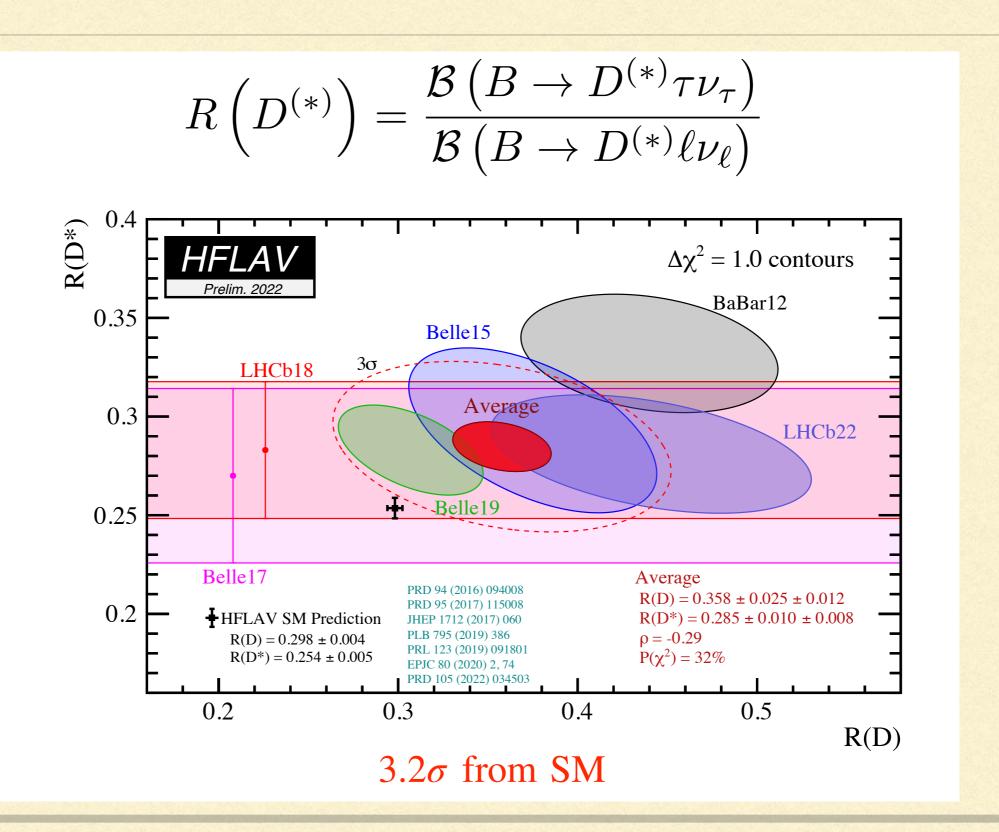


Jung & Straub, 1801.01112



Differential distributions constrain NP strongly, SMEFT interpretation incompatible with LEP data: Crivellin, Pokorski, Jung, Straub...

VIOLATION OF LFU with TAUS



see Jung & Harrison talk

INCLUSIVE SEMILEPTONIC B DECAYS

Inclusive observables are double series in Λ/m_b and α_s

$$M_{i} = M_{i}^{(0)} + \frac{\alpha_{s}}{\pi} M_{i}^{(1)} + \left(\frac{\alpha_{s}}{\pi}\right)^{2} M_{i}^{(2)} + \left(M_{i}^{(\pi,0)} + \frac{\alpha_{s}}{\pi} M_{i}^{(\pi,1)}\right) \frac{\mu_{\pi}^{2}}{m_{b}^{2}} + \left(M_{i}^{(G,0)} + \frac{\alpha_{s}}{\pi} M_{i}^{(G,1)}\right) \frac{\mu_{G}^{2}}{m_{b}^{2}} + M_{i}^{(D,0)} \frac{\rho_{D}^{3}}{m_{b}^{3}} + M_{i}^{(LS,0)} \frac{\rho_{LS}^{3}}{m_{b}^{3}} + \dots$$

Global shape parameters (first moments of the distributions, with various lower cuts on E_I) tell us about m_b , m_c and the B structure, total rate about $|V_{cb}|$

OPE parameters describe universal properties of the B meson and of the quarks: they are useful in many applications (rare decays, V_{ub},...)

Reliability of the method depends on our control of higher order effects. Quark-hadron duality violation would manifest itself as inconsistency in the fit.

Kinetic scheme fit includes all corrections $O(\alpha_s^2, \alpha_s/m_b^2, 1/m_b^3)$, m_c constraint from sum rules/lattice, and recent $O(\alpha_s^3)$ contribution to width.

3LOOP CALCULATIONS

Fael, Schoenwald, Steinhauser, 2011.11655, 2011.13654

3loop and 2loop charm mass effects in relation between kinetic and \overline{MS} b mass

$$m_b^{kin}(1\text{GeV}) = \left[4163 + 259_{\alpha_s} + 78_{\alpha_s^2} + 26_{\alpha_s^3}\right] \text{MeV} = (4526 \pm 15) \text{MeV}$$

Using FLAG $\overline{m}_b(\overline{m}_b) = 4.198(12) \text{GeV}$ one gets $m_b^{kin}(1 \text{GeV}) = 4.565(19) \text{GeV}$

3loop correction to total semileptonic width

$$\Gamma_{sl} = \Gamma_0 f(\rho) \left[0.9255 - 0.1162_{\alpha_s} - 0.0350_{\alpha_s^2} - 0.0097_{\alpha_s^3} \right]$$

in the kin scheme with $\mu=1{\rm GeV}$ and $\overline{m}_c(3{\rm GeV})=0.987\,{\rm GeV}$, $\mu_{\alpha_s}=m_b^{kin}$

$$\Gamma_{sl} = \Gamma_0 f(\rho) \left[0.9255 - 0.1140_{\alpha_s} - 0.0011_{\alpha_s^2} + 0.0103_{\alpha_s^3} \right]$$

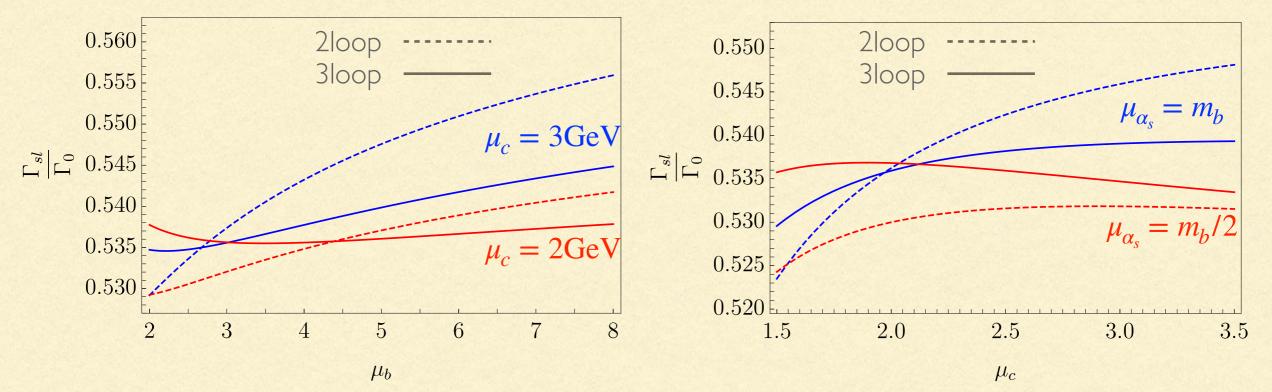
in the kin scheme with $\mu=1{\rm GeV}$ and $\overline{m}_c(2{\rm GeV})=1.091\,{\rm GeV}$, $\mu_{\alpha_s}=m_b^{kin}/2$

(d)
(d)
(e)
(e)
(e)
(e)
(e)
(e)
(f)

3loop correction tends to lower Γ_{sl} and therefore pushes $|V_{cb}|$ slightly up (~0.5%)

RESIDUAL UNCERTAINTY on Γ_{sl}

Bordone, Capdevila, PG, 2107.00604



Similar reduction in μ_{kin} dependence. Purely perturbative uncertainty $\pm 0.7\,\%$ (max spread), central values at $\mu_c=2{\rm GeV}, \mu_{\alpha_s}=m_b/2$.

