Event generators for (high energy) Heavy Ion Collisions

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Who am I? & motivation of a heavy perspective

- Researcher at Lund University, PhD 2017, MCnet student.
  - Pythia (soft physics: strings, multiparton interactions, heavy ion collisions, space–time structure of collisions).
  - Rivet (heavy ion functionality, flow measurements).
- Research interest: Where heavy ions meet proton–proton .

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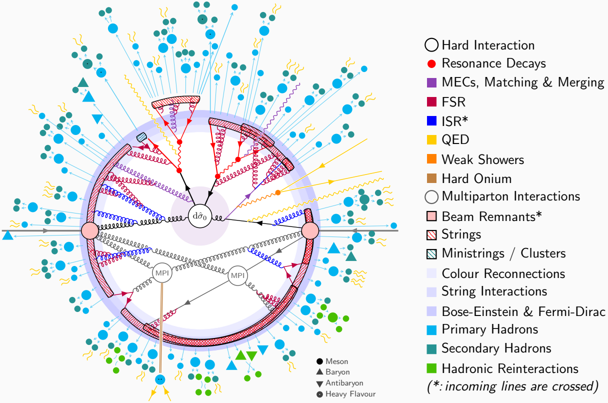
- Why? Heavy ions are The Wild West compared to pp.
- Order-of-magnitude effects vs. percent or per-mille corrections.

Proton collisions are the reference

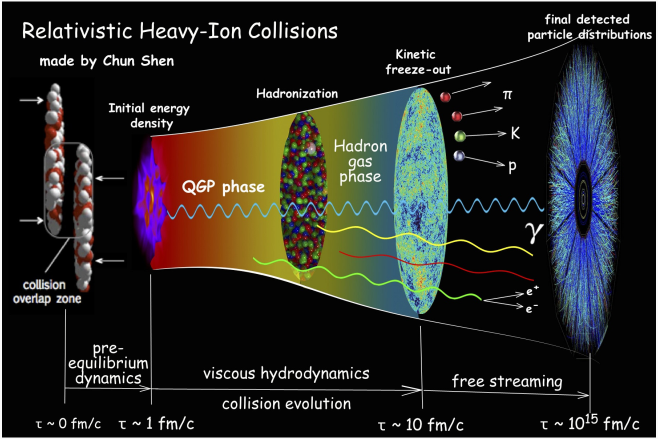
- They are complex beasts by themselves!



- But we think we have a general purpose prescription.
- Jet universality a cornerstone.

Standard model of heavy ion physics

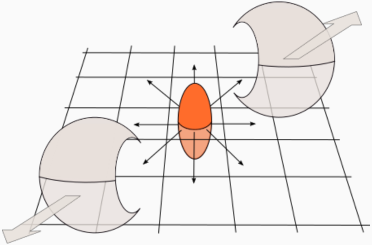
- Heavy ions traditionally viewed very differently.

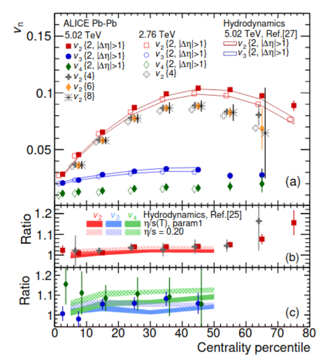


- Experimentally focused on properties of the QGP, viscosity, temperature, mean-free-path.

Flow: the collective behaviour of heavy ions

- Staple measurement: often modeled with hydrodynamics.
- Several MCEG treatments exist.





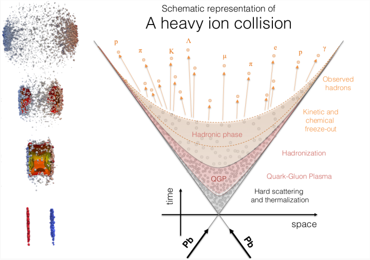
(ALICE: 1602.01119)

Fourier series decomposition of ϕ distribution:

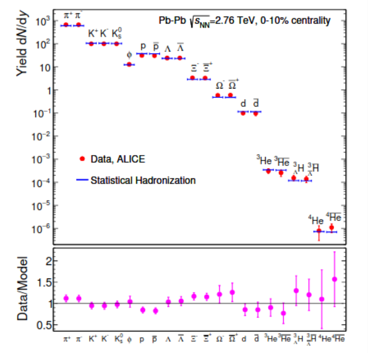
$$\frac{dN}{d\phi} \propto 1 + 2\sum_{n=1}^{\infty} v_n \cos\left[n(\phi - \Psi_n)\right]$$

Hadron abundances: a QGP thermometer

- The temperature when QGP ends: statistical hadronization.
- Describes total yields well with few parameters.



(Figure: D. Chinellato)

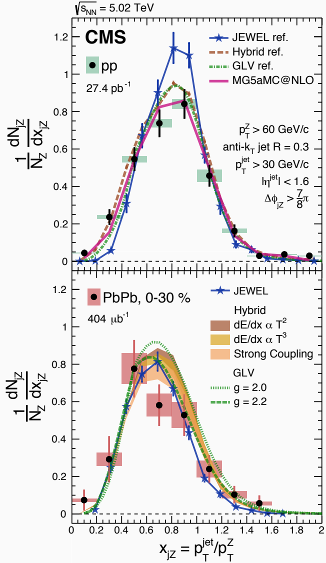


(Andronic et al: 1710.09425)

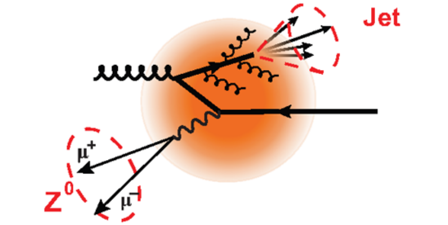
- No first principles dynamics. Must be included "by hand" in an MCEG.

Jet quenching (arXiv:1702.01060)

- Jet evolution affected by presence of QGP.

