Collider Phenomenology (2)

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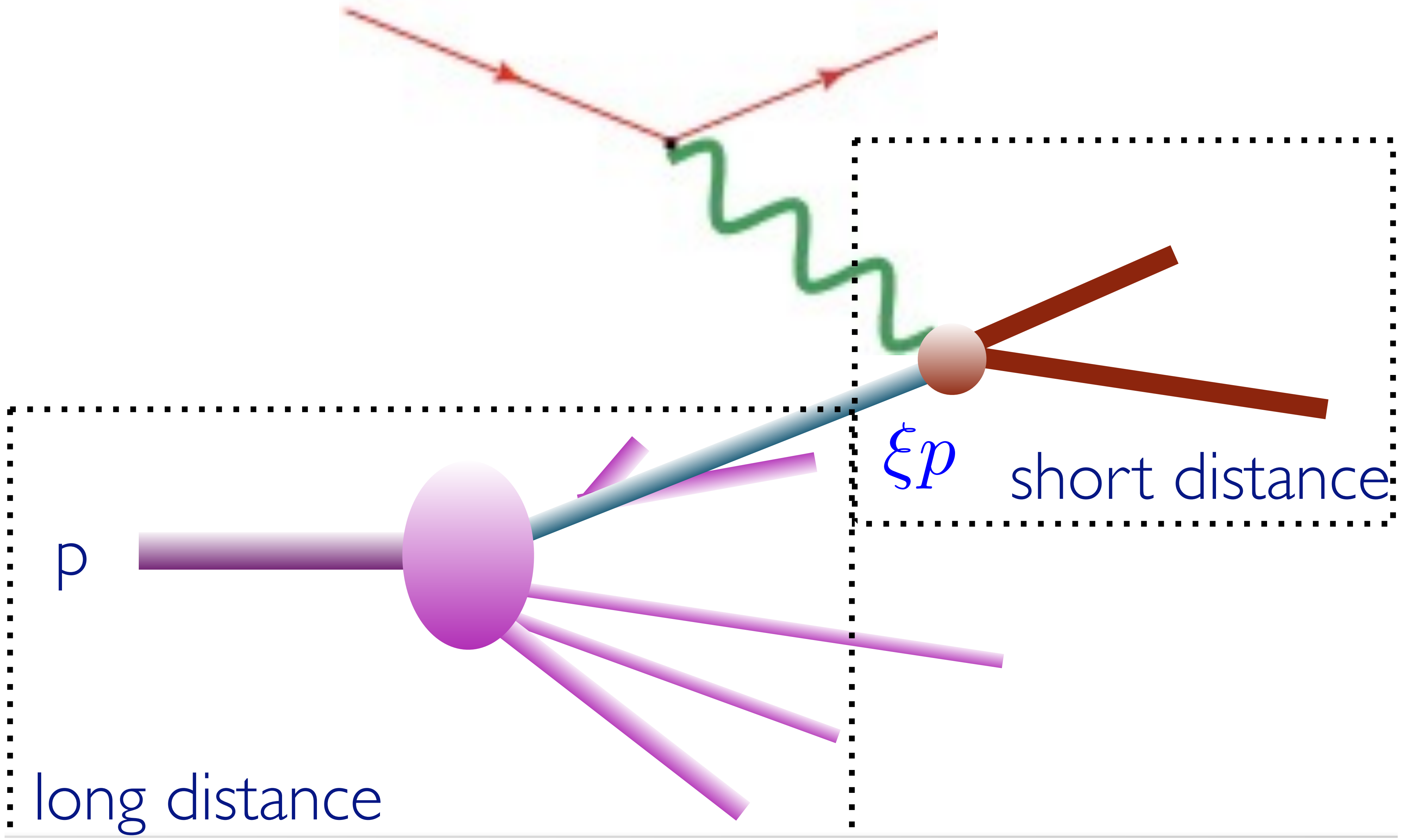
Plan for the lectures

- Basics of collider physics
- Basics of QCD
  - DIS and the Parton Model
  - Higher order corrections
  - Asymptotic freedom
  - QCD improved parton model
- State-of-the-art computations for the LHC
- Monte Carlo generators
- Higgs phenomenology
- Top phenomenology
- Searching for New Physics: EFT

Factorisation

Breit picture frame allows us to assume partons are free within proton:

$$\frac{d^2\sigma}{dx dQ^2} = \int_0^1 \frac{d\xi}{\xi} \sum_i f_i(\xi) \frac{d^2\hat{\sigma}}{dx dQ^2}\left(\frac{x}{\xi}, Q^2\right)$$



$f_i(\xi)$ Probability of finding parton i in hadron carrying momentum fraction ξ

$\frac{d^2\hat{\sigma}}{dx dQ^2}$ Cross-section for parton-photon scattering

DIS cross-section

Comparing our inclusive cross-section:

$$\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4}\left\{[1+(1-y)^2]F_1(x,Q^2) + \frac{1-y}{x}\left[F_2(x,Q^2) - 2xF_1(x,Q^2)\right]\right\}$$

Factorised cross-section in the parton model:

$$\frac{d^2\sigma}{dxdQ^2} = \int_0^1 \frac{d\xi}{\xi}\sum_i f_i(\xi)\frac{d^2\sigma}{d\hat{x}dQ^2}(\frac{x}{\xi},Q^2) \quad \text{with} \quad \frac{d^2\hat{\sigma}}{dQ^2dx} = \frac{4\pi\alpha^2}{Q^4}\frac{1}{2}\left[1+(1-y)^2\right]e_q^2\,\delta(x-\xi)$$

We can express the structure functions as:

$$F_2(x) = 2xF_1 = \sum_{i=q,\bar{q}}\int_0^1 d\xi\, f_i(\xi)\, x e_q^2 \delta(x-\xi) = \sum_{i=q,\bar{q}} e_q^2\, x f_i(x)$$

