The NLL accurate parton shower Alaric in Sherpa

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[arXiv:2208.06057]

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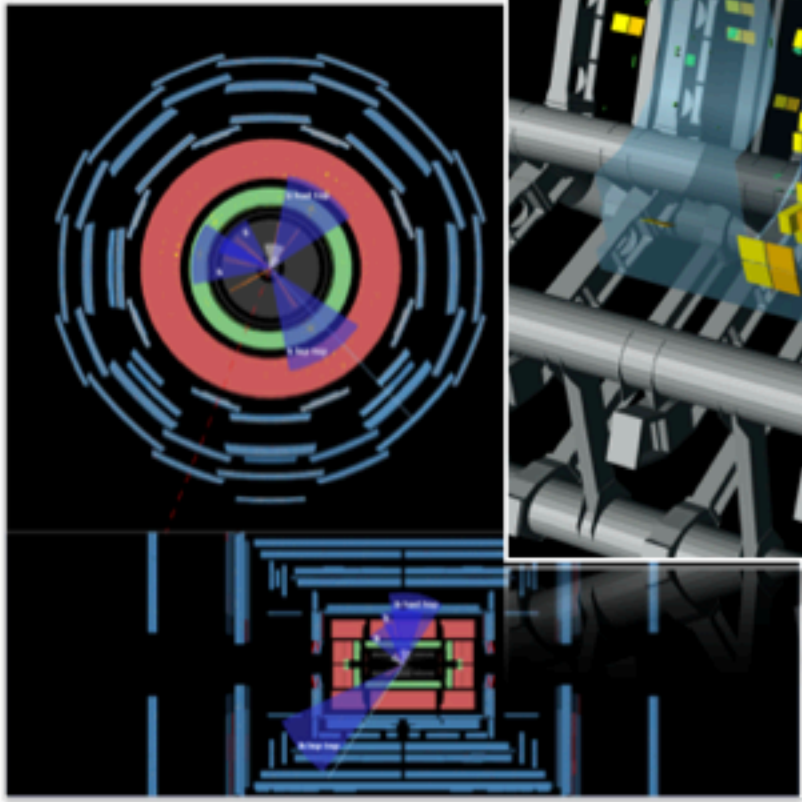
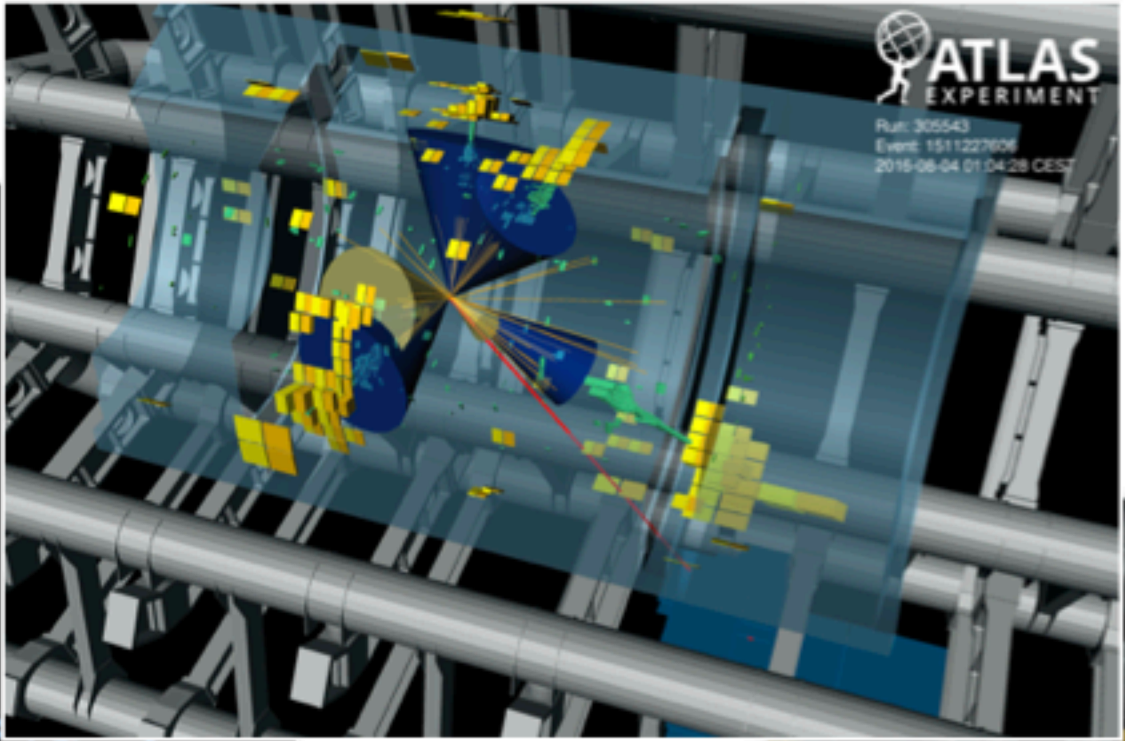
colliders in the real world