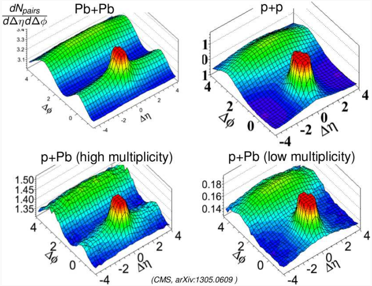


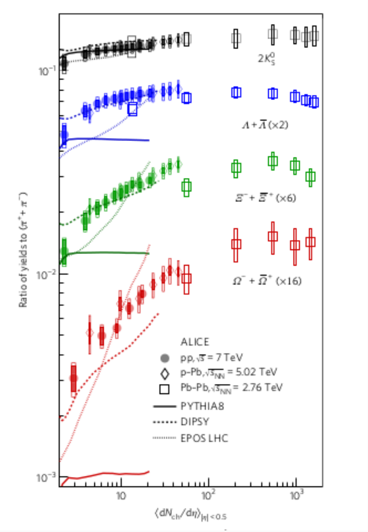
- Boson as calibrated reference.
- Fixed anti-$k_\perp$ R, jet broadens/softens.
- "Underlying event" difficult.
- Not found in small systems, intensive search.
- Will not be covered in this lecture.



Not so clear division!

- Heavy-ion like effects in pp collisions: Most surprising discovery of LHC .





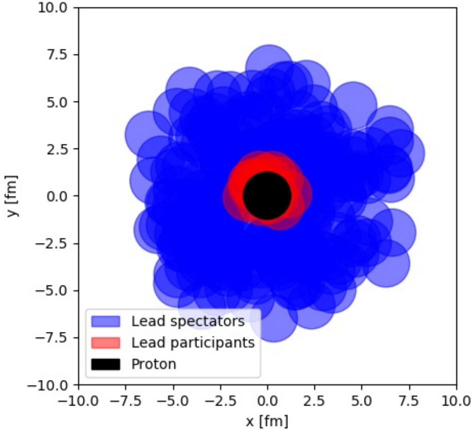
- Transition is smooth!
- Fully general purpose MCEG ($e^+e^- \rightarrow$ AA) more active than ever.

This lecture

- *The initial state*
  - The Glauber model.
  - Effective theory: The color glass condensate (CGC).
- *Total multiplicities*
  - HIJING/AMPT.
  - The Pythia/Angantyr treatment.
  - Color glass + HERWIG & PYTHIA.
- *Collective effects*
  - Parton shower modifications.
  - Some soft collective effects.
  - Hadronic rescattering.
- Not a complete overview, but my curated selection.
- Focus on concepts, details in bonus material + references.

The Glauber model

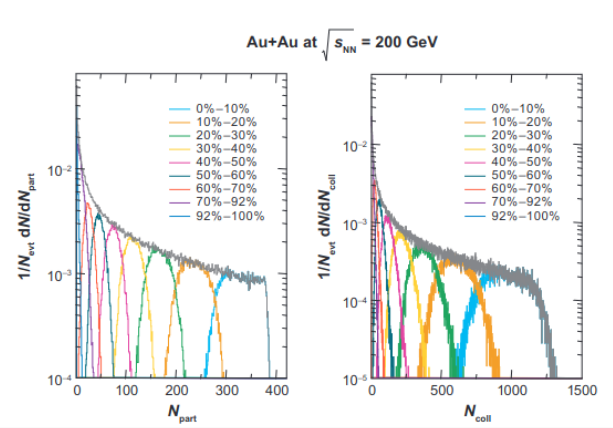
Nucleon size: $r_p = \sqrt{\sigma^{NN}_{\text{inel}}/4\pi}$



Participants and subcollisions

Basic geometric quantities readily available.

Not directly measurable, don't believe what they tell you!



Source of "centrality" binning.
Works fine in AA, ambiguous in *p*A.

(arXiv:0701025)

Scaling behaviours

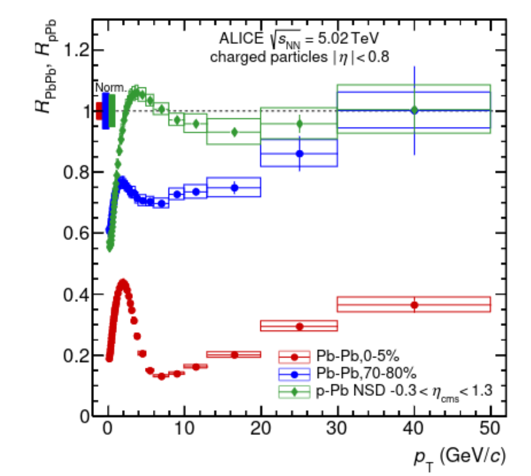
- Multiplicity scaling, observation (1970s, since formalized):
  - ♠ low $p_\perp$: scaling with N_{part}.
  - ♣ high $p_\perp$: scaling with N_{coll}.
- Formation time argument: In $p_L = 0$ frame $\tau_0 \geq 1/m_\perp$.

$$\tau_{\mathrm{lab}} = \gamma\tau_0 = \frac{E}{m_\perp^2} = \frac{\cosh y}{m_\perp}$$

- Minimal resolution scale $\lambda \geq v\tau_{\mathrm{lab}} = \frac{\sinh y}{m_\perp}$.
- Only fast particles can resolve individual partons in sub-collisions.
- Total multiplicity scales with number of wounded sources (N_{part}).

Nuclear modification factor

- Simple, scaled observables – no effect in pPb, what about pp?



(ALICE: JHEP11(2018)013)

- Percentages are centrality intervals

$$R_{AA} = \frac{\mathrm{d}N^{AA}/\mathrm{d}p_\perp}{\langle N_{coll}\rangle \mathrm{d}N^{pp}/\mathrm{d}p_\perp},$$

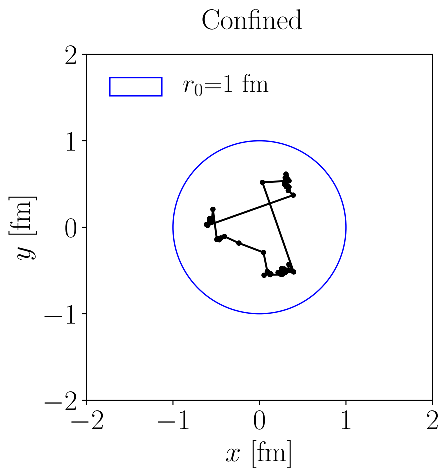
$R_{AA} > 1$: enhancement

$R_{AA} = 1$: no effect

$R_{AA} < 1$: suppression

Cross section fluctuations (arXiv:1907.12871, arXiv:1607.04434)

- Because protons are not just static balls.
- Substructure event by event → modified Glauber calculation (details in bonus material).



