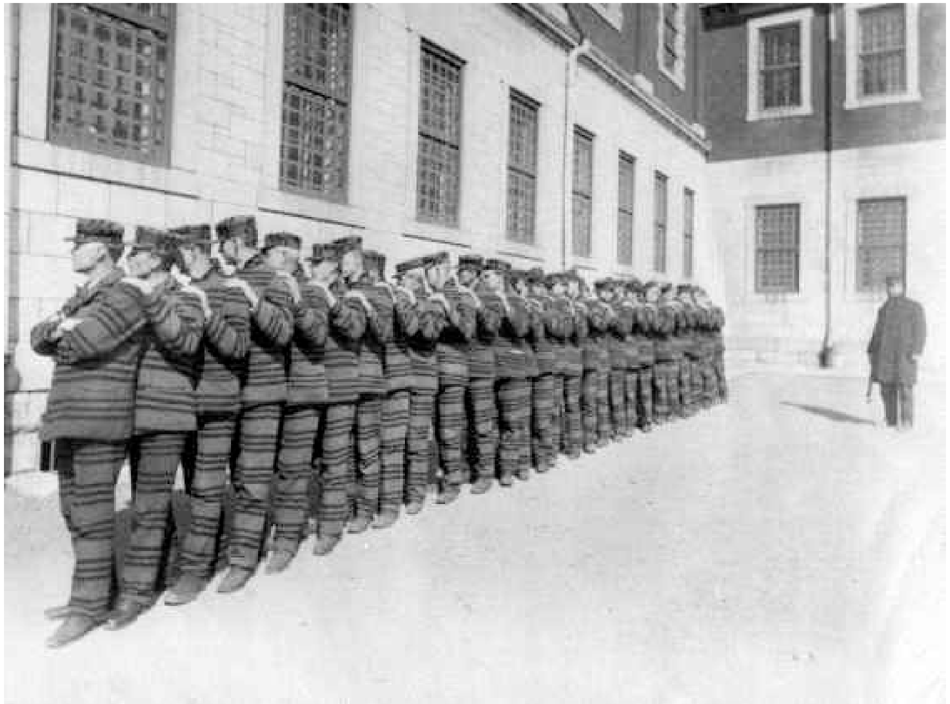
Status of the Confinement Problem



Jeff Greensite

Annual Theory Meeting

Durham, England

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We would like to also *understand* QCD, i.e. how it does what it does.

In particular we would like to understand confinement.

In this area progress has been slow, and there is still no general agreement about how confinement comes about.

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There is even disagreement about what we are trying to explain...

Outline

1. What *is* Confinement?

   Is it distinguished from "non-Confinement" by some symmetry?

2. Relevance of the gauge-Higgs model, and the ambiguity of spontaneous gauge symmetry breaking.

3. Order parameters and center symmetry

4. Status of current approaches (briefly!)

   vortices, monopoles, calorons, Dyson-Schwinger eqns., vacuum wavefunctionals...

   (for AdS/CFT, see Zarembo/Mateos/Wiedemann...)

5. Can we still be surprised?

6. Is there now a proof?

What is Confinement?

Juliet:

"What's in a name? That which we call a rose
By any other name would smell as sweet."

Romeo and Juliet (II, ii, 1-2)

What are people trying to prove, in order to “prove” confinement?
And what do they *mean* by that word?

1. linear static quark potential, rising to infinity ← most order parameters

2. colorless asymptotic particle states ← common terminology

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against #2 - asymptotic particle states are also colorless in a Higgs theory, where there is no linear potential at all.

so are broken gauge theories confining?

The Fradkin-Shenker-Osterwalder-Seiler Theorem

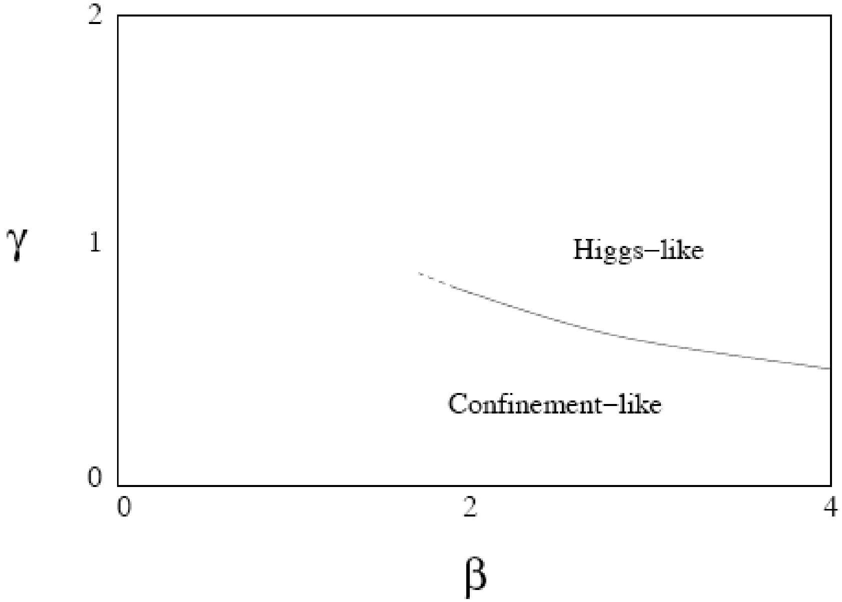
Consider an SU(2) gauge-Higgs theory with lattice action

$$S = \beta \sum_{plaq} \frac{1}{2}\mathrm{Tr}[UUU^\dagger U^\dagger] + \gamma \sum_{x,\mu} \frac{1}{2}\mathrm{Tr}[\phi^\dagger(x) U_\mu(x) \phi(x+\widehat{\mu})]$$

It has a phase diagram something like this:

The theorem says that there is no complete separation between the Higgs-like and the confinement-like regions.

No abrupt change from colorless to colored spectrum.



So what about spontaneous gauge symmetry breaking?

Elitzur's Theorem:

Local gauge symmetries do not break spontaneously. In the absence of gauge fixing, $\langle \varphi \rangle = 0$ regardless of the shape of the Higgs potential.

However, one can always fix to some gauge, e.g. Landau or Coulomb, having some residual gauge symmetry. These residual symmetries *can* break spontaneously.