DIS cross-section

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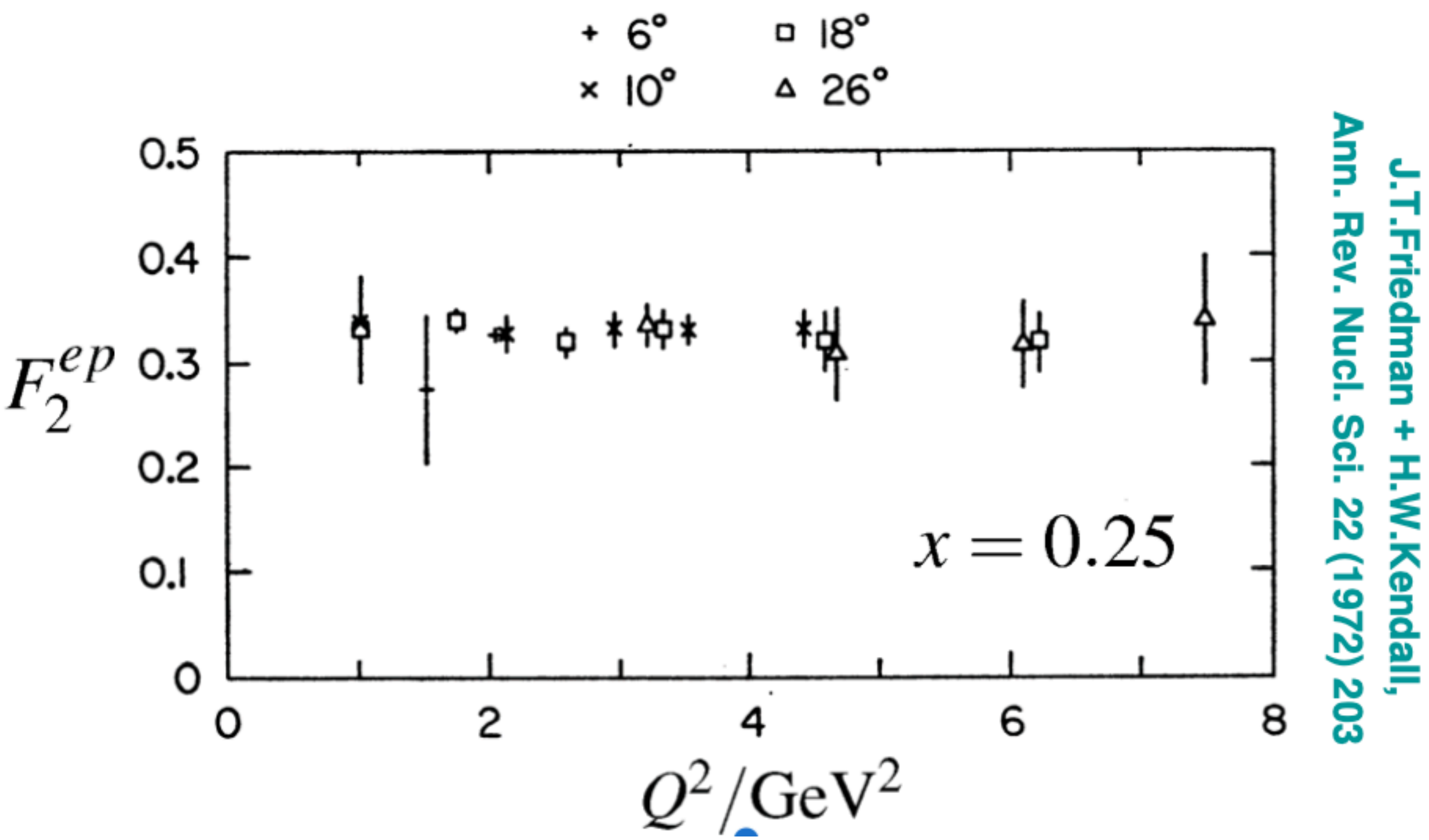
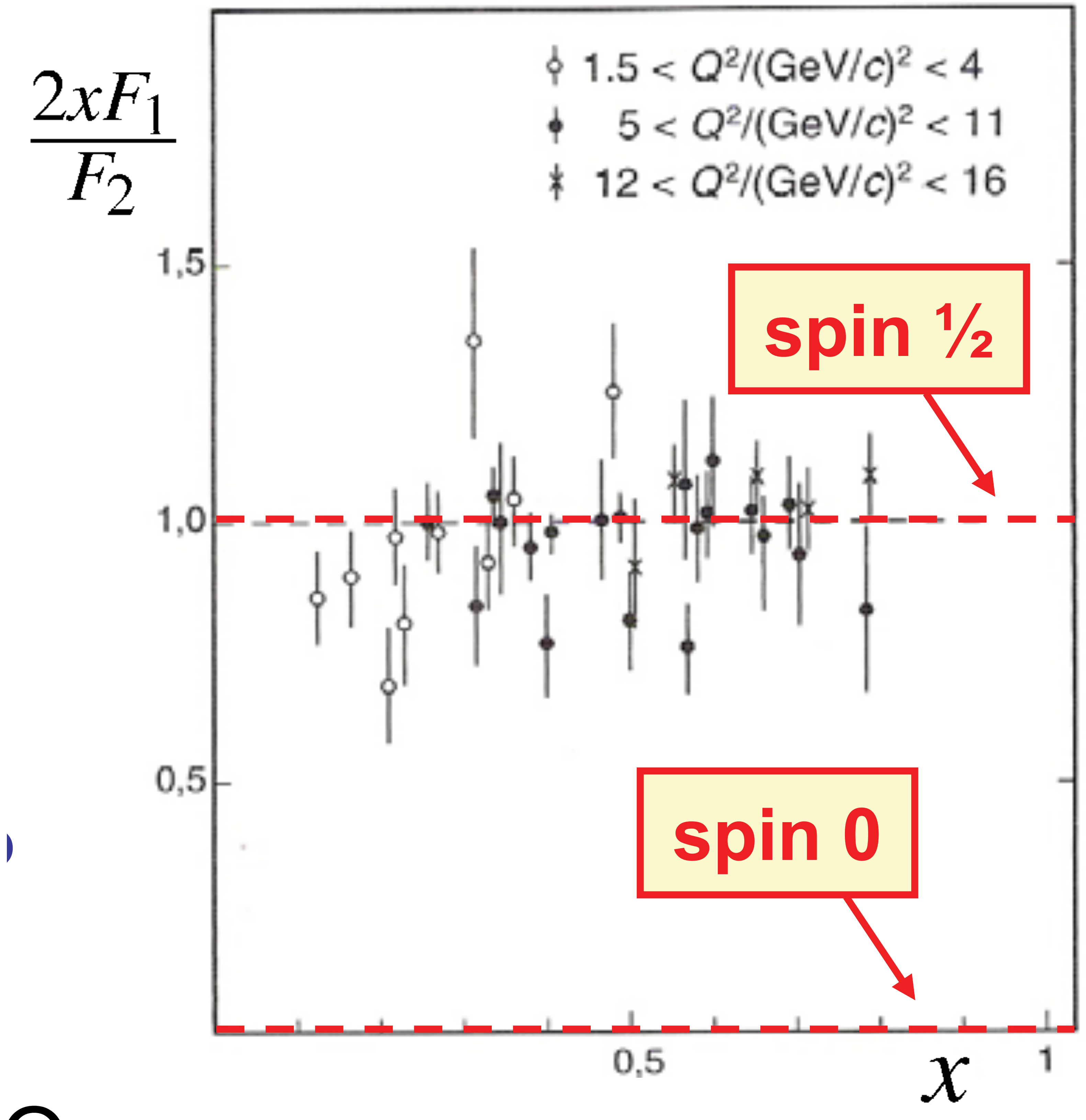
Quarks and anti-quarks enter together. How can we separate them?

