Flavour Physics with Domain Wall Fermions

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Introduction

The plenary programme at Lattice 2007 opened with reviews from 5 major collaborations/groupings using different formulations of lattice QCD (lattice fermions in particular):

- 2+1 flavour Domain Wall Fermion simulations by the RBC and UKQCD Collaborations; P.A.Boyle, arXiv:0710.5337 [hep-lat]
- Exploring chiral regime with dynamical overlap fermions; H.Matsufuru (JLQCD) , arXiv:0710.4225 [hep-lat]
- Lattice QCD with two light Wilson quarks and maximally twisted mass; C.Urbach (ETM) , arXiv:0710.1517 [hep-lat]
- Dynamical Wilson quark simulations toward the physical point; Y.Kuramashi, arXiv:0711.3938 [hep-lat]
- Lattice QCD with Staggered Quarks: Why, Where, and How (Not) A.Kronfeld (MILC and friends), arXiv:0711.0699 [hep-lat]

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Selection of RBC/UKQCD Talks at Lattice 2007

1. Computational Requirements of the Rational Hybrid Monte Carlo Algorithm;
2. Chiral Limit and Light Quark Masses in 2+1 Flavour Domain Wall Fermions; arXiv:0710.0536 [hep-lat]
3. The Kaon Bag Parameter from 2+1 Flavour Domain Wall Fermion Lattices;
4. Lattice Results for Vector Meson Couplings and Parton Distribution Amplitudes; arXiv:0710.0869 [hep-lat]
5. $K_{\ell 3}$ Form-Factor with $N_F = 2+1$ Domain Wall Fermions;
6. Nucleon Form Factors and Structure Functions with $N_f = 2+1$ dynamical domain wall fermions; arXiv:0710.0422 [hep-lat]
7. Kaon weak matrix elements in 2+1 flavor DWF QCD; arXiv:0710.3414 [hep-lat]
8. B - $\bar{B}$ Mixing with Domain Wall Fermions in the Static Approximation;
9. Status of 2+1 Flavor, $32^3 \times 64$ Domain Wall Fermion Simulations; arXiv:0710.5337 [hep-lat]
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Introduction

- The focus of the first part of the talk will be the determination of V_{us} and the precision of the unitarity relation

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 . \qquad (*)$$

(Since $|V_{ub}|^2 \simeq 1\text{-}2 \times 10^{-5}$, today the unitarity relation is effectively

$$|V_{ud}|^2 + |V_{us}|^2 = 1 . \;)$$

- Marciano in *Implications of CKM Unitarity* - $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$, [Kaon2007]:

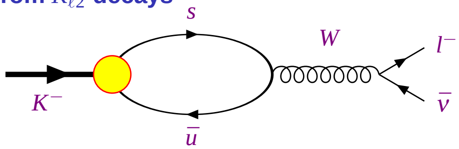
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9990(5)_{V_{ud}}(2)_{K_{\ell 3}}(5)_{f^+} = 0.9990(7) .$$

"Future is in the hands of lattice ($f_+(0), f_K/f_\pi, m_s$)"

- The main motivation for studying $(*)$ is to provide a very stringent test of the universality of weak interactions between quarks and leptons.

$(*)$ assumes this universality.

Determination of V_{us} from $K_{\ell 2}$ decays



All QCD effects are contained in a single constant, f_K, the kaon's *(leptonic) decay constant.*

$$\langle 0|\bar{s}\gamma^\mu\gamma^5 u|K(p)\rangle = if_K\, p^\mu \ . \qquad (f_\pi \simeq 132\,\mathrm{MeV})$$

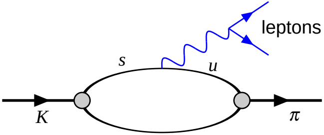
$$\frac{\Gamma(K\to\mu\bar{\nu}(\gamma))}{\Gamma(\pi\to\mu\bar{\nu}(\gamma))} = \frac{|V_{us}|^2}{|V_{ud}|^2}\,\frac{f_K^2}{f_\pi^2}\,\frac{m_K\left(1-\frac{m_\mu^2}{m_K^2}\right)}{m_\pi\left(1-\frac{m_\mu^2}{m_\pi^2}\right)}\times 0.9930(35)$$

From the experimental ratio of the widths we get:

$$\frac{|V_{us}|^2}{|V_{ud}|^2}\,\frac{f_K^2}{f_\pi^2} = 0.07602(23)_{\mathrm{exp}}(27)_{\mathrm{RC}}\,, \qquad \text{PDG2006}$$

so that a precise determination of f_K/f_π will yield V_{us}/V_{ud} .

Determination of V_{us} from $K_{\ell 3}$ decays



$$\langle \pi(p_\pi)|\bar{s}\gamma_\mu u|K(p_K)\rangle = f_0(q^2)\,\frac{M_K^2 - M_\pi^2}{q^2} q_\mu + f_+(q^2)\left[(p_\pi + p_K)_\mu - \frac{M_K^2 - M_\pi^2}{q^2} q_\mu\right]$$

where $q \equiv p_K - p_\pi$.

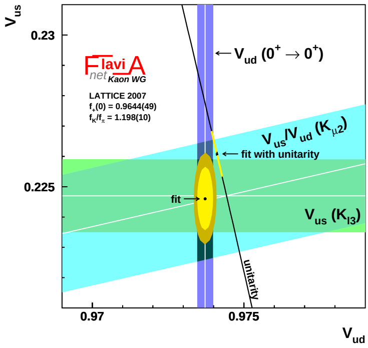
$$\Gamma_{K\to\pi\ell\nu} = C_K^2 \frac{G_F^2 m_K^5}{192\pi^3} I\, S_{\mathrm{EW}}[1 + 2\Delta_{\mathrm{SU(2)}} + \Delta_{EM}]\,|V_{us}|^2\,|f_+(0)|^2$$

From the experimental measurement of the width we get:

$$|V_{us}|f_+(0) = 0.2169(9)\,, \qquad \text{PDG2006}$$

so that a precise determination of $f_+(0)$ will yield V_{us}.