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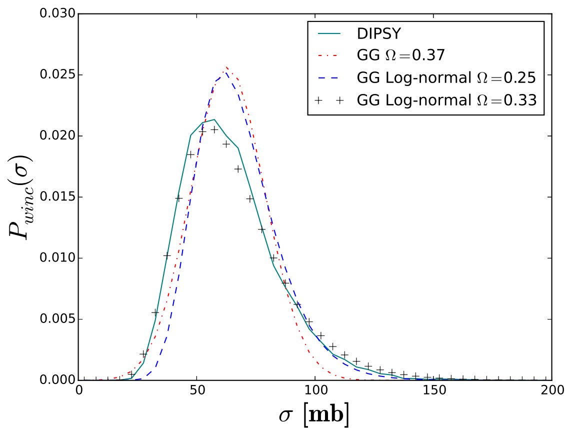

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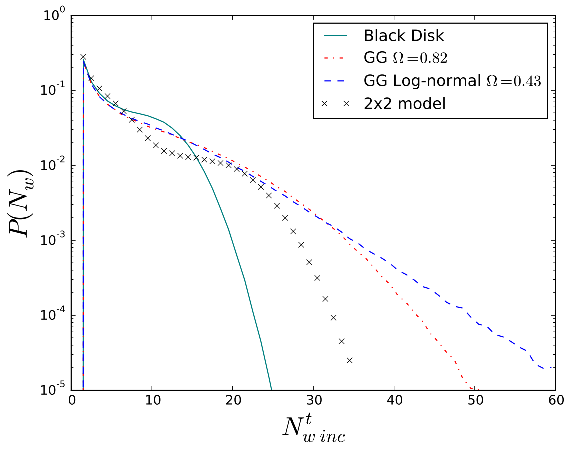

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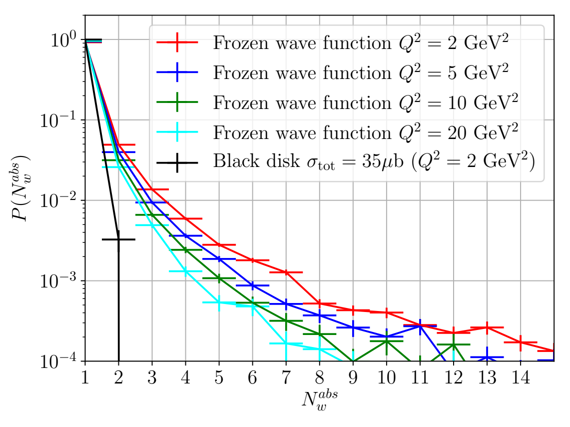
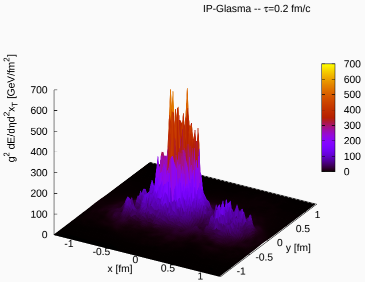
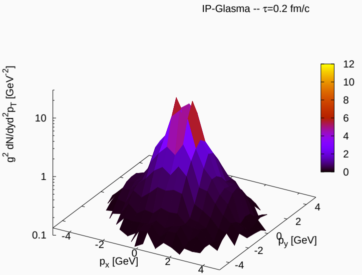


The color glass condensate (CGC)

- Treat incoming nuclei as classical colour fields.
- Evolved using "B-JIMWLK" (ask...), includes gluon saturation ($gg \to g$).
- DGLAP: gluon density increases with decreasing x, no limit.





(arXiv:2012.08493)

- But what to do with the fields or wounded nuclei? Stay tuned!

Particle production: HIJING and AMPT

- Both relies heavily on Pythia for nucleon-nucleon interactions.
- HIJING: No explicity (soft, hot) QGP effects:
  - Glauber initial state, no cross section fluctuations, nuclear PDFs.
  - NN cross section suppressed with geometrical shadowing factor
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  - Stack Pythia events, optional models for jet quenching.

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- AMPT = HIJING + extras = Pythia + extra extras.
  - ♠ Let strings melt, recover "partons" (fuzzy concept here).
  - ♣ Parton rescattering in final state.

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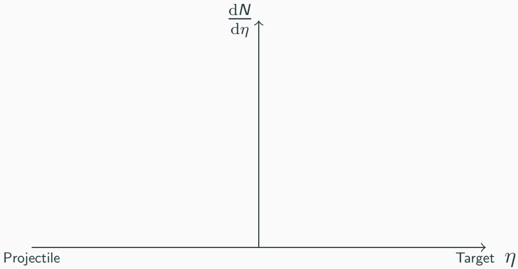
- Pythia + corrections: representative of many HI MC generators.
- Corrections may be very large!

Particle production: The Angantyr model (arXiv:1806.10820)

- Emission $F(\eta)$ per wounded nucleon
$$\rightarrow \frac{\mathrm{d}N}{\mathrm{d}\eta} = n_t F(\eta) + n_p F(-\eta).$$
- $F(\eta)$ modelled with even gaps in rapidity, as diffraction.
- Tuned to reproduce pp in the $n_t = n_p = 1$ case.
- No tunable parameters for AA – though some freedom in choices along the way.



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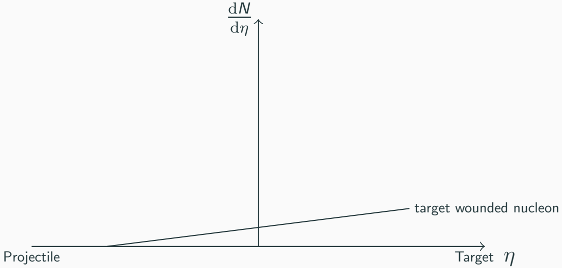

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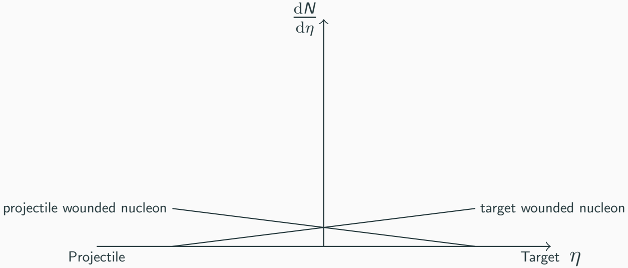

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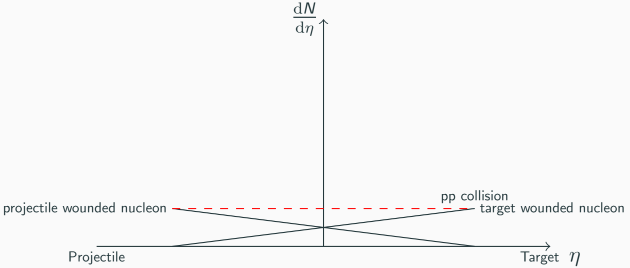

