(Re)interpretation of the LHC results for new physics
Durham University

Testing the Standard Model and beyond with $B \to PP$ decays

29th August 2023

Adam Szabelski

NCBJ

Nice to be back

Outline

$B \to K\pi$ puzzle

NCBJ

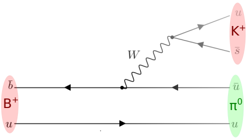
Newest LHCb results:
$B^+ \to K^+\pi^0$ - Phys. Rev. Lett. 126(9), 091802 (2021)
$B^0 \to K^+\pi^-$ - JHEP 03, 075 (2021)
World average:

$$\Delta A_{CP}(K\pi) = A_{CP}(B^+ \to K^+\pi^0) - A_{CP}(B^0 \to K^+\pi^-) = 0.114 \pm 0.014$$

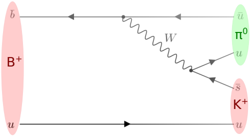
$$\Delta A_{CP}(K\pi) = 0 \quad \text{isospin symmetry}$$

$$\Delta A_{CP}(K\pi) = (0.018^{+0.041}_{-0.032}) \quad \text{QCD factorisation}$$

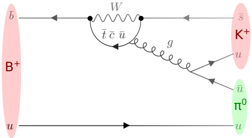
Over 8σ different from 0



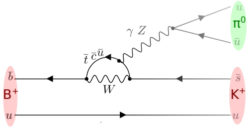
(a) tree colour-allowed



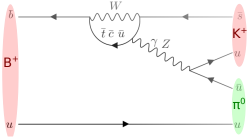
(b) tree colour-suppressed



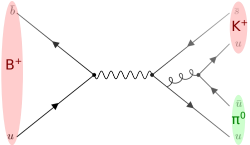
(c) QCD penguin



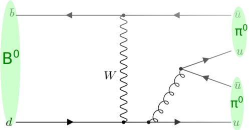
(d) EW penguin colour-allowed



(e) EW penguin colour-suppressed



(f) annihilation



(g) W-exchange

Operator product expansion

$B \to \pi K \; - \; b \to s, \; \Delta U = \Delta C = 0 \qquad (\bar{p}q)_{V\pm A} \equiv \bar{p}\gamma^\mu(1 \pm \gamma_5)q$

NCBJ

$$Q_1^\alpha \equiv (\bar{b}_x \alpha_y)_{V-A}(\bar{\alpha}_y s_x)_{V-A}, \quad Q_2^\alpha \equiv (\bar{b}\alpha)_{V-A}(\bar{\alpha}s)_{V-A};$$

Current-current operators(tree operators)

$$Q_3 \equiv (\bar{b}s)_{V-A}\sum_q (\bar{q}q)_{V-A},$$
$$Q_4 \equiv (\bar{b}_x s_y)_{V-A}\sum_q (\bar{q}_y q_x)_{V-A},$$
$$Q_5 \equiv (\bar{b}s)_{V-A}\sum_q (\bar{q}q)_{V+A},$$
$$Q_6 \equiv (\bar{b}_x s_y)_{V-A}\sum_q (\bar{q}_y q_x)_{V+A};$$

Gluonic-penguin operators

$$Q_7 \equiv \frac{3}{2}(\bar{b}s)_{V-A}\sum_q (e_q \bar{q}q)_{V+A},$$
$$Q_8 \equiv \frac{3}{2}(\bar{b}_x s_y)_{V-A}\sum_q (e_q \bar{q}_y q_x)_{V+A},$$
$$Q_9 \equiv \frac{3}{2}(\bar{b}s)_{V-A}\sum_q (e_q \bar{q}q)_{V-A},$$
$$Q_{10} \equiv \frac{3}{2}(\bar{b}_x s_y)_{V-A}\sum_q (e_q \bar{q}_y q_x)_{V-A};$$

Electroweak-penguin operators

Parameterisation

NCBJ

$$\tilde{T} = \lambda^3 A R_b(\mathcal{T} - \mathcal{P}_{tu} + \mathcal{E} - \mathcal{PA}_{tu}),$$
$$\tilde{C} = \lambda^3 A R_b(\mathcal{C} + \mathcal{P}_{tu} - \mathcal{E} + \mathcal{PA}_{tu}),$$
$$P = \lambda^3 A(\mathcal{P}_t - \mathcal{P}_c),$$

$$r_X = \frac{X}{P} \quad e.g.\ r_T = \frac{\tilde{T}}{P}$$
$$\sqrt{2}\mathcal{A}(B^+ \to \pi^0\pi^+) = -P\left[e^{i\gamma}(r_T + r_C) + e^{-i\beta}\tilde{q}(r_T + r_C)\right],$$
$$\mathcal{A}(B^0 \to \pi^-\pi^+) = P(1 - r_T e^{i\gamma})$$
$$\sqrt{2}\mathcal{A}(B^0 \to \pi^0\pi^0) = P(1 + r_C e^{i\gamma} + e^{-i\beta}\tilde{q}(r_T + r_C))$$

$$\tilde{q} \equiv \left|\frac{P_{EW} + P^C_{EW}}{T + C}\right|$$

$B \to K\pi$

$$A(B^+ \to \pi^+ K^0) = -P' \left[1 + r_{\rho_c} e^{i\gamma} - \frac{1}{3} a_C \boldsymbol{q e^{i\omega} e^{i\phi}} (r'_T + r'_C)\right]$$

$$\sqrt{2} A(B^+ \to \pi^0 K^+) = P' \left[1 + r_{\rho_c} e^{i\gamma} - \left\{e^{i\gamma} - \left(1 - \frac{1}{3} a_C\right) \boldsymbol{q e^{i\omega} e^{i\phi}}\right\} (r'_T + r'_C)\right]$$

$$A(B_d^0 \to \pi^- K^+) = P' \left[1 + r_{\rho_c} e^{i\gamma} + \frac{2}{3} a_C \boldsymbol{q e^{i\omega} e^{i\phi}} (r'_T + r'_C) - r_T e^{i\gamma}\right]$$

$$\sqrt{2} A(B_d^0 \to \pi^0 K^0) = -P' \left[1 + r_{\rho_c} e^{i\gamma} - \left(1 - \frac{2}{3} a_C\right) \boldsymbol{q e^{i\omega} e^{i\phi}} (r'_T + r'_C) + r'_C e^{i\gamma}\right]$$

