Constraints for hadronic matrix elements in rare $\boldsymbol{B}$ decays

Nico Gubernari

Based on
arXiv:2011.09813, 2206.03797, 23xx.xxxxx
in collaboration with
Danny van Dyk, Javier Virto, and Méril Reboud

Beyond the Flavour Anomalies IV
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DFG Particle Physics Phenomenology after the Higgs Discovery
Collaborative Research Center TRR 257

Universität Siegen

Center for Particle Physics Siegen

Theoretical framework

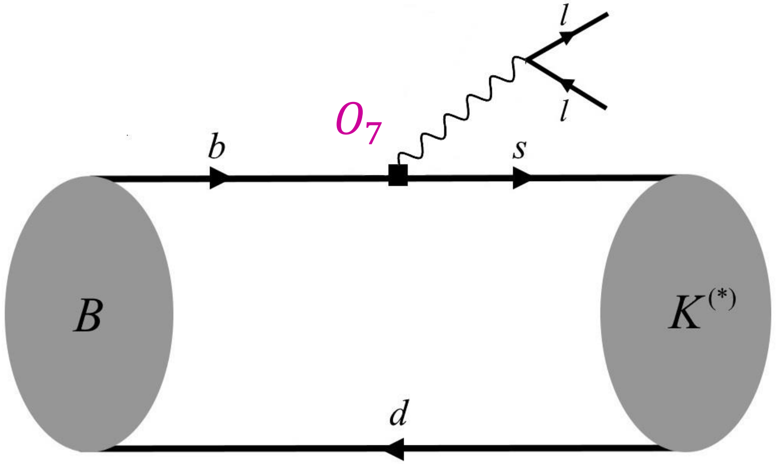
$b \to s\ell^+\ell^-$ effective Hamiltonian

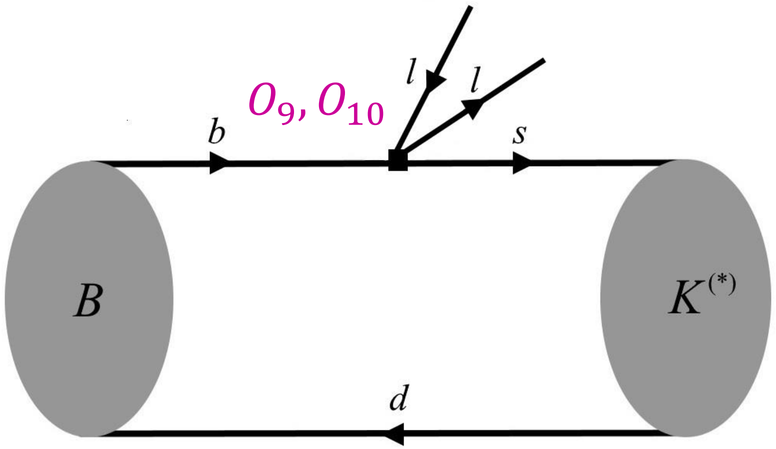
transitions described by the **effective Hamiltonian**

$$\mathcal{H}(b \to s\ell^+\ell^-) = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu) \qquad \mu = m_b$$

main contributions to $B_{(s)} \to \{K^{(*)}, \phi\}\ell^+\ell^-$ in the SM given by local operators O_7, O_9, O_{10}

$$O_7 = \frac{e}{16\pi^2} m_b(\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu} \qquad O_9 = \frac{e^2}{16\pi^2}(\bar{s}_L \gamma^\mu b_L) \sum_\ell (\bar{\ell}\gamma_\mu \ell) \qquad O_{10} = \frac{e^2}{16\pi^2}(\bar{s}_L \gamma^\mu b_L) \sum_\ell (\bar{\ell}\gamma_\mu \gamma_5 \ell)$$

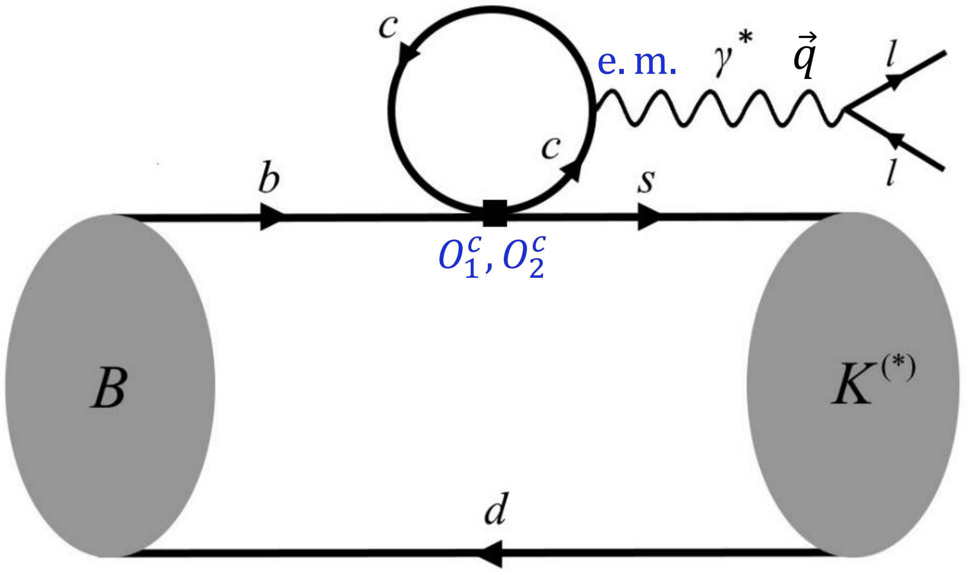




Charm loop in $B \to K^{(*)}\ell^+\ell^-$

additional non-local contributions come from O_1^c and O_2^c combined with the e.m. current (charm-loop contribution)

