

Neutrino Mixing and Oscillations: Current Status and Future Prospects

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Compelling Evidences for ν -Oscillations

– ν_{atm} : **SK** UP-DOWN ASYMMETRY

θ_{z-} , L/E - dependences of μ -like events

Dominant $\nu_{\mu} \rightarrow \nu_{\tau}$ K2K, MINOS; CNGS (OPERA)

– ν_{\odot} : Homestake, Kamiokande, SAGE, GALLEX/GNO

Super-Kamiokande, SNO, BOREXINO; KamLAND

Dominant $\nu_e \rightarrow \nu_{\mu, \tau}$ BOREXINO; KamLAND; ... LowNu

– LSND

Dominant $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_e$; MiniBOONE 11/04/07: **negative result**

$$\nu_{lL} = \sum_{j=1} U_{lj} \nu_{jL} \quad l = e, \mu, \tau.$$

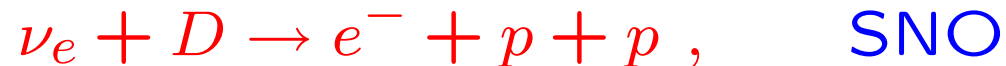
Solar Neutrinos ν_e , $E \sim 1$ MeV: B. Pontecorvo 1946



R. Davis et al., 1967 - 1996: 615 t C_2Cl_4 ; 0.5 Ar atoms/day, exposure 60 days.



Kamiokande (1986-1994), Super-Kamiokande (1996 -), SNO (2000 - 2006), BOREXINO (2007 -);



Super-Kamiokande: 50000t ultra-pure water, ~ 11000 PM

SNO: 1000t heavy water (D_2O)



SAGE (60t), 1990-; GALLEX/GNO (30t, LNGS), 1991-2003

Atmospheric Neutrinos $\nu_\mu, \bar{\nu}_\mu, \nu_e, \bar{\nu}_e, E \sim 1$ GeV (0.20 - 100 GeV); accelerator $\nu_\mu, \bar{\nu}_\mu, E \sim 3$ (20) GeV (K2K, MINOS, OPERA,...)



Reactor $\bar{\nu}_e, E \sim 2$ MeV: CHOOZ, KamLAND (2-8 MeV)



Neutrinos: “Identity Crisis”

R. Davis et al., Kamiokande, SAGE, GALLEX/GNO,
Super-K, SNO, KamLAND: solar ν_e oscillate!

$$\nu_e \rightarrow \nu_\mu + \nu_\tau$$

Kamiokande, Super-K: atmospheric $\nu_\mu, \bar{\nu}_\mu$ oscillate!

$$\nu_\mu \rightarrow \nu_\tau$$

Oscillations of Neutrinos: $m(\nu) \neq 0$

Hypothesis of Neutrino Oscillations:

B. Pontecorvo, 1957; 1958; 1967

Z. Maki, M. Nakagawa, S. Sakata, 1962

V. Gribov, B. Pontecorvo, 1969; S.M. Bilenky, B. Pontecorvo, 1976; 1978

Vast literature: H. Fritzsch, P. Minkowski, 1976; B. Kayser, 1981; etc.

Neutrino Oscillations: QM in Action

Suppose at $t = 0$ in vacuum

$$|\nu_e\rangle = |\nu_1\rangle \cos\theta + |\nu_2\rangle \sin\theta,$$

$$|\nu_{\mu(\tau)}\rangle = -|\nu_1\rangle \sin\theta + |\nu_2\rangle \cos\theta; \quad \nu_{1,2}: m_{1,2} \neq 0$$

After time t in vacuum

$$|\nu_e\rangle_t = e^{-iE_1 t} |\nu_1\rangle \cos\theta + e^{-iE_2 t} |\nu_2\rangle \sin\theta, \quad E_{1,2} = \sqrt{p^2 + m_{1,2}^2}$$

$$A(\nu_e \rightarrow \nu_{\mu}; t) = \langle \nu_{\mu} | \nu_e \rangle_t = \frac{1}{2} \sin 2\theta (e^{-iE_2 t} - e^{-iE_1 t})$$

$$P(\nu_e \rightarrow \nu_{\mu}; t) = \frac{1}{2} \sin^2 2\theta (1 - \cos((E_2 - E_1)t))$$

$$P(\nu_e \rightarrow \nu_e; t) \equiv P_{ee} = 1 - P(\nu_e \rightarrow \nu_{\mu}; t)$$

V. Gribov, B. Pontecorvo, 1969

Neutrinos are relativistic: $t \cong L$, $E_2 - E_1 \cong (m_2^2 - m_1^2)/(2p)$

$$(E_2 - E_1)t \cong (m_2^2 - m_1^2)L/(2p) = 2\pi \frac{L}{L_{osc}^{vac}}, \quad L_{osc}^{vac} \equiv \frac{4\pi E}{\Delta m^2}$$

$$P(\nu_e \rightarrow \nu_\mu; t) = \frac{1}{2} \sin^2 2\theta \left(1 - \cos 2\pi \frac{L}{L_{osc}^{vac}}\right), \quad L_{osc}^{vac} \equiv \frac{4\pi E}{\Delta m^2}$$

$$L_{osc}^{vac} \cong 2.48 \text{ m} \frac{E[\text{MeV}]}{\Delta m^2[\text{eV}^2]}$$

$$E \cong 3 \text{ MeV}, \quad \Delta m^2[\text{eV}^2] \cong 8 \times 10^{-5} : \quad L_{osc}^{vac} \cong 100 \text{ km}$$

$$E \cong 1 \text{ GeV}, \quad \Delta m^2[\text{eV}^2] \cong 2.5 \times 10^{-3} : \quad L_{osc}^{vac} \cong 1000 \text{ km}$$

Effects of oscillations observable if

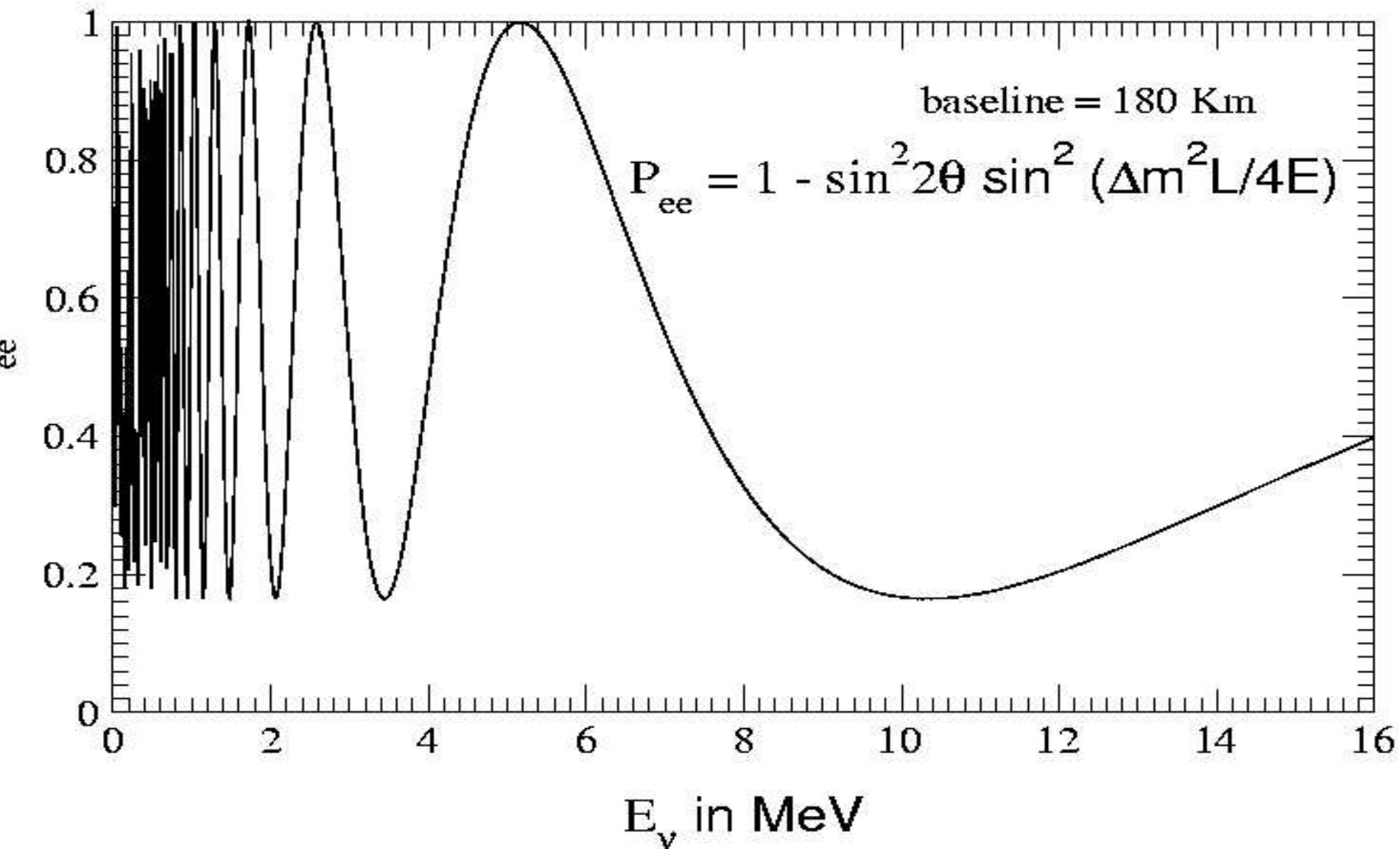
$$\sin^2 2\theta - \text{sufficiently large}, \quad L \gtrsim L_{osc}^{vac}$$

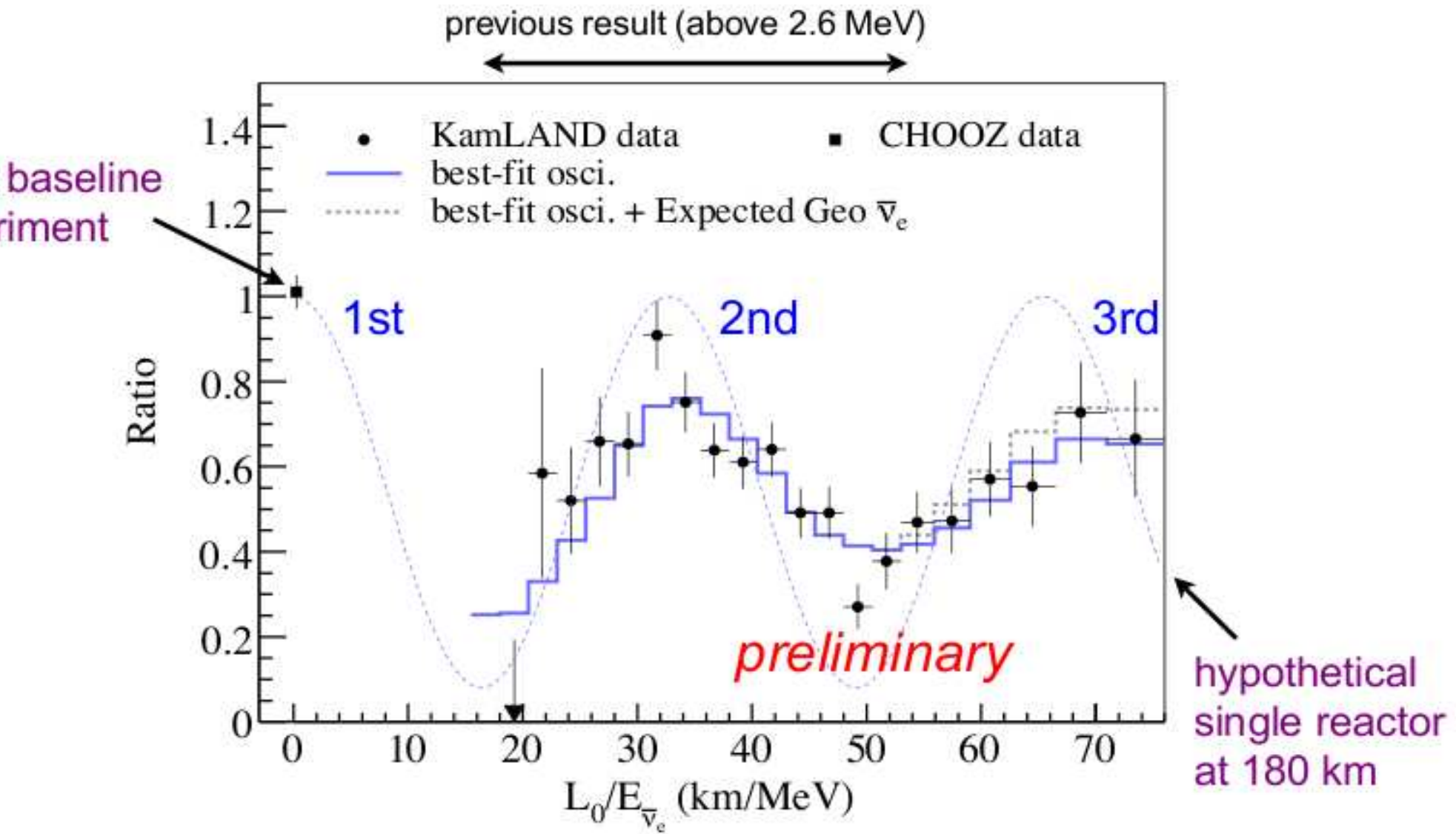
Two basic parameters: $\sin^2 2\theta$, Δm^2

SK, K2K, MINOS; CNGS (OPERA): dominant $\nu_\mu \rightarrow \nu_\tau$

KamLAND: $\bar{\nu}_e \rightarrow \bar{\nu}_e$; $\bar{\nu}_e \rightarrow (\bar{\nu}_\mu + \bar{\nu}_\tau)/\sqrt{2}$

$\nu_e \rightarrow \nu_e$





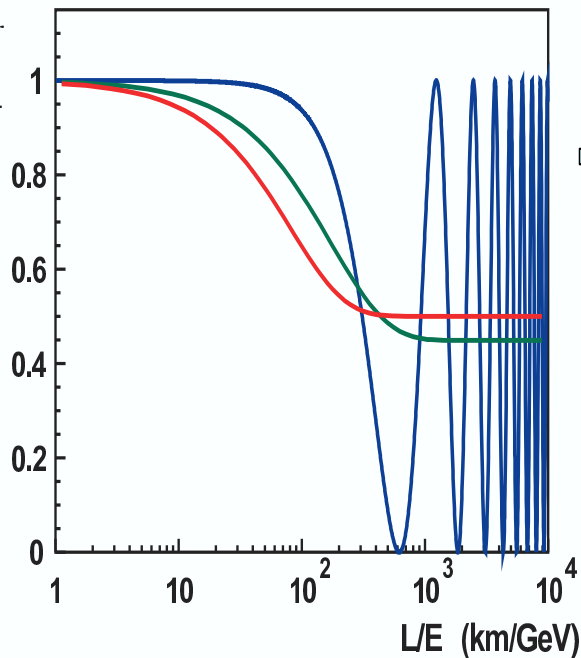
KamLAND: L/E -Dependence

L/E analysis

Neutrino oscillation : $P_{\mu\mu} = 1 - \sin^2 2\theta \sin^2\left(1.27 \frac{\Delta m^2 L}{E}\right)$

Neutrino decay : $P_{\mu\mu} = \left(\cos^2\theta + \sin^2\theta \times \exp\left(-\frac{m}{2\tau} \frac{L}{E}\right)\right)^2$

Neutrino decoherence : $P_{\mu\mu} = 1 - \frac{1}{2} \sin^2 2\theta \times \left(1 - \exp\left(-\gamma_0 \frac{L}{E}\right)\right)$



