## Machine Learning for Event Generation

Sapienta ex machina?

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MCnet School 2023 - Durham
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## Plan of attack

1. Machine learning for particle physics?
2. Normalizing flows
3. MadNIS
4. Summary and discussion

## PartI

## Machine learning for particle physics?

Why care about ML in particle physics?

## Why do we care about ML?



4
We care about machine learning because it can improve data analysis, simulation and modeling, lead to new discoveries, and foster cross-disciplinary collaboration

ChatGPT

## How machine learning often feels like

## Aim of this lecture:

Giving you the ideas to use it right!


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Giving you the ideas to use it right!

## Be aware!

The core of machine learning is to find structure in data - no more no less!


Thanks to machine-learning algorithms, the robot apocalypse was short-lived.

## LHC analysis (oversimplified)

Fundamental Theory


Simulation


## LHC analysis + ML






## ML improved simulations



## ML improved simulations



## ML improved simulations



## ML improved simulations



## Monte Carlo integration



## Monte Carlo integration



## Monte Carlo integration



## Importance sampling - VEGAS

Factorize probability

$$
p(x)=p\left(x_{1}\right) \cdots p\left(x_{n}\right)
$$

Fit bins with equal probability and varying width


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$\downarrow$
Fit bins with equal probability and varying width

$\rightarrow \oplus$ Computationally cheap
$\ominus$ High-dim and rich peaking functions
$\rightarrow$ slow convergence
$\ominus$ Peaks not aligned with grid axes
$\rightarrow$ phantom peaks


## How can we do better?

## Part II

Normalizing Flows

## Normalizing flow - Basics



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Conservation of probability: $p(x) \mathrm{d} x=p_{z}(z) \mathrm{d} z \quad$ with $\quad z=G(z) \quad x=\bar{G}(x)$

## Normalizing flow - Basics



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Change-of-variables formula: $p_{\theta}(x)=p_{z}\left(z=G_{\theta}(x)\right) \cdot\left|\frac{\partial G_{\theta}(x)}{\partial x}\right|$

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## Normalizing flow - Basics



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p(x) \mathrm{d} x=p_{z}(z) \mathrm{d} z \quad \text { with } \quad z=G(z) \quad x=\bar{G}(x)
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Change-of-variables formula: $\quad \log p_{\theta}(x)=\log p_{z}\left(z=G_{\theta}(x)\right)+\log \left|\frac{\partial G_{\theta}(x)}{\partial x}\right|$

## How to train it?

## Normalizing flow — Training

forward $G$


$$
\log p_{\theta}(x)=\log p_{z}\left(z=G_{\theta}(x)\right)+\log \left|\frac{\partial G_{\theta}(x)}{\partial x}\right|
$$

## Normalizing flow — Training

forward $G$


$$
\log p_{\theta}(x)=\log p_{z}\left(z=G_{\theta}(x)\right)+\log \left|\frac{\partial G_{\theta}(x)}{\partial x}\right|
$$

$\longrightarrow$ Match $p_{\theta}(x)$ with $p_{\text {data }}(x)$

## Normalizing flow — Training

forward $G$


$$
\log p_{\theta}(x)=\log p_{z}\left(z=G_{\theta}(x)\right)+\log \left|\frac{\partial G_{\theta}(x)}{\partial x}\right|
$$

Kullback-Leibler divergence:

$$
\begin{aligned}
\operatorname{KL}\left(p_{\text {data }}(x) \mid p_{\theta}(x)\right) & =\int \mathrm{d} x p_{\text {data }}(x) \log \frac{p_{\text {data }}(x)}{p_{\theta}(x)} \\
& =-\int \mathrm{d} x p_{\text {data }}(x) \log p_{\theta}(x)+\int \mathrm{d} x p_{\text {data }}(x) p_{\text {data }}(x)
\end{aligned}
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\end{aligned}
$$

Negative log-likelihood loss:

$$
L_{\mathrm{NLL}}=-\int \mathrm{d} x p_{\mathrm{data}}(x) \log p_{\theta}(x) \approx\left\langle-\log p_{\theta}(x)\right\rangle_{x \sim p_{\text {data }}}
$$

## Tractable Jacobian?

$\log p_{\theta}(x)=\log p_{z}\left(z=G_{\theta}(x)\right)+\log \left|\frac{\partial G_{\theta}(x)}{\partial x}\right| \rightarrow$ Requires tractable Jacobian!