$$P' \equiv \frac{\lambda^3 A}{\sqrt{\epsilon}} (\mathcal{P}'_t - \mathcal{P}'_c)$$

$$r_{\rho_c} \equiv \frac{\rho_c e^{i\theta_c}}{P'} \equiv \left(\frac{\lambda^2 R_b}{1 - \lambda^2}\right) \left[\frac{\mathcal{P}'_t - \tilde{\mathcal{P}}'_u - \mathcal{A}'}{\mathcal{P}'_t - \mathcal{P}'_c}\right] \qquad B^+ \to K^+ \overline{K}^0 : \quad \rho_c = 0.03 \pm 0.01, \quad \theta_c = (2.6 \pm 4.6)^\circ$$

$$a_C \equiv \frac{\hat{\mathcal{P}}'^C_{EW}}{\hat{\mathcal{P}}'_{EW} + \hat{\mathcal{P}}'^C_{EW}}$$

$$\boldsymbol{q e^{i\phi} e^{i\omega} \equiv -\left(\frac{\hat{\mathcal{P}}'_{EW} + \hat{\mathcal{P}}'^C_{EW}}{\hat{T}' + \hat{C}'}\right)}$$

In SM: $\boldsymbol{q e^{i\phi} e^{i\omega} \equiv \frac{-3}{2\lambda^2 R_b} \left[\frac{C_9(\mu) + C_{10}(\mu)}{C_1(\mu) + C_2(\mu)}\right] R_q = (0.64 \pm 0.05) R_q}$

$R_q = 1 \pm 0.05$ - compensates for possible $SU(3)$ violation

$B \to K\pi$

NCBJ

$$A(B^+ \to \pi^+ K^0) = -P' \left[1 + r_{\rho_c} e^{i\gamma} - \frac{1}{3} a_C \mathbf{q} e^{i\omega} e^{i\phi} (r'_T + r'_C)\right]$$

$$\sqrt{2} A(B^+ \to \pi^0 K^+) = P' \left[1 + r_{\rho_c} e^{i\gamma} - \left\{e^{i\gamma} - \left(1 - \frac{1}{3} a_C\right) \mathbf{q} e^{i\omega} e^{i\phi}\right\} (r'_T + r'_C)\right]$$

$$A(B_d^0 \to \pi^- K^+) = P' \left[1 + r_{\rho_c} e^{i\gamma} + \frac{2}{3} a_C \mathbf{q} e^{i\omega} e^{i\phi} (r'_T + r'_C) - r_T e^{i\gamma}\right]$$

$$\sqrt{2} A(B_d^0 \to \pi^0 K^0) = -P' \left[1 + r_{\rho_c} e^{i\gamma} - \left(1 - \frac{2}{3} a_C\right) \mathbf{q} e^{i\omega} e^{i\phi} (r'_T + r'_C) + r'_C e^{i\gamma}\right]$$

$$P' \equiv \frac{\lambda^3 A}{\sqrt{\epsilon}} (\mathcal{P}'_t - \mathcal{P}'_c)$$

$$r_{\rho_c} \equiv \frac{\rho_c e^{i\theta_c}}{P'} \equiv \left(\frac{\lambda^2 R_b}{1 - \lambda^2}\right) \left[\frac{\mathcal{P}'_t - \tilde{\mathcal{P}}'_u - \mathcal{A}'}{\mathcal{P}'_t - \mathcal{P}'_c}\right]$$

$B^+ \to K^+ \overline{K}^0$: $\rho_c = 0.03 \pm 0.01, \quad \theta_c = (2.6 \pm 4.6)^\circ$

$$a_C \equiv \frac{\hat{\mathcal{P}}'^C_{EW}}{\hat{\mathcal{P}}'_{EW} + \hat{\mathcal{P}}'^C_{EW}}$$

Isospin relation

$$\sqrt{2}\mathcal{A}(B^+ \to K^0 \pi^0) + \mathcal{A}(B^+ \to K^+ \pi^-) = \sqrt{2}\mathcal{A}(B^+ \to K^+ \pi^0) + \mathcal{A}(B^0 \to K^0 \pi^+)$$

$$q e^{i\phi} e^{i\omega} \equiv -\left(\frac{\hat{\mathcal{P}}'_{EW} + \hat{\mathcal{P}}'^C_{EW}}{\hat{T}' + \hat{C}'}\right)$$

In SM: $q e^{i\phi} e^{i\omega} \equiv \frac{-3}{2\lambda^2 R_b} \left[\frac{C_9(\mu) + C_{10}(\mu)}{C_1(\mu) + C_2(\mu)}\right] R_q = (0.64 \pm 0.05) R_q$

$R_q = 1 \pm 0.05$ - compensates for possible $SU(3)$ violation

U-spin relation

NCBJ

Connection between amplitudes describing $B \to \pi\pi$ and $B \to K\pi$ decays

$$B \to \pi\pi \text{ - } b \to d \iff B \to K\pi \text{ - } b \to s(')$$

$$R_{T+C} = \left|\frac{T' + C'}{T + C}\right| \equiv \left|\frac{(r'_T + r'_C)P'}{\epsilon(r_T + r_C)P}\right| = 1.21 \pm 0.015 \quad \text{QCDF}$$

$$R_{T+C} = 1.2 \pm 0.2 \quad \text{safe estimate}$$

$$\epsilon \equiv \frac{\lambda^2}{1-\lambda^2} = 0.0535 \pm 0.0002$$

$$arg(r'_T) - arg(r_T) = 0 \pm 20^\circ \quad arg(r'_C) - arg(r_C) = 0 \pm 20^\circ$$

same as arxiv:1806.08783 R. Fleischer et al.