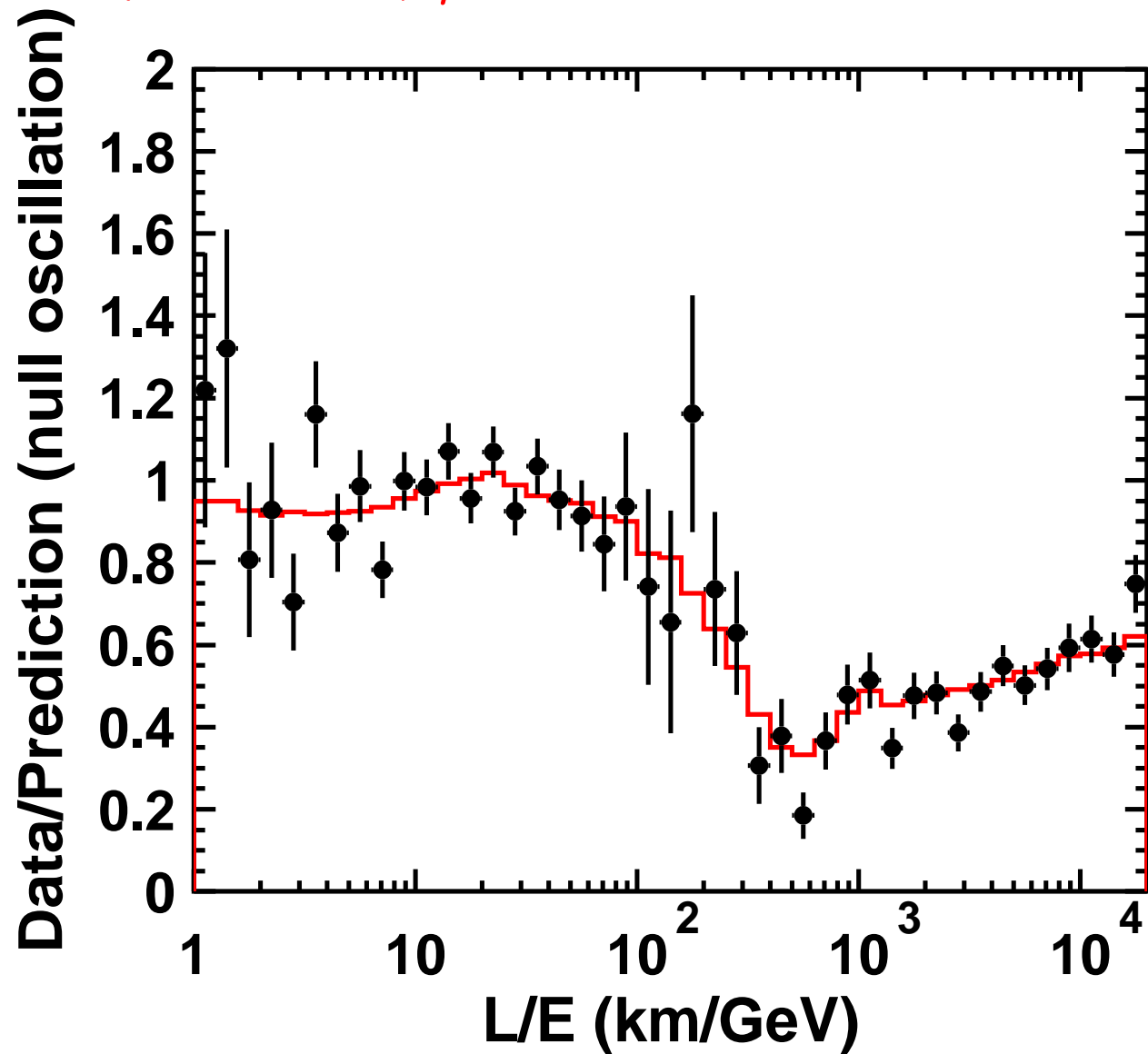
Use events with high resolution in L/E



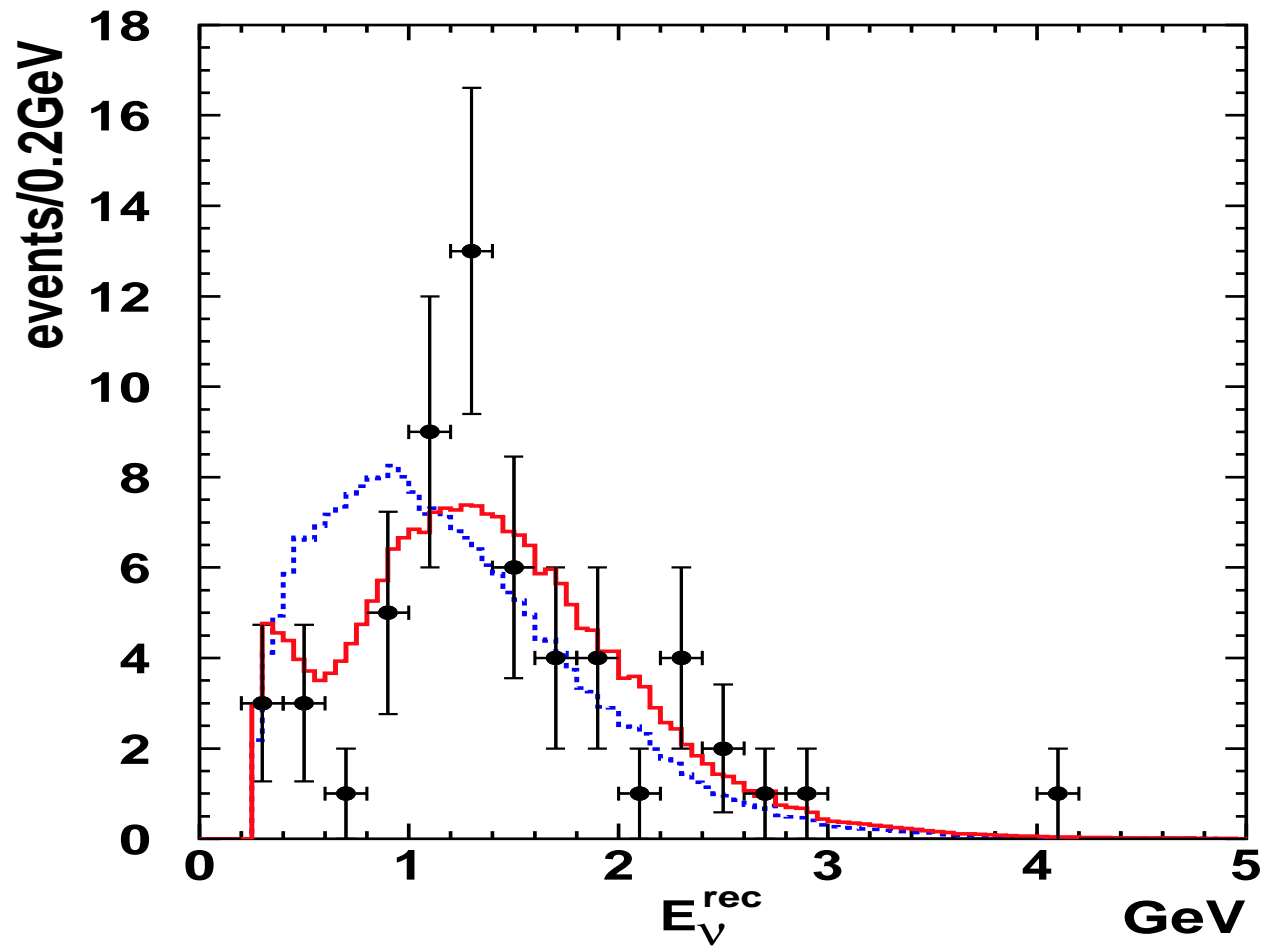
The first dip can be observed

- Direct evidence for oscillations
- Strong constraint to oscillation parameters, especially Δm^2 value

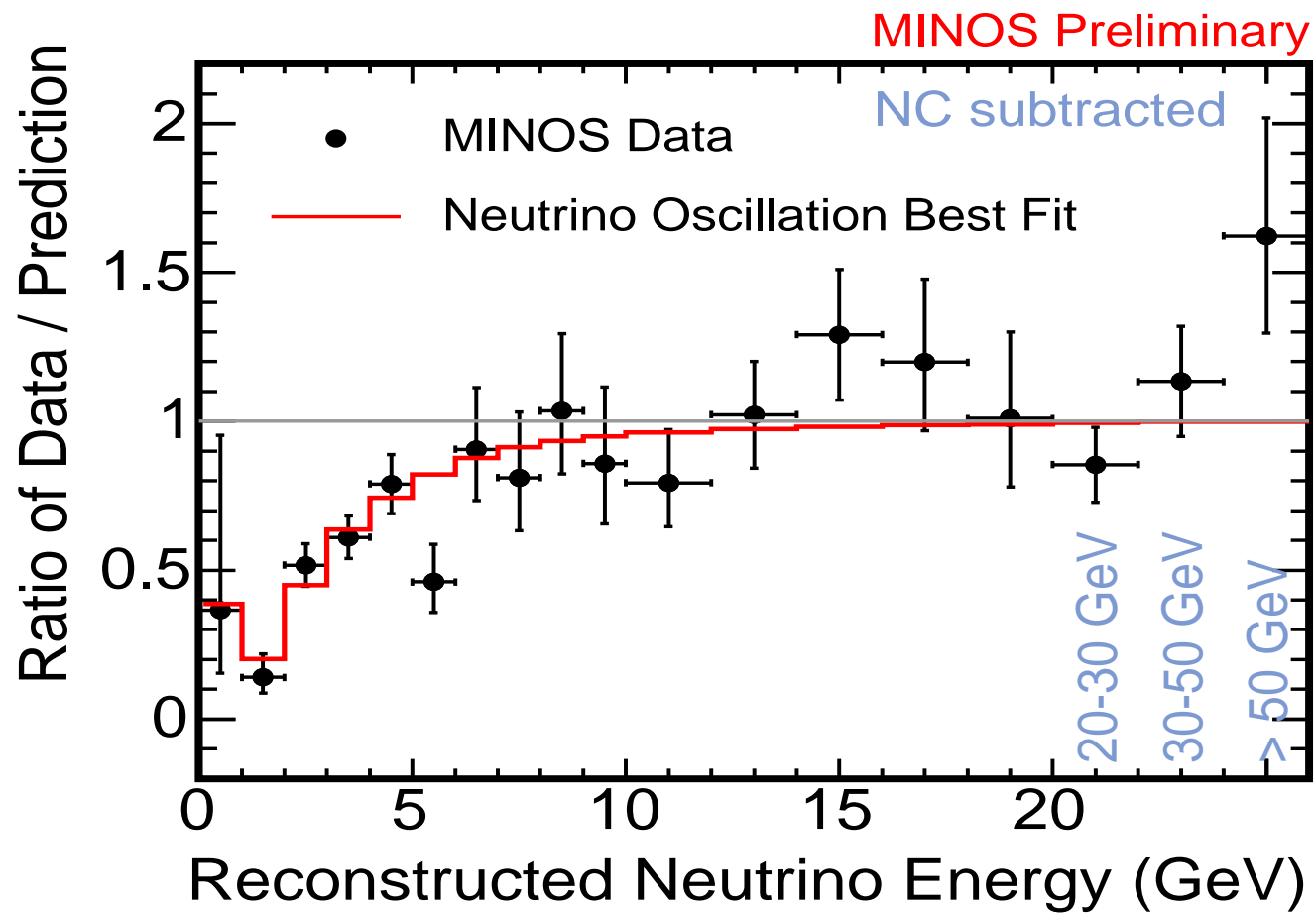
SK: L/E Dependence, μ -Like Events



K2K: ν_μ Spectrum



MINOS: ν_μ Spectrum



L. Wolfenstein, 1978; V. Barger et al., 1980;
S.P. Mikheyev, A.Yu. Smirnov, 1985; etc.

Matter can affect strongly ν -oscillations:

ν **coherent** scattering on e^- , p , n - effective potential
(index of refraction)

$$V_{e\mu} = V(\nu_e) - V(\nu_\mu) = \sqrt{2}G_F N_e$$

$$\bar{V}_{e\mu} = V(\bar{\nu}_e) - V(\bar{\nu}_\mu) = -\sqrt{2}G_F N_e$$

$$V_{\mu\tau} = V(\nu_\mu) - V(\nu_\tau) = 0 \text{ (leading order)}$$

$$V_{e\mu} \neq \bar{V}_{e\mu}: \text{ CP, CPT violated}$$

Earth: $\bar{N}_e^{mant} \sim 2.3 N_A \text{ cm}^{-3}$, $\bar{N}_e^{core} \sim 6.0 N_A \text{ cm}^{-3}$

$$P^m(\nu_e \rightarrow \nu_\mu; t) = \frac{1}{2} \sin^2 2\theta_m \left(1 - \cos 2\pi \frac{L}{L_{osc}^m}\right), \quad L_{osc}^m \sim L_{osc}^{vac}$$

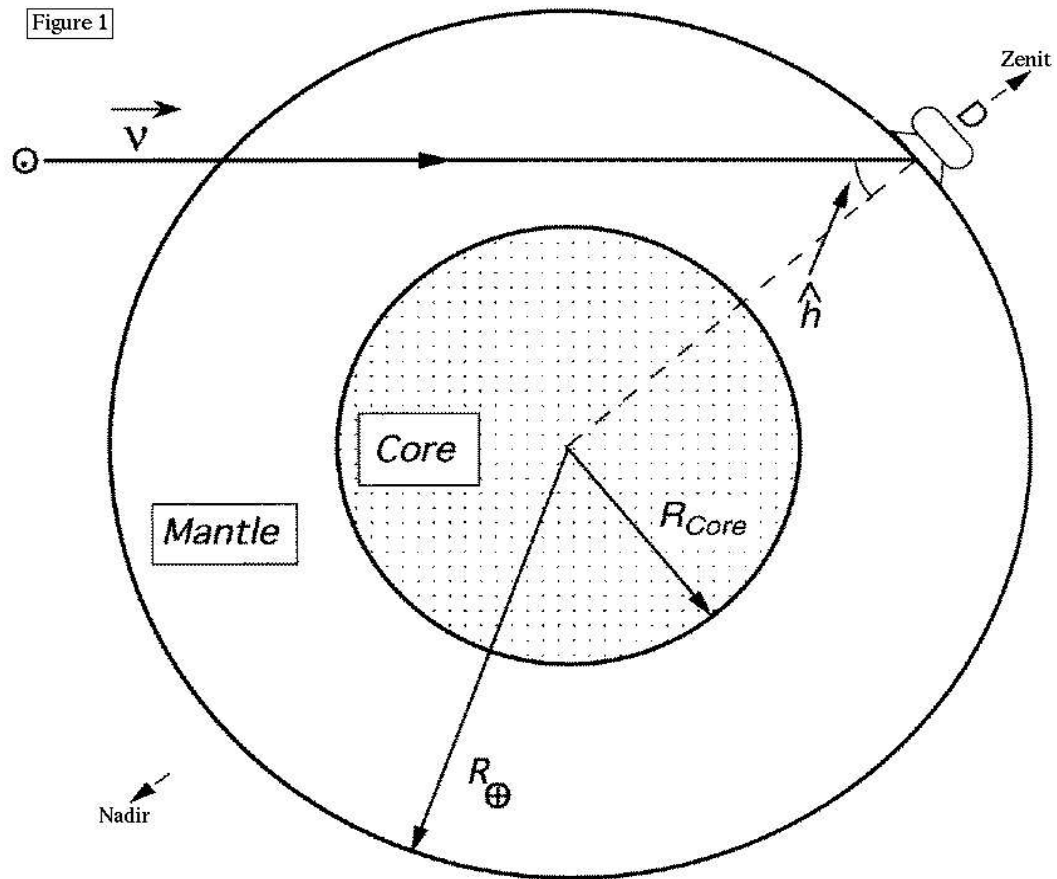
$$\sin^2 2\theta_m = \frac{\sin^2 2\theta}{\left(1 - \frac{N_e}{N_e^{res}}\right)^2 \cos^2 2\theta + \sin^2 2\theta}, \quad N_e^{res} \equiv \frac{\Delta m^2 \cos 2\theta}{2E\sqrt{2}G_F}$$

$N_e = N_e^{res}$: MSW resonance

$\Delta m^2 \cos 2\theta > 0$: $\nu_e \rightarrow \nu_\mu$

$\Delta m^2 \cos 2\theta < 0$: $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$

The Earth



Earth: $R_{core} = 3446 \text{ km}$, $R_{mant} = 2885 \text{ km}$

Earth: $\bar{N}_e^{mant} \sim 2.3 N_A \text{ cm}^{-3}$, $\bar{N}_e^{core} \sim 6.0 N_A \text{ cm}^{-3}$

The Earth

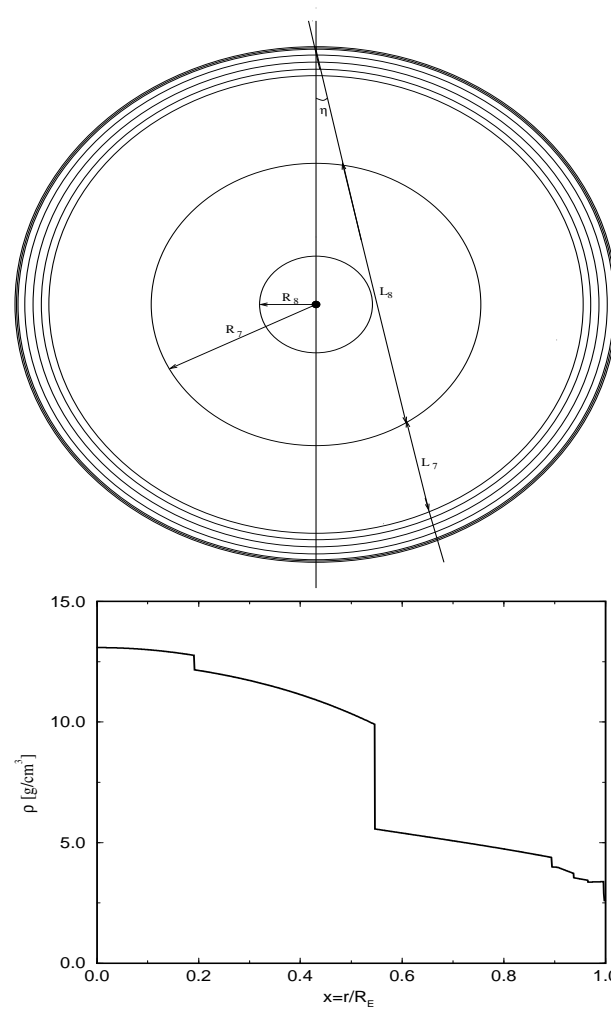
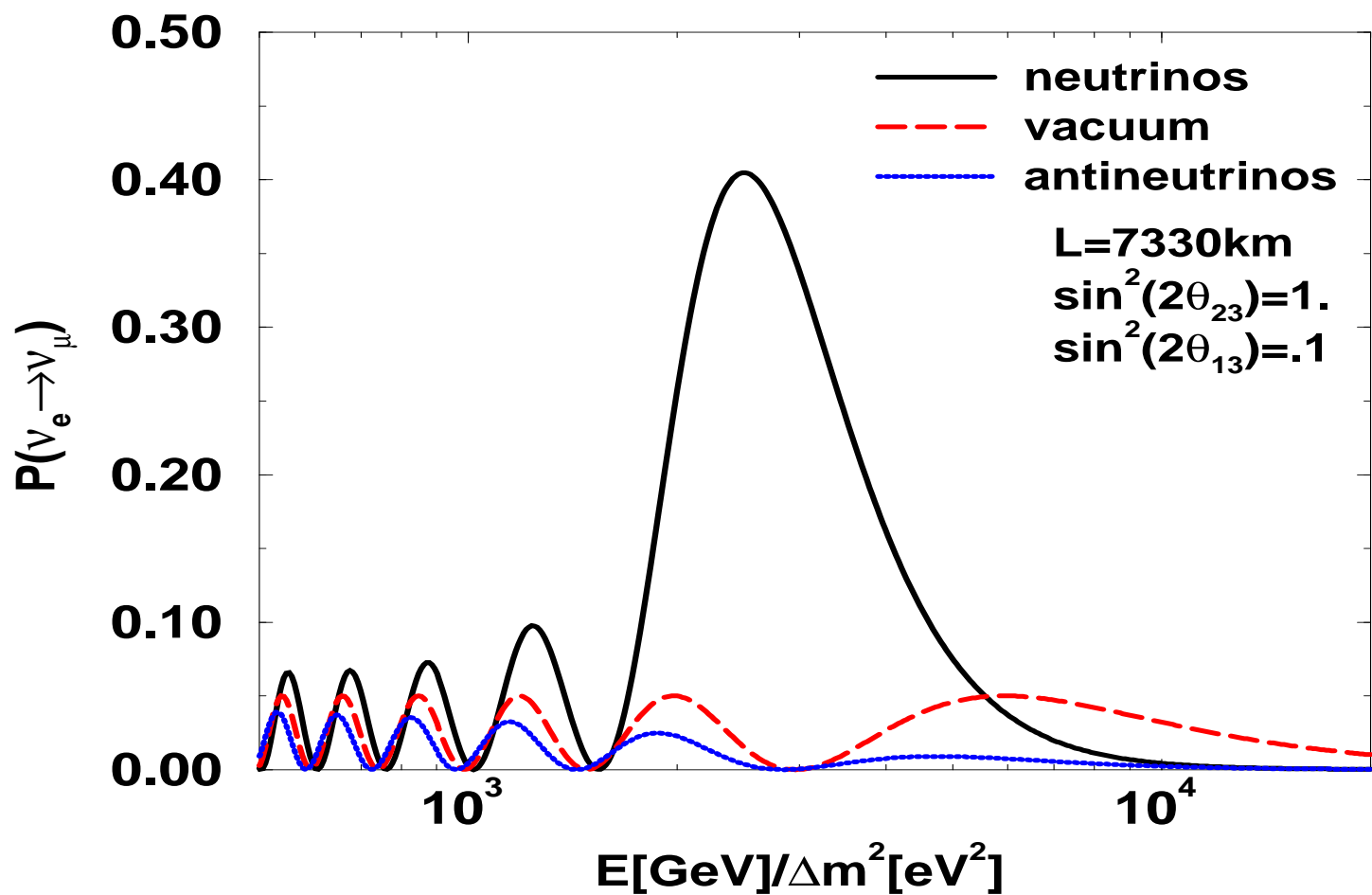