Summary of Results – A.Jüttner – Lattice 2007



$$
\begin{aligned}
f_+^{K\pi}(0) &= 0.9644(33)(34) \\
&\Rightarrow |V_{us}| = 0.2247(12)
\end{aligned}
$$

$$
\begin{aligned}
\frac{f_K}{f_\pi} &= 1.198(10) \\
&\Rightarrow |V_{us}| = 0.2241(24)
\end{aligned}
$$

A.Jüttner, Lattice 2007

Our final result from the $K_{\ell 3}$ project is

$$f_+^{K\pi}(0) = 0.964(5)\,.$$

P.A.Boyle et al. [RBC&UKQCD Collaborations – arXiv:0710.5136 [hep-lat]]

Authors

- The work described below is part of the programme of the RBC & UKQCD collaborations.
- **UKQCD Members:**
  C. Allton, D. Antonio, P. Boyle, M. Clark, L.Del Debbio, M.Donnellan, J. Flynn, A. Hart, R.Horsley, B.Joo, A. Juettner, A. Kennedy, R. Kenway, C. Kim, C. Maynard, J. Noaki, H. Pedrosa de Lima, B. Pendleton, C. Sachrajda, C.Torres, A. Trivini, R. Tweedie, J. Wennekers, A. Yamaguchi, J. Zanotti
- **RBC Members:**
  Y. Aoki, C. Aubin, T. Blum, M. Cheng, N. Christ, S. Cohen, C. Dawson, T. Doi, K. Hashimoto, T. Ishikawa, T. Izubuchi, C. Jung, M. Li, S. Li, M. Lightman, H. Lin, M. Lin, O. Loktik, R. Mawhinney, S. Ohta, S. Sasaki, E. Scholz, A. Soni, T. Yamazaki
- A summary of the overall programme was given at Lattice 2007 by Peter Boyle:
  2+1 flavour Domain Wall Fermion simulations by the RBC and UKQCD collaborations arXiv:0710.5880 [hep-lat]

Introduction Cont.

- We use two datasets of DWF with the Iwasaki Gauge Action with a lattice spacing of about 0.114 fm:
  - $24^3 \times 64 \times 16 \quad (L \simeq 2.74\,\text{fm})$
  - $16^3 \times 32 \times 16 \quad (L \simeq 1.83\,\text{fm})$
- On the 24^3 lattice measurements have been made with 4 values of the light-quark mass:

$$ma = 0.03\ (m_\pi \simeq 670\,\text{MeV}); \qquad ma = 0.02\ \ (m_\pi \simeq 555\,\text{MeV});$$
$$ma = 0.01\ (m_\pi \simeq 415\,\text{MeV}); \qquad ma = 0.005\ (m_\pi \simeq 330\,\text{MeV})\,.$$

  - (Using partial quenching the lightest pion in our analysis has a mass of about 240 MeV .)

  On the 16^3 lattice results were obtained with $ma = 0.03, 0.02$ and 0.01.
- For the (sea) strange quark we take $m_s a = 0.04$, although a posteriori we see that this is a little too large.

Chiral Fits

- Lattice simulations are performed for fixed bare input parameters $g(a)$, $m_u = m_d$ (in the isospin limit) and m_s .
  Three physical quantities are therefore needed to determine the *physical* values of these bare parameters (we take m_π, m_K and m_{Ω^-}).
- Simulations are performed with m_{ud} larger than the physical values and the results are extrapolated to the physical limit.
  Increased computing resources and improvements in algorithms $\Rightarrow$ now dynamical simulations with $m_\pi \simeq 300$ MeV are becoming the norm and the situation is constantly improving.
- m_s can be kept at the physical value (after tuning).
- Chiral Perturbation Theory (χPT) is a key ingredient in performing the extrapolation in m_{ud}, raising the questions of:
  - How reliable is it?
  - What are the values of the *Low Energy Constants*?
  - $SU(3) \times SU(3)$ or $SU(2) \times SU(2)$?
- The use of *Partially Quenched* simulations, in which the masses of the valence and sea quarks are different $\Rightarrow$ the use of PQχPT.

S.R.Sharpe and N.Shoresh, [hep-lat/0006017]

χPT

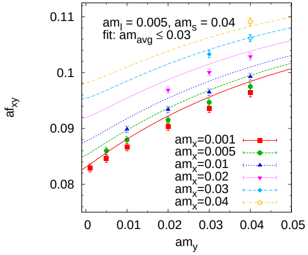
- Approximate chiral symmetry of QCD $\Rightarrow$
effective theory of pseudo-goldstone bosons of chiral symmetry breaking $\Rightarrow$
systematic expansion in powers of $M^2_{(\pi,K,\eta)}/\Lambda^2_\chi$ (up to *chiral logarithms*).
- For example, at one-loop order:

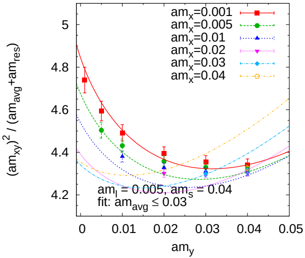
$$
\begin{aligned}
m_\pi^2 &= \chi_{ud}\Bigg\{1+\frac{48}{f_0^2}(2L_6-L_4)\bar{\chi}+\frac{16}{f_0^2}(2L_8-L_5)\chi_{ud} \\
&\qquad +\frac{1}{24\pi^2 f_0^2}\left(\frac{3}{2}\chi_{ud}\log\left[\frac{\chi_{ud}}{\Lambda_\chi^2}\right]-\frac{1}{2}\chi_\eta\log\left[\frac{\chi_\eta}{\Lambda_\chi^2}\right]\right)\Bigg\}, \\
f_\pi &= f_0\Bigg\{1+\frac{24}{f_0^2}L_4\bar{\chi}+\frac{8}{f_0^2}L_5\chi_{ud} \\
&\qquad -\frac{1}{16\pi^2 f_0^2}\left(2\chi_{ud}\log\left[\frac{\chi_{ud}}{\Lambda_\chi^2}\right]+\frac{\chi_{ud}+\chi_s}{2}\log\left[\frac{\chi_{ud}+\chi_s}{2\Lambda_\chi^2}\right]\right)\Bigg\}.
\end{aligned}
$$

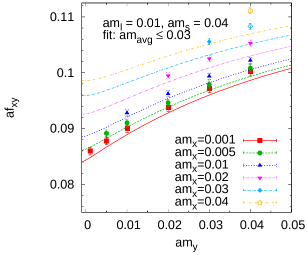
where $\chi_i = 2B_0 m_i$ ($i = ud, s$), $\chi_\eta = \frac{1}{3}(\chi_{ud}+2\chi_s)$ and $\bar{\chi} = \frac{1}{3}(2\chi_{ud}+\chi_s)$.