Effective Hamiltonian - $\phi_{SM} = 0$

[Matthias Neubert and Jonathan L. Rosner]

$$H_{eff} = \frac{G_F}{\sqrt{2}}\left[\sum_{i=1,2} C_i(\lambda_u Q_i^u + \lambda_c Q_i^c) + \lambda_t \sum_{i=3}^{10} C_i Q_i\right] + \text{h.c.}$$

$$\lambda_\alpha = V_{\alpha b}^* V_{\alpha s} \quad \lambda_u + \lambda_c + \lambda_t = 0$$

$$C_7, C_8 \ll C_9, C_{10} + \text{Fierz identity} \implies Q_9^{\Delta I=1} \sim Q_1^u - Q_1^d \quad , Q_{10}^{\Delta I=1} \sim Q_2^u - Q_2^d$$

When the operators are linearly dependent the weak phase between them $\phi_{SM} = 0$
The weak interaction can break $SU(3)$ but this reasoning there is a strong constrain on the weak phase of $qe^{i(\phi+\omega)}$ in the SM.

$qe^{i\omega}$ in $SU(3)$ decomposition

[Michael Gronau, Dan Pirjol, Tung-Mow Yan]

NCBJ

Operators: $\overline{q}_1\overline{q}_3 q_2 \simeq (\overline{b}q_1)(\overline{q}_2 q_3)$ form a SU(3) group that can be decomposed into:

$$\mathbf{\overline{3}} \otimes \mathbf{\overline{3}} \otimes \mathbf{3} = \mathbf{\overline{3}} \oplus \mathbf{\overline{3}} \oplus \mathbf{6} \oplus \mathbf{\overline{15}}$$

$$\mathcal{A}^{tree}(B^0 \to \pi^- K^+) + \sqrt{2}\mathcal{A}^{tree}(B^0 \to \pi^0 K^0) = T' + C' = -\lambda_u^{(s)}\frac{\sqrt{10}}{3}(C_1 + C_2)\langle 27||\overline{15}_{I=1}||3\rangle$$

$qe^{i\omega}$ in $SU(3)$ decomposition

[Michael Gronau, Dan Pirjol, Tung-Mow Yan]

Operators: $\overline{q}_1 \overline{q}_3 q_2 \simeq (\overline{b} q_1)(\overline{q}_2 q_3)$ form a SU(3) group that can be decomposed into:

$$\mathbf{\overline{3}} \otimes \mathbf{\overline{3}} \otimes \mathbf{3} = \mathbf{\overline{3}} \oplus \mathbf{\overline{3}} \oplus \mathbf{6} \oplus \mathbf{\overline{15}}$$

$$\mathcal{A}^{tree}(B^0 \to \pi^- K^+) + \sqrt{2}\mathcal{A}^{tree}(B^0 \to \pi^0 K^0) = T' + C' = -\lambda_u^{(s)} \frac{\sqrt{10}}{3}(C_1 + C_2)\langle 27 || \overline{15}_{I=1} || 3 \rangle$$

$$\mathcal{P}^{EW}(B^+ \to \pi^+ K^0) + \sqrt{2}\mathcal{P}^{EW}(B^+ \to \pi^0 K^+) = \mathcal{P}'_{EW} + \mathcal{P}'^{C}_{EW} = -\lambda_t^{(s)} \sqrt{\frac{5}{2}}(C_9 + C_{10})\langle 27 || \overline{15}_{I=1} || 3 \rangle$$

$qe^{i\omega}$ in $SU(3)$ decomposition

[Michael Gronau, Dan Pirjol, Tung-Mow Yan]

Operators: $\overline{q}_1\overline{q}_3 q_2 \simeq (\overline{b}q_1)(\overline{q}_2 q_3)$ form a SU(3) group that can be decomposed into:

$$\overline{\mathbf{3}} \otimes \overline{\mathbf{3}} \otimes \mathbf{3} = \overline{\mathbf{3}} \oplus \overline{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{15}}$$

$$\mathcal{A}^{tree}(B^0 \to \pi^- K^+) + \sqrt{2}\mathcal{A}^{tree}(B^0 \to \pi^0 K^0) = T' + C' = -\lambda_u^{(s)} \frac{\sqrt{10}}{3}(C_1 + C_2)\langle 27||\overline{15}_{I=1}||3\rangle$$

$$\mathcal{P}^{EW}(B^+ \to \pi^+ K^0) + \sqrt{2}\mathcal{P}^{EW}(B^+ \to \pi^0 K^+) = \mathcal{P}'_{EW} + \mathcal{P}'^{C}_{EW} = -\lambda_t^{(s)}\sqrt{\frac{5}{2}}(C_9 + C_{10})\langle 27||\overline{15}_{I=1}||3\rangle$$

Combining the two:

$$\mathcal{P}'_{EW} + \mathcal{P}'^{C}_{EW} = \frac{3}{2}\frac{\lambda_t^{(s)}}{\lambda_u^{(s)}}\frac{C_9 + C_{10}}{C_1 + C_2}(T' + C')$$

$qe^{i\omega}$ in $SU(3)$ decomposition

[Michael Gronau, Dan Pirjol, Tung-Mow Yan]

NCBJ

Operators: $\bar{q}_1 \bar{q}_3 q_2 \simeq (\bar{b} q_1)(\bar{q}_2 q_3)$ form a SU(3) group that can be decomposed into:

$$\mathbf{\bar{3}} \otimes \mathbf{\bar{3}} \otimes \mathbf{3} = \mathbf{\bar{3}} \oplus \mathbf{\bar{3}} \oplus \mathbf{6} \oplus \mathbf{\overline{15}}$$

$$\mathcal{A}^{tree}(B^0 \to \pi^- K^+) + \sqrt{2}\mathcal{A}^{tree}(B^0 \to \pi^0 K^0) = T' + C' = -\lambda_u^{(s)} \frac{\sqrt{10}}{3} (C_1 + C_2) \langle 27 || \overline{15}_{I=1} || 3 \rangle$$

$$\mathcal{P}^{EW}(B^+ \to \pi^+ K^0) + \sqrt{2}\mathcal{P}^{EW}(B^+ \to \pi^0 K^+) = \mathcal{P'}_{EW} + \mathcal{P'}^{C}_{EW} = -\lambda_t^{(s)} \sqrt{\frac{5}{2}} (C_9 + C_{10}) \langle 27 || \overline{15}_{I=1} || 3 \rangle$$

Combining the two:

$$\mathcal{P'}_{EW} + \mathcal{P'}^{C}_{EW} = \frac{3}{2} \frac{\lambda_t^{(s)}}{\lambda_u^{(s)}} \frac{C_9 + C_{10}}{C_1 + C_2} (T' + C')$$

