Introduction to Event Generators

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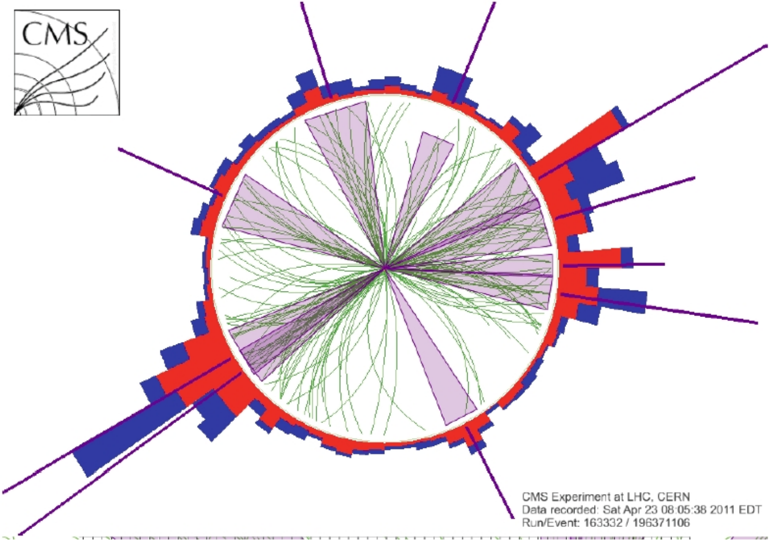
Motivation: jets

[Google Images]

Motivation: jets (at LHC of course)



[CMS 2011]

Why Monte Carlos?

We want to understand

$$\mathscr{L}_{\text{int}} \longleftrightarrow \text{Final states}\ .$$

Why Monte Carlos?

LHC experiments require
sound understanding of signals and *backgrounds*.

↑

Full detector simulation.

↑

Fully exclusive hadronic final state.

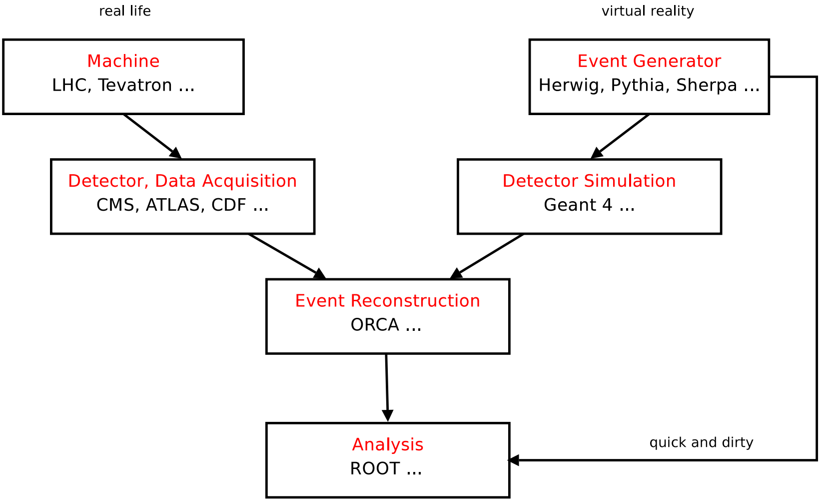
↑

Monte Carlo event generator with
parton shower, hadronization model, decays of unstable particles.

↑

Parton level computations.

Experiment and Simulation



Monte Carlo Event Generators

- Complex final states in full detail (jets).
- Arbitrary observables and cuts from final states.
- Studies of new physics models.

- Rates and topologies of final states.
- Background studies.
- Detector Design.
- Detector Performance Studies (Acceptance).

- *Obvious* for calculation of observables on the quantum level

$$|A|^2 \longrightarrow \text{Probability.}$$

pp Event Generator

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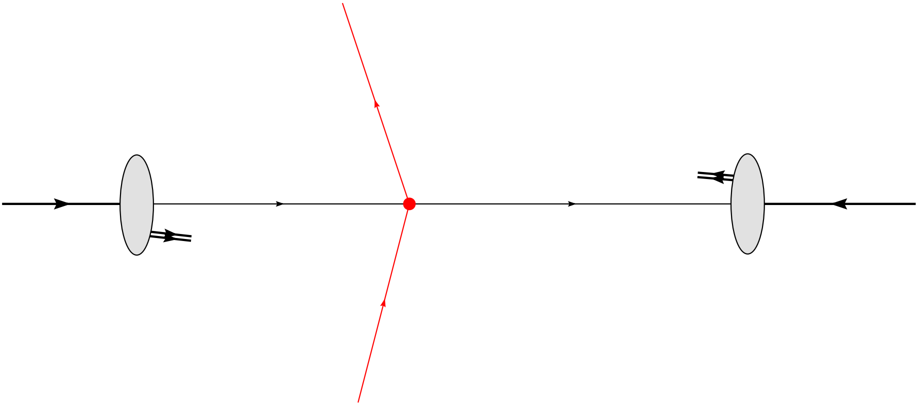

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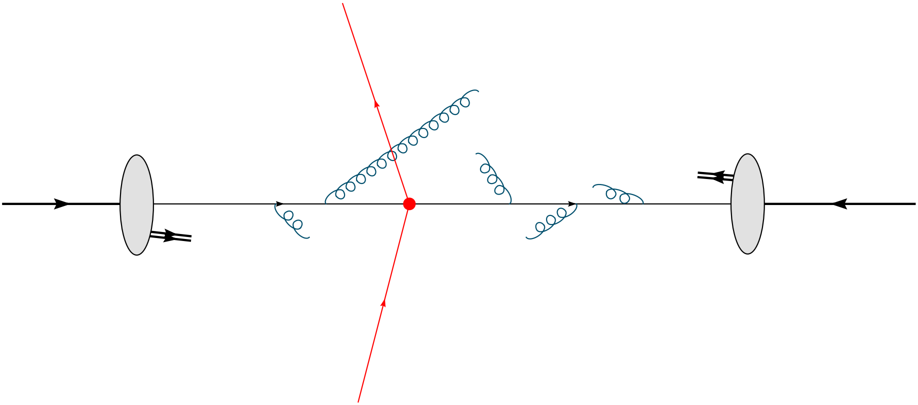

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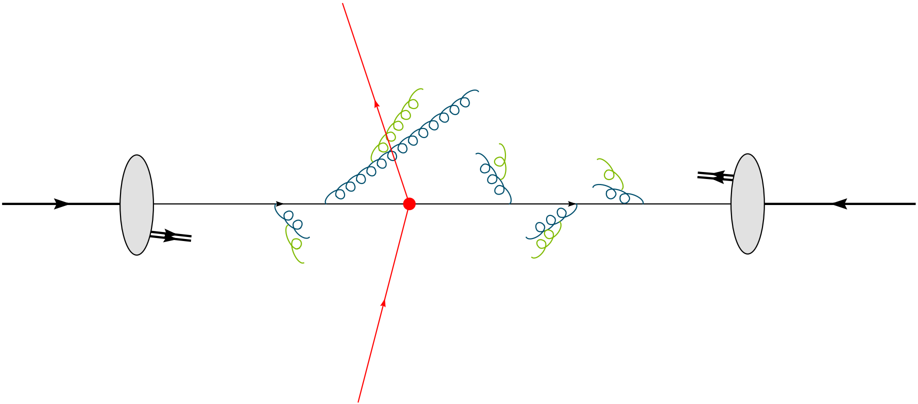

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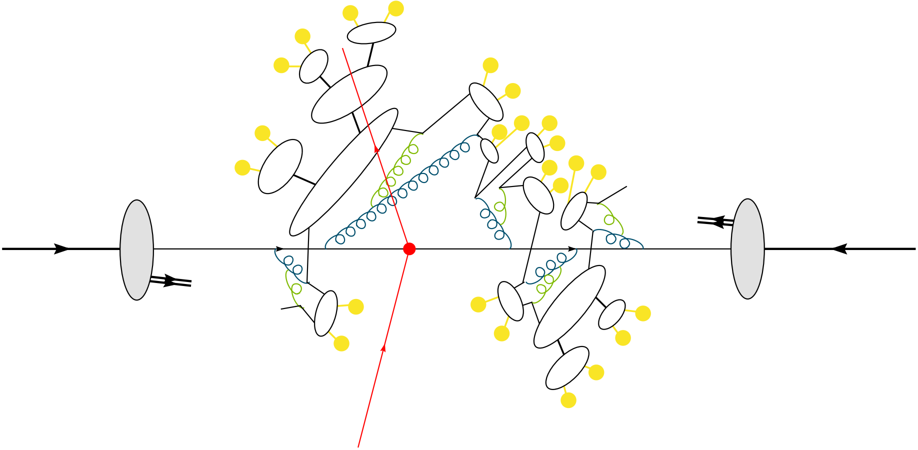

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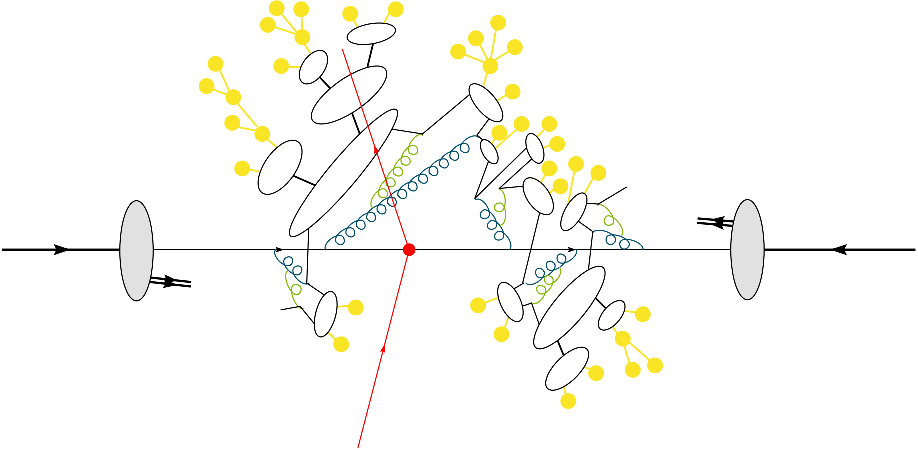

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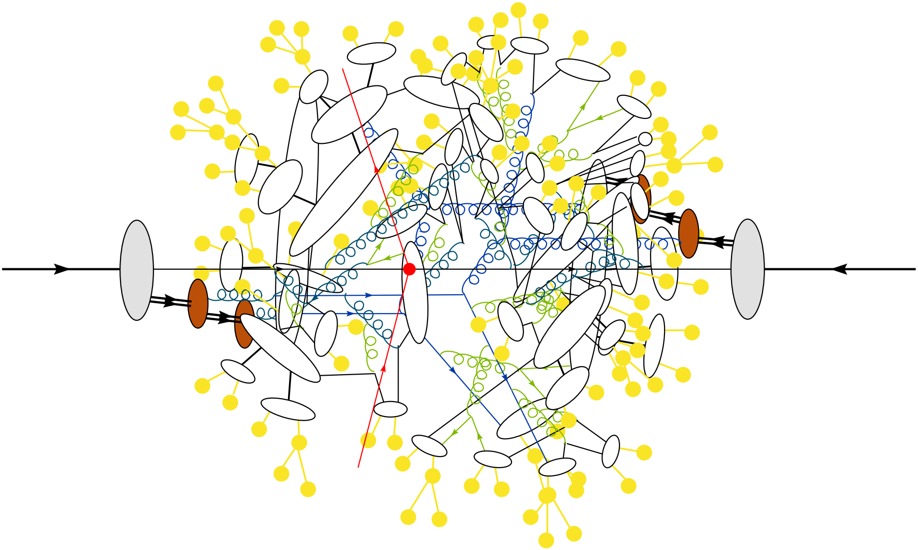


Divide and conquer

Partonic cross section from Feynman diagrams

$$\mathrm{d}\sigma = \mathrm{d}\sigma_{\mathrm{hard}} \mathrm{d}P(\text{partons} \to \text{hadrons})$$

$$\begin{aligned}
\mathrm{d}P(\text{partons} \to \text{hadrons}) = {} & \mathrm{d}P(\text{resonance decays}) && [\Gamma > Q_0] \\
& \times \mathrm{d}P(\text{parton shower}) && [\text{TeV} \to Q_0] \\
& \times \mathrm{d}P(\text{hadronisation}) && [\sim Q_0] \\
& \times \mathrm{d}P(\text{hadronic decays}) && [O(\text{MeV})]
\end{aligned}$$

Underlying event from multiple partonic interactions

$$\mathrm{d}\sigma \longleftarrow \mathrm{d}\sigma(\text{QCD } 2 \to 2)$$

Plan for these lectures

- Monte Carlo Methods
- Hard Scattering
- Parton Showers
- Hadronization and Hadronic Decays
- Underlying Event
- Multiple Parton Interactions (MPI) Modelling

Monte Carlo Methods

Monte Carlo Methods

Introduction to the most important MC sampling (= integration) techniques.

1. Hit and miss.
2. Simple MC integration.
3. (Some) methods of variance reduction.
4. Adaptive MC, VEGAS.
5. Multichannel.
6. Mini event generator in particle physics.

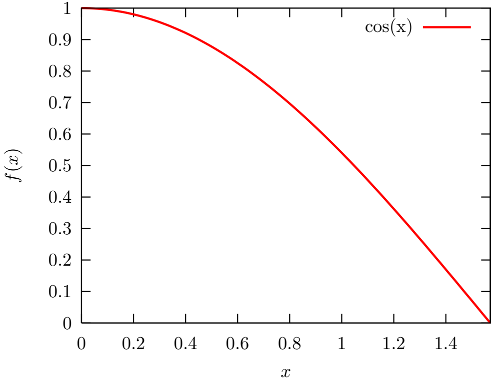
Probability

Example: $f(x) = \cos(x)$.

Probability density:

$$dP = f(x)\,dx$$

is probability to find value x.



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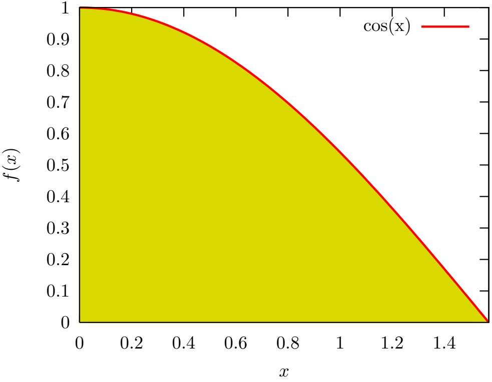

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$$F(x) = \int_{x_0}^{x} f(x)\,dx$$

is called *probability distribution*.

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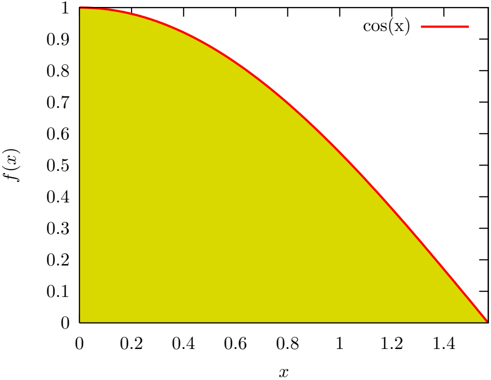

Probability ∼ Area

Hit and Miss

Hit and miss method:

- throw N random points (x, y) into region.
- Count hits N_{hit}, i.e. whenever $y < f(x)$.

Then

$$I \approx V \frac{N_{\text{hit}}}{N}.$$

approaches 1 again in our example.

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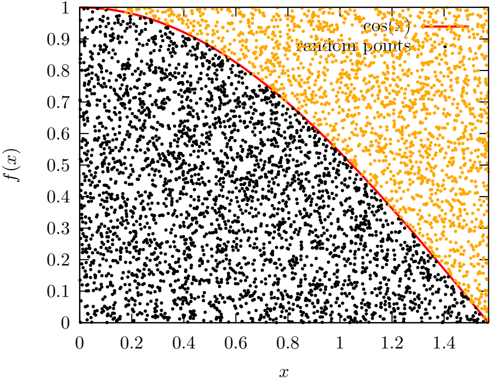

Hit and Miss

Example: $f(x) = \cos(x)$.

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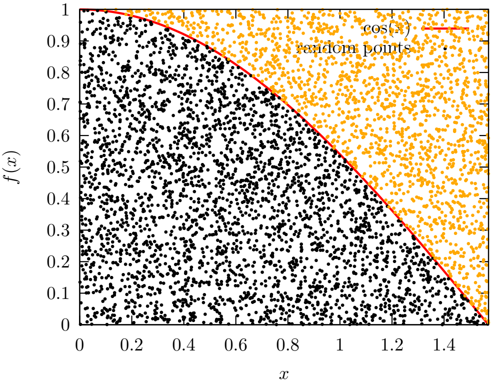
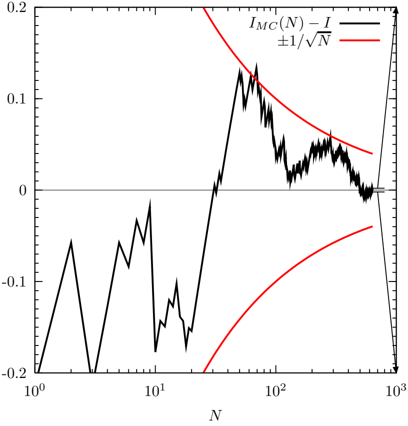


Every accepted value of x can be considered an event in this picture. As $f(x)$ is the 'histogram' of x, it seems obvious that the x values are distributed as $f(x)$ from this picture.

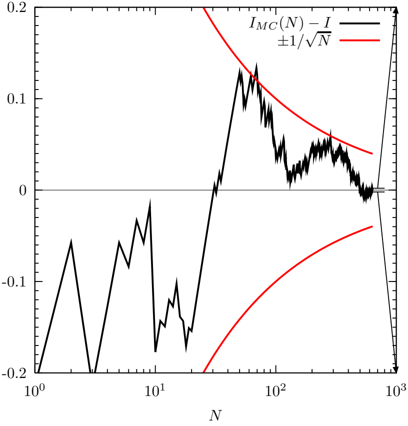
Hit and Miss



How well does it converge?

Error $1/\sqrt{N}$.

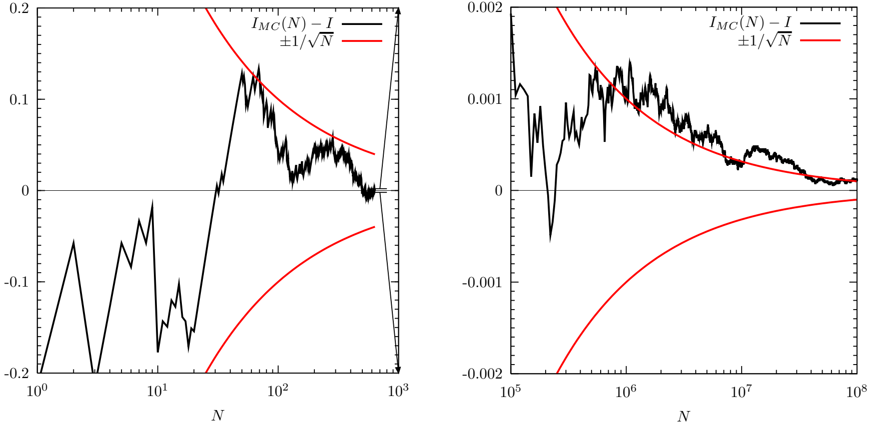
Hit and Miss



More points, zoom in…

Error $1/\sqrt{N}$.

Hit and Miss



Error $1/\sqrt{N}$.

Hit and Miss

This method is used in many event generators. However, it is not sufficient as such.

- Can handle any density $f(x)$,
  however wild and unknown it is.
- $f(x)$ should be bounded from above.
- Sampling will be very *inefficient* whenever $\mathrm{Var}(f)$ is large.

Improvements go under the name variance reduction as they improve the error of the crude MC at the same time.

Simple MC integration

Mean value theorem of integration:

$$
\begin{aligned}
I &= \int_{x_0}^{x_1} f(x)\,dx \\
&= (x_1 - x_0)\langle f(x)\rangle
\end{aligned}
$$

(Riemann integral).

Simple MC integration

Mean value theorem of integration:

$$\begin{aligned} I &= \int_{x_0}^{x_1} f(x)\,dx \\ &= (x_1 - x_0)\langle f(x)\rangle \\ &\approx (x_1 - x_0)\frac{1}{N}\sum_{i=1}^{N} f(x_i) \end{aligned}$$

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Sum doesn't depend on ordering
$\longrightarrow$ randomize x_i.

Simple MC integration

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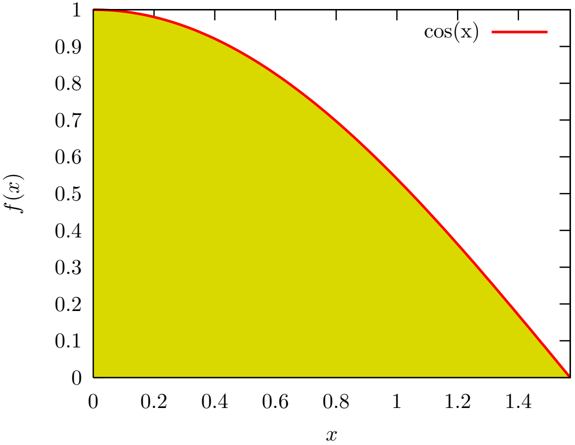
Sum doesn't depend on ordering
$\longrightarrow$ randomize x_i.

Yields a flat distribution of events x_i,
but weighted with *weight* $f(x_i)$ ($\rightarrow$ unweighting).

Simple MC integration

Pictorially:

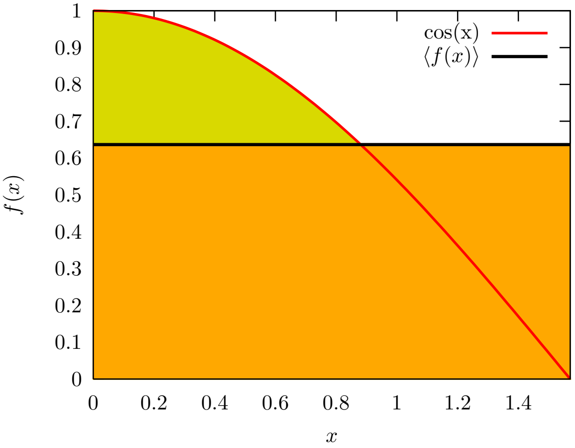
$$I = \int_{x_0}^{x_1} f(x)\,dx$$
$$= (x_1 - x_0)\langle f(x)\rangle$$



Simple MC integration

Pictorially:

$$I = \int_{x_0}^{x_1} f(x)\,dx$$
$$= (x_1 - x_0)\langle f(x) \rangle$$

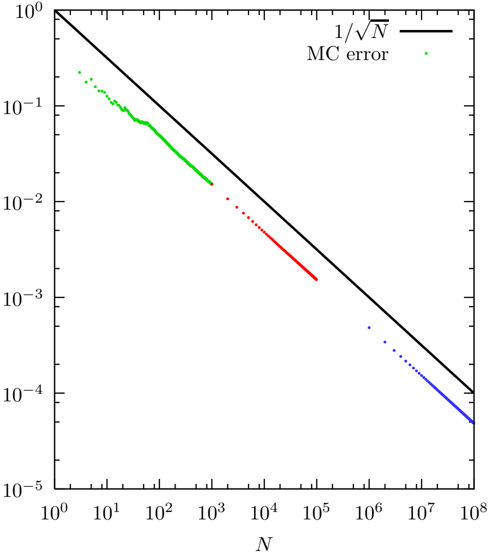


Simple MC integration

What's the error?

Again, looks like

$$\sigma \sim \frac{1}{\sqrt{N}}$$



Simple MC integration

What's the error?

We can calculate it (central limit theorem for the average):

In general: *Crude MC*

$$
\begin{aligned}
I &= \int f dV \\
&\approx V\langle f\rangle \pm V\sqrt{\frac{\langle f\rangle^2 - \langle f^2\rangle}{N}} \\
&\approx V\langle f\rangle \pm V\frac{\sigma}{\sqrt{N}}
\end{aligned}
$$

Simple MC integration

What's the error?

We can calculate it (central limit theorem for the average):

Our example: $\cos(x), 0 \leq x \leq \pi/2$,
compute σ_{MC} from

$$\langle f \rangle = \frac{1}{N} \sum_{i=1}^{N} f(x_i)$$
$$\langle f^2 \rangle = \frac{1}{N} \sum_{i=1}^{N} f^2(x_i).$$

Simple MC integration

What's the error?

We can calculate it (central limit theorem for the average):

Compute σ directly ($V = \pi/2$):

$$\langle f \rangle = \int_0^{\pi/2} \cos(x)\,dx = 1$$
$$\langle f^2 \rangle = \int_0^{\pi/2} \cos^2(x)\,dx = \frac{\pi}{4}$$

then

$$\sigma = \sqrt{1^2 - \frac{\pi}{4}} \approx 0.4633.$$

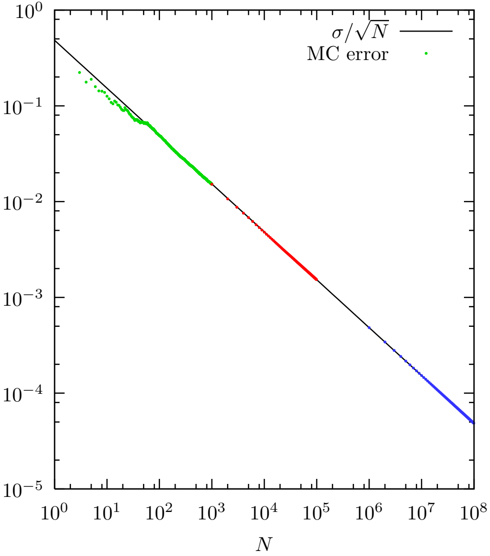
Simple MC integration

What's the error?

Now, compare

$$\sigma_{MC} = \frac{0.4633}{\sqrt{N}}$$

with error estimate from MC.



Simple MC integration

What's the error?

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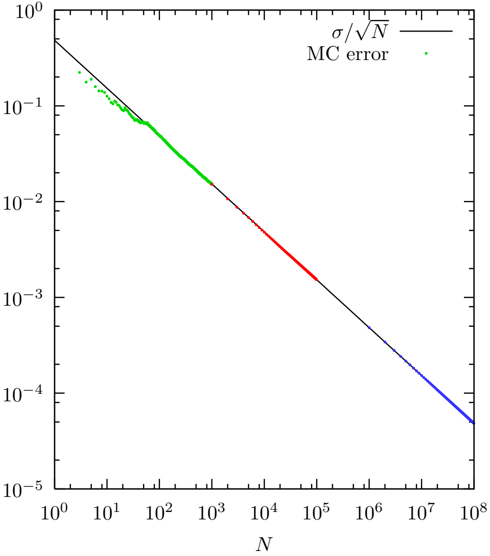
Spot on.



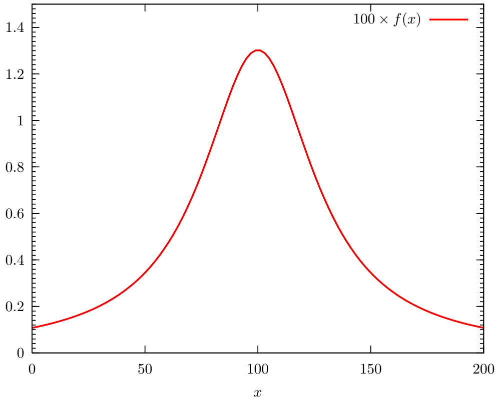
Inverting the Integral

Another basic MC method, based on the observation that

Probability $\sim$ *Area*

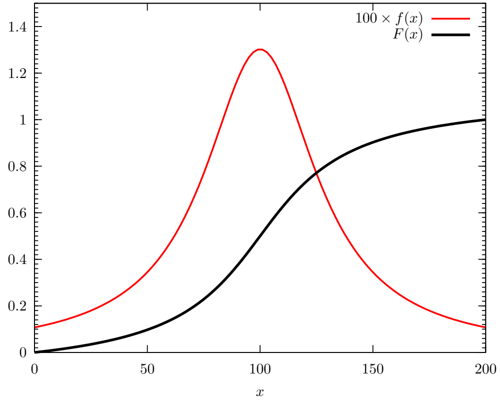
Inverting the Integral

- Probability density $f(x)$. Not necessarily normalized.