Gauge Condition	Residual symmetry	unbroken realization required by the
Landau gauge:	g(x,t) = g	Kugo-Ojima confinement criterion
Coulomb gauge:	g(x,t) = g(t)	Coulomb confinement scenario (symmetry unbroken on any time-slice)

Kugo-Ojima criterion (covariant gauges)

Says that $\langle \text{phys}|Q^a|\text{phys}\rangle = 0$ if a certain operator condition is satisfied, *and* if the remnant gauge symmetry in Landau gauge is unbroken. In the gauge-Higgs model it requires

$$\langle \phi \rangle = 0$$

Coulomb confinement (Coulomb gauge)

Confining color Coulomb potential. Gribov and Zwanziger
(measured from the correlator of timelike links)

The scenario implies unbroken remnant gauge symmetry in Coulomb gauge

$$\left\langle \frac{1}{L^3} \sum_{\mathbf{x}} \mathrm{Tr}[U_0(\mathbf{x},t)] \right\rangle \to 0$$

Marinari, Paciello, Parisi, Taglienti

Either criterion can work in real QCD, so is this what we mean by confinement?

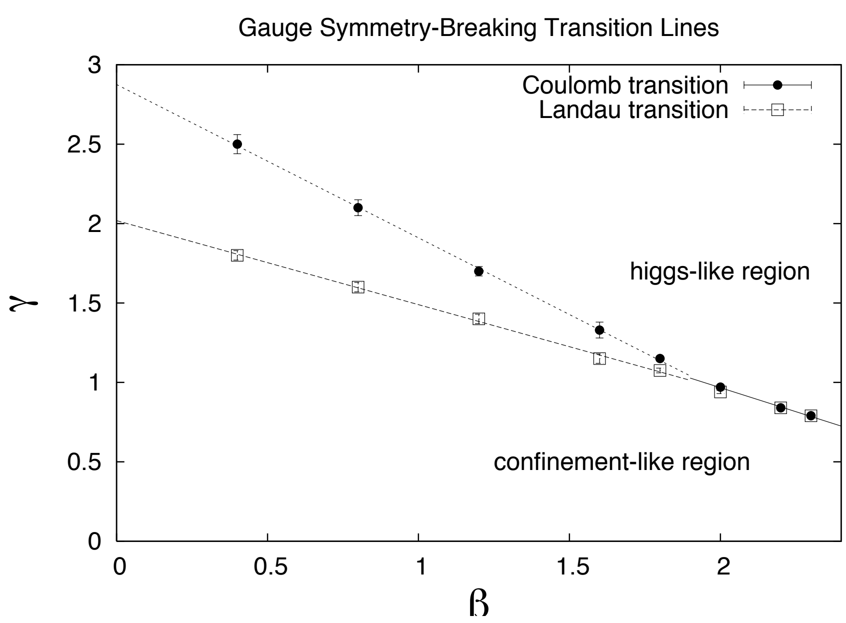
The problem is:

1. these residual symmetries *break in different places*, and

2. they break in the absence of any other abrupt change in the physical state (Fradkin-Shenker)

Not a good criterion for confinement!

(affects the Kugo-Ojima, Coulomb, and “monopole condensate” criteria)



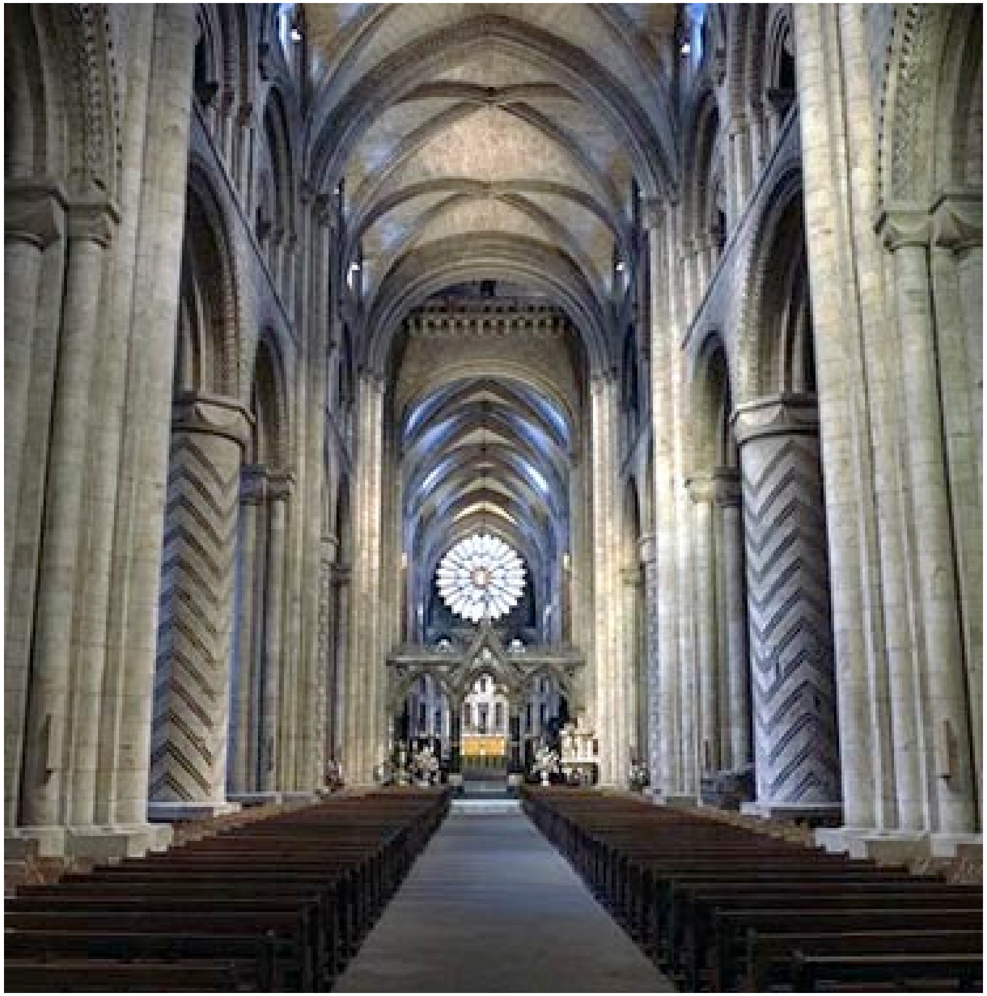
Caudy & JG (07)

If not (remnant) gauge symmetry, is there any *other* symmetry which distinguishes confined from non-confined phases?

The answer depends on what is meant by "confinement".

In gauge-Higgs theory, as $\gamma \to 0$, string-breaking goes away, and a non-gauge global symmetry appears, known as *center symmetry*.