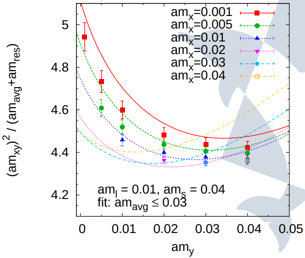
Do such formulae represent our data?

Results – NLO $SU(3) \times SU(3)$ fit is bad for cut $am_{avg} < 0.03$

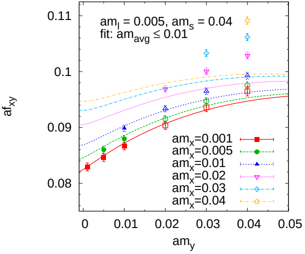


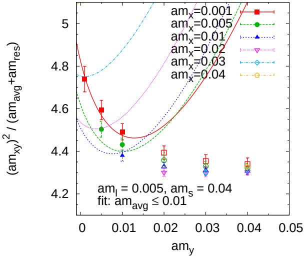


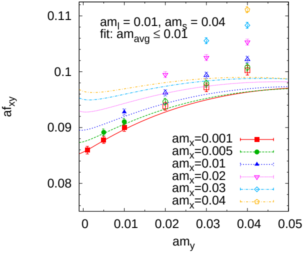


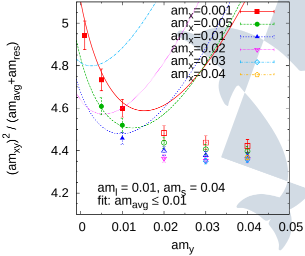


Results – NLO $SU(3) \times SU(3)$ fit is good for cut $am_{avg} < 0.01$









$SU(3) \times SU(3)$ fits

- $SU(3) \times SU(3)$ chiral fits to the pseudoscalar masses and decay constants work well, but only at very light masses.
- Perhaps going to NNLO would increase the range of the good fits, but the number of new LECs is too large for the data which we have (other collaborations are trying to use at least the analytical terms).
- We find for $\Lambda_\chi = m_\rho$ (preliminary)

$2L_8 - L_5$	L_5	$2L_6 - L_4$	L_4
$2.4(5) \cdot 10^{-4}$	$8.7(10) \cdot 10^{-4}$	$0.0(4) \cdot 10^{-4}$	$1.4(8) \cdot 10^{-4}$

  and $af_0 = 0.054(4)$ and $aB_0 = 2.35(16)$.
- The fits can also be performed using $SU(2) \times SU(2)$ chiral perturbation theory in the range $m_{avg} < 0.01$. This treats the heavy strange quark mass correctly.

	aB	af	$\bar{l}_3$	$\bar{l}_4$
$SU(2) \times SU(2)$	2.41(6)	0.067(2)	3.1(3)	4.4(2)
$SU(3) \times SU(3)$ conv.	2.46(8)	0.066(2)	2.9(3)	4.1(1)

$\bar{l}_3$ and $\bar{l}_4$

$$m_\pi^2 = \chi_l\left\{1+\frac{\chi_l}{16\pi^2 f^2}\left(64\pi^2 l_3^r+\log\left[\frac{\chi_l}{\Lambda_\chi^2}\right]\right)\right\} \equiv \chi_l\left\{1-\frac{\chi_l}{16\pi^2 f^2}\bar{l}_3\right\}$$

$$f_\pi = f\left\{1+\frac{m_\pi^2}{8\pi^2 f^2}\left(16\pi^2 l_4^r-\log\left[\frac{m_\pi^2}{\Lambda_\chi^2}\right]\right)\right\} \equiv f\left\{1+\frac{m_\pi^2}{8\pi^2 f^2}\bar{l}_4\right\}$$

- “Phenomenological Indirect Determinations":

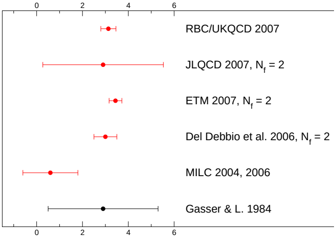
  $\bar{l}_3 = 2.9 \pm 2.4$, Gasser&Leutwyler (1984); $\bar{l}_4 = 4.4 \pm 0.2$, Colangelo, Gasser, Leutwyler (2001)

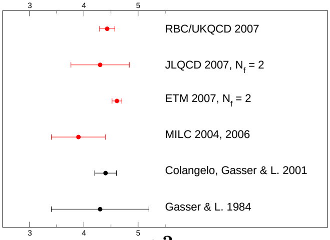
  G.Colangelo – Kaon2007

- Lattice Determinations:

Collaboration	Paper	$\bar{l}_3$	$\bar{l}_4$
MILC	hep-lat/0611024	0.6 ± 0.12	3.9 ± 0.5
Lüscher et al.	hep-lat/0610059	$3.5 \pm 0.5 \pm 0.1$	–
ETM	hep-lat/0701012	3.65 ± 0.12	4.52 ± 0.06
RBC/UKQCD	Lin & Scholz (Lattice 2007)	3.13 ± 0.33	4.43 ± 0.14

$\overline{l}_3$ and $\overline{l}_4$

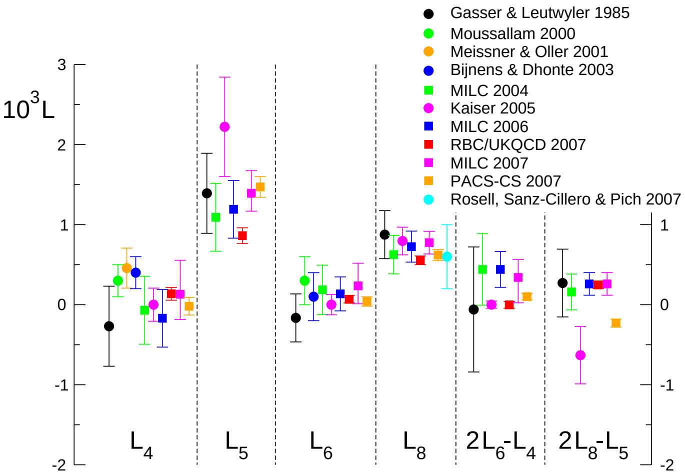




$$\bar{\ell}_3 = \ln \frac{\Lambda_3^2}{M_\pi^2} \qquad \bar{\ell}_4 = \ln \frac{\Lambda_4^2}{M_\pi^2}$$