$$O_1^c = (\bar{s}_L\gamma^\mu c_L)(\bar{c}_L\gamma_\mu b_L) \qquad O_2^c = \left(\bar{s}_L^j\gamma^\mu c_L^i\right)\left(\bar{c}_L^i\gamma_\mu b_L^j\right)$$



Decay amplitude for $B \to K^{(*)}\ell^+\ell^-$ decays

calculate decay amplitudes precisely to probe the SM
$b \to s\mu^+\mu^-$ anomalies: NP or underestimated systematic uncertainties?
(analogous formulas apply to $B_s \to \phi\ell^+\ell^-$ decays)

$$\mathcal{A}(B \to K^{(*)}\ell^+\ell^-) = \mathcal{N}\left[\left(C_9 L_V^\mu + C_{10} L_A^\mu\right) \mathcal{F}_\mu - \frac{L_V^\mu}{q^2}\left(C_7\, \mathcal{F}_{T,\mu} + \mathcal{H}_\mu\right)\right]$$

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local hadronic matrix elements

$$\mathcal{F}_\mu = \langle K^{(*)}(k)|O_{7,9,10}^{\mathrm{had}}|B(k+q)\rangle$$

non-local hadronic matrix elements

$$\mathcal{H}_\mu = i\int d^4x\, e^{iq\cdot x}\langle K^{(*)}(k)|T\{j_\mu^{\mathrm{em}}(x), (C_1 O_1^c + C_2 O_2^c)(0)\}|B(k+q)\rangle$$

goal of this talk: **study and combine model independent constraints for hadronic matrix elements**

Form factors definitions

form factors (FFs) parametrize hadronic matrix elements

FFs are functions of the momentum transfer squared q^2

local FFs

$$\mathcal{F}_\mu(k,q) = \sum_\lambda \mathcal{S}_\mu^\lambda(k,q)\,\mathcal{F}_\lambda(q^2)$$

computed with lattice QCD and light-cone sum rules with good precision $3\% - 20\%$

non-local FFs

$$\mathcal{H}_\mu(k,q) = \sum_\lambda \mathcal{S}_\mu^\lambda(k,q)\mathcal{H}_\lambda(q^2)$$

calculated using an Operator Product Expansion (OPE) or QCD factorization or …
(variety of approaches, most of them model-dependent)

large uncertainties → reduce uncertainties for a better understanding of rare B decays

Local form factors

Methods to compute FFs

non-perturbative techniques are needed to compute FFs

1. Lattice QCD (LQCD)
   numerical evaluation of correlators in a finite and discrete space-time
   more efficient usually at **high** q^2
   reducible systematic uncertainties

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   need universal non-perturbative inputs (**light-meson or B-meson** distribution amplitudes)
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complementary approaches to calculate FFs
in the long run LQCD will dominate the theoretical predictions (smaller and reducible syst unc.)

Local form factors predictions

available theory calculations for local FFs $\mathcal{F}_\lambda$

$B \to K$:

- LQCD calculations at **high** q^2
  [HPQCD 2013/2023] [FNAL/MILC 2015]
  and in the **whole** semileptonic region
  [HPQCD 2023] (see Will's talk)
- LCSR at **low** q^2
  [Khodjamirian/Rusov 2017] [NG/Kokulu/van Dyk 2018]

$B \to K^*$ and $B_s \to \phi$:

- LQCD calculations at **high** q^2
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$B \to K$ **FFs excellent** status (need independent calculation at low q^2)

more LQCD results needed for vector states (for high precision K^* width cannot be neglected)

how to **combine** different calculations for the same channel?
how to obtain result in the **whole** semileptonic region if not available from LQCD?

Parametrization for $\mathcal{F}_\lambda$

obtain local FFs $\mathcal{F}_\lambda$ in the whole semileptonic region by either

- extrapolating LQCD calculations to low q^2
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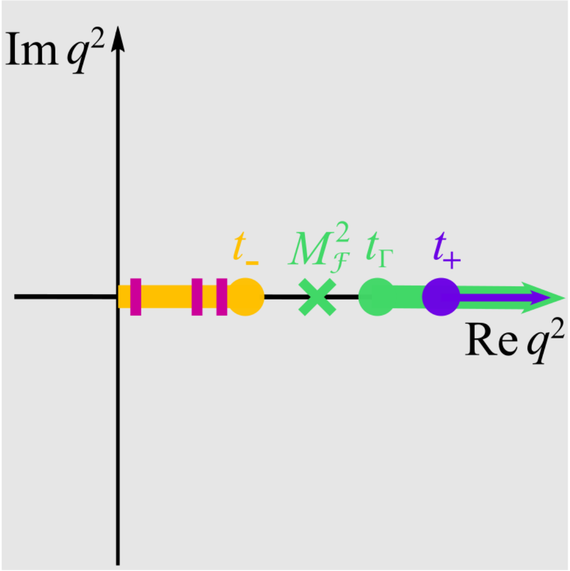
branch cut differs from the pair production threshold:

$t_\Gamma \neq t_+ = \left(M_B + M_{K^{(*)}}\right)^2$ contrary to, e.g., $B \to \pi$



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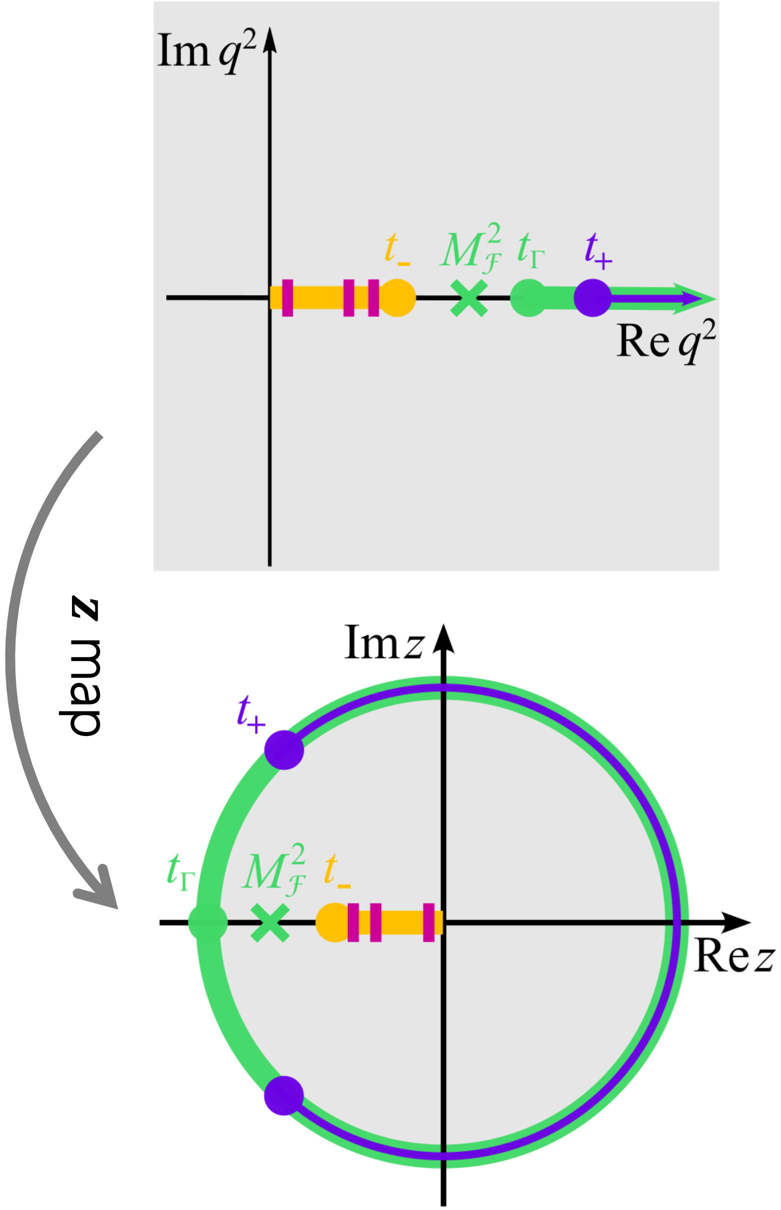
define the map

$$z(q^2) = \frac{\sqrt{t_\Gamma - q^2} - \sqrt{t_\Gamma}}{\sqrt{t_\Gamma - q^2} + \sqrt{t_\Gamma}}$$



previous works on $B \to K^{(*)}$ local FFs always approximated $t_\Gamma = t_+$
non-quantifiable systematic uncertainties (see Javier's talk)

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[Boyd/Grinstein/Lebed 1997]

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GvDV parametrization $\Longrightarrow$ valid also for $t_\Gamma \neq t_+$, generalization of BGL, polynomials orthonormal on the arc of the unit circle (alternative implementation of this parametrization in Flynn/Jüttner/Tsang 2023) [NG/van Dyk/Virto 2020]

$$\mathcal{F}_\lambda = \frac{1}{\mathcal{P}(z)\phi(z)} \sum_{k=0}^{\infty} c_k\, p_k(z) \qquad \sum_{k=0}^{\infty} |c_k|^2 < 1$$

fit this parametrization to LQCD (and LCSR) results and use new improved bounds in Javier's talk

Non-local form factors

Obtaining theoretical predictions for $\mathcal{H}_\lambda$

1. compute the non-local FFs $\mathcal{H}_\lambda$ using a light-cone OPE at negative q^2

$$\mathcal{H}_\lambda(q^2) = \mathcal{C}_\lambda(q^2)\mathcal{F}_\lambda(q^2) + \tilde{\mathcal{C}}_\lambda(q^2)\mathcal{V}_\lambda(q^2) + \cdots$$