 $O(\alpha_s/m_b^2, \alpha_s/m_b^3)$ effects in the width are known. Additional uncertainty from higher power corrections, soft charm effects of $O(\alpha_s/m_b^3m_c)$, duality violation.

Conservatively: 1.2% overall theory uncertainty in Γ_{sl} (a ~50% reduction)

Interplay with fit to semileptonic moments, known only to $O(\alpha_s^2, \alpha_s \Lambda^2/m_b^2)$

INCLUSIVE SEMILEPTONIC FITS

Bordone, Capdevila, PG, 2107.00604

m_b^{kin}	$\overline{m}_c(2{ m GeV})$	μ_π^2	$ ho_D^3$	$\mu_G^2(m_b)$	$ ho_{LS}^3$	$BR_{c\ell\nu}$	$10^3 V_{cb} $
4.573	1.092	0.477	0.185	0.306	-0.130	10.66	42.16
0.012	1.092 0.008	0.056	0.031	0.050	0.092	0.15	0.51

Higher power corrections see a proliferation of parameters. We use the Lowest Lying State Saturation Approximation (Mannel, Turczyk, Uraltsev 1009.4622) as loose constraint or priors (60% gaussian uncertainty, dimensional estimate for vanishing matrix elements) in a fit including higher powers.

$$|V_{cb}| = 42.00(53) \times 10^{-3}$$

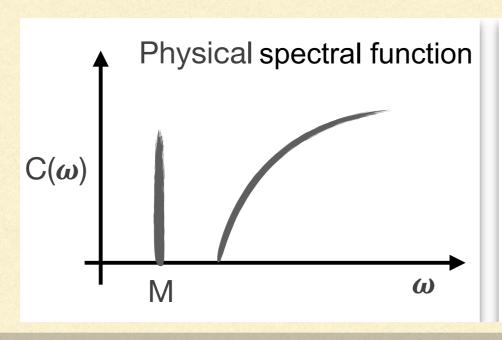
Update of 1606.06174

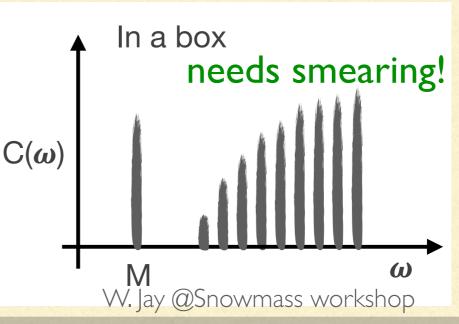
PROSPECTS for INCLUSIVE Vcb

- Theoretical uncertainties no longer dominate, we are now close to 1% accuracy
- $O(\alpha_s \rho_D^3/m_b^3)$ calculation completed for width (Mannel, Pivovarov) in progress for the moments (S. Nandi, PG)
- Electroweak (QED) corrections require attention, a study is under way
- New observables: FB asymmetry (Turczyk) could be measured already by Babar and Belle now, new q² moments measurements by Belle (2109.01685) and Belle II (2205.06372) not yet included in our fit
- Reparametrisation invariance implies that q^2 moments depend on a smaller set of HQE parameters (Fael, Mannel, Vos), 8 at $O(1/m_b^4)$, but using only the q^2 moments: $|V_{cb}| = 41.99(65) \cdot 10^{-3}$ using the same BR inputs we employ (2205.10274)
- Lattice QCD calculations: HQE determination of matrix elements (PG, Melis, Simula 1704.06105) or direct inclusive calculation

INCLUSIVE DECAYS ON THE LATTICE

- Inclusive processes impractical to treat directly on the lattice. Vacuum current correlators computed in euclidean space-time are related to $e^+e^- \rightarrow$ hadrons or τ decay via analyticity. In our case the correlators have to be computed in the B meson, but analytic continuation more complicated: two cuts, decay occurs only on a portion of the physical cut.
- While the lattice calculation of the spectral density of hadronic correlators is an ill-posed problem, the spectral density is accessible after smearing, as provided by phase-space integration Hansen, Meyer, Robaina, Hansen, Lupo, Tantalo, Bailas, Hashimoto, Ishikawa