Courtesy of H.Leutwyler

Gasser-Leutwyler Coefficients



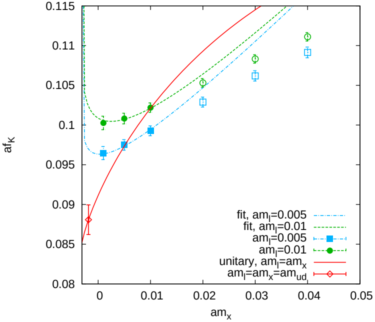
Courtesy of H.Leutwyler

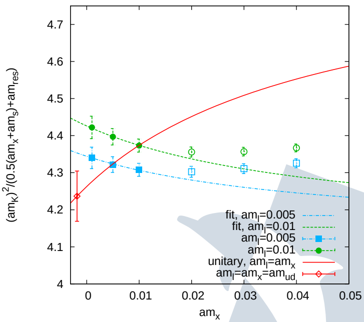
Kaon χPT

- Applying $SU(2) \times SU(2)$ χPT transformations to kaons, only the u and d quarks transform $\Rightarrow$ χPT formalism must be extended.
- Roessl has introduced the corresponding Lagrangian for the interactions of kaons and pions in order to study $K\pi$ scattering near threshold.
  A.Roessl, hep-ph/9904230
- There are overlaps with Heavy Meson Chiral Perturbation Theory, but an important difference is that $m_{K^*} \neq m_K$, whereas in the heavy quark limit $m_{B^*} = m_B$.
  M.B.Wise, Phys.Rev **D45** (1992) 2188
  G.Burdman and J.Donoghue, Phys.Lett. **B280** (1992) 287
- We have derived the chiral behaviour of m_K^2, f_K and B_K in the unitary and partially quenched theories and have used the results in our phenomenological studies. UKQCD/RBC Collaboration – In Preparation
- m_s is considered to be of $O(\Lambda_{\mathrm{QCD}})$ so that the expansion is in m_π^2/m_K^2 as well as m_π^2/Λ_χ^2.
  m_K^2/Λ_χ^2 effects however, are fully absorbed into the LECs of $SU(2) \times SU(2)$ χPT.

Chiral Behaviour of m_K^2 and f_K

- For f_K and m_K^2 we use PQ $SU(2) \times SU(2)$ χPT keeping the light valence quark $am_{ud} < 0.01$ and $am_s = 0.04$.



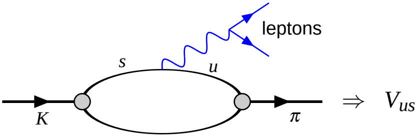


Our preliminary result is

$$f_K/f_\pi = 1.205(18)$$

to be compared with A.Jüttner's best lattice value of 1.198(10) .

$K_{\ell 3}$ Decays



$$\langle \pi(p_\pi) | \bar{s}\gamma_\mu u | K(p_K) \rangle = f_0(q^2)\,\frac{M_K^2 - M_\pi^2}{q^2} q_\mu + f_+(q^2) \left[(p_\pi + p_K)_\mu - \frac{M_K^2 - M_\pi^2}{q^2} q_\mu \right]$$

where $q \equiv p_K - p_\pi$.

To be useful in extracting V_{us} we require $f_0(0) = f_+(0)$ to better than about 1% precision.

$$\chi\text{PT} \Rightarrow f_+(0) = 1 + f_2 + f_4 + \cdots \quad \text{where} \quad f_n = O(M^n_{K,\pi,\eta}).$$

Reference value $f_+(0) = 0.961 \pm 0.008$ where $f_2 = -0.023$ is relatively well known from χPT and $f_4, f_6, \cdots$ are obtained from models. Leutwyler & Roos (1984)

1% precision of $f^+(0)$ is conceivable because it is actually $1 - f^+(0)$ which is computed:

Bećirević et al. [hep-ph/0403217] based on S.Hashimoto et al. [hep-ph/9906376] for $B \to D$ Decays

- The starting point is the evaluation of the matrix elements at $q^2_{\rm max}$, i.e. with the pion and kaon at rest:

$$\frac{\langle\pi|\bar{s}\gamma_4 u|K\rangle\langle K|\bar{u}\gamma_4 s|\pi\rangle}{\langle\pi|\bar{u}\gamma_4 u|\pi\rangle\langle K|\bar{s}\gamma_4 s|K\rangle} = \left[f_0(q^2_{\rm max})\right]^2 \frac{(m_K+m_\pi)^2}{4m_K m_\pi}\,.$$

$f_0(q^2_{\rm max})$ is obtained with excellent precision.

	$16^3\times 32$		$24^3\times 64$	
am_{ud}	$q^2_{\rm max}$ (GeV^2)	$f_0(q^2_{\rm max})$	$q^2_{\rm max}$ (GeV^2)	$f_0(q^2_{\rm max})$
0.03	0.00233(4)	1.00035(3)	0.00235(4)	1.00029(6)
0.02	0.01178(24)	1.00241(19)	0.01152(20)	1.00192(34)
0.01	0.03475(66)	1.01436(81)	0.03524(62)	1.00887(89)
0.005	–	–	0.06070(107)	1.02143(132)