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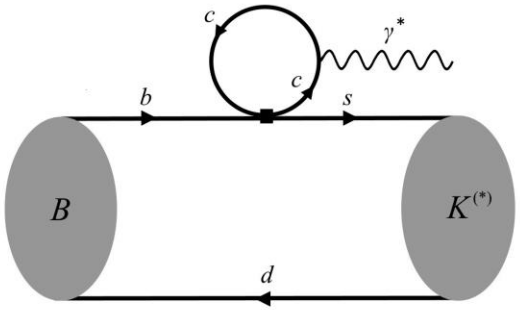
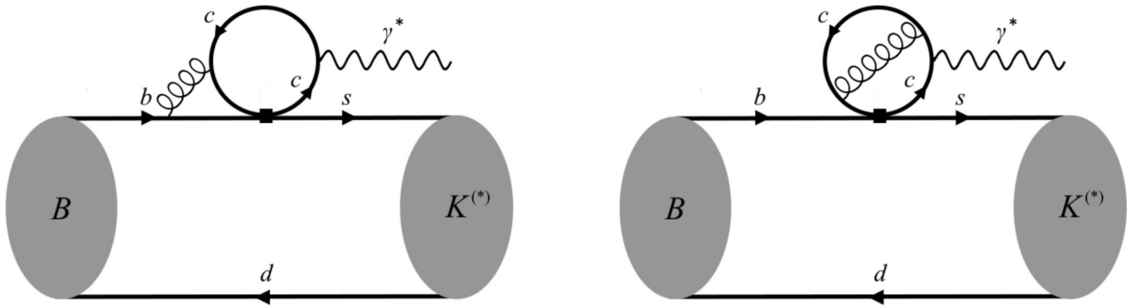


+ hard gluons (α_s) corrections



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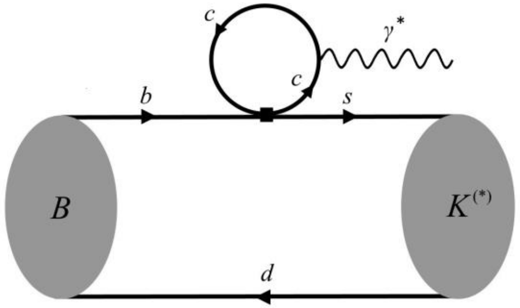

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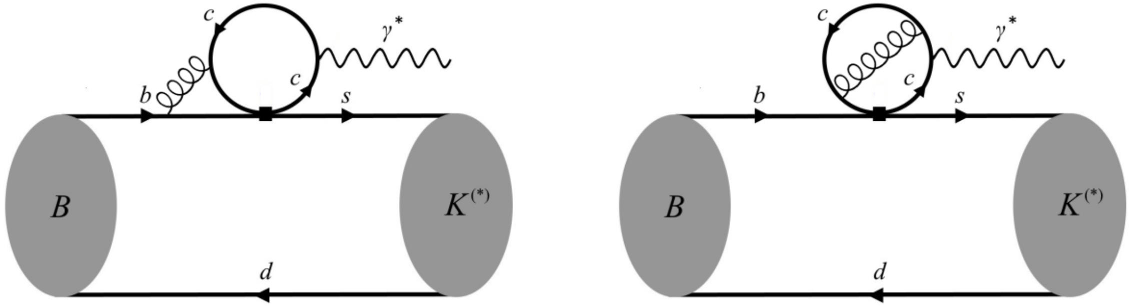

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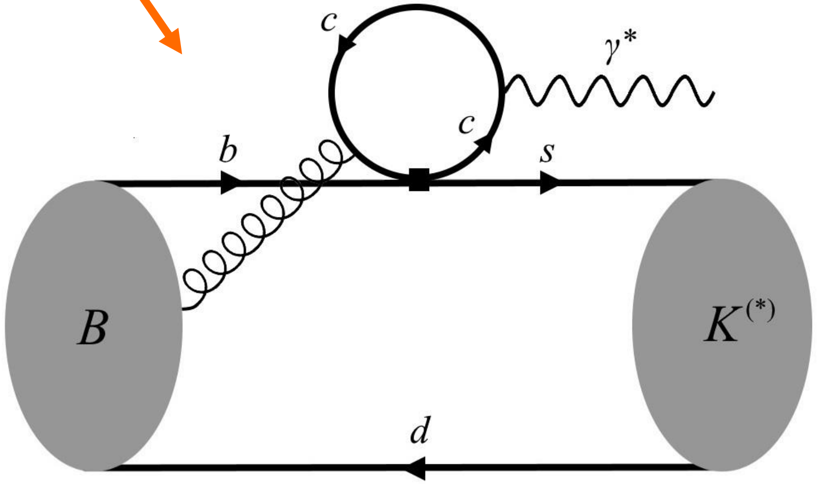
soft gluon correction
non-perturbative
$\Longrightarrow$ not α_s suppressed



[Khodjamirian et al. 2010]
[NG/van Dyk/Virto 2020]