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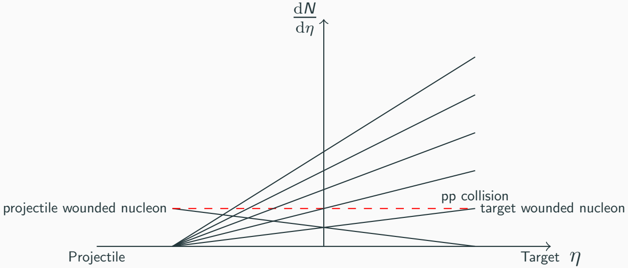

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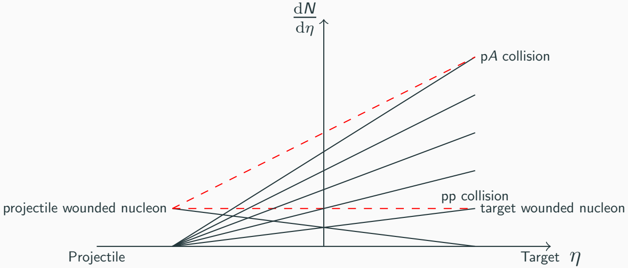

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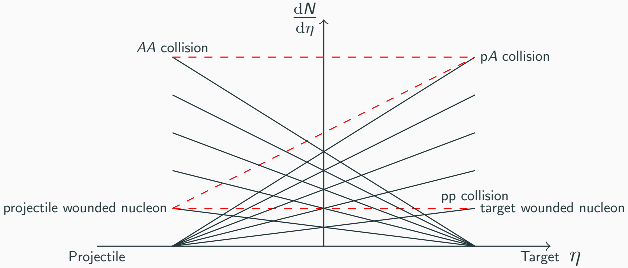
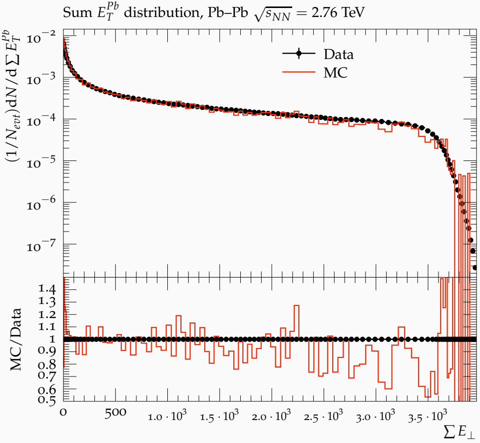


Angantyr results

- Reduces to normal Pythia in pp. In pA and AA:
  - ♠ Centrality measures & multiplicities.
  - ♣ Fluctuations more important in pA.



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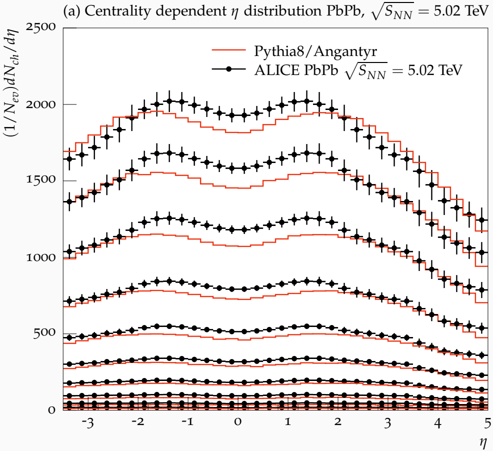

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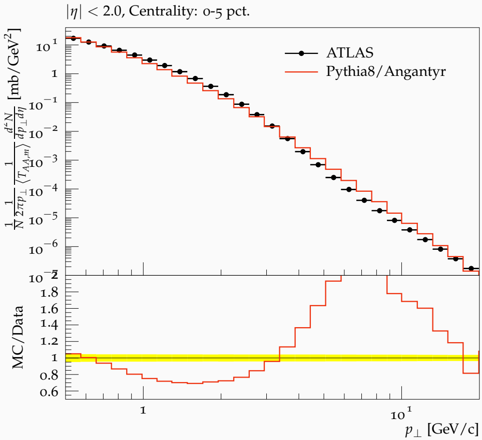

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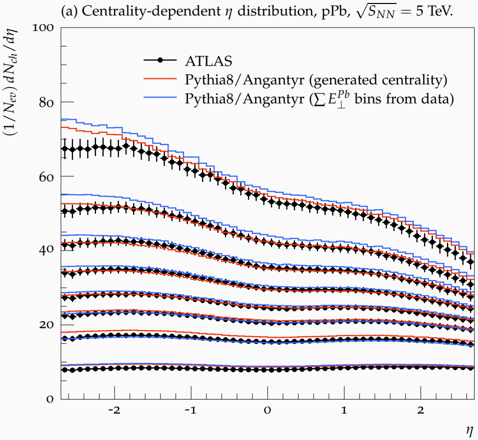

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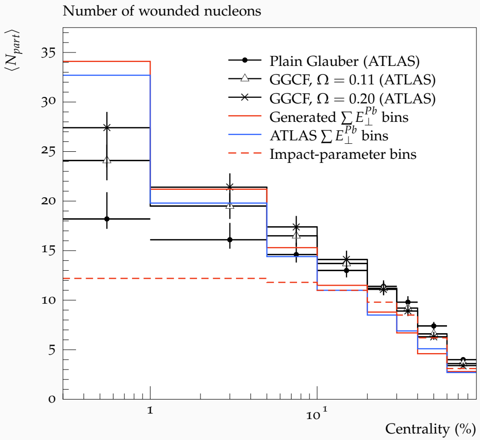
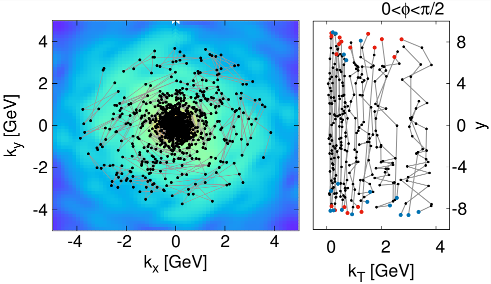


Particle production with CGC (arXiv:2012.08493, arXiv:1607.02496)

- A long way from classical fields to hadrons.
  - ♠ Standard path: decay to plasma → hydrodynamic expandision → hadronic freezeout.
  - ♣ Interesting development: Sample gluons (Weizsäcker-Williams) → hadronize with HERWIG or PYTHIA.
  - ♥ Retains correlations from initial state.
  - ♦ Colour connections (& energy density) are points of tension.