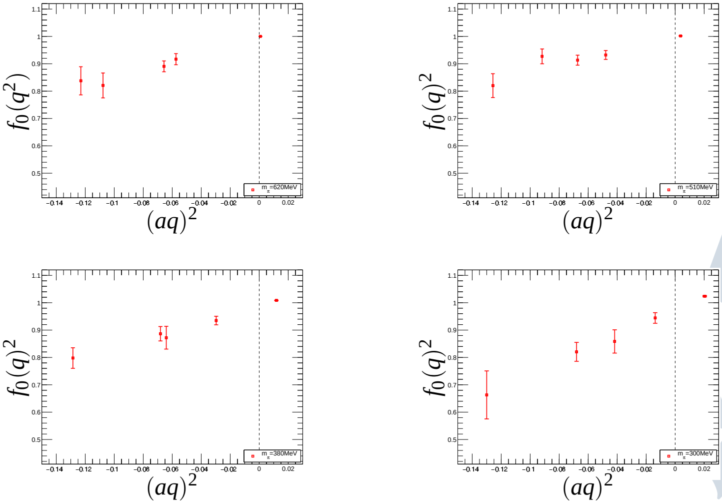
	$16^3 \times 32$		$24^3 \times 64$	
am_{ud}	$q^2_{\rm max}$ (GeV^2)	$f_0(q^2_{\rm max})$	$q^2_{\rm max}$ (GeV^2)	$f_0(q^2_{\rm max})$
0.03	0.00233(4)	1.00035(3)	0.00235(4)	1.00029(6)
0.02	0.01178(24)	1.00241(19)	0.01152(20)	1.00192(34)
0.01	0.03475(66)	1.01436(81)	0.03524(62)	1.00887(89)
0.005	–	–	0.06070(107)	1.02143(132)

- Having obtained $f_0(q^2_{\rm max})$ we need to extrapolate in q^2 and m_{ud}.
- Note that for heavier values of m_{ud}, $q^2_{\rm max}$ is close to zero.
- In the conventional approach, the q^2 extrapolation is done by calculating the form factors with

$$|\vec{p}_K| \quad \text{or} \quad |\vec{p}_\pi| \quad = \quad p_{\rm min} \quad \text{or} \quad \sqrt{2}p_{\rm min}\,,$$

where $p_{\rm min} = 2\pi/L$ and L is the spatial extent of the lattice.

$f_0(q^2)$ for the four values of m_{ud}



q^2 and Chiral Extrapolations

- There are a number of ChiPT-motivated extrapolation ansatze available. For our central values we use a simultaneous fit to the q^2 and chiral behaviour:

$$f_0(q^2, m_\pi^2, m_K^2) = \frac{1 + f_2 + (m_K^2 - m_\pi^2)^2 (A_0 + A_1 (m_K^2 + m_\pi^2))}{1 - q^2/(M_0 + M_1 (m_K^2 + m_\pi^2))^2},$$

  where f_2 is known and A_0, A_1, M_0, M_1 are fit parameters.

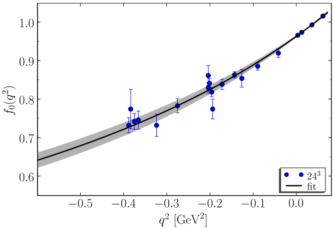
- The spread of results obtained with this simultaneous above, the polynomial fit

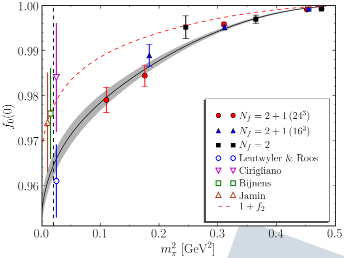
$$\begin{aligned} f_0(q^2, m_\pi^2, m_K^2) = 1 + f_2 + (m_K^2 - m_\pi^2)^2 (A_0 + A_1 + A_2 (m_K^2 + m_\pi^2)) \\ + (A_3 + (2A_0 + A_1)(m_K^2 + m_\pi^2))\, q^2 + (A_4 - A_0 + A_5 (m_K^2 + m_\pi^2))\, q^4, \end{aligned}$$

  and the *z-fit* form of Hill (hep-ph/0607108) are used to estimate the systematic errors.

- It would be particularly interesting to have the NNLO results in a form useful for these extrapolations. Bijnens et al. – Work in Progress.

q^2 and Chiral Extrapolations – Cont.





- Simultaneous pole fit to our 24^3 data.
- $f_0(q^2, m_\pi^{\rm latt}, m_K^{\rm latt}) - f_0(q^2, m_\pi^{\rm phys}, m_K^{\rm phys})$ has been subtracted from the lattice data.
- Fit shown is at physical masses.

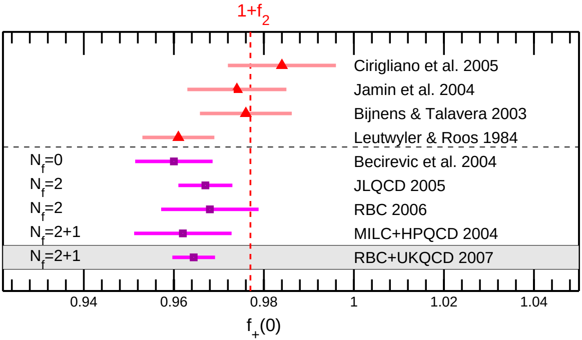
- $f_0(0)$ as a function of the pion masses with the simultaneous fit.
- $1+f_2$ is not a good approximation to $f_0(0)$.

Comparison with Other Calculations

Ref.	$f_+(0)$	Δf	m_π [GeV]	a [fm]	N_f
Leutwyler & Roos (1984)	0.961(8)	-0.016(8)			
Bijnens& Talavera (2003)	0.978(10)	+0.001(10)			
Cirigliano et al. (2005)	0.984(12)	+0.007(12)			
Jamin, Oller & Pich (2004)	0.974(11)	-0.003(11)			
Becirevic et al. (2005)	0.960(5)(7)	-0.017(5)(7)	$\gtrsim 0.5$	0.07	0
Dawson et al. (2006)	0.968(9)(6)	-0.009(9)(6)	$\gtrsim 0.49$	0.12	2
Okamoto et al. (2004)	0.962(6)(9)†	-0.015(6)(9)†	‡	‡	2+1
Tsutsui et al. (2005)	0.967(6)†	-0.010(6)†	$\gtrsim 0.55$	0.09	2
Brommel et al. (2007)	0.965(2)$^\dagger_{\mathrm{stat}}$	-0.012(2)$^\dagger_{\mathrm{stat}}$	$\gtrsim 0.5$	0.08	2
This work	0.964(5)	-0.013(5)	$\gtrsim 0.33$	0.114	2+1

- Summary of ChPT-based and lattice results.
- † Results in conference proceedings only.
- ‡ Information not provided.