In SM: $qe^{i\phi} e^{i\omega} \equiv \frac{-3}{2\lambda^2 R_b} \left[\frac{C_9(\mu) + C_{10}(\mu)}{C_1(\mu) + C_2(\mu)} \right] R_q = (0.64 \pm 0.05) R_q$

In $SU(3)$ limit: $\omega = 0$

P_{EW} and P_{EW}^C

$$b_3 = \lambda_t^{(s)} \frac{3}{2}(C_9 - C_{10})\langle \mathbf{8}||\mathbf{6}||\mathbf{3}\rangle$$

$$b_4 = \lambda_t^{(s)} \frac{1}{2}(C_9 + C_{10})\langle \mathbf{8}||\overline{\mathbf{15}}||\mathbf{3}\rangle$$

$$b_5 = \lambda_t^{(s)} \frac{1}{2}(C_9 + C_{10})\langle \mathbf{27}||\overline{\mathbf{15}}||\mathbf{3}\rangle$$

$$P'_{EW} + P'^{C}_{EW} = -\sqrt{\frac{5}{2}}b_5 = \frac{3}{2}\frac{\lambda_t^{(s)}}{\lambda_u^{(s)}}e^{i\gamma}\frac{C_9 + C_{10}}{C_1 + C_2}(T + C) = -q e^{i\phi}(T' + C')$$

$$P'^{C}_{EW} = \frac{1}{2}\sqrt{\frac{3}{5}}b_3 + \frac{3}{2}\sqrt{\frac{3}{5}}b_4 - \frac{3}{2}\sqrt{\frac{2}{5}}b_5$$

$$= \frac{3}{4}\frac{\lambda_t^{(s)}}{\lambda_u^{(s)}}e^{i\gamma}\left(-\frac{C_9 - C_{10}}{C_1 - C_2}(T' - C') + \frac{C_9 + C_{10}}{C_1 + C_2}(T' + C')\right)$$

$$\frac{C_9 - C_{10}}{C_1 - C_2} \simeq \frac{C_9 + C_{10}}{C_1 + C_2}$$

$$P'^{C}_{EW} = -q e^{i\phi} C' \qquad P'_{EW} = -q e^{i\phi} T'$$

$$P_{EW} + P_{EW}^C = -\frac{\sqrt{5}}{2}b_5 = \lambda^2 R_b \left|\frac{V_{td}}{V_{ub}}\right| q e^{i(\phi-\beta)}(T' + C')\frac{f_\pi}{f_K}$$

Exchange and annihilation diagrams neglected.

P_{EW} and P_{EW}^C

$$b_3 = \lambda_t^{(s)} \frac{3}{2}(C_9 - C_{10})\langle \mathbf{8} || \mathbf{6} || \mathbf{3} \rangle$$

$$b_4 = \lambda_t^{(s)} \frac{1}{2}(C_9 + C_{10})\langle \mathbf{8} || \overline{\mathbf{15}} || \mathbf{3} \rangle$$

$$b_5 = \lambda_t^{(s)} \frac{1}{2}(C_9 + C_{10})\langle \mathbf{27} || \overline{\mathbf{15}} || \mathbf{3} \rangle$$

$$P'_{EW} + P'^{C}_{EW} = -\sqrt{\frac{5}{2}} b_5 = \frac{3}{2} \frac{\lambda_t^{(s)}}{\lambda_u^{(s)}} e^{i\gamma} \frac{C_9 + C_{10}}{C_1 + C_2}(T + C) = -q e^{i\phi}(T' + C')$$

$$P'^{C}_{EW} = \frac{1}{2}\sqrt{\frac{3}{5}} b_3 + \frac{3}{2}\sqrt{\frac{3}{5}} b_4 - \frac{3}{2}\sqrt{\frac{2}{5}} b_5$$

$$= \frac{3}{4} \frac{\lambda_t^{(s)}}{\lambda_u^{(s)}} e^{i\gamma} \left(-\frac{C_9 - C_{10}}{C_1 - C_2}(T' - C') + \frac{C_9 + C_{10}}{C_1 + C_2}(T' + C') \right)$$

$$\frac{C_9 - C_{10}}{C_1 - C_2} \simeq \frac{C_9 + C_{10}}{C_1 + C_2}$$

$$P'^{C}_{EW} = -q e^{i\phi} C' \qquad P'_{EW} = -q e^{i\phi} T'$$

$$P_{EW} + P_{EW}^{C} = -\frac{\sqrt{5}}{2} b_5 = \lambda^2 R_b \left| \frac{V_{td}}{V_{ub}} \right| q e^{i(\phi - \beta)}(T' + C') \frac{f_\pi}{f_K}$$

New Physics

Exchange and annihilation diagrams neglected.

P_{EW} and P_{EW}^C

NCBJ

$$b_3 = \lambda_t^{(s)} \frac{3}{2}(C_9 - C_{10})\langle \mathbf{8}||\mathbf{6}||\mathbf{3}\rangle$$

$$b_4 = \lambda_t^{(s)} \frac{1}{2}(C_9 + C_{10})\langle \mathbf{8}||\overline{\mathbf{15}}||\mathbf{3}\rangle$$

$$b_5 = \lambda_t^{(s)} \frac{1}{2}(C_9 + C_{10})\langle \mathbf{27}||\overline{\mathbf{15}}||\mathbf{3}\rangle$$

model-dependent

$$P'_{EW} + P'^{C}_{EW} = -\sqrt{\frac{5}{2}}b_5 = \frac{3}{2}\frac{\lambda_t^{(s)}}{\lambda_u^{(s)}}e^{i\gamma}\frac{C_9 + C_{10}}{C_1 + C_2}(T + C) = -qe^{i\phi}(T' + C')$$

New Physics

$$P'^{C}_{EW} = \frac{1}{2}\sqrt{\frac{3}{5}}b_3 + \frac{3}{2}\sqrt{\frac{3}{5}}b_4 - \frac{3}{2}\sqrt{\frac{2}{5}}b_5$$

$$= \frac{3}{4}\frac{\lambda_t^{(s)}}{\lambda_u^{(s)}}e^{i\gamma}\left(-\frac{C_9 - C_{10}}{C_1 - C_2}(T' - C') + \frac{C_9 + C_{10}}{C_1 + C_2}(T' + C')\right)$$

$$\frac{C_9 - C_{10}}{C_1 - C_2} \simeq \frac{C_9 + C_{10}}{C_1 + C_2}$$

$$P'^{C}_{EW} = -qe^{i\phi}C' \qquad P'_{EW} = -qe^{i\phi}T'$$

$$P_{EW} + P_{EW}^{C} = -\frac{\sqrt{5}}{2}b_5 = \lambda^2 R_b\left|\frac{V_{td}}{V_{ub}}\right|qe^{i(\phi-\beta)}(T' + C')\frac{f_\pi}{f_K}$$

Exchange and annihilation diagrams neglected.