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Solution: Autoregressive transformations $\quad z=\left(\begin{array}{c}z_{1} \\ \vdots \\ z_{d}\end{array}\right) \quad x=\left(\begin{array}{c}x_{1} \\ \vdots \\ x_{d}\end{array}\right)$

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$$
\begin{aligned}
& z_{1} \equiv z_{1}\left(x_{1}\right) \\
& z_{2} \equiv z_{2}\left(x_{1}, x_{2}\right) \\
& \vdots \\
& z_{d} \equiv z_{d}\left(x_{1}, x_{2}, \ldots, x_{d}\right)
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\end{aligned}
$$

$$
\longrightarrow J_{i j}(x)=\left(\begin{array}{cccc}
\frac{\partial z_{1}}{\partial x_{1}} & \frac{\partial z_{2}}{\partial x_{1}} & \cdots & \frac{\partial z_{z_{i}}}{\partial x_{1}} \\
0 & \frac{\partial z_{2}}{\partial x_{2}} & \cdots & \frac{\partial z_{d}}{\partial x_{1}} \\
\vdots & \ddots & & \vdots \\
0 & \ldots & 0 & \frac{\partial z_{d}}{\partial x_{d}}
\end{array}\right)
$$

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\end{array}\right) \longrightarrow \operatorname{det} J=\prod_{i} J_{i i} \sim \mathcal{O}(d)
$$

## Coupling block



Forward pass: $\begin{aligned} & z^{A}=C\left(x^{A} ; f_{\theta}\left(x^{B}\right)\right) \\ & z^{B}=x^{B}\end{aligned}$

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$$
J_{i j}(x)=\left(\begin{array}{cc}
\frac{\partial C}{\partial x^{A}} & \frac{\partial C}{\partial f_{\theta}} \frac{\partial f_{\theta}}{\partial x^{B}} \\
0 & I_{m}
\end{array}\right)
$$

## Coupling block



Forward pass: $\begin{aligned} & z^{A}=C\left(x^{A} ; f_{\theta}\left(x^{B}\right)\right) \\ & z^{B}=x^{B}\end{aligned}$
Inverse pass: $\begin{aligned} & x^{A}=C^{-1}\left(z^{A} ; f_{\theta}\left(z^{B}\right)\right) \\ & x^{B}=z^{B}\end{aligned}$

## Coupling block



## Coupling block



$$
\begin{aligned}
& \text { Forward pass: } \begin{array}{l}
z^{A}=C\left(x^{A} ; f_{\theta}\left(x^{B}\right)\right) \\
z^{B}=x^{B}
\end{array} \\
& \text { Inverse pass: } \begin{array}{l}
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$$

Affine $\quad C^{A}=\alpha_{\theta}\left(x^{B}\right) \cdot x^{A}+\mu_{\theta}\left(x^{B}\right)$ [1605.08803]


## Coupling block



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J_{i j}(x)=\left(\begin{array}{cc}
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Quadratic

$$
C=a x^{2}+b x+c
$$

Rational
quadratic
[1906.04032]

$$
C=\frac{a x^{2}+b x+c}{d x^{2}+e x+f}
$$

## Coupling block



$$
\begin{aligned}
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z^{A}=C\left(x^{A} ; f_{\theta}\left(x^{B}\right)\right) \\
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$$

$$
\begin{array}{|rl|}
\hline \text { Affine } & C^{A}=\alpha_{\theta}\left(x^{B}\right) \cdot x^{A}+\mu_{\theta}\left(x^{B}\right) \\
\text { Quadratic } & C=a x^{2}+b x+c \\
\text { [1600.088865] }
\end{array}
$$

Rational
quadratic
[1906.04032]

$$
C=\frac{a x^{2}+b x+c}{d x^{2}+e x+f}
$$

## Part III — MadNIS

## Neural Importance Sampling

## MadNIS - Basic functionality

$$
I=\sum_{i}\left\langle\alpha_{i}(x) \frac{f(x)}{g_{i}(x)}\right\rangle_{x \sim g_{i}(x)}
$$

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[^0]
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$$
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Use physics knowledge to construct channel and mappings