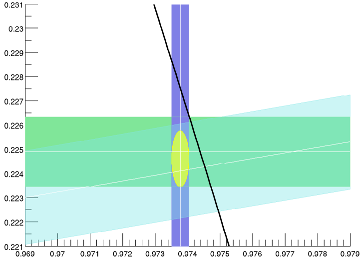
Comparison with Other Calculations - Cont.



- Our final answer:

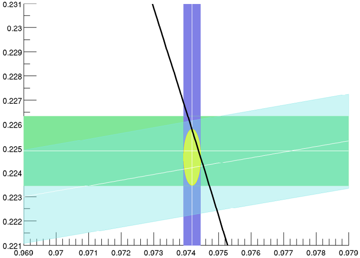
$$f_0(0) = 0.964(5)$$

V_{ud}



$$V_{ud} = 0.97372(10)(15)(19)$$

W.Marciano, Kaon2007



$$V_{ud} = 0.97418(26)$$

I.Towner and J.Hardy, arXiv:0710.3181 [nucl-th]

Courtesy of Flavianet Kaon WG and A.Jüttner

Improvements – Eliminating the Interpolation in q^2

- The momentum resolution with conventional methods is very poor:

  On the present lattice:

$$L = 24a \quad \text{with} \quad a^{-1} = 1.73\,\text{GeV} \quad \Rightarrow \quad \frac{2\pi}{L} = .45\,\text{GeV}$$

- Using twisted boundary conditions

$$q(x_i + L) = e^{i\theta_i} q(x_i)$$

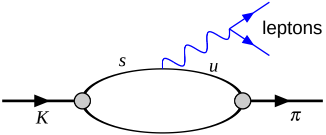
  the momentum spectrum is modified (relative to periodic bcs)

$$p_i = n_i \frac{2\pi}{L} + \frac{\theta_i}{L}\,.$$

- For quantities which do not involve Final State Interactions (e.g. masses, decay constants, form-factors) the Finite-Volume corrections are exponentially small also with Twisted BC's. CTS & G. Villadoro (2004)
- Moreover they are also exponentially small for *partially twisted boundary conditions* in which the sea quarks satisfy periodic BC's but the valence quarks satisfy twisted BC's. CTS & G. Villadoro (2004); Bedaque & Chen (2004)

We do not need to perform new simulations for every choice of $\{\theta_i\}$.

Improvements – Eliminating the Interpolation in q^2 Cont.



$\Rightarrow \quad V_{us}$

- By tuning the twisting angles appropriately it is possible to calculate the matrix element at $q^2 = 0$ directly (or at any other required value of q^2).

  P.A.Boyle, J.M.Flynn, A.Jüttner, CTS, and J.M.Zanotti, [hep-lat/0703005]

- By calculating:

$$\langle \pi(\vec{0}) \,|\, V_4 \,|\, K(\vec{\theta}_K) \rangle \quad \text{with} \quad |\vec{\theta}_K| = L\sqrt{\left[\frac{(m_K^2 + m_\pi^2)}{2m_\pi}\right]^2 - m_K^2}$$

and

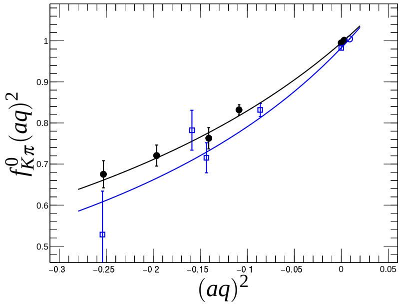
$$\langle \pi(\vec{\theta}_\pi) \,|\, V_4 \,|\, K(\vec{0}) \rangle \quad \text{with} \quad |\vec{\theta}_\pi| = L\sqrt{\left[\frac{(m_K^2 + m_\pi^2)}{2m_K}\right]^2 - m_\pi^2}$$

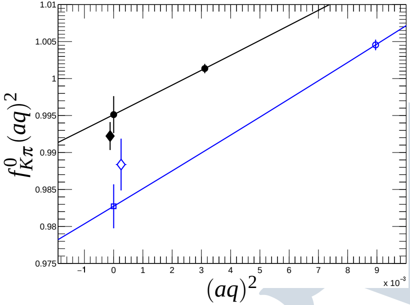
we obtain the form factors directly at $q^2 = 0$.

Improvements – Eliminating the Interpolation in q^2 Cont.

- The feasibility of this method was demonstrated on a subset of configurations on a $16^3 \times 32$ lattice at two values of m_{ud}.

P.A.Boyle, J.M.Flynn, A.Jüttner, CTS, and J.M.Zanotti, [hep-lat/0703005]

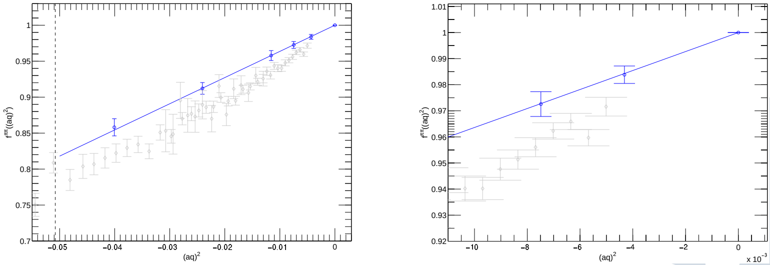




Improvements – Eliminating the Interpolation in q^2 Cont.

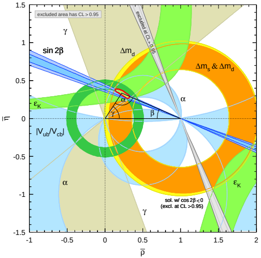
- We are currently using partially twisted boundary conditions to get $f_0(0)$ for our lightest quark mass ($ma = 0.005$) directly at $q^2 = 0$.
- Twisted boundary conditions were previously applied to $K_{\ell 3}$ decays (although not directly at $q^2 = 0$) in a quenched simulation.

D.Guadagnoli, F.Mescia and S,Simula, [hep-lat/0512020]



Pion Electromagnetic Form Factor at Small q^2 .

B_K



- $|\varepsilon_K| = (2.232 \pm 0.007)\,10^{-3}$
  0.3% precision
- B_K known to 16% precision
  $\Rightarrow$ physics information severely limited by theoretical uncertainty.