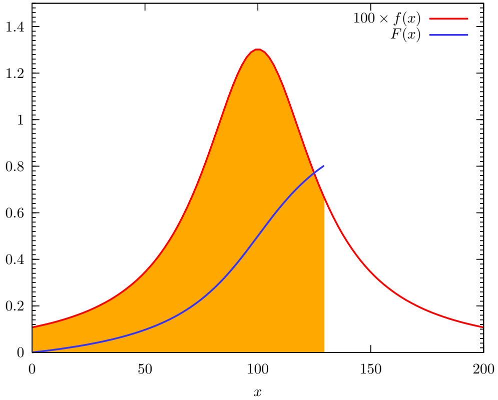
Inverting the Integral

- Probability density $f(x)$. Not necessarily normalized.
- Integral $F(x)$ known,



Inverting the Integral

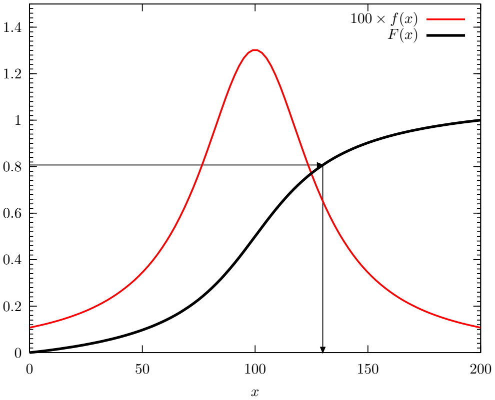
- Probability density $f(x)$. Not necessarily normalized.
- Integral $F(x)$ known,
- $P(x < x_s) = F(x_s)$.



Inverting the Integral

- Probability density $f(x)$. Not necessarily normalized.
- Integral $F(x)$ known,
- $P(x < x_s) = F(x_s)$.
- Probability = 'area', distributed evenly,

$$\int_{x_0}^{x} dP = r$$



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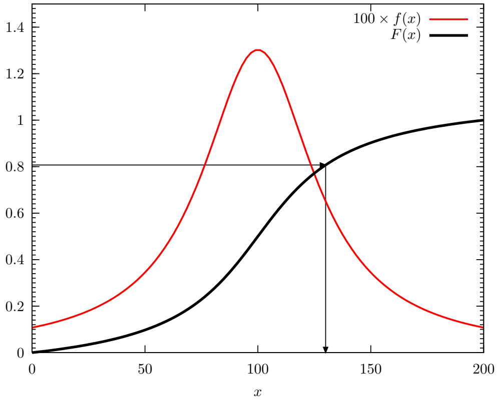

Sample x according to $f(x)$ with

$$x = F^{-1}\left[F(x_0) + r\big(F(x_1) - F(x_0)\big)\right] .$$

Inverting the Integral

Another basic MC method, based on the observation that

Probability $\sim$ *Area*

Sample x according to $f(x)$ with

$$x = F^{-1}\left[F(x_0) + r\big(F(x_1) - F(x_0)\big)\right] .$$

Optimal method, but we need to know

- The integral $F(x) = \int f(x)\,\mathrm{d}x$,
- It's inverse $F^{-1}(y)$.

That's rarely the case for real problems.

But very powerful in combination with other techniques.

Importance sampling

Error on Crude MC $\sigma_{MC} = \sigma/\sqrt{N}$.
$\Longrightarrow$ Reduce error by reducing variance of integrand.

Importance sampling

Error on Crude MC $\sigma_{MC} = \sigma/\sqrt{N}$.
$\Longrightarrow$ Reduce error by reducing variance of integrand.

Idea: *Divide out the singular structure.*

$$I = \int f \, \mathrm{d}V = \int \frac{f}{p} p \mathrm{d}V \approx \left\langle \frac{f}{p} \right\rangle \pm \sqrt{\frac{\langle f^2/p^2 \rangle - \langle f/p \rangle^2}{N}} \; .$$

where we have chosen $\int p \, \mathrm{d}V = 1$ for convenience.

Note: need to sample flat in $p \, \mathrm{d}V$, so we better know $\int p \mathrm{d}V$ and it's inverse.

Importance sampling

Consider error term:

$$E = \left\langle \frac{f^2}{p^2} \right\rangle - \left\langle \frac{f}{p} \right\rangle^2 = \int \frac{f^2}{p^2} p \mathrm{d}V - \left[\int \frac{f}{p} p \mathrm{d}V \right]^2$$

$$= \int \frac{f^2}{p} \mathrm{d}V - \left[\int f \, \mathrm{d}V \right]^2 .$$

Importance sampling

Consider error term:

$$E = \int \frac{f^2}{p} \, \mathrm{d}V - \left[\int f \, \mathrm{d}V \right]^2 \; .$$

Best choice of p? Minimises $E \to$ functional variation of error term with (normalized) p:

$$0 = \delta E = \delta \left(\int \frac{f^2}{p} \, \mathrm{d}V - \left[\int f \, \mathrm{d}V \right]^2 + \lambda \int p \, \mathrm{d}V \right)$$

$$= \int \left(-\frac{f^2}{p^2} + \lambda \right) \mathrm{d}V \delta p \; ,$$

Importance sampling

Consider error term:

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Best choice of p? Minimises $E \rightarrow$ functional variation of error term with (normalized) p:

$$0 = \delta E = \int \left(-\frac{f^2}{p^2} + \lambda \right) \mathrm{d}V \delta p ,$$

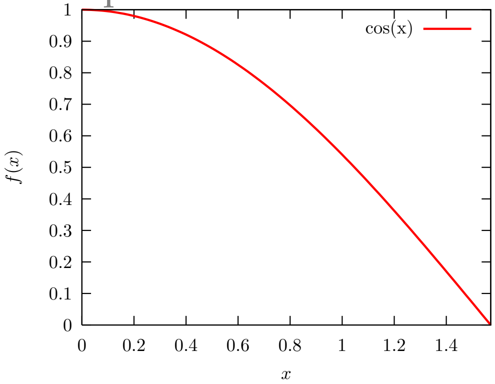
hence

$$p = \frac{|f|}{\sqrt{\lambda}} = \frac{|f|}{\int |f| \, \mathrm{d}V} .$$

Choose p as close to f as possible.

Importance sampling — example

Improving $\cos(x)$ sampling,



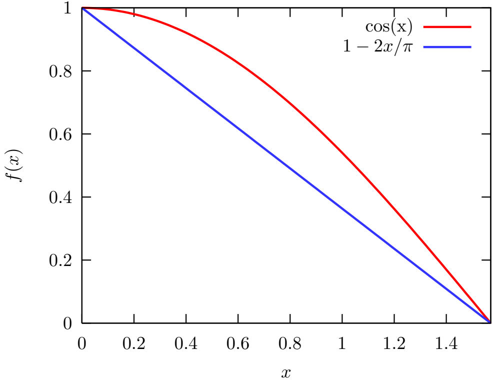
Importance sampling — example

Improving $\cos(x)$ sampling,

$$I = \int_0^{\pi/2} \cos(x)\, dx$$

$$= \int_0^{\pi/2} \frac{\cos(x)}{1 - \frac{2}{\pi}x} \left(1 - \frac{2}{\pi}x\right) dx$$

$$= \int_0^{1} \left. \frac{\cos(x)}{1 - \frac{2}{\pi}x} \right|_{x=x(\rho)} d\rho \ .$$



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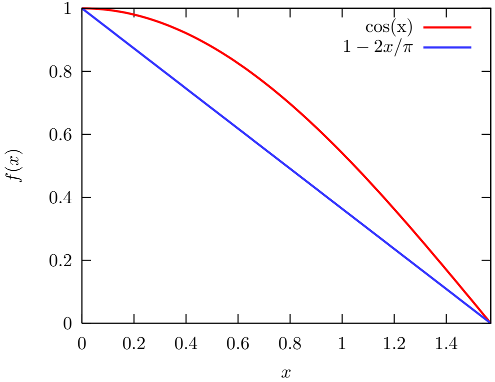
$$= \int_0^{\pi/2} \frac{\cos(x)}{1 - \frac{2}{\pi}x} \left(1 - \frac{2}{\pi}x\right) dx$$
$$= \int_0^1 \left. \frac{\cos(x)}{1 - \frac{2}{\pi}x} \right|_{x=x(\rho)} d\rho \; .$$



Sample x with *inverting the integral* technique (flat random number ρ),

$$x = \frac{\pi}{2}\left(1 - \sqrt{1-\rho}\right) \hat{=} \frac{\pi}{2}\left(1 - \sqrt{\rho}\right) \quad \left(I = \int_0^1 \frac{\cos\left(\frac{\pi}{2}\left(1-\sqrt{\rho}\right)\right)}{\sqrt{\rho}}\, d\rho . \right)$$

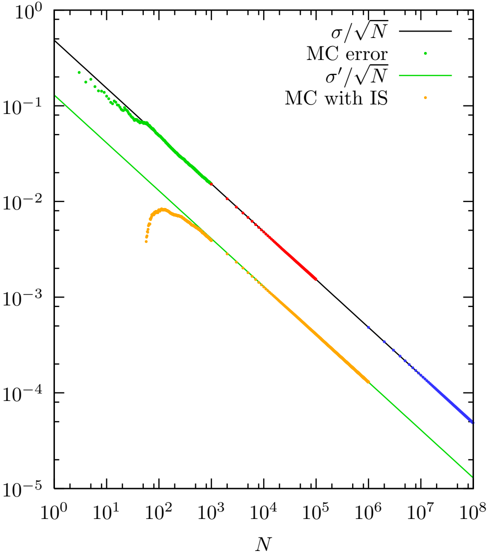
Importance sampling — example

Improving $\cos(x)$ sampling,

much better convergence,

about 80% “accepted events”.

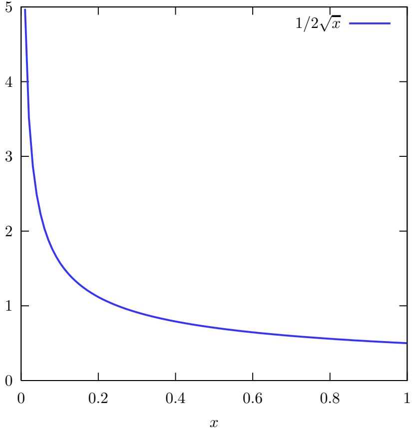
Reduced variance ($\sigma' = 0.027$)
$\Rightarrow$ better efficiency.



Importance sampling — better example

More interesting for divergent integrands, eg

$$\frac{1}{2\sqrt{x}} \, ,$$



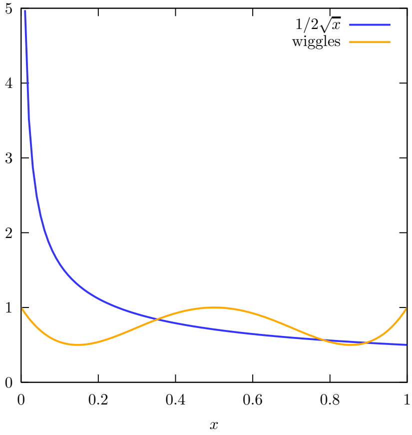
Importance sampling — better example

More interesting for divergent integrands, eg

$$\frac{1}{2\sqrt{x}}\ ,$$

with some wiggles,

$$p(x) = 1 - 8x + 40x^2 - 64x^3 + 32x^4\ .$$



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with some wiggles,

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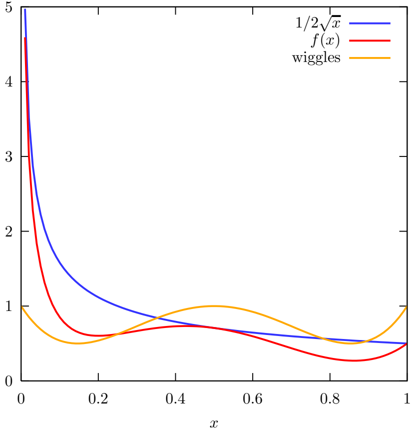
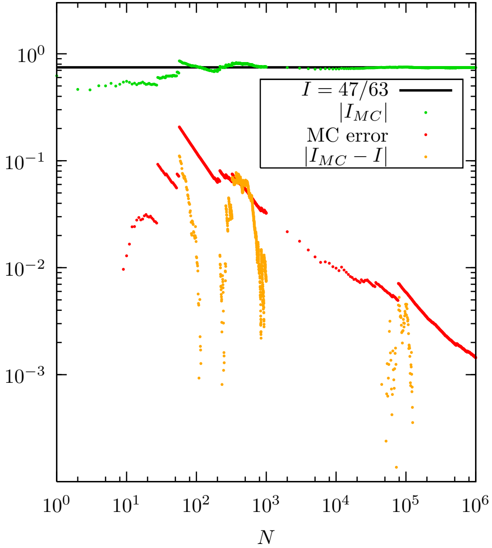
i.e. we want to integrate

$$f(x) = \frac{p(x)}{2\sqrt{x}}\ .$$



Importance sampling — better example

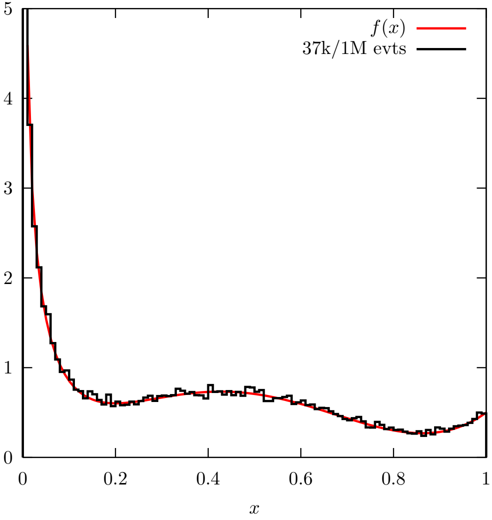
- Crude MC gives result in reasonable 'time'.
- Error a bit unstable.
- Event generation with maximum weight $w_{\max} = 20$. (that's arbitrary.)
- hit/miss/events with $(w > w_{\max}) =$ 36566/963434/617 with 1M generated events.



Importance sampling — better example

Want events:
use hit+mass variant here:

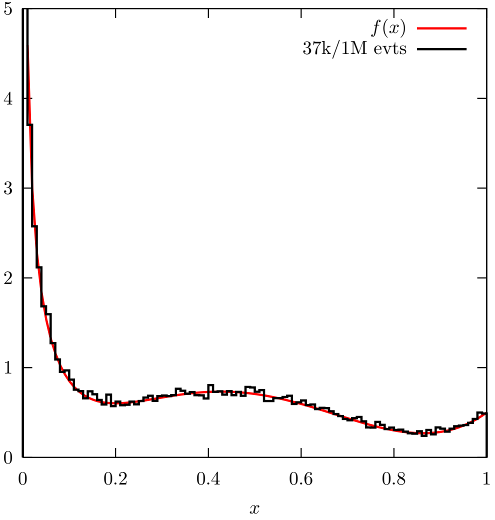
- Choose new random number r
- $w = f(x)$ in this case.
- if $r < w/w_{\max}$ then "hit".
- MC efficiency = hit$/N$.



Importance sampling — better example

Want events:
use hit+mass variant here:

- Choose new random number r
- $w = f(x)$ in this case.
- if $r < w/w_{\max}$ then "hit".
- MC efficiency = hit$/N$.
- Efficiency for MC events only 3.7%.
- Note the wiggly histogram.



Importance sampling — better example

Now importance sampling, i.e. divide out $1/2\sqrt{x}$.

$$\int_0^1 \frac{p(x)}{2\sqrt{x}}\,dx = \int_0^1 \left(\frac{p(x)}{2\sqrt{x}} \Big/ \frac{1}{2\sqrt{x}} \right) \frac{dx}{2\sqrt{x}}$$
$$= \int_0^1 p(x)\,d\sqrt{x}$$
$$= \int_0^1 p(x(\rho))\,d\rho$$
$$= \int_0^1 1 - 8\rho^2 + 40\rho^4 - 64\rho^6 + 32\rho^8\,d\rho$$

so,

$$\rho = \sqrt{x}, \qquad d\rho = \frac{dx}{2\sqrt{x}}$$

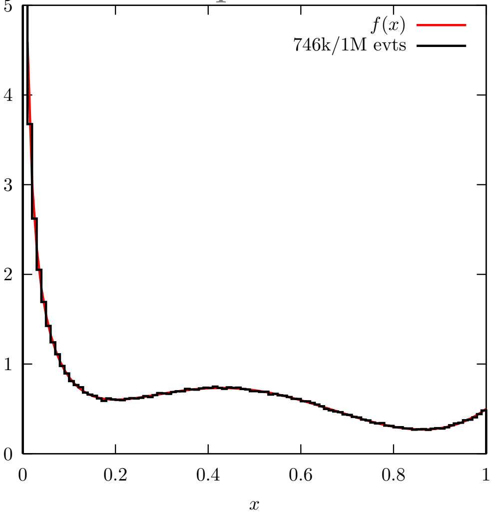
x sampled with *inverting the integral* from flat random numbers ρ, $x = \rho^2$.

Importance sampling — better example

$$\int_0^1 \frac{p(x)}{2\sqrt{x}}\,dx = \int_0^1 p(x(\rho))\,d\rho$$

with

$$\rho = \sqrt{x}, \qquad d\rho = \frac{dx}{2\sqrt{x}}$$



Events generated with $w_{\text{max}} = 1$, as $p(x) \leq 1$, no guesswork needed here! Now, we get 74.6% MC efficiency.

Importance sampling — better example

$$\int_0^1 \frac{p(x)}{2\sqrt{x}}\,dx = \int_0^1 p(x(\rho))\,d\rho$$

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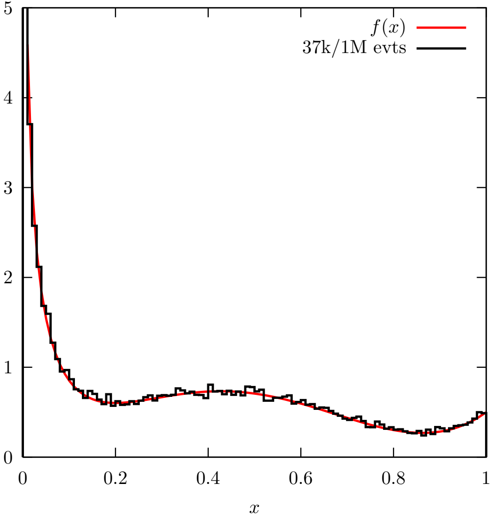
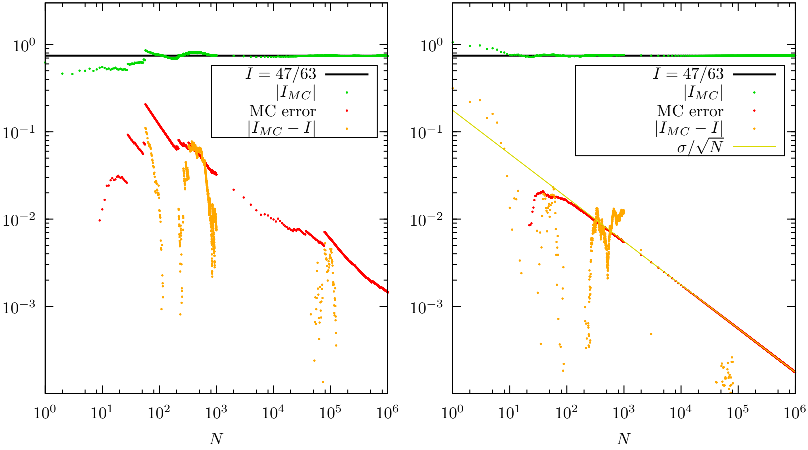


Events generated with $w_{\max} = 1$, as $p(x) \leq 1$, no guesswork needed here! Now, we get 74.6% MC efficiency.

…as opposed to 3.7%.

Importance sampling — better example

Crude MC vs Importance sampling.



100× more events needed to reach same accuracy.

Importance sampling — another useful example

Breit–Wigner peaks appear in many realistic MEs for cross sections and decays.

$$I = \int_{s_0}^{s_1} \frac{ds}{(s-m^2)^2 + m^2\Gamma^2}$$

Importance sampling — another useful example

Breit–Wigner peaks appear in many realistic MEs for cross sections and decays.

$$I = \int_{s_0}^{s_1} \frac{ds}{(s-m^2)^2 + m^2\Gamma^2} = \frac{1}{m\Gamma} \int_{y_0}^{y_1} \frac{dy}{y^2+1} \qquad \left(y = \frac{s-m^2}{m\Gamma}\right)$$

$$= \frac{1}{m\Gamma} \arctan \frac{s-m^2}{m\Gamma} \bigg|_{s_0}^{s_1}$$

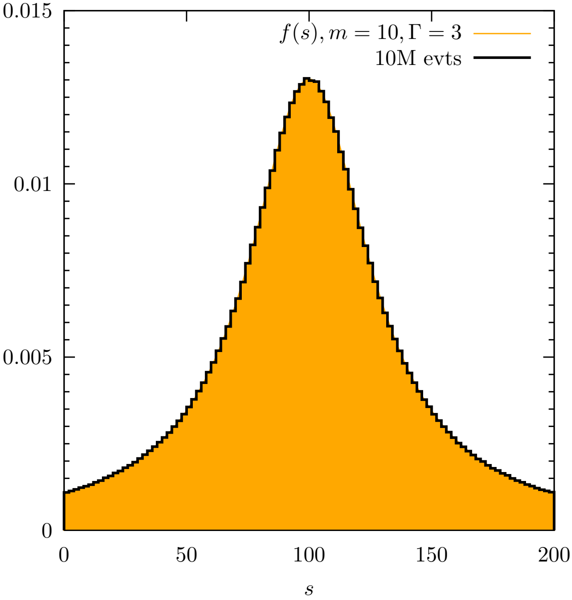
Inverting the integral gives ("tan mapping").

$$f(s) = \frac{m\Gamma}{(s-m^2)^2 + m^2\Gamma^2} \ ,$$

$$F(s) = \arctan \frac{s-m^2}{m\Gamma} = \rho \ ,$$

$$F^{-1}(\rho) = m^2 + m\Gamma \tan \rho \ .$$

Importance sampling — another useful example



VEGAS

- Classic algorithm.
- Automatic impotance sampling.
- Adopt grid size.
- Often used for multidimensional integration.
- Very robust.

VEGAS

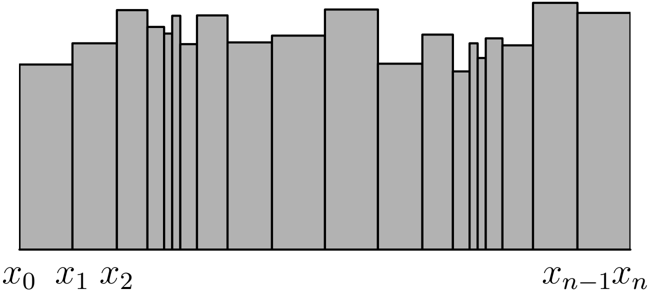
- start with equidistant grid $x_0, x_1, \ldots, x_N$.
- Sample a number of points $\big(x_{s,i}, f(x_{s,i})\big)$, compute first estimate of integral as $\langle f \rangle$.
- Resize grid:
  choose x_i' such that contribution from partial areas inside $x_i < x < x_{i+1}$ to integral is $\langle f \rangle / N$.
- Remember, optimal $p(x) \sim |f(x)|$.
- Sample again with same number of points into every bin $x_i < x < x_{i+1}$. Results in step weight function with steps

$$p_i = \frac{1}{N(x_i - x_{i-1})} , \qquad x_i < x < x_{i+1} .$$

- $\Rightarrow$ Sample often where density is high.

VEGAS

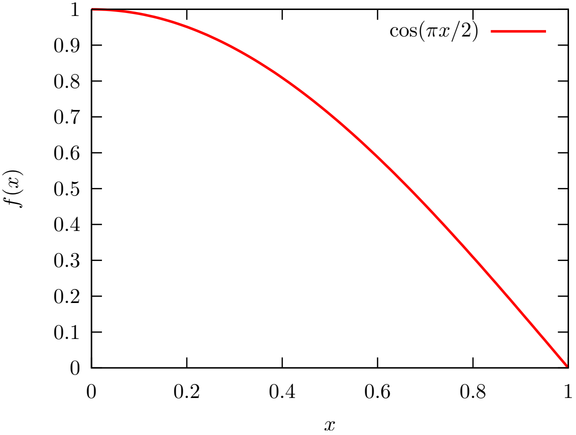
Rebinning:



[from T. Ohl, VAMP]

VEGAS

Example: $\cos(\frac{\pi x}{2})$
$N_{\text{grid}} = 20, 100$
Convergence improved.