No dependence on Q: **Scaling**

$f_i(x)$ are the parton distribution functions which describe the probabilities of finding specific partons in the proton carrying momentum fraction x

Scaling and Callan-Gross relation



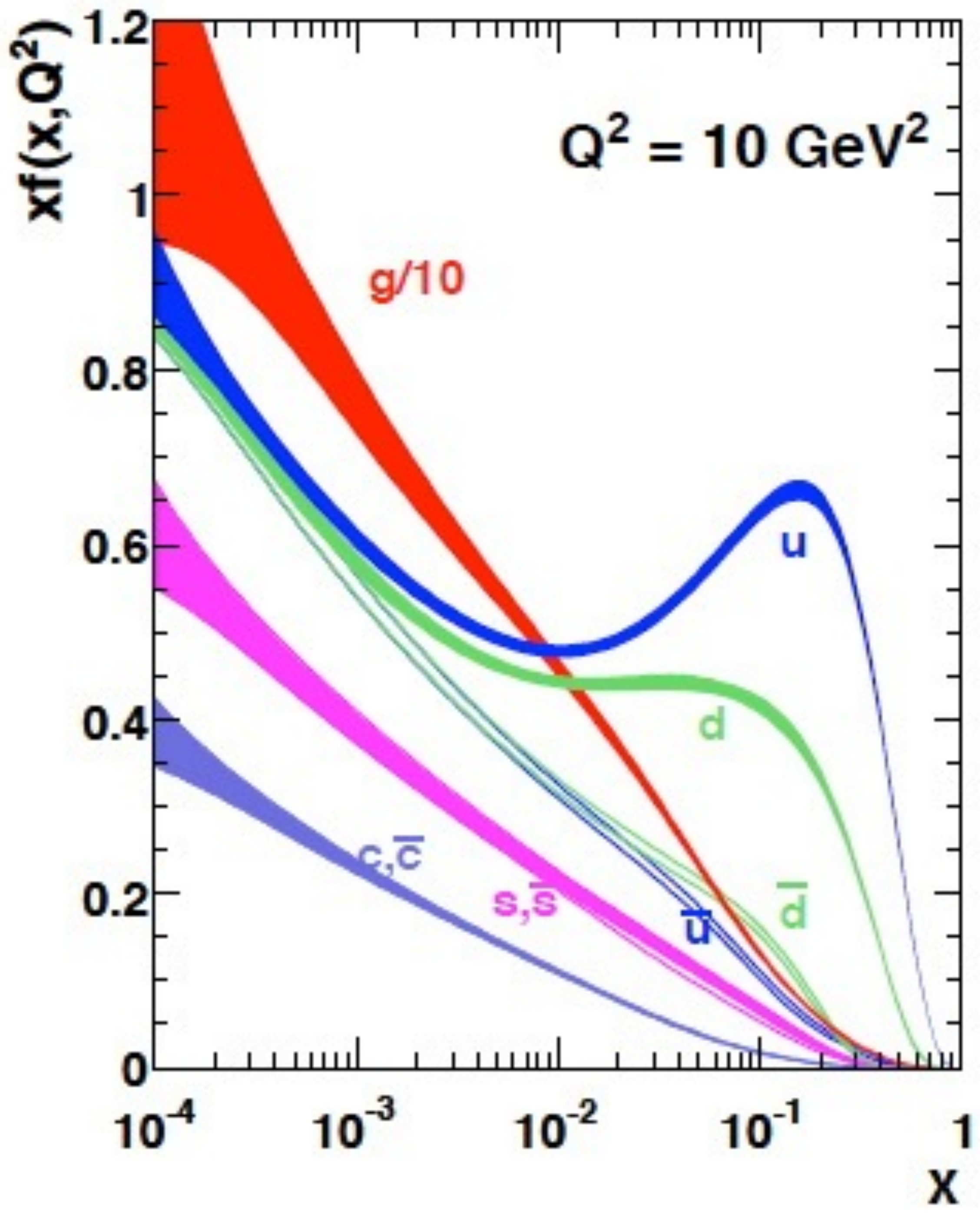


Scaling: Structure function does not depend on Q

Callan-Gross relation

Quarks are spin-1/2 particles!