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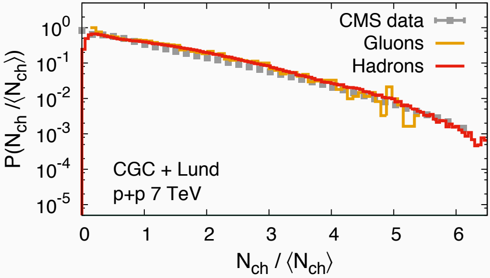

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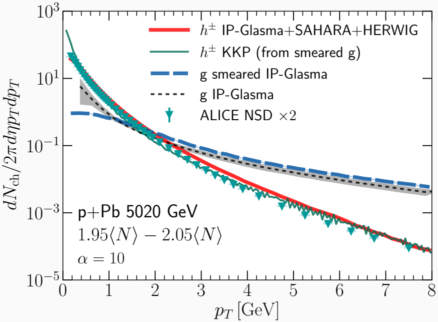


Collective effects

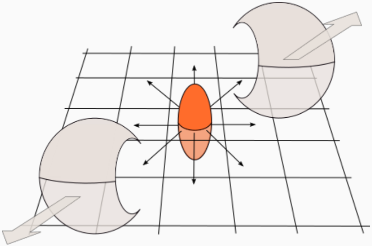
- Here: Umbrella term covering all effects arising from final state interactions, influenced by event geometry .
- Other people may have other definitions. Beware.
- Today:
  - ♠ Hydrodynamic expansion.
  - ♣ String interactions.
  - ♥ Hadronic rescattering.

Hydrodynamic expansion

- Thermalization → perfect fluid. Enegy-momentum tensor:

$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu - Pg^{\mu\nu}$ P is pressure, ε energy density, u^μ 4-velocity of fluid element.

- EOMs from cons. laws: $\partial_\mu T^{\mu\nu} = 0$ + Equation of state.
- Equation of state good for intuition:



- State–of–the art: 3+1D incl viscous terms. EOS with lattice input.
- MCEG: IP-Glasma + MUSIC + URQMD.

- Freeze-out when energy density is low enough.

Pythia: No QGP, just interacting strings

- Contrast to PYTHIA: Let us see how far just strings can take us.
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$\tau \approx 0.6$ **fm:** Parton shower ends. Depending on "diluteness", strings may shove each other around.

$\tau \approx 1$ **fm:** Strings at full transverse extension. Shoving effect maximal.

$\tau \approx 2$ **fm:** Strings will hadronize. Possibly as a colour rope.

$\tau > 2$ **fm:** Possibility of hadronic rescatterings.

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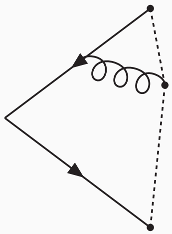

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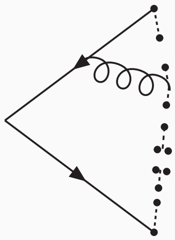
Flavour by tunnelling

$\mathcal{P} \propto \exp\left(-\frac{\pi m_{\perp}^{2}}{\kappa}\right)$, where m is the quark mass $\rightarrow$ parameter.



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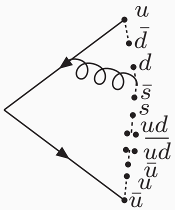
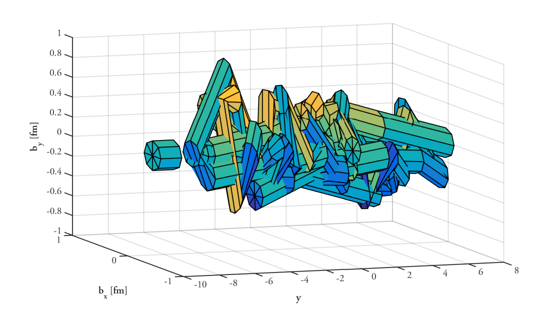
- Non-perturbative fragmentation, Lund strings, $\kappa \approx 1$ GeV/fm.

Flavour by tunnelling

$\mathcal{P} \propto \exp\left(-\frac{\pi m_\perp^2}{\kappa}\right)$, where m is the quark mass → parameter.

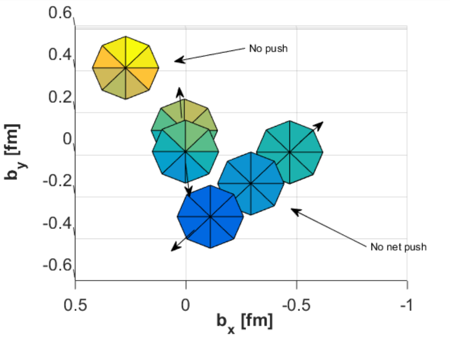




But many strings overlap in pp collisions!

Shoving: The cartoon picture (arXiv:1710.09725, arXiv:2010.07595)

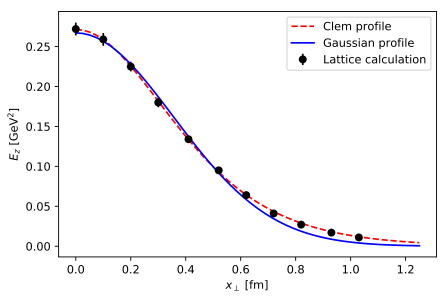
- Strings push each other in transverse space.
- Colour-electric fields → classical force.



👍 Transverse-space geometry.
👍 Particle production mechanism.
?? String radius and shoving force

MIT bag model, dual superconductor or lattice?

- Easier analytic approaches, eg. bag model:
  $\kappa = \pi R^2[(\Phi/\pi R^2)^2/2 + B]$
- Bad R 1.7 and dual sc. 0.95 respectively, shape of field is input.
- Lattice can provide shape, but uncertain R.



- Solution: Keep shape fixed, but R ballpark-free.

The shoving force

- Energy in field, in condensate and in magnetic flux.
- Let g determine fraction in field, and normalization N is given:

$$E = N \exp(-\rho^2/2R^2)$$

- Interaction energy calculated for transverse separation $d_\perp$, giving a force:

$$f(d_\perp) = \frac{g\kappa d_\perp}{R^2} \exp\left(-\frac{d_\perp^2}{4R^2}\right)$$

- Distance calculated in "shoving frame", resolved as two-string interactions.