Center Symmetry
and Order Parameters for Confinement



Traditional order parameters for confinement

A. finite asymptotic string tension $\sigma > 0$ (implies linear potential)

$$W(C) = \left\langle P \exp[i \oint_C dx^\mu A_\mu] \right\rangle \sim \exp[-\sigma \text{Area}(C)]$$

B. vanishing Polyakov lines (isolated charge has infinite energy)

$$P(\vec{x}) = \left\langle P \exp[i \int_0^T dt\ A_0(\vec{x}, t)] \right\rangle = 0$$

C. 't Hooft loop (center vortex creation operator)

$$B(C) \sim \exp[-\mu \text{Perimeter(C)}]$$

D. center vortex free energy:

if $F_v = L_z L_t \exp[-\sigma' L_x L_y]$ then $\sigma \geq \sigma'$

None of these conditions are satisfied if global center symmetry is broken spontaneously (deconfinement) *or explicitly* (quarks).

Center symmetry on the lattice is the global transformation

$$U_0(\vec{x}, t_0) \to z U_0(\vec{x}, t_0) \quad , \quad z \in Z_N \quad , \quad \text{all } \vec{x}$$

where $z = e^{2\pi i n/N}$ is an element of the center subgroup of SU(N)

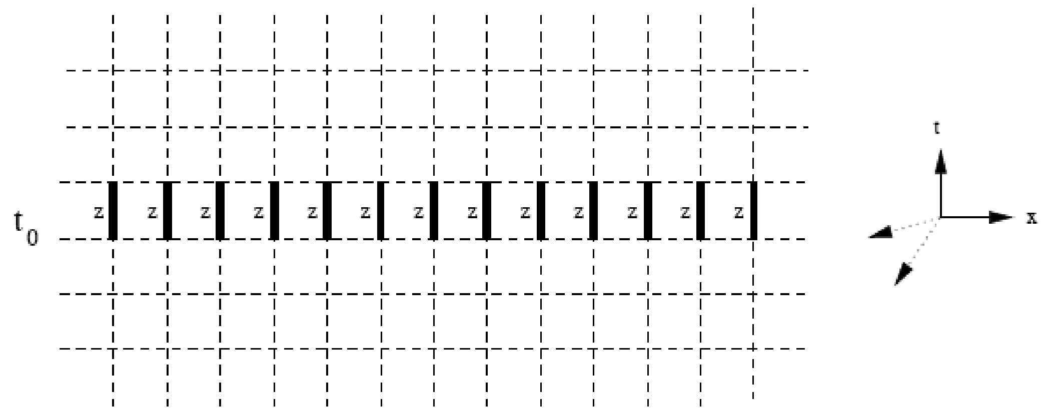


Figure 4: The global center transformation. Each of the indicated links in the t-direction, at $t = t_0$, is multiplied by a center element z. The lattice action is left unchanged by this operation.

This transformation does not change plaquettes or Wilson loops, but Polyakov lines are multiplied by the center element **z** .

$\langle P \rangle = 0$ **iff center symmetry is unbroken.**

So, what’s in a name?

What is really being tested by the traditional order parameters for confinement is not really the color neutrality of the spectrum, but rather a certain property of the vacuum: magnetic disorder.

Magnetic Disorder

the existence of vacuum fluctuations strong enough to induce an area-law falloff in Wilson loops at arbitrarily large scales.

If we take “Confinement” to mean “Magnetic Disorder”, then

Confinement is the phase of unbroken center symmetry.

Current Approaches

I. “Topology” - special field configurations

a. Center Vortices and Monopoles
b. Calorons

II. “Propagators”

a. Gribov-Zwanziger scenario
b. Dyson-Schwinger Eqns (DSE)

III. Vacuum Wavefunctionals

IV. AdS/CFT (talks by Zarembo/Mateos/Wiedemann)

Center Vortices

't Hooft (1978)

Motivations:

1. The asymptotic string tension in pure gauge theories depends only on the N-ality of the static charges

   (i.e. how the charges transform under the center subgroup of the gauge group)

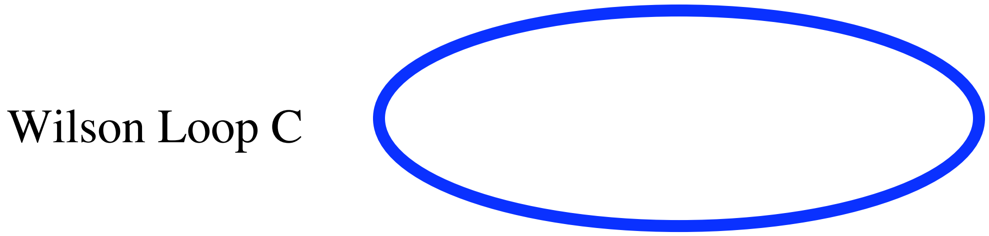
2. All of the unambiguous order parameters for confinement indicate that confinement is the phase of unbroken center symmetry.

The only scenario I know of, which explains point 1 in terms of vacuum field configurations, is the center vortex mechanism.

A center vortex is a loop of quantized magnetic flux which sweeps out a (thick) sheet as it propagates in time.

Creation of a center vortex, topologically linked to a Wilson loop, multiplies the Wilson loop by a center element.