Fitting procedure

Observables:

CP asymmetries direct $A_{CP} = \frac{\Gamma(\bar{i}\to\bar{f})-\Gamma(i\to f)}{\Gamma(\bar{i}\to\bar{f})+\Gamma(i\to f)} = \frac{|\overline{\mathcal{A}}|^2-|\mathcal{A}|^2}{|\overline{\mathcal{A}}|^2+|\mathcal{A}|^2}$

and time-dependent S_{CP}: $A(t) = A_{CP}\cos\phi_d t + S_{CP}\sin\phi_d t$

Branching fraction $\mathcal{B} = \frac{\Gamma(\bar{i}\to\bar{f})+\Gamma(i\to f)}{2} = \frac{|\overline{\mathcal{A}}|^2+|\mathcal{A}|^2}{2}$

Fitting procedure

Observables:

CP asymmetries direct $A_{CP} = \frac{\Gamma(\bar{i}\to\bar{f})-\Gamma(i\to f)}{\Gamma(\bar{i}\to\bar{f})+\Gamma(i\to f)} = \frac{|\overline{\mathcal{A}}|^2-|\mathcal{A}|^2}{|\overline{\mathcal{A}}|^2+|\mathcal{A}|^2}$

and time-dependent S_{CP}: $A(t) = A_{CP}\cos\phi_d t + S_{CP}\sin\phi_d t$

Branching fraction $\mathcal{B} = \frac{\Gamma(\bar{i}\to\bar{f})+\Gamma(i\to f)}{2} = \frac{|\overline{\mathcal{A}}|^2+|\mathcal{A}|^2}{2}$

χ^2:

asymmetries $\chi^2 = \sum_i \frac{((A_{CP}^{\text{exp}})_i-(A_{CP}^{\text{theory}})_i)^2}{(\sigma_{\text{exp}})_i^2}$

ratios of BF $\chi^2 = \sum_{ij}(R_i^{\text{exp}} - R_i^{\text{theory}})\,Cov_{ij}^{-1}(R_j^{\text{exp}} - R_j^{\text{theory}})$

$R = \frac{\mathcal{B}_x}{\mathcal{B}_y}$

Fitting procedure

Observables:

CP asymmetries direct $A_{CP} = \frac{\Gamma(\bar{i}\to\bar{f})-\Gamma(i\to f)}{\Gamma(\bar{i}\to\bar{f})+\Gamma(i\to f)} = \frac{|\bar{\mathcal{A}}|^2-|\mathcal{A}|^2}{|\bar{\mathcal{A}}|^2+|\mathcal{A}|^2}$

and time-dependent S_{CP}: $A(t) = A_{CP}\cos\phi_d t + S_{CP}\sin\phi_d t$

Branching fraction $\mathcal{B} = \frac{\Gamma(\bar{i}\to\bar{f})+\Gamma(i\to f)}{2} = \frac{|\bar{\mathcal{A}}|^2+|\mathcal{A}|^2}{2}$

parameters (including $\boldsymbol{q}$ and $\boldsymbol{\phi}$)

χ^2:

asymmetries $\chi^2 = \sum_i \frac{((A_{CP}^{\text{exp}})_i-(A_{CP}^{\text{theory}})_i)^2}{(\sigma_{\text{exp}})_i^2}$

ratios of BF $\chi^2 = \sum_{ij}(R_i^{\text{exp}} - R_i^{\text{theory}})\,Cov_{ij}^{-1}(R_j^{\text{exp}} - R_j^{\text{theory}})$

$R = \frac{\mathcal{B}_x}{\mathcal{B}_y}$

Experimental data

PDG world averages. Mainly BaBar, BELLE(2) and LHCb.

Observable	experimental value	source
$A^{\pi^+\pi^-}$	0.314 ± 0.030	PDG22
$A^{\pi^+\pi^0}$	0.01 ± 0.04	PDG22, BELLE 2 CONF (2022)
$A^{\pi^0\pi^0}$	0.33 ± 0.22	PDG22
$S^{\pi^+\pi^-}$	-0.670 ± 0.030	PDG22
$\mathcal{B}(\pi^+\pi^-)$	$(5.16 \pm 0.19)10^{-6}$	PDG22, BELLE 2 CONF (2021)
$\mathcal{B}(\pi^+\pi^0)$	$(5.6 \pm 0.4)10^{-6}$	PDG22, BELLE 2 CONF (2022)
$\mathcal{B}(\pi^0\pi^0)$	$(1.48 \pm 0.24)10^{-6}$	PDG22, BELLE 2 CONF (2021)

Observable	experimental value	source
$A^{\pi^+K^-}$	-0.0837 ± 0.0032	PDG22, BELLE 2 CONF (2021)
$A^{\pi^+K^0}$	-0.017 ± 0.016	PDG22, BELLE 2 CONF (2021)
$A^{\pi^0K^+}$	0.030 ± 0.013	PDG22, BELLE 2 CONF (2022)
$A^{\pi^0K^0}$	-0.054 ± 0.121	PDG22, BELLE 2 CONF (2022)
$S^{\pi^0K^0}$	0.58 ± 0.17	PDG22
$\mathcal{B}(\pi^+K^-)$	$(1.94 \pm 0.05)10^{-5}$	PDG22, BELLE 2 CONF (2021)
$\mathcal{B}(\pi^+K^0)$	$(2.35 \pm 0.08)10^{-5}$	PDG22, BELLE 2 CONF (2021)
$\mathcal{B}(\pi^0K^+)$	$(1.32 \pm 0.05)10^{-5}$	PDG22, BELLE 2 CONF (2022)
$\mathcal{B}(\pi^0K^0)$	$(10.0 \pm 0.5)10^{-6}$	PDG22, BELLE 2 CONF (2022)