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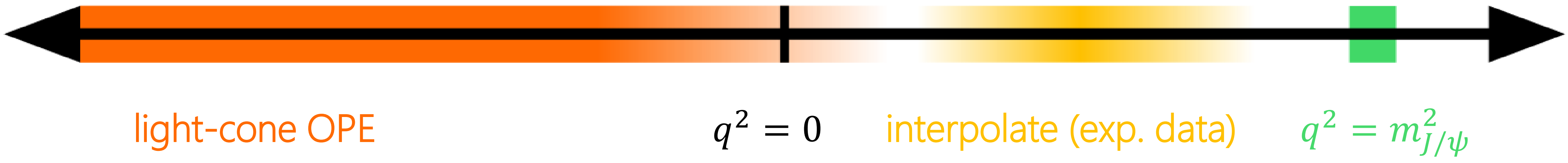

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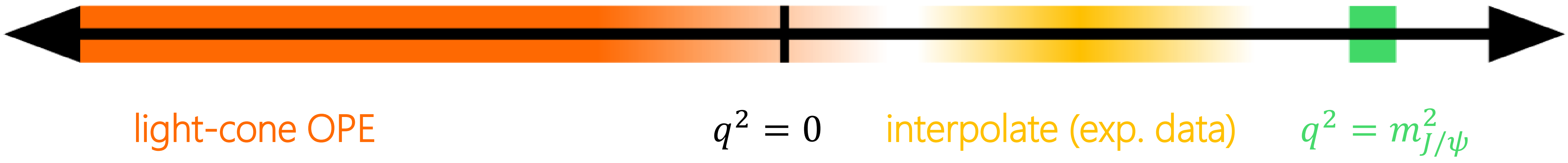
need a parametrization to interpolate $\mathcal{H}_\lambda$: which is the optimal parametrization?



Dispersive bound for $\mathcal{H}_\lambda$

similar situation with respect to $\mathcal{F}_\lambda$

$\mathcal{H}_\lambda$ analytic functions of q^2 except for isolated $c\bar{c}$ poles (J/ψ and $\psi(2S)$)
and a branch cut for $q^2 > \hat{t}_\Gamma = 4M_D^2$

branch cut differs from the pair production threshold:

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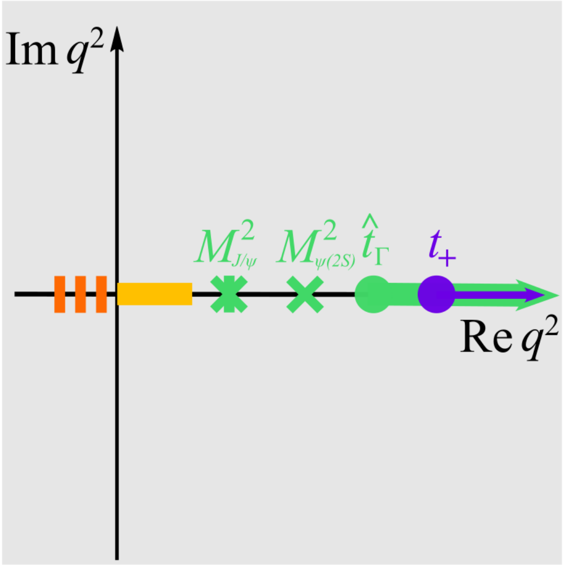

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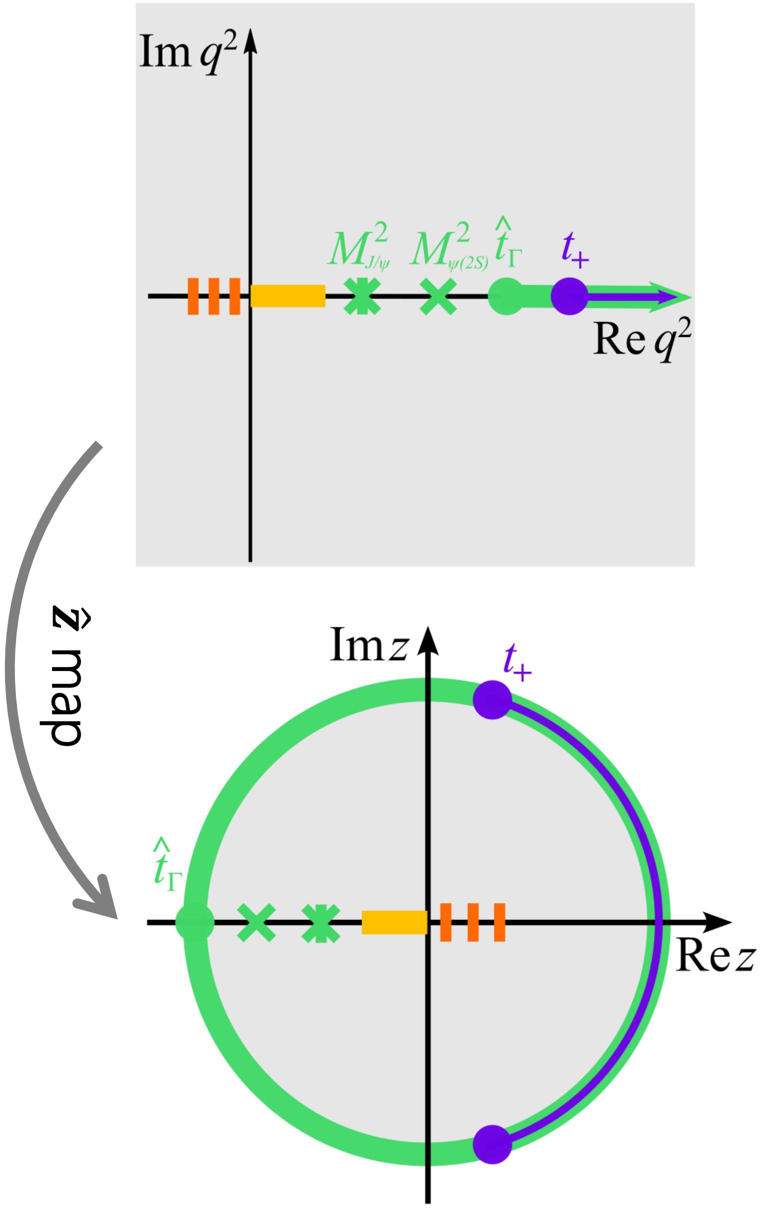
$$\hat{z}(q^2) = \frac{\sqrt{\hat{t}_\Gamma - q^2} - \sqrt{\hat{t}_\Gamma}}{\sqrt{\hat{t}_\Gamma - q^2} + \sqrt{\hat{t}_\Gamma}}$$