$$U(C) = P \exp\left[i \oint_C dx^\mu \ A_\mu]\right]$$



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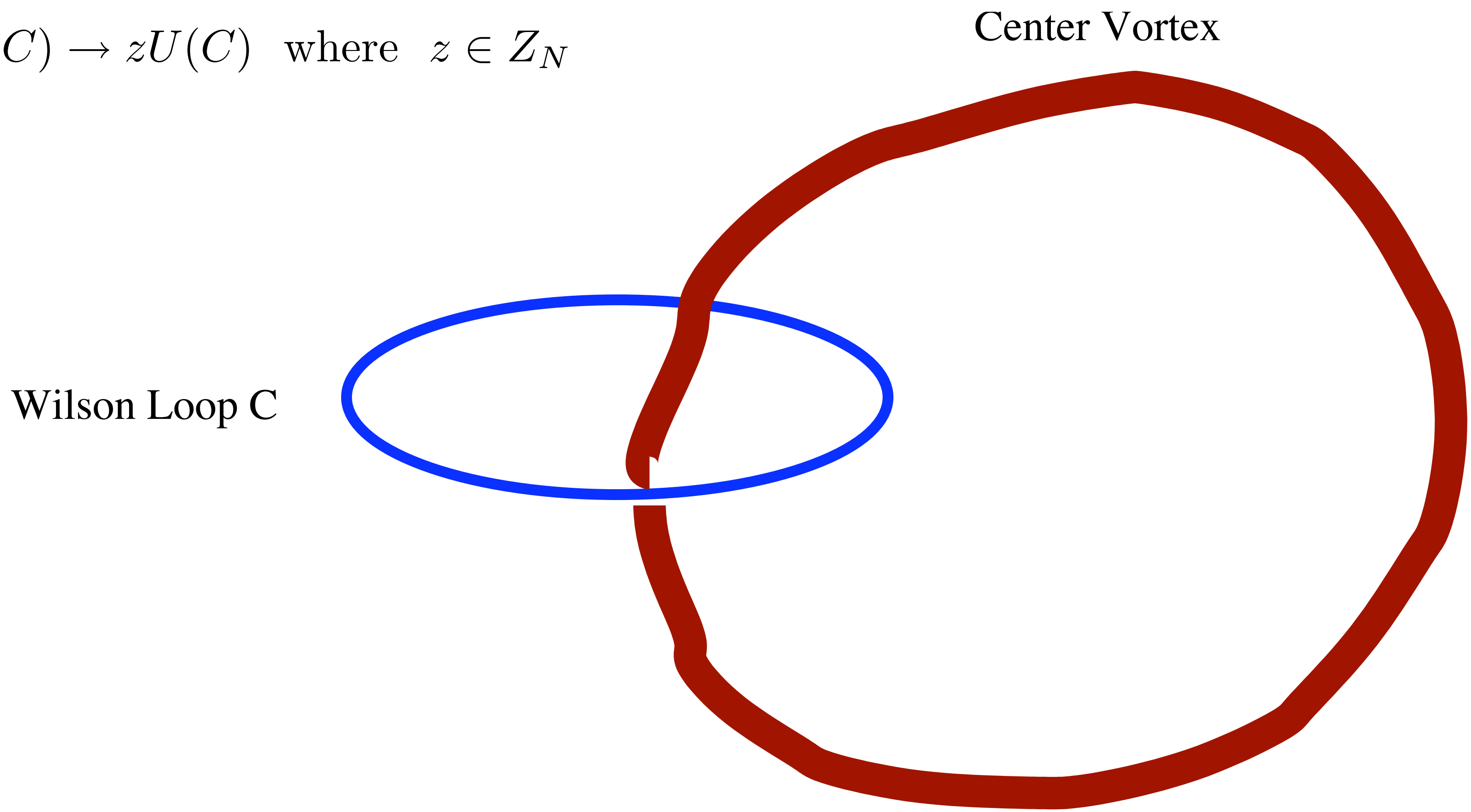
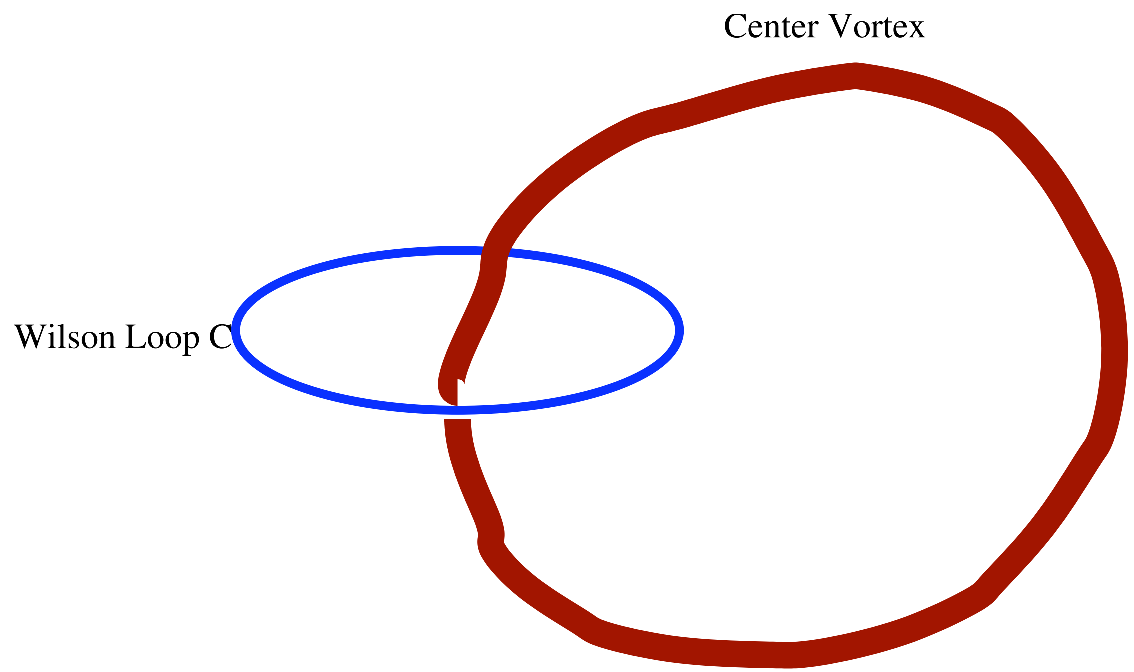
$$U(C) \to zU(C) \quad \text{where} \quad z \in Z_N$$





Area law $W(C) \sim \exp[-\mathrm{Area}(C)]$ is due to fluctuations in the number of vortices linked to loop C.

Asymptotic string tension depends only on N-ality.

There is a lot of numerical evidence in favor of this picture
based on methods, developed in 1997-98, for locating vortices in lattice configurations

1. Vortex linking number is correlated with the sign of the Wilson loop;
2. Vortices by themselves give about the right string tension;
3. Plaquette action is high on vortex surfaces;
4. Vortex density scales according to asymptotic freedom
5. when vortices are removed from lattice configurations
   a. the string tension vanishes, and
   b. chiral symmetry breaking goes away
6. vortex thickness agrees with independent estimates (adjoint string breaking, vortex free energy measurements)

Faber, Olejnik, & JG,
Tubingen group: Reinhardt, Engelhardt,Langfeld...
ITEP group: Polikarpov, Gubarev, Zakharov,...
de Forcrand et al.

$$V_r(R) = \frac{C_r}{C_F} V_F(R)$$

N-ality dependence is fine for the asymptotic string tension.