$$R^{\pi^+\pi^-} \equiv 2\frac{M_{B^+}}{M_{B_d^0}}\frac{\Phi(m_\pi/M_{B_d^0}, m_\pi/M_{B_d^0})}{\Phi(m_{\pi^0}/M_{B^+}, m_\pi/M_{B^+})}\left[\frac{\mathcal{B}(B^+\to\pi^+\pi^0)}{\mathcal{B}(B_d^0\to\pi^+\pi^-)}\right]\frac{\tau_{B_d^0}}{\tau_{B^+}}$$

$$R^{\pi^0\pi^0} \equiv 2\frac{\Phi(m_\pi/M_{B_d^0}, m_\pi/M_{B_d^0})}{\Phi(m_{\pi^0}/M_{B_d^0}, m_{\pi^0}/M_{B_d^0})}\left[\frac{\mathcal{B}(B_d^0\to\pi^0\pi^0)}{\mathcal{B}(B_d^0\to\pi^+\pi^-)}\right]$$

$$\Phi(X, Y) = \sqrt{[1-(X+Y)^2][1-(X-Y)^2]}$$

$$R \equiv \left[\frac{BF^{corr}(B_d^0\to\pi^-K^+)}{BF^{corr}(B^+\to\pi^+K^0)}\right]$$

$$R_c \equiv \left[\frac{BF^{corr}(B^+\to\pi^0K^+)}{BF^{corr}(B^+\to\pi^+K^0)}\right]$$

$$R_n \equiv \left[\frac{BF^{corr}(B_d^0\to\pi^-K^+)}{BF^{corr}(B_d^0\to\pi^0K^0)}\right]$$

$$R^{\pi\pi-\pi K} \equiv \left[\frac{BF^{corr}(B^+\to\pi^+\pi^0)}{BF^{corr}(B^+\to\pi^0K^+)}\right]$$

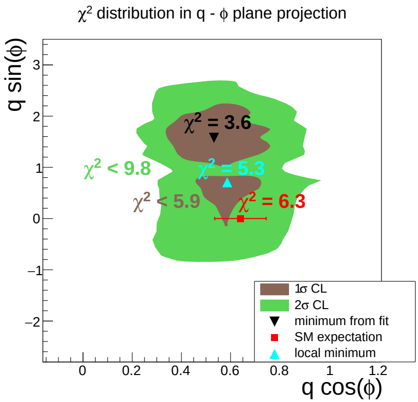
BELLE 2 Update

NCBJ

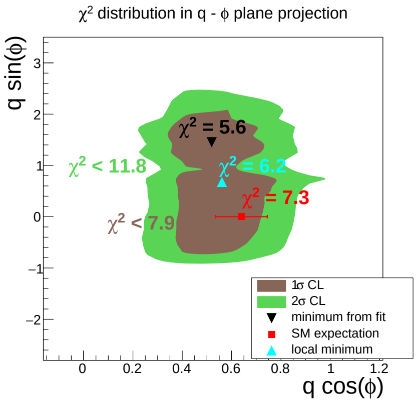
Observable	PDG2022+BELLE 2(2022)	PDG 2022+BELLE 2(2023)
$\mathcal{B}(\pi^0 K^0)$	10.00 ± 0.48	9.96 ± 0.44
$A(\pi^0 K^0)$	-0.05 ± 0.12	-0.005 ± 0.092
$S(\pi^0 K^0)$	0.58 ± 0.17	0.64 ± 0.14

fits with/without BELLE2023 update on $B^0 \to \pi^0 K^0$

w/o $B^0 \to \pi^0 K^0$



with $B^0 \to \pi^0 K^0$



Solving the 'old' $B \to K\pi$ puzzle

$$\Delta A_{CP}(K\pi) = A_{CP}(B^+ \to K^+\pi^0) - A_{CP}(^0\to K^+\pi^-)$$
$$\simeq -2(\Im(r'_C))\sin\gamma + 2\Im(r'_T + r'_C)\,q\sin\phi$$
$$\Delta A_{CP}(K\pi) = 0.114 \pm 0.014$$

BELLE 2 2023 Update: $\Delta A_{CP}(K\pi) = 0.110 \pm 0.013$

	SM scenario	NP scenario
ϕ	0	$(68.1 \pm 4.3)^\circ$
q	0.614 ± 0.051	1.41 ± 0.16
γ	$(65.3 \pm 1.3)^\circ$	$(65.6 \pm 1.3)^\circ$
$\Im r'_C$	-0.0569 ± 0.0075	0.0043 ± 0.0061
$\Im r'_T$	0.036 ± 0.010	0.0443 ± 0.0021
$\boldsymbol{\Delta A_{CP}}$	$\mathbf{0.108 \pm 0.012}$	$\mathbf{0.107 \pm 0.012}$

Summary

NCBJ

- ✔ Diagrammatic approach is a helpful tool to study $B \to PP$ decays.
- ✔ The ad-hoc parameters $qe^{i\phi}$ are used to search for NP impact.
- ✔ The operator product expansion together with the $SU(3)$ decomposition brings relations between the EW penguin and tree amplitudes reducing the number of parameters in the problem.
- ✔ The joint fit to the $B \to \pi\pi$ and $B \to \pi K$ data does not exclude neither the SM nor the NP scenario.
- ✔ The fit is very sensitive to the measurement of $A_{CP}^{\pi^0 K^0}$ and $S_{CP}^{\pi^0 K^0}$ as well as $\mathcal{B}(\pi^0 K^0)$.
- ✔ New results from BELLE 2 can shed more light to understand the situation (update: it did!).