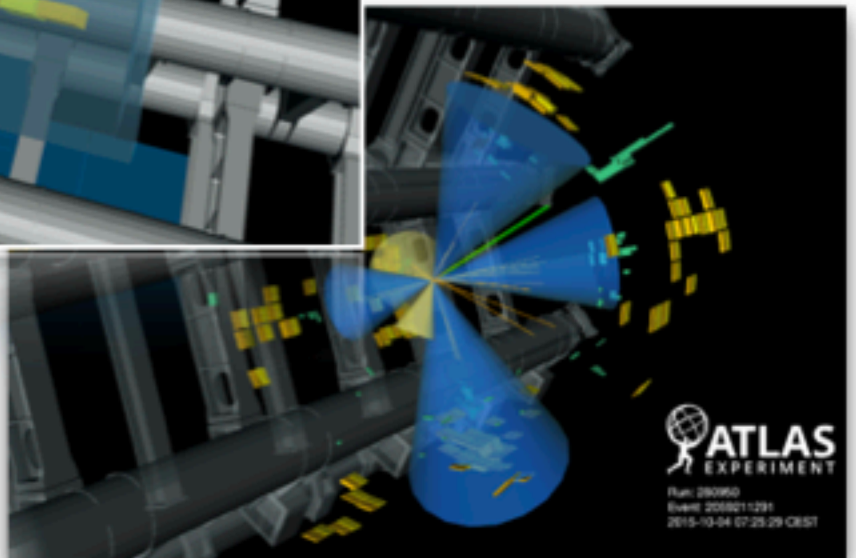




Example displays

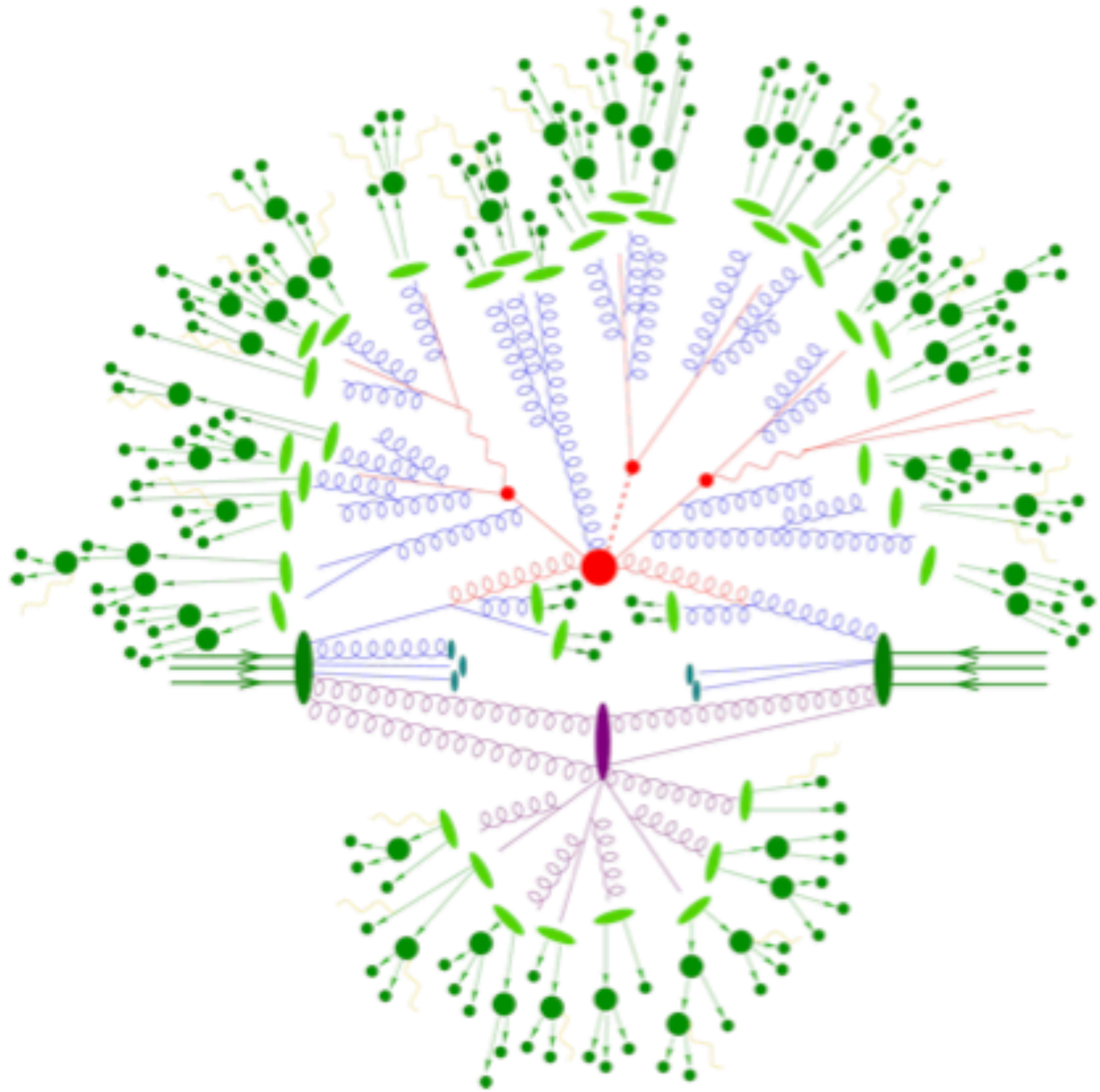
of $t\bar{t}H(\to b\bar{b})$

events



colliders for theorists

- Event simulation factorised into
  - Hard Process
  - Parton Shower
  - Underlying event
  - Hadronisation
  - QED radiation
  - Hadron Decays



colliders for theorists

- Event simulation factorised into
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This Talk:

Why?

- parton showers resum large logs ~ NLL, but open questions on actual accuracy
- starting work towards NNLL → probably better resolve this first

  needed for fully consistent NNLO matching

- recent formal discussion → current dipole showers need reworking

colliders for theorists

- Event simulation factorised into
  - Hard Process
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This Talk:

- Go over resummation in CAESAR framework and relation to PS accuracy
- Formal problems in current parton showers
- Alaric
  - construction principles
  - proof of NLL accuracy
  - first pheno and data comparison

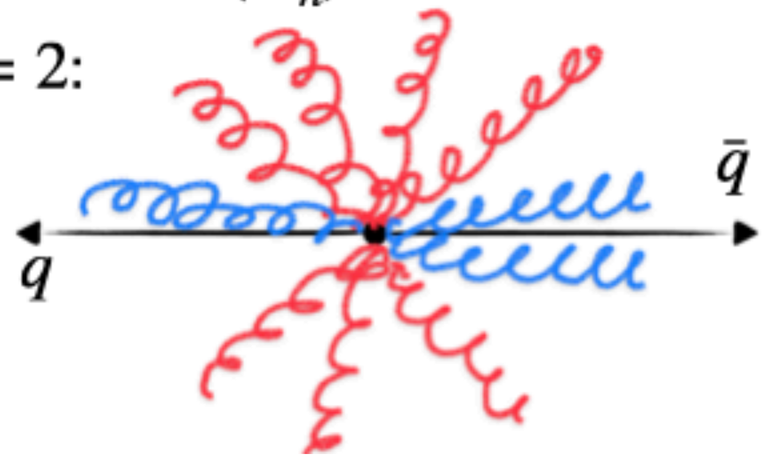
NLL resummation

- CAESAR [Banfi, Salam, Zanderighi '05] → 'direct' QCD, easy to compare to what parton showers are doing

- looking for cross sections with an upper cut on some IRC safe observable,

$$\Sigma(v) = \int d\Phi \frac{d\sigma}{d\Phi} \Theta(v - V(\Phi)) = \sigma \left(1 - \int d\Phi \frac{d\sigma}{\sigma d\Phi} \Theta(V(\Phi) - v) \right)$$

- assume there is some multiplicity n where $V(\Phi_n) = 0$, so we measure effect of additional particles, e.g. with $n = 2$:



LL single emission

- factorisation in the soft limit ('Eikonal')

$$d\sigma_{n+1} = d\sigma_n \otimes d\Phi_{+1} \frac{\alpha_s}{2\pi} \sum_{k,i} \mathbb{T}_k \mathbb{T}_i \frac{p_k p_i}{(p_k q)(p_i q)}$$

$$\frac{p_k p_i}{(p_k q)(p_i q)} \sim 1/k_t^2$$

- integrate over triangle in "Lund plane"

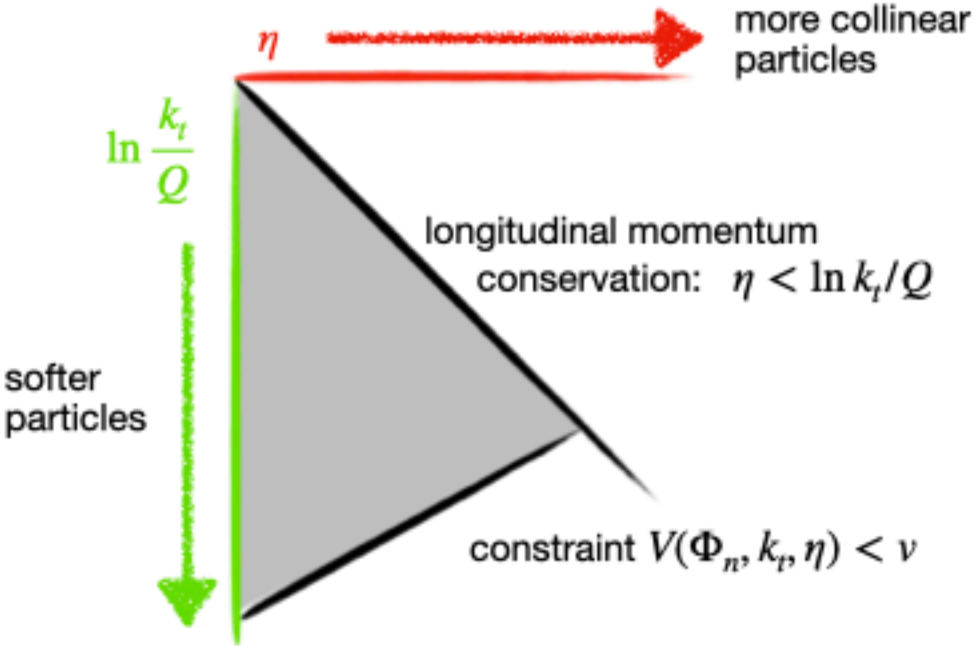
single emission phase space

transverse momentum

rapidity

azimuthal angle

$$d\Phi_{+1} \sim dk_t^2 \, d\eta \, d\phi$$



e.g. take $V(k_t, \eta) = k_t/Q$

$$\rightarrow \frac{\alpha_s}{2\pi} \int_{vQ}^{Q} \frac{dk_t}{k_t} \int_0^{\ln k_t/Q} d\eta \sim \frac{\alpha_s}{2\pi} \ln^2 1/v$$

multiple emissions

QCD coherence: emissions separated in η and k_t are independent

$$\Sigma(v) = \sigma_n \left(1 - \sum_m \int d\Phi_{n+m} \frac{d\sigma_{n+m}}{\sigma_n d\Phi_{n+m}} \Theta(V(\Phi_{n+m}) - v) \right)$$

$$= \sigma_n \left(1 - \int d\Phi_n \frac{d\sigma_n}{\sigma_n d\Phi_n} \sum_m \int d\Phi_{+1} \frac{d\sigma_{+1}}{d\Phi_{+1}} \Theta(V(\Phi_{n+m}) - v) \right)$$

$$\int d\Phi_{+1} \frac{d\sigma_{+1}}{d\Phi_{+1}} \Theta(V(\Phi_{+1}) - v) + \int d\Phi_{+1} \frac{d\sigma_{+1}}{d\Phi_{n+1}} \Theta(\epsilon v - V(\Phi_{+1}))$$

single emissions, all larger than v

negligible emissions, contributing $< \epsilon v$

$$+ \int d\Phi_{+1} \frac{d\sigma_{+1}}{d\Phi_{n+1}} \Theta(V(\Phi_{n+m}) - v) \Theta(V(\Phi_{+1}) - \epsilon v)) \Theta(v - V(\Phi_{+1}))$$

multiple emissions, together contributing $> v$

multiple emissions

- how to extract NLL observable independent (i.e. without additional information)?