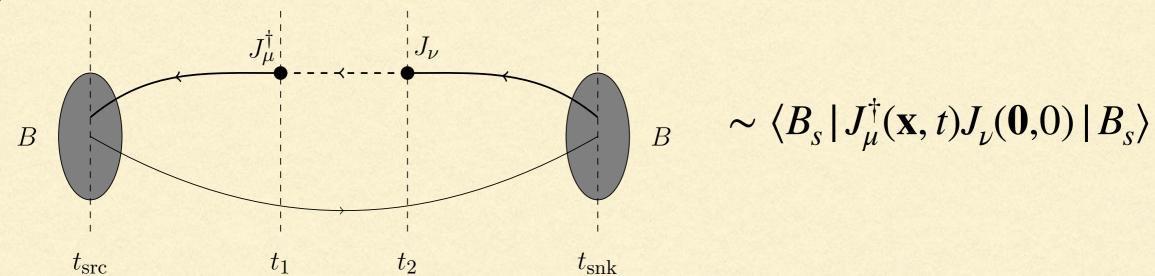




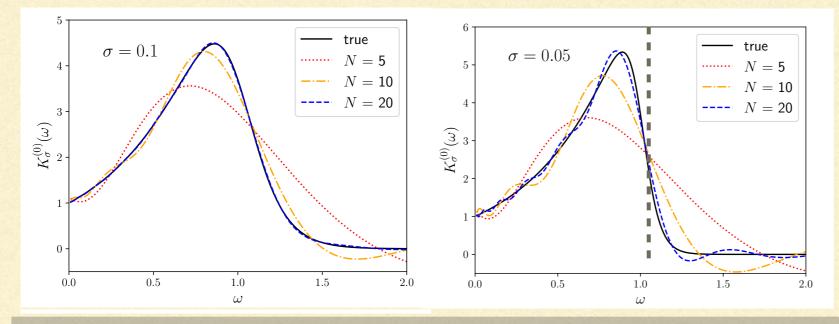
A NEW APPROACH

Hashimoto, PG 2005.13730

4point functions on the lattice are related to the hadronic tensor in euclidean



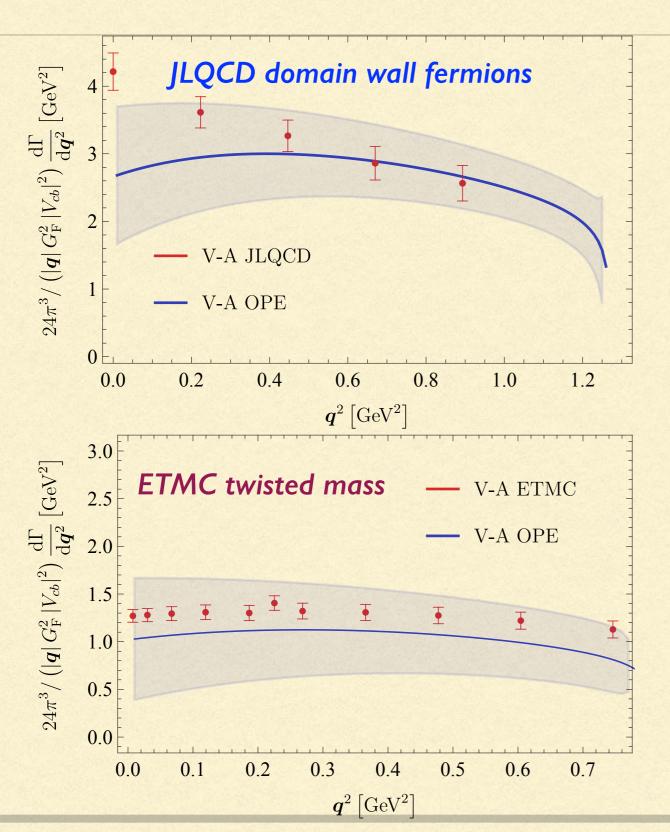
The necessary smearing is provided by phase space integration over the hadronic energy, which however is cut by a θ with a sharp hedge: sigmoid $1/(1+e^{x/\sigma})$ can be used to replace kinematic $\theta(x)$ for $\sigma \to 0$. Larger number of polynomials needed for small σ



important:

$$\lim_{\sigma \to 0} \lim_{V \to \infty} \bar{X}_{\sigma}$$

LATTICE VS OPE



m_b^{kin} (JLQCD)	2.70 ± 0.04
$\overline{m}_c(2 \text{ GeV}) \text{ (JLQCD)}$	1.10 ± 0.02
m_b^{kin} (ETMC)	2.39 ± 0.08
$\overline{m}_c(2 \text{ GeV}) \text{ (ETMC)}$	1.19 ± 0.04
μ_π^2	0.57 ± 0.15
$ ho_D^3$	0.22 ± 0.06
$\mu_G^2(m_b)$	0.37 ± 0.10
$ ho_{LS}^3$	-0.13 ± 0.10
$\alpha_s^{(4)}(2 \text{ GeV})$	0.301 ± 0.006

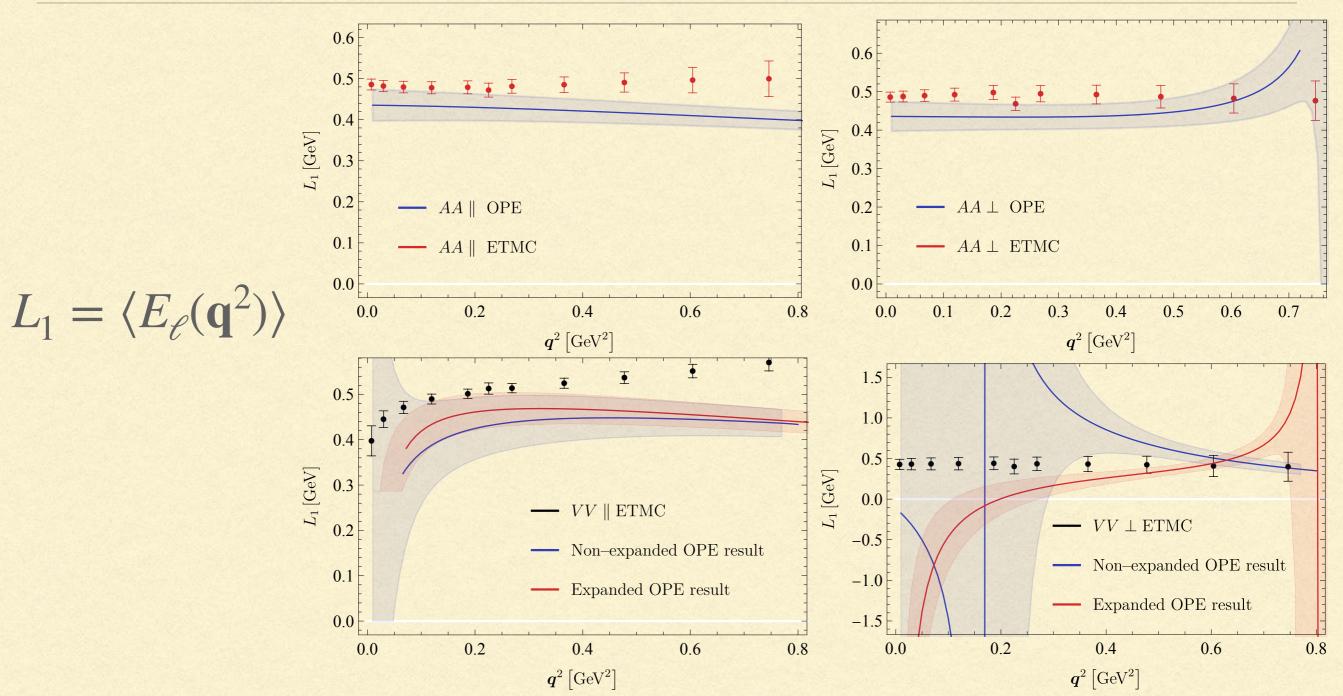
OPE inputs from fits to exp data (physical m_b), HQE of meson masses on lattice 1704.06105, J.Phys.Conf.Ser. 1137 (2019) 1,012005

We include $O(1/m_b^3)$ and $O(\alpha_s)$ terms Hard scale $\sqrt{m_c^2+{\bf q}^2}\sim 1-1.5\,{\rm GeV}$ We do not expect OPE to work at high $|{\bf q}|$

Twisted boundary conditions allow for any value of \vec{q}^2 Smaller statistical uncertainties

MOMENTS

PG, Hashimoto, Maechler, Panero, Sanfilippo, Simula, Smecca, Tantalo, 2203. I 1762



smaller errors, cleaner comparison with OPE, individual channels AA, VV, parallel and perpendicular polarization, could help extracting its parameters