Parton distribution functions



$$\begin{aligned} u(x) &= u_V(x) + \bar{u}(x) \\ d(x) &= d_V(x) + \bar{d}(x) \\ s(x) &= \bar{s}(x) \end{aligned}$$

$$\int_0^1 dx\, u_V(x) = 2\,, \quad \int_0^1 dx\, d_V(x) = 1$$

$$\sum_q \int_0^1 dx\, x[q(x) + \bar{q}(x)] \simeq 0.5$$

Quarks carry only 50% of the proton momentum

Evidence for gluons!

Parton model summary

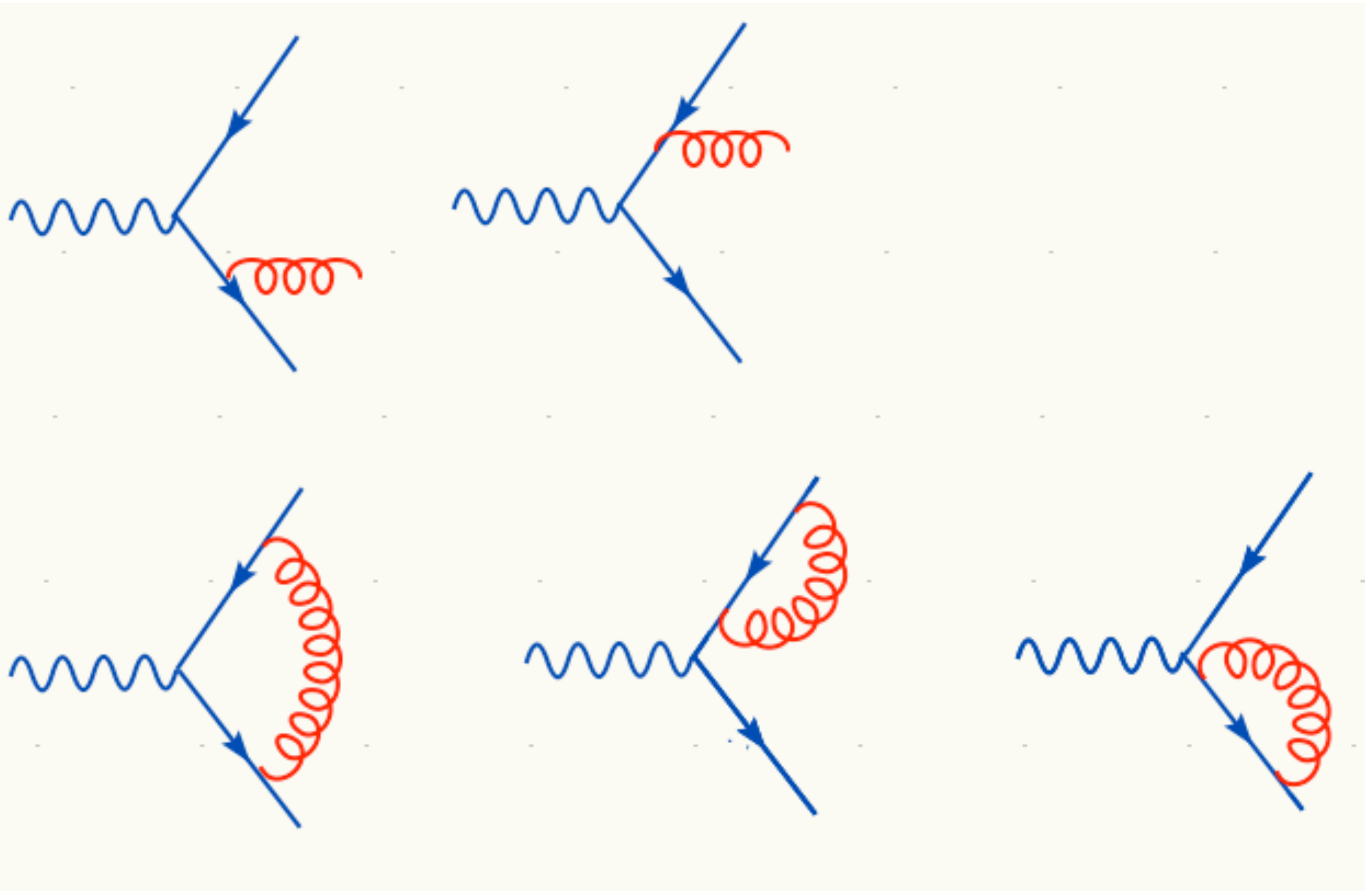
DIS experiments show that virtual photon scatters off massless, free, point like, spin-1/2 quarks

One can **factorise** the short- and long-distance physics entering this process. Long-distance physics absorbed in PDFs. Short distance physics described by the hard scattering of the parton with the virtual photon.

$$\sum_{a,b} \int dx_1 dx_2 \, d\Phi_{PS} \, f_a(x_1) f_b(x) \, \hat{\sigma}(\hat{s})$$