only difference between $\mathcal{F}_\lambda$ and $\mathcal{H}_\lambda$ is the threshold $\hat{t}_\Gamma$ and the poles due to more complicate structure of the operator



Parametrizations for $\mathcal{H}_\lambda$

naïve q^2 parametrization [Jäger/Camalich 2012, Ciuchini et al. 2015]

$$\mathcal{H}_\lambda(q^2) = \mathcal{H}_\lambda^{\mathrm{QCDF}}(q^2) + \mathcal{H}_\lambda^{\mathrm{rest}}(0) + \frac{q^2}{M_B^2}\mathcal{H}_\lambda^{\mathrm{rest},\prime}(0) + \frac{(q^2)^2}{M_B^4}\mathcal{H}_\lambda^{\mathrm{rest},\prime\prime}(0) + \cdots$$

dispersion relation [Khodjamirian et al. 2010]

$$\mathcal{H}_\lambda(q^2) = \mathcal{H}_\lambda(0) + \sum_{\psi=J/\psi,\psi(2S)} \frac{f_\psi \mathcal{A}_\psi}{M_\psi^2\left(M_\psi^2 - q^2\right)} + \int_{4M_D^2}^{\infty} dt\, \frac{\rho(t)}{t(t-q^2)}$$

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GvDV parametrization ⟹ new (bounded) parametrization, $\hat{z}$ polynomials [NG/van Dyk/Virto 2020]

$$\mathcal{H}_\lambda(\hat{z}) = \frac{1}{\mathcal{P}(z)\phi(z)} \sum_{k=0}^{\infty} c_k p_k(\hat{z}) \qquad \sum_{k=0}^{\infty} |c_k|^2 < 1$$

fit this parametrization to OPE result and $B \to K^{(*)} J/\psi$ data in Javier's talk

Derivation of the bound

define the correlator

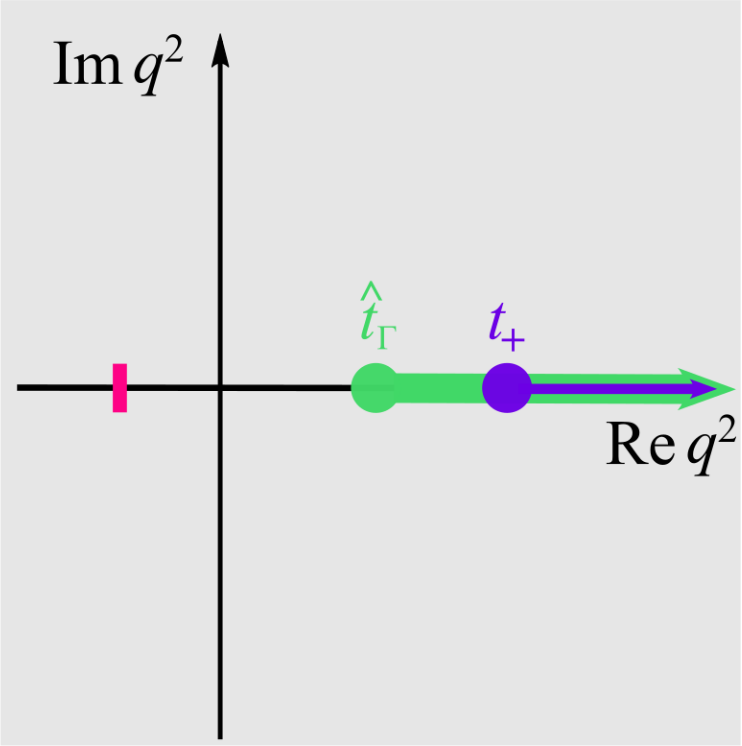
$$\Pi(k,q) = i\int \mathrm{d}^4x\, e^{ikx}\langle 0|T\{\mathcal{O}^\mu(x),\mathcal{O}_\mu(y)\}|0\rangle$$