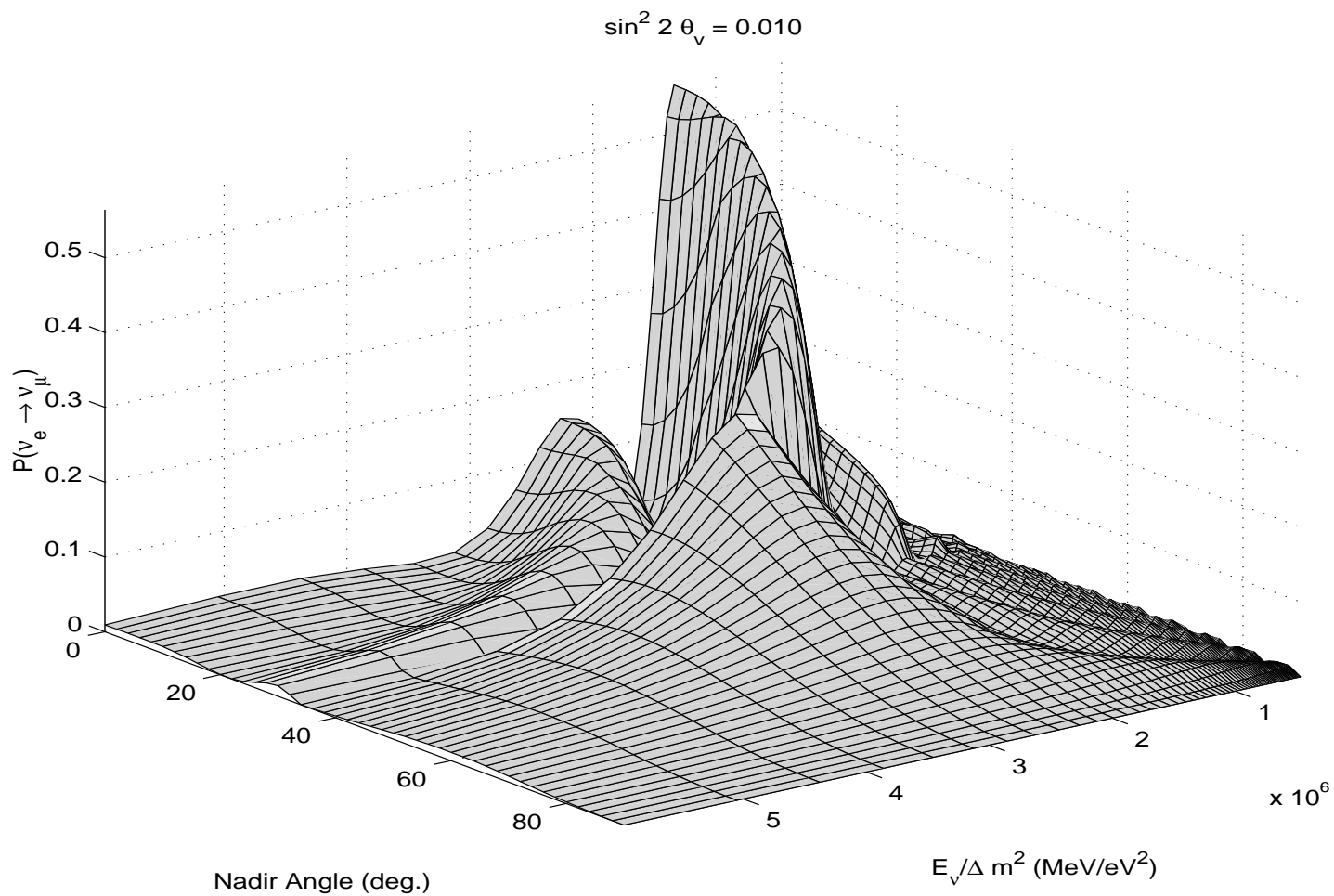
FIG. 1. Density profile of the Earth.

$$R_c = 3446 \text{ km}, R_m = 2886 \text{ km}; \bar{N}_e^{mant} \sim 2.3 N_A \text{ cm}^{-3}, \bar{N}_e^{core} \sim 6.0 N_A \text{ cm}^{-3}$$

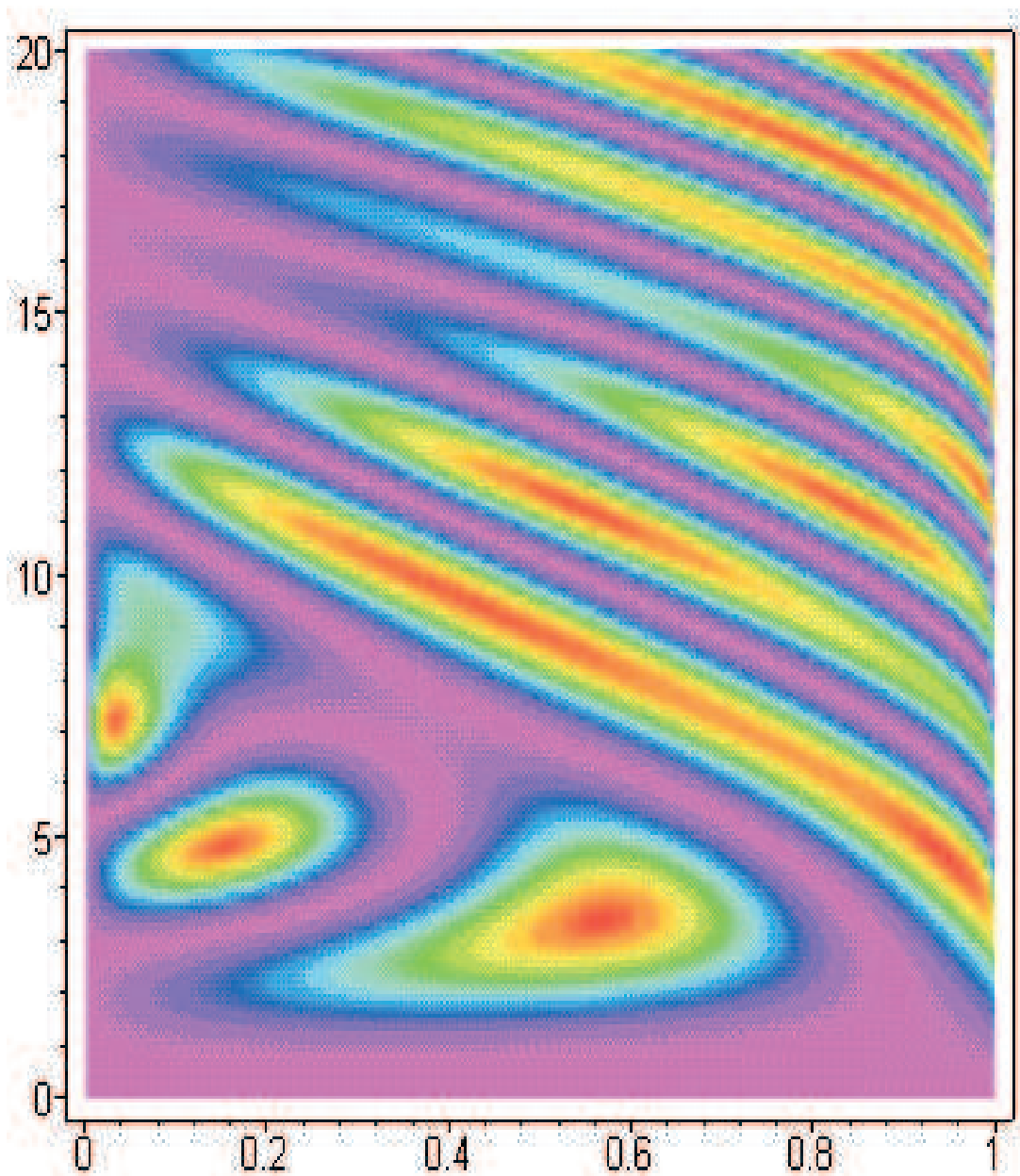
Earth matter effect in $\nu_\mu \rightarrow \nu_e, \bar{\nu}_\mu \rightarrow \bar{\nu}_e$ (MSW)



Earth matter effects in $\nu_\mu \rightarrow \nu_e, \bar{\nu}_\mu \rightarrow \bar{\nu}_e$ (NOLR)

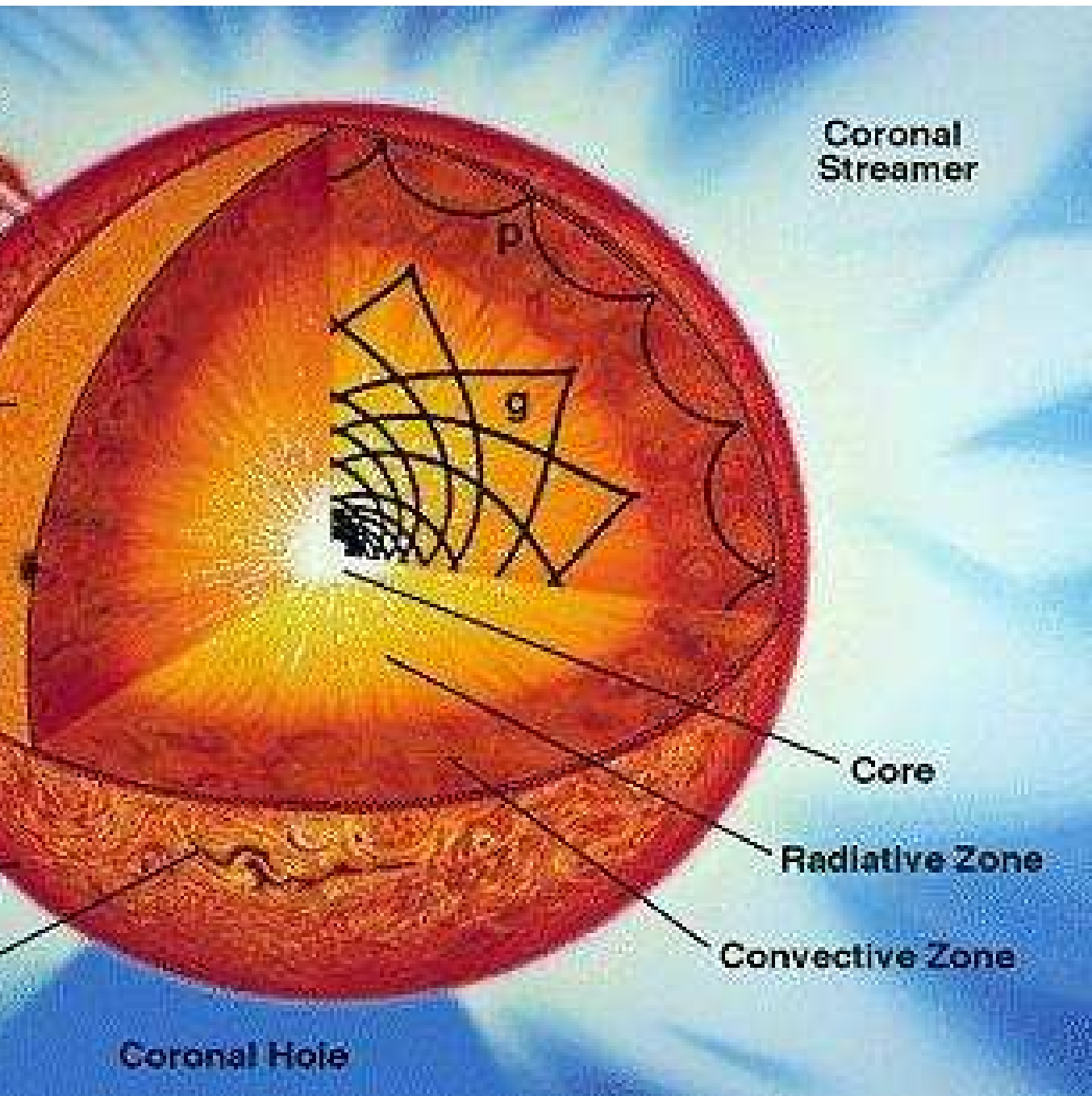


S.T.P., 1998; M. Chizhov, M. Maris, S.T.P., 1998;
M. Chizhov, S.T.P., 1999



Vertical axis: $\Delta m^2/E$ [$10^{-7} eV^2/MeV$]; horizontal axis: $\sin^2 2\theta$; $\theta_n = 0$ (NOLR)

M. Chizhov, S.T.P., 1999



Sun: $N_e(x) \cong N_e(x_0)e^{-\frac{x}{r_0}}$, $r_0 \cong 0.1R_\odot$, $R_\odot \cong 7 \times 10^5$ km

Region of ν_\odot production:

$$20 N_A \text{ cm}^{-3} \lesssim N_e(x_0) \lesssim 100 N_A \text{ cm}^{-3}$$

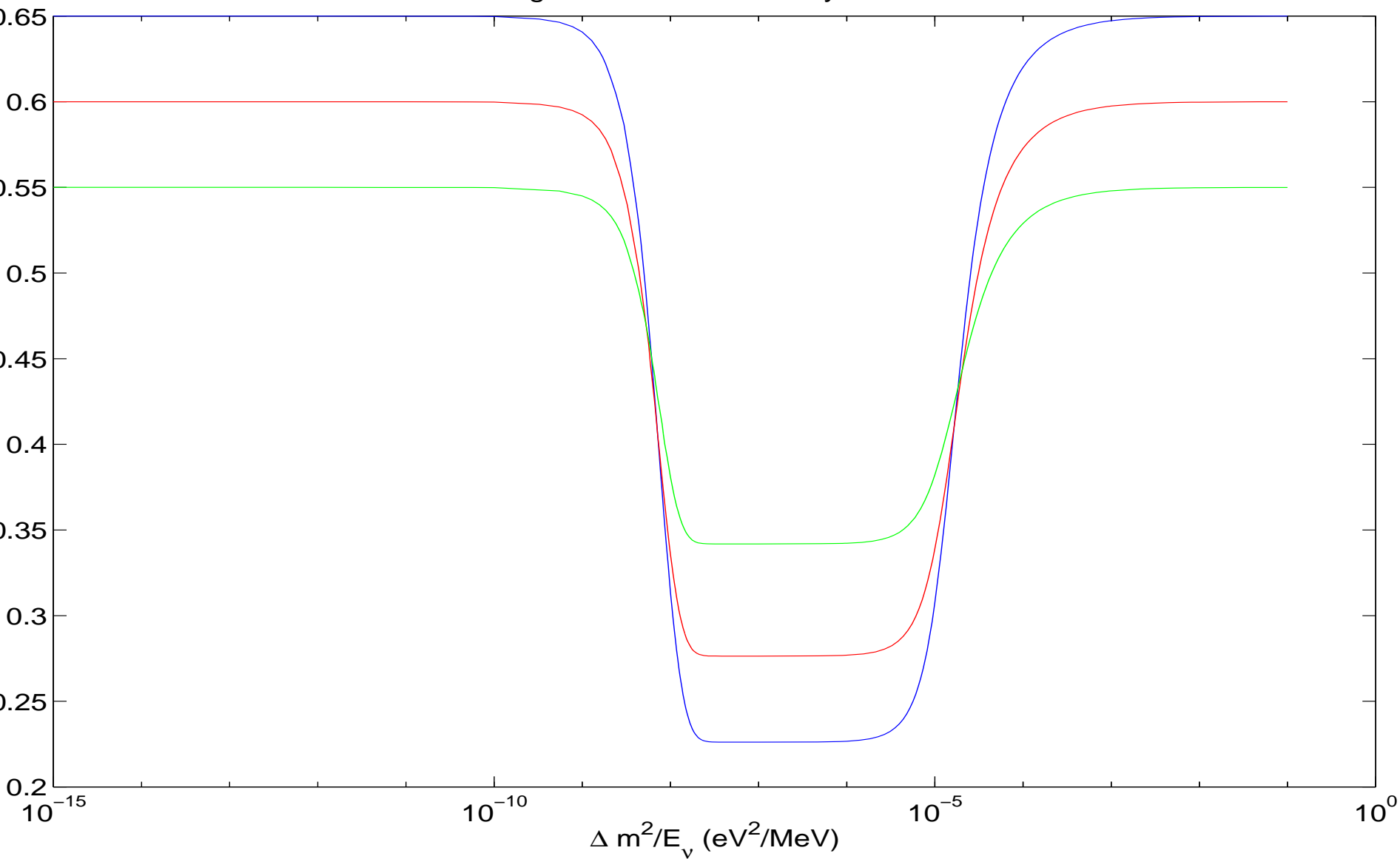
The solar ν_e survival probability:

$$\bar{P}(\nu_e \rightarrow \nu_e) = \frac{1}{2} + \left(\frac{1}{2} - P'\right) \cos 2\theta_m^0 \cos 2\theta,$$

$$P' = \frac{e^{-2\pi r_0 \frac{\Delta m^2}{2E}} \sin^2 \theta - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}{1 - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}$$

$$\nu_e \rightarrow \nu_e$$

Averaged Survival Probability in the Sun



MSW Transitions of Solar Neutrinos in the Sun and the Hydrogen Atom

$$i \frac{d}{dt} \begin{pmatrix} A_\alpha(t, t_0) \\ A_\beta(t, t_0) \end{pmatrix} = \begin{pmatrix} -\epsilon(t) & \epsilon'(t) \\ \epsilon'(t) & \epsilon(t) \end{pmatrix} \begin{pmatrix} A_\alpha(t, t_0) \\ A_\beta(t, t_0) \end{pmatrix} \quad (1)$$

where $\alpha = \nu_e$, $\beta = \nu_{\mu(\tau)}$,

$$\epsilon(t) = \frac{1}{2} \left[\frac{\Delta m^2}{2E} \cos 2\theta - \sqrt{2} G_F N_e(t) \right],$$

$$\epsilon'(t) = \frac{\Delta m^2}{4E} \sin 2\theta, \text{ with } \Delta m^2 = m_2^2 - m_1^2.$$