- method from [Banfi, Salam, Zanderighi '05]: need explicit implementation of soft-collinear limit*:

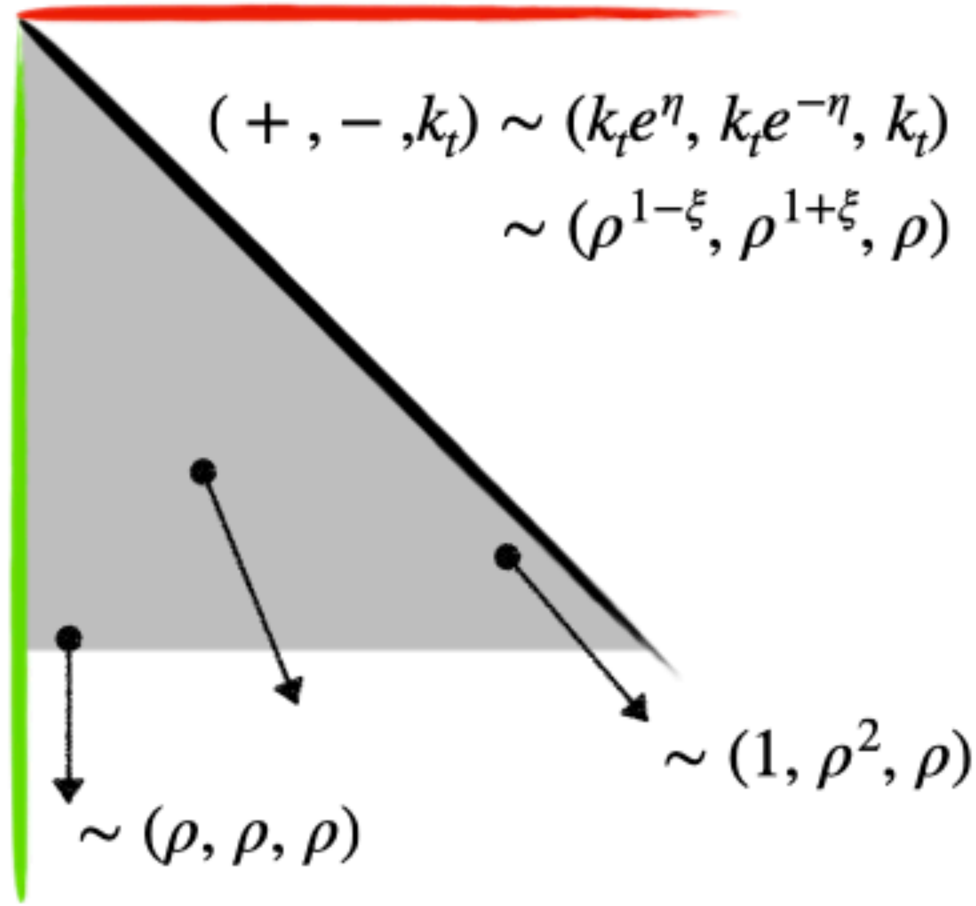
$$k_t^\rho = k_t \rho \qquad \xi = \frac{\eta}{\eta_{\max}}$$

$$\eta^\rho = \eta - \xi \ln \rho$$

and assume

$$V(k_i^\rho) = \rho V(k_i)$$

→ numerically evaluate integrals in this limit

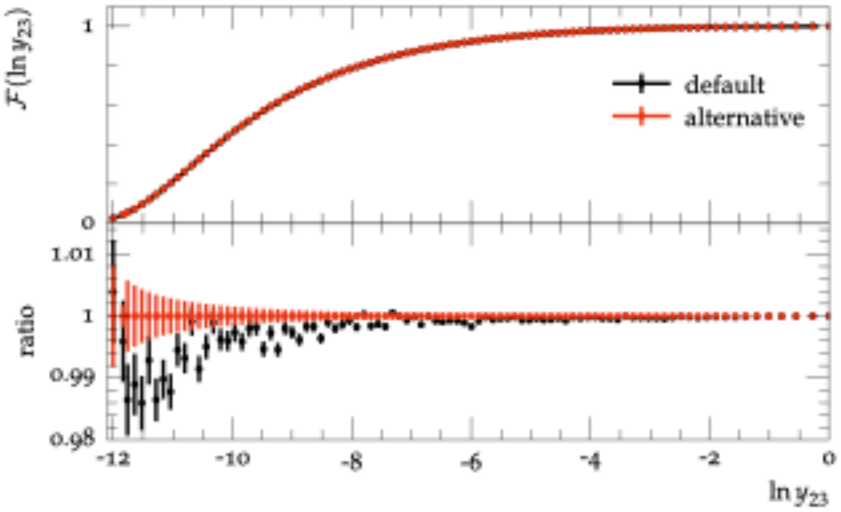


* again assume $V(k_t, \eta) \sim k_t/Q$ for brevity

example: jet resolution scales

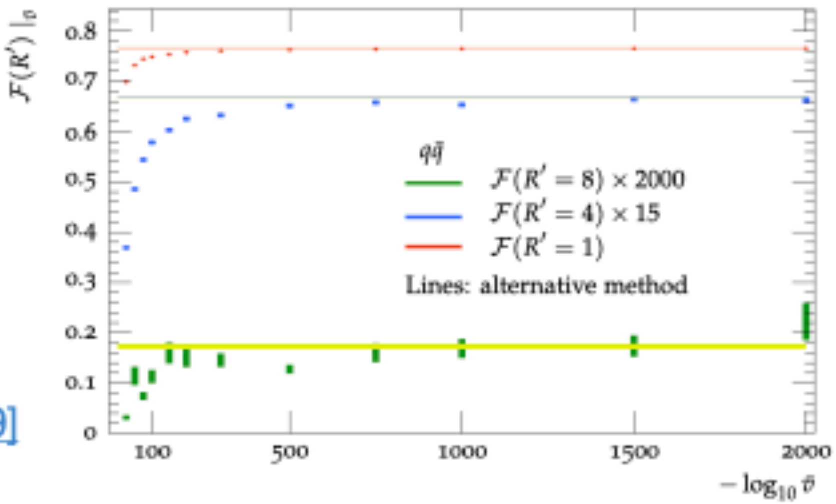
- consider Durham jet cluster algorithm:

$$y_{i,j} = \frac{\min\left[E_i^2, E_j^2\right]}{Q^2}(1-\cos\theta_{ij}) \sim k_t^2/Q^2$$



- $y_{n\,n+1} \rightarrow$ min scale to resolve $(n+1)$ jets

- little analytic insight → numerical evaluation needed



- y_{23} first in [Banfi, Salam, Zanderighi '02]

- higher rates in [Baberuxki, Preuss, DR, Schumann '19]