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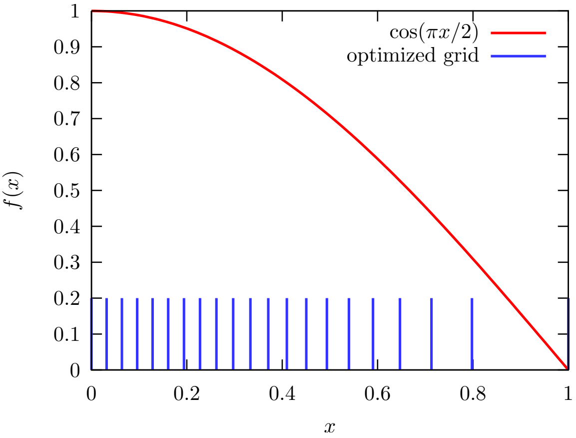

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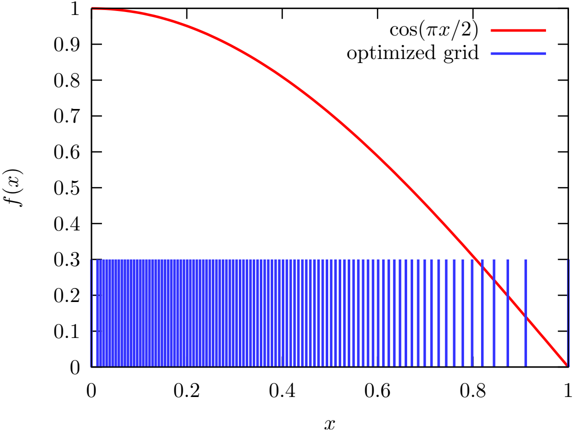

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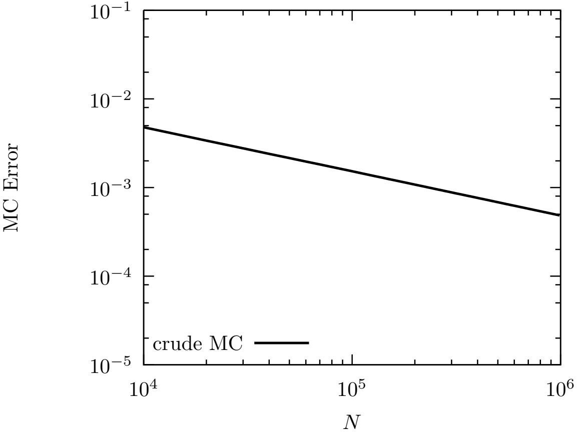

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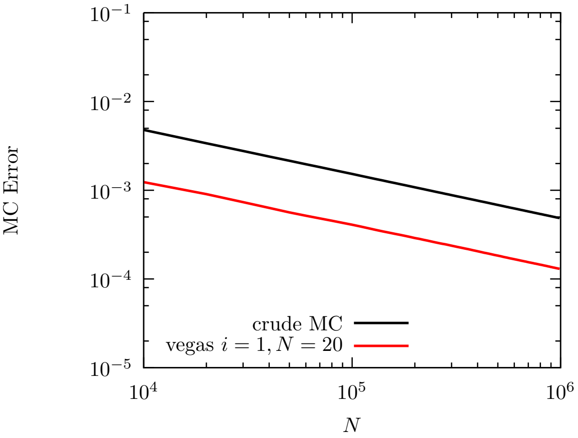

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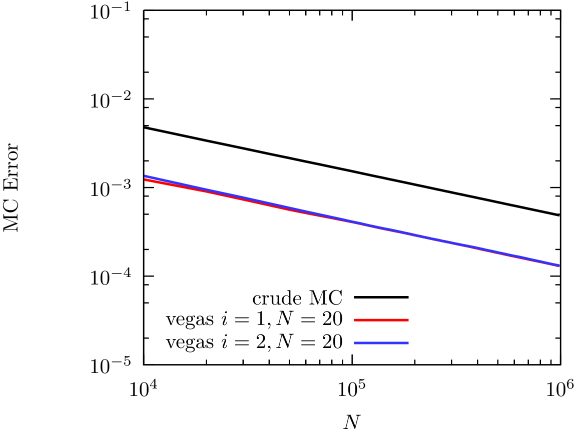

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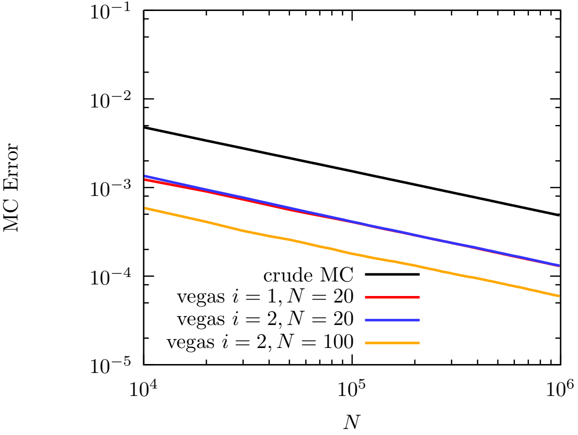

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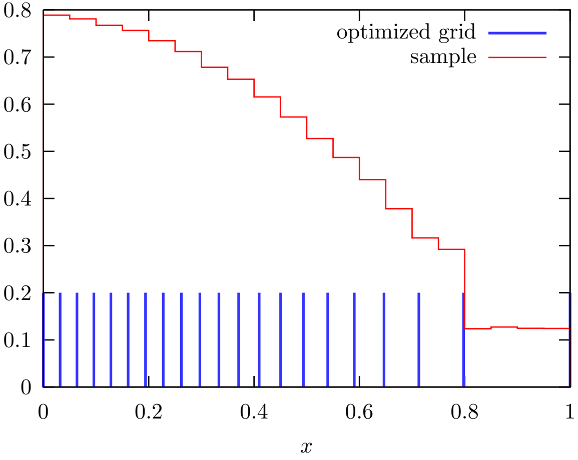

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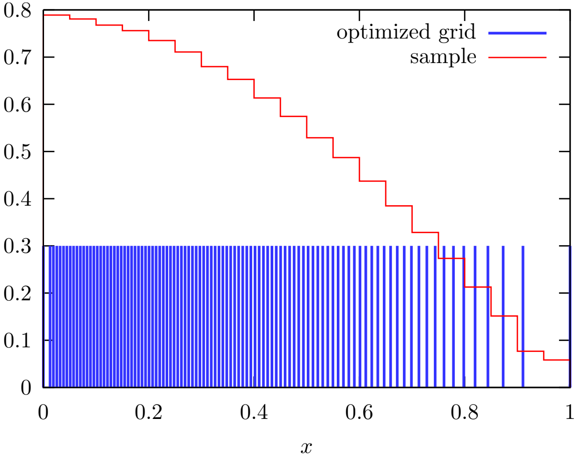
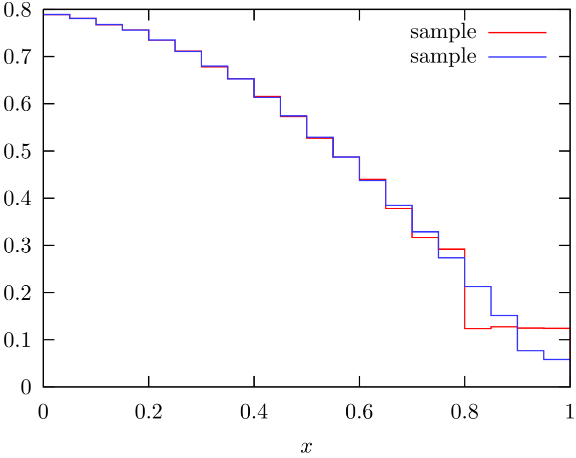


VEGAS



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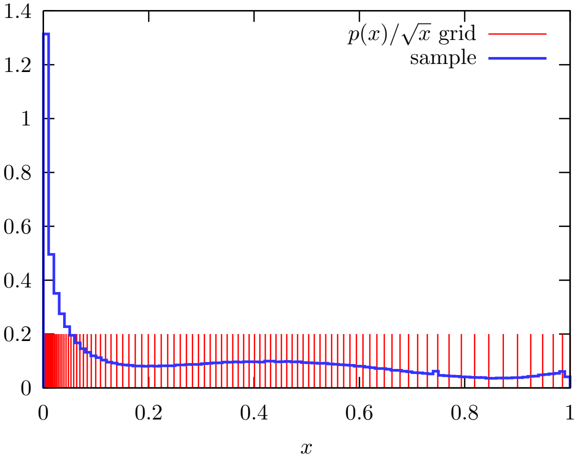
VEGAS

Second example:
$p(x)/\sqrt{x}$
(divergence with wiggles)



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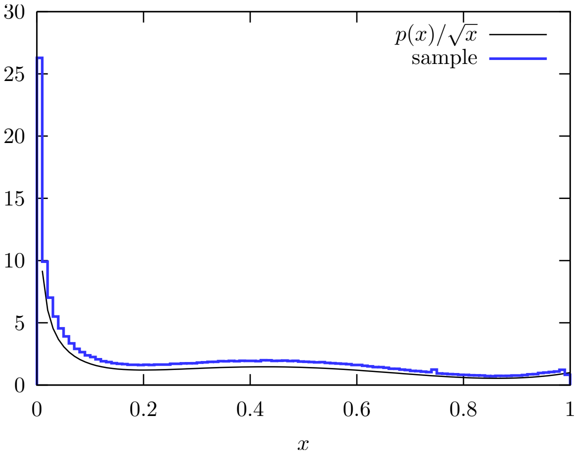

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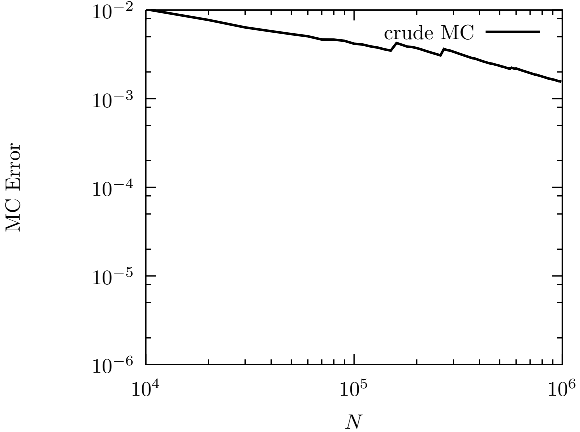

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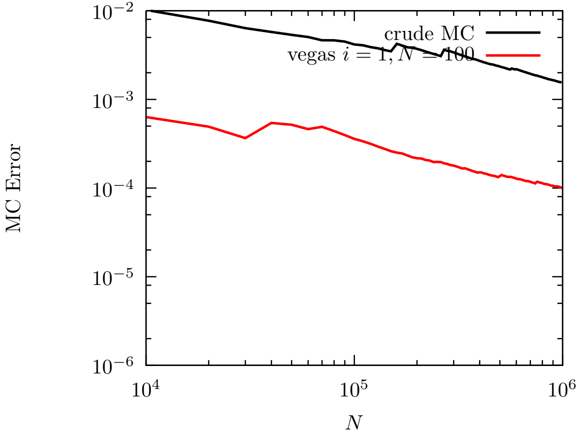

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Acc 10^{-4} after $N = 10^6$ comparable with 'inverting the integral'.

VEGAS

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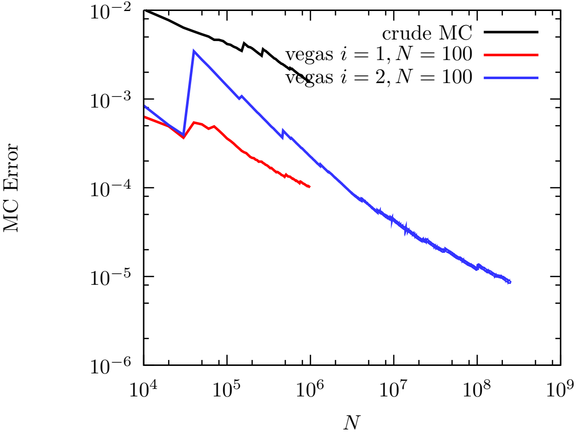
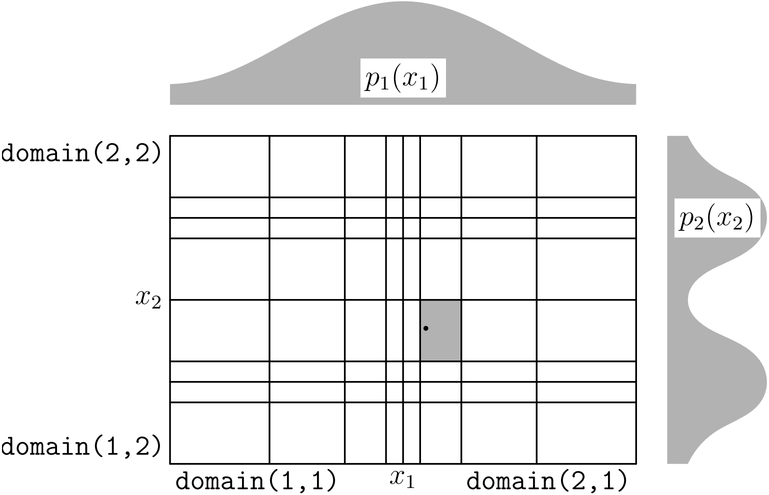


VEGAS

Problem to adapt in multiple dimensions:

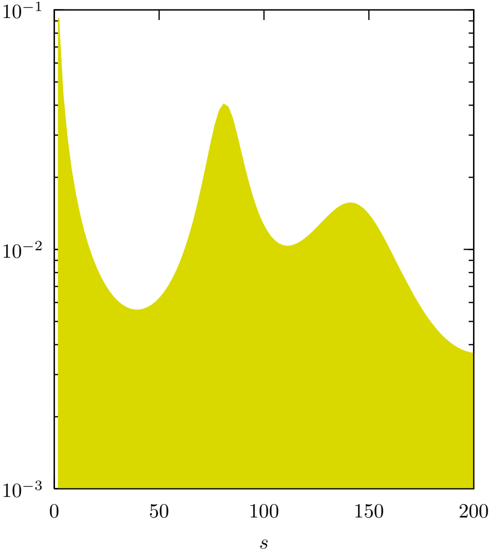


[from T. Ohl, VAMP]

Multichannel MC

Typical problem:

- $f(s)$ has multiple peaks ($\times$ wiggles from ME).



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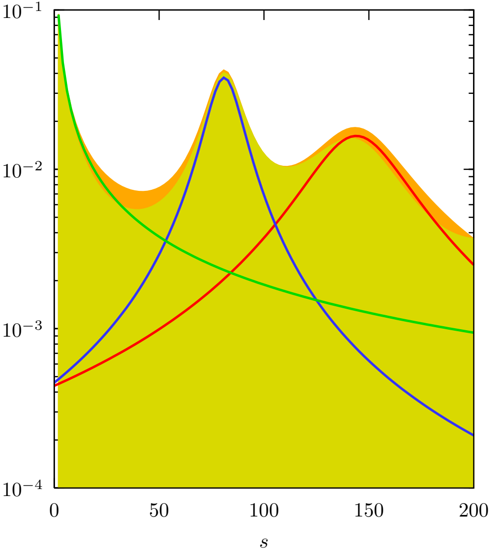
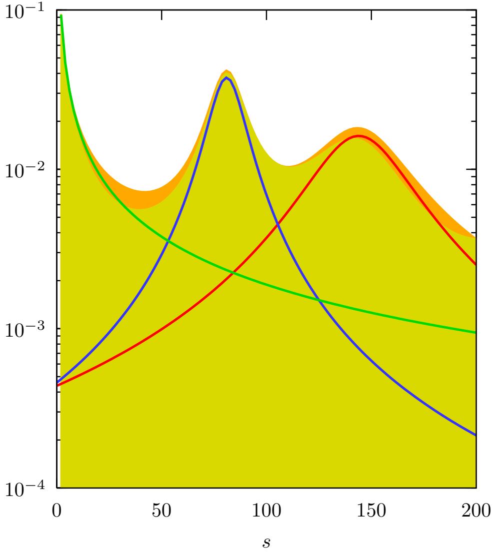


Multichannel MC

Typical problem:

- $f(s)$ has multiple peaks ($\times$ wiggles from ME).
- Usually have some idea of the peak structure.
- Encode this in sum of sample functions $g_i(s)$ with weights $\alpha_i, \sum_i \alpha_i = 1$.

$$g(s) = \sum_i \alpha_i g_i(s) \ .$$



Multichannel MC

Now rewrite

$$\int_{s_0}^{s_1} f(s)\,ds = \int_{s_0}^{s_1} \frac{f(s)}{g(s)} g(s)\,ds$$
$$= \int_{s_0}^{s_1} \frac{f(s)}{g(s)} \sum_i \alpha_i g_i(s)\,ds$$
$$= \sum_i \alpha_i \int_{s_0}^{s_1} \frac{f(s)}{g(s)} g_i(s)\,ds$$

Now $g_i(s)\,ds = d\rho_i$ (inverting the integral).

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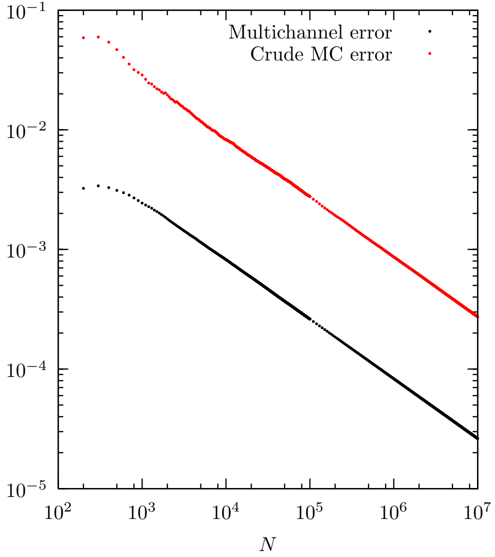
Now $g_i(s)\,ds = d\rho_i$ (inverting the integral).

Select the distribution $g_i(s)$ you'd like to sample next event from acc to weights α_i.

α_i can be optimized after a number of trials.

Multichannel MC

Works quite well:



Hard Scattering

Hard scattering

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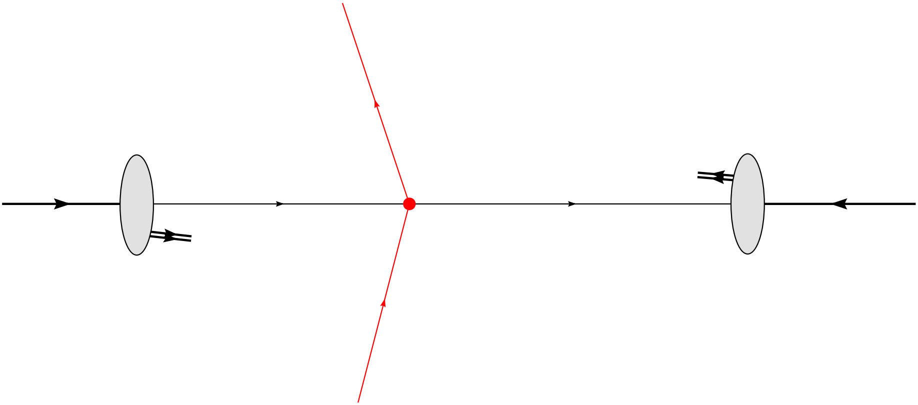
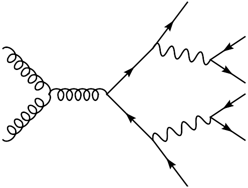


Matrix elements

- Perturbation theory/Feynman diagrams give us (fairly accurate) final states for a few number of legs ($\mathcal{O}(1)$).



- OK for very inclusive observables.

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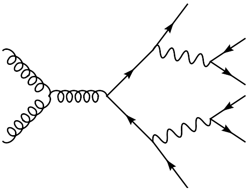

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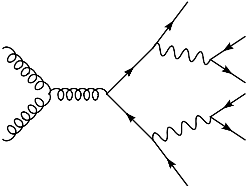

- OK for very inclusive observables.
- Starting point for further simulation.
- Want exclusive final state at the LHC ($O(100)$).
- Want arbitrary cuts.
- $\rightarrow$ use Monte Carlo methods.

Matrix elements

Where do we get (LO) $|M|^2$ from?

- Most/important simple processes (SM and BSM) are 'built in'.
- Calculate yourself (≤ 3 particles in final state).
- Matrix element generators:
  - MadGraph/MadEvent.
  - Comix/AMEGIC (part of Sherpa).
  - HELAC/PHEGAS.
  - Whizard.
  - CalcHEP/CompHEP.

  generate code or event files that can be further processed.
- $\rightarrow$ FeynRules interface to ME generators.

Also NLO mostly automatically available.
See "Matching and Merging".

Cross section formula

From Matrix element, we calculate

$$\sigma = \int f_i(x_1, \mu^2) f_j(x_2, \mu^2) \frac{1}{F} \overline{\sum} |M|^2 \qquad\qquad dx_1 dx_2 d\Phi_n \ ,$$

Cross section formula

From Matrix element, we calculate

$$\sigma = \int f_i(x_1, \mu^2) f_j(x_2, \mu^2) \frac{1}{F} \overline{\sum} |M|^2 \, \Theta(\text{cuts}) \, \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}\Phi_n \ ,$$

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now,

$$\frac{1}{F} \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}\Phi_n = J(\vec{x}) \prod_{i=1}^{3n-2} \mathrm{d}x_i \qquad \left(\mathrm{d}\Phi_n = (2\pi)^4 \delta^{(4)}(\ldots) \prod_{i=1}^{n} \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3 2E_i} \right)$$

such that

$$\sigma = \int g(\vec{x}) \, \mathrm{d}^{3n-2}\vec{x} \ , \qquad \left(g(\vec{x}) = J(\vec{x}) f_i f_j \overline{\sum} |M|^2 \Theta(\text{cuts}) \right)$$

$$= \frac{1}{N} \sum_{i=1}^{N} \frac{g(\vec{x}_i)}{p(\vec{x}_i)} = \frac{1}{N} \sum_{i=1}^{N} w_i \ .$$

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We generate events $\vec{x}_i$ with weights w_i.

Mini event generator

- We generate pairs $(\vec{x}_i, w_i)$.

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Generate events with same frequency as in nature!

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where $w_{\max}$ has to be chosen sensibly.
$\rightarrow$ reweighting, when $\max(w_i) = \bar{w}_{\max} > w_{\max}$, as

$$P_i = \frac{w_i}{\bar{w}_{\max}} = \frac{w_i}{w_{\max}} \cdot \frac{w_{\max}}{\bar{w}_{\max}} \; ,$$

i.e. reject events with probability $(w_{\max}/\bar{w}_{\max})$ afterwards.

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Some comments:

- Use common Monte Carlo techniques to generate events efficiently. Goal: small variance in w_i distribution!

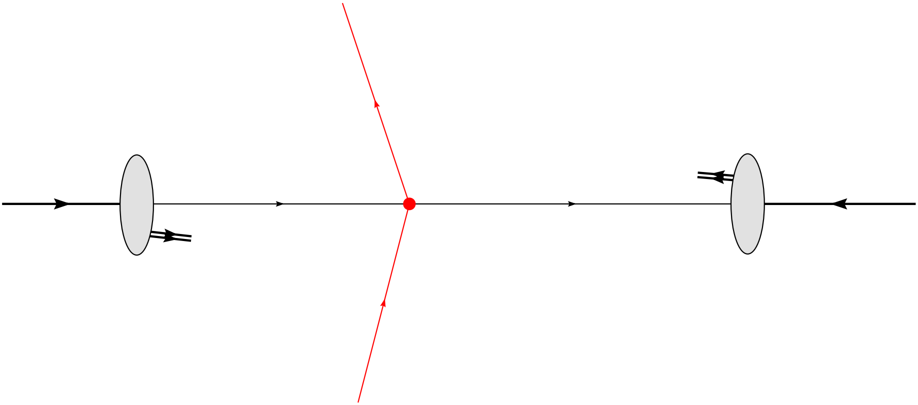
Matrix elements

Some comments:

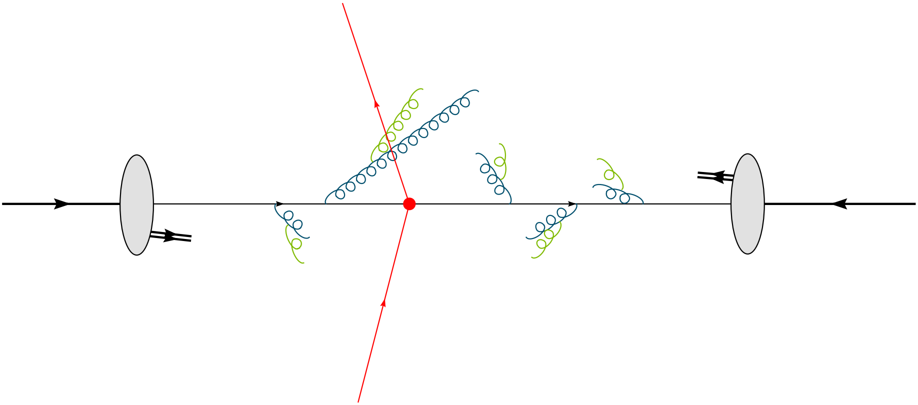
- Use common Monte Carlo techniques to generate events efficiently. Goal: small variance in w_i distribution!
- Efficient generation closely tied to knowledge of $f(\vec{x}_i)$, *i.e.* the matrix element's propagator structure.
  $\rightarrow$ build phase space generator already while generating ME's automatically.

Parton Showers

Hard matrix element



Hard matrix element → parton showers



Parton showers

Quarks and gluons in final state, pointlike.

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- Know short distance (short time) fluctuations from matrix element/Feynman diagrams: $Q \sim$ few GeV to O(TeV).

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Dominated by large logs, terms

$$\alpha_S^n \log^{2n} \frac{Q}{Q_0} \sim 1 \; .$$

Generated from emissions *ordered* in Q.

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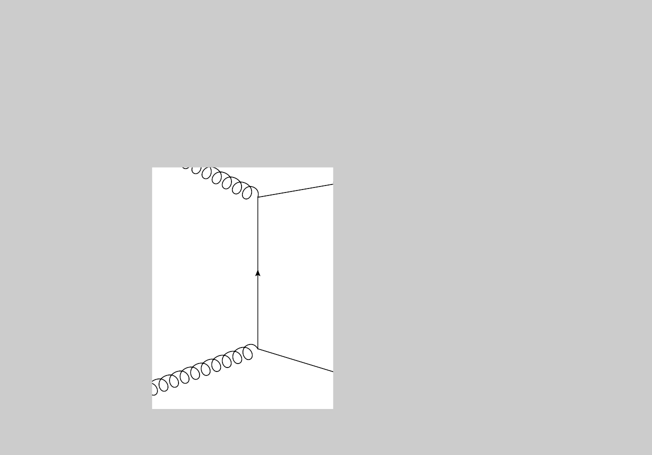
Dominated by large logs, terms

$$\alpha_S^n \log^{2n} \frac{Q}{Q_0} \sim 1 \,.$$

Generated from emissions *ordered* in Q.
Soft and/or collinear emissions.

ME approximated by parton cascade

Evolution in scale, typically $Q \sim 1\,\text{TeV}$ down to $Q \sim 1\,\text{GeV}$.



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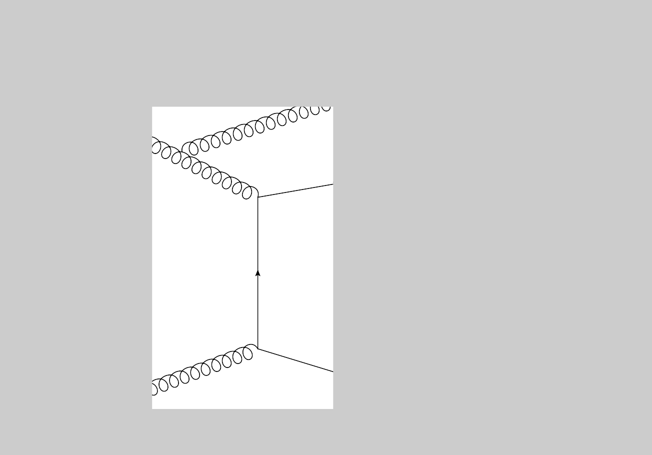

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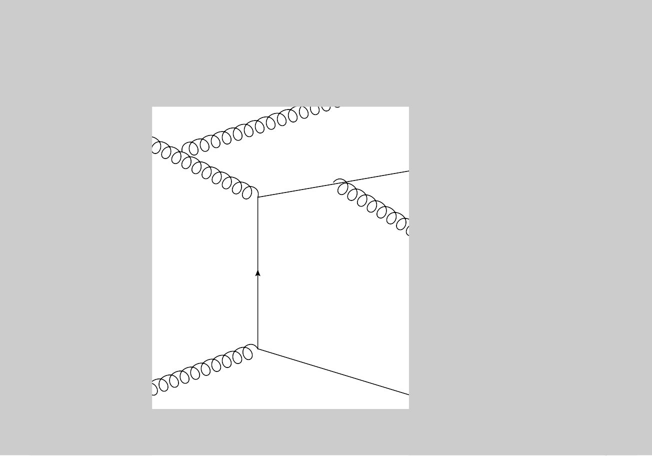

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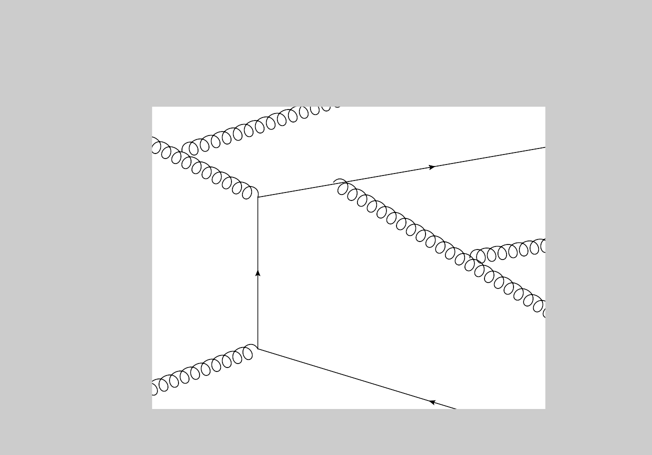

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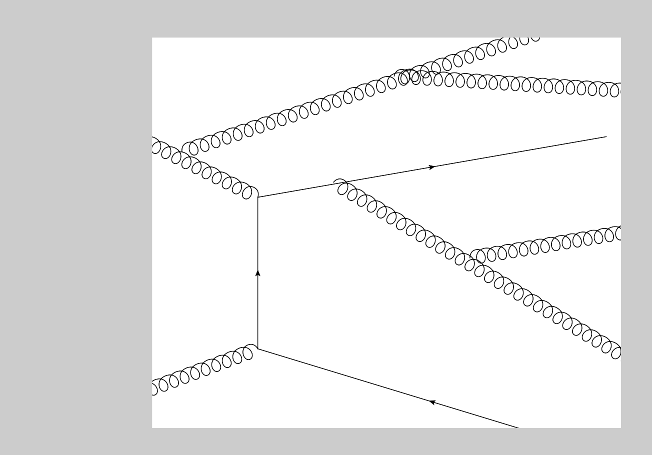


e^+e^- annihilation

Good starting point: $e^+e^- \to q\bar{q}g$:

Final state momenta in one plane (orientation usually averaged).
Write momenta in terms of

$$x_i = \frac{2p_i \cdot q}{Q^2} \quad (i = 1,2,3)\ ,$$

$$0 \le x_i \le 1\ , x_1 + x_2 + x_3 = 2\ ,$$

$$q = (Q,0,0,0)\ ,$$

$$Q \equiv E_{cm}\ .$$

$(x_1, x_2) = (x_q, x_{\bar{q}})$ –plane:

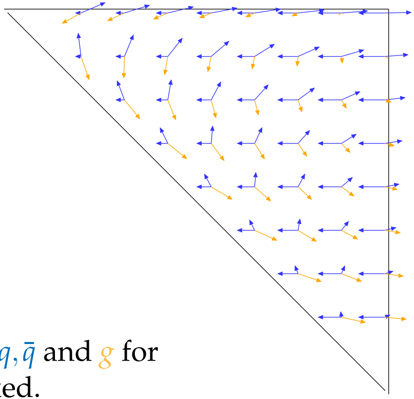


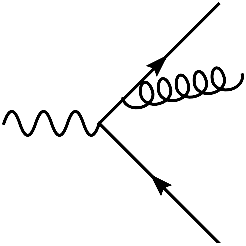
Fig: momentum configuration of $q, \bar{q}$ and g for given point (x_1, x_2), $\bar{q}$ direction fixed.