• Standard Solar Models

$$N_e(t) = N_e(t_0) \exp \left\{ -\frac{t-t_0}{r_0} \right\}, \quad r_0 \sim 0.1 R_\odot, \quad R_\odot = 6.96 \times 10^5 \text{ km}$$

Introducing the dimensionless variable

$$Z = ir_0\sqrt{2}G_F N_e(t_0)e^{-\frac{t-t_0}{r_0}}, \quad Z_0 = Z(t = t_0),$$

and making the substitution

$$A_e(t, t_0) = (Z/Z_0)^{c-a} e^{-(Z-Z_0)+i\int_{t_0}^t \epsilon(t')dt'} A'_e(t, t_0),$$

$A'_e(t, t_0)$ satisfies the confluent hypergeometric equation (CHE):

$$\left\{ Z \frac{d^2}{dZ^2} + (c - Z) \frac{d}{dZ} - a \right\} A'_e(t, t_0) = 0,$$

where

$$a = 1 + ir_0 \frac{\Delta m^2}{2E} \sin^2 \theta, \quad c = 1 + ir_0 \frac{\Delta m^2}{2E}.$$

The confluent hypergeometric equation describing the ν_e oscillations in the Sun, coincides in form with the **Schroedinger (energy eigenvalue) equation obeyed by the radial part, $\psi_{kl}(r)$, of the non-relativistic wave function of the hydrogen atom,**

$$\Psi(\vec{r}) = \frac{1}{r} \psi_{kl}(r) Y_{lm}(\theta', \phi'),$$

r , θ' and ϕ' are the spherical coordinates of the electron in the proton's rest frame, l and m are the orbital momentum quantum numbers ($m = -l, \dots, l$), k is the quantum number labeling (together with l) the electron energy (the principal quantum number is equal to $(k+l)$), E_{kl} ($E_{kl} < 0$), and $Y_{lm}(\theta', \phi')$ are the spherical harmonics. The function

$$\psi'_{kl}(Z) = Z^{-c/2} e^{Z/2} \psi_{kl}(r)$$

satisfies the confluent hypergeometric equation in which the variable Z and the parameters a and c are in this case related to the physical quantities characterizing the hydrogen atom:

$$Z = 2 \frac{r}{a_0} \sqrt{-E_{kl}/E_I}, \quad a \equiv a_{kl} = l+1 - \sqrt{-E_I/E_{kl}}, \quad c \equiv c_l = 2(l+1),$$

$a_0 = \hbar/(m_e e^2)$ is the Bohr radius and $E_I = m_e e^4/(2\hbar^2) \cong 13.6 \text{ eV}$ is the ionization energy of the hydrogen atom.

Quite remarkably, the behavior of such different physical systems as solar neutrinos undergoing MSW transitions in the Sun and the non-relativistic hydrogen atom are governed by one and the same differential equation.

S.T.P., 1997

Compelling Evidences for ν -Oscillations: 3- ν mixing

$$\nu_{lL} = \sum_{j=1}^3 U_{lj} \nu_{jL} \quad l = e, \mu, \tau.$$

B. Pontecorvo, 1957; 1958; 1967;
Z. Maki, M. Nakagawa, S. Sakata, 1962;

Three Neutrino Mixing

$$\nu_{lL} = \sum_{j=1}^3 U_{lj} \nu_{jL} .$$

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix,

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix}$$

- U - $n \times n$ unitary:

	n	2	3	4
mixing angles:	$\frac{1}{2}n(n-1)$	1	3	6

CP-violating phases:

ν_j - Dirac:	$\frac{1}{2}(n-1)(n-2)$	0	1	3
ν_j - Majorana:	$\frac{1}{2}n(n-1)$	1	3	6

$n = 3$: 1 Dirac and

2 additional CP-violating phases, Majorana phases

Majorana Neutrinos

Can be defined in QFT using fields or states.

Fields: $\chi_k(x)$ - 4 component (spin 1/2), complex, m_k

Majorana condition:

$$C (\bar{\chi}_k(x))^T = \xi_k \chi_k(x), \quad |\xi_k|^2 = 1, \quad C^{-1} \gamma_\mu C = -\gamma_\mu^T; \quad C^T = -C, \quad C^{-1} = C^\dagger$$

- Invariant under proper Lorentz transformations.
- Reduces by 2 the number of components in $\chi_k(x)$.

Implications:

$$U(1) : \chi_k(x) \rightarrow e^{i\alpha} \chi_k(x) - \text{impossible}$$

- $\chi_k(x)$ cannot absorb phases.
- $Q_{U(1)} = 0 : Q_{el} = 0, L_l = 0, L = 0, \dots$
- $\chi_k(x)$: 2 spin states of a spin 1/2 absolutely neutral particle
- $\chi_k \equiv \bar{\chi}_k$

Propagators: $\Psi(x)$ –Dirac, $\chi(x)$ –Majorana

$$\langle 0|T(\Psi_\alpha(x)\bar{\Psi}_\beta(y))|0\rangle = S_{\alpha\beta}^F(x-y) ,$$

$$\langle 0|T(\Psi_\alpha(x)\Psi_\beta(y))|0\rangle = 0 , \quad \langle 0|T(\bar{\Psi}_\alpha(x)\bar{\Psi}_\beta(y))|0\rangle = 0 .$$

$$\langle 0|T(\chi_\alpha(x)\bar{\chi}_\beta(y))|0\rangle = S_{\alpha\beta}^F(x-y) ,$$

$$\langle 0|T(\chi_\alpha(x)\chi_\beta(y))|0\rangle = -\xi^* S_{\alpha\kappa}^F(x-y) C_{\kappa\beta} ,$$

$$\langle 0|T(\bar{\chi}_\alpha(x)\bar{\chi}_\beta(y))|0\rangle = \xi C_{\alpha\kappa}^{-1} S_{\kappa\beta}^F(x-y)$$

$$U_{CP} \chi(x) U_{CP}^{-1} = \eta_{CP} \gamma_0 \chi(x') , \quad \eta_{CP} = \pm i .$$

PMNS Matrix: Standard Parametrization

$$U = V \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin \theta_{ij}$, $c_{ij} \equiv \cos \theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,
- δ - Dirac CP-violation phase, $\delta = [0, 2\pi]$,
- α_{21} , α_{31} - the two Majorana CP-violation phases.
- $\Delta m_{\odot}^2 \equiv \Delta m_{21}^2 \cong 8.0 \times 10^{-5} \text{ eV}^2 > 0$, $\sin^2 \theta_{12} \cong 0.30$, $\cos 2\theta_{12} \gtrsim 0.28$ (2σ),
- $|\Delta m_{\text{atm}}^2| \equiv |\Delta m_{31}^2| \cong 2.5 \times 10^{-3} \text{ eV}^2$, $\sin^2 2\theta_{23} \cong 1$,
- θ_{13} - the CHOOZ angle: $\sin^2 \theta_{13} < 0.033$ (0.050) 2σ (3σ).

A.Bandyopadhyay, S.Choubey, S.Goswami, S.T.P., D.P.Roy, hep-ph/0406328 (updated);

T. Schwetz, hep-ph/0606060.

3- ν Mixing Analysis: $\Delta m_{\odot}^2 \ll |\Delta m_{\text{atm}}^2|$

$$P_{\odot}^{3\nu} \cong \sin^4 \theta_{13} + \cos^4 \theta_{13} P_{\odot}^{2\nu},$$

$$P_{\odot}^{2\nu} = \bar{P}_{\odot}^{2\nu} + P_{\odot}^{2\nu}{}_{\text{osc}},$$

$$\bar{P}_{\odot}^{2\nu} = \frac{1}{2} + \left(\frac{1}{2} - P'\right) \cos 2\theta_{12}^m(t_0) \cos 2\theta_{12} \quad (\theta_{12} \equiv \theta_{\odot}),$$

S. Parke, W. Haxton, 1986

$$N_e \rightarrow N_e \cos^2 \theta_{13},$$

$$P' = \frac{e^{-2\pi r_0 \frac{\Delta m^2}{2E}} \sin^2 \theta - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}{1 - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}$$

S.T.P., 1988

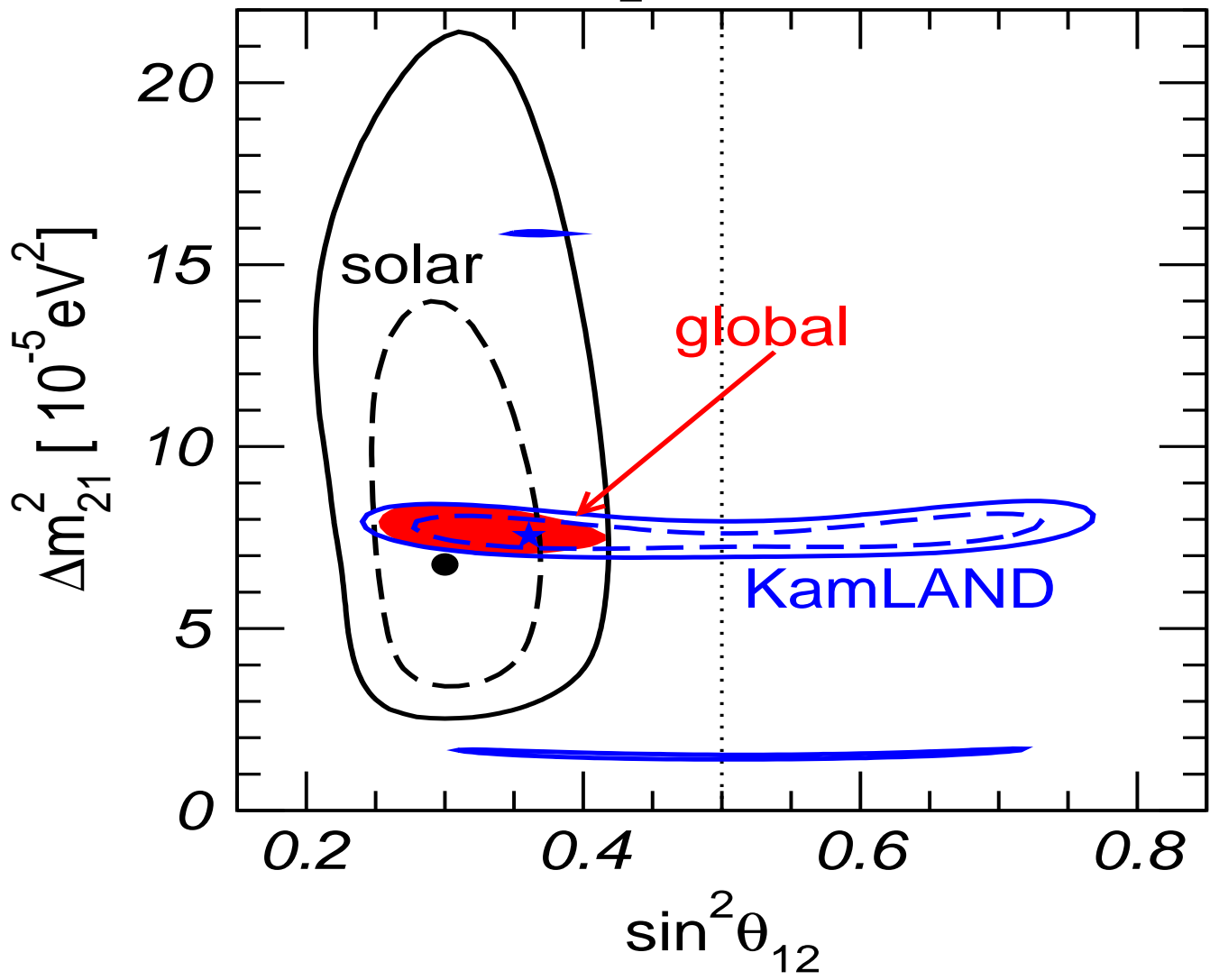
$$\text{LMA: } P' \ll 1, \quad \langle P_{\odot}^{2\nu}{}_{\text{osc}} \rangle \cong 0$$

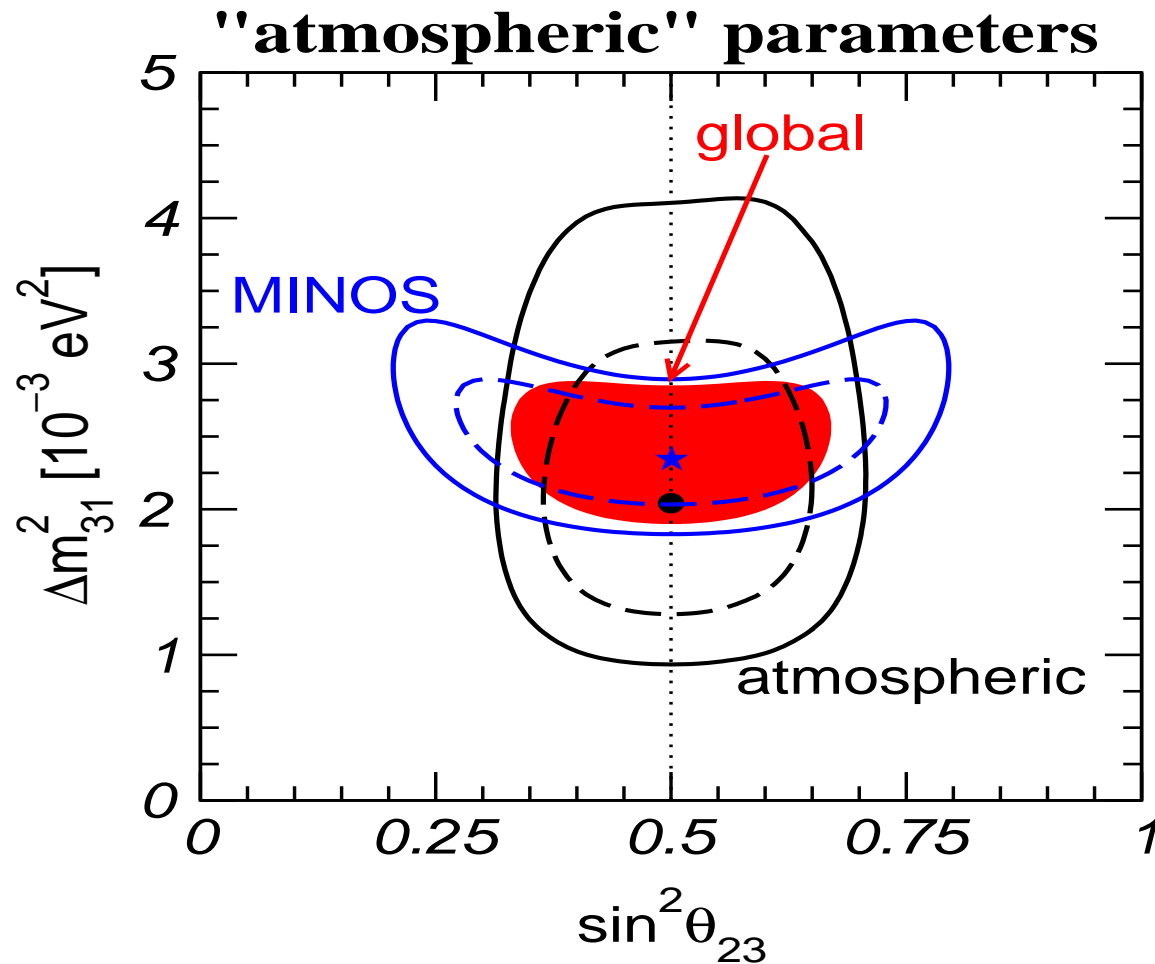
J. Rich, S.T.P., 1988

$$P_{\text{KL}}^{3\nu} \cong \sin^4 \theta_{13} + \cos^4 \theta_{13} \left[1 - \sin^2 2\theta_{12} \sin^2 \left(\frac{\Delta m_{\odot}^2}{4E} L \right) \right]$$

$$P_{\text{CHOOZ}}^{3\nu} \cong 1 - \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m_{\text{atm}}^2}{4E} L \right)$$