NLL wrap up

- Additional components: collinear terms form DGLAP splitting kernels, running coupling and CMW scheme for α_s evolution → relevant for single emission only
- Master formula $$\Sigma(v) = \int d\Phi_n e^{-R(v)} \mathscr{F}(v) \sim \exp(L g_1(\alpha_s L) + g_1(\alpha_s L))$$

$$L \equiv \ln 1/v$$

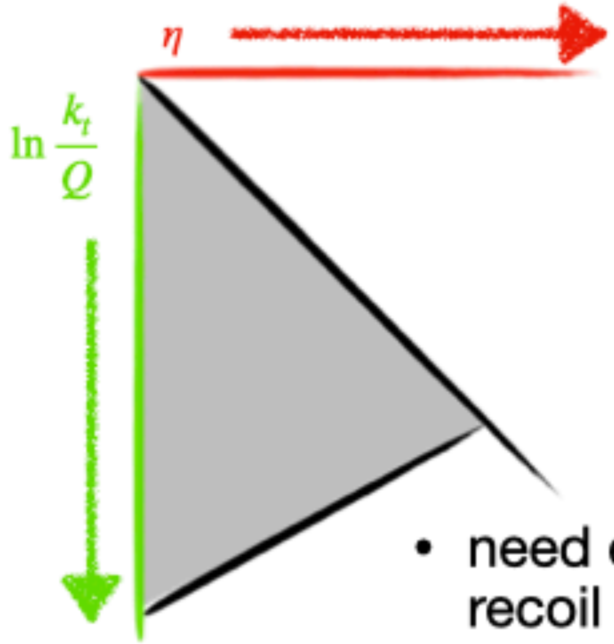
$$g_i \equiv \sum_k \alpha_s^k L^k$$

$$R(v) = \int d\Phi_{+1} \frac{d\sigma_{+1}}{d\Phi_{+1}} \Theta(V(\Phi_{+1}) - v)$$

$$\mathscr{F}(v) = \lim_{\epsilon \to 0} \epsilon^{R'} \sum_m \int d\Phi_{+1} \frac{d\sigma_{+1}}{d\Phi_{+1}} \Theta\left(1 - \lim_{\rho \to 0} \frac{V(k_i^\rho)}{\rho v}\right)$$

shower intro

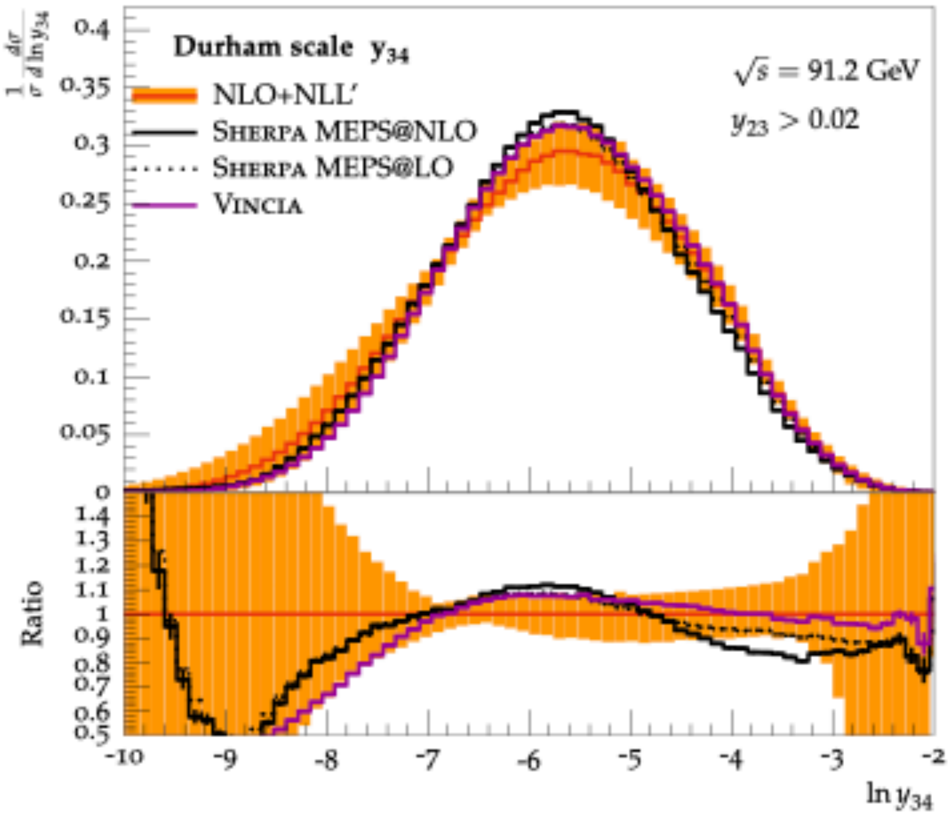
- Start from no-emission probability $\sim e^{-R(v)}$ with some choice for evolution variable
- generate splittings according to this, iteratively generate effective $\mathscr{F}(v)$ from multiple emissions
- no explicit soft limit → generate momenta and distribute transverse recoil among other particles in event

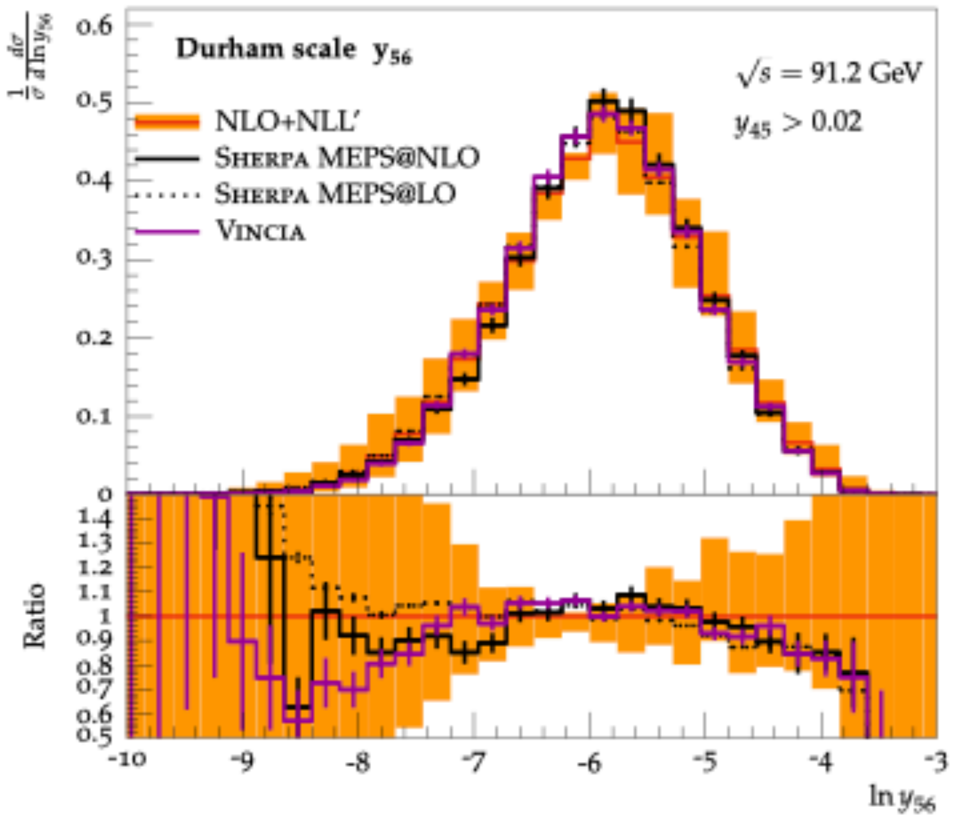


- need explicit choice of recoil scheme, for example default dipole shower in Sherpa:

$$p_i = z\tilde{p}_{ij} + (1-z)y\tilde{p}_k + k_\perp$$
$$p_j = (1-z)\tilde{p}_{ij} + zy\tilde{p}_k - k_\perp$$
$$p_k = (1-y)\tilde{p}_k \ .$$