Phase-space integral Parton density functions Parton-level cross section

R-ratio@NLO



Real

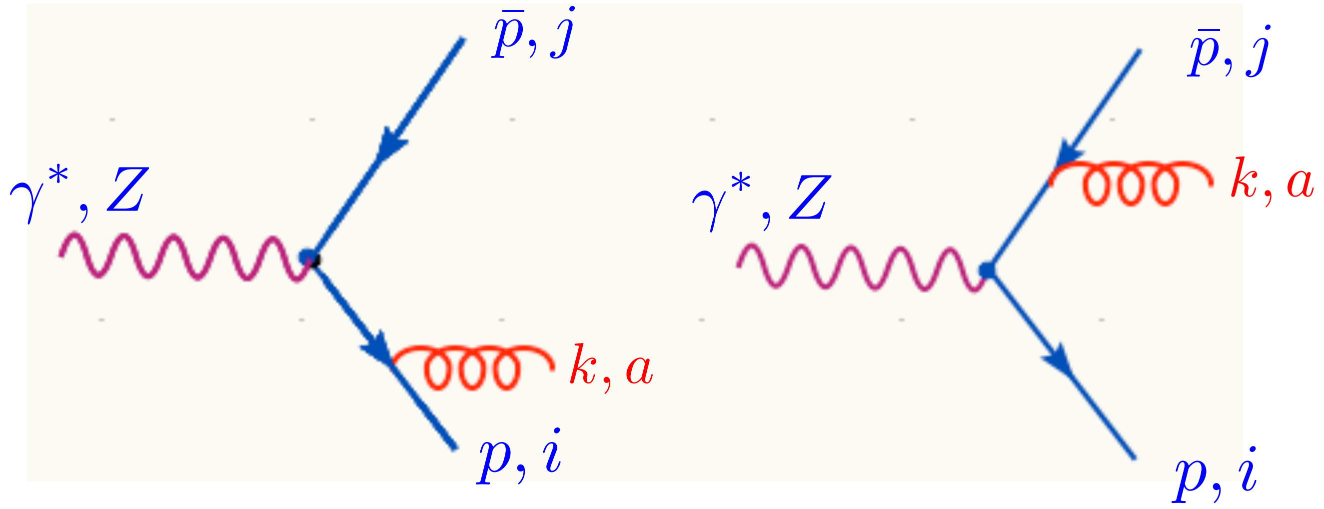
Virtual

$$\sigma_{NLO} = \sigma_{LO} + \int_R |M_{real}|^2 \, d\Phi_3 + \int_V 2\mathrm{Re}(M_0 M_{vir}^*) \, d\Phi_2$$

QCD in the final state

R-ratio@NLO

Real corrections:



$$A = \bar{u}(p)\not{\epsilon}(-ig_s)\frac{-i}{\not{p}+\not{k}}\Gamma^\mu v(\bar{p})t^a + \bar{u}(p)\Gamma^\mu \frac{i}{\not{\bar{p}}+\not{k}}(-ig_s)\not{\epsilon}v(\bar{p})t^a$$

$$= -g_s\left[\frac{\bar{u}(p)\not{\epsilon}(\not{p}+\not{k})\Gamma^\mu v(\bar{p})}{2p\cdot k} - \frac{\bar{u}(p)\Gamma^\mu(\not{\bar{p}}+\not{k})\not{\epsilon}v(\bar{p})}{2\bar{p}\cdot k}\right]t^a$$

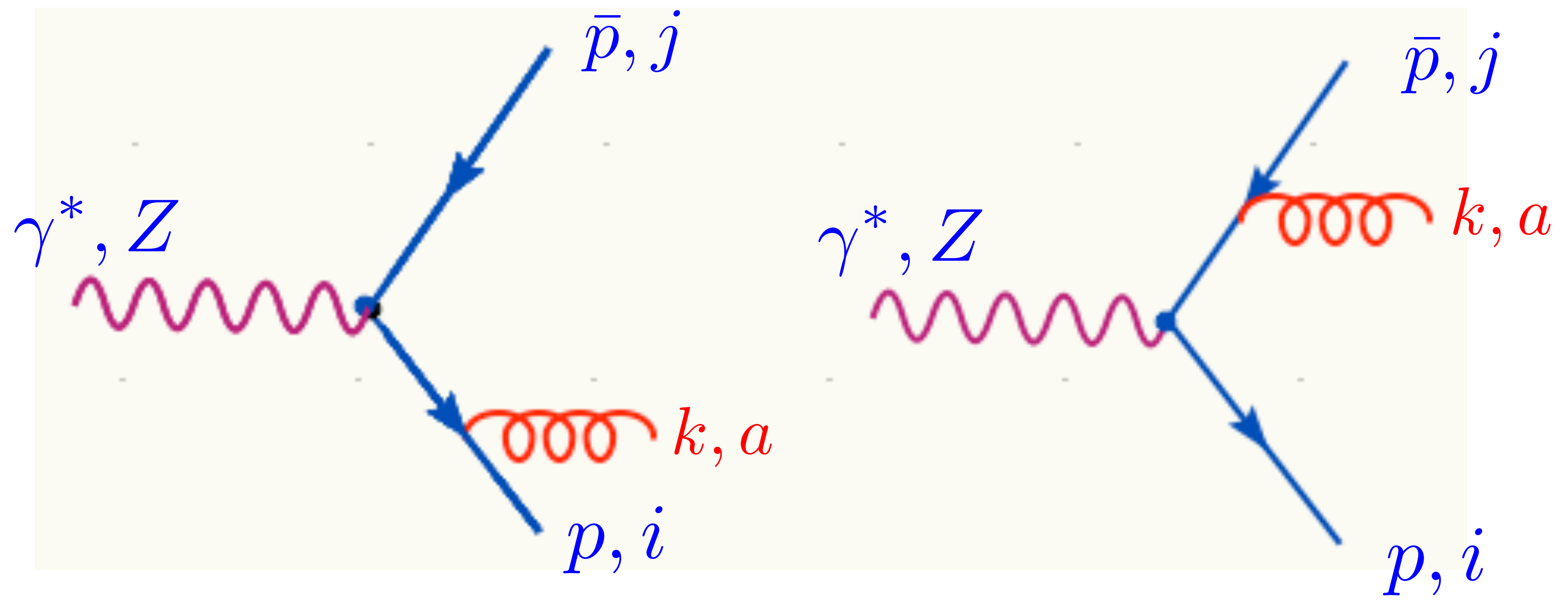
What are those denominators?

$$p \cdot k = p_0 k_0 (1 - \cos\theta)$$

What happens when the gluon is soft ($k_0 \to 0$) or collinear ($\theta \to 0$) to the quark?