"solar" parameters





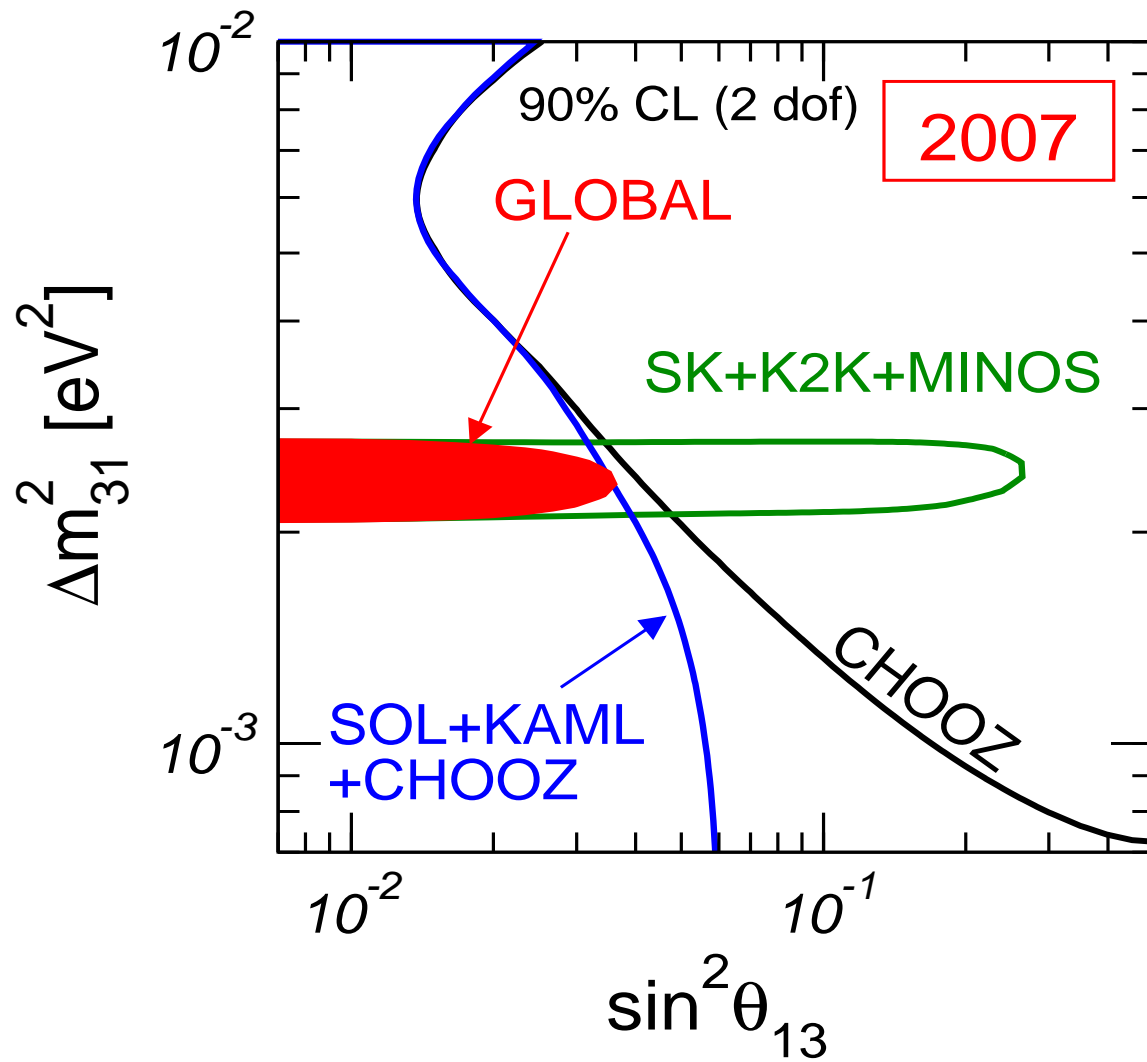
T. Schwetz, arXiv:0710.5027[hep-ph]

- sign of Δm_{atm}^2 not determined;

3- ν mixing: $\Delta m_{31}^2 > 0$, $m_1 < m_2 < m_3$ (normal ordering (NO));

$\Delta m_{31}^2 < 0$, $m_3 < m_1 < m_2$ (inverted ordering (IO)).

- If $\theta_{23} \neq \frac{\pi}{4}$: θ_{23} , $(\frac{\pi}{4} - \theta_{23})$ ambiguity.



- $\sin^2 \theta_{13} < 0.033$ (0.050) at 95% (99.73%) C.L.

Neutrino Oscillation Parameters

parameter	bf	1σ acc.	2σ range	3σ range
Δm_{21}^2 [10^{-5} eV ²]	7.6	3%	7.3 – 8.1	7.1 – 8.3
$ \Delta m_{31}^2 $ [10^{-3} eV ²]	2.4	6%	2.1 – 2.7	2.0 – 2.8
$\sin^2 \theta_{12}$	0.32	9%	0.28 – 0.37	0.26 – 0.40
$\sin^2 \theta_{23}$	0.50	16%	0.38 – 0.63	0.34 – 0.67
$\sin^2 \theta_{13}$	–	–	≤ 0.033	≤ 0.050

Best fit values (bf), relative accuracies at 1σ , and 2σ and 3σ allowed ranges of three-flavor neutrino oscillation parameters from a combined analysis of global data.

ν_\odot , Δm_{atm}^2 , CHOOZ Data:

- $\theta_{12} = \theta_\odot \cong \frac{\pi}{6}$, $\theta_{23} = \theta_{\text{atm}} \cong \frac{\pi}{4}$, $\theta_{13} < \frac{\pi}{12}$

$$U_{\text{PMNS}} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & \epsilon \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & -\frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} .$$

Very different from the CKM-matrix!

- $\cos \theta_{12} \cong \cos\left(\frac{\pi}{4} - \frac{\pi}{12}\right) = \frac{1}{\sqrt{2}}(1 + \lambda)$, $\sin \theta_{12} \cong \frac{1}{\sqrt{2}}(1 - \lambda)$,
- $\lambda \cong (0.20 - 0.25)$: $\theta_\odot + \theta_c = \pi/4$?

Natural Possibility:

$$U = U_{\text{lep}}^\dagger(\lambda) U_{\text{bim(tri)}}$$

with

$$U_{\text{bim}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad U_{\text{tri}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}.$$

- $U_{\text{lep}}^\dagger(\lambda)$ - from diagonalization of the l^- mass matrix,
- $U_{\text{bim(tri)}}$ - from diagonalization of the ν -mass matrix

Further, $\Delta m_{\odot}^2 \ll |\Delta m_{\text{atm}}^2|$.

- U_{bim} can be associated with a symmetry:

$$L' = L_e - L_\mu - L_\tau$$

S.T.P., 1982

- $U_{\text{bim(tri)}}$ can be associated with a $\mu - \tau$ symmetry of M_ν
T. Fukuyama, H. Nishiura, 1997; R.N. Mohapatra, S. Nussinov, 1999;...

These symmetries cannot be exact.

For $\sin^\ell \theta_{ij} \equiv \lambda_{ij}$ “small”, $\lambda_{12} \gg \lambda_{13}$ (natural),

$$\sin^2 \theta_{12} = \frac{1}{2} - \sin \theta_{13} \cos \phi, \quad U_{\text{bim}},$$

ϕ is the Dirac CPV phase,

$$\sin^2 \theta_{12} = \frac{1}{3} - 2 \frac{\sqrt{2}}{3} \sin \theta_{13} \cos \phi, \quad U_{\text{tri}}.$$

P. Frampton, S.T.P., W. Rodejohann, 2004;
S. King, 2005; S. Antusch, S. King, 2005; I. Masina, 2006;
K. Hochmuth, S.T.P., W. Rodejohann, 2007

Can be tested experimentally.

- $\text{sgn}(\Delta m_{\text{atm}}^2) = \text{sgn}(\Delta m_{31}^2)$ not determined

$$\Delta m_{\text{atm}}^2 \equiv \Delta m_{31}^2 > 0, \quad \text{normal mass ordering}$$

$$\Delta m_{\text{atm}}^2 \equiv \Delta m_{32}^2 < 0, \quad \text{inverted mass ordering}$$

Convention: $m_1 < m_2 < m_3$ - **NMO**, $m_3 < m_1 < m_2$ - **IMO**

$$m_1 \ll m_2 \ll m_3, \quad \text{NH,}$$

$$m_3 \ll m_1 < m_2, \quad \text{IH,}$$

$$m_1 \cong m_2 \cong m_3, \quad m_{1,2,3}^2 \gg \Delta m_{\text{atm}}^2, \quad \text{QD; } m_j \gtrsim 0.10 \text{ eV.}$$

- Majorana phases α_{21}, α_{31} :

– $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

S.M. Bilenky, J. Hosek, S.T.P., 1980;

P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987

– $|\langle m \rangle|$ in $(\beta\beta)_{0\nu}$ -decay depends on α_{21}, α_{31} ;

– $\Gamma(\mu \rightarrow e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;

– BAU, leptogenesis scenario: $\alpha_{21,31}$!

Future Progress

- Determination of the nature - Dirac or Majorana, of ν_j .
- Determination of $\text{sgn}(\Delta m_{\text{atm}}^2)$, type of ν - mass spectrum

$$m_1 \ll m_2 \ll m_3, \quad \text{NH,}$$

$$m_3 \ll m_1 < m_2, \quad \text{IH,}$$

$$m_1 \cong m_2 \cong m_3, \quad m_{1,2,3}^2 \gg \Delta m_{\text{atm}}^2, \quad \text{QD; } m_j \gtrsim 0.10 \text{ eV.}$$

- Determining, or obtaining significant constraints on, the absolute scale of ν_j -masses, or $\min(m_j)$.
- Status of the CP-symmetry in the lepton sector: violated due to δ (Dirac), and/or due to α_{21}, α_{31} (Majorana)?
- High precision determination of $\Delta m_{\odot}^2, \theta_{\odot}, \Delta m_{\text{atm}}^2, \theta_{\text{atm}}$.
- Measurement of, or improving by at least a factor of (5 - 10) the existing upper limit on, $\sin^2 \theta_{13}$.
- Searching for possible manifestations, other than ν_l -oscillations, of the non-conservation of $L_l, l = e, \mu, \tau$, such as $\mu \rightarrow e + \gamma, \tau \rightarrow \mu + \gamma$, etc. decays.

- Understanding at fundamental level the mechanism giving rise to the ν - masses and mixing and to the L_l -non-conservation. Includes understanding
 - the origin of the observed patterns of ν -mixing and ν -masses ;
 - the physical origin of CPV phases in U_{PMNS} ;
 - Are the observed patterns of ν -mixing and of $\Delta m_{21,31}^2$ related to the existence of a new symmetry?
 - Is there any relations between q -mixing and ν - mixing? Is $\theta_{12} + \theta_c = \pi/4$?
 - Is $\theta_{23} = \pi/4$, or $\theta_{23} > \pi/4$ or else $\theta_{23} < \pi/4$?
 - Is there any correlation between the values of CPV phases and of mixing angles in U_{PMNS} ?
- Progress in the theory of ν -mixing might lead to a better understanding of the origin of the BAU.
 - Can the Majorana and/or Dirac CPVP in U_{PMNS} be the leptogenesis CPV parameters at the origin of BAU?

HOW?

- ν_{\odot} –, ν_{atm} – experiments

SK (ν_{atm});

INO (ν_{atm}); MEMPHYS

MINOS (ν_{μ}^{atm}); ATLAS, CMS (ν_{μ}^{atm}) (?)

SNO (2006)

SAGE

BOREXINO

LowNu (XMASS, LENS,...)

- Reactor Experiments \sim (1 – 180) km (SKGd)

- Accelerator Experiments

MINOS 732 km

CNGS (OPERA) 732 km

- Super Beams

T2K, SK (HK) 295 km

NO ν A \sim 800 km

SPL+ β -beams, MEMPHYS (0.5 megaton):
CERN-Frejus \sim 140 km

ν -Factories \sim 3000, 7000 km

- $(\beta\beta)_{0\nu}$ -Decay, ^3H β -Decay

- Astrophysics, Cosmology

Rephasing Invariants Associated with CPVP

Dirac phase δ :

$$J_{CP} = \text{Im} \{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \} .$$

C. Jarlskog, 1985 (for quarks)

CP-, T- violation effects in neutrino oscillations

P. Krastev, S.T.P., 1988

Majorana phases α_{21}, α_{31} :

$$S_1 = \text{Im} \{ U_{e1} U_{e3}^* \}, \quad S_2 = \text{Im} \{ U_{e2} U_{e3}^* \} \quad (\text{not unique}); \quad \text{or}$$
$$S'_1 = \text{Im} \{ U_{\tau 1} U_{\tau 2}^* \}, \quad S'_2 = \text{Im} \{ U_{\tau 2} U_{\tau 3}^* \}$$

J.F. Nieves and P. Pal, 1987, 2001

G.C. Branco et al., 1986

J.A. Aguilar-Saavedra and G.C. Branco, 2000

CP-violation: both $\text{Im} \{ U_{e1} U_{e3}^* \} \neq 0$ and $\text{Re} \{ U_{e1} U_{e3}^* \} \neq 0$.

S_1, S_2 appear in $|\langle m \rangle|$ in $(\beta\beta)_{0\nu}$ -decay.

In general, J_{CP}, S_1 and S_2 are independent.

Dirac CP-Nonconservation: δ in U_{PMNS}

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'} , \quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'} , \quad l, l' = e, \mu, \tau$$

- not sensitive to Majorana CPVP α_{21}, α_{31}

CP-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}) , \quad l \neq l' = e, \mu, \tau$$

N. Cabibbo, 1978
S.M. Bilenky, J. Hosek, S.T.P., 1980;
V. Barger et al., 1980.

CPT-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l)$$

$$l = l' : \quad P(\nu_l \rightarrow \nu_l) = P(\bar{\nu}_l \rightarrow \bar{\nu}_l)$$

T-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_{l'} \rightarrow \nu_l), \quad l \neq l'$$

3ν -mixing:

$$A_{\text{CP}}^{(l,l')} \equiv P(\nu_l \rightarrow \nu_{l'}) - P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}) , \quad l \neq l' = e, \mu, \tau$$

$$A_{\text{T}}^{(l,l')} \equiv P(\nu_l \rightarrow \nu_{l'}) - P(\nu_{l'} \rightarrow \nu_l), \quad l \neq l'$$

$$A_{\text{T}}^{(e,\mu)} = A_{\text{T}}^{(\mu,\tau)} = -A_{\text{T}}^{(e,\tau)}$$

In vacuum:

$$A_{\text{CP(T)}}^{(e,\mu)} = J_{\text{CP}} F_{\text{osc}}^{\text{vac}}$$

$$J_{\text{CP}} = \text{Im} \{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta$$

$$F_{\text{osc}}^{\text{vac}} = \sin\left(\frac{\Delta m_{21}^2 L}{2E}\right) + \sin\left(\frac{\Delta m_{32}^2 L}{2E}\right) + \sin\left(\frac{\Delta m_{13}^2 L}{2E}\right)$$

P.I. Krastev, S.T.P., 1988

In matter: Matter effects violate

$$\text{CP} : P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'})$$

$$\text{CPT} : P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l)$$

P. Langacker et al., 1987

Can conserve the T-invariance (**Earth**)

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_{l'} \rightarrow \nu_l), \quad l \neq l'$$

In matter with constant density: $A_{\text{T}}^{(e,\mu)} = J_{\text{CP}}^{\text{mat}} F_{\text{osc}}^{\text{mat}}$

$$J_{\text{CP}}^{\text{mat}} = J_{\text{CP}}^{\text{vac}} R_{\text{CP}}$$

R_{CP} does not depend on θ_{23} and δ ; $|R_{\text{CP}}| \lesssim 2.5$

P.I. Krastev, S.T.P., 1988

P. Harrison, S. Scott, 2000

HOW?

- Reactor Experiments at $L \sim 1$ km: D-CHOOZ, Daya Bay,....;

MINOS, CNGS (OPERA), $L \sim 730$ km:

$$\sin^2 \theta_{13}$$

- Super Beams: $\theta_{13}, \delta, \dots$

JHF (T2K), SK (HK) 295 km

NuMI (NO ν A) ~ 800 km

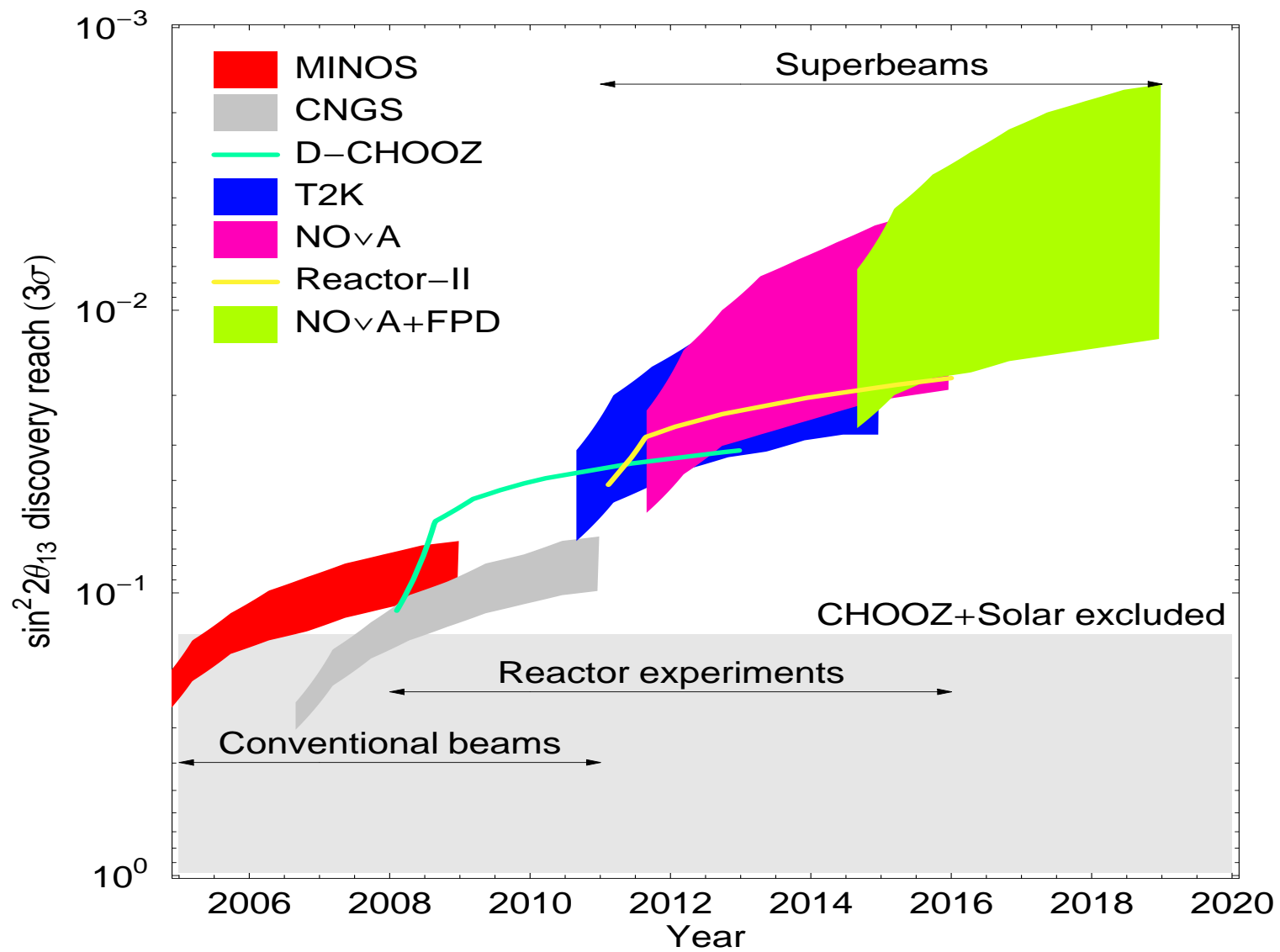
SPL+ β -beams, UNO (1 megaton):
CERN-Frejus ~ 140 km