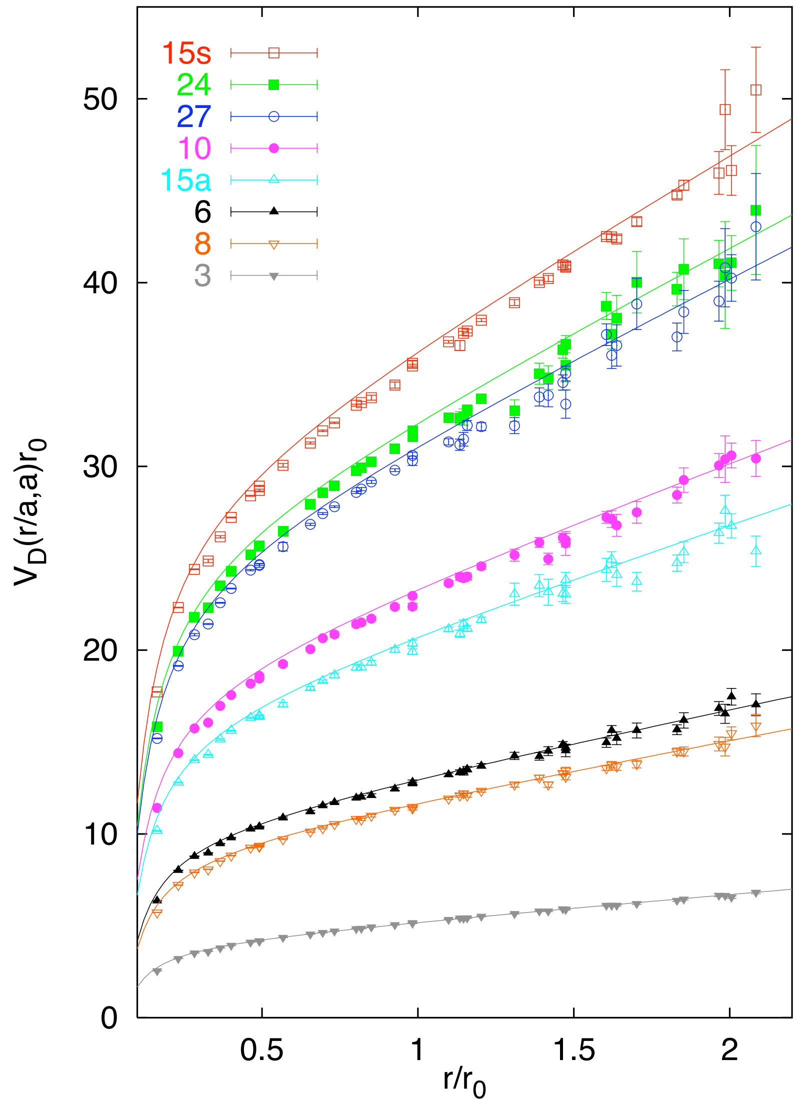
However, at intermediate scales, string tensions are proportional to the quadratic Casimir of the color charge representation, not the N-ality.

"Casimir Scaling"

In the vortex scenario, this can be explained in terms of

a. finite vortex thickness
b. fluctuations within vortices

Langfeld, Olejnik, Reinhardt, Tok & JG (2006)



Monopoles

lattice investigations by the
Kanazawa group (Suzuki et al.)
Pisa group (di Giacomo et al.)
ITEP group (Polikarpov et al.)
among others...

Motivations:

1. "Dual Superconductivity" ('t Hooft & Mandelstam)

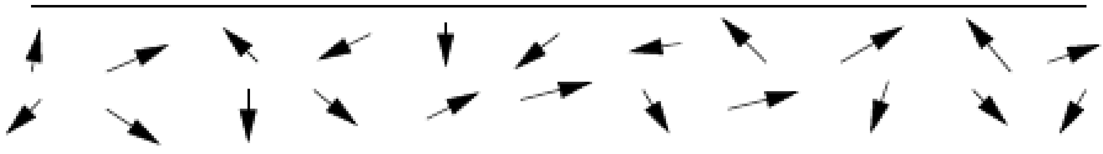
2. Compact U(1) in 2+1 dimensions (Polyakov)

3. Witten-Seiberg model

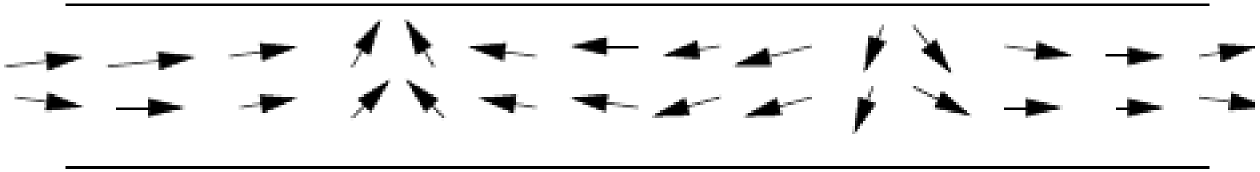
In the absence of a Higgs field in the adjoint representation, it is necessary to single out an abelian subgroup using an "abelian projection" gauge. ('t Hooft, '80)

It turns out that in abelian projection gauges, the abelian monopole worldlines lie on vortex sheets...

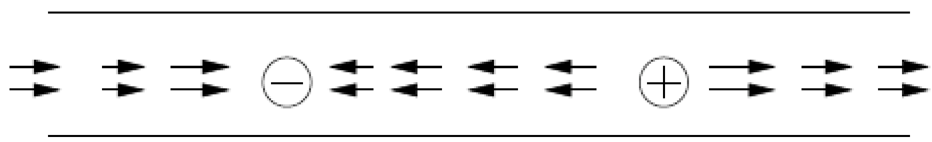
In the absence of gauge-fixing, the vortex field B^a points in random directions in the Lie algebra



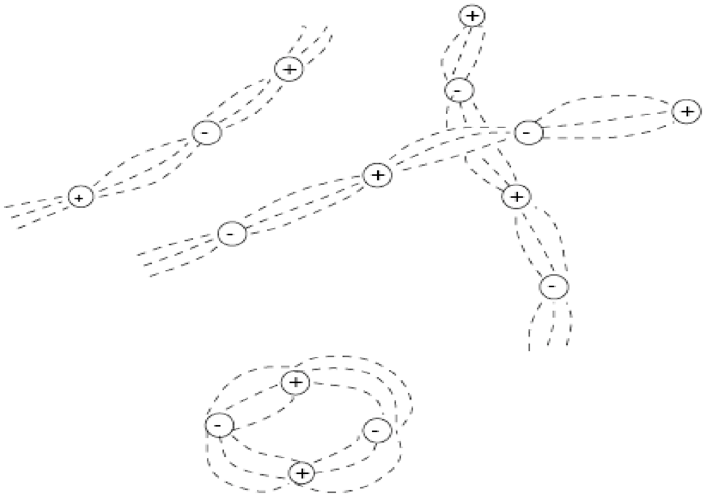
For the SU(2) gauge group, fixing to maximal abelian gauge, the field tends to line up in the $\pm\sigma_3$ direction. But there will still be regions where the B-field rotates in group space, from $+\sigma_3$ to $-\sigma_3$.



If we keep only the diagonal part of the link variables ("abelian projection"), a center vortex appears as a ***monopole-antimonopole chain***, with flux running between a monopole and neighboring antimonopole

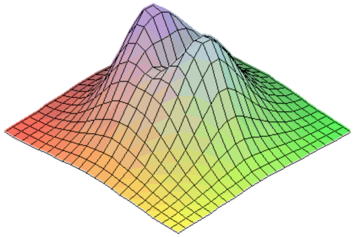


Then a typical vacuum configuration at a fixed time, after abelian projection, looks something like this:



This is the picture found in lattice simulations.