QCD in the final state

R-ratio@NLO



What happens when the gluon is soft ($k_0 \to 0$) or collinear ($\theta \to 0$) to the quark?

$$A_{soft} = -g_s t^a \boxed{\left(\frac{p \cdot \epsilon}{p \cdot k} - \frac{\bar{p} \cdot \epsilon}{\bar{p} \cdot k} \right)} A_{Born}$$

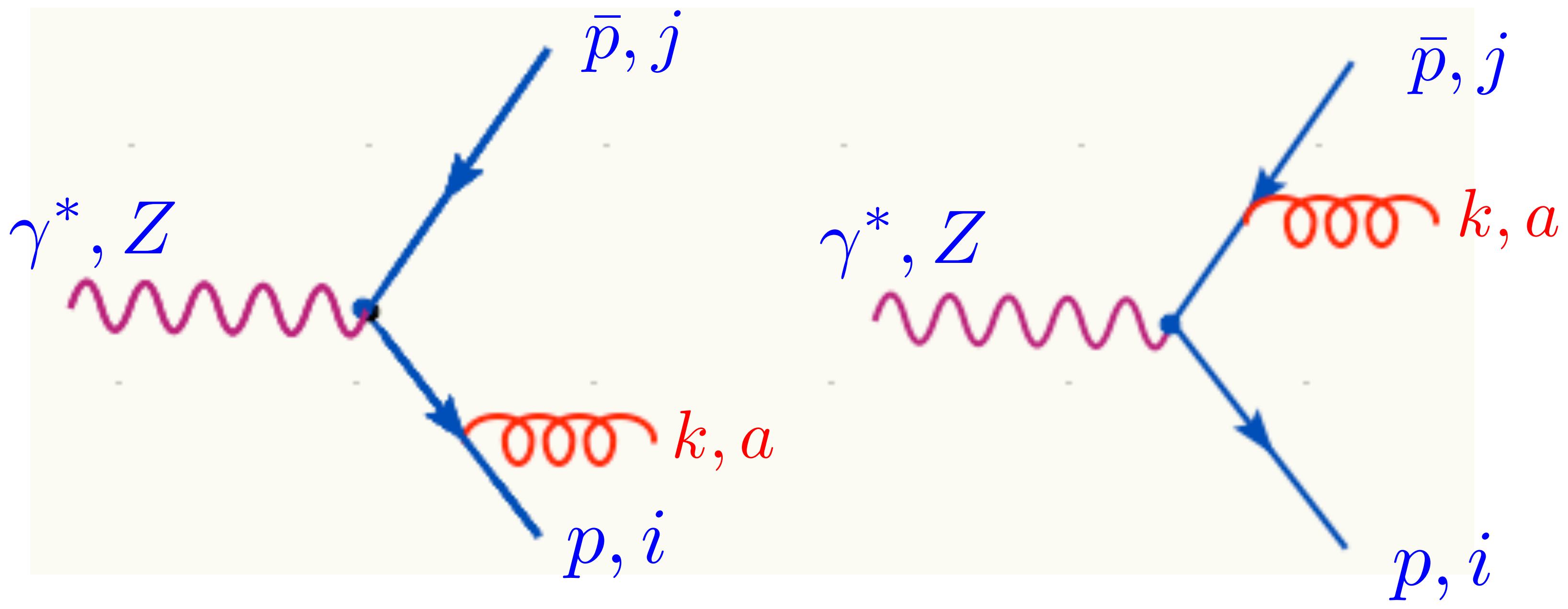
Very important property of QCD

Factorisation of long-wavelength (soft) emission from the short-distance (hard) scattering!

Soft emission factor is universal!

QCD in the final state

R-ratio@NLO



$$\sigma_{NLO} = \sigma_{LO} + \int_R |M_{real}|^2 \, d\Phi_3 + \int_V 2\mathrm{Re}(M_0 M^*_{vir}) \, d\Phi_2$$

$$A_{soft} = -g_s t^a \boxed{\left(\frac{p \cdot \epsilon}{p \cdot k} - \frac{\bar{p} \cdot \epsilon}{\bar{p} \cdot k} \right)} A_{Born}$$