## MadNIS - Basic functionality

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I=\sum_{i}\left\langle\alpha_{i}(x) \frac{f(x)}{g_{i}(x)}\right\rangle_{x \sim g_{i}(x)}
$$

Use physics knowledge to construct channel and mappings


## MadNIS - Basic functionality

Phase space
$\Phi \subseteq \mathbb{R}^{N}$


Unit hypercube $U=[0,1]^{N}$


## Single channel $i$

## MadNIS - Basic functionality

## Phase space




## MadNIS - Overview

Basic functionality


## MadNIS - Overview

Basic functionality



## VEGAS initialization

Basic functionality



## VEGAS initialization


$\sqrt{7}$

Combine advantages:
Pre-trained VEGAS grid as starting point for flow training

## VEGAS initialization



## Bin reduction



64 VEGAS bins

## Bin reduction



64 VegAs bins


10 RQS bins

## Buffered training

Basic functionality



## Online Training



## Buffered training

## Training algorithm

generate new samples, train on them, save samples
$\downarrow$
train on saved samples $n$ times
$\downarrow$
repeat

Reduction in training statistics by

$$
R_{@}=n+1
$$



## LHC examples

Basic functionality


## LHC example I — Drell-Yan



## LHC example I — Drell-Yan



## LHC example I - Drell-Yan



## LHC example I — Drell-Yan



## LHC example I - Drell-Yan



## LHC example II — VBS



## LHC example II — VBS




Unweighting efficiency improved up to factor ~9 compared to VEGAS

## LHC example II — VBS



Unweighting efficiency improved up to factor $\sim 9$ compared to VEGAS


Big improvement from VEGAS initialization

## LHC example II — VBS

Significant improvement from trained channel weights


Unweighting efficiency improved up to factor $\sim 9$ compared to VEGAS


Big improvement from VEGAS initialization

## LHC example II — VBS

Buffered training: small effect on performance, much faster training


Unweighting efficiency improved up to factor $\sim 9$ compared to VEGAS

Significant improvement from trained channel weights


Big improvement from
VEGAS initialization

## LHC example III - W + 2 jets

Process has small interference terms
$\rightarrow$ no significant improvement from trained channel weights



Otherwise similar to results for VBS

## Summary and Outlook

## Take-home message

- Fast and precise predictions with ML-based simulations
- Normalizing flows provide statistically well-defined likelihoods for inference
- Account for uncertainties with Bayesian neural networks


## Future exercises

- Full integration of ML-based simulations into standard tools $\rightarrow$ MadGraph,....
- Make everything run on the GPU and differentiable (MadJax - Heinrich et al. [2203.00057])



## Summary and Outlook

Machine learning and LHC event generation
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## Abstract

First-principle simulations are at the heart of the high-energy physics research program. They link the vast data output of multi-purpose detectors with fundamental the ory predictions and interpretation. This review illustrates a wide range of applications of modern machine learning to event generation and simulation-based inference, includ ing conceptional developments driven by the specific requirements of particle physics. New ideas and tools developed at the interface of particle physics and machine learning will improve the speed and precision of forward simulations, handle the complexity of will improve the speed and precision of forward simulaions, hande the complexity of collision data, and enhance inference as an inverse simulation problem.


## Summary and Outlook



## Future exercises

－Full integration of ML－based simulations into standard tools $\rightarrow$ MadGraph，．．．．
－Make everything run on the GPU and differentiable（MadJax－Heinrich et al．［2203．000577）
－More details in our Snowmass report
－Stay tuned for many other ML4HEP applications

Modern machine learning technizues，including deep learning，is rapidly being apolied，adapted，and developed for higio energy
physics．The goal of this document s to provide a nearly comprehensive list of citations for those developing and appliving these physics．The goal of this document is to provide a nearly comprehensive list of citations for those developing and applying thes possible to incorporate the latest developments．A list of proper（unchanging）revieris can be found within．Papers are grouped into a small set of topics to be as usceful as possibic．Suggestions are mast welcome．

Expand all sections Collapse all sections
Reviews
－Modern roviews
I Specialized reviews
I．Classical papers
Furction Approximation
Symbolic Regressia


[^0]:    Use physics knowledge to construct channel and mappings