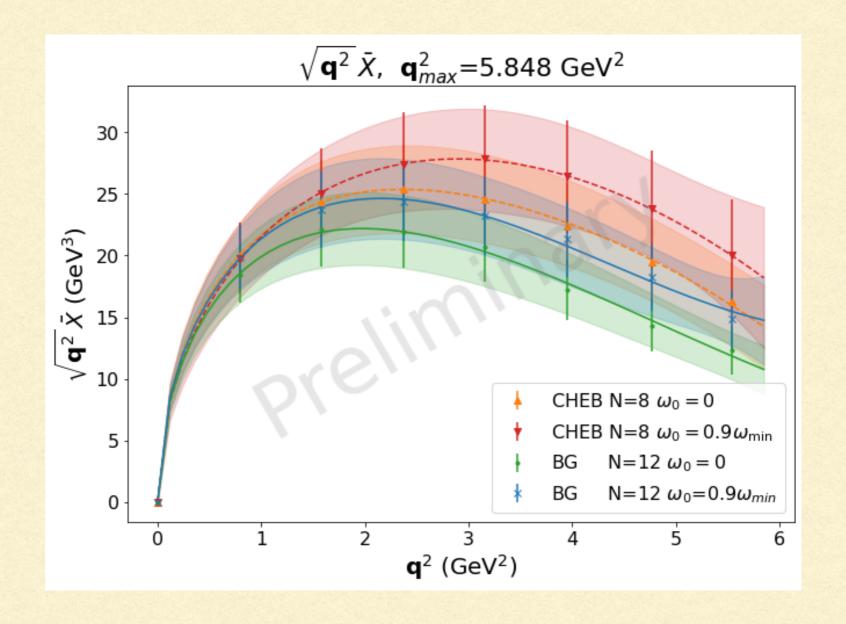
First results at the physical b mass

Relativistic heavy quark effective action for b

B_s decays

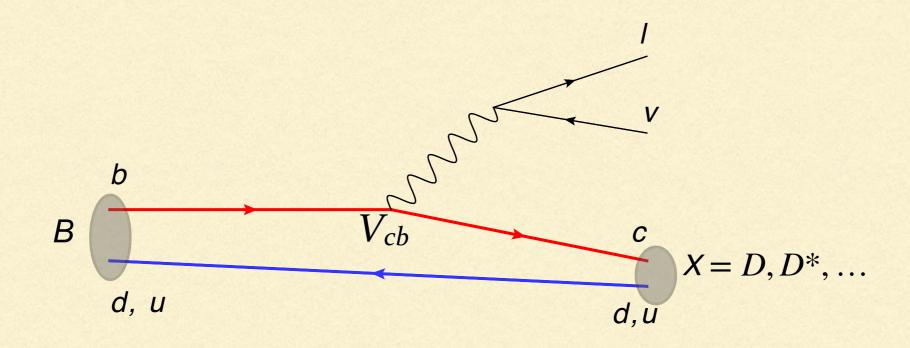
domain wall fermions

~10% determination of total width possibly compare with partial width at low q²



Barone, Hashimoto, Juttner, Kaneko, Kellermann, Lattice 2022

EXCLUSIVE DECAYS



There are I(2) and 3(4) FFs for D and D* for light (heavy) leptons, for instance

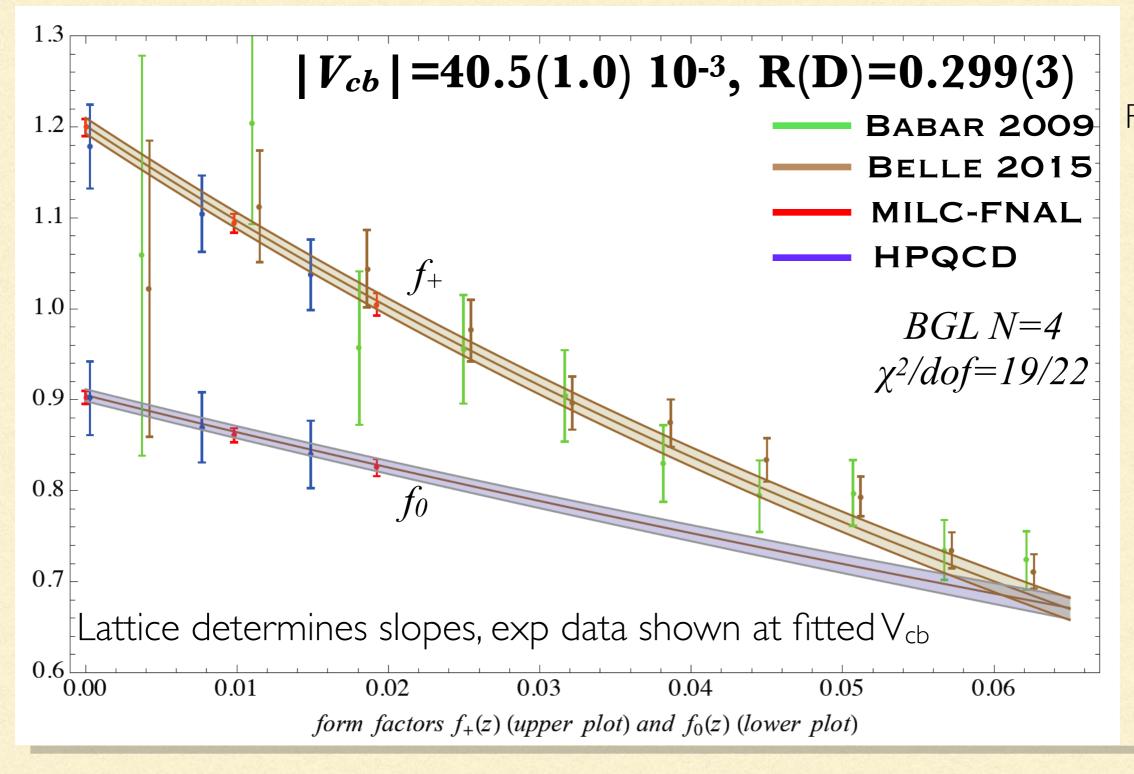
$$\langle D(k) | \, \bar{c} \gamma^{\mu} b \, | \bar{B}(p) \rangle = \left[(p+k)^{\mu} - \frac{M_B^2 - M_D^2}{q^2} q^{\mu} \right] f_+^{B \to D}(q^2) + \frac{M_B^2 - M_D^2}{q^2} q^{\mu} f_0^{B \to D}(q^2)$$

Information on FFs from LQCD (at high q^2), LCSR (at low q^2), HQE, exp, extrapolation, unitarity constraints, ...

A model independent parametrization is necessary

LATTICE + EXP BGL FIT for $B \to D\ell\nu$

Bigi, PG 1606.08030



R(D) = 0.299(3)

 1.3σ from exp

FLAG has very similar results

CLN cannot fit both ff

D'AGOSTINI BIAS

Standard χ^2 fits sometimes lead to paradoxical results

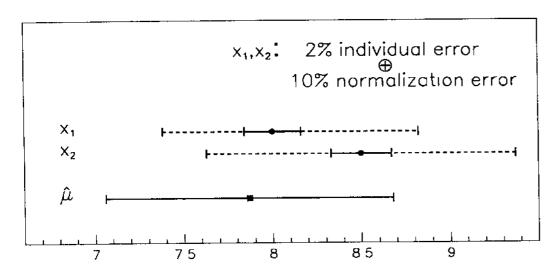


Fig. 1. Best estimate of the true value from two correlated data points, using in the χ^2 the empirical covariance matrix of the meaurements. The error bars show individual and total errors.

$$\hat{k} = \frac{x_1 \sigma_2^2 + x_2 \sigma_1^2}{\sigma_1^2 + \sigma_2^2 + (x_1 - x_2)^2 \sigma_f^2},$$

Many exp systematics are highly correlated. Bias is stronger with more bins

On the use of the covariance matrix to fit correlated data

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(Received 10 December 1993; revised form received 18 February 1994)