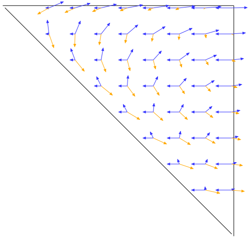
e^+e^- annihilation

Differential cross section:

$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 \frac{C_F \alpha_S}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

Collinear singularities: $x_1 \to 1$ or $x_2 \to 1$. Soft singularity: $x_1, x_2 \to 1$.





e^+e^- annihilation

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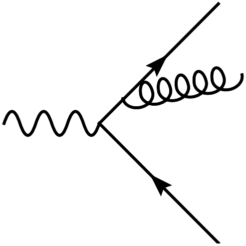
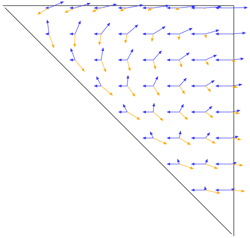


Collinear singularities: $x_1 \to 1$ or $x_2 \to 1$. Soft singularity: $x_1, x_2 \to 1$.

Rewrite in terms of x_3 and $\theta = \angle(q,g)$:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta \mathrm{d}x_3} = \sigma_0 \frac{C_F \alpha_S}{2\pi} \left[\frac{2}{\sin^2\theta} \frac{1+(1-x_3)^2}{x_3} - x_3 \right]$$



Singular as $\theta \to 0$ and $x_3 \to 0$.

e^+e^- annihilation

Can separate into two jets as

$$\frac{2\mathrm{d}\cos\theta}{\sin^2\theta} = \frac{\mathrm{d}\cos\theta}{1-\cos\theta} + \frac{\mathrm{d}\cos\theta}{1+\cos\theta}$$
$$= \frac{\mathrm{d}\cos\theta}{1-\cos\theta} + \frac{\mathrm{d}\cos\bar{\theta}}{1-\cos\bar{\theta}}$$
$$\approx \frac{\mathrm{d}\theta^2}{\theta^2} + \frac{\mathrm{d}\bar{\theta}^2}{\bar{\theta}^2}$$

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$$\approx \frac{\mathrm{d}\theta^2}{\theta^2} + \frac{\mathrm{d}\bar{\theta}^2}{\bar{\theta}^2}$$

So, we rewrite $\mathrm{d}\sigma$ in collinear limit as

$$\mathrm{d}\sigma = \sigma_0 \sum_{\text{jets}} \frac{\mathrm{d}\theta^2}{\theta^2} \frac{\alpha_S}{2\pi} C_F \frac{1+(1-z)^2}{z} \mathrm{d}z$$

e^+e^- annihilation

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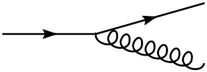
$$= \sigma_0 \sum_{\text{jets}} \frac{\mathrm{d}\theta^2}{\theta^2} \frac{\alpha_S}{2\pi} P(z) \mathrm{d}z$$

with DGLAP splitting function $P(z)$.

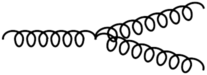
Collinear limit

Universal DGLAP splitting kernels for collinear limit:

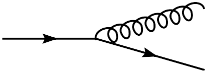
$$d\sigma = \sigma_0 \sum_{\text{jets}} \frac{d\theta^2}{\theta^2} \frac{\alpha_S}{2\pi} P(z) dz$$



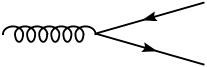
$$P_{q\to qg}(z) = C_F \frac{1+z^2}{1-z}$$



$$P_{g\to gg}(z) = C_A \frac{(1-z(1-z))^2}{z(1-z)}$$



$$P_{q\to gq}(z) = C_F \frac{1+(1-z)^2}{z}$$



$$P_{g\to qq}(z) = T_R(1-2z(1-z))$$

Collinear limit

Universal DGLAP splitting kernels for collinear limit:

$$\mathrm{d}\sigma = \sigma_0 \sum_{\text{jets}} \frac{\mathrm{d}\theta^2}{\theta^2} \frac{\alpha_S}{2\pi} P(z) \mathrm{d}z$$

Note: Other variables may equally well characterize the collinear limit:

$$\frac{\mathrm{d}\theta^2}{\theta^2} \sim \frac{\mathrm{d}Q^2}{Q^2} \sim \frac{\mathrm{d}p_\perp^2}{p_\perp^2} \sim \frac{\mathrm{d}\tilde{q}^2}{\tilde{q}^2} \sim \frac{\mathrm{d}t}{t}$$

whenever $Q^2, p_\perp^2, t \to 0$ means "collinear".

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Universal DGLAP splitting kernels for collinear limit:

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whenever $Q^2, p_\perp^2, t \to 0$ means "collinear".

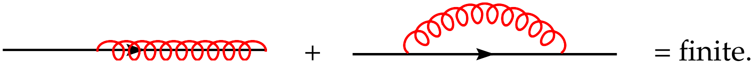
- θ: HERWIG
- Q^2: PYTHIA ≤ 6.3, SHERPA.
- $p_\perp$: PYTHIA ≥ 6.4, ARIADNE, Catani–Seymour showers.
- $\tilde{q}$: Herwig++.

Resolution

Need to introduce resolution t_0, e.g. a cutoff in $p_\perp$. Prevent us from the singularity at $\theta \to 0$.

Emissions below t_0 are unresolvable.

Finite result due to virtual corrections:



unresolvable + virtual emissions are included in Sudakov form factor via unitarity (see below!).

Towards multiple emissions

Starting point: factorisation in collinear limit, single emission.

$$\sigma_{2+1}(t_0) = \sigma_2(t_0) \int_{t_0}^{t} \frac{\mathrm{d}t'}{t'} \int_{z_-}^{z_+} \mathrm{d}z \, \frac{\alpha_S}{2\pi} \hat{P}(z) \; = \sigma_2(t_0) \int_{t_0}^{t} \mathrm{d}t \, W(t) \; .$$

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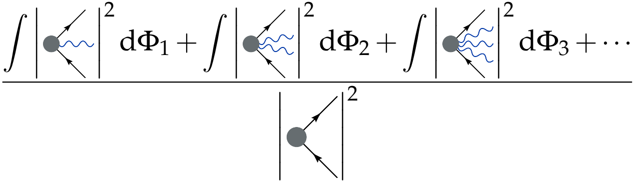
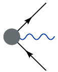
Simple example:
Multiple photon emissions, strongly ordered in t.
We want

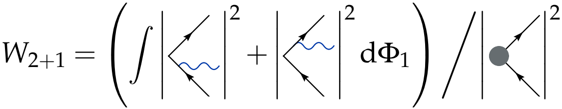
$$W_{\mathrm{sum}} = \sum_{n=1} W_{2+n} =$$



for any number of emissions.

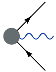
Towards multiple emissions

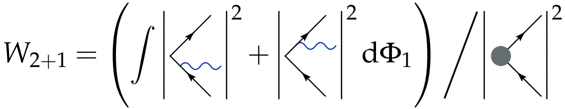
$(n = 1)$ 



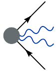
$$= \frac{2}{1!}\int_{t_0}^{t} \mathrm{d}t\, W(t)\ .$$

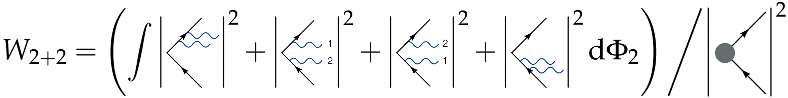
Towards multiple emissions

$(n=1)$ 



$$= \frac{2}{1!}\int_{t_0}^{t} \mathrm{d}t\, W(t)\ .$$

$(n=2)$ 

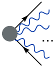


$$= 2^2 \int_{t_0}^{t} \mathrm{d}t' \int_{t_0}^{t'} \mathrm{d}t''\, W(t')W(t'') = \frac{2^2}{2!}\left(\int_{t_0}^{t} \mathrm{d}t\, W(t)\right)^2\ .$$

We used

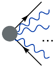
$$\int_{t_0}^{t} \mathrm{d}t_1 \ldots \int_{t_0}^{t_{n-1}} \mathrm{d}t_n\ W(t_1)\ldots W(t_n) = \frac{1}{n!}\left(\int_{t_0}^{t} \mathrm{d}t\, W(t)\right)^n\ .$$

Towards multiple emissions

Easily generalized to n emissions  by induction. *i.e.*

$$W_{2+n} = \frac{2^n}{n!}\left(\int_{t_0}^{t} \mathrm{d}t\, W(t)\right)^n$$

Towards multiple emissions

Easily generalized to n emissions  by induction. *i.e.*

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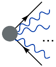
So, in total we get

$$\sigma_{>2}(t_0) = \sigma_2(t_0) \sum_{k=1}^{\infty} \frac{2^k}{k!} \left(\int_{t_0}^{t} \mathrm{d}t\, W(t) \right)^k = \sigma_2(t_0) \left(\mathrm{e}^{2\int_{t_0}^{t} \mathrm{d}t\, W(t)} - 1 \right)$$

Towards multiple emissions

Easily generalized to n emissions  by induction. *i.e.*

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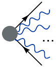
$$= \sigma_2(t_0) \left(\frac{1}{\Delta^2(t_0, t)} - 1 \right)$$

Sudakov Form Factor

$$\Delta(t_0, t) = \exp\left[- \int_{t_0}^{t} \mathrm{d}t\, W(t) \right]$$

Towards multiple emissions

Easily generalized to n emissions  by induction. *i.e.*

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$$= \sigma_2(t_0) \left(\frac{1}{\Delta^2(t_0, t)} - 1 \right)$$

Sudakov Form Factor in QCD

$$\Delta(t_0, t) = \exp\left[- \int_{t_0}^{t} \mathrm{d}t\, W(t) \right] = \exp\left[- \int_{t_0}^{t} \frac{\mathrm{d}t}{t} \int_{z_-}^{z_+} \frac{\alpha_S(z,t)}{2\pi} \hat{P}(z,t) \mathrm{d}z \right]$$

Sudakov form factor

Note that

$$\sigma_{\text{all}} = \sigma_2 + \sigma_{>2} = \sigma_2 + \sigma_2 \left(\frac{1}{\Delta^2(t_0,t)} - 1 \right) ,$$

$$\Rightarrow \Delta^2(t_0,t) = \frac{\sigma_2}{\sigma_{\text{all}}} .$$

$$\text{Two jet rate} = \Delta^2 = P^2(\text{No emission in the range } t \to t_0) .$$

$$\text{Sudakov form factor} = \text{No emission probability} .$$

Often $\Delta(t_0,t) \equiv \Delta(t)$.

- Hard scale t, typically CM energy or $p_\perp$ of hard process.
- Resolution t_0, two partons are resolved as two entities if inv mass or relative $p_\perp$ above t_0.
- P^2 (not P), as we have two legs that evolve independently.

Sudakov form factor from Markov property

Unitarity

$$P(\text{“some emission”}) + P(\text{“no emission”}) \\ = P(0 < t \le T) + \bar{P}(0 < t \le T) = 1\ .$$

Multiplication law (no memory)

$$\bar{P}(0 < t \le T) = \bar{P}(0 < t \le t_1)\bar{P}(t_1 < t \le T)$$

Sudakov form factor from Markov property

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Multiplication law (no memory)

$$\bar{P}(0 < t \leq T) = \bar{P}(0 < t \leq t_1)\bar{P}(t_1 < t \leq T)$$

Then subdivide into n pieces: $t_i = \frac{i}{n}T, 0 \leq i \leq n$.

$$\bar{P}(0 < t \leq T) = \lim_{n\to\infty}\prod_{i=0}^{n-1}\bar{P}(t_i < t \leq t_{i+1}) = \lim_{n\to\infty}\prod_{i=0}^{n-1}\left(1 - P(t_i < t \leq t_{i+1})\right)$$

$$= \exp\left(-\lim_{n\to\infty}\sum_{i=0}^{n-1}P(t_i < t \leq t_{i+1})\right) = \exp\left(-\int_0^T \frac{\mathrm{d}P(t)}{\mathrm{d}t}\mathrm{d}t\right)\ .$$

Sudakov form factor

Again, no–emission probability!

$$\bar{P}(0 < t \le T) = \exp\left(-\int_0^T \frac{\mathrm{d}P(t)}{\mathrm{d}t}\mathrm{d}t\right)$$

So,

$$\begin{aligned}\mathrm{d}P(\text{first emission at } T) &= \mathrm{d}P(T)\bar{P}(0 < t \le T) \\ &= \mathrm{d}P(T)\exp\left(-\int_0^T \frac{\mathrm{d}P(t)}{\mathrm{d}t}\mathrm{d}t\right)\end{aligned}$$

That's what we need for our parton shower! Probability density for next emission at t:

$$\mathrm{d}P(\text{next emission at } t) =$$
$$\frac{\mathrm{d}t}{t}\int_{z_-}^{z_+}\frac{\alpha_S(z,t)}{2\pi}\hat{P}(z,t)\mathrm{d}z\ \exp\left[-\int_{t_0}^{t}\frac{\mathrm{d}t}{t}\int_{z_-}^{z_+}\frac{\alpha_S(z,t)}{2\pi}\hat{P}(z,t)\mathrm{d}z\right]$$

Parton shower Monte Carlo

Probability density:

$$\mathrm{d}P(\text{next emission at } t) = \frac{\mathrm{d}t}{t}\int_{z_-}^{z_+}\frac{\alpha_S(z,t)}{2\pi}\hat{P}(z,t)\mathrm{d}z\,\exp\left[-\int_{t_0}^{t}\frac{\mathrm{d}t}{t}\int_{z_-}^{z_+}\frac{\alpha_S(z,t)}{2\pi}\hat{P}(z,t)\mathrm{d}z\right]$$

Conveniently, the probability distribution is $\Delta(t)$ itself.

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Conveniently, the probability distribution is $\Delta(t)$ itself.
Hence, parton shower very roughly from (HERWIG):

1. Choose flat random number $0 \le \rho \le 1$.
2. If $\rho < \Delta(t_{\max})$: no resolbable emission, stop this branch.
3. Else solve $\rho = \Delta(t_{\max})/\Delta(t)$
   (= no emission between $t_{\max}$ and t) for t.
   Reset $t_{\max} = t$ and goto 1.

Determine z essentially according to integrand in front of exp.

Parton shower Monte Carlo

Probability density:

$$\mathrm{d}P(\text{next emission at } t) =$$
$$\frac{\mathrm{d}t}{t}\int_{z_-}^{z_+}\frac{\alpha_S(z,t)}{2\pi}\hat{P}(z,t)\mathrm{d}z\ \exp\left[-\int_{t_0}^{t}\frac{\mathrm{d}t}{t}\int_{z_-}^{z_+}\frac{\alpha_S(z,t)}{2\pi}\hat{P}(z,t)\mathrm{d}z\right]$$

Conveniently, the probability distribution is $\Delta(t)$ itself.

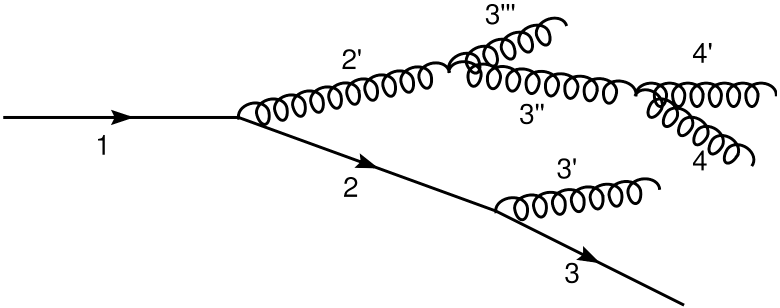
- That was old HERWIG variant. Relies on (numerical) integration/tabulation for $\Delta(t)$.
- Pythia, now also Herwig++, use the Veto Algorithm.
- Method to sample x from distribution of the type

$$\mathrm{d}P = F(x)\exp\left[-\int^{x}\mathrm{d}x'\,F(x')\right]\mathrm{d}x\ .$$

Simpler, more flexible, but slightly slower.

Parton cascade

Get tree structure, ordered in evolution variable t:



Here: $t_1 > t_2 > t_3$; $t_2 > t_{3'}$ etc.
Construct four momenta from (t_i, z_i) and (random) azimuth ϕ.

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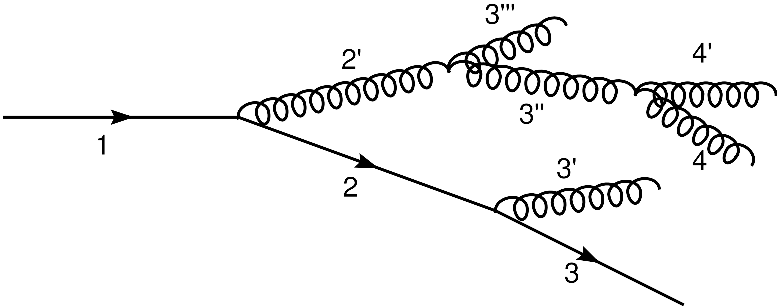

Here: $t_1 > t_2 > t_3$; $t_2 > t_{3'}$ etc.
Construct four momenta from (t_i, z_i) and (random) azimuth ϕ.

Not at all unique!
Many (more or less clever) choices still to be made.

Parton cascade

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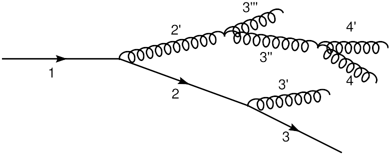

- t can be θ, Q^2, $p_\perp$, ...
- Choice of hard scale $t_{\max}$ not fixed. “Some hard scale”.
- z can be light cone momentum fraction, energy fraction, ...
- Available parton shower phase space.
- Integration limits.
- Regularisation of soft singularities.
- ...

Good choices needed here to describe wealth of data!

Soft emissions

- Only *collinear* emissions so far.
- Including *collinear+soft*.
- *Large angle+soft* also important.

Soft emissions

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Soft emission: consider *eikonal factors*,
here for $q(p+q) \to q(p)g(q)$, soft g:

$$u(p) \not{\varepsilon} \frac{\not{p} + \not{q} + m}{(p+q)^2 - m^2} \longrightarrow u(p) \frac{p \cdot \varepsilon}{p \cdot q}$$

soft factorisation. Universal, *i.e.* independent of emitter.
In general:

$$d\sigma_{n+1} = d\sigma_n \frac{d\omega}{\omega} \frac{d\Omega}{2\pi} \frac{\alpha_S}{2\pi} \sum_{ij} C_{ij} W_{ij} \quad \text{(“QCD–Antenna”)}$$

with

$$W_{ij} = \frac{1 - \cos\theta_{ij}}{(1 - \cos\theta_{iq})(1 - \cos\theta_{qj})} \, .$$

Soft emissions

We define

$$W_{ij} = \frac{1-\cos\theta_{ij}}{(1-\cos\theta_{iq})(1-\cos\theta_{qj})} \equiv W_{ij}^{(i)} + W_{ij}^{(j)}$$

with

$$W_{ij}^{(i)} = \frac{1}{2}\left(W_{ij} + \frac{1}{1-\cos\theta_{iq}} - \frac{1}{1-\cos\theta_{qj}}\right) .$$

$W_{ij}^{(i)}$ is only collinear divergent if $q \| i$ etc .

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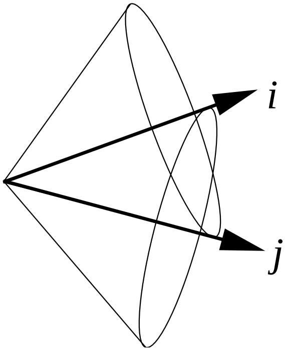
$W_{ij}^{(i)}$ is only collinear divergent if $q \| i$ etc .
After integrating out the azimuthal angles, we find

$$\int \frac{d\phi_{iq}}{2\pi} W_{ij}^{(i)} = \begin{cases} \dfrac{1}{1-\cos\theta_{iq}} & (\theta_{iq} < \theta_{ij}) \\ 0 & \text{otherwise} \end{cases}$$

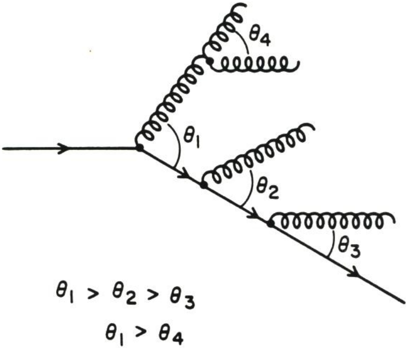
That's angular ordering.

Angular ordering

Radiation from parton *i* is bound to a cone, given by the colour partner parton *j*.



Results in angular ordered parton shower and suppresses soft gluons viz. hadrons in a jet.



Colour coherence from CDF

Events with 2 hard (> 100 GeV) jets and a soft 3rd jet (∼ 10 GeV)

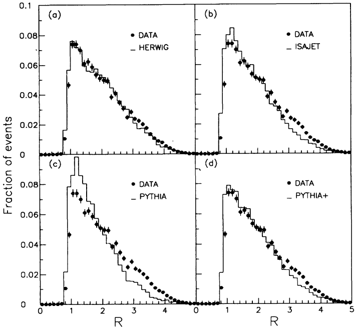


FIG. 14. Observed R distribution compared to the predictions of (a) HERWIG; (b) ISAJET; (c) PYTHIA; (d) PYTHIA+.

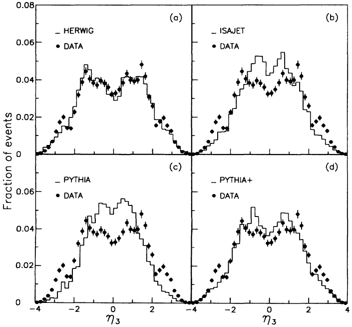


FIG. 13. Observed η_3 distribution compared to the predictions of (a) HERWIG; (b) ISAJET; (c) PYTHIA; (d) PYTHIA+.

F. Abe *et al.* [CDF Collaboration], Phys. Rev. D **50** (1994) 5562.

Best description with angular ordering.

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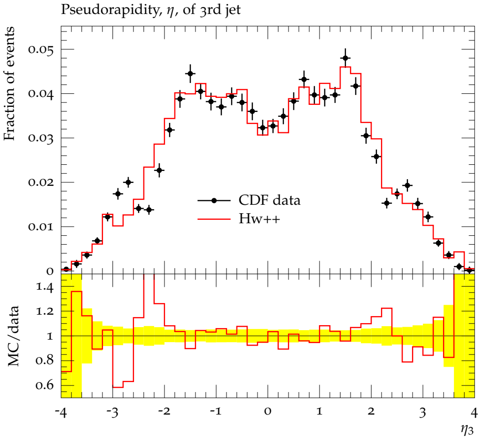

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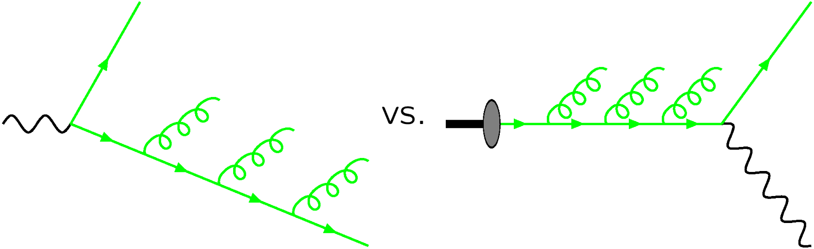
Best description with angular ordering.

Initial state radiation



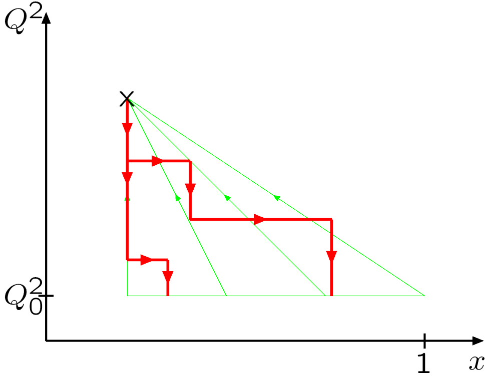
Similar to final state radiation. Sudakov form factor ($x' = x/z$)

$$\Delta(t, t_{\max}) = \exp\left[-\sum_b \int_t^{t_{\max}} \frac{\mathrm{d}t}{t} \int_{z_-}^{z_+} \mathrm{d}z \frac{\alpha_S(z,t)}{2\pi} \frac{x' f_b(x',t)}{x f_a(x,t)} \hat{P}_{ba}(z,t)\right]$$

Have to divide out the pdfs.

Initial state radiation

Evolve backwards from hard scale Q^2 *down* towards cutoff scale Q_0^2. Thereby increase x.

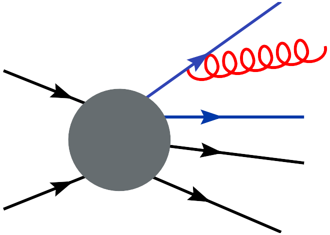


With parton shower we *undo* the DGLAP evolution of the pdfs.

Dipoles

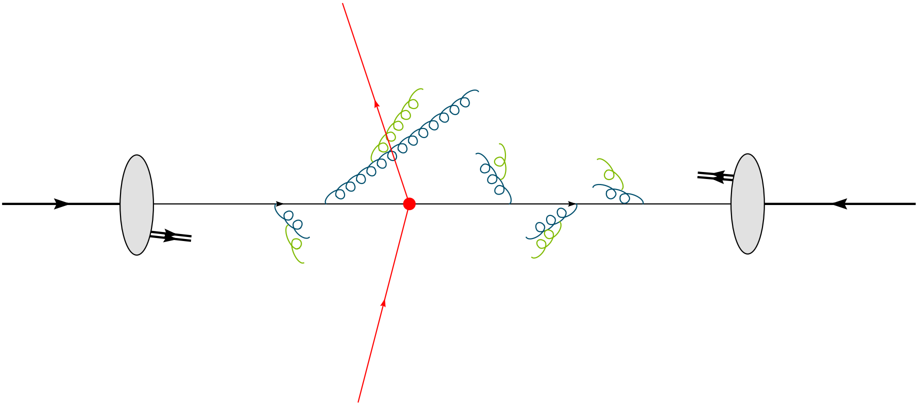
Exact kinematics when recoil is taken by spectator(s).

- Dipole showers.
- Ariadne.
- Recoils in Pythia.
- New dipole showers, based on
    - Catani Seymour dipoles.
    - QCD Antennae.
    - Herwig, Sherpa, Vincia, Dire, . . .
    - Goal: matching with NLO.
- Generalized to IS–IS, IS–FS.