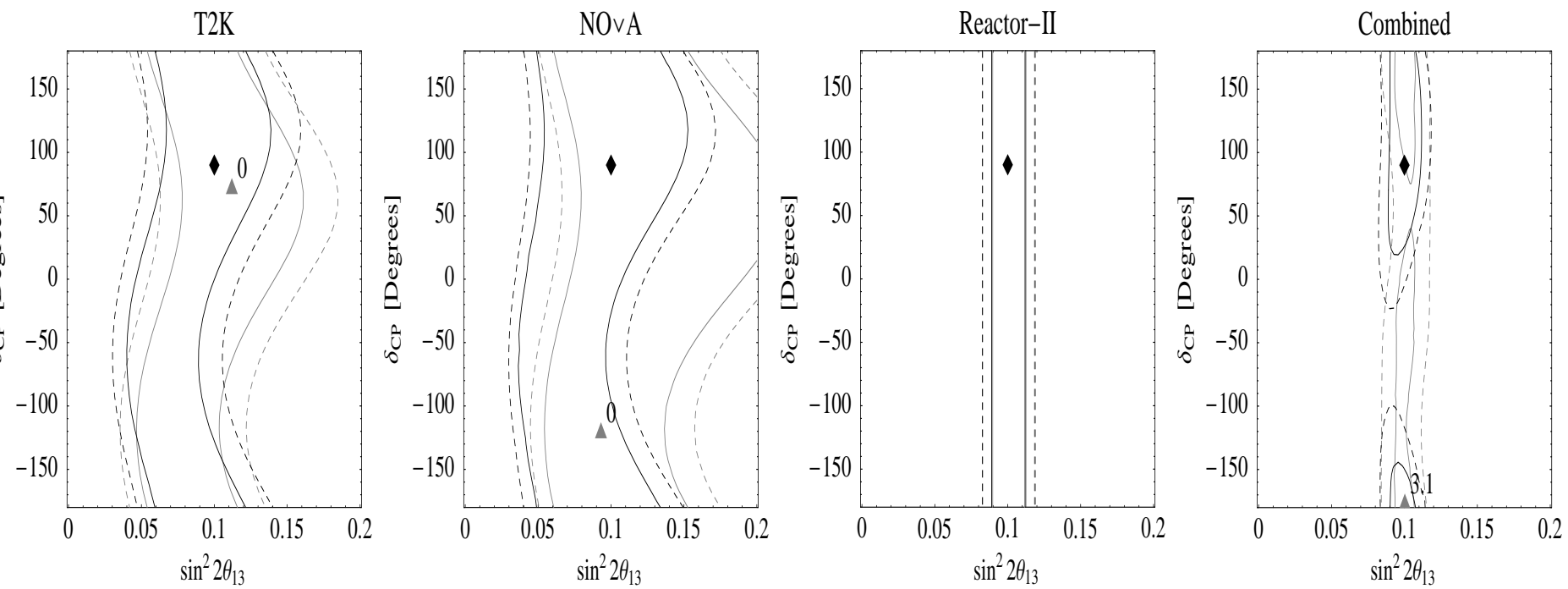
ν -Factories $\sim 3000, 7000$ km



(c) 2000 ESRI

420.0 mi / 675.8 km across





T. Schwetz, arXiv:0710.5027[hep-ph]

If ν_j – Majorana particles, U_{PMNS} contains (3- ν mixing)

δ -Dirac, α_{21} , α_{31} - Majorana physical CPV phases

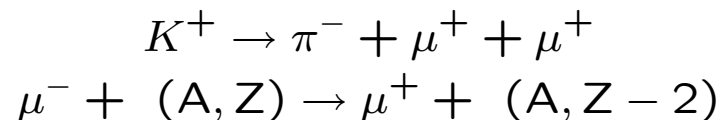
ν -oscillations $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l, l' = e, \mu, \tau$,

- are not sensitive to the nature of ν_j ,

S.M. Bilenky et al., 1980;
P. Langacker et al., 1987

- provide information on $\Delta m_{jk}^2 = m_j^2 - m_k^2$, but not on the absolute values of ν_j masses.

The Majorana nature of ν_j can manifest itself in the existence of $\Delta L = \pm 2$ processes:



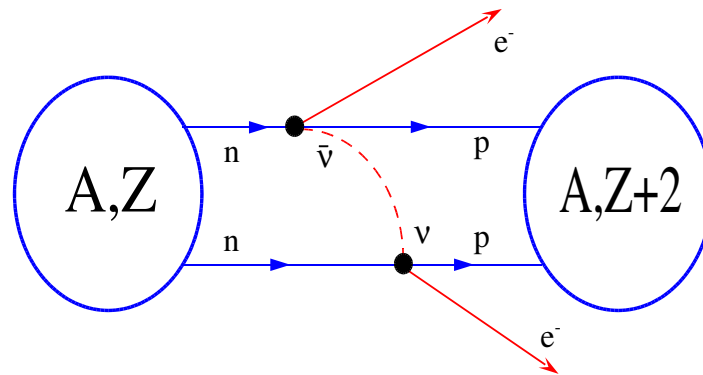
The process most sensitive to the possible Majorana nature of ν_j - $(\beta\beta)_{0\nu}$ -decay



of even-even nuclei, ^{48}Ca , ^{76}Ge , ^{82}Se , ^{100}Mo , ^{116}Cd , ^{130}Te , ^{136}Xe , ^{150}Nd .

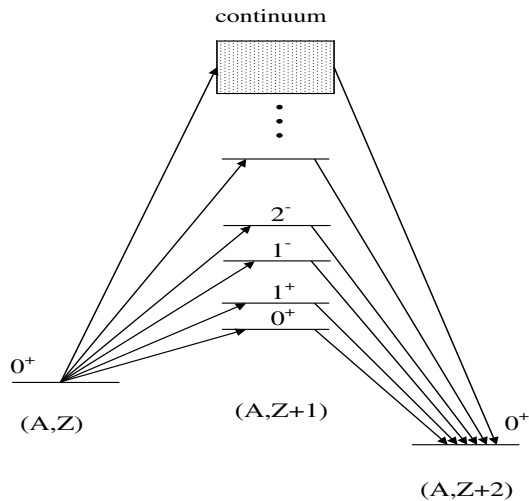
$2n$ from (A, Z) exchange a virtual Majorana ν_j (via the CC weak interaction) and transform into $2p$ of $(A, Z+2)$ and two free e^- .

Nuclear $0\nu\beta\beta$ -decay



strong in-medium modification of the basic process

$$dd \rightarrow uue^-e^-(\bar{\nu}_e\bar{\nu}_e)$$



virtual excitation
of states of all multiplicities
in $(A, Z+1)$ nucleus

$(\beta\beta)_{0\nu}$ –Decay Experiments:

- Majorana nature of ν_j
- Type of ν –mass spectrum (NH, IH, QD)
- Absolute neutrino mass scale

^3H β -decay , cosmology: m_{ν} (QD, IH)

- CPV due to Majorana CPV phases

ν_j – Dirac or Majorana particles, fundamental problem

ν_j –Dirac: **conserved lepton charge exists**, $L = L_e + L_\mu + L_\tau$, $\nu_j \neq \bar{\nu}_j$

ν_j –Majorana: **no lepton charge is exactly conserved**, $\nu_j \equiv \bar{\nu}_j$

The observed patterns of ν –mixing and of Δm_{atm}^2 and Δm_{\odot}^2 can be related to Majorana ν_j and an **approximate** symmetry:

$$L' = L_e - L_\mu - L_\tau$$

S.T.P., 1982

See-saw mechanism: ν_j – Majorana

Establishing that ν_j are Majorana particles would be as important as the discovery of ν – oscillations.

$$A(\beta\beta)_{0\nu} \sim \langle m \rangle M(A,Z), \quad M(A,Z) - \text{NME},$$

$$|\langle m \rangle| = |m_1 |U_{e1}|^2 + m_2 |U_{e2}|^2 e^{i\alpha_{21}} + m_3 |U_{e3}|^2 e^{i\alpha'_{31}}|$$

$$= |m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{i\alpha_{21}} + m_3 s_{13}^2 e^{i\alpha'_{31}}|, \quad \theta_{12} \equiv \theta_{\odot}, \theta_{13} - \text{CHOOZ}$$

α_{21}, α_{31} - the two Majorana CPVP of the PMNS matrix; $\alpha'_{31} \equiv \alpha_{31} - 2\delta$

CP-invariance: $\alpha_{21} = 0, \pm\pi, \alpha_{31} = 0, \pm\pi$;

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1, \quad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1$$

relative CP-parities of ν_1 and ν_2 , and of ν_1 and ν_3 .

L. Wolfenstein, 1981;

S.M. Bilenky, N. Nedelcheva, S.T.P., 1984;

B. Kayser, 1984.

$$|\langle m \rangle| : m_j, \theta_\odot \equiv \theta_{12}, \theta_{13}, \alpha_{21,31}$$

$m_{1,2,3}$ - in terms of $\min(m_j)$, Δm_{atm}^2 , Δm_\odot^2

S.T.P., A.Yu. Smirnov, 1994

Convention: $m_1 < m_2 < m_3$ - **NMO**, $m_3 < m_1 < m_2$ - **IMO**

$$\Delta m_\odot^2 \equiv \Delta m_{21}^2, \quad m_2 = \sqrt{m_1^2 + \Delta m_\odot^2},$$

while either

$$\Delta m_{\text{atm}}^2 \equiv \Delta m_{31}^2 > 0, \quad m_3 = \sqrt{m_1^2 + \Delta m_{\text{atm}}^2}, \quad \text{normal mass ordering, or}$$

$$\Delta m_{\text{atm}}^2 \equiv \Delta m_{32}^2 < 0, \quad m_1 = \sqrt{m_3^2 + |\Delta m_{\text{atm}}^2| - \Delta m_\odot^2}, \quad \text{inverted mass ordering}$$

The neutrino mass spectrum –

Normal hierarchical (NH) if $m_1 \ll m_2 \ll m_3$,

Inverted hierarchical (IH) if $m_3 \ll m_1 \cong m_2$,

Quasi-degenerate (QD) if $m_1 \cong m_2 \cong m_3 = m$, $m_j^2 \gg |\Delta m_{\text{atm}}^2|$; $m_j \gtrsim 0.1$ eV

Given $|\Delta m_{\text{atm}}^2|$, Δm_\odot^2 , θ_\odot , θ_{13} ,

$$|\langle m \rangle| = |\langle m \rangle| (m_{\text{min}}, \alpha_{21}, \alpha_{31}; \mathbf{S}), \quad \mathbf{S} = \text{NO(NH)}, \text{IO(IH)}.$$

$$A(\beta\beta)_{0\nu} \sim \langle m \rangle M(A,Z), \quad M(A,Z) - \text{NME},$$

$$|\langle m \rangle| \cong \left| \sqrt{\Delta m_{\odot}^2} \sin^2 \theta_{12} e^{i\alpha} + \sqrt{\Delta m_{31}^2} \sin^2 \theta_{13} e^{i\beta} \right|, \quad m_1 \ll m_2 \ll m_3 \text{ (NH)},$$

$$|\langle m \rangle| \cong \sqrt{m_3^2 + \Delta m_{13}^2} |\cos^2 \theta_{12} + e^{i\alpha} \sin^2 \theta_{12}|, \quad m_3 < (\ll) m_1 < m_2 \text{ (IH)},$$

$$|\langle m \rangle| \cong m |\cos^2 \theta_{12} + e^{i\alpha} \sin^2 \theta_{12}|, \quad m_{1,2,3} \cong m \gtrsim 0.10 \text{ eV (QD)},$$

$$\theta_{12} \equiv \theta_{\odot}, \theta_{13} \text{-CHOOZ}; \quad \alpha \equiv \alpha_{21}, \beta + 2\delta \equiv \alpha_{31}.$$

CP-invariance: $\alpha = 0, \pm\pi, \beta_M = 0, \pm\pi;$

$$|\langle m \rangle| \lesssim 5 \times 10^{-3} \text{ eV, NH};$$

$$\sqrt{\Delta m_{13}^2} \cos 2\theta_{12} \cong 0.013 \text{ eV} \lesssim |\langle m \rangle| \lesssim \sqrt{\Delta m_{13}^2} \cong 0.055 \text{ eV, IH};$$

$$m \cos 2\theta_{12} \lesssim |\langle m \rangle| \lesssim m, \quad m \gtrsim 0.10 \text{ eV, QD}.$$

Best sensitivity: Heidelberg-Moscow ^{76}Ge experiment.

Claim for a positive signal at $> 3\sigma$:

H. Klapdor-Kleingrothaus et al., PL B586 (2004),

$|\langle m \rangle| = (0.1 - 0.9) \text{ eV}$ (99.73% C.L.).

IGEX ^{76}Ge : $|\langle m \rangle| < (0.33 - 1.35) \text{ eV}$ (90% C.L.).

Taking data - NEMO3 (^{100}Mo), CUORICINO (^{130}Te):

$|\langle m \rangle| < (0.7 - 1.2) \text{ eV}$, $|\langle m \rangle| < (0.18 - 0.90) \text{ eV}$ (90% C.L.).

Large number of projects: $|\langle m \rangle| \sim (0.01 - 0.05) \text{ eV}$

CUORE - ^{130}Te ,

GERDA - ^{76}Ge ,

SuperNEMO,

COBRA - ^{116}Cd ,

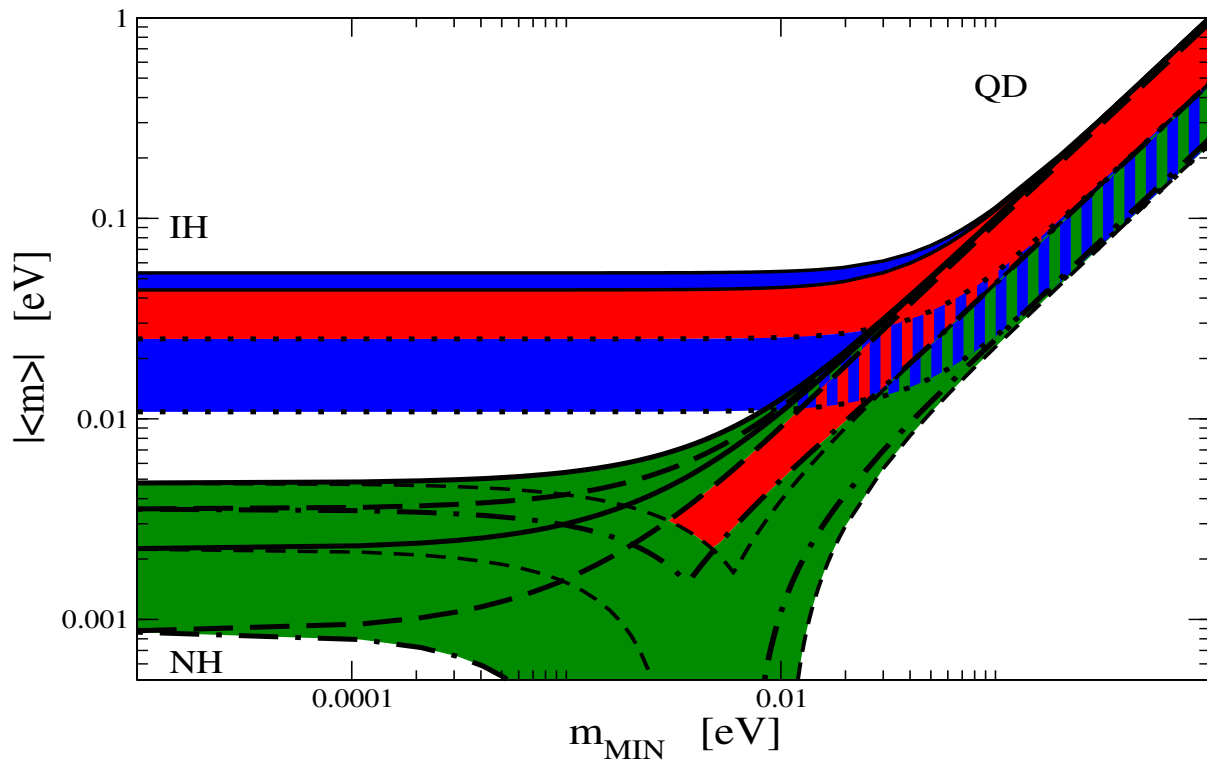
EXO - ^{136}Xe ,

MAJORANA - ^{76}Ge ,

MOON - ^{100}Mo ,

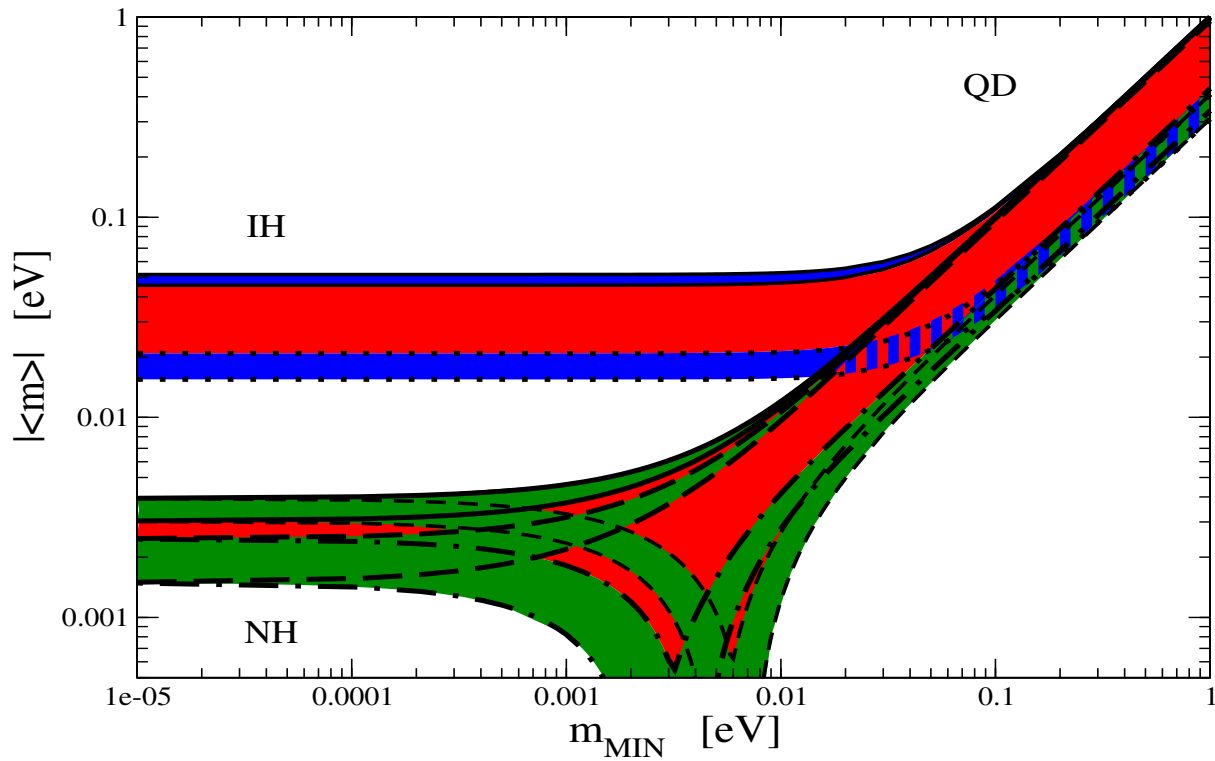
CANDLES - ^{48}Ca ,

XMASS - ^{136}Xe .