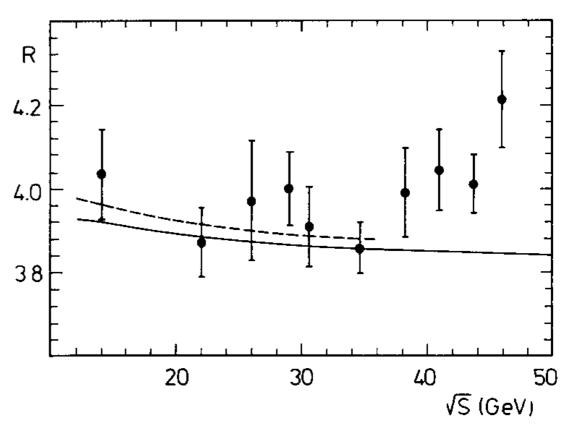
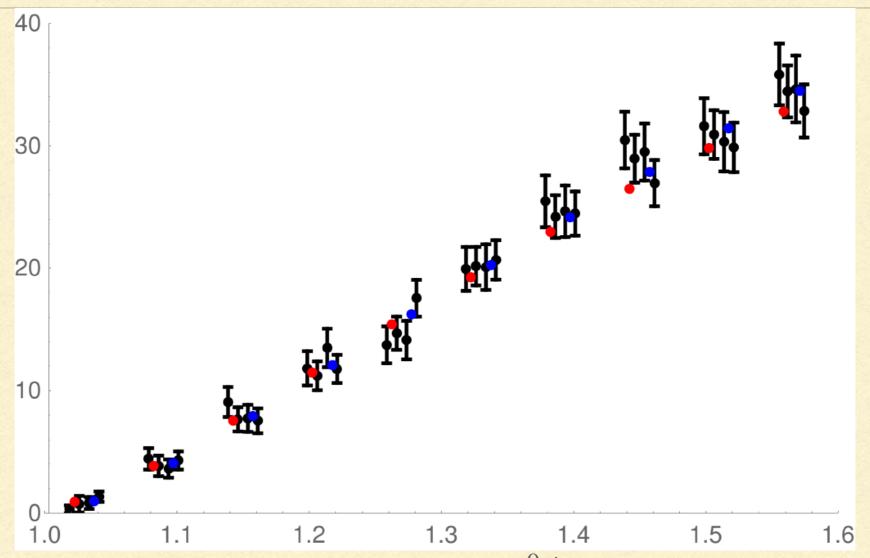


Fig. 2. R measurements from PETRA and PEP experiments with the best fits of QED+QCD to all the data (full line) and only below 36 GeV (dashed line). All data points are correlated (see text).

w DISTRIBUTION for $B \to D\ell\nu$



Belle 2015 consider 4 channels $(B^{0,+},e,\mu)$ for each bin. Average (red points) usually lower than all central values. Bias? Blue points are average of normalised bins.

Standard fit to Belle I 5+FNAL+HPQCD: $|V_{cb}|=40.9(1.2)\,10^{-3}$ Jung, PG Fit to normalised bins Belle I 5+FNAL+HPQCD: $|V_{cb}|=41.9(1.2)\,10^{-3}$

$|V_{cb}|$ from $B \rightarrow D^*/V$

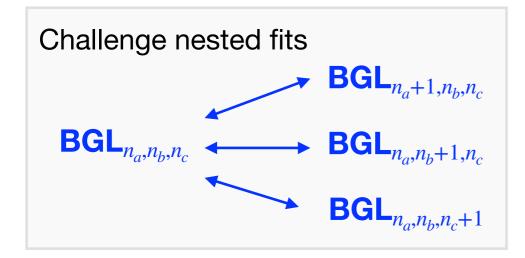
More complicated: 4 FFs, angular spectra, D* unstable. Present status unclear.

- 1. Parametrisations matter and the related uncertainties require careful consideration. Belle 2017 dataset analysed with BGL or CLN leads to 6-8% difference in $|V_{cb}|$. Bigi, PG, Schacht, Grinstein, Kobach Discard old exp results obtained with CLN and provide data in a parametrisation independent way.
- 2. Despite recent progress, *lattice calculations* are indecisive. Tension between *Fermilab/MILC* 2021 and HPQCD 2023 results at non-zero recoil and *Belle* untagged 2018 data, while *JLQCD* preliminary results give a consistent picture.
- 3. **Problems** in Belle 2018 analysis (D'Agostini bias, μ/e 4σ tension in the FB asymmetry) PG, Jung, Schacht & Bobeth, Bordone, van Dyk, Gubernari, Jung new Belle tagged analysis leads to higher |V_{cb}|, Babar 2019 to low |V_{cb}|, LHCb to high |V_{cb}|. New Belle II untagged analysis presented at Moriond. Data not yet available.

See also Jung's talk

Z. Ligeti, D. Robinson, M. Papucci, FB [arXiv:1902.09553, PRD100,013005 (2019)]

Use a **nested hypothesis test (NHT)** to determine optimal truncation order



Test statistics & Decision boundary

$$\Delta \chi^2 = \chi_N^2 - \chi_{N+1}^2 \qquad \Delta \chi^2 > 1$$

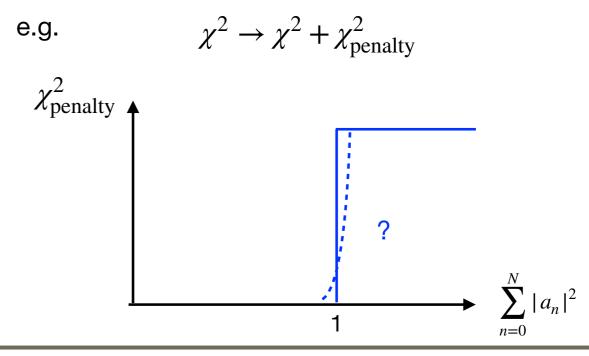
Distributed like a χ^2 -distribution with 1 dof (Wilk's theorem)

Gambino, Jung, Schacht

[arXiv:1905.08209, PLB]

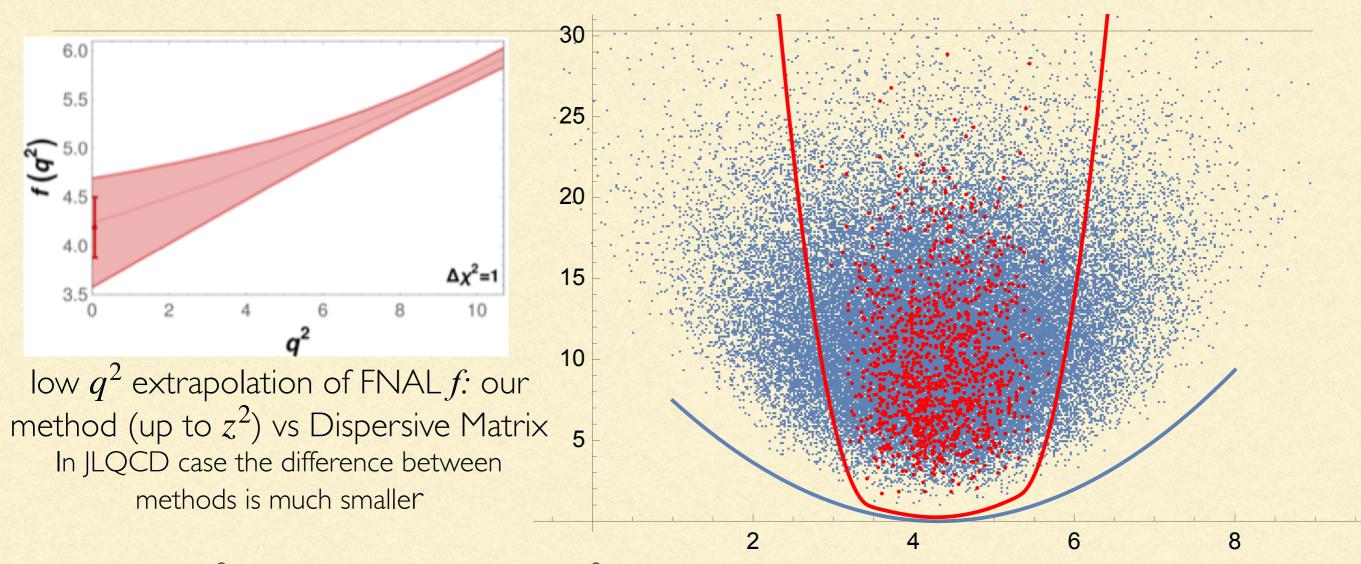
Constrain contributions from higher order coefficients using unitarity bounds

$$\sum_{n=0}^{N} |a_n|^2 \le 1 \qquad \sum_{n=0}^{N} \left(|b_n|^2 + |c_n|^2 \right) \le 1$$