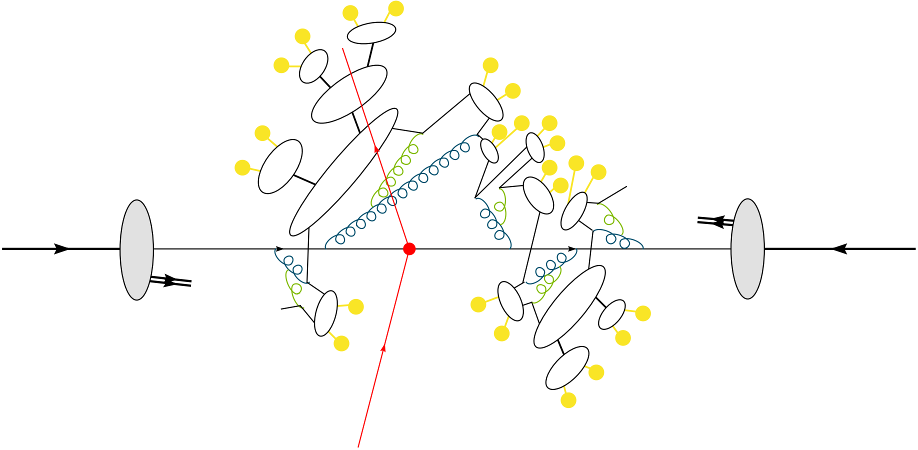


Hadronization

Parton shower



Parton shower ⟶ hadrons

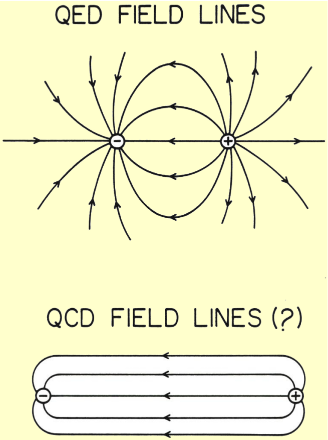


Parton shower $\longrightarrow$ hadrons

- Parton shower terminated at t_0 = lower end of PT.
- Can't measure quarks and gluons.
- Degrees of freedom in the detector are hadrons.
- Need a description of confinement.

Physical input

Self coupling of gluons
$\leftrightarrow$ "attractive field lines"



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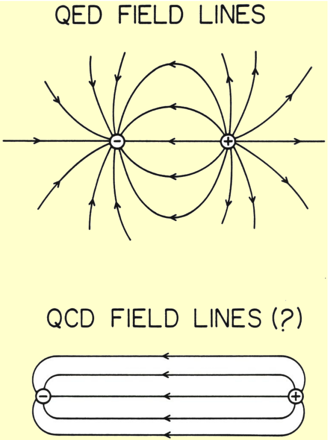
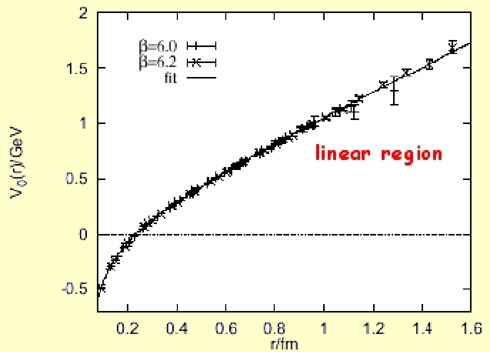


Linear static potential $V(r) \approx \kappa r$.



Supported by lattice QCD,
hadron spectroscopy.

Hadronization models

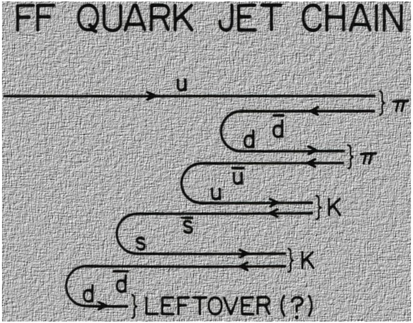
Older models:

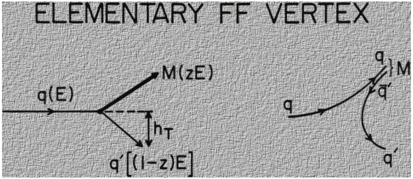
- Flux tube model.
- Independent fragmentation.

Today's models.

- Lund string model (Pythia).
- Cluster model (Herwig).

Independent fragmentation

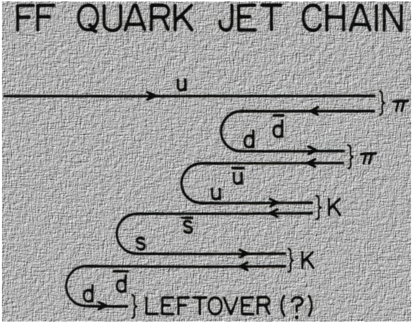


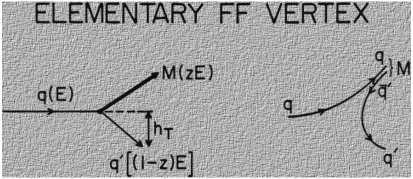


Feynman–Field fragmentation ('78).

- $q\bar{q}$ pairs created from vacuum to dress bare quarks.
- Fragmentation function $f_{q\to h}(z) =$ density of momentum fraction z carried away by hadron h from quark q.
- Gaussian $p_\perp$ distribution.

Independent fragmentation





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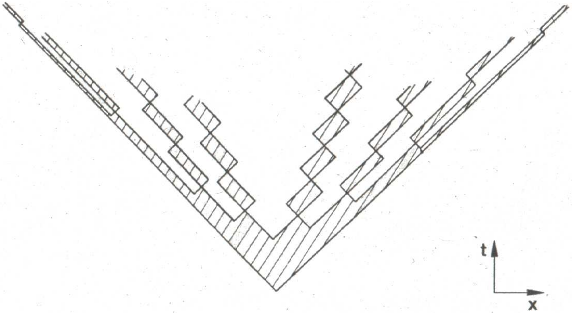
- $q\bar{q}$ pairs created from vacuum to dress bare quarks.
- Fragmentation function $f_{q\to h}(z)$ = density of momentum fraction z carried away by hadron h from quark q.
- Gaussian $p_\perp$ distribution.
- Problems:
  - "last quark".
  - not Lorentz invariant.
  - infrared safety.
  - …
- Good at that time.
- Still usefull for inclusive descriptions.

Lund string model

String energy $\sim$ intense chromomagnetic field.
$\longrightarrow$ Additional $q\bar{q}$ pairs created by QM tunneling.

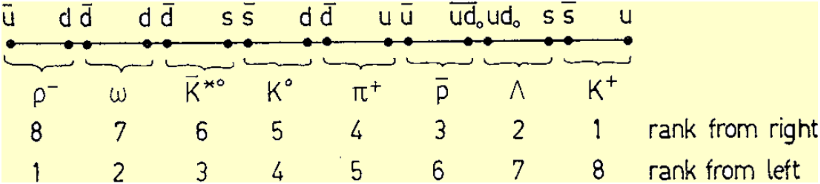
$$\frac{\mathrm{dProb}}{\mathrm{d}x\mathrm{d}t} \sim \exp\left(-\pi m_q^2/\kappa\right) \qquad \kappa \sim 1\,\mathrm{GeV}\,.$$



String breaking expected long before yoyo point.

Lund string model

Ajacent breaks form hadrons.



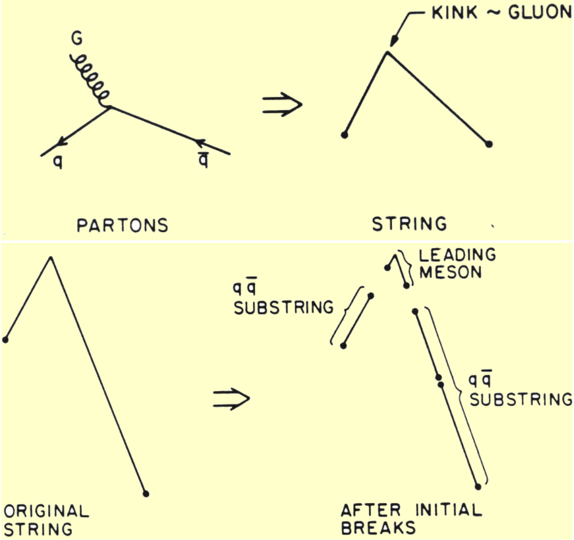
Works in both directions (symmetry).
Lund symmetric fragmentation function

$$f(z, p_\perp) \sim \frac{1}{z}(1-z)^a \exp\left(-\frac{b(m_h^2 + p_\perp^2)}{z}\right)$$

a, b, m_h^2 main adjustable parameters.
Note: diquarks $\to$ baryons.

Lund string model

gluon = kink on string = motion pushed into the $q\bar{q}$ system.



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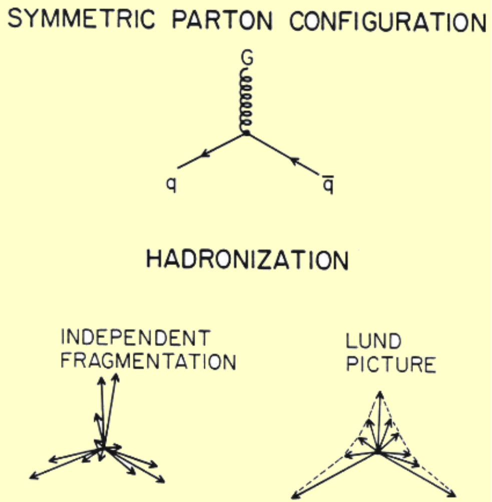

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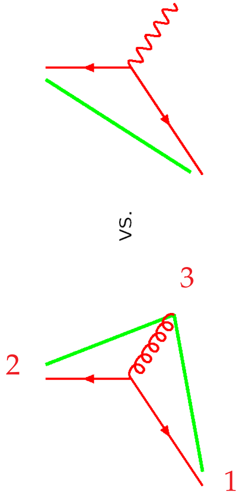
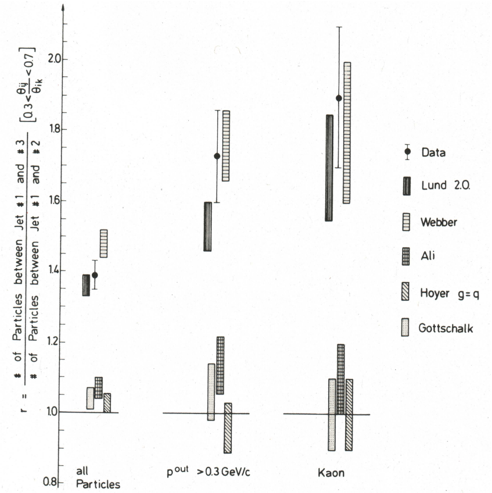


“String effect”



Lund string model

Some remarks:

- Originally invented without parton showers in mind.

Lund string model

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- Originally invented without parton showers in mind.
- Stong physical motivation.
- Very successful desription of data.
- Universal desription of data
  (fit at e^+e^-, transfer to hadron-hadron).
- Many parameters, ~ 1 per hadron.
- Too easy to hide errors in perturbative description?

Lund string model

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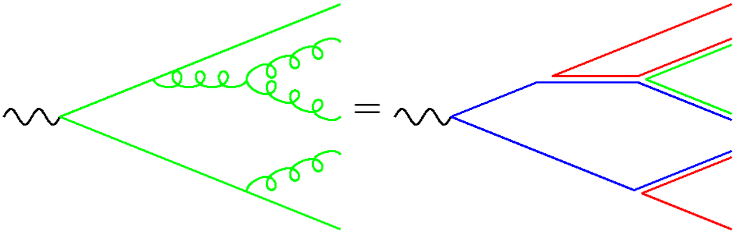
$\longrightarrow$ try to use more QCD information/intuition.

Colour preconfinement

Large N_C limit $\longrightarrow$ planar graphs dominate.
Gluon = colour — anticolourpair



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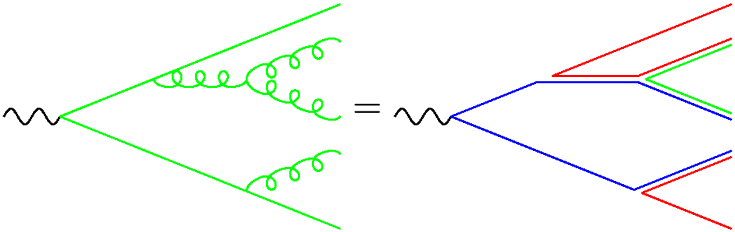
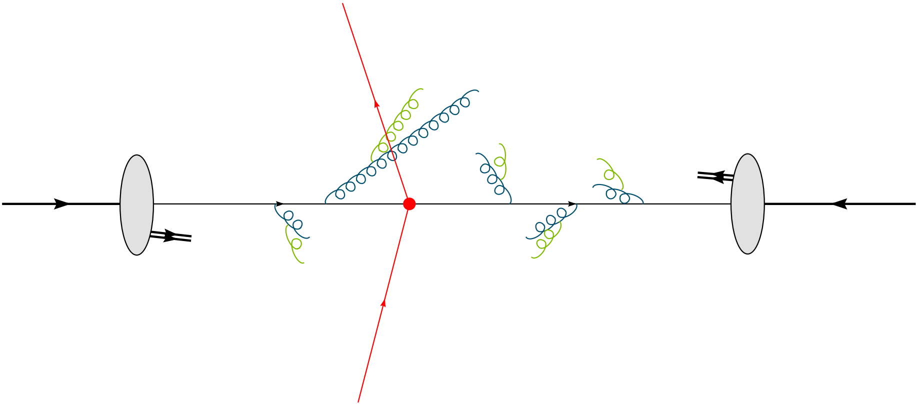


Parton shower organises partons in colour space. Colour partners (=colour singlet pairs) end up close in phase space.

$\longrightarrow$ Cluster hadronization model

Cluster hadronization



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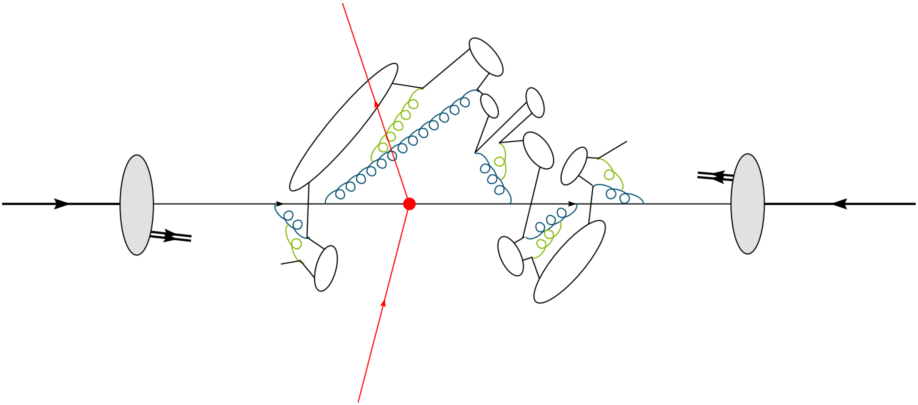

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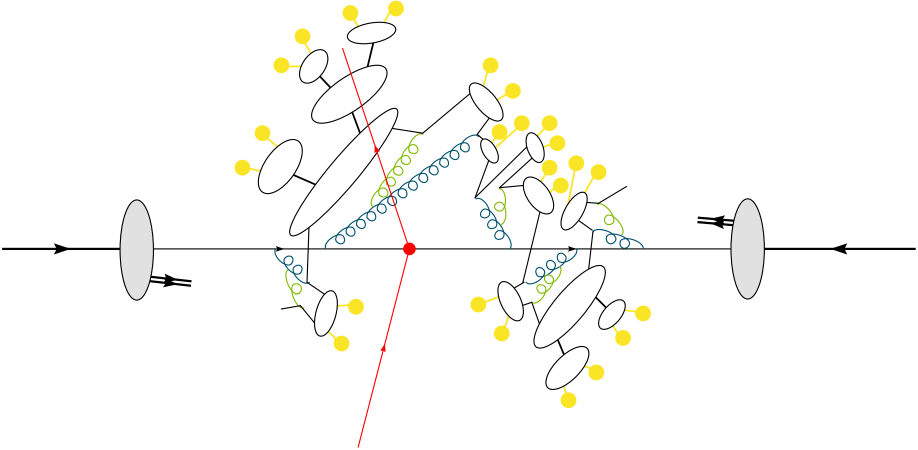

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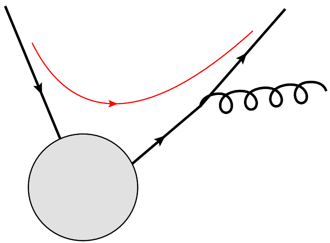
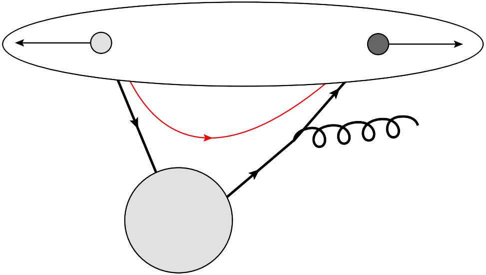


After parton shower, partons on constituent mass shell
Find colour singlets as 3-$\bar{3}$ pairs $\rightarrow$ cluster
Colour neighbours $\sim$ neighbours in momentum space

Cluster Hadronization



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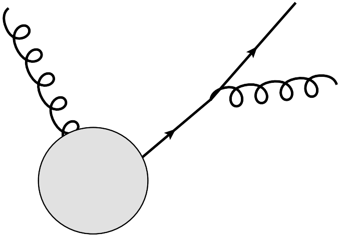
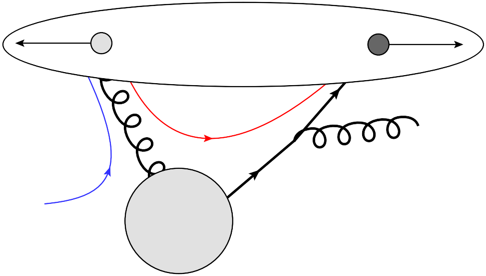


But gluons are not just 3 or $\bar{3}$!

Cluster Hadronization



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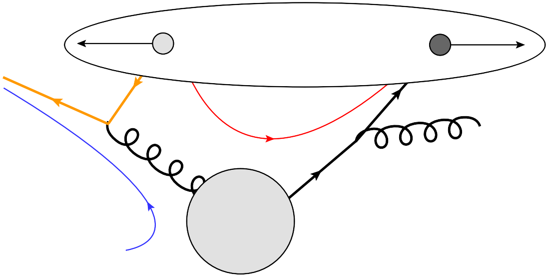
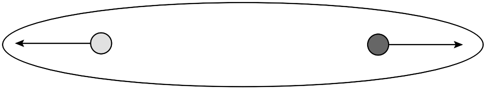


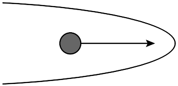
But gluons are not just 3 or $\bar{3}$!
non-perturbative gluon splitting
$m_g > 2m_q$
kinematics from masses
quarks and diquarks possible

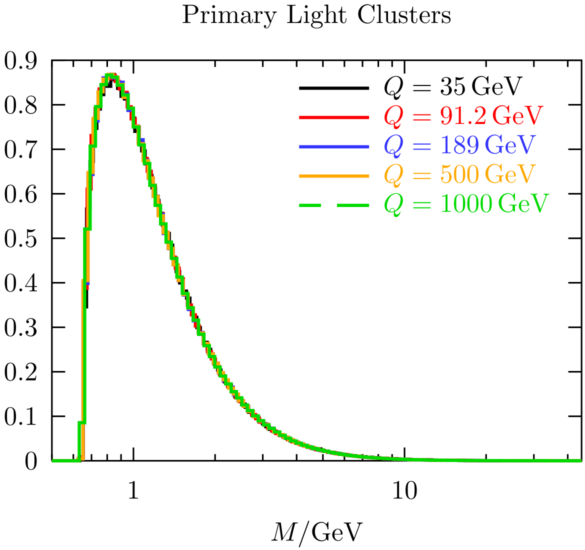
Cluster Hadronization



Cluster carries net momentum of its constituents
Spectrum determined by final state of parton shower
Independent of hard scales
Tail of *heavy clusters*, still large scale available

Cluster Hadronization





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on shower

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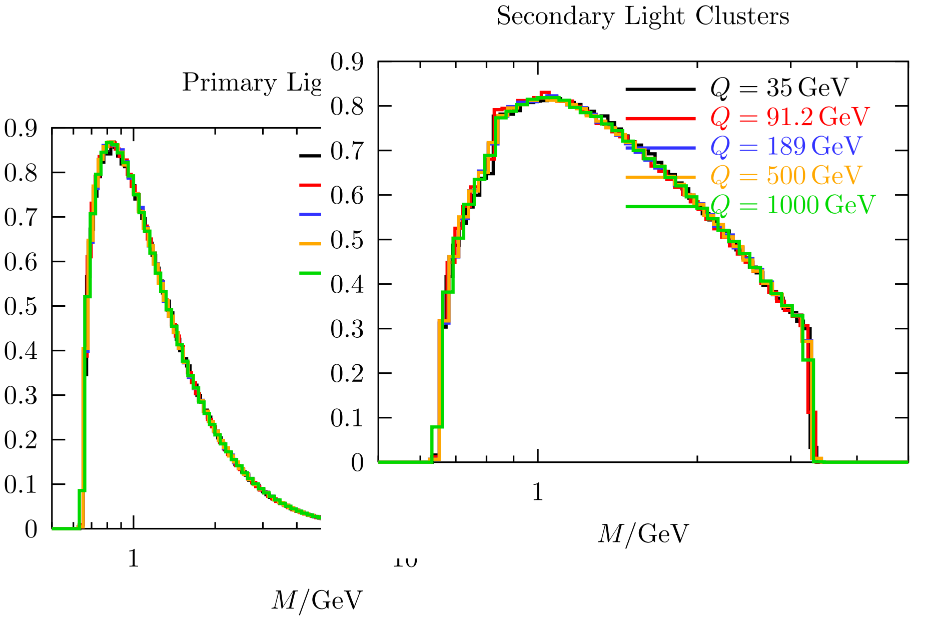
Cluster Hadronization



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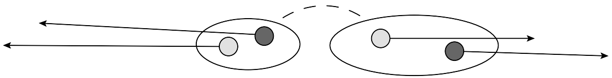
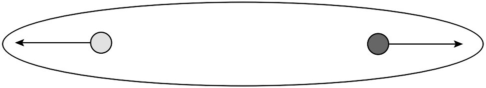


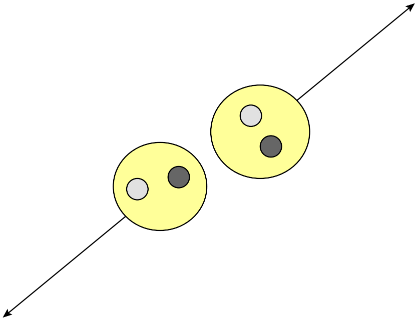
Binary fission along quarks' direction of motion
Flavour introduced in $q\bar{q}$ pairs
Mass $\rightarrow$ multiplicity, momentum
Beam remnant clusters split off as very light clusters

Cluster Hadronization



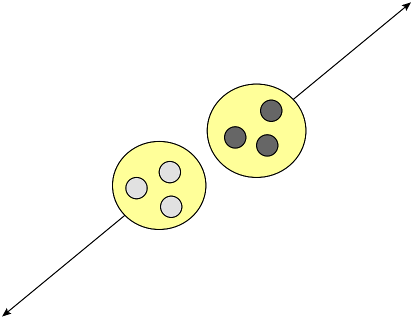
End up with fairly light clusters
too light? Decay into single hadron
Exchange momentum with neighbour

Cluster Hadronization



Decay isotropically into hadron pairs
Individual Hadrons get weight according to flavour multiplet, CM momentum, spin multiplicity etc.

Cluster Hadronization



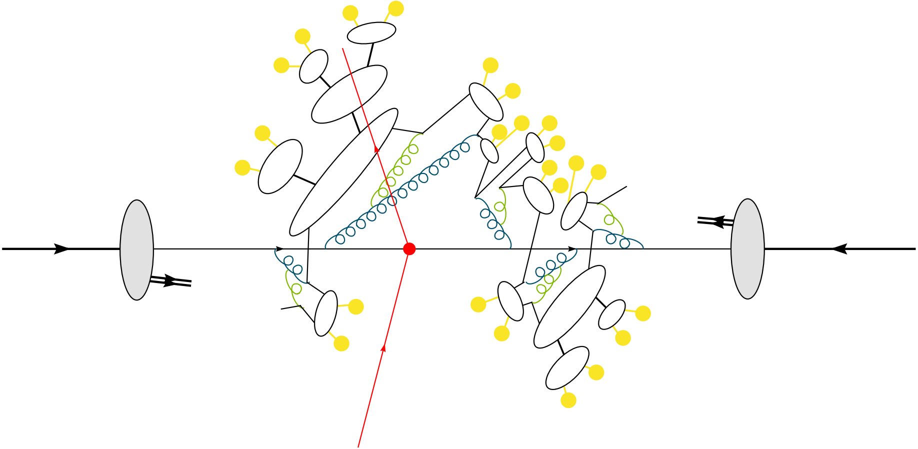
Baryon pairs possible
usually appear from clusters with 1 or 2 diquarks
could also emerge in pairs from mesonic clusters

Hadronization

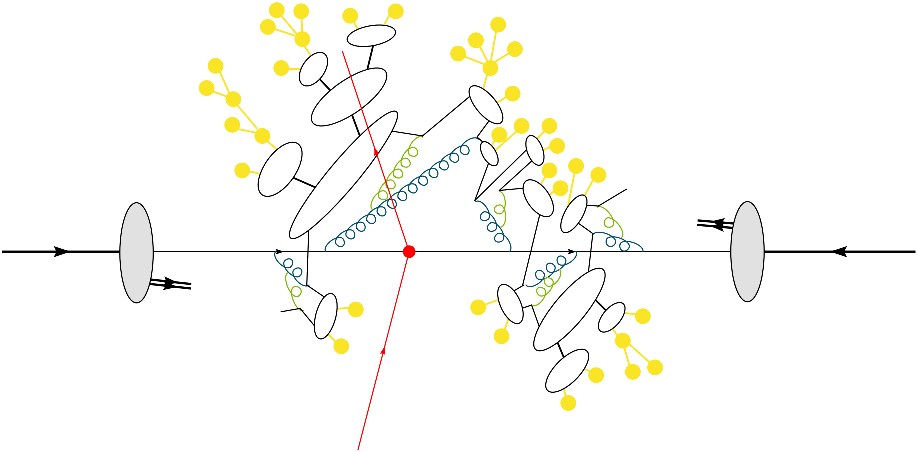
- Only string and cluster models used in recent MC programs.
  Independent fragmentation only for inclusive observables.
- Strings started non–perturbatively,
  improved by parton shower.
- Cluster model started mostly on perturbative side,
  improved by string like cluster fission.

Hadronic Decays

Hadronic decays



Hadronic decays



Hadronic decays

Many aspects:

$$
\begin{aligned}
B^{*0} \to \gamma B^0 & \\
& \hookrightarrow \bar{B}^0 \\
& \quad \hookrightarrow e^- \bar{\nu}_e D^{*+} \\
& \qquad \hookrightarrow \pi^+ D^0 \\
& \qquad\quad \hookrightarrow K^- \rho^+ \\
& \qquad\qquad \hookrightarrow \pi^+ \pi^0 \\
& \qquad\qquad\quad \hookrightarrow e^+ e^- \gamma
\end{aligned}
$$

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\end{aligned}
$$

EM decay.

Hadronic decays

Many aspects:

$$B^{*0} \to \gamma B^0$$
$$\hookrightarrow \bar{B}^0$$
$$\hookrightarrow e^- \bar{\nu}_e D^{*+}$$
$$\hookrightarrow \pi^+ D^0$$
$$\hookrightarrow K^- \rho^+$$
$$\hookrightarrow \pi^+ \pi^0$$
$$\hookrightarrow e^+ e^- \gamma$$

Weak mixing.

Hadronic decays

Many aspects:

$$B^{*0} \to \gamma B^0$$
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Weak decay.