Rope Hadronization (arXiv:1412.6259 – explored heavily in 80's and 90's!)

- Overlapping strings combine into multiplet with effective string tension $\tilde{\kappa}$.

Effective string tension from the lattice

$$\kappa \propto C_2 \Rightarrow \frac{\tilde{\kappa}}{\kappa_0} = \frac{C_2(\text{multiplet})}{C_2(\text{singlet})}.$$

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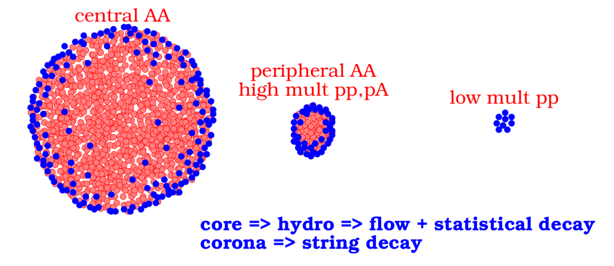
Strangeness enhanced by:

$$\rho_{LEP} = \exp\left(-\frac{\pi(m_s^2 - m_u^2)}{\kappa}\right) \to \tilde{\rho} = \rho_{LEP}^{\kappa_0/\kappa}$$

- QCD + geometry extrapolation from LEP.
- Can never do better than LEP initial conditions!

EPOS: The core-corona model (arXiv:0704.1270, https://klaus.pages.in2p3.fr/epos4/)

- In the same event:
  - ♠ Single-string treatment at low densities.
  - ♣ Full QGP treatment at high densities.

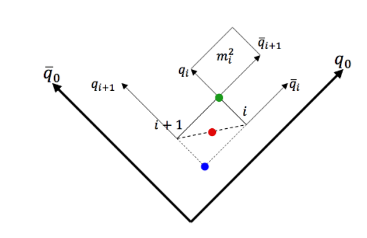


(Figure credit: Klaus Werner)

- Geometric interpolation between two extremes.
- Ambitious MCEG, closest to general purpose on market.

Hadronic Rescattering (arXiv:2103.09665, arXiv:2005.05658, arXiv:1808.04619)

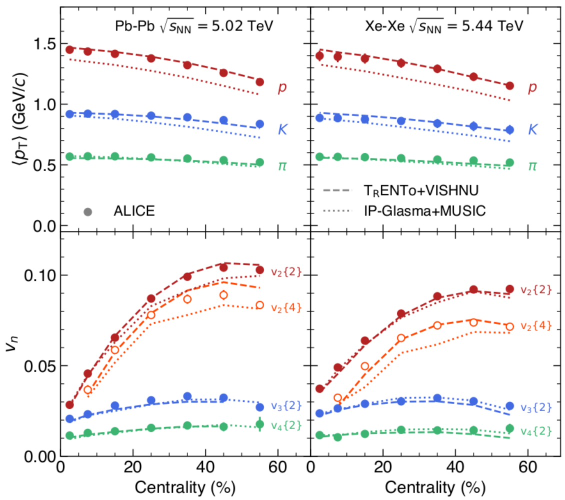
- Several implementations, (URQMD is standard reference) – here Pythia.
- Rescattering requires hadron space–time vertices.
- Key difference to existing approaches: Earlier hadronization $\tau \approx 2$ fm.
- Momentum-space to space-time breakup vertices through string EOM: $v_i = \frac{\hat{x}_i^+ p^+ + \hat{x}_i^- p^-}{\kappa}$
- Hadron located between vertices: $v_i^h = \frac{v_i + v_{i+1}}{2} \left(\pm \frac{p_h}{2\kappa} \right)$



- Formalism also handles complex topologies.
- Hadron cross sections from Regge theory or data.

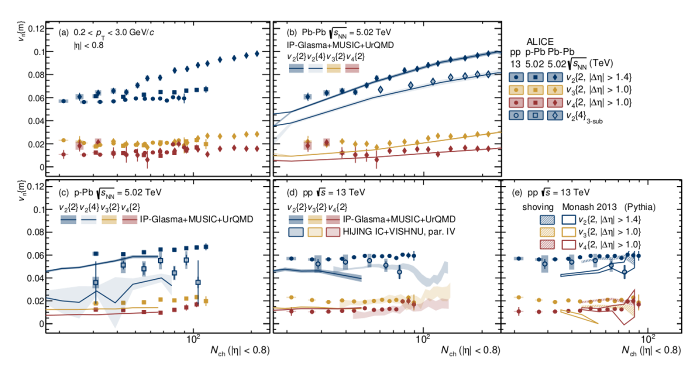
Hydrodynamics does very well for flow (arXiv:2211.04384)

- Special purpose "generators", different hydro implementations.



String shoving competetive in small systems (arXiv:2211.04384)

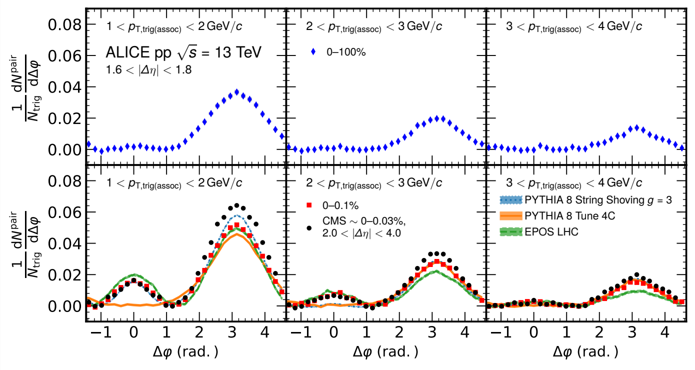
- Probably cannot distinguish models with such inclusive observables.



- In Pythia, download and play around.

Add a hard probe? (arXiv:2101.03110)

- Changes to the UE, must be modelled correctly.
- Cannot be done by special purpose EGs.