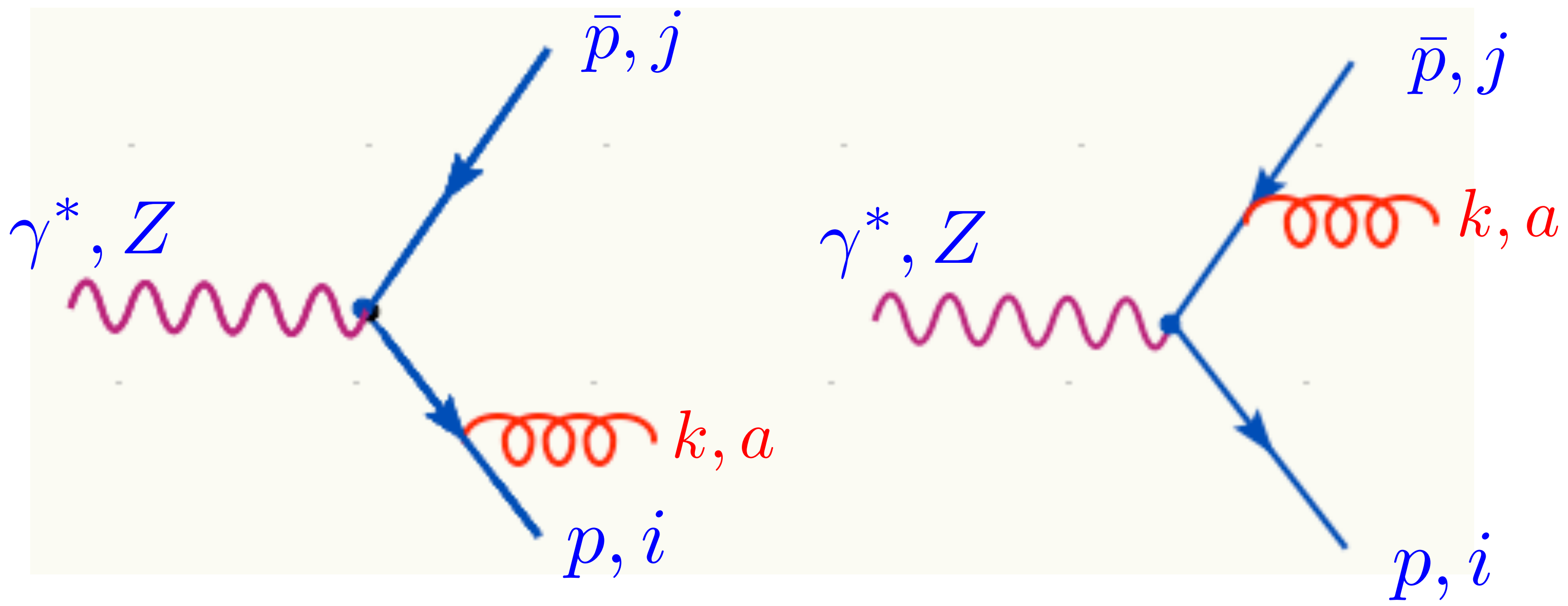
What does that mean for the NLO cross-section?

$$\sigma^{\mathrm{REAL}}_{q\bar{q}g} = C_F g_s^2 \sigma^{\mathrm{Born}}_{q\bar{q}} \int \frac{d^3k}{2k^0(2\pi)^3} 2 \frac{p \cdot \bar{p}}{(p \cdot k)(\bar{p} \cdot k)}$$

$$= C_F \frac{\alpha_S}{2\pi} \sigma^{\mathrm{Born}}_{q\bar{q}} \int d\cos\theta \, \frac{dk^0}{k^0} \frac{4}{(1-\cos\theta)(1+\cos\theta)}$$

QCD in the final state

R-ratio@NLO



$$\sigma^{\text{REAL}}_{q\bar{q}g} = C_F g_s^2 \sigma^{\text{Born}}_{q\bar{q}} \int \frac{d^3k}{2k^0(2\pi)^3} 2 \frac{p\cdot\bar{p}}{(p\cdot k)(\bar{p}\cdot k)}$$