Hadronic decays

Many aspects:

$$
\begin{aligned}
B^{*0} \to \gamma B^0 & \\
& \hookrightarrow \bar{B}^0 \\
& \quad \hookrightarrow e^- \bar{\nu}_e D^{*+} \\
& \qquad \hookrightarrow \pi^+ D^0 \\
& \qquad\quad \hookrightarrow K^- \rho^+ \\
& \qquad\qquad \hookrightarrow \pi^+ \pi^0 \\
& \qquad\qquad\quad \hookrightarrow e^+ e^- \gamma
\end{aligned}
$$

Strong decay.

Hadronic decays

Many aspects:

$$
\begin{aligned}
B^{*0} \to \gamma B^0 \\
\hookrightarrow \bar{B}^0 \\
\hookrightarrow e^- \bar{\nu}_e D^{*+} \\
\hookrightarrow \pi^+ D^0 \\
\hookrightarrow K^- \rho^+ \\
\hookrightarrow \pi^+ \pi^0 \\
\hookrightarrow e^+ e^- \gamma
\end{aligned}
$$

Weak decay, ρ^+ mass smeared.

Hadronic decays

Many aspects:

$$B^{*0} \to \gamma B^0$$
$$\hookrightarrow \bar{B}^0$$
$$\hookrightarrow e^- \bar{\nu}_e D^{*+}$$
$$\hookrightarrow \pi^+ D^0$$
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$$\hookrightarrow \pi^+ \pi^0$$
$$\hookrightarrow e^+ e^- \gamma$$

ρ^+ polarized, angular correlations.

Hadronic decays

Many aspects:

$$B^{*0} \to \gamma B^0$$
$$\hookrightarrow \bar{B}^0$$
$$\hookrightarrow e^- \bar{\nu}_e D^{*+}$$
$$\hookrightarrow \pi^+ D^0$$
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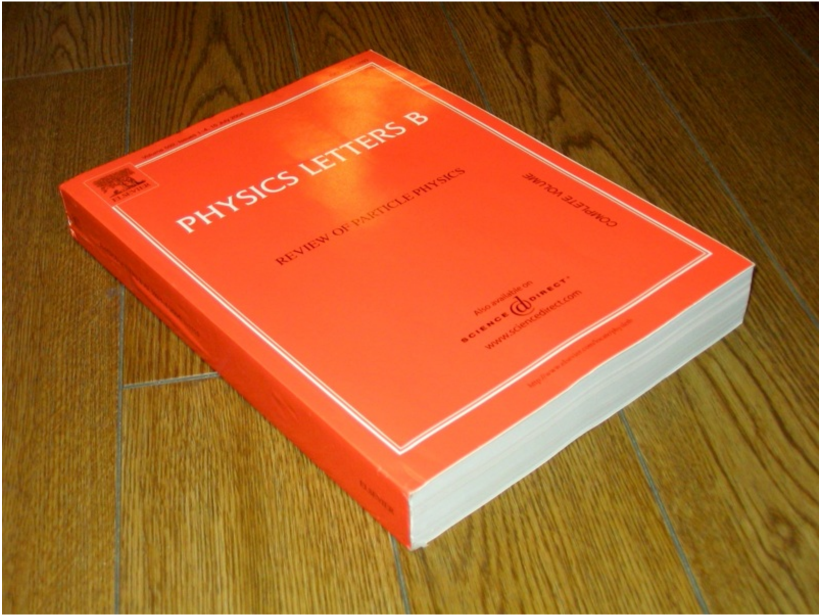
$$\hookrightarrow e^+ e^- \gamma$$

Dalitz decay, m_{ee} peaked.

Hadronic decays

Tedious.
100s of different particles, 1000s of decay modes, phenomenological matrix elements with parametrized form factors…

Hadronic decays



A few plots

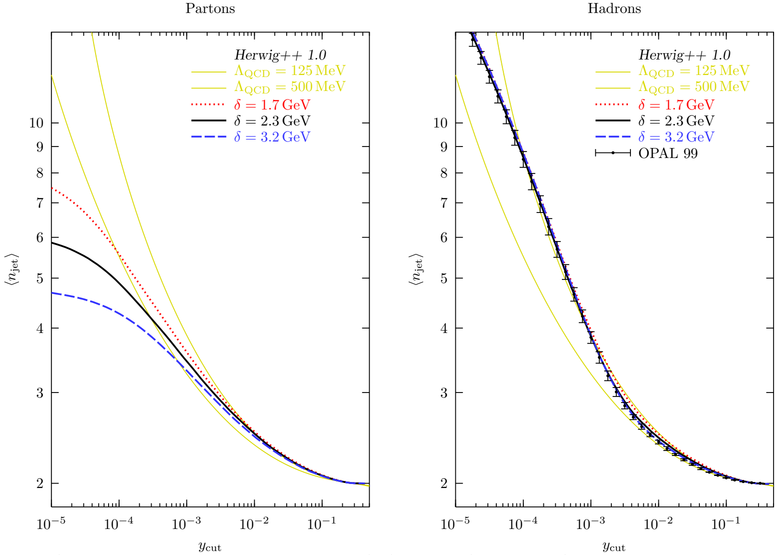
How well does it work?

- $e^+e^- \to$ hadrons, mostly at LEP.
- Jet shapes, jet rates, event shapes, identified particles…
- 'Tuning' of parameters.
- Use *all* analyses available in Rivet.
- Want to get *everything* right with *one* parameter set.
- Compare to literally $\approx$ 20000 plots.

- Check out `http://herwig.hepforge.org` ($\to$ Plots) for many more and comparisons with the latest release.

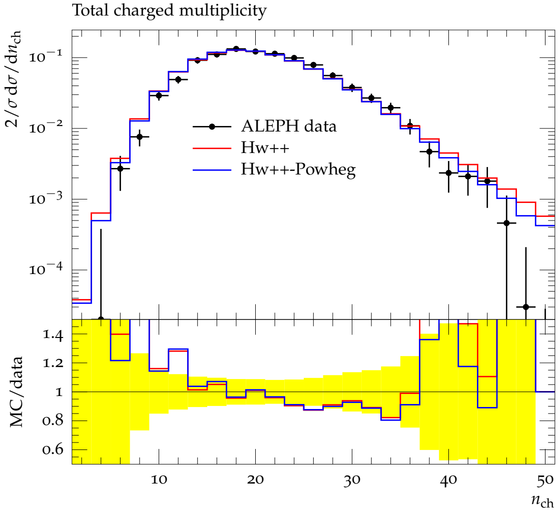
How well does it work?

Smooth interplay between shower and hadronization.



How well does it work?

N_{ch} at LEP. Crucial for t_0 (Herwig++ 2.5.2)

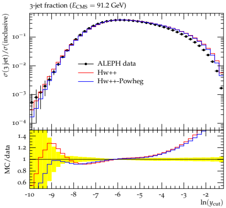


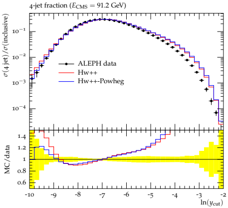
How well does it work?

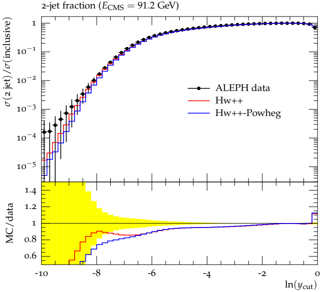
Jet rates at LEP.

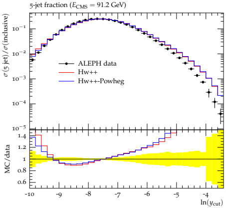
$$R_n = \sigma(n\text{–jets})/\sigma(\text{jets})$$
$$R_6 = \sigma(> 5\text{–jets})/\sigma(\text{jets})$$

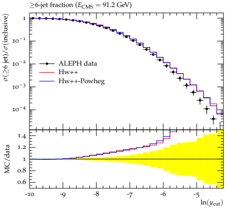
(Herwig++ 2.5.2)j





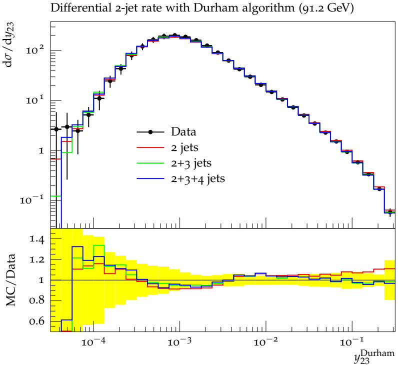


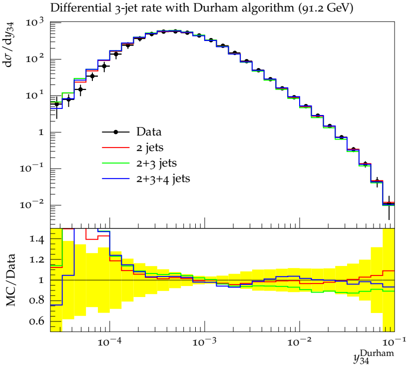




How well does it work?

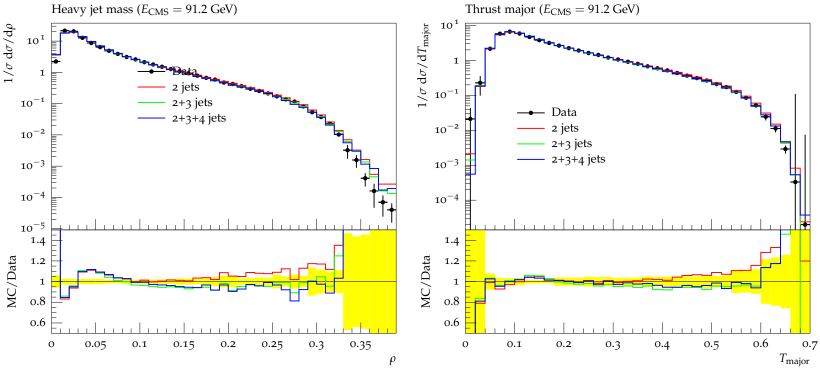
Differential Jet Rates at LEP (Herwig++ pre-3.0).
Dipole shower + some merging





How well does it work?

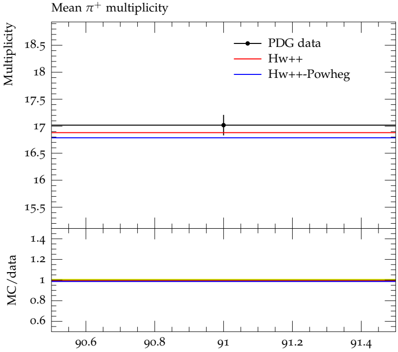
Event Shapes at LEP (Herwig++ pre-3.0). Dipole shower + some merging

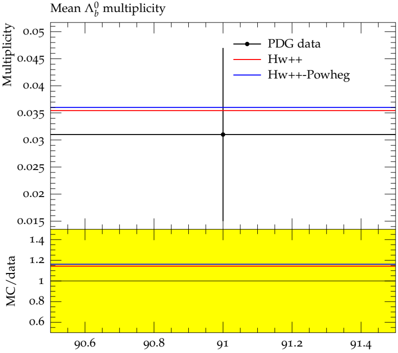


Parton showers do very well, today!

How well does it work?

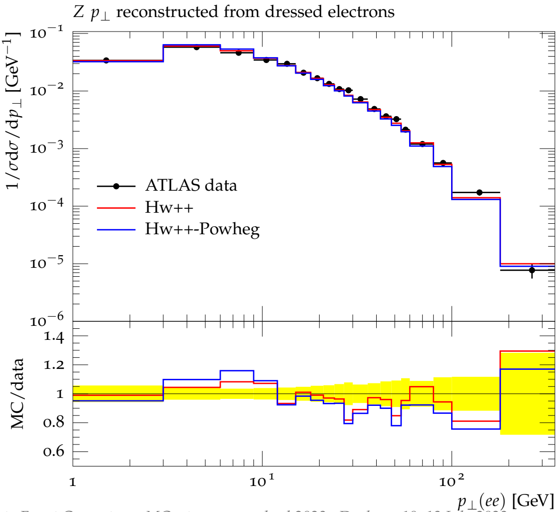
Hadron Multiplicities at LEP (e.g. π^+, Λ_b^0).



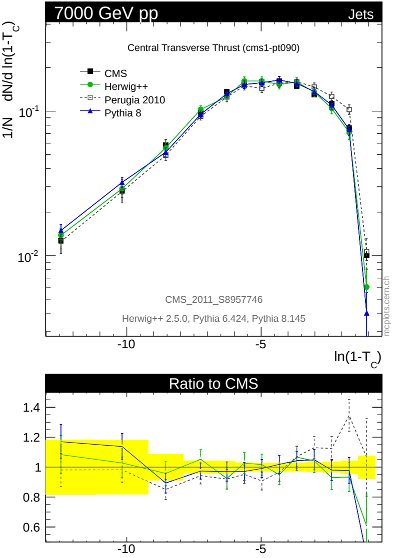


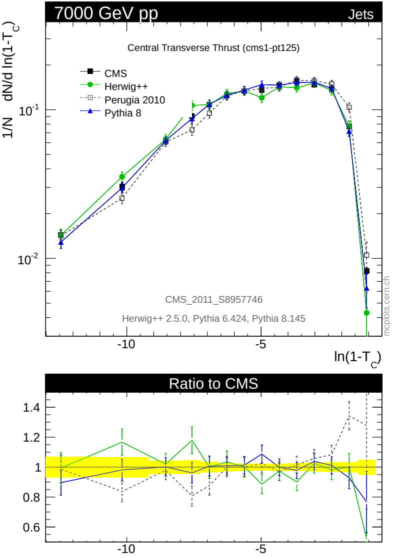
How well does it work?

$p_\perp(Z^0) \to$ intrinsic $k_\perp$ (LHC 7 TeV).
See also in context of matching/marging.



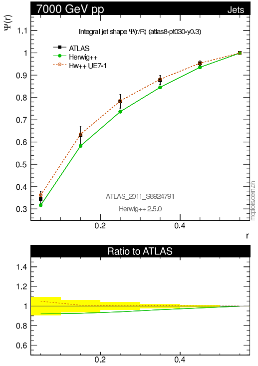
Transverse thrust

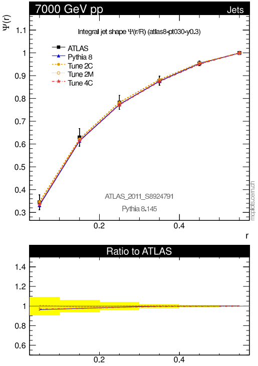


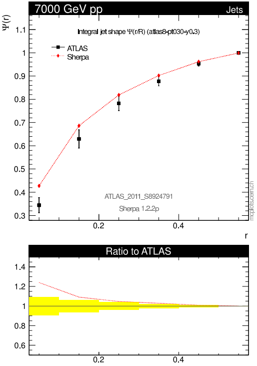


Integral jet shapes

not too hard, central $(30 < p_T/\mathrm{GeV} < 40; 0 < |y| < 0.3)$

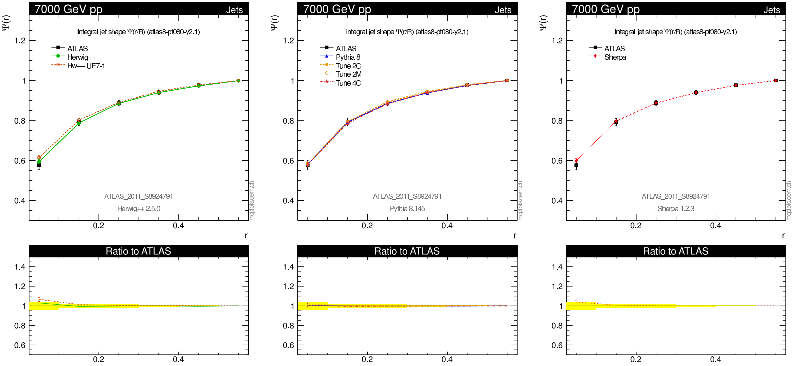






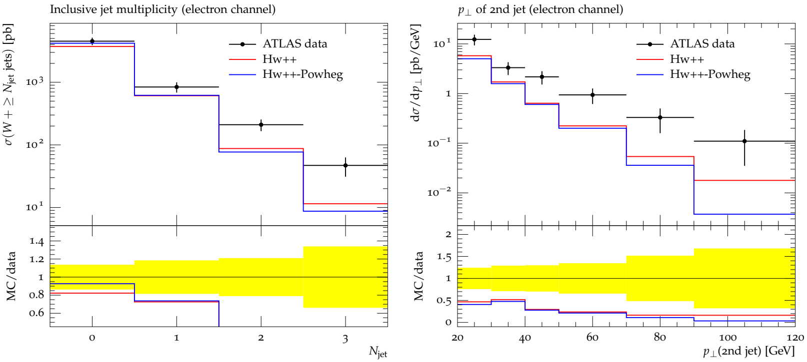
Integral jet shapes

harder, more forward ($80 < p_T/\mathrm{GeV} < 110; 1.2 < |y| < 2.1$)



Limits of parton showers

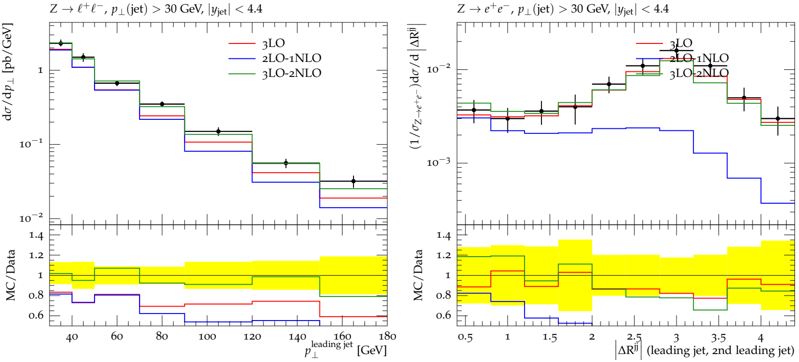
$W + \text{jets}$, LHC 7 TeV.



Higher jets not covered by parton shower only → merging.

Unitarized Matching/Merging

Preliminary example: Z production, jet-jet correlation.

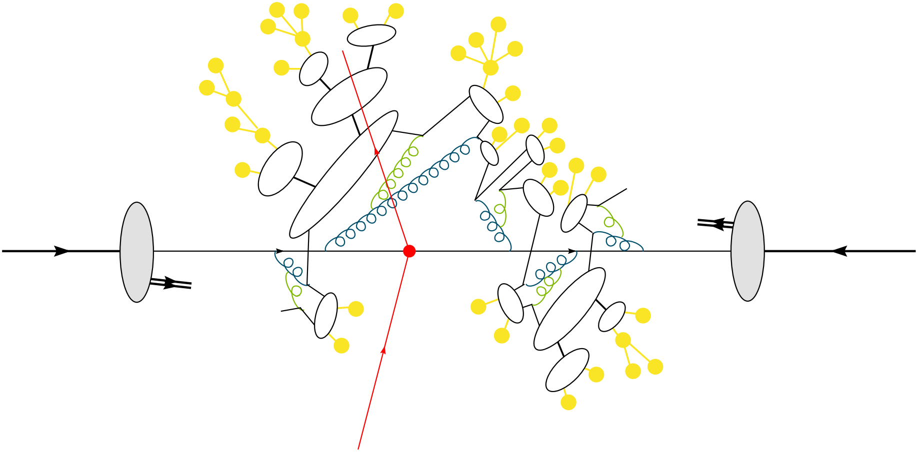


[J. Bellm, KIT]

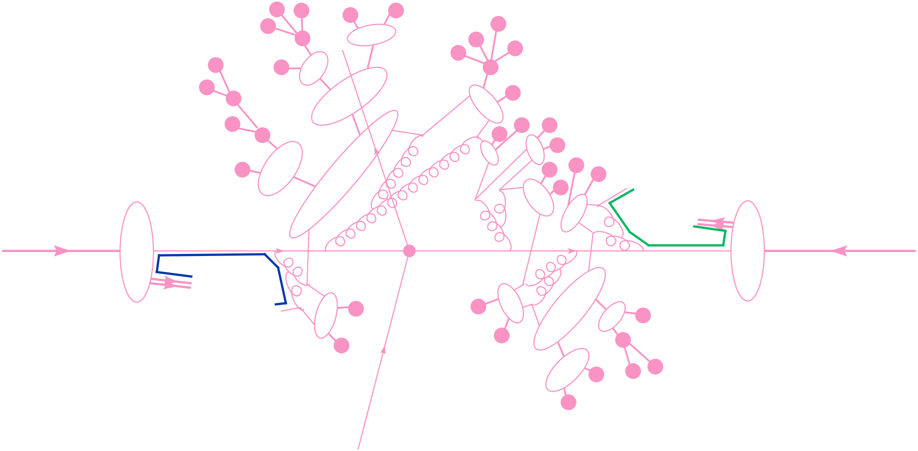
3LO-2NLO = Z+0, 1, 2 (tree) and Z+0,1 NLO (virtual).

Min Bias/Underlying event in data

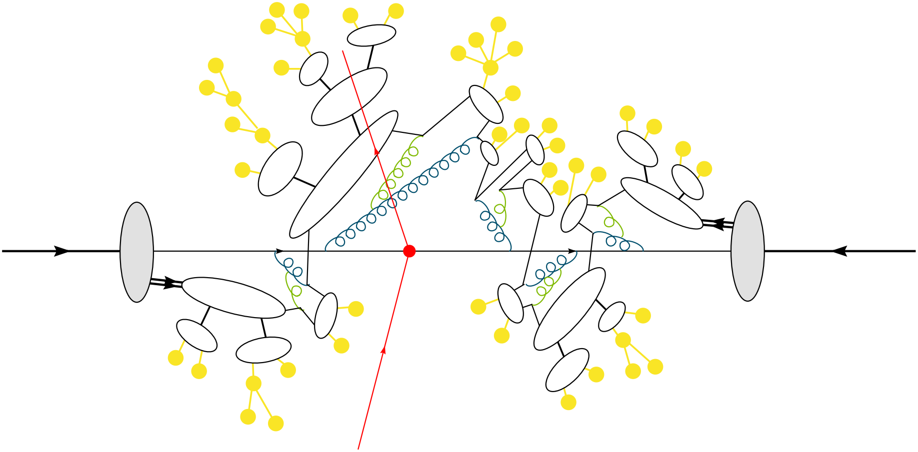
pp Event Generator



pp Event Generator



pp Event Generator



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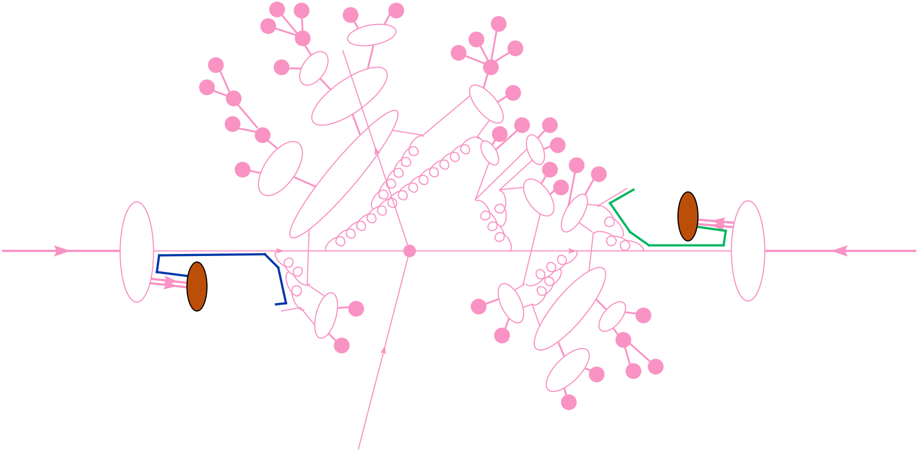

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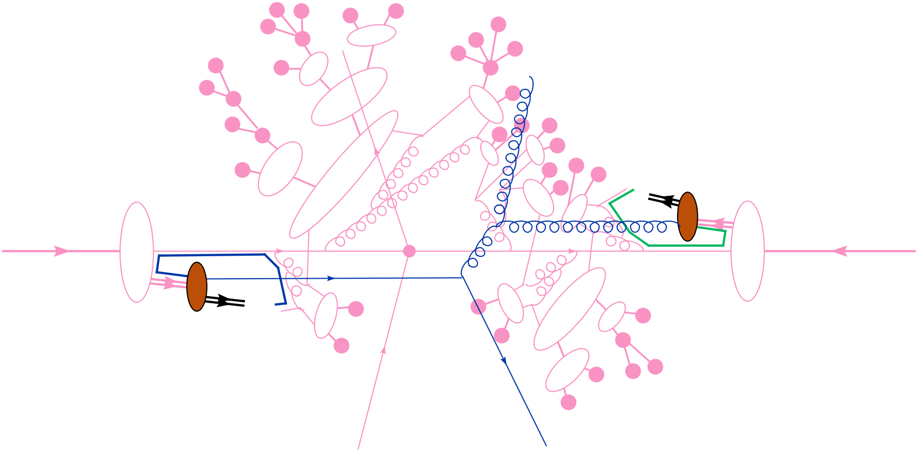

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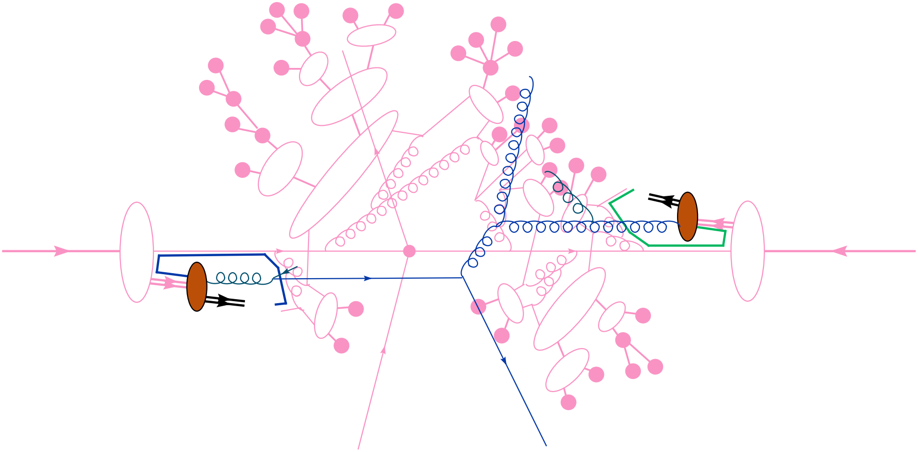

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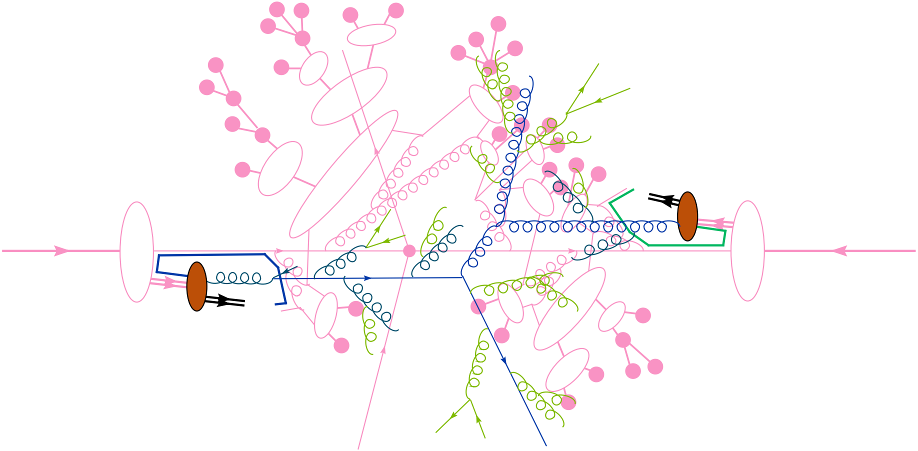