S. Pascoli, S.T.P., 2006

The current 2σ ranges of values of the parameters used.



$\sin^2 \theta_{13} = 0.015 \pm 0.006$; $1\sigma(\Delta m_{\odot}^2) = 4\%$, $1\sigma(\sin^2 \theta_{\odot}) = 4\%$, $1\sigma(|\Delta m_{\text{atm}}^2|) = 6\%$;

$2\sigma(|\langle m \rangle|)$ used.

Majorana CPV Phases and $|\langle m \rangle|$

CPV can be established provided

- $|\langle m \rangle|$ measured with $\Delta \lesssim 15\%$;
- Δm_{atm}^2 (IH) or m_0 (QD) measured with $\delta \lesssim 10\%$;
- $\xi \lesssim 1.5$;
- α_{21} (QD): in the interval $\sim [\frac{\pi}{4} - \frac{3\pi}{4}]$, or $\sim [\frac{5\pi}{4} - \frac{3\pi}{2}]$;
- $\tan^2 \theta_{\odot} \gtrsim 0.40$.

S. Pascoli, S.T.P., W. Rodejohann, 2002

S. Pascoli, S.T.P., L. Wolfenstein, 2002

S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226

No “No-go for detecting CP-Violation via $(\beta\beta)_{0\nu}$ -decay”

V. Barger *et al.*, 2002

Absolute Neutrino Mass Measurements

The Troitzk and Mainz ${}^3\text{H}$ β -decay experiments

$$m_{\nu_e} < 2.3 \text{ eV} \quad (95\% \text{ C.L.})$$

There are prospects to reach sensitivity

$$\text{KATRIN :} \quad m_{\nu_e} \sim 0.2 \text{ eV}$$

Cosmological and astrophysical data: the WMAP result combined with data from large scale structure surveys (2dFGRS, SDSS)

$$\sum_j m_j \equiv \Sigma < (0.4 - 1.7) \text{ eV}$$

The WMAP and future PLANCK experiments can be sensitive to

$$\sum_j m_j \cong 0.4 \text{ eV}$$

Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP and PLANCK experiments may allow to determine

$$\sum_j m_j : \quad \delta \cong 0.04 \text{ eV.}$$

M_ν from the See-Saw Mechanism

P. Minkowski, 1977.

M. Gell-Mann, P. Ramond, R. Slansky, 1979;

T. Yanagida, 1979;

R. Mohapatra, G. Senjanovic, 1980.

- Explains the smallness of ν -masses.
- Through **leptogenesis theory** links the ν -mass generation to the generation of baryon asymmetry of the Universe Y_B .

S. Fukugita, T. Yanagida, 1986.

- In SUSY GUT's with see-saw mechanism of ν -mass generation, the LFV decays

$$\mu \rightarrow e + \gamma, \quad \tau \rightarrow \mu + \gamma, \quad \tau \rightarrow e + \gamma, \quad \text{etc.}$$

are predicted to take place with rates within the reach of present and future experiments.

F. Borzumati, A. Masiero, 1986.

- The ν_j are **Majorana particles**; $(\beta\beta)_{0\nu}$ -decay is allowed.

See-Saw: Dirac ν -mass m_D + Majorana mass M_R for N_R

The See-Saw Lagrangian

$$\mathcal{L}^{\text{lep}}(x) = \mathcal{L}_{\text{CC}}(x) + \mathcal{L}_Y(x) + \mathcal{L}_M^{\text{N}}(x),$$

$$\mathcal{L}_{\text{CC}} = -\frac{g}{\sqrt{2}} \bar{l}_L(x) \gamma_\alpha \nu_{lL}(x) W^{\alpha\dagger}(x) + h.c.,$$

$$\mathcal{L}_Y(x) = \lambda_{il} \bar{N}_{iR}(x) H^\dagger(x) \psi_{lL}(x) + Y_l H^c(x) \bar{l}_R(x) \psi_{lL}(x) + h.c.,$$

$$\mathcal{L}_M^{\text{N}}(x) = -\frac{1}{2} M_i \bar{N}_i(x) N_i(x).$$

ψ_{lL} - LH doublet, $\psi_{lL}^T = (\nu_{lL} \ l_L)$, l_R - RH singlet, H - Higgs doublet.

Basis: $M_R = (M_1, M_2, M_3)$; $D_N \equiv \text{diag}(M_1, M_2, M_3)$, $D_\nu \equiv \text{diag}(m_1, m_2, m_3)$.

m_D generated by the Yukawa interaction:

$$-\mathcal{L}_Y^\nu = \lambda_{il} \bar{N}_{iR} H^\dagger(x) \psi_{lL}(x), \quad v = 174 \text{ GeV}, \quad v \lambda = m_D - \text{complex}$$

For M_R - sufficiently large,

$$m_\nu \simeq v^2 \lambda^T M_R^{-1} \lambda = U_{\text{PMNS}}^* m_\nu^{\text{diag}} U_{\text{PMNS}}^\dagger.$$

$Y_\nu \equiv \lambda = \sqrt{D_N} R \sqrt{D_\nu} (U_{\text{PMNS}})^\dagger / v_u$, all at M_R ; R -complex, $R^T R = \mathbf{1}$.

J.A. Casas and A. Ibarra, 2001

In GUTs, $M_R < M_X$, $M_X \sim 10^{16}$ GeV;

in GUTs, e.g., $M_R = (10^9, 10^{12}, 10^{15})$ GeV, $m_D \sim 1$ GeV.

Leptogenesis

$$Y_B = \frac{n_B - n_{\bar{B}}}{S} \sim 8.6 \times 10^{-11} \quad (n_\gamma: \sim 6.3 \times 10^{-10})$$

$$Y_B \cong -10^{-2} \quad \varepsilon \kappa$$

W. Buchmüller, M. Plümacher, 1998;

W. Buchmüller, P. Di Bari, M. Plümacher, 2004

κ - efficiency factor; $\kappa \sim 10^{-1} - 10^{-3}$: $\varepsilon \gtrsim 10^{-7}$.

ε : CP -, L - violating asymmetry generated in out of equilibrium N_{Rj} -decays in the early Universe,

$$\varepsilon_1 = \frac{\Gamma(N_1 \rightarrow \Phi^- \ell^+) - \Gamma(N_1 \rightarrow \Phi^+ \ell^-)}{\Gamma(N_1 \rightarrow \Phi^- \ell^+) + \Gamma(N_1 \rightarrow \Phi^+ \ell^-)}$$

M.A. Luty, 1992;

L. Covi, E. Roulet and F. Vissani, 1996;

M. Flanz *et al.*, 1996;

M. Plümacher, 1997;

A. Pilaftsis, 1997.

$\kappa = \kappa(\tilde{m})$, \tilde{m} - determines the rate of wash-out processes:

$\Phi^+ + \ell^- \rightarrow N_1$, $\ell^- + \Phi^+ \rightarrow \Phi^- + \ell^+$, etc.

W. Buchmüller, P. Di Bari and M. Plümacher, 2002;

G. F. Giudice *et al.*, 2004

Low Energy Leptonic CPV and Leptogenesis

Assume: $M_1 \ll M_2 \ll M_3$

Individual asymmetries:

$$\varepsilon_{1l} = -\frac{3M_1}{16\pi v^2} \frac{\text{Im} \left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U_{lj}^* U_{lk} R_{1j} R_{1k} \right)}{\sum_j m_j |R_{1j}|^2}, \quad v = 174 \text{ GeV}$$

$$\widetilde{m}_l \equiv \frac{|\lambda_{1l}|^2 v^2}{M_1} = \left| \sum_k R_{1k} m_k^{1/2} U_{lk}^* \right|^2, \quad l = e, \mu, \tau.$$

The “one-flavor” approximation - $\mathbf{Y}_{e,\mu,\tau}$ - “small”:

Boltzmann eqn. for $n(N_1)$ and $\Delta L = \Delta(L_e + L_\mu + L_\tau)$.

$Y_l H^c(x) \bar{l}_R(x) \psi_{lL}$ - out of equilibrium at $T \sim M_1$.

One-flavor approximation: $M_1 \sim T > 10^{12} \text{ GeV}$

$$\varepsilon_1 = \sum_l \varepsilon_{1l} = -\frac{3M_1}{16\pi v^2} \frac{\text{Im} \left(\sum_{j,k} m_j^2 R_{1j}^2 \right)}{\sum_k m_k |R_{1k}|^2},$$

$$\widetilde{m}_1 = \sum_l \widetilde{m}_l = \sum_k m_k |R_{1k}|^2.$$

Two-Flavour Regime

At $M_1 \sim T \sim 10^{12}$ GeV: Y_τ - in equilibrium, $Y_{e,\mu}$ - not; dynamics changes: τ_R^-, τ_L^+

$\tau_R^- + N_1 \rightarrow \nu_L + \tau_R^-$, $N_1 + \nu_L \rightarrow \tau_R^- + \tau_L^+$, etc.

$\varepsilon_{1\tau}$ and $(\varepsilon_{1e} + \varepsilon_{1\mu}) \equiv \varepsilon_2$ evolve independently.

Three-Flavour Regime

At $M_1 \sim T \sim 10^9$ GeV: Y_τ, Y_μ - in equilibrium, Y_e - not.

$\varepsilon_{1\tau}, \varepsilon_{1e}$ and $\varepsilon_{1\mu}$ evolve independently.

Thus, at $M_1 \sim 10^9 - 10^{12}$ GeV: $L_\tau, \Delta L_\tau$ - distinguishable;

$L_e, L_\mu, \Delta L_e, \Delta L_\mu$ - individually not distinguishable;

$L_e + L_\mu, \Delta(L_e + L_\mu)$

A. Abada et al., 2006; E. Nardi et al., 2006

A. Abada et al., 2006

Individual asymmetries:

Assume: $M_1 \ll M_2 \ll M_3$, $10^9 \lesssim M_1 (\sim T) \lesssim 10^{12}$ GeV,

$$\epsilon_{1l} = -\frac{3M_1}{16\pi v^2} \frac{\text{Im} \left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U_{lj}^* U_{lk} R_{1j} R_{1k} \right)}{\sum_j m_j |R_{1j}|^2}$$

$$\widetilde{m}_l \equiv \frac{|\lambda_{1l}|^2 v^2}{M_1} = \left| \sum_k R_{1k} m_k^{1/2} U_{lk}^* \right|^2, \quad l = e, \mu, \tau.$$

The baryon asymmetry is

$$Y_B \simeq -\frac{12}{37g_*} \left(\epsilon_2 \eta \left(\frac{417}{589} \widetilde{m}_2 \right) + \epsilon_\tau \eta \left(\frac{390}{589} \widetilde{m}_\tau \right) \right),$$

$$\eta(\widetilde{m}_l) \simeq \left(\left(\frac{\widetilde{m}_l}{8.25 \times 10^{-3} \text{ eV}} \right)^{-1} + \left(\frac{0.2 \times 10^{-3} \text{ eV}}{\widetilde{m}_l} \right)^{-1.16} \right)^{-1}.$$

$$Y_B = -(12/37) (Y_2 + Y_\tau),$$

$$Y_2 = Y_{e+\mu}, \quad \epsilon_2 = \epsilon_{1e} + \epsilon_{1\mu}, \quad \widetilde{m}_2 = \widetilde{m}_{1e} + \widetilde{m}_{1\mu}$$

A. Abada et al., 2006; E. Nardi et al., 2006

A. Abada et al., 2006

Real (Purely Imaginary) R : $\varepsilon_{1l} \neq 0$, CPV from U

$$\varepsilon_{1e} + \varepsilon_{1\mu} + \varepsilon_{1\tau} = \varepsilon_2 + \varepsilon_{1\tau} = 0,$$

$$\begin{aligned} \varepsilon_{1\tau} &= -\frac{3M_1}{16\pi v^2} \frac{\text{Im} \left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U_{\tau j}^* U_{\tau k} R_{1j} R_{1k} \right)}{\sum_j m_j |R_{1j}|^2} \\ &= -\frac{3M_1}{16\pi v^2} \frac{\sum_{j,k>j} m_j^{1/2} m_k^{1/2} (m_k - m_j) R_{1j} R_{1k} \text{Im} (U_{\tau j}^* U_{\tau k})}{\sum_j m_j |R_{1j}|^2}, R_{1j} R_{1k} = \pm |R_{1j} R_{1k}|, \end{aligned}$$

S. Pascoli, S.T.P., A. Riotto, 2006.

CP-Violation: $\text{Im} (U_{\tau j}^* U_{\tau k}) \neq 0$, $\text{Re} (U_{\tau j}^* U_{\tau k}) \neq 0$;

$$Y_B = -\frac{12}{37} \frac{\varepsilon_{1\tau}}{g_*} \left(\eta \left(\frac{390}{589} \widetilde{m}_\tau \right) - \eta \left(\frac{417}{589} \widetilde{m}_2 \right) \right)$$

$m_1 \ll m_2 \ll m_3, M_1 \ll M_{2,3}; R_{12}R_{13} - \text{real}; m_1 \cong 0, R_{11} \cong 0 (N_3 \text{ decoupling})$

$$\varepsilon_{1\tau} = - \frac{3M_1 \sqrt{\Delta m_{31}^2}}{16\pi v^2} \left(\frac{\Delta m_{\odot}^2}{\Delta m_{31}^2} \right)^{\frac{1}{4}} \frac{|R_{12}R_{13}|}{\left(\frac{\Delta m_{\odot}^2}{\Delta m_{31}^2} \right)^{\frac{1}{2}} |R_{12}|^2 + |R_{13}|^2} \times \left(1 - \frac{\sqrt{\Delta m_{\odot}^2}}{\sqrt{\Delta m_{31}^2}} \right) \text{Im}(U_{\tau 2}^* U_{\tau 3})$$

$$\text{Im}(U_{\tau 2}^* U_{\tau 3}) = -c_{13} \left[c_{23}s_{23}c_{12} \sin\left(\frac{\alpha_{32}}{2}\right) - c_{23}^2 s_{12}s_{13} \sin\left(\delta - \frac{\alpha_{32}}{2}\right) \right]$$

$\alpha_{32} = \pi, \delta = 0: \text{Re}(U_{\tau 2}^* U_{\tau 3}) = 0, \text{CPV due to } R$

S. Pascoli, S.T.P., A. Riotto, 2006.

Low Energy Leptonic CPV and Leptogenesis: Summary

Leptogenesis: see-saw mechanism; N_j - heavy RH ν 's;

N_j, ν_k - Majorana particles

$$N_j: M_1 \ll M_2 \ll M_3$$

The observed value of the baryon asymmetry of the Universe can be generated

A. CP-violation due to the Dirac phase δ in U_{PMNS} , no other sources of CPV (Majorana phases in U_{PMNS} equal to 0, etc.); requires $M_1 \gtrsim 10^{11}$ GeV.