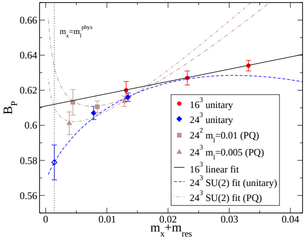
G.Sciolla (Kaon 2007)

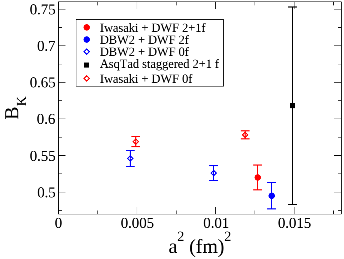
- Flavour and Chiral symmetry properties of DWF well suited to this calculation.
- $\Delta S = 2$ operator renormalises multiplicatively and is renormalized nonperturbatively.
- Again it is found that $SU(2)_L \times SU(2)_R$ (PQ)ChPT should be used:

$$B_K = B_0^{(K)} \left\{ 1 + \frac{b_1 \chi_l}{f^2} + \frac{b_2 \chi_x}{f^2} - \frac{\chi_l}{32\pi^2 f^2} \log \frac{\chi_x}{\Lambda_\chi^2} \right\}$$

- Kaons with $m_s \neq m_d$ are used and the chiral behaviour in m_d is fit successfully.

B_K

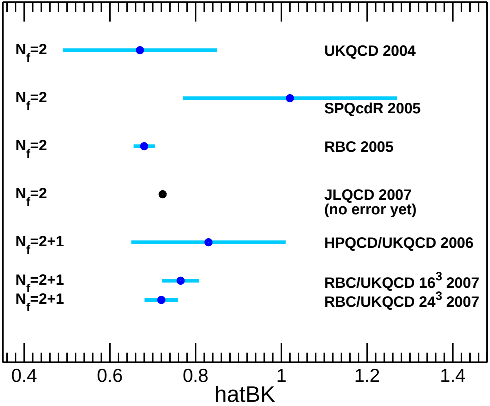




$$B_K^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 0.524(10)(28), \qquad \hat{B}_K = 0.720(13)(37)\,.$$

RBC/UKQCD, hep-ph/0702042 (PRL - in press)

- Systematic error includes estimates of finite-volume effects, discretization errors, interpolation to the physical strange quark mass and ChPT.
- Calculation at a second lattice spacing (in progress) will reduce the estimated 4% discretization error (which is the largest component of the quoted systematic error).



A.Jüttner, Lattice 2007, arXiv:0711.1239 [hep-lat]

Conclusions

- I presented only a very small selection of the phenomenological lattice studies being undertaken in flavour physics.
- We are beginning to make strong contact with Chiral Perturbation Theory and to determine the *low energy constants* with unprecedented precision.
- The RBC/UKQCD research programme will now move on to a finer lattice $\Rightarrow$ information about the continuum extrapolation.
- We will continue to extend the range of quantities being computed: $\{K \to \pi\pi$ decays; Heavy Quark Physics; Hadron Structure; Proton Decay/Guts ; Technicolour/Susy $\cdots .\}$
- In the medium term we plan to move onto the Blue Gene family of machines $\Rightarrow$ a target simulation of $a = 0.06\,\mathrm{fm}$, $L = 4\,\mathrm{fm}$, $m_\pi = 195$ MeV.

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- Selected Physics Results:

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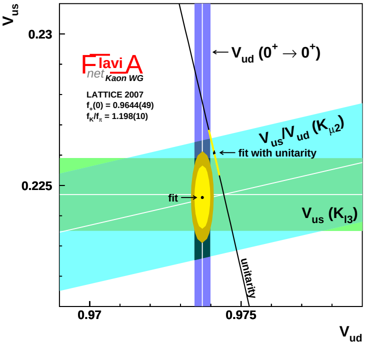
Conclusions Cont.



$$f_+^{K\pi}(0) \quad = \quad 0.9644(33)(34)$$

$$\Rightarrow \quad |V_{us}| = 0.2247(12)$$

$$\frac{f_K}{f_\pi} \quad = \quad 1.198(10)$$

$$\Rightarrow \quad |V_{us}| = 0.2241(24)$$

A.Jüttner, Lattice 2007

Our final result from the $K\ell3$ project is

$$f_+^{K\pi}(0) = 0.964(5)\,.$$

P.A.Boyle et al. [RBC&UKQCD Collaborations – arXiv:0710.5136 [hep-lat]]

AUXILIARY SLIDES

Estimates of Lattice Errors in 2015

V.Lubicz

Measured Quantity	CKM Matrix Element	Hadronic Matrix Element	Current Error	Estimated Error in 2015
$K \to \pi \ell \nu$	$\lvert V_{us}\rvert$	$f^+_{K\pi}(0)$	0.9% 22% on $1 - f^+_{K\pi}(0)$	<0.1% 2.4% on $1 - f^+_{K\pi}(0)$
ε_K	Im V^2_{td}	B_K	11%	1%
$B \to \ell \nu$	$\lvert V_{ub}\rvert$	f_B	14%	1-2%
Δm_d	$\lvert V_{td}\rvert$	$f_{B_d}\sqrt{B_{B_d}}$	14%	1-2%
$\Delta m_d / \Delta m_s$	$\lvert V_{td}/V_{ts}\rvert$	ξ	5% 25% on $\xi - 1$	0.5-0.8% 3–4% on $\xi - 1$
$B \to D/D^* \ell \nu$	$\lvert V_{cb}\rvert$	$\mathscr{F}_{B \to D/D^* \ell \nu}$	4% 40% on $1 - \mathscr{F}$	0.5% 5% on $1 - \mathscr{F}$
$B \to \pi/\rho \ell \nu$	$\lvert V_{ub}\rvert$	$f^+_{B\pi}, \cdots$	11%	2–3%
$B \to K^*/\rho\,(\gamma, \ell^+\ell^-)$	$\lvert V_{td}/V_{ts}\rvert$	T_1	13%	3–4%