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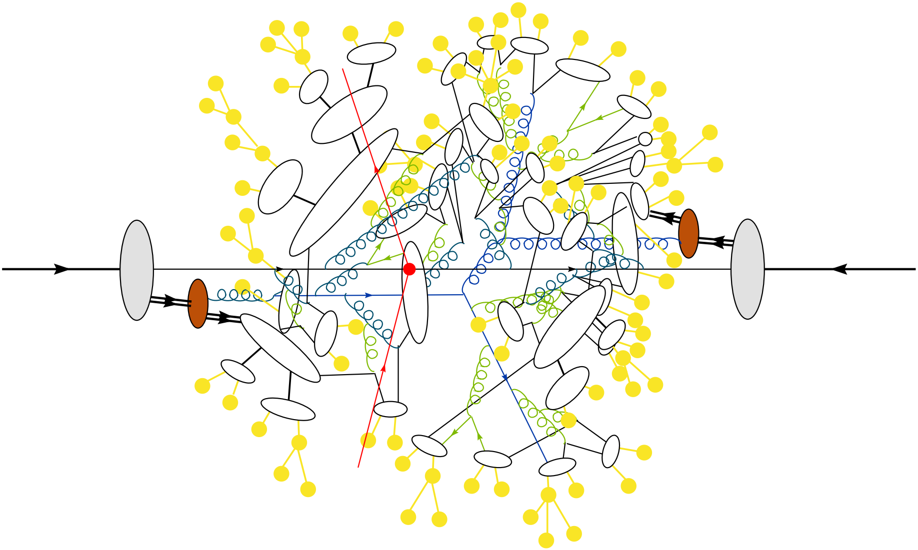

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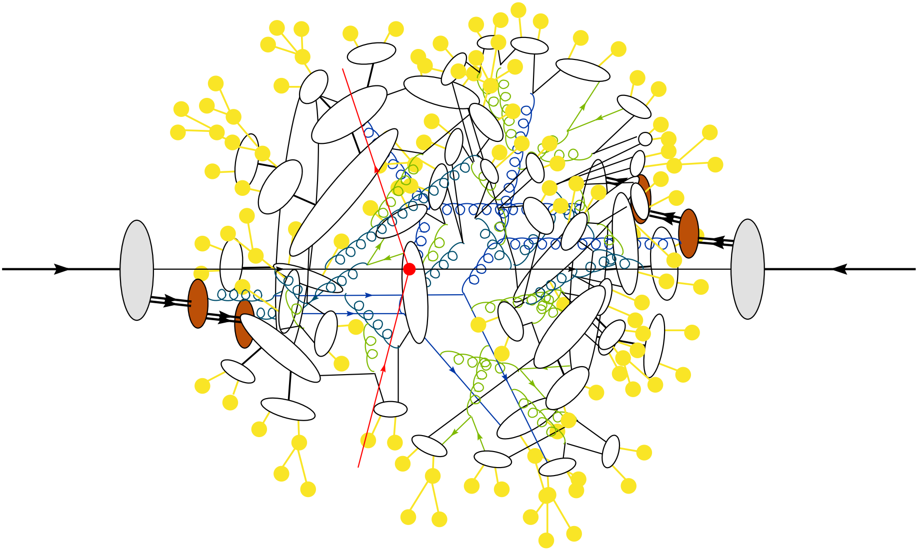
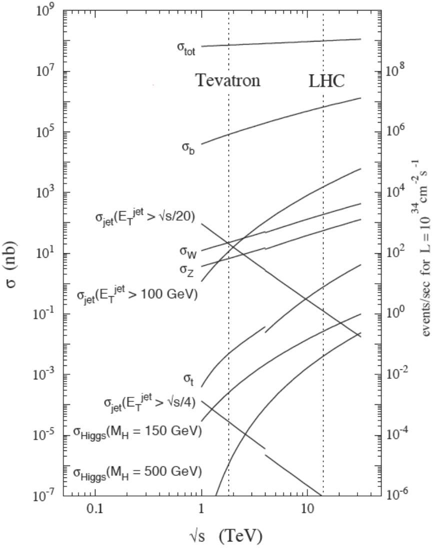


Collider cross sections

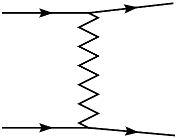
$$\sigma_{\rm tot} = \sigma_{\rm el} + \underbrace{\sigma_{\rm SD} + \overbrace{\sigma_{\rm DD}}^{}}_{\sigma_{\rm Diff}} \overbrace{+ \underbrace{\sigma_{\rm soft} + \sigma_{\rm hard}}_{\sigma_{\rm ND}}}^{\sigma_{\rm NSD}}$$

Collider cross sections



What is the Underlying event?

$$\sigma_{\mathrm{tot}} = \sigma_{\mathrm{el}} + \sigma_{\mathrm{SD}} + \overbrace{\sigma_{\mathrm{DD}} + \underbrace{\left(\sigma_{\mathrm{soft}} + \sigma_{\mathrm{hard}}\right)}_{\sigma_{\mathrm{ND}}}}^{\sigma_{\mathrm{NSD}}}$$



elastic

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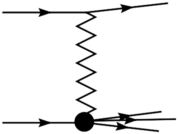
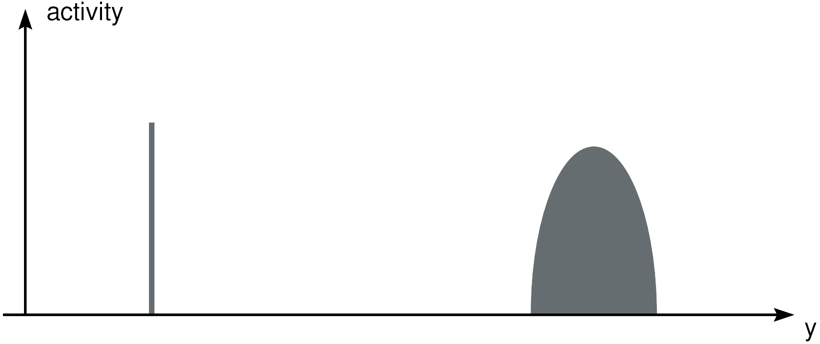
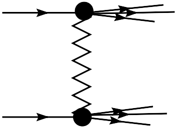


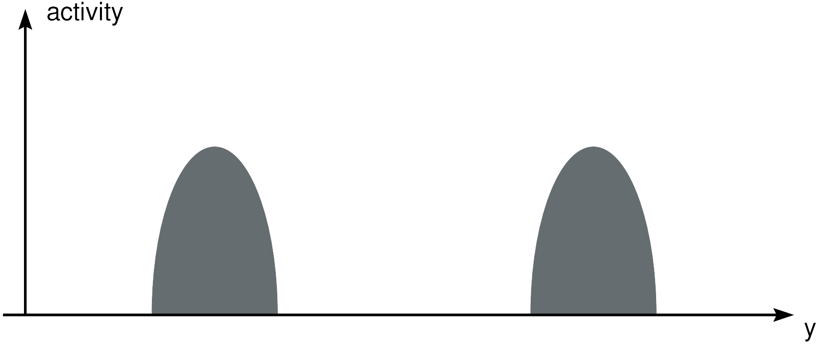


single diffractive

What is the Underlying event?

$$\sigma_{\mathrm{tot}} = \sigma_{\mathrm{el}} + \sigma_{\mathrm{SD}} + \overbrace{\color{red}{\sigma_{\mathrm{DD}}} + \underbrace{\left(\sigma_{\mathrm{soft}} + \sigma_{\mathrm{hard}}\right)}_{\sigma_{\mathrm{ND}}}}^{\sigma_{\mathrm{NSD}}}$$





double diffractive

What is the Underlying event?

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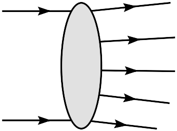

(multiple/soft) interactions

What is the Underlying event?

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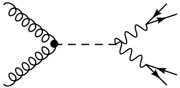
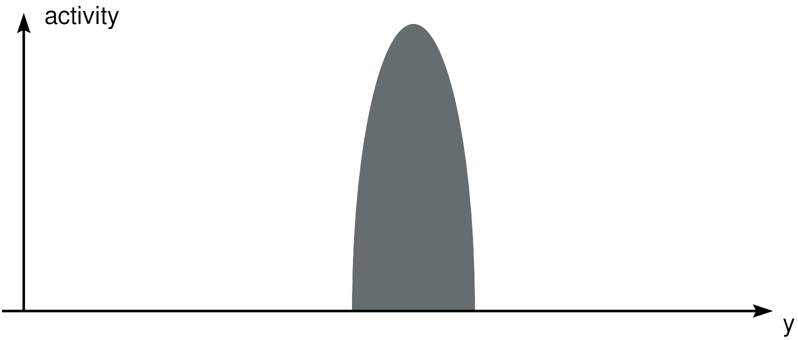
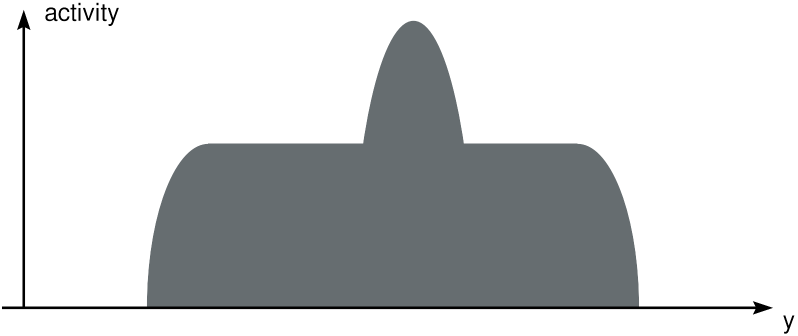




hard scattering

What is the Underlying event?

$$\sigma_{\text{tot}} = \sigma_{\text{el}} + \sigma_{\text{SD}} + \overbrace{\sigma_{\text{DD}} + \underbrace{(\sigma_{\text{soft}} + \sigma_{\text{hard}})}_{\sigma_{\text{ND}}}}^{\sigma_{\text{NSD}}}$$



hard scattering + underlying event

What is the Underlying event?

"Everything except the process of interest."

- Experimentalist: "includes parton showers etc."
- MC author: "everything on top of primary hard process."

The Underlying event (UE) is everywhere in the detector.

- Cannot select UE
- May spoil measurements.
- What characteristics?
- Hard?
- Soft?

Why should I learn about it?

- UE comes with every event.
- Can't trigger/select it away.
- Gives additional tracks and calorimeter hits, in the same cells as your signal.
- Jet energy scale determination.
- Important systematic error.
- Jets where your signal shouldn't give any (VBF).

Triggers

- Zero bias
  - *Every* event in a perfect 4π detector.

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- Minimum bias (MB)
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  - At least have to distinguish from noise/cosmics.
  - small number of tracks of charged tracks (e.g. 1, 2, 6),
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  - Often want non–single–diffractive

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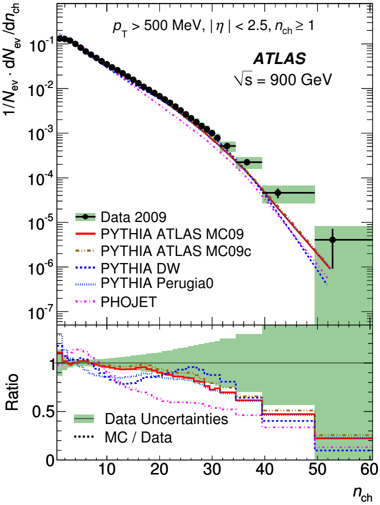
Physics in MB and UE very similar.

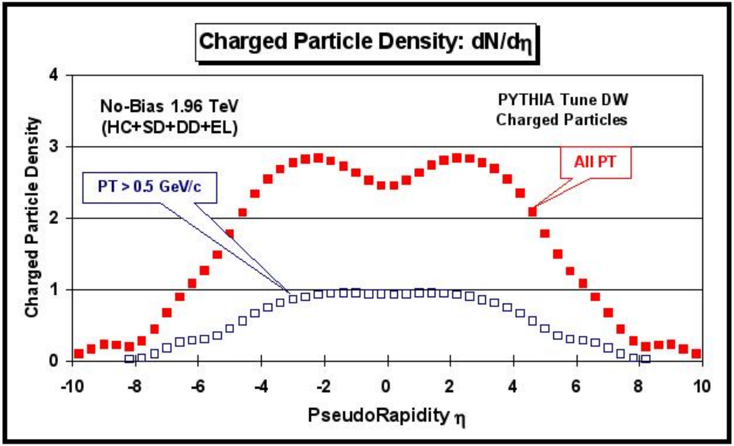
Charakteristics of MB events

N_{ch}



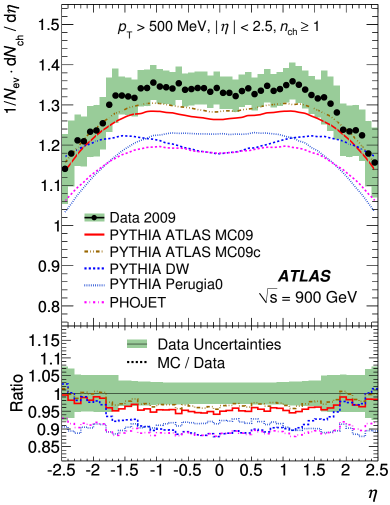
Charakteristics of MB events

$dN/d\eta$ Zero bias vs min bias (Tevatron)



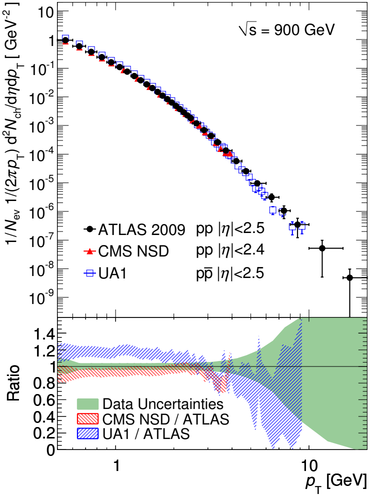
Charakteristics of MB events

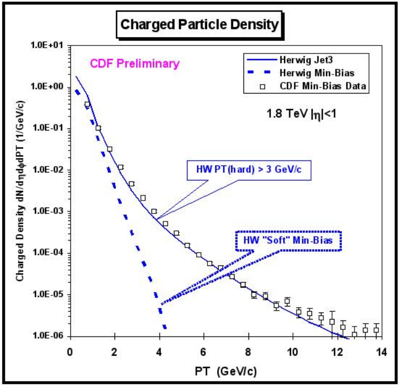
$dN/d\eta$ ATLAS



Charakteristics of MB events

$p_\perp$ spectra of all particles





Charakteristics of MB events

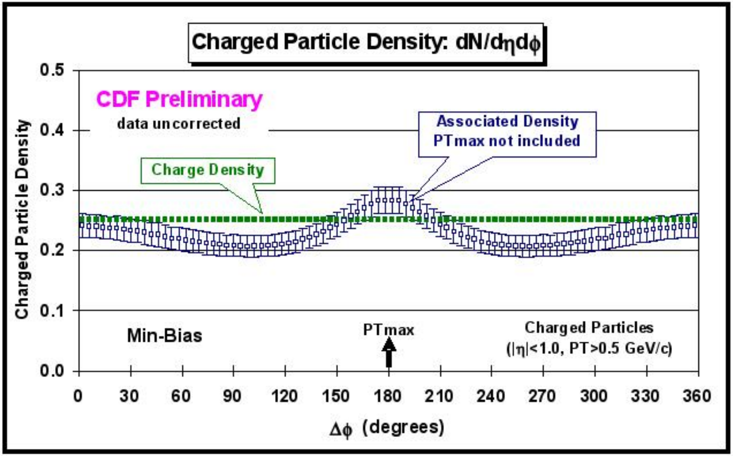
- Inclusive quantities have to be correct, of course.
- Already show, that soft component is important in modelling.

Charakteristics of MB events

- Inclusive quantities have to be correct, of course.
- Already show, that soft component is important in modelling.
- Don't tell much about morphology of event.
- $\rightarrow$ look at distributions inside detector.
- $\rightarrow$ leading particles.

Azimuthal distributions

Measure $\Delta\phi$ relative to leading particle/jet/track.



Azimuthal distributions

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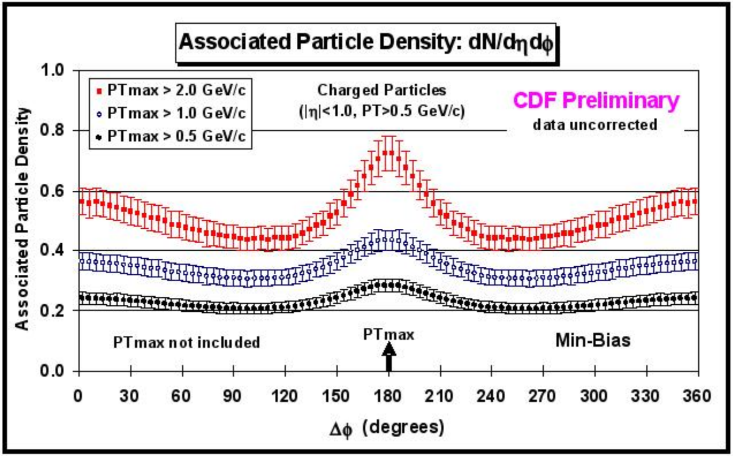


Azimuthal distributions

Observation:

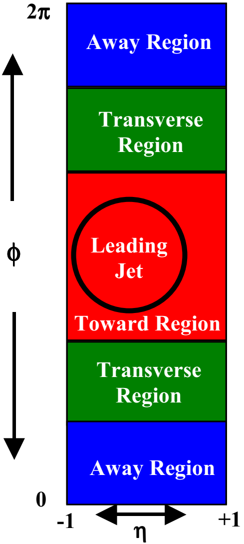
- Events not flat. Have 'leading object'.
- Harder leading object:
  - $\rightarrow$ harder recoil.
  - $\rightarrow$ more activity everywhere, also transverse.

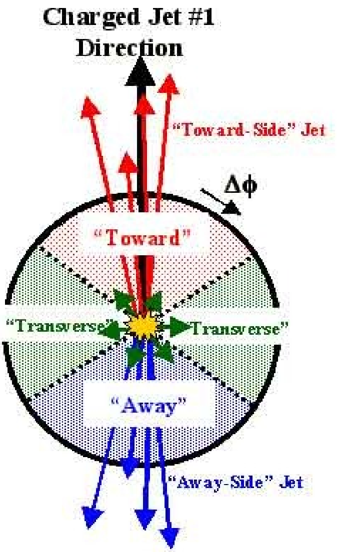
Trigger: The harder leading object, the more jets are inclusively just below this threshold (pedestal effect).

Closer look at transverse region!

"Rick Field analysis".

Towards, away, transverse

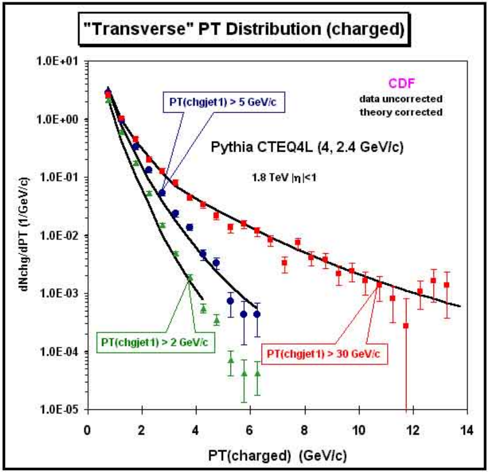




Measurements of the UE: separate from hard bit of event.

- How big is the 'activity' in the different regions?
- How does it depend on the leading object?
- If UE is really *underlying*,
  should decouple from leading event.

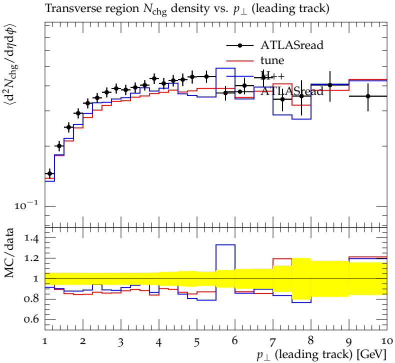
Spectrum in transverse region

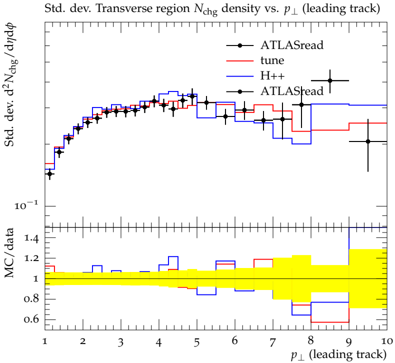


Not only average important. The UE has a jetty substructure!

Underlying Event (ATLAS 900 GeV)

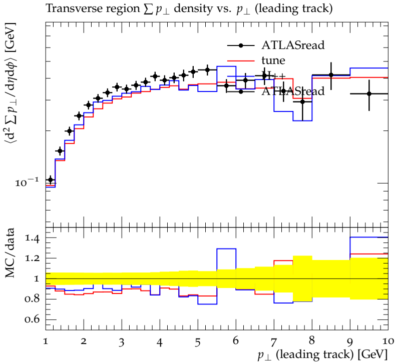
⟨"activity"⟩ and 1 σ deviation

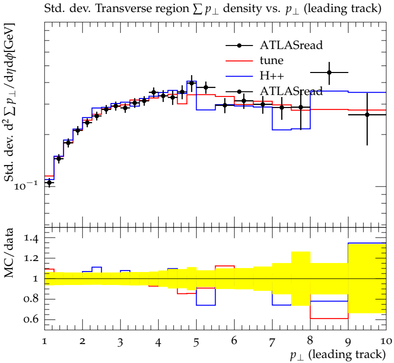




Underlying Event (ATLAS 900 GeV)

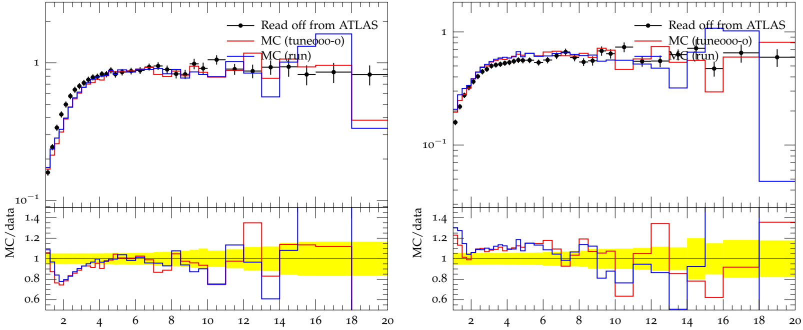
⟨"activity"⟩ and $1\,\sigma$ deviation





Underlying Event (ATLAS 7 TeV)

N_{ch} /StdDev transverse vs p_t^{lead} /GeV.

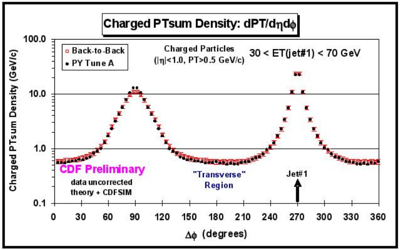


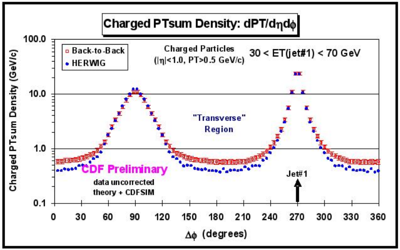
So far

- Idea of decoupling UE from hard event seems to hold.
- UE has jetty structure.
- Must contain hard physics as well.

More azimuthal distributions

Require at least two nearly b2b jets.
Dominated by hard physics.

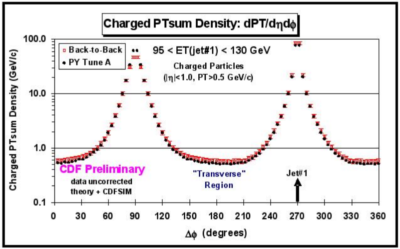


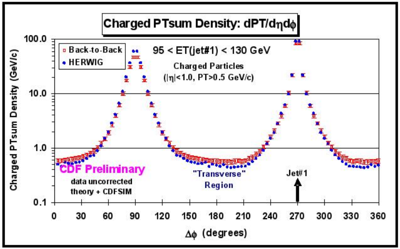


Old Herwig soft model not sufficient.

More azimuthal distributions

Require at least two nearly b2b jets.
Dominated by hard physics.

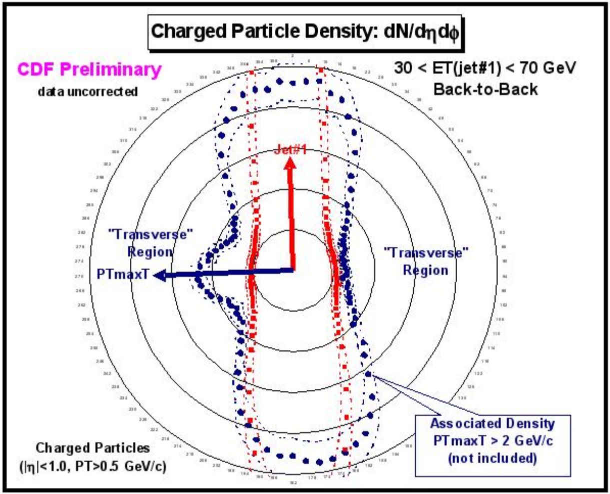




Better with harder jets.

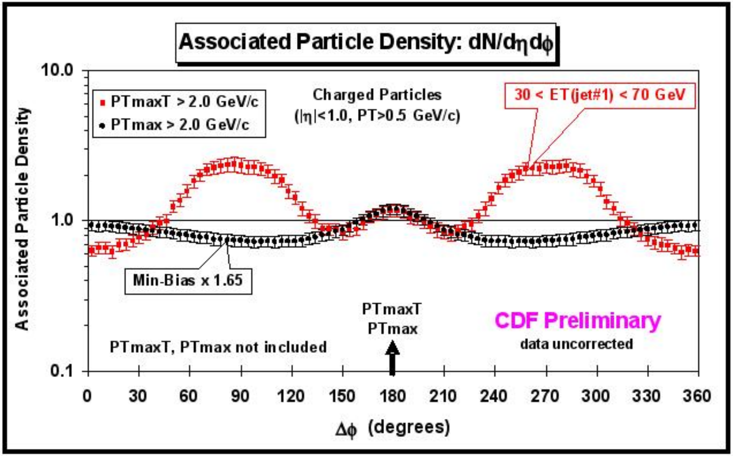
More azimuthal distributions

Now select the hardest of the two transverse regions only (TransMAX): associated distribution:



More azimuthal distributions

Now select the hardest of the two transverse regions only (TransMAX): associated distribution:



Birth of 3rd jet ~ leading jet in MinBias

Towards modelling

- Leading jet in Minimum bias $\sim$ 3rd jet in back–to–back sample.
- UE and MB really seem to reflect the same physics.
- Hard component important.
- Hard jets not sufficient
  (but well described $\to$ D0 dijet angular decorrelation).

Hard jets in the UE via multiple interactions?

- Additional Partonic $2 \to 2$ interactions (MPI).
- No correlation with hard event.

Indirect evidence for MPI

N_{ch} distribution (vs UA5; Sjöstrand, van Zijl (1987))

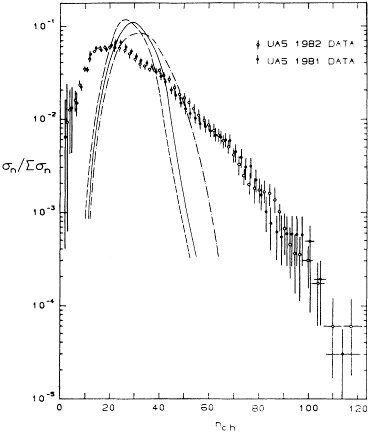


FIG. 3. Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs simple models: dashed low p_T only, full including hard scatterings, dash-dotted also including initial- and final-state radiation.