where

$$\mathcal{O}_\mu \propto \int d^4x\, e^{iq\cdot x}\, T\{j_\mu^{em}(x),(C_1O_1 + C_2O_2)(0)\}$$

use a subtracted dispersion relation

$$\chi(s) \equiv \frac{1}{2}\left(\frac{d}{ds}\right)^2 \Pi(s) \propto \int_{t_+}^{\infty} dq^2\, \frac{\mathrm{Disc}_{bs}\Pi(q^2)}{(q^2-s)^3}$$

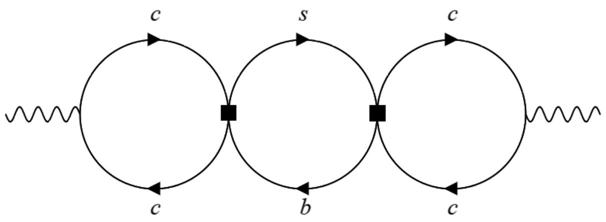


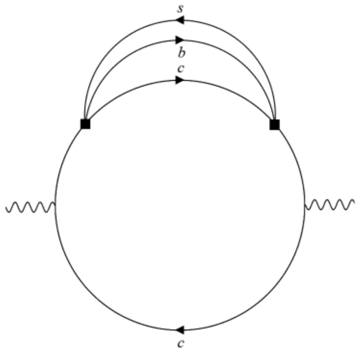
calculate χ perturbatively and $\mathrm{Disc}_{bs}\Pi$ using unitarity

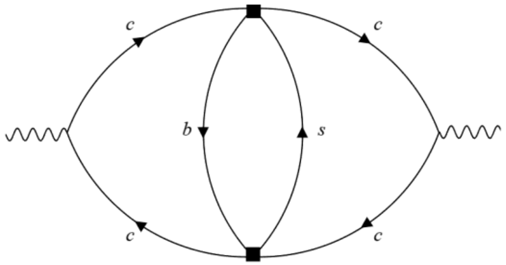
χ calculation very involved while latter is trivial $\mathrm{Disc}_{bs}\Pi \propto |\mathcal{H}_\lambda|^2$

Derivation of the bound

calculate 3-loop diagrams to obtain χ^{OPE}







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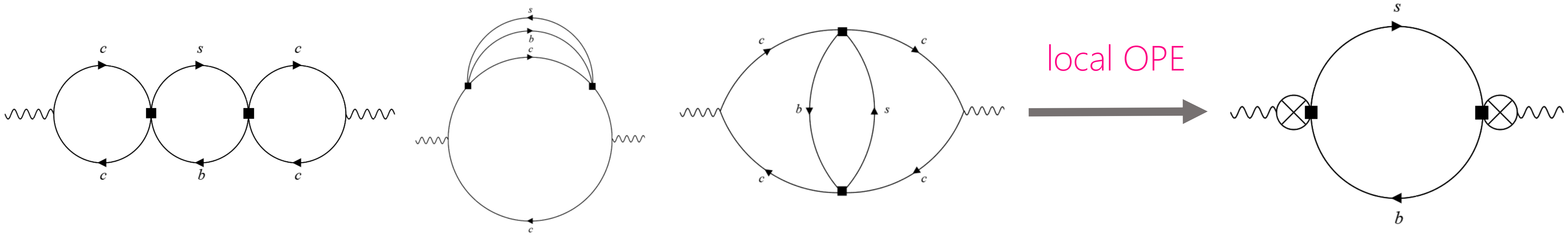


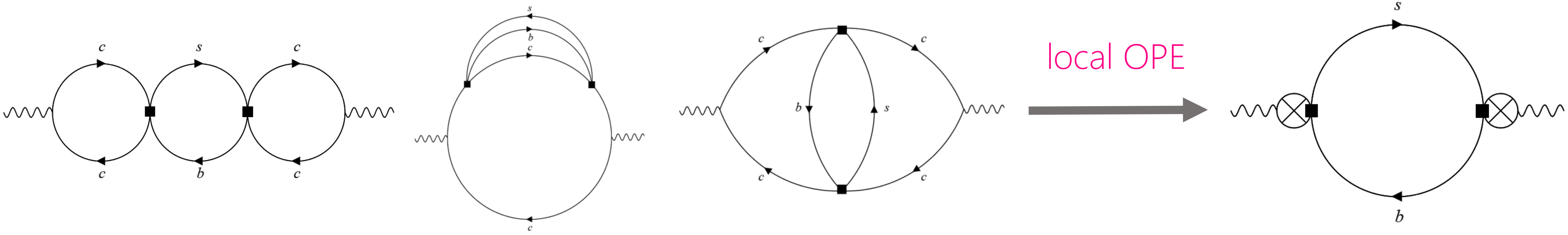
simplify calculation by using the local OPE for $|q^2| \gtrsim m_b^2$ (including α_s corrections)
we obtain at $s = -m_b^2$

$$\chi^{\mathrm{OPE}}(-m_b^2) = (1.81 \pm 0.02) \cdot 10^{-4}\mathrm{GeV}^{-2}$$

[NG/van Dyk/Virto 2020]