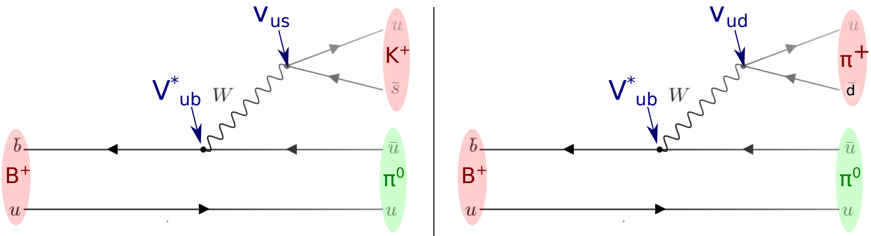
Ideas for the future

- ✔ Use this analysis to test NP models.
- ✔ Check the possible impact of the SMEFT operators.
- ✔ See if it is feasible to put constraints on the SMEFT Wilson coefficients.

NCBJ

Thank you!

CKM matrix elements

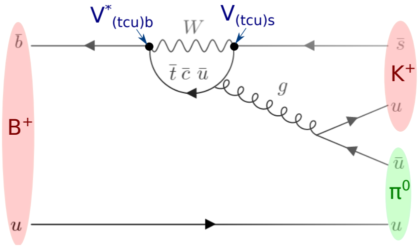


$$V_{ub}^* V_{us} = A\lambda^4(\rho + i\eta)$$

$$V_{ub}^* V_{ud} = A\lambda^3 \left(1 - \frac{\lambda^2}{2}\right)(\rho + i\eta)$$

$$\epsilon = \frac{\lambda^2}{1-\lambda^2} \quad \gamma = arg\left[-\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}}\right] \quad \beta = arg\left[-\frac{V_{cb}^* V_{cd}}{V_{tb}^* V_{td}}\right]$$

Gluonic penguin



$$P'_{QCD} = V^*_{tb} V_{ts} P'_t + V^*_{cb} V_{cs} P'_c + V^*_{ub} V_{us} P'_u$$

UT: $V^*_{tb} V_{ts} + V^*_{cb} V_{cs} + V^*_{ub} V_{us} = 0$

$$P'_{QCD} = -V^*_{cb} V^*_{cs} P'_{tc} - V^*_{ub} V_{us} P'_{tu} = A\lambda^3 (P'_{tc} - e^{i\gamma} R_b P'_{tu})$$

$$R_b \equiv \left(1 - \frac{\lambda^2}{2}\right) \frac{1}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right| = 0.390 \pm 0.030$$

$SU(3)$

NCBJ

$$\sqrt{2}\mathcal{A}(B \to \pi^+\pi^0) = -e^{i\gamma}(T' + C') \overset{SU(3)}{=} -\frac{V_{ud}}{V_{us}}\frac{f_\pi}{f_K}(C+T)$$

$\frac{f_\pi}{f_K} = 1.22 \pm 0.01$ - leading (i.e., factorisable) $SU(3)$ - breaking corrections
prime $'$ - labels $b \to d$ transitions.
Bose-Einstein statics brings $I(\pi^0\pi^+) = 2 \;\Rightarrow \Delta I = \frac{3}{2}$
$\overline{Q}_1' - \overline{Q}_2'$ contribute to $\Delta I = \frac{1}{2}$
$\Rightarrow$ Only $\overline{Q}_1' + \overline{Q}_2'$ contributes to $B^+ \to \pi^0\pi^+$
assuming $SU(3)$ only $\overline{Q}_1 + \overline{Q}_2$ contributes to $A_{3/2}$ in $B \to \pi K$

$$\frac{\langle \pi K(I=\frac{3}{2})|\,\overline{Q}_1 - \overline{Q}_2\,|B^+\rangle}{\langle \pi K(I=\frac{3}{2})|\,\overline{Q}_1 + \overline{Q}_2\,|B^+\rangle} \equiv -\delta_{SU(3)}\, e^{i\Delta\varphi}$$

general factorisation hypothesis - $\delta_{SU(3)} \approx 1 - 3\%$ and $\Delta\varphi$

Isospin decomposition

NCBJ

$$H_{\Delta I=0} = \frac{G_F}{\sqrt{2}} \left\{ \sum_{i=1,2} \left[\frac{\lambda_u}{2} C_i (Q_i^u + Q_i^d) + \lambda_c Q_i^c \right] + \lambda_t \sum_{i=3}^{10} C_i Q_i - \lambda_t \sum_{i=7}^{10} C_i Q_i^{\Delta I=1} \right\} + h.c.$$

$$H_{\Delta I=1} = \frac{G_F}{\sqrt{2}} \left[\sum_{i=1,2} \frac{\lambda_u}{2} C_i (Q_i^u - Q_i^d) + \lambda_t \sum_{i=7}^{10} C_i Q_i^{\Delta I=1} \right] + h.c.$$

$$A_{3/2} = \sqrt{\frac{1}{3}} \left\langle \frac{3}{2}, \pm\frac{1}{2} \middle| \mathcal{H}_{\Delta I=1} \middle| \frac{1}{2}, \pm\frac{1}{2} \right\rangle$$

$$A_{1/2} = \pm\sqrt{\frac{2}{3}} \left\langle \frac{1}{2}, \pm\frac{1}{2} \middle| \mathcal{H}_{\Delta I=1} \middle| \frac{1}{2}, \pm\frac{1}{2} \right\rangle$$

$$B_{1/2} = \sqrt{\frac{2}{3}} \left\langle \frac{1}{2}, \pm\frac{1}{2} \middle| \mathcal{H}_{\Delta I=0} \middle| \frac{1}{2}, \pm\frac{1}{2} \right\rangle$$

$$\mathcal{A}_{-+} = \mathcal{A}(B^0 \to \pi^- K^+) = A_{3/2} + A_{1/2} - B_{1/2}$$

$$\mathcal{A}_{+0} = \mathcal{A}(B^+ \to \pi^+ K^0) = A_{3/2} + A_{1/2} + B_{1/2}$$

$$\mathcal{A}_{00} = \sqrt{2}\mathcal{A}(B^0 \to \pi^0 K^0) = 2A_{3/2} - A_{1/2} + B_{1/2}$$

$$\mathcal{A}_{0+} = \sqrt{2}\mathcal{A}(B^+ \to \pi^0 K^+) = 2A_{3/2} - A_{1/2} - B_{1/2}$$