example: jet resolution scales



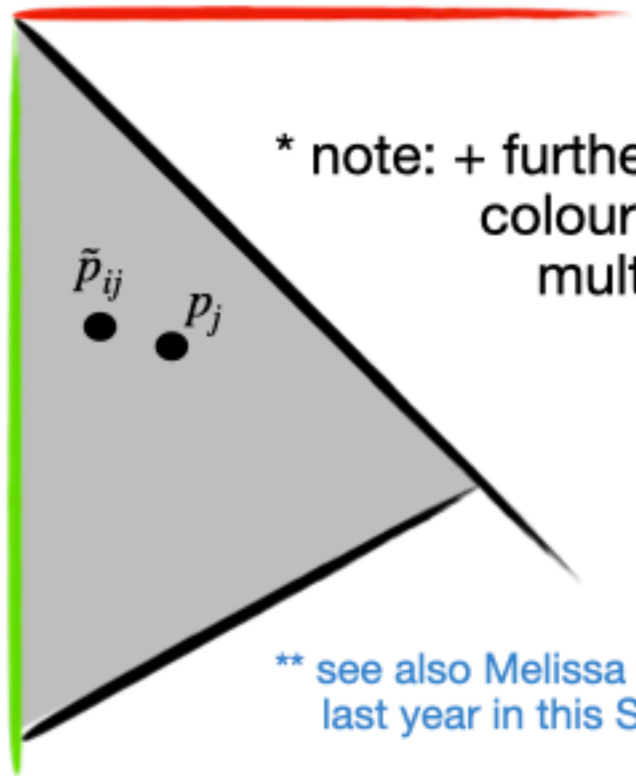


effect of recoil on accuracy

- question: do recoil effects indeed vanish in soft limit (i.e. $\rho \to 0$)?*
  [Dasgupta,Dreyer,Hamilton,Monni,Salam '18]**
- consider situation where we first emit $\tilde{p}_{ij}$ from p_a, p_b, then emit p_j, $\tilde{p}_{ij} \to p_i, p_j$
- transverse momentum of p_i will be $\sim k_t^{ij} + k_t^j$

$$\Rightarrow \frac{\Delta k_t^{ij}}{k_t^{ij}} \to \frac{\rho k_t^j}{\rho k_t^{ij}} = \mathcal{O}(1)$$

$$p_i = z\tilde{p}_{ij} + (1-z)y\tilde{p}_k + k_\perp$$
$$p_j = (1-z)\tilde{p}_{ij} + zy\tilde{p}_k - k_\perp$$
$$p_k = (1-y)\tilde{p}_k \ .$$



* note: + further problems for colour assignment in multiple emissions

** see also Melissa van Beekveld's talk last year in this Seminar

Alaric intro, splitting of Eikonal

Starting point: again eikonal $\frac{p_i p_k}{(p_i p_j)(p_j p_k)} = \frac{1}{E_j^2} \frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{jk})} \equiv \frac{W_{ik,j}}{E_j^2}$

naive implementation leads to soft double counting need to split into ij and kj collinear terms [Marchesini, Webber '88]

Option 1:

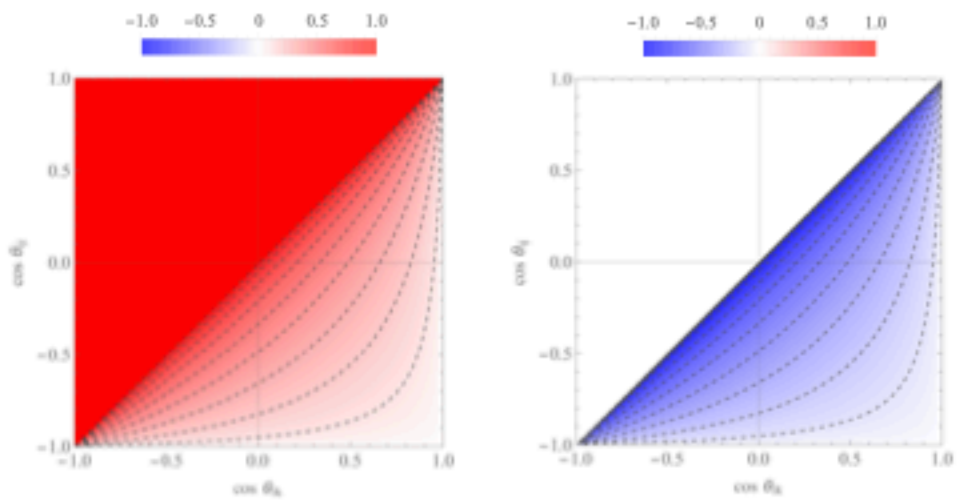
$$W_{ik,j} = \tilde{W}^i_{ik,j} + \tilde{W}^k_{ki,j} \,, \qquad \text{where} \qquad \tilde{W}^i_{ik,j} = \frac{1}{2}\left(\frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{jk})} + \frac{1}{1-\cos\theta_{ij}} - \frac{1}{1-\cos\theta_{jk}}\right)$$

Option 1a: Angular ordered shower

- downside: problems with NGLs

Option 1b: differential

- downside: cancellation between positive and negative contributions



Alaric intro, splitting of Eikonal

Starting point: again eikonal $\dfrac{p_i p_k}{(p_i p_j)(p_j p_k)} = \dfrac{1}{E_j^2} \dfrac{1 - \cos\theta_{ik}}{(1 - \cos\theta_{ij})(1 - \cos\theta_{jk})} \equiv \dfrac{W_{ik,j}}{E_j^2}$

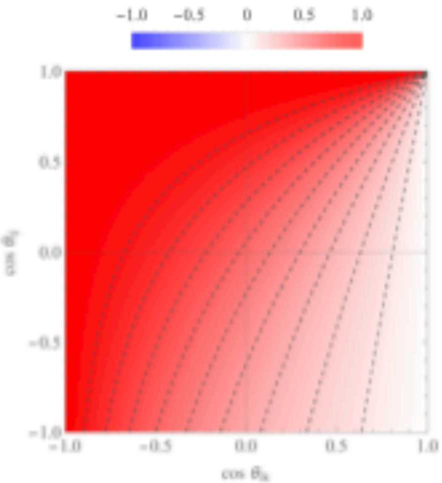
naive implementation leads to soft double counting need to split into ij and kj collinear terms [Marchesini, Webber '88]

Option 2: follow [Catani, Seymour '97]

$$W_{ik,j} = \bar{W}^i_{ik,j} + \bar{W}^k_{ki,j}\ , \qquad \text{where} \qquad \bar{W}^i_{ik,j} = \frac{1 - \cos\theta_{ik}}{(1 - \cos\theta_{ij})(2 - \cos\theta_{ij} - \cos\theta_{jk})}$$

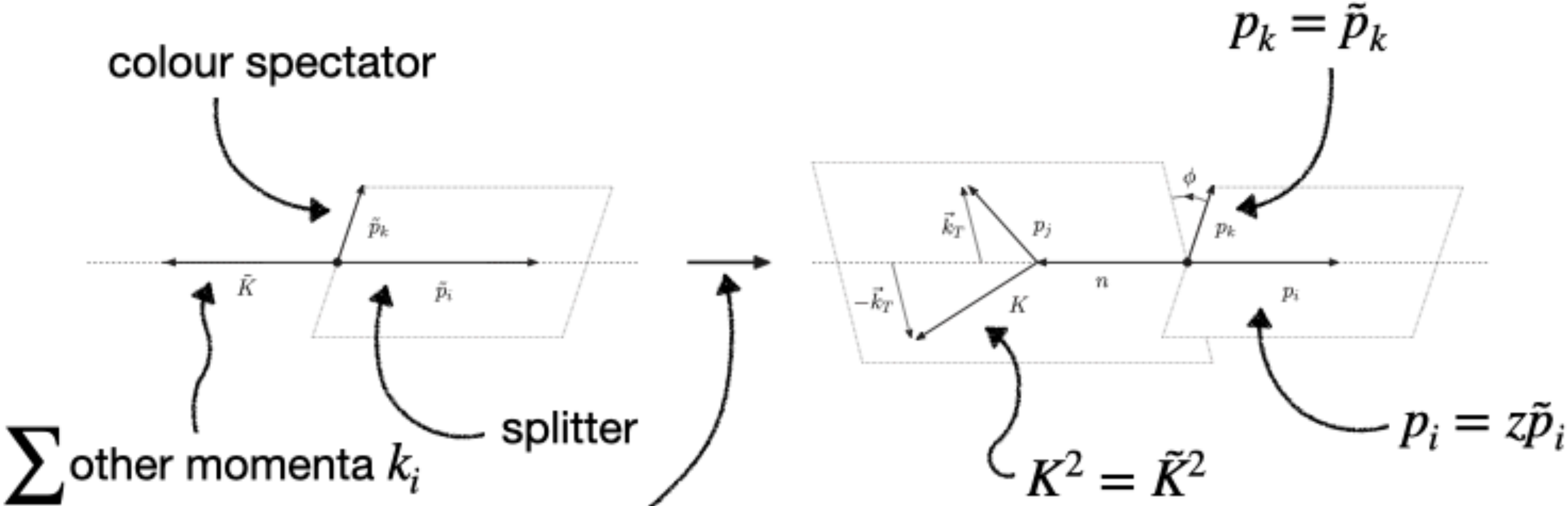
- full phase space coverage
- splitting functions remain