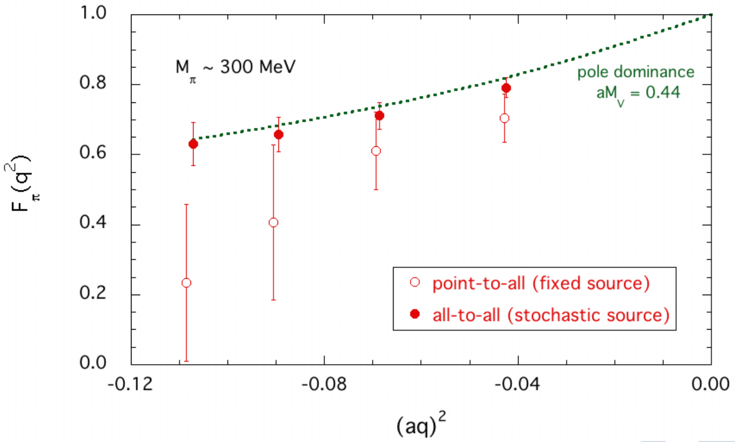
Improvements – Using Propagators with Stochastic Sources

- Traditionally quark propagators are generated from an arbitrary lattice point to a fixed *source*; e.g. point-to-point propagators $\{S(x,0)\ \forall\ x\}$.
- It is now being proposed to exploit *volume averaging* and to evaluate all-to-all propagators, albeit with less precision. This is done by using stochastic sources.
- It then becomes a balance between the gain of precision from volume-averaging vs loss of precision because of the stochastic noise.

The lattice community is currently building up experience with these methods.

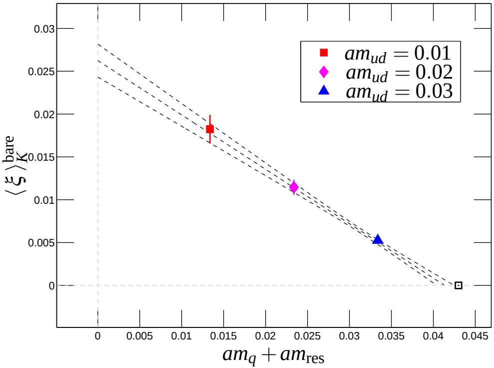
- The calculations described above with twisted boundary conditions were obtained using stochastic sources.
- For some form factors at least, it seems as though there is a significant statistical gain.

Improvements – Using Propagators with Stochastic Sources



S.Simula (ETMC Collaboration), Lattice 2007; arXiv:0710:0097 [hep-lat]

Results for $\langle \xi \rangle_K^{\text{bare}}$ (16^3 results - χ extrapolation)



$$\langle \xi \rangle_K^{\text{bare}} = 0.0262(23)$$

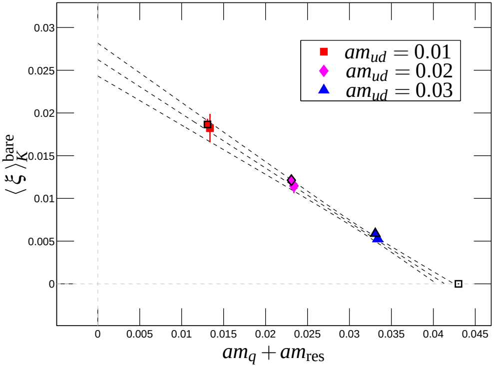
P.A.Boyle et al. [hep-lat/0607018]

- At lowest non trivial order in χPT: $\langle \xi \rangle_K \propto (m_s - m_q)$ (no logs).

  Chen and Stewart, [hep-ph/0311285]

- Our result shows clear signs of partonic SU(3)-breaking effects and extrapolation is compatible with iso-spin symmetry ($\langle \xi \rangle_\pi = 0$).

Results for $\langle \xi \rangle_K^{\mathrm{bare}}$ (16^3 and 24^3 results)

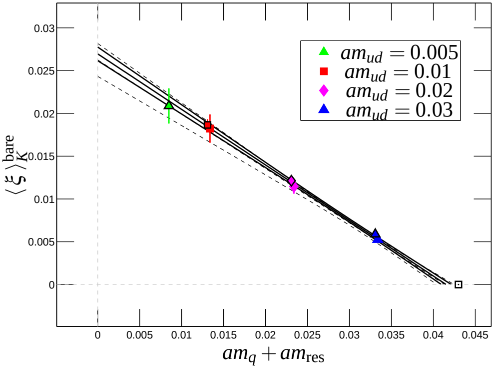


$$\langle \xi \rangle_K^{\mathrm{bare}} = 0.0262(23)$$

P.A.Boyle et al. [hep-lat/0607018]

- No evidence for significant finite-volume effects.

Results for $\langle \xi \rangle_K^{\mathrm{bare}}$ (16^3 and 24^3 results - χ extrapolation)



$$\langle \xi \rangle_K^{\mathrm{bare}} = 0.0270(8)$$

- Quadratic Fit - $\langle \xi \rangle_K^{\mathrm{bare}} = 0.0271(26)$.
- Renormalization Factor computed perturbatively $\Rightarrow$

$$\langle \xi \rangle_K^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 0.029(2)\,.$$

- Error dominated by the perturbative renormalization constant.

Distribution Amplitudes - Summary of Results

- Our (preliminary) results are:

$$\langle \xi \rangle_K^{\overline{\rm MS}}(2\,{\rm GeV}) = 0.029(2)\,,\ \langle \xi^2 \rangle_\pi^{\overline{\rm MS}}(2\,{\rm GeV}) = 0.28(3)\,,$$

$$\langle \xi^2 \rangle_K^{\overline{\rm MS}}(2\,{\rm GeV}) = 0.27(2)\,.$$

Lattice calculations can be performed with an excellent precision.

- Important improvements will be to perform the non-perturbative renormalization and to repeat the calculation at a finer lattice spacing.
- Our results are in agreement with those of an $N_f = 2$ study using Improved Wilson fermions:

$$\langle \xi \rangle_K^{\overline{\rm MS}}(2\,{\rm GeV}) = 0.0272(5)\,,\ \langle \xi^2 \rangle_\pi^{\overline{\rm MS}}(2\,{\rm GeV}) = 0.269(39)\,,$$

$$\langle \xi^2 \rangle_K^{\overline{\rm MS}}(2\,{\rm GeV}) = 0.260(6)\,.$$

- Analysis of moments of PDAs of vector mesons is in progress.