$A_{3/2}$

NCBJ

$$\mathcal{A}_{-+} = \mathcal{A}(B^0 \to \pi^- K^+) = A_{3/2} + A_{1/2} - B_{1/2}$$

$$\mathcal{A}_{00} = \sqrt{2}\mathcal{A}(B^0 \to \pi^0 K^0) = 2A_{3/2} - A_{1/2} + B_{1/2}$$

$$\mathcal{A}_{-+} = -Te^{i\gamma} + P_{tc} - P'_{uc}e^{i\gamma} - \frac{2}{3}P^C_{EW}$$

$$\mathcal{A}_{00} = -Ce^{i\gamma} - P_{tc} + P_{uc}e^{i\gamma} - P_{EW} - \frac{1}{3}P^C_{EW}$$

$$\mathcal{A}_{-+} + \mathcal{A}_{00} = 3A_{3/2} = -(C+T)e^{i\gamma} - P_{EW} - P^C_{EW} = -(C+T)(e^{i\gamma} - qe^{i\omega}e^{i\phi})$$

$$qe^{i(\phi+\omega)} \equiv -\frac{P_{EW} + P^C_{EW}}{C+T}$$

Hamiltonian decomposition

NCBJ

$$\mathcal{H}_T = \frac{G_F}{\sqrt{2}}\Big(\lambda_u^{(s)}[\frac{1}{2}(c_1-c_2)(-\bar{\mathbf{3}}^{(a)}_{I=0}-\mathbf{6}_{I=1})+\frac{1}{2}(c_1+c_2)(-\overline{\mathbf{15}}_{I=1}-\frac{1}{\sqrt{2}}\overline{\mathbf{15}}_{I=0}+\frac{1}{\sqrt{2}}\bar{\mathbf{3}}^{(s)}_{I=0})$$

$$+\lambda_u^{(d)}[\frac{1}{2}(c_1-c_2)(\mathbf{6}_{I=\frac{1}{2}}-\bar{\mathbf{3}}^{(a)}_{I=\frac{1}{2}})+\frac{1}{2}(c_1+c_2)(-\frac{2}{\sqrt{3}}\overline{\mathbf{15}}_{I=\frac{3}{2}}-\frac{1}{\sqrt{6}}\overline{\mathbf{15}}_{I=\frac{1}{2}}+\frac{1}{\sqrt{2}}\bar{\mathbf{3}}^{(s)}_{I=\frac{1}{2}})\Big)$$

$$\mathcal{H}_{EWP} \simeq -\lambda_t^{(s)}\left(c_9 Q_9^{(s)}+c_{10}Q_{10}^{(s)}\right)-\lambda_t^{(d)}\left(c_9 Q_9^{(d)}+c_{10}Q_{10}^{(d)}\right)=$$

$$-\frac{\lambda_t^{(s)}}{2}\left(\frac{c_9-c_{10}}{2}(3\cdot\mathbf{6}_{I=1}+\bar{\mathbf{3}}^{(a)}_{I=0})+\frac{c_9+c_{10}}{2}(-3\cdot\overline{\mathbf{15}}_{I=1}-\frac{3}{\sqrt{2}}\overline{\mathbf{15}}_{I=0}-\frac{1}{\sqrt{2}}\bar{\mathbf{3}}^{(s)}_{I=0})\right)$$

$$-\frac{\lambda_t^{(d)}}{2}\left(\frac{c_9-c_{10}}{2}(-3\cdot\mathbf{6}_{I=\frac{1}{2}}+\bar{\mathbf{3}}^{(a)}_{I=\frac{1}{2}})+\frac{c_9+c_{10}}{2}(-\sqrt{\frac{3}{2}}\cdot\overline{\mathbf{15}}_{I=\frac{1}{2}}-2\sqrt{3}\cdot\overline{\mathbf{15}}_{I=\frac{3}{2}}-\frac{1}{\sqrt{2}}\bar{\mathbf{3}}^{(s)}_{I=\frac{1}{2}})\right)$$

NCBJ

$$A_{3/2} = \sqrt{\frac{1}{3}} \left\langle \frac{3}{2}, \pm\frac{1}{2} \right| \mathcal{H}_{\Delta I=1} \left| \frac{1}{2}, \pm\frac{1}{2} \right\rangle$$

$$= \sqrt{\frac{1}{3}} \left\langle \frac{3}{2}, \pm\frac{1}{2} \right| \frac{1}{2} \left(1 - \frac{3}{2}\frac{\lambda_t(C_9 + C_{10})}{\lambda_u(C_1 + C_2)}\right) \lambda_u(C_1 + C_2)(\overline{Q}_1 + \overline{Q}_2) \left| \frac{1}{2}, \pm\frac{1}{2} \right\rangle$$

$$A_{3/2} = -\frac{1}{3}(C + T)(e^{i\gamma} - qe^{i\omega}e^{i\phi})$$

If $\left\langle \frac{3}{2}, \pm\frac{1}{2} \right| \frac{\sqrt{3}}{2}\lambda_u(C_1 + C_2)(\overline{Q}_1 + \overline{Q}_2) \left| \frac{1}{2}, \pm\frac{1}{2} \right\rangle = (T + C)e^{i\gamma}$

$$q = -\frac{3}{2\lambda^2 R_b}\frac{C_9 + C_{10}}{C_1 + C_2} = 0.64 \pm 0.05$$

$\frac{\lambda_u}{\lambda_t} \approx -\lambda^2 R_b e^{i\gamma}$ $\lambda \approx 0.22$ and $R_b = \frac{1}{\lambda}\frac{V_{ub}}{V_{cb}} \approx 0.41 \pm 0.07$

$SU(3)$ breaking

[Benjamin Grinstein and Richard F. Lebec

Let us now consider SU(3)-breaking corrections to the lowest-order Hamiltonian. The simplest such breaking originates through insertions of the strange quark mass,

$$\mathcal{H}_s = m_s \bar{s} s, \tag{4.3}$$

which transforms as an $I = 0$, $Y = 0$ octet plus singlet in SU(3). Clearly neither piece changes the isospin of the Hamiltonian; this would be accomplished by insertions of the up or down masses, which are much smaller. Let us consider SU(3) breaking linear in m_s. In the case of $B \to PP$, the Hamiltonian contains pieces transforming under

$$(\bar{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{15}}) \otimes (\mathbf{1} \oplus \mathbf{8}) = \bar{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{15}} \oplus \mathbf{24} \oplus \overline{\mathbf{42}}, \tag{4.4}$$