  positive definite



Alaric kinematics - global recoil scheme

- Before splitting:
- After splitting:



$$\tilde{K} + \tilde{p}_i = K + p_i + p_j$$

$$k_i^\mu \to \Lambda^\mu{}_\nu k_i^\nu$$

[Catani, Seymour '97]

$$\Lambda^\mu{}_\nu = g^\mu{}_\nu - \frac{(K+\tilde{K})^\mu (K+\tilde{K})_\nu}{K\cdot\tilde{K} + \tilde{K}^2} + 2\frac{K^\mu \tilde{K}_\nu}{\tilde{K}^2} \;\to\; \Lambda^\mu{}_\nu \tilde{K}^\nu = K^\mu$$

Alaric analytic proof of accuracy

$$\Lambda^{\mu}_{\ \nu}(K,\tilde{K}) = g^{\mu}_{\nu} + \tilde{K}^{\mu}A_{\nu} + X^{\mu}B_{\nu}$$

vanishes in soft limit

work out $\rho \to 0$ limit: $A^{\nu} \xrightarrow{\rho\to 0} 2\,\frac{\tilde{K}X}{\tilde{K}^2}\,\frac{\tilde{K}^{\nu}}{\tilde{K}^2} - \frac{X^{\nu}}{\tilde{K}^2}\,,$ and $B^{\nu} \xrightarrow{\rho\to 0} \frac{\tilde{K}^{\nu}}{\tilde{K}^2}$

apply to soft momentum p_l:

$$\frac{\Delta p_l^{0,3}}{p_l^{0,3}} \sim \rho^{1-\max(\xi_i,\xi_j)}$$

$$\frac{\Delta p_l^{1,2}}{p_l^{1,2}} \sim \rho^{(1-\xi_l)(\max(\xi_i,\xi_j)-\xi_l)}$$

compare to $\Delta k_t \sim \mathcal{O}(1) \Rightarrow \frac{\Delta k_t}{k_t}$ from local dipole scheme

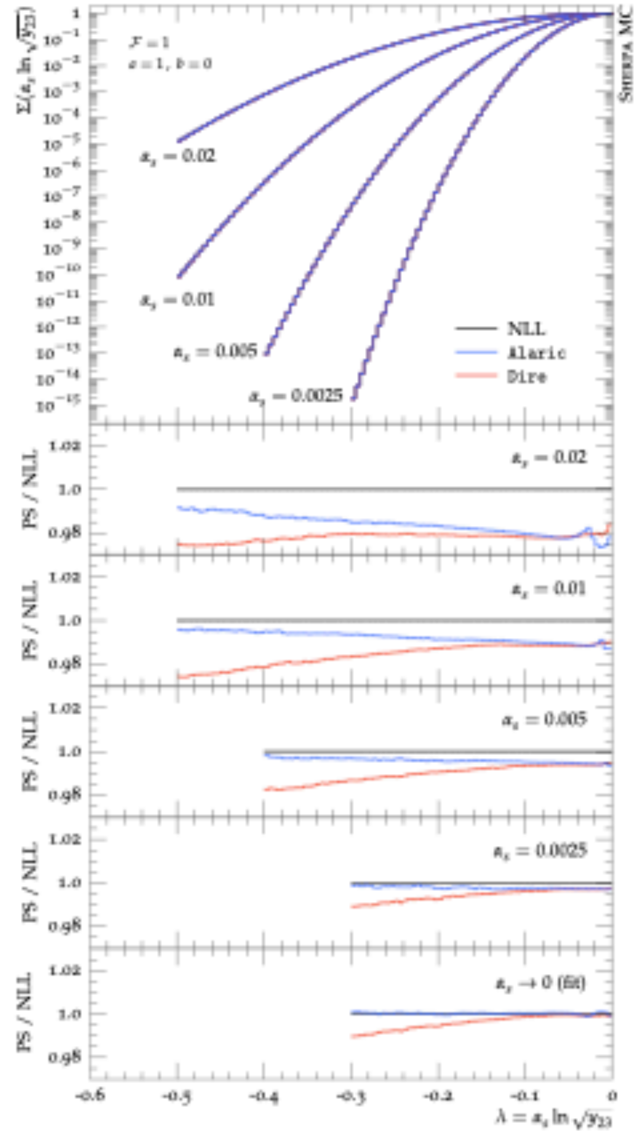
Alaric numerical validation

- Limit $\alpha_s \to 0$ with $\lambda = \alpha_s L =$ const. of

$$\frac{\Sigma^{\text{Shower}}}{\Sigma^{\text{NLL}}} \sim \exp\left(f^{LL}_{\text{Shower}} - Lg_1(\alpha_s^n L^n)\right)$$
$$\times \exp\left(f^{NLL}_{\text{Shower}} - g_2(\alpha_s^n L^n)\right)$$
$$\times \exp\left(\mathcal{O}(\alpha_s^{n+1} L^n)\right)$$
$$\to 1$$

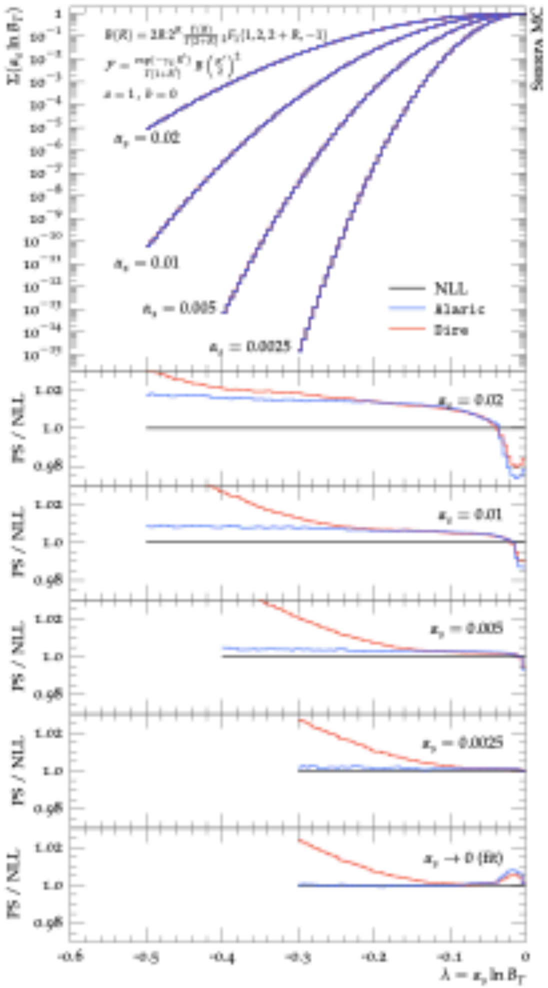
if shower reproduces LL, NLL logs

- Observable: jet resolution y_{23} in Cambridge jet measure, $\mathscr{F} = 1 \to$ only largest emission matters, check that additional shower emissions vanish