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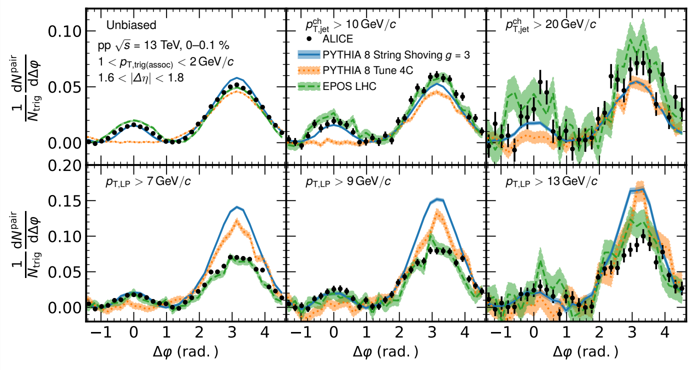

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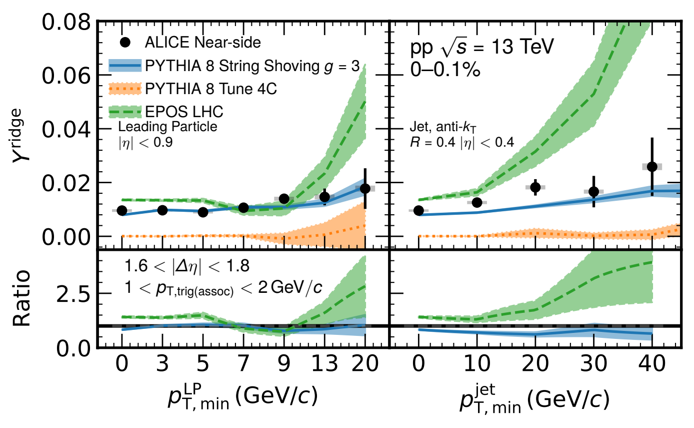
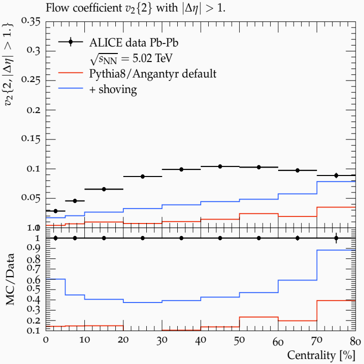
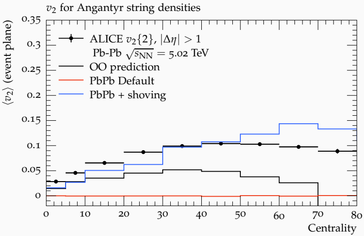


String shoving in large systems (arXiv:2010.07595)

- We are getting there, but slowly.



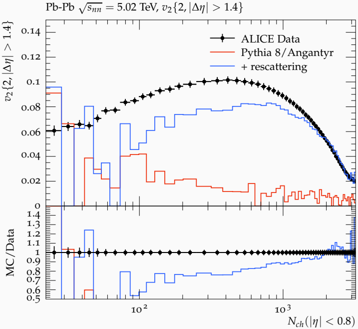


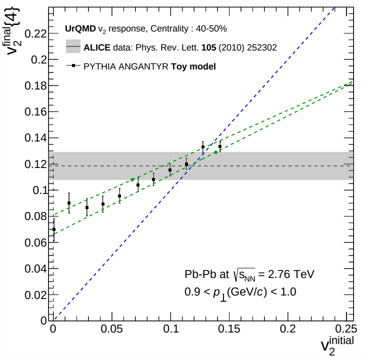
Toy configuration, not real events.

- Goal: A full microscopic description, across all systems.
- These results without hadronic rescattering.

Hadronic rescattering (arXiv:2002.10236, arXiv:2103.09665)

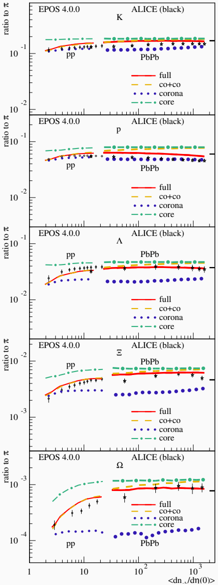
- Crucial for large systems, very sensitive to system lifetime.



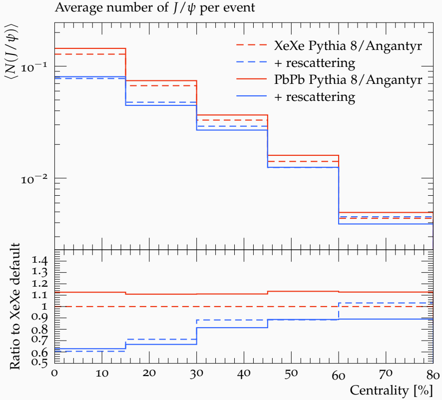


- Not trivial to combine effects!

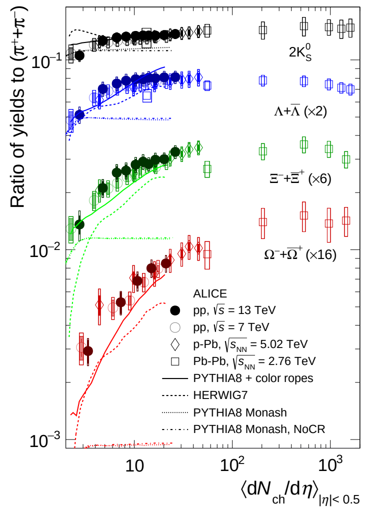
Hadronic rescattering and flavour (arXiv:2306.10277, arXiv:2103.09665)



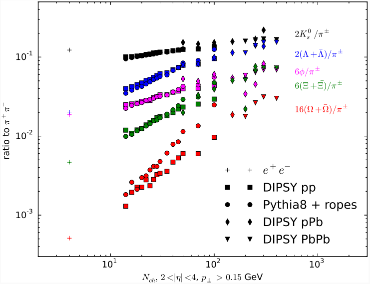
- Crucial for large systems, very sensitive to system lifetime.
- EPOS left, uses URQMD.
- Pythia below, heavy flavour.



Rope hadronization from small to large (arXiv:2003.02394, arXiv:1807.05271)



- Rope production works in pp, download Pythia and play.
- Extension to pA and AA is still work in progress.



How to continue from here? (arXiv:2003.10997)

- Many different models on the market, each with their niche.
- Messy models, difficult to place limits and get on with your life.
- Rivet + global χ^2 = profit?
  - ♠ model uncertainties not under control.
  - ♣ most are special purpose calculations.
  - ♥ attempts (Bayesian) exist, and might eventually be succesful.
- Another route: Qualitative differences.