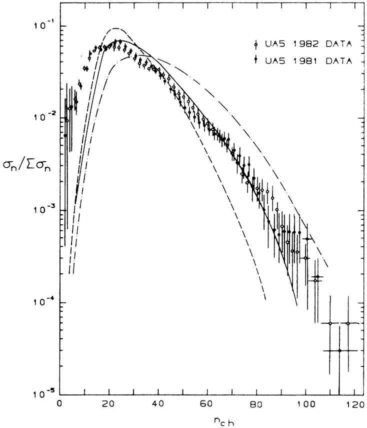


FIG. 5. Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs impact-parameter-independent multiple-interaction model: dashed line, $p_{T\min}=2.0$ GeV; solid line, $p_{T\min}=1.6$ GeV; dashed-dotted line, $p_{T\min}=1.2$ GeV.

no MPI (left)/MPI (right).

Indirect evidence for MPI

FB correlation in η bins (vs UA5; Sjöstrand, van Zijl (1987))

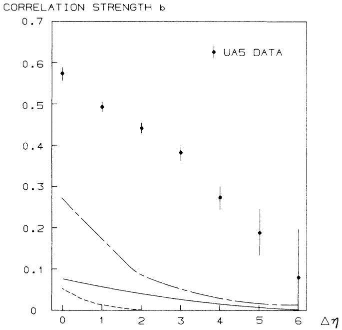


FIG. 4. Forward-backward multiplicity correlation at 540 GeV, UA5 results (Ref. 33) vs simple models; the latter models with notation as in Fig. 3.

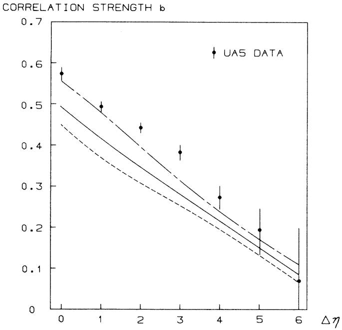
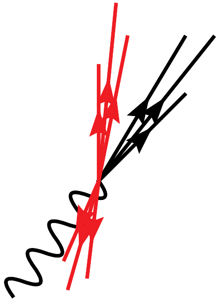


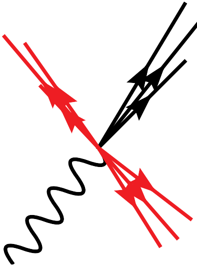
FIG. 6. Forward-backward multiplicity correlation at 540 GeV, UA5 results (Ref. 33) vs impact-parameter-independent multiple-interaction model; the latter with notation as in Fig. 5.

no MPI (left)/MPI (right).

Evidence for MPI

Angle ϕ from 4 final state objects (jets, γ).

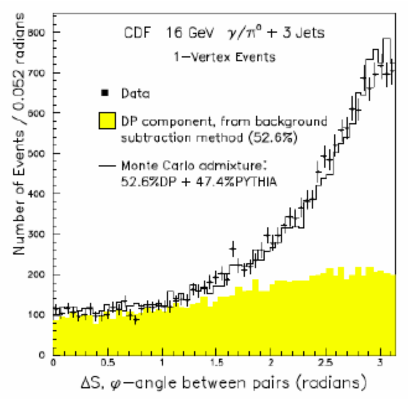




Evidence for MPI

Angle ϕ from 4 final state objects (jets, γ). Latest: CDF ('97).

$$\phi = \angle(\vec{p}_1 \pm \vec{p}_2, \vec{p}_3 \pm p_4)$$

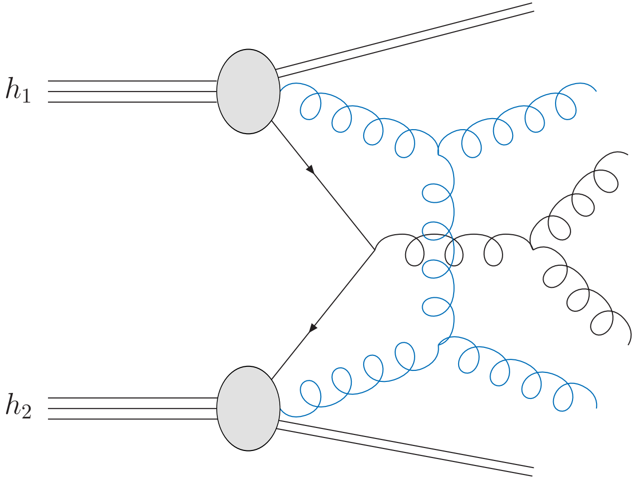


53% double parton scattering needed!

Modelling MPI (in Herwig)

Eikonal model basics

Mulitple hard interactions



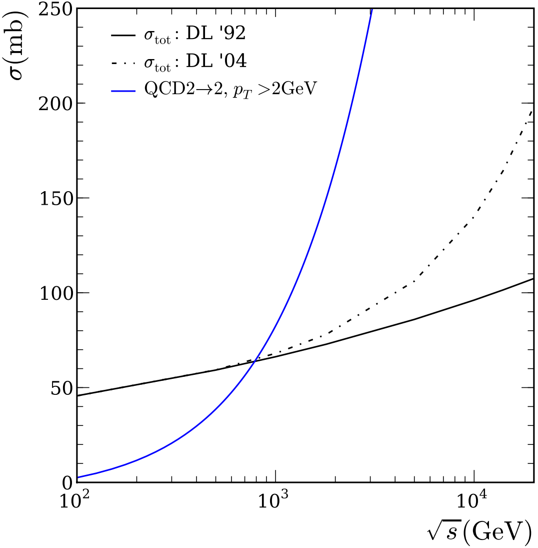
Eikonal model basics

Starting point: hard inclusive jet cross section.

$$\sigma^{\rm inc}(s;p_t^{\rm min}) = \sum_{i,j} \int_{{p_t^{\rm min}}^2} {\rm d}p_t^2 f_{i/h_1}(x_1,\mu^2) \otimes \frac{{\rm d}\hat{\sigma}_{i,j}}{{\rm d}p_t^2} \otimes f_{j/h_2}(x_2,\mu^2)\,,$$

$\sigma^{\rm inc} > \sigma_{\rm tot}$ eventually (for moderately small $p_t^{\rm min}$).

Eikonal model basics



Eikonal model basics

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$$\sigma^{\rm inc}(s;p_t^{\rm min}) = \sum_{i,j} \int_{p_t^{\rm min\,2}} {\rm d}p_t^2 f_{i/h_1}(x_1,\mu^2) \otimes \frac{{\rm d}\hat\sigma_{i,j}}{{\rm d}p_t^2} \otimes f_{j/h_2}(x_2,\mu^2)\,,$$

$\sigma^{\rm inc} > \sigma_{\rm tot}$ eventually (for moderately small $p_t^{\rm min}$).

Interpretation: $\sigma^{\rm inc}$ counts *all* partonic scatters that happen during a single *pp* collision ⇒ more than a single interaction.

$$\sigma^{\rm inc} = \bar{n}\sigma_{\rm inel}\,.$$

Eikonal model basics

Use eikonal approximation (= independent scatters). Leads to Poisson distribution of number m of additional scatters,

$$P_m(\vec{b},s) = \frac{\bar{n}(\vec{b},s)^m}{m!} \mathrm{e}^{-\bar{n}(\vec{b},s)} \ .$$

Then we get σ_{inel}:

$$\sigma_{\mathrm{inel}} = \int \mathrm{d}^2\vec{b} \sum_{m=1}^{\infty} P_m(\vec{b},s) = \int \mathrm{d}^2\vec{b} \left(1 - \mathrm{e}^{-\bar{n}(\vec{b},s)}\right) \ .$$

Eikonal model basics

Use eikonal approximation (= independent scatters). Leads to Poisson distribution of number m of additional scatters,

$$P_m(\vec{b},s) = \frac{\bar{n}(\vec{b},s)^m}{m!} \mathrm{e}^{-\bar{n}(\vec{b},s)} \; .$$

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Cf. σ_{inel} from scattering theory in eikonal approx. with scattering amplitude $a(\vec{b},s) = \frac{1}{2i}(\mathrm{e}^{-\chi(\vec{b},s)} - 1)$

$$\sigma_{\mathrm{inel}} = \int \mathrm{d}^2\vec{b} \left(1 - \mathrm{e}^{-2\chi(\vec{b},s)}\right) \qquad \Rightarrow \qquad \chi(\vec{b},s) = \tfrac{1}{2}\bar{n}(\vec{b},s) \; .$$

$\chi(\vec{b},s)$ is called *eikonal* function.

Eikonal model basics

Calculation of $\bar{n}(\vec{b}, s)$ from parton model assumptions:

$$\bar{n}(\vec{b}, s) = L_{\text{partons}}(x_1, x_2, \vec{b}) \otimes \sum_{ij} \int \mathrm{d}p_t^2 \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}p_t^2}$$

$$= \sum_{ij} \frac{1}{1+\delta_{ij}} \int \mathrm{d}x_1 \mathrm{d}x_2 \int \mathrm{d}^2\vec{b}' \int \mathrm{d}p_t^2 \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}p_t^2}$$

$$\times D_{i/A}(x_1, p_t^2, |\vec{b}'|) D_{j/B}(x_2, p_t^2, |\vec{b} - \vec{b}'|)$$

Eikonal model basics

Calculation of $\bar{n}(\vec{b},s)$ from parton model assumptions:

$$
\begin{aligned}
\bar{n}(\vec{b},s) &= L_{\text{partons}}(x_1,x_2,\vec{b}) \otimes \sum_{ij} \int \mathrm{d}p_t^2 \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}p_t^2} \\
&= \sum_{ij} \frac{1}{1+\delta_{ij}} \int \mathrm{d}x_1 \mathrm{d}x_2 \int \mathrm{d}^2\vec{b}' \int \mathrm{d}p_t^2 \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}p_t^2} \\
&\qquad \times D_{i/A}(x_1,p_t^2,|\vec{b}'|) D_{j/B}(x_2,p_t^2,|\vec{b}-\vec{b}'|) \\
&= \sum_{ij} \frac{1}{1+\delta_{ij}} \int \mathrm{d}x_1 \mathrm{d}x_2 \int \mathrm{d}^2\vec{b}' \int \mathrm{d}p_t^2 \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}p_t^2} \\
&\qquad \times f_{i/A}(x_1,p_t^2) G_A(|\vec{b}'|) f_{j/B}(x_2,p_t^2) G_B(|\vec{b}-\vec{b}'|) \\
&= A(\vec{b}) \sigma^{\text{inc}}(s;p_t^{\text{min}}) \ .
\end{aligned}
$$

Eikonal model basics

Calculation of $\bar{n}(\vec{b},s)$ from parton model assumptions:

$$\bar{n}(\vec{b},s) = L_{\text{partons}}(x_1,x_2,\vec{b}) \otimes \sum_{ij} \int \mathrm{d}p_t^2 \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}p_t^2}$$

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$$\times D_{i/A}(x_1,p_t^2,|\vec{b}'|) D_{j/B}(x_2,p_t^2,|\vec{b}-\vec{b}'|)$$

$$= \sum_{ij} \frac{1}{1+\delta_{ij}} \int \mathrm{d}x_1 \mathrm{d}x_2 \int \mathrm{d}^2\vec{b}' \int \mathrm{d}p_t^2 \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}p_t^2}$$

$$\times f_{i/A}(x_1,p_t^2) G_A(|\vec{b}'|) f_{j/B}(x_2,p_t^2) G_B(|\vec{b}-\vec{b}'|)$$

$$= A(\vec{b}) \sigma^{\text{inc}}(s;p_t^{\text{min}}) \,.$$

$$\Rightarrow \quad \chi(\vec{b},s) = \tfrac{1}{2}\bar{n}(\vec{b},s) = \tfrac{1}{2}A(\vec{b})\sigma^{\text{inc}}(s;p_t^{\text{min}}) \,.$$

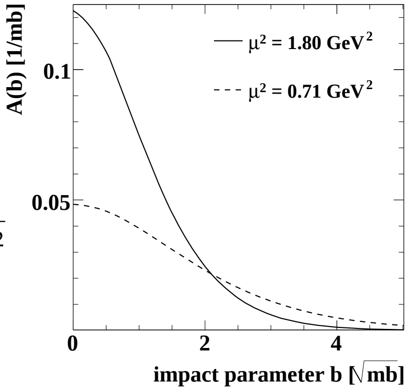
Overlap function

$$A(b) = \int d^2\vec{b}' G_A(|\vec{b}'|) G_B(|\vec{b}-\vec{b}'|)$$

$G(\vec{b})$ from electromagnetic FF:

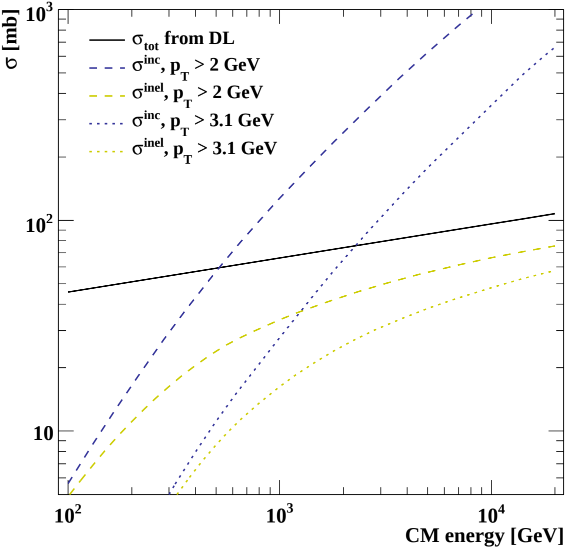
$$G_p(\vec{b}) = G_{\bar{p}}(\vec{b}) = \int \frac{d^2\vec{k}}{(2\pi)^2} \frac{e^{i\vec{k}\cdot\vec{b}}}{(1+\vec{k}^2/\mu^2)^2}$$

But μ^2 *not fixed* to the *electromagnetic* 0.71 GeV^2.
Free for colour charges.



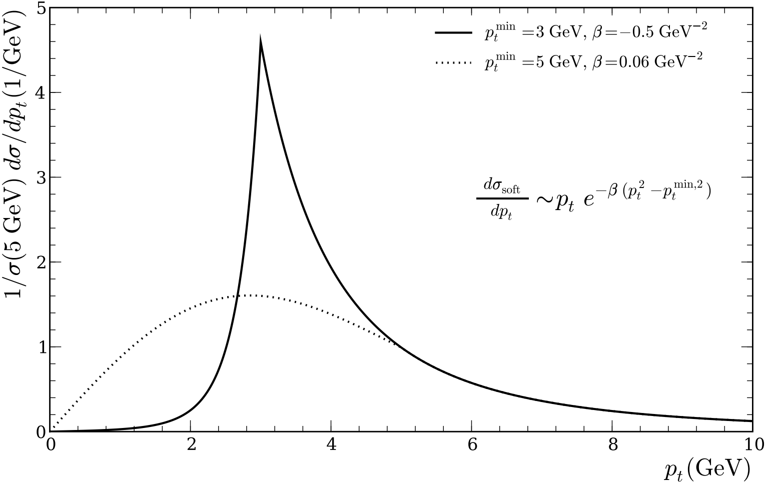
⇒ Two main parameters: $\mu^2, p_t^{\min}$.

Unitarized cross sections



Extending into the soft region

Continuation of the differential cross section into the soft region $p_t < p_t^{\min}$ (here: p_t integral kept fixed)



Hot Spot model

Fix the two parameters μ_{soft} and $\sigma^{\text{inc}}_{\text{soft}}$ in

$$\chi_{\text{tot}}(\vec{b},s) = \frac{1}{2}\left(A(\vec{b};\mu)\sigma^{\text{inc}}\text{hard}(s;p_t^{\text{min}}) + A(\vec{b};\mu_{\text{soft}})\sigma^{\text{inc}}_{\text{soft}}\right)$$

from two constraints. Require simultaneous description of σ_{tot} and b_{el} (measured/well predicted),

$$\sigma_{\text{tot}}(s) \overset{!}{=} 2\int \mathrm{d}^2\vec{b}\left(1 - \mathrm{e}^{-\chi_{\text{tot}}(\vec{b},s)}\right) ,$$

$$b_{\text{el}}(s) \overset{!}{=} \int \mathrm{d}^2\vec{b}\,\frac{b^2}{\sigma_{\text{tot}}}\left(1 - \mathrm{e}^{-\chi_{\text{tot}}(\vec{b},s)}\right) .$$

Diffractive final states

Strictly low mass diffraction only. Allow M^2 large nonetheless.

M^2 power-like, t exponential (Regge).

$$pp \to \text{(baryonic cluster)} + p \; .$$

Hadronic content from cluster fission/decay $C \to hh\ldots$
Cluster may be quite light. If very light, use directly

$$pp \to \Delta + p \; .$$

Also double diffraction implemented.

$$pp \to \text{(cluster)} + \text{(cluster)} \qquad pp \to \Delta + \Delta \; .$$

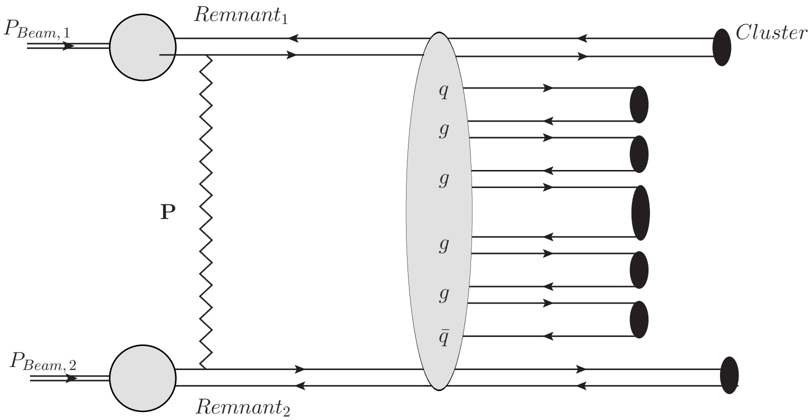
Technically: new MEs for diffractive processes set up.

Soft particle production model in Herwig

- #ladders = N_{soft} (MPI).
- N particles from Poissonian, width $\langle N \rangle$.
  Model parameter $1/\ln C \equiv n_{\text{ladder}} \rightarrow$ tuned.
- x_i smeared around $\langle x \rangle$ (calculated).
- $p_\perp$ from Gaussian acc to soft MPI model.
- particles are q, g, see figure.
  Symmetrically produced from both remnants.
- Colour connections between neighboured particles.

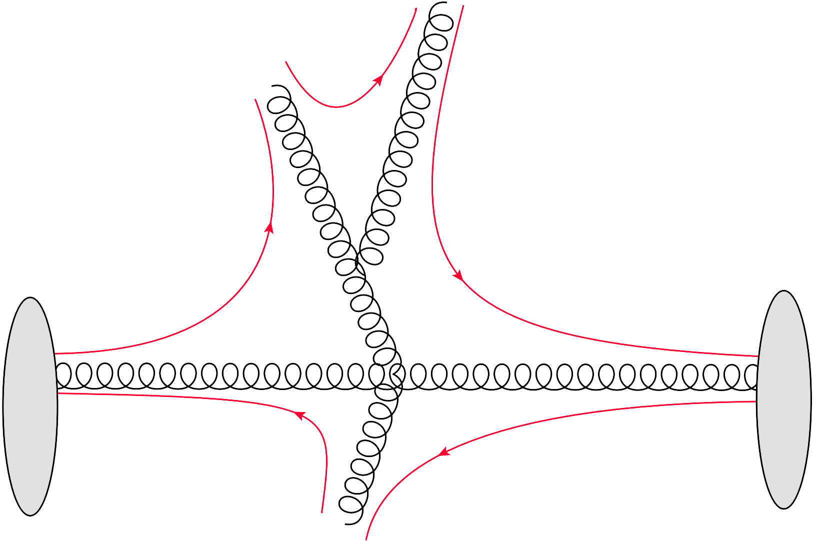
Soft particle production model in Herwig

Single soft ladder with MinBias initiating process.



Further hard/soft MPI scatters possible.

Colour correlations in hadronic collisions



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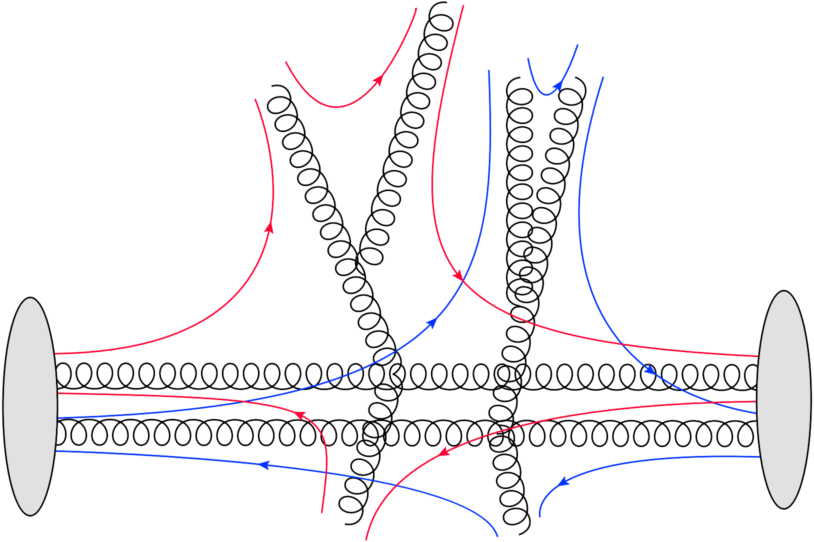
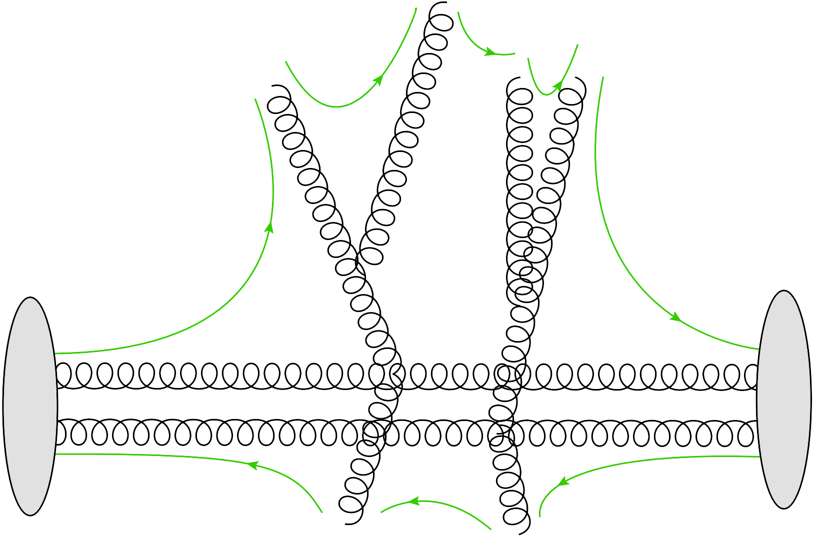
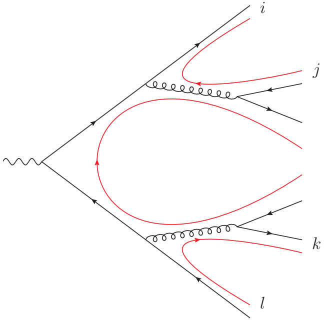


Colour correlations in hadronic collisions



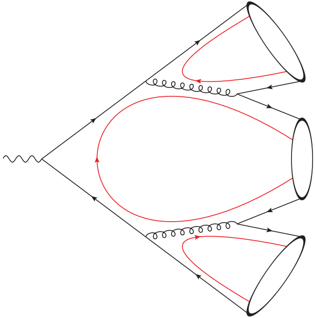
Colour reconnection (CR) in Herwig



Extend cluster hadronization:

- QCD parton showers provide *pre-confinement* ⇒ colour-anticolour pairs

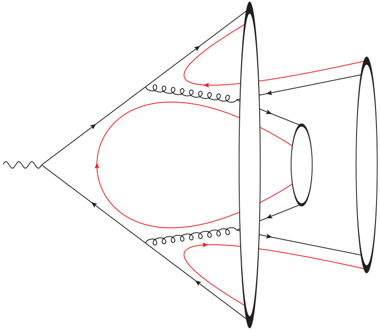
Colour reconnection (CR) in Herwig



Extend cluster hadronization:

- QCD parton showers provide *pre-confinement* $\Rightarrow$ colour-anticolour pairs
- $\rightarrow$ *clusters*

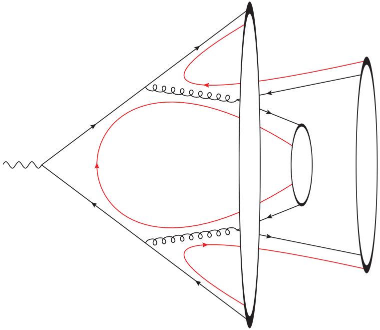
Colour reconnection (CR) in Herwig



Extend cluster hadronization:

- QCD parton showers provide *pre-confinement* $\Rightarrow$ colour-anticolour pairs
- $\rightarrow$ *clusters*
- CR in the cluster hadronization model: allow *reformation* of clusters, *e.g.* $(il) + (jk)$

Colour reconnection (CR) in Herwig



Extend cluster hadronization:

- QCD parton showers provide *pre-confinement* ⇒ colour-anticolour pairs
- → *clusters*
- CR in the cluster hadronization model: allow *reformation* of clusters, *e.g.* $(il) + (jk)$

Plain CR, iterate cluster pairs in "random order":

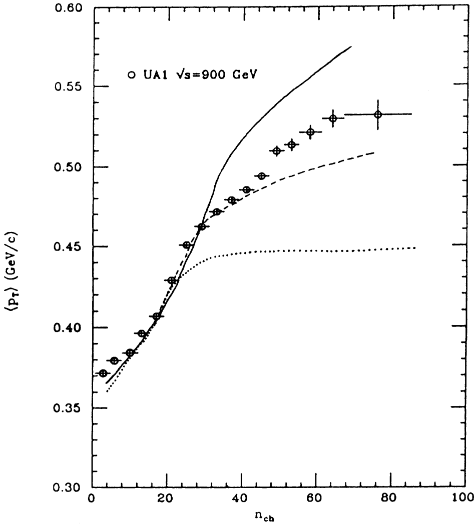
- Allow CR if the cluster mass decreases,

$$M_{il} + M_{kj} < M_{ij} + M_{kl},$$

- Accept alternative clustering with probability p_{reco} (model parameter) ⇒ this allows to switch on CR smoothly
- Alternative **Statistical CR** (Metropolis)

[SG, C. Röhr, A. Siodmok, EPJ C72 (2012) 2225]

Colour reconnections

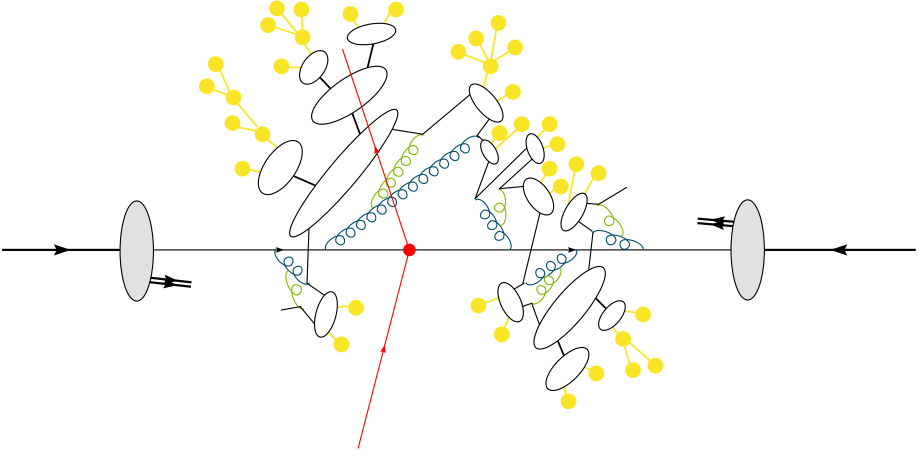


- Sensitivity to CR already known since UA1.
- (From Sjöstrand / van Zijl)

MPI Summary

- MPI (with colour reconnections) currently model of choice.
- Describes averages *and* fluctuations.
- Not always universal, but all models tunable.
- soft component needed for MB modelling.
- Constraints from inclusive cross sections.
- Different emphasis on hard/soft modelling between generators.
- Many details still only models.

Brief graphical summary



Brief graphical summary