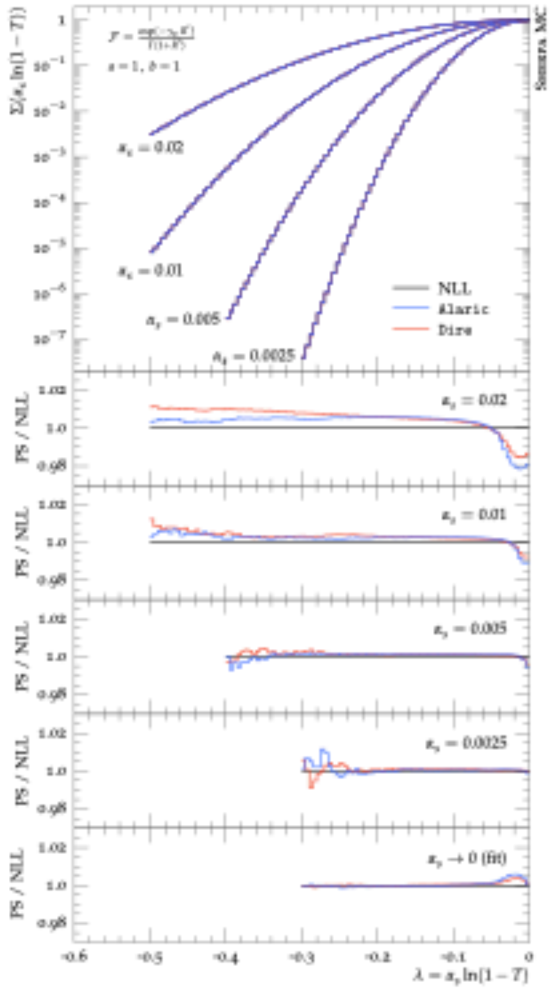


Alaric numerical validation

- total broadening $B_T = B_L + B_R$
- scaling k_t like, similar to y_{23}
- but non-trivial $\mathscr{F}$ function

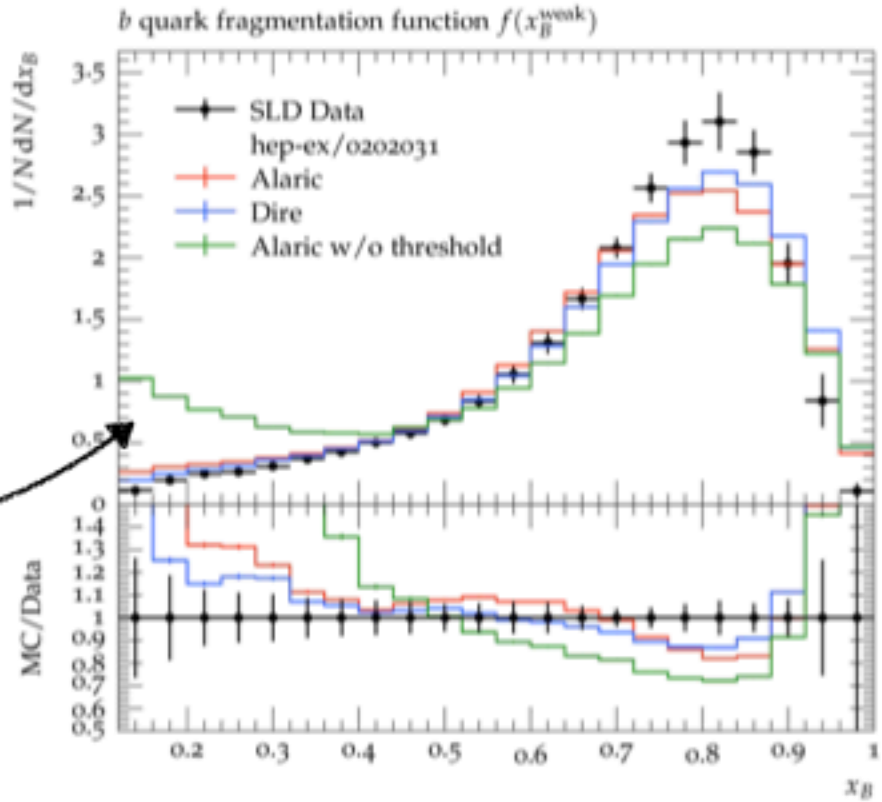


- thrust $\tau = 1 - t$
- scaling like virtuality $k_t e^{-\eta}$
- standard function
$$\mathscr{F} = \frac{\exp(-\gamma_E R)}{\Gamma(1 + R')}$$
- no evidence for NLL violation even for standard showers



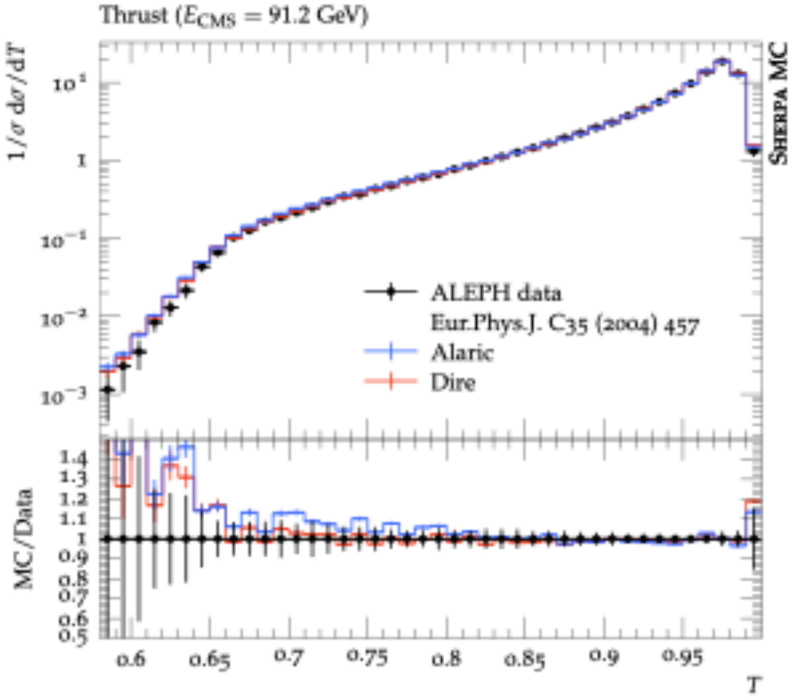
Alaric pheno, details and b fragmentation

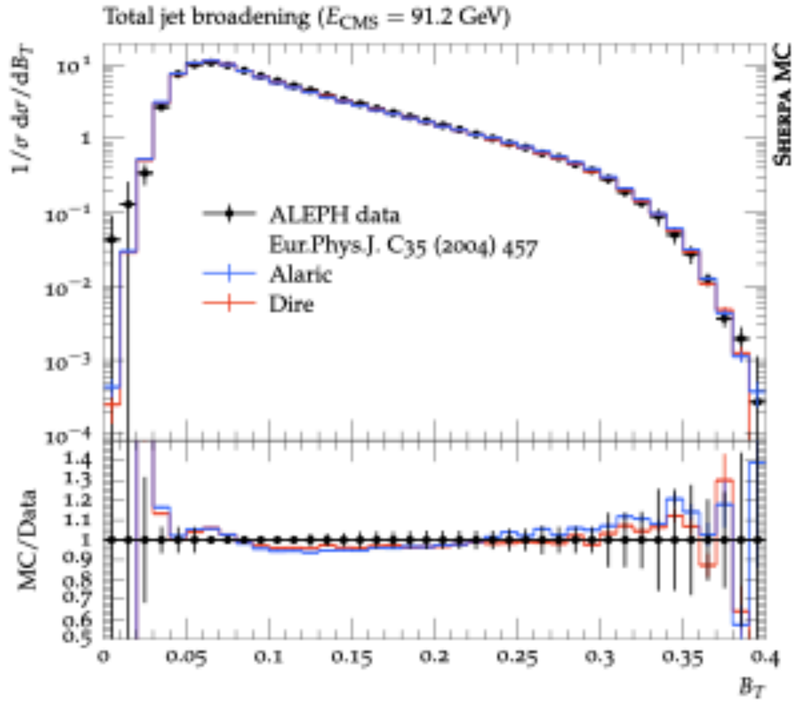
- first caveat: no quark masses implemented yet
- problem for cluster hadronisation
  → use Lund model via Pythia
- + need flavour threshold for $g \to b\bar{b}/g \to c\bar{c}$ splittings
- Dire parton shower as implemented in Sherpa as reference