Ambjorn, Giedt, & JG, 2000
de Forcrand & Pepe, 2001



Calorons

van Baal, Bruckman, Diakonov, Ilgenfritz, Mueller-Preussker, Gattringer, Garcia-Perez...

Calorons are instantons at finite temperature (volumes with finite time extent).

Kraan-van Baal-Lee-Lu calorons have non-trivial (i.e. non-center) Polyakov lines P_∞ asymptotically far from the caloron, and have monopole constituents.

There is evidence of calorons on cooled/smeared lattices at low temperatures.

A confinement mechanism at low temperature ??

In principle certain types of calorons (with $\mathrm{Tr}[P_\infty] = 0$) can give the correct N-ality dependence for Polyakov line correlators.

However...

1. Getting N-ality right for spacelike Wilson loops seems more problematic; I see no real reason this should happen.
   (see, however, Gerhold et al., hep-ph/0607315)

2. No explanation for the spacelike string tension above the deconfinement transition; a different mechanism is needed.

The other main approach is

"Propagators"

Effects of the Gribov Horizon

In Landau gauge and Coulomb gauge there exist Gribov copies, all satisfying the given gauge-fixing condition.

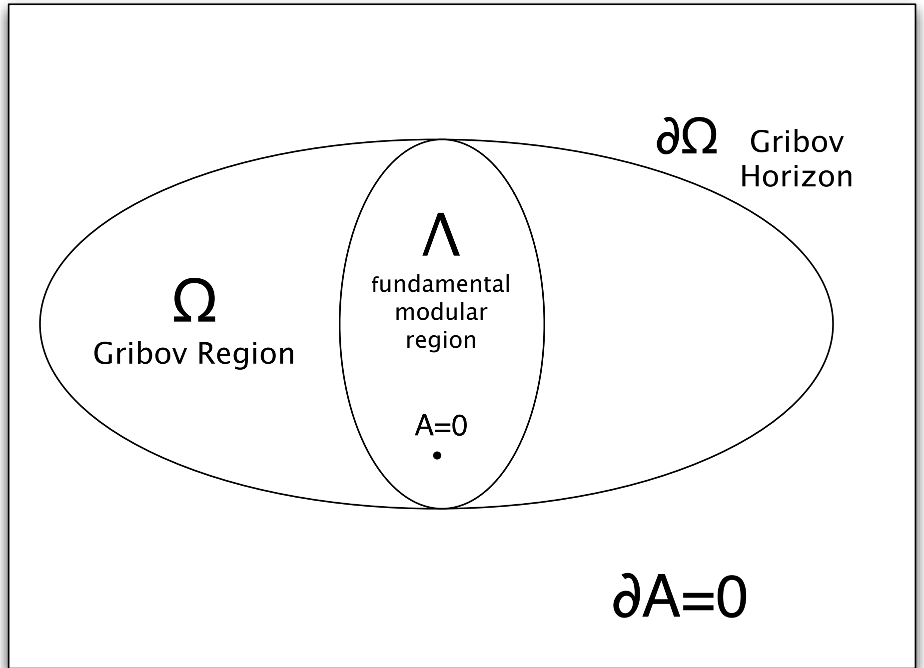
For BRST invariant actions, there is

Neuberger's Theorem

$$\langle Q \rangle = \frac{\int DU\, Dc D\overline{c}\; Q[U] e^{-(S+S_{gf})}}{\int DU\, Dc D\overline{c}\; e^{-(S+S_{gf})}} = \frac{0}{0}$$

Related to: In lattice regularization, there are even numbers of Gribov copies on a gauge orbit, half having a positive sign for the Faddeev-Popov determinant, half negative, so the sum over all copies vanishes.

Because of Gribov copies, the functional integration in Landau and Coulomb gauges should be restricted to the Fundamental Modular Region Λ, where $\partial A = 0$ and ||A|| is minimized.



Gribov and Zwanziger argue that most of the volume within Λ (and Ω) is concentrated near the boundary $\partial\Omega$, *the Gribov Horizon*, where the Fadeev-Popov operator $\partial \cdot D$ has a zero eigenvalue.

Dressed ghost and gluon propagators in Landau gauge

$$D_{\mu\nu}(p) = \frac{Z(p^2)}{p^2}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \quad , \quad D_{ghost}(p) = -\frac{G(p^2)}{p^2}$$

the Zwanziger Horizon Conditions are that at $p^2 \to 0$

$$\begin{aligned} D_{\mu\nu}(p) &\to 0 \\ G(p^2) &\to \infty \end{aligned}$$

"Gluon Confinement"

Kugo-Ojima condition

Ties in nicely with both Kugo-Ojima and the Dyson-Schwinger Equation (DSE) approach.

In Coulomb gauge, where the color Coulomb potential is related to the operator

$$\frac{1}{\nabla \cdot D}(-\nabla^2)\frac{1}{\nabla \cdot D}$$

the proximity to the Gribov horizon can, in principle, enhance the potential in the infrared.

In fact, Monte Carlo measurements of the color Coulomb potential find that it *does* rise linearly

Olejnik & JG, 2003

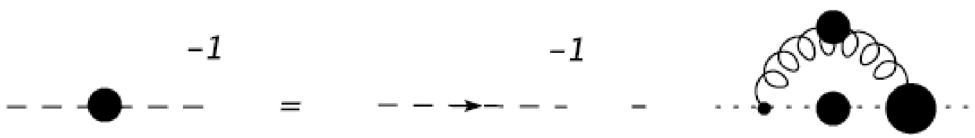
$$V_{coul}(R) \sim \sigma_{coul} R \quad \text{with} \quad \sigma_{coul} \approx 3\sigma$$

albeit with a slope which is three times larger than the asymtotic string tension.

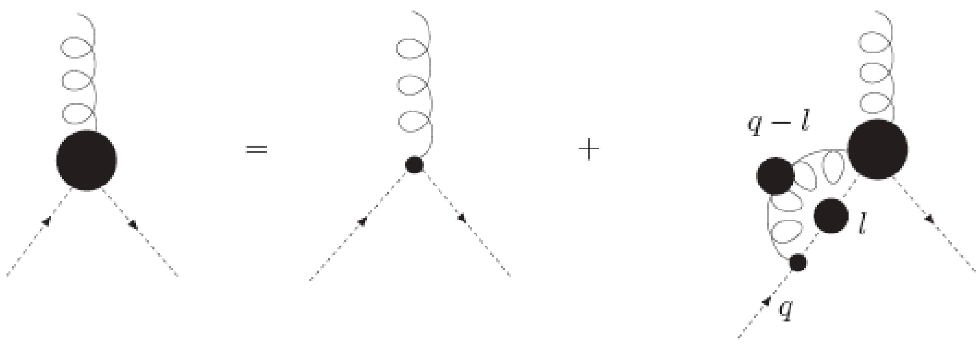
Dyson-Schwinger Equations

Alkofer, Fischer, Krassnig, Maris,
Maas, Roberts, Watson,
von Smekal...