Other approaches: Dispersive Matrix Lellouch, Martinelli, Simula, Vittorio and similar Bayesian inference method by Flynn, Juttner, Tsang 2303.11285

UNITARITY CONSTRAINTS and UNCERTAINTY

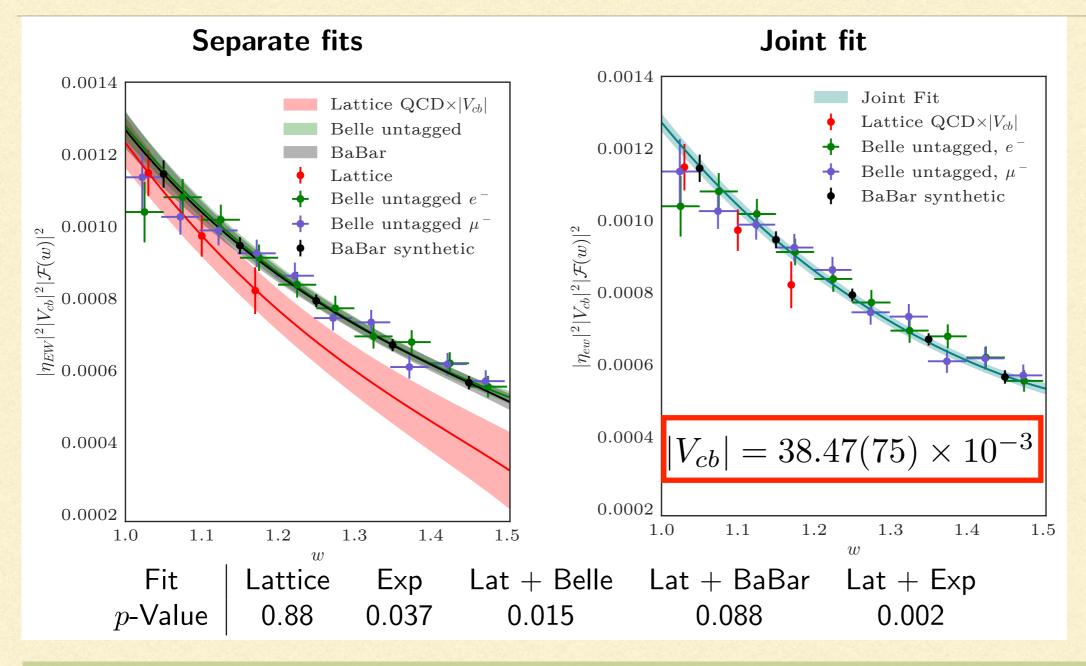


In **blue** the χ^2_{min} profile as a function of $f(q^2=0)$ without imposing unitarity. In red with minimum complying with unitarity. Our $\delta_{\pm}f(0)$ corresponds to $\Delta\chi^2_{min}=1$ from the absolute minimum.

Blue points are generated according to FNAL covariance matrix. Red points survive unitarity filtering: their distribution is much narrower but the points at its edge correspond to small fluctuations in FNAL data. Our $\delta_{\pm}f(0)$ reflect this and are always larger than its standard deviation. The curves at the edge of our band are consistent with unitarity and represent $\sim 1\sigma$ fluctuations in FNAL data