$$x_B \sim \frac{E_{B\text{–Hadron}}}{E_{\text{tot.}}/2}$$

Alaric pheno, LEP observables



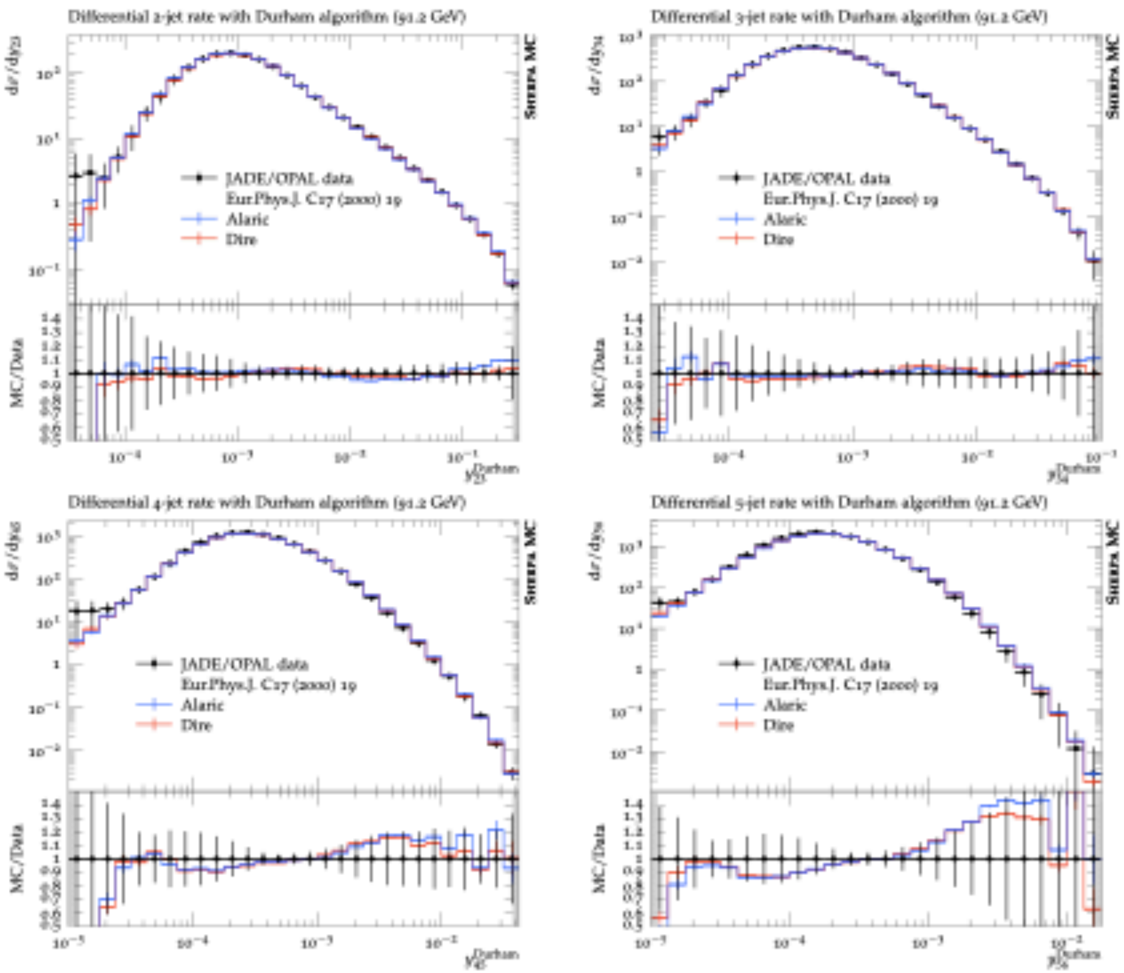


Thrust:
- Note this is T, not 1-T: soft physics is to the right
- Note there is no matching, relevant for small T

Total Broadening:
- soft physics is left hand side
- some deviations from data, but similar to Dire

Alaric pheno, LEP observables

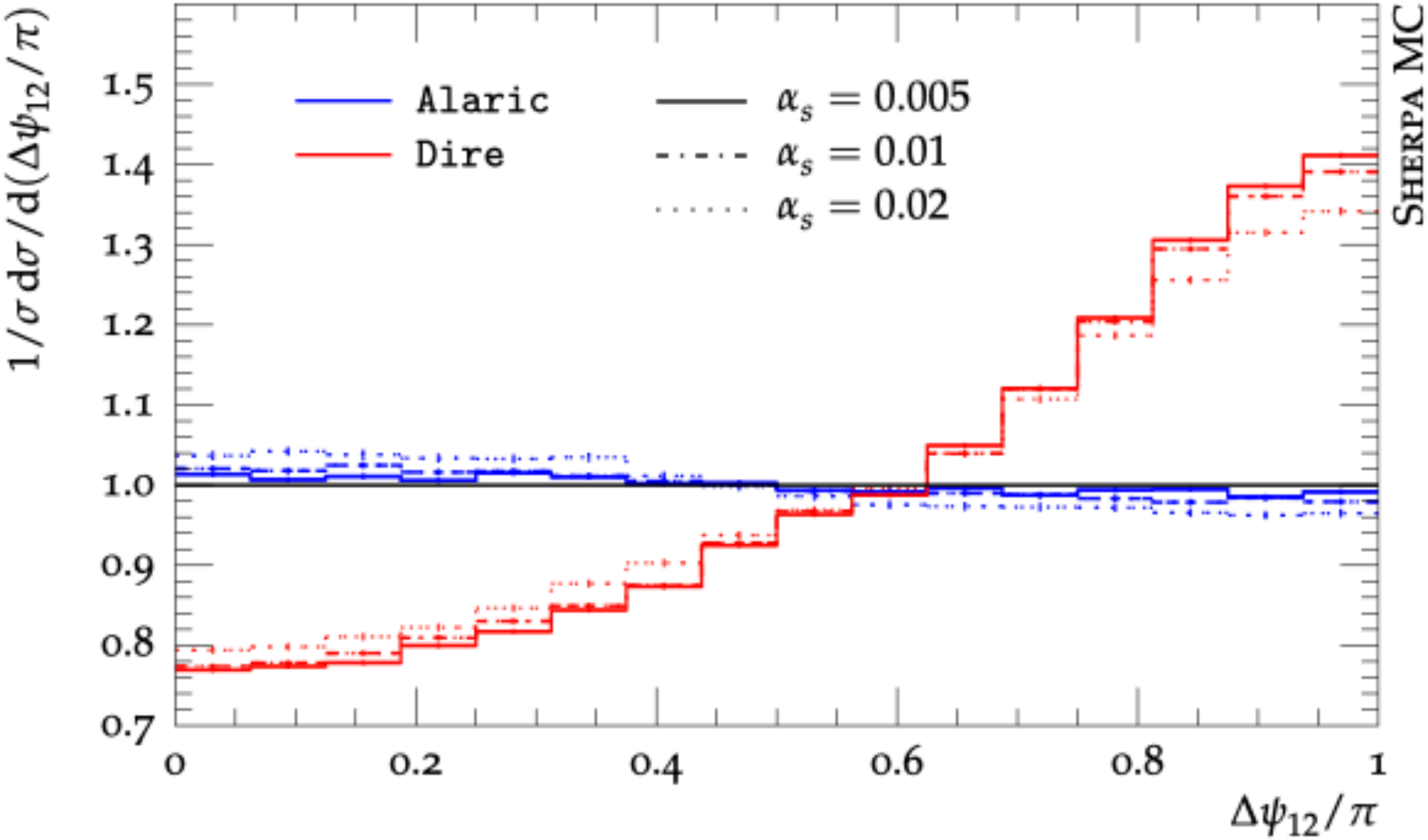
- Durham resolution scales $y_{n,n+1} \sim k_t^2/Q^2$
- higher Born multiplicities → sensitivity to multiple emissions increased
- again, note no matching/merging involved



Conclusion

- NLL resummation in CAESAR formalism as definition and validation of parton shower accuracy
- New parton shower Alaric
  - partial fractioning of eikonal → positive definite splitting function with full phase space coverage
  - global kinematics scheme enables analytic proof of NLL accuracy + numerical validation
  - included in Sherpa framework and first pheno results

Backup



$1/\sigma\, d\sigma/d(\Delta\psi_{12}/\pi)$
Alaric
Dire
$\alpha_s = 0.005$
$\alpha_s = 0.01$
$\alpha_s = 0.02$
SHERPA MC
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0
0.2
0.4
0.6
0.8
1
$\Delta\psi_{12}/\pi$