The idea is that DSE's for n-point functions may be soluble in the infrared. Look for power-law behavior. Diagramatically,



ghost propagator



ghost-gluon vertex

$$D_{\mu\nu}(p) = \frac{Z(p^2)}{p^2}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \quad , \quad D_{ghost}(p) = -\frac{G(p^2)}{p^2}$$

self-consistency requires

$$Z(p^2) \sim (p^2)^{2\kappa} \text{ and } G(p^2) \sim (p^2)^{-\kappa}$$

and (assuming a bare ghost-gluon vertex)

$$\kappa \approx 0.595353$$

in agreement with the Zwanziger Horizon conditions. The general expectation is $0.5 < \kappa < 0.6$.

OK so far, but a problem has recently emerged...

Lattice data on *huge* lattices (27 fm)^4 does not agree that the gluon propagator vanishes in the infrared...

Cuccieri & Mendes 07

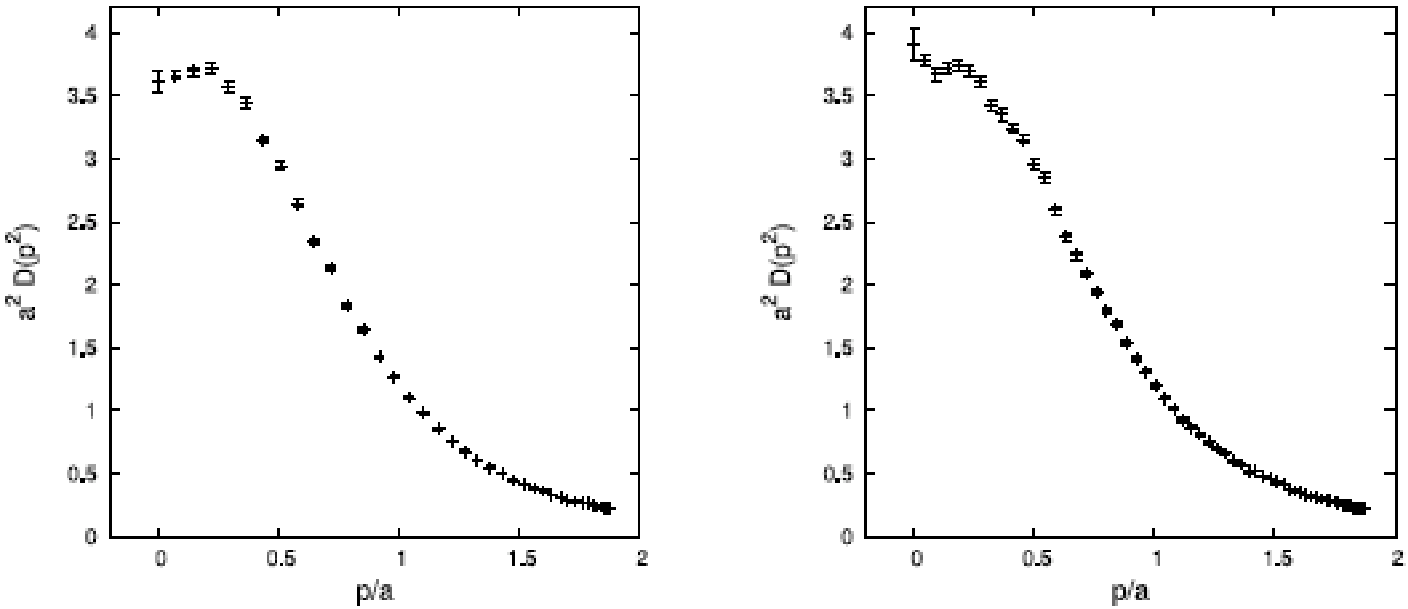


Figure 2: Unrenormalized gluon propagator $a^2D(p^2)$ (in GeV^{-2}) as a function of the momentum p/a (in GeV) for lattice volumes $V = 80^4$ (left) and $V = 128^4$ (right) at $\beta = 2.2$.

...nor is the ghost propagator more singular than a pole, so the situation is a bit unclear, at present.

Yang-Mills Vacuum Wavefunctionals

The problem is to solve $$H\Psi_0 = E_0\Psi_0$$

to see if anything can be learned about confinement and the mass gap. Currently there are several approaches.

1. **Coulomb Gauge** (Reinhardt et al., Szczepaniak et al.)

Use a gaussian ansatz and determine the kernel by minimizing <H>

$$\Psi_0[A] = \exp\left[-\int A_i(x)K^{ij}_{xy}A_j(y)\right]$$

Some success in arriving at a linear Coulomb potential.

2. Temporal Gauge D=2+1 (Olejnik & JG)

$$\Psi_0[A] = \exp\left[-\frac{1}{2}\int d^2x d^2y \; B^a(x)\left(\frac{1}{\sqrt{-D^2-\lambda_0+m^2}}\right)^{ab}_{xy} B^b(y)\right]$$

adjust m^2 to get the string tension right, then we find that the mass gap comes out right.

3. New Variables D=2+1 (Karabali & Nair, Leigh et al.)

change variables from A^a_μ to gauge-invariant J^a , the tradeoff is local gauge invariance for local holomorphic invariance under

$$\overline{\partial} J \rightarrow h(z)\overline{\partial} J h^{-1}(z)$$

The Hamiltonian has the form

$$H = m\left(\int_x J^a(x)\frac{\delta}{\delta J^a(x)} + \int_{z,w}\Omega^{ab}(z,w,J)\frac{\delta}{\delta J^a(z)}\frac{\delta}{\delta J^b(w)}\right) + \frac{\pi}{mc_A}\int_x \bar{\partial}J^a\bar{\partial}J^a$$

holomorphic-invariant ground-state wavefunctional:

$$\Psi_0 = \exp\left(-\frac{\pi}{2c_A m^2}\int \bar{\partial}J\ K\left(\frac{\Delta}{m^2}\right)\bar{\partial}J + \ldots\right)$$

Leigh et al claim to have an exact expression for the kernel, and calculate a string tension, and a spectrum of glueball masses that are in impressive agreement with the lattice results.