FERMILAB/MILC CALCULATION

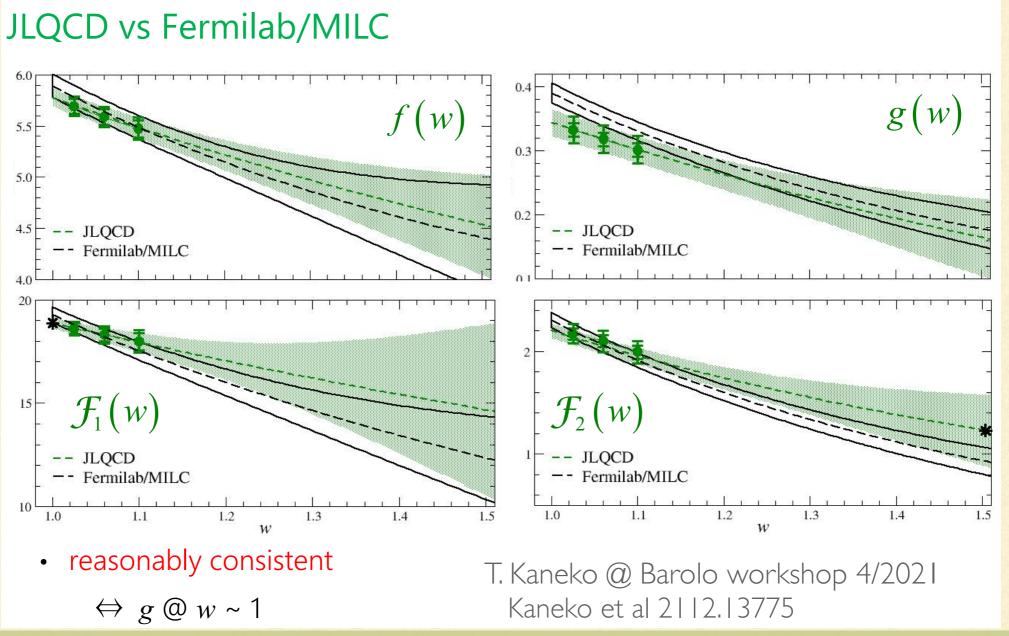
2105.14019



First lattice calculation beyond zero recoil for this mode

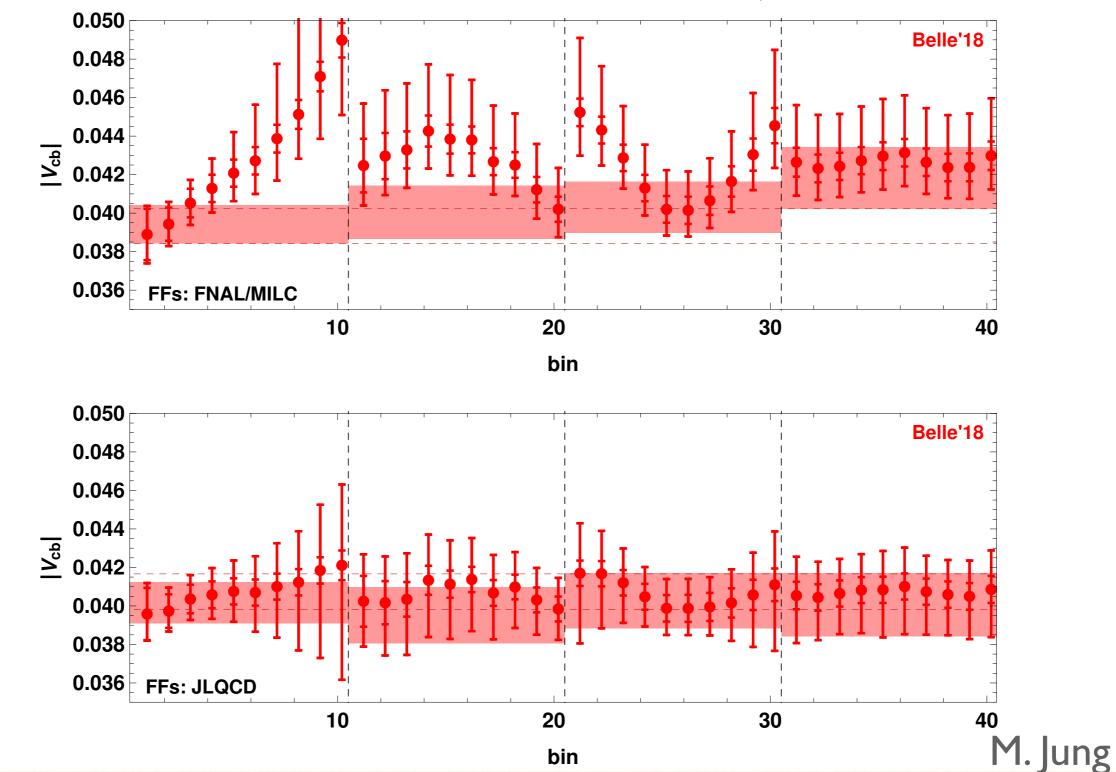
Our analysis of same exp+lattice data (Jung, PG): $|V_{cb}|=39.4(9)\ 10^{-3}(\chi^2_{min}=50)$ using only total rate $|V_{cb}|=42.2^{+2.8}_{-1.7}\ 10^{-3}$

JLQCD PRELIMINARY RESULTS



Our analysis of same exp (Belle I 8)+ JLQCD data (Jung, PG): $|V_{cb}|=40.7(9)~10^{-3}~(\chi^2_{min}=33)$ using only total rate $|V_{cb}|=40.8^{+1.8}_{-2.3}~10^{-3}$

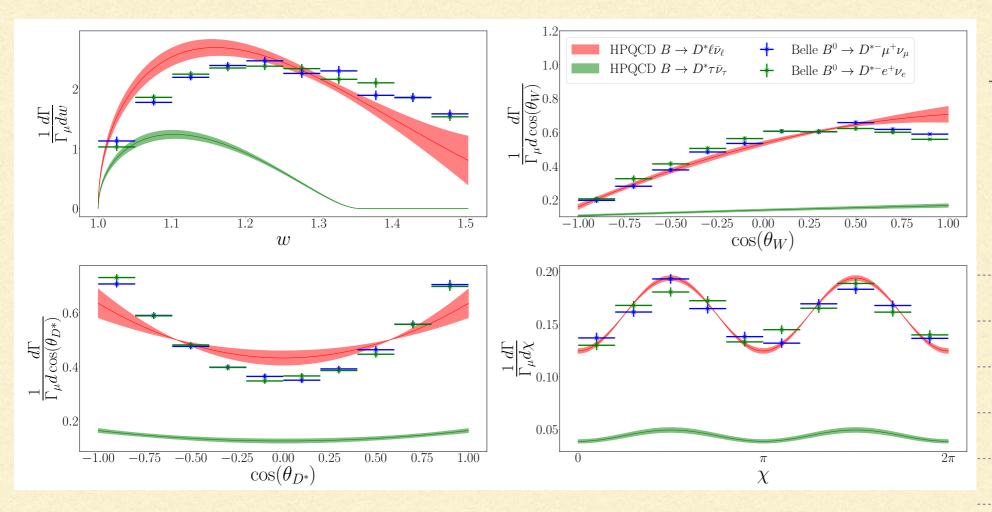
Binned V_{cb} from Belle'18 data: FNAL/MILC vs JLQCD



Extracting V_{cb} from each bin, FFs only determined by lattice QCD

NEW HPQCD FFS CALCULATION

2304.03137



Tension with Belle 2018 data similar to FNAL

Belle I 8+HPQCD

BGL exp	\mathcal{X}^2	$ V_{cb} $
0001	78	41.0(8)
0101	68	41.2(8)
0111	57	40.8(8)
1111	57	40.8(8)
1121	54	40.6(8)
1222	52	40.6(8)
2222	50	40.4(8)

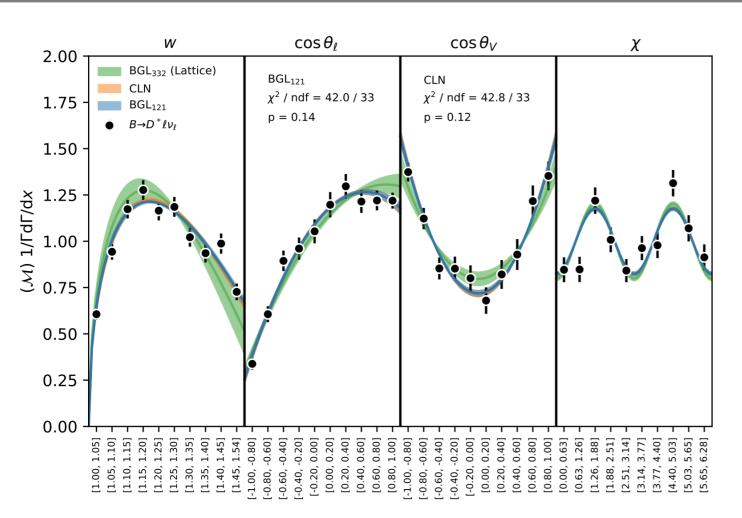
Extrapolation in mh, data cover the whole w region

Our analysis of same exp (Belle I 8) + HPQCD data (Jung, PG): $|V_{cb}|=40.4(8)~10^{-3}$ using only total rate $|V_{cb}|=44.4\pm1.6~10^{-3}$

2232 50 40.4(8) 3333 50 40.4(8)

HPQCD and FNAL are not really compatible: adding 16 FNAL points increases χ^2 by 35

Measurement of Differential Distributions of $B \to D^* \ell \nu_{\ell}$ and Determination of $|V_{cb}|$



Measured Shapes + External Branching Ratio Input

$a_0 \times 10^3$				ation	Correla	Value	BGL(121)	
$b_1 \times 10^3$ -11.93 ± 12.72 -0.21 -0.01 1.00 0.25	-0.30	0.26	-0.21	0.25	1.00	24.93 ± 1.41	$a_0 \times 10^3$	
	-0.62	-0.01	-0.01	1.00	0.25	13.11 ± 0.18		
$c_1 \times 10^3$ $-6.87 \pm 0.87 + 0.26 -0.01 + 0.25 + 1.00$	0.25 -0.48	0.25	1.00	-0.01	-0.21	-11.93 ± 12.72	$b_1 \times 10^3$	
	-0.49	1.00	0.25	-0.01	0.26		$c_1 \times 10^3$	
$ V_{cb} \times 10^3$ 40.77 ± 0.92 -0.30 -0.62 -0.48 -0.49	.49 1.00	-0.49	-0.48	-0.62	-0.30	40.77 ± 0.92	$ V_{cb} \times 10^3$	