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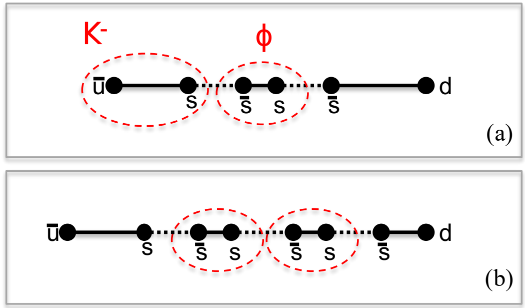

Summary

- There is no single general purpose MC for heavy ions. (Yet. EPOS comes quite close).
- Myriad of models to describe same effects: event generators allow for honest comparisons .
- Border between small and large systems is vanishing quickly.
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  - ♣ jet quenching, HBT, thermal charm, flow correlations, critical point searches, thermal photons, statistical hadronization, kinetic theory, nuclear PDFs, etc...
- Best student resources on conference "student days" or dedicated summer schools. Ask if interested.
- Thank you for your attention!
- Thank you for nice nightcap discussions!

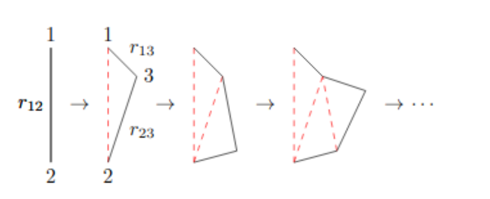
Bonus material

1. B-JIMWLK from dipoles.
2. Glauber model with fluctuating cross sections and frozen projectiles.
3. Strings with very soft gluon kinks.

BFKL, B-JIMWLK and all that...

- Start with Mueller dipole branching probability:

$$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}y} = \mathrm{d}^2\vec{r}_3 \; \frac{N_c\alpha_s}{2\pi^2}\frac{r_{12}^2}{r_{13}^2 r_{23}^2} \equiv \mathrm{d}^2\vec{r}_3 \; \kappa_3.$$



- Evolve any observable $O(y) \to O(y + \mathrm{d}y)$ in rapidity:

$$\bar{O}(y{+}\mathrm{d}y) = \mathrm{d}y \int \mathrm{d}^2\vec{r}_3\, \kappa_3 \left[O(r_{13}) \otimes O(r_{23})\right] + O(r_{12})\left[1 - \mathrm{d}y \int \mathrm{d}^2\vec{r}_3 \; \kappa_3\right]$$

$$\to \frac{\partial\bar{O}}{\partial y} = \int \mathrm{d}^2\vec{r}_3 \; \kappa_3 \left[O(r_{13}) \otimes O(r_{23}) - O(r_{12})\right].$$

A powerful formalism!

- Example: S-matrix (eikonal approximation, b-space):

$$O(r_{13}) \otimes O(r_{23}) \to S(r_{13})S(r_{23})$$

- Change to $T \equiv 1 - S$:

$$\frac{\partial \overline{\langle T \rangle}}{\partial y} = \int \mathrm{d}^2 \vec{r}_3 \; \kappa_3 \left[\langle T_{13} \rangle + \langle T_{23} \rangle - \langle T_{12} \rangle - \langle T_{13} T_{23} \rangle \right].$$

- B-JIMWLK equation, but could be written with other observables.
- Example: Average dipole coordinate ($\langle z \rangle$):

$$\frac{\partial \overline{\langle z \rangle}}{\partial y} = \int \mathrm{d}^2 \vec{r}_3 \kappa_3 \left(\frac{1}{3} z_3 - \frac{1}{6}(z_1 + z_2) \right).$$

Good–Walker & cross sections

- Cross sections from $T(\vec{b})$ with normalizable particle wave functions:

$$\sigma_{\rm tot} = 2\int \mathrm{d}^2\vec{b}\,\Gamma(\vec{b}) = 2\int \mathrm{d}^2\vec{b}\,\langle T(\vec{b})\rangle_{p,t}$$

$$\sigma_{\rm el} = \int \mathrm{d}^2\vec{b}\,|\Gamma(\vec{b})|^2 = \int \mathrm{d}^2\vec{b}\,\langle T(\vec{b})\rangle^2_{p,t}$$

$$B_{\rm el} = \frac{\partial}{\partial t}\log\left(\frac{\mathrm{d}\sigma_{\rm el}}{\mathrm{d}t}\right)\Big|_{t=0} = \frac{\int \mathrm{d}^2\vec{b}\; b^2/2\,\langle T(\vec{b})\rangle_{p,t}}{\int \mathrm{d}^2\vec{b}\,\langle T(\vec{b})\rangle_{p,t}}$$

- Or with photon wave function:

$$\sigma^{\gamma^* \rm p}(s) = \int_0^1 \mathrm{d}z \int_0^{r_{\rm max}} r\mathrm{d}r \int_0^{2\pi} \mathrm{d}\phi \left(|\psi_L(z,r)|^2 + |\psi_T(z,r)|^2\right)\sigma_{\rm tot}(z,\vec{r})$$

Cross section colour fluctuations

- Cross section fluctuates event by event: important for pA, $\gamma^* A$ and less AA.
- Projectile remains frozen through the passage of the nucleus.
- Consider fixed state (k) projectile scattered on single target nucleon:

$$\Gamma_k(\vec{b}) = \langle \psi_S | \psi_I \rangle = \langle \psi_k, \psi_t | \hat{T}(\vec{b}) | \psi_k, \psi_t \rangle =$$
$$(c_k)^2 \sum_t |c_t|^2 T_{tk}(\vec{b}) \langle \psi_k, \psi_t | \psi_k, \psi_t \rangle =$$
$$(c_k)^2 \sum_t |c_t|^2 T_{tk}(\vec{b}) \equiv \langle T_{tk}(\vec{b}) \rangle_t$$

- And the relevant amplitude becomes $\langle T_{t_i,k}^{(nN_i)}(\vec{b}_{ni}) \rangle_t$

Fluctuating nucleon-nucleon cross sections

- Let nucleons collide with total cross section $2\langle T\rangle_{p,t}$
- Inserting frozen projectile recovers total cross section.
- Consider instead inelastic collisions only (color exchange, particle production):

$$\frac{\mathrm{d}\sigma_{\mathrm{inel}}}{\mathrm{d}^2\vec{b}} = 2\langle T(\vec{b})\rangle_{p,t} - \langle T(\vec{b})\rangle^2_{p,t}.$$

- Frozen projectile will not recover original expression, but requre target average first.

$$\frac{\mathrm{d}\sigma_w}{\mathrm{d}^2\vec{b}} = 2\langle T_k(\vec{b})\rangle_p - \langle T_k^2(\vec{b})\rangle_p = 2\langle T(\vec{b})\rangle_{t,p} - \langle\langle T(\vec{b})\rangle^2_t\rangle_p$$

- Increases fluctuations! But pp can be parametrized.

Strings with very soft gluon kinks

- String geometries can get quite complicated!