Derivation of the bound

calculate 3-loop diagrams to obtain χ^{OPE}



simplify calculation by using the local OPE for $|q^2| \gtrsim m_b^2$ (including α_s corrections)
we obtain at $s = -m_b^2$

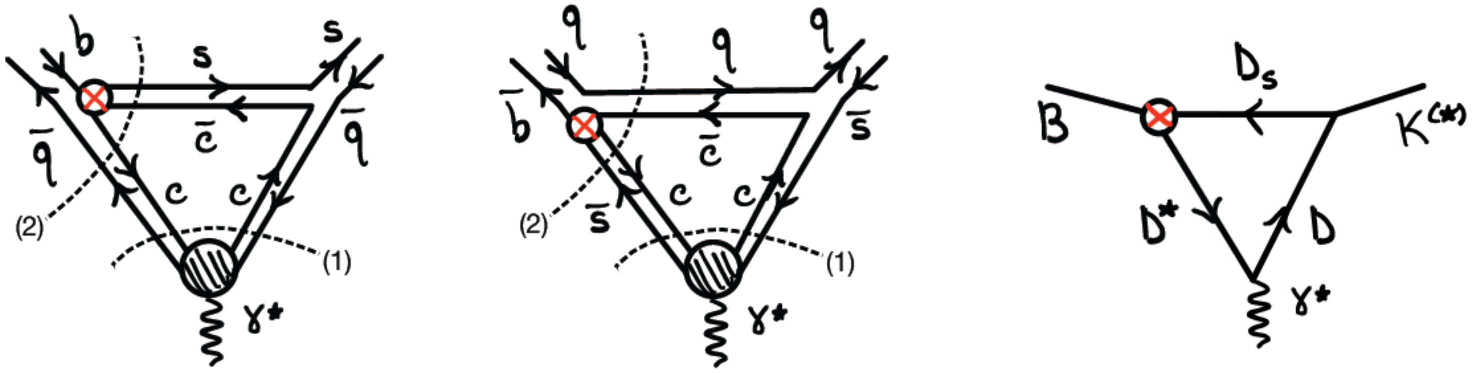
$$\chi^{\text{OPE}}(-m_b^2) = (1.81 \pm 0.02) \cdot 10^{-4}\text{GeV}^{-2}$$ [NG/van Dyk/Virto 2020]

apply $\hat{z}$ mapping, expansion $\mathcal{H}_\lambda(\hat{z}) \propto \sum_k c_k p_k$ to recast the bound in a simple form

$$\chi^{\text{OPE}}(s) \equiv \frac{1}{2}\left(\frac{d}{ds}\right)^2 \Pi(s) \propto \int_{t_+}^{\infty} dq^2 \frac{\text{Disc}_{bs}\Pi(q^2)}{(q^2 - s)^3} \quad \Longrightarrow \quad \sum_{k=0}^{\infty} |c_k|^2 < 1$$

first dispersive bound for non-local FFs = model independent constraints

Missing something?



Ciuchini et al. 2022 (also way before) claim that $B \to \bar{D}D_s \to K^{(*)}\ell^+\ell^-$ rescattering might have a sizable contribution $O(20\%)$

is a **mesonic** estimate the best way to go? (many states contributing, interferences even harder to compute)

partonic calculation doesn't yield large contribution (LP OPE and NLO α_s) [Asatrian/Greub/Virto 2019]

$$\mathcal{H}_\lambda(q^2) = C_\lambda(q^2)\mathcal{F}_\lambda(q^2) + \tilde{C}_\lambda(q^2)\mathcal{V}_\lambda(q^2) + \cdots$$

C_λ is complex valued for any q^2 value due to branch cut in $p^2 = M_B^2$ as expected

large duality violations? large NLP OPE or α_s^2 corrections? spectator scattering?

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   combine theory inputs in new dispersive analysis of the local FFs $\boldsymbol{\mathcal{F}_\lambda}$ [see Javier's talk]

Summary and conclusion

1. reassess BGL parametrization for local FFs $\mathcal{F}_\lambda$ to consider below threshold branch cut and obtain more constraining dispersive bound

   combine theory inputs in new dispersive analysis of the local FFs $\boldsymbol{\mathcal{F}_\lambda}$ [see Javier's talk]

2. new approach for non-local FFs $\mathcal{H}_\lambda$ that combines our OPE calculation at $q^2 < 0$, experimental data for $B \to K^{(*)} J/\psi$, and a dispersive bound

   first dispersive bound for non-local FFs = model independent constraints

   dispersive bound allows to control truncation error

   $\mathcal{H}_\lambda$ uncertainties can be systematically reduced [see Javier's talk]

   major issue for $\mathcal{H}_\lambda$ is $B \to \bar{D} D_s \to K^{(*)} \ell^+ \ell^-$ rescattering
   w.i.p. different groups but no complete estimate yet

Thank you!