CLN	Value	Correlation			
$ ho^2$	1.25 ± 0.09	1.00	0.56	-0.89	0.38
$R_1(1)$	1.32 ± 0.08	0.56	1.00	-0.63	-0.03
$R_2(1)$	0.85 ± 0.07	-0.89	-0.63	1.00	-0.15
$ V_{cb} \times 10^3$	40.30 ± 0.86	0.38	-0.03	-0.15	1.00

Based on the lattice input at zero-recoil:

$$h_{A_1}(1) = 0.906 \pm 0.013$$

2301.07529

$B^0 \to D^{*-} \mathcal{C}^+ \nu$ untagged (189/fb) preliminary [to be submitted to Phys. Rev. D] Belle ||

BGL fit result

BGL truncation order determined by Nested Hypothesis Test [Phys. Rev. D100, 013005]

	Values		Correlations		χ^2/ndf
	0.89 ± 0.05				
$\tilde{b}_0 \times 10^3$	0.54 ± 0.01 -0.44 ± 0.34	0.26	1.00 - 0.41	-0.46	40 /21
$\tilde{b}_1 \times 10^3$	-0.44 ± 0.34	-0.27	-0.41 1.00	0.56	40/31
$\tilde{c}_1 \times 10^3$	-0.05 ± 0.03	0.07	-0.46 0.56	1.00	

Preliminary

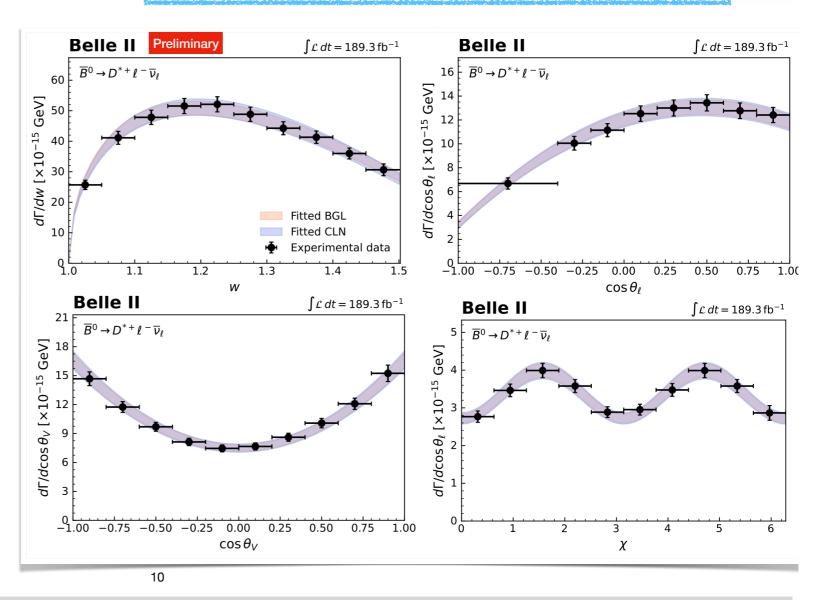
Relative	uncertainty	(%)	Preliminary

	\tilde{a}_0	$ ilde{b}_0$	$ ilde{b}_1$	\tilde{c}_1
Statistical	3.3	0.7	44.8	35.4
Finite MC samples	3.0	0.7	39.4	33.0
Signal modelling	3.0	0.4	40.0	30.8
Background subtraction	1.2	0.4	24.8	18.1
Lepton ID efficiency	1.5	0.3	3.1	2.5
Slow pion efficiency	1.5	1.5	18.4	22.0
Tracking of K , π , ℓ	0.5	0.5	0.6	0.5
$N_{B\overline{B}}$	0.8	0.8	1.1	0.8
$f_{+-}/f00$	1.3	1.3	1.7	1.3
$\mathcal{B}(D^{*+} \to D^0 \pi^+)$	0.4	0.4	0.5	0.4
$\mathcal{B}(D^0 \to K^- \pi^+)$	0.4	0.4	0.5	0.4
B^0 lifetime	0.1	0.1	0.2	0.1
Total	6.1	2.5	78.3	64.1

LQCD used only for normalisation at zero recoil (w = 1)

$$|V_{cb}|_{\eta_{\rm EW}} \mathcal{F}(1) = \frac{1}{\sqrt{m_B m_{D^*}}} \left(\frac{|\tilde{b}_0|}{P_f(0)\phi_f(0)} \right) \qquad \qquad \mathcal{F}(1) = 0.906 \pm 0.013$$

$$|\mathbf{V}_{cb}|_{\rm BGL} = \textbf{(40.9 \pm 0.3}_{\rm stat} \pm \textbf{1.0}_{\rm syst} \pm \textbf{0.6}_{\rm theo} \textbf{)} \times \textbf{10}^{-3}$$

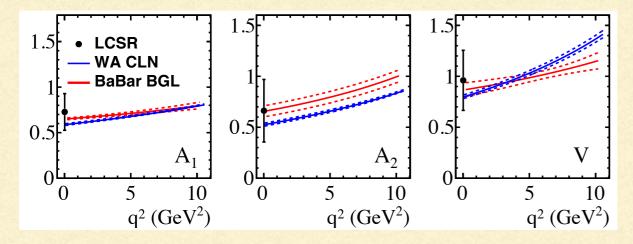


$$|V_{cb}|_{CLN} = (40.4 \pm 0.3_{stat} \pm 1.0_{syst} \pm 0.6_{theo}) \times 10^{-3}$$

RESULTS BY BABAR AND LHCb

1903.10002, 2001.03225

Reanalysis of tagged B⁰ and B⁺ data, unbinned 4 dimensional fit with simplified BGL and CLN About 6000 events
No data provided yet



No clear BGL(111)/CLN difference but disagreement with HFLAV CLN ffs

 $V_{cb} = 0.0384(9)$



Measurement of $|V_{cb}|$ with $B_s^0 o D_s^{(*)-} \mu^+
u_\mu ext{ decays}$

$$\mathcal{R} \equiv \frac{\mathcal{B}(B_s^0 \to D_s^- \mu^+ \nu_{\mu})}{\mathcal{B}(B^0 \to D^- \mu^+ \nu_{\mu})},$$

$$\mathcal{R}^* \equiv \frac{\mathcal{B}(B_s^0 \to D_s^{*-} \mu^+ \nu_{\mu})}{\mathcal{B}(B^0 \to D^{*-} \mu^+ \nu_{\mu})}$$

 $V_{cb}=0.0414(16)$ CLN $V_{cb}=0.0423(17)$ BGL(222)

Fit to exp data and lattice FFs based on HFLAV BRs, employs BGL⁽²²²⁾

SUMMARY

- Despite many new theoretical and exp results, the V_{cb} puzzle persists. However, the level of activity gives hope.
- Inclusive $b \rightarrow c$: new 3loop calculations show pert effects under control, 1.2% accuracy on $|V_{cb}|$
- New method to study inclusive semileptonic meson decays on the lattice. Exploratory calculations for $m_b \sim 2.5 \, \mathrm{GeV}$ in good agreement with OPE. Promising way to complement/validate the OPE, but still a long way to go
- **Exclusive** $b \to c$: uncertainties have been underestimated; several lattice groups are computing necessary FFs at non-zero recoil and new exp analyses are under way but the **situation is still unclear**. FNAL & HPQCD in tension with exp spectra, JLQCD gives a more consistent picture with reduced tension with inclusive.