$m_1 \ll m_2 \ll m_3$ (NH):

$$|\sin \theta_{13} \sin \delta| \gtrsim 0.09, \quad \sin \theta_{13} \gtrsim 0.09; \quad |J_{\text{CP}}| \gtrsim 2.0 \times 10^{-2}$$

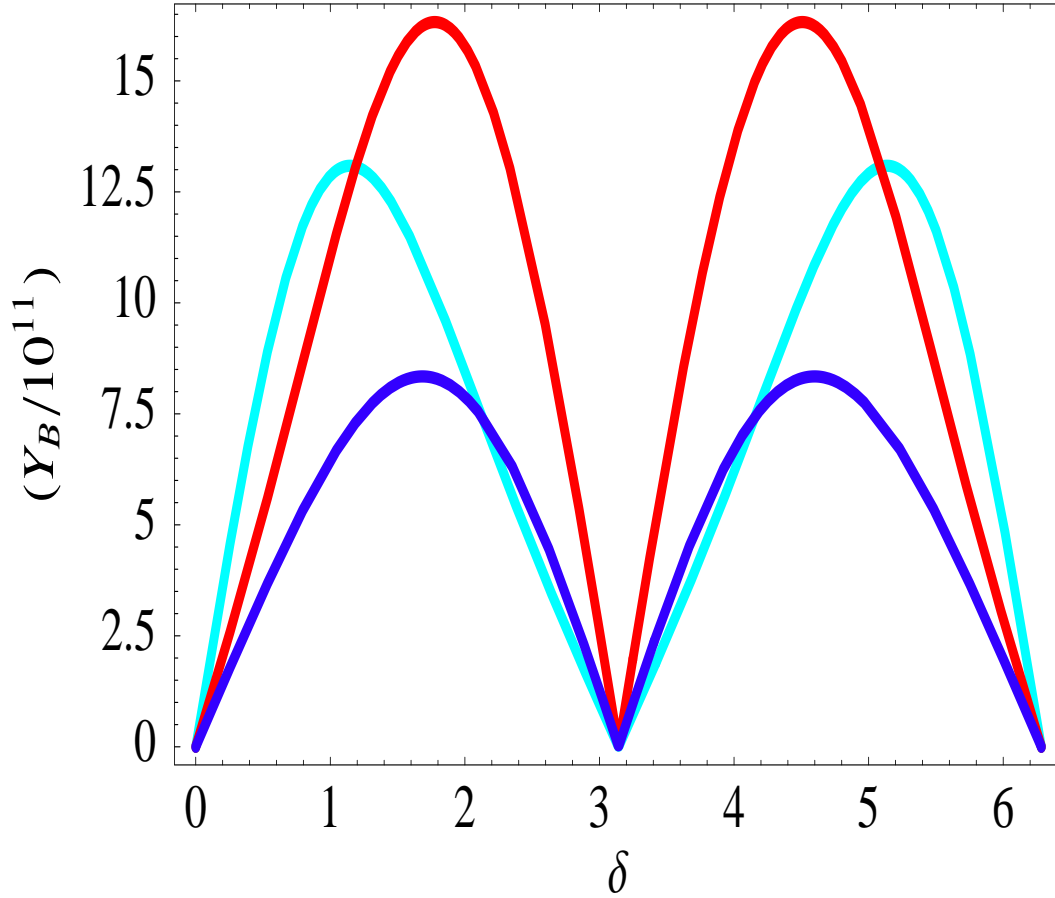
$m_3 \ll m_1 < m_2$ (IH):

$$|\sin \theta_{13} \sin \delta| \gtrsim 0.02, \quad \sin \theta_{13} \gtrsim 0.02; \quad |J_{\text{CP}}| \gtrsim 4.6 \times 10^{-3}$$

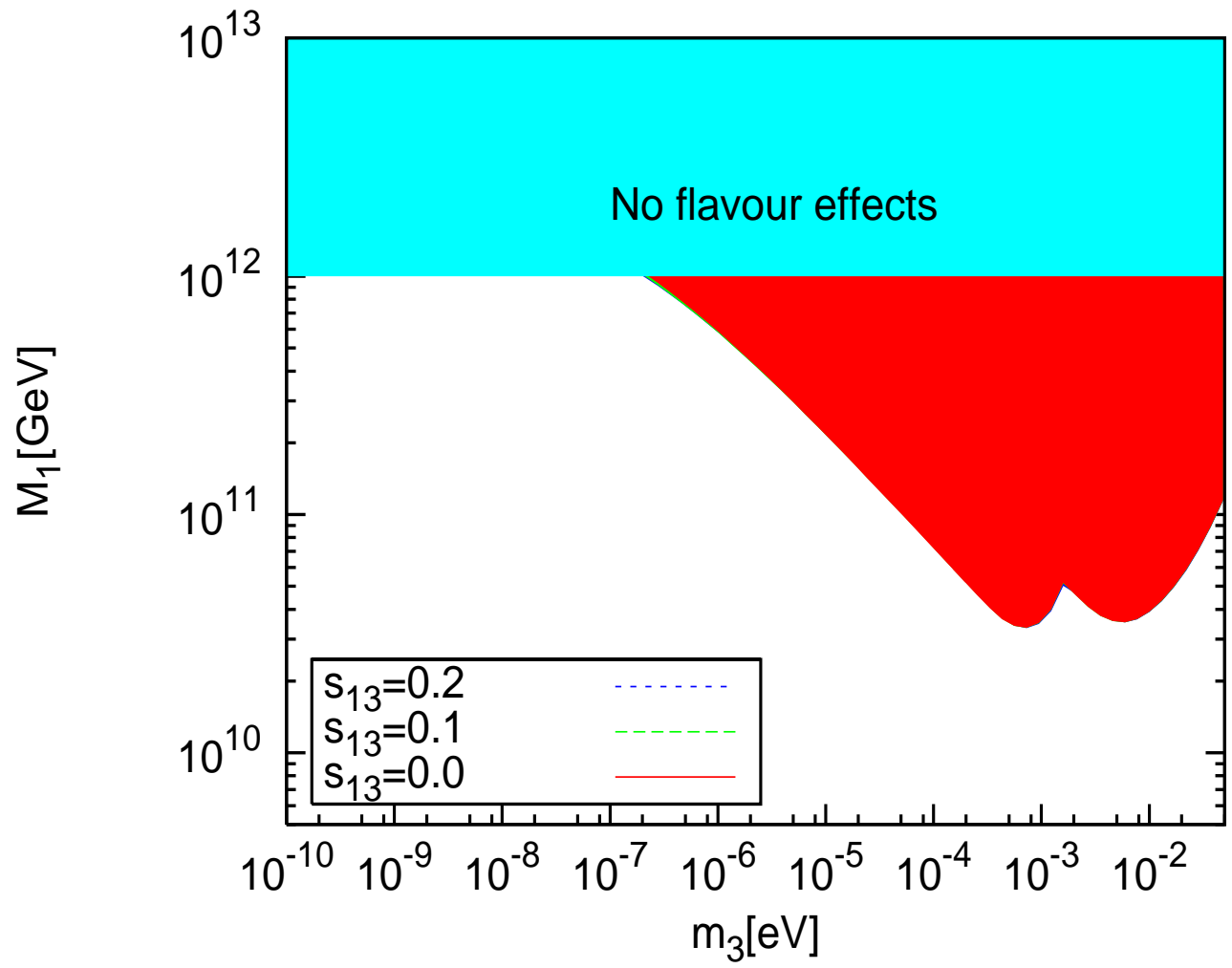
B. CP-violation due to the Majorana phases in U_{PMNS} , no other sources of CPV (Dirac phase in U_{PMNS} equal to 0, etc.); requires $M_1 \gtrsim 3.5 \times 10^{10}$ GeV.

C. CP-violation due to both Dirac and Majorana phases in U_{PMNS} .

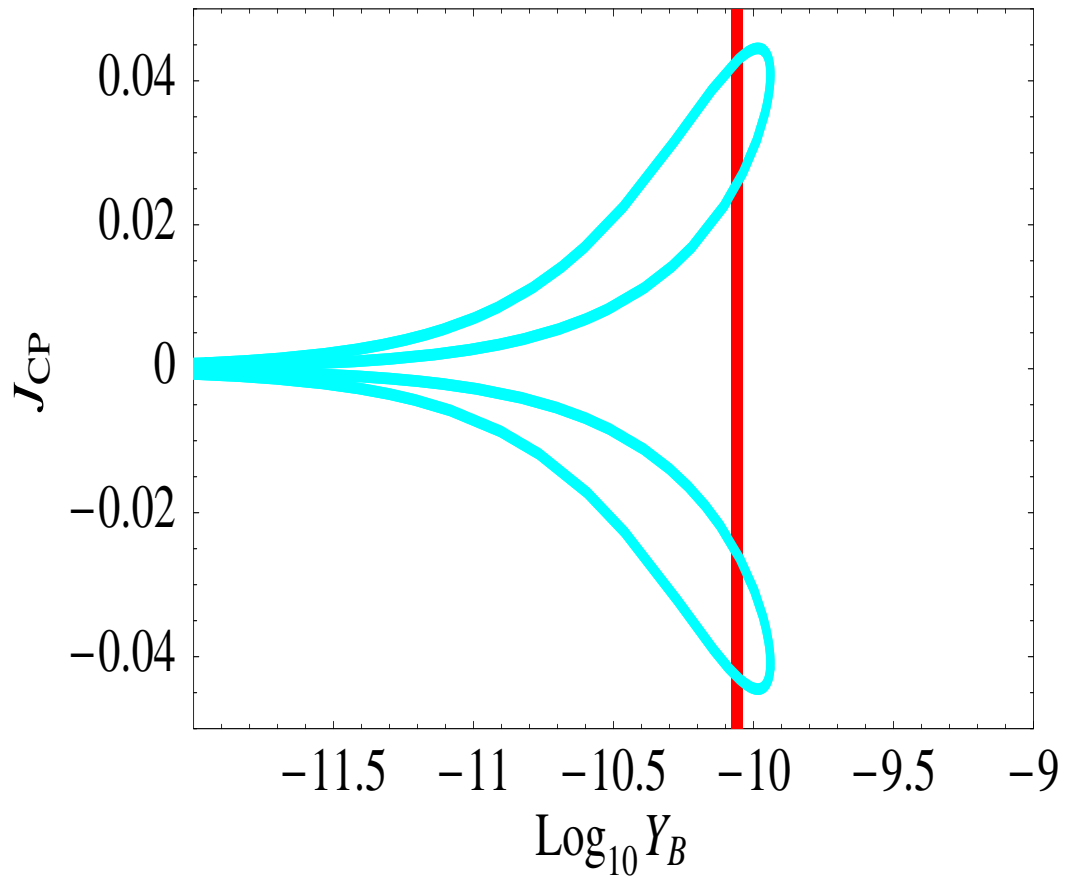
D. Y_B can depend non-trivially on $\min(m_j) \sim (10^{-5} - 10^{-2})$ eV.



$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$; Dirac CP-violation, $\alpha_{32} = 0$; 2π ;
 real R_{12} , R_{13} , $|R_{12}|^2 + |R_{13}|^2 = 1$, $|R_{12}| = 0.86$, $|R_{13}| = 0.51$, $\text{sign}(R_{12}R_{13}) = +1$;
 i) $\alpha_{32} = 0$ ($\kappa' = +1$), $s_{13} = 0.2$ (red line) and $s_{13} = 0.1$ (dark blue line);
 ii) $\alpha_{32} = 2\pi$ ($\kappa' = -1$), $s_{13} = 0.2$ (light blue line);
 $M_1 = 5 \times 10^{11}$ GeV.

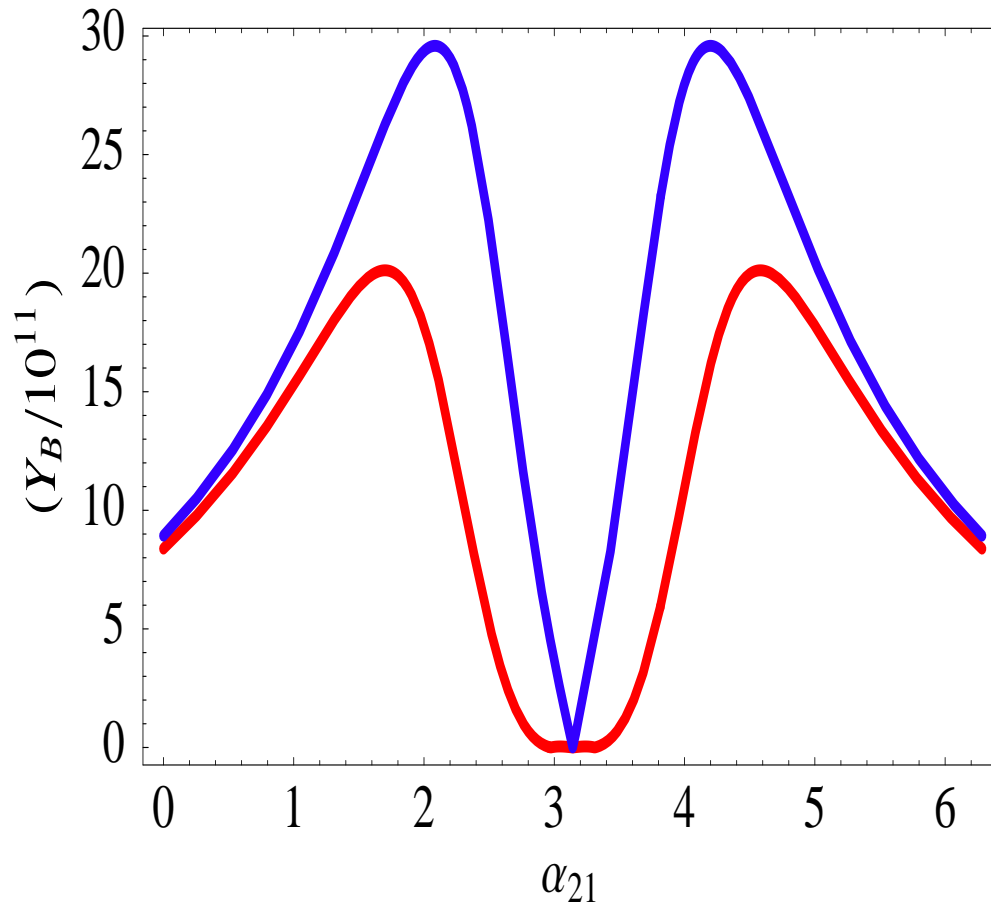


$m_3 < m_1 < m_2$, $M_1 \ll M_2 \ll M_3$, real R_{1j} ; $M_1 = (10^9 - 10^{12})$ GeV, $s_{13} = 0.2; 0.1; 0$;
 R_{1j} varied within $|R_{13}|^2 + |R_{12}|^2 + |R_{11}|^2 = 1$; $\alpha_{21}, \alpha_{31}, \delta$ varied in $[0, 2\pi]$;
 min(M_1) for given m_3 : $|Y_B| = 8.6 \times 10^{-11}$; absolute minima of M_1 :
 $m_3 \cong 5.5 \times 10^{-4}$; 5.9×10^{-3} eV, $\alpha_{32} \cong \pi/2$, $M_1 = 3.4$ (3.5) $\times 10^{10}$ GeV.

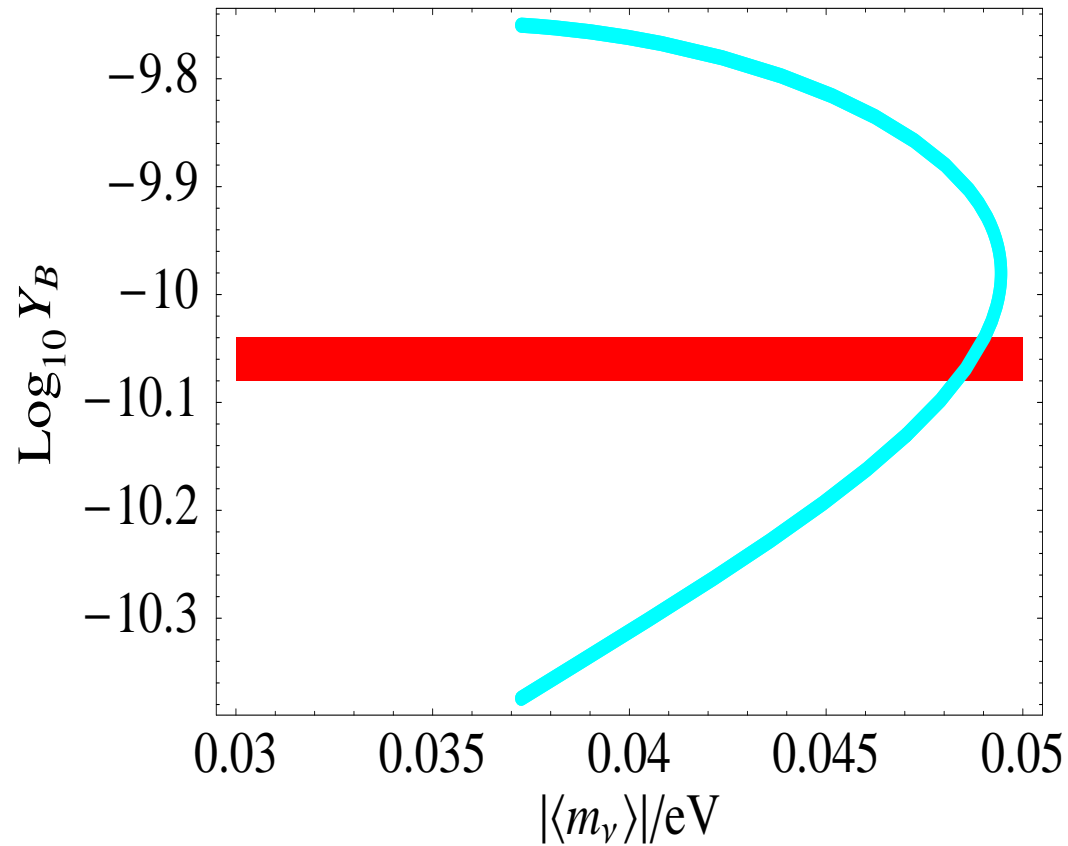


$M_1 \ll M_2 \ll M_3, m_1 \ll m_2 \ll m_3; M_1 = 5 \times 10^{11} \text{ GeV};$
 Dirac CP-violation, $\alpha_{32} = 0 \text{ (} 2\pi \text{)}$;
 $|R_{12}| = 0.86, |R_{13}| = 0.51, \text{sign}(R_{12}R_{13}) = +1 \text{ (-1)} \text{ (}\beta_{23} = 0 \text{ (}\pi\text{), } \kappa' = +1\text{)}$;
 The red region denotes the 2σ allowed range of Y_B .

S. Pascoli, S.T.P., A. Riotto, 2006.



$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$; $M_1 = 2 \times 10^{11}$ GeV;
 Majorana CP-violation, $\delta = 0$;
 purely imaginary $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = -1$, $|R_{11}|^2 - |R_{12}|^2 = 1$, $|R_{11}| = 1.2$;
 $s_{13} = 0$ (blue line) and 0.2 (red line).



$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$; $M_1 = 2 \times 10^{11}$ GeV;
 Majorana CP-violation, $\delta = 0$, $s_{13} = 0$;
 purely imaginary $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = +1$ $|R_{11}|^2 - |R_{12}|^2 = 1$, $|R_{11}| = 1.05$.
 The Majorana phase α_{21} is varied in the interval $[-\pi/2, \pi/2]$.

S. Pascoli, S.T.P., A. Riotto, 2006.

Instead of Conclusions

We are at the beginning of the Road...