(however - these predictions entail ignoring the J-dependence of the holomorphic-covariant kernel)

Can we still be surprised?

Is there any chance that we are *really* wrong in our expectations; e.g. that flux tubes

1) have finite width, and

2) finite energy density everwhere?

Gubarev & Boyko (arXiv: 0704.1203) have presented data on (up to) 36^4 lattices which challenges these assumptions.

string width decreases as lattice spacing decreases

central action density extrapolates to infinity in the continuum limit

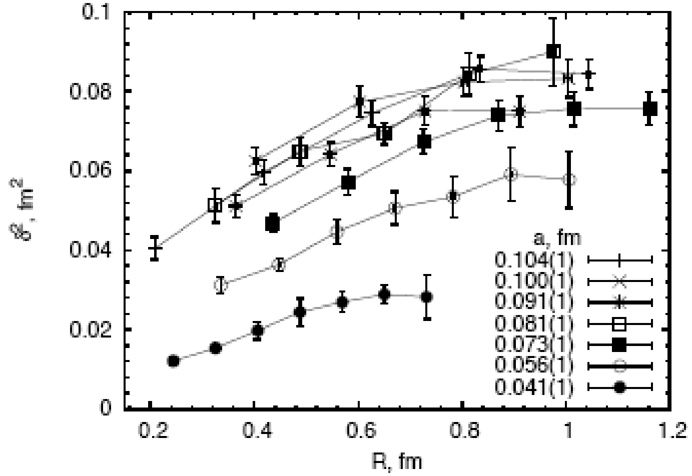


FIG. 2: Squared string width δ^2 versus the string length R. Graph includes only the points for which both $T \to \infty$ and Gaussian fits are reliable. Data for $a = 0.062(1)$ fm is omitted for readability reasons.

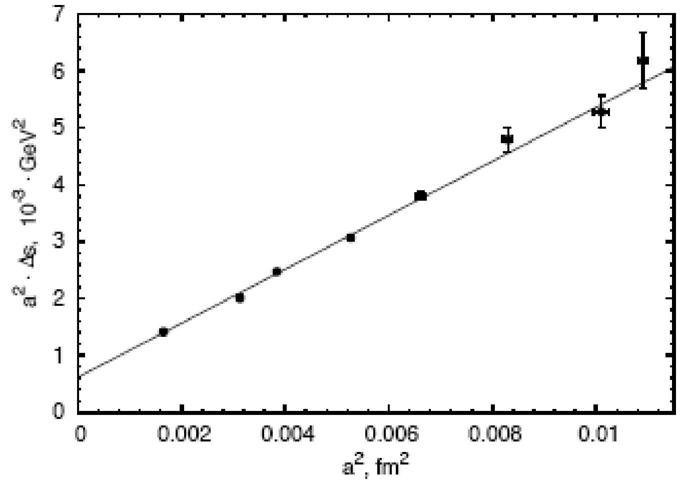


FIG. 3: Lattice spacing dependence of the action density at the string geometrical center in the limit of large quark-antiquark separation.

Probably wrong, but should be checked!

Is there now a proof?

Tomboulis, arXiv: 0707.2179

We can insert a center vortex into a finite volume using twisted, rather than periodic boundary conditions.

Let Z_- denote the SU(2) partition function with t.b.c, and
F_v denote the center vortex free energy

then

$$e^{-F_v} = \frac{Z_-}{Z}$$

Confinement is proven if $F_v = cL_z L_t \exp[-\sigma' L_x L_y]$

for a vortex sheet in the z-t plane.

Migdal-Kadanoff blocking

This is an RG decimation scheme, involving an uncontrolled approximation, which takes a lattice action with spacing ***a*** to a lattice action with spacing ***2a*** .

The idea is to take a 2^4 hypercube, move the interior plaquettes to the exterior faces and integrate out some of the link variables.

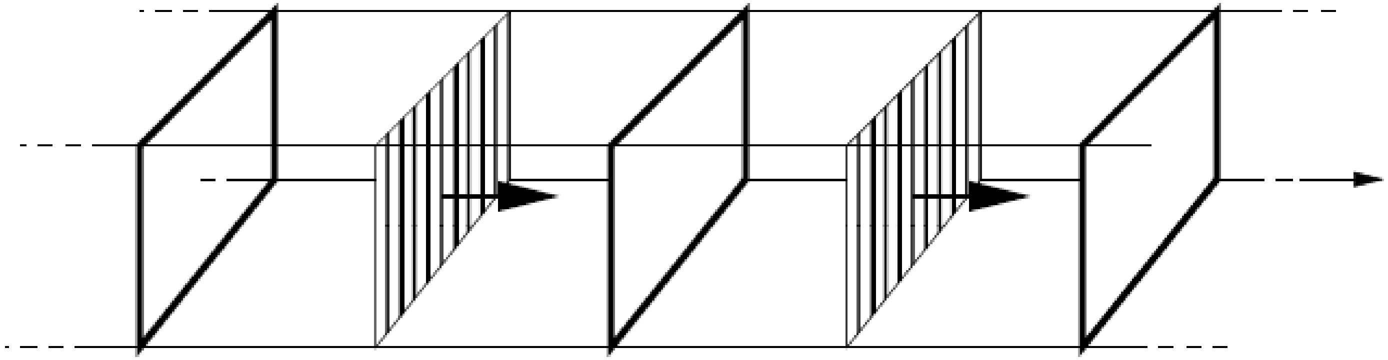


Figure 1: Basic plaquette moving operation, $b = 2$

Tomboulis's idea is to use to the MK procedure to prove an inequality, after ***n*** blocking steps,

$$\frac{Z_-}{Z} \geq \frac{Z_-^{MK}(n)}{Z^{MK}(n)}$$

If ***n*** is large enough, the rhs can be evaluated by strong-coupling methods, and confinement is proved.

This comes the closest to a proof that I've seen...

...but I think there is one crucial step in the argument which has not yet been shown to be true.

The point has been made in a very recent article by Ito & Seiler, arXiv:0711.4930.

Conclusions

Until asymptotically-free pure gauge theories are solved analytically in the infrared, there is likely to be disagreement about the structure of the vacuum, the origin of confinement and the origin of the mass gap.

Several approaches - not necessarily compatible with each other - seem promising. So far, however:

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Until asymptotically-free pure gauge theories are solved analytically in the infrared, there is likely to be disagreement about the structure of the vacuum, the origin of confinement and the origin of the mass gap.

Several approaches - not necessarily compatible with each other - seem promising. So far, however:

The confinement problem remains open

(and that is a serious challenge to our understanding of non-abelian gauge theories).

One last thing...

Thanks to our kind organizers!