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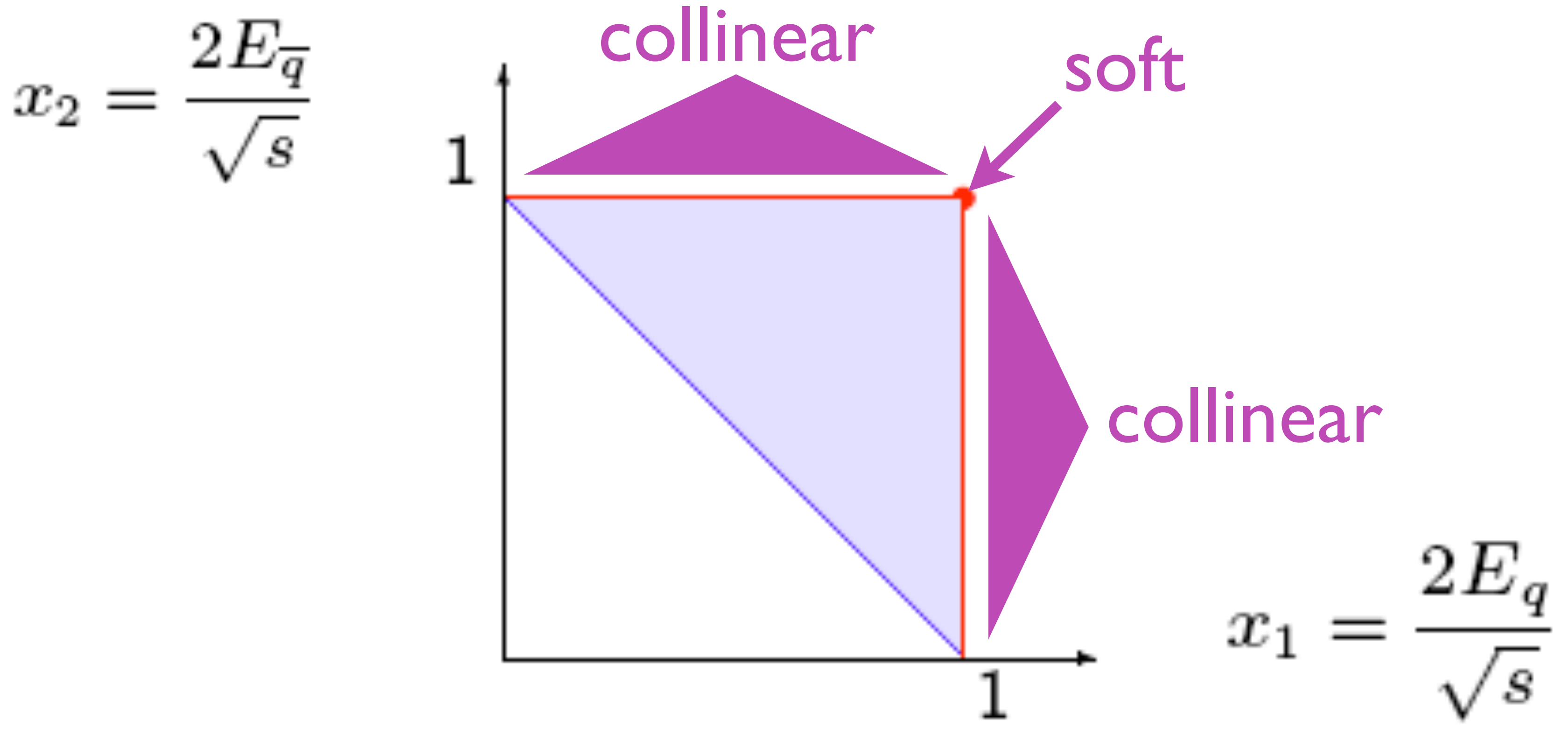
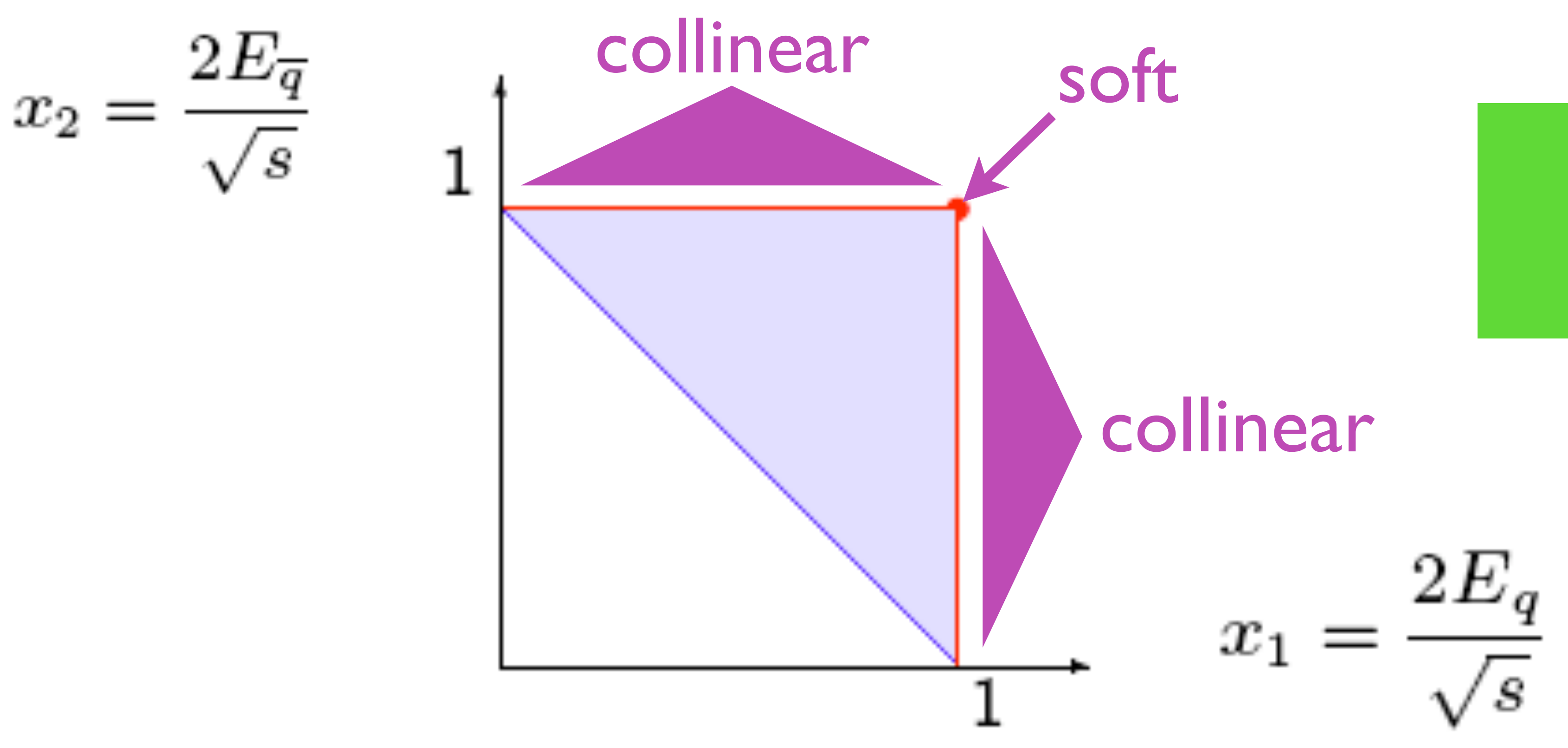
Soft divergence Collinear divergence

$$x_1 = 1 - x_2 x_3 (1-\cos\theta_{23})/2$$
$$x_2 = 1 - x_1 x_3 (1-\cos\theta_{13})/2$$
$$x_1 + x_2 + x_3 = 2$$
$$0 \le x_1, x_2 \le 1, \quad \text{and} \quad x_1 + x_2 \ge 1$$



Divergences

$$x_1 = 1 - x_2 x_3 (1 - \cos\theta_{23})/2$$
$$x_2 = 1 - x_1 x_3 (1 - \cos\theta_{13})/2$$
$$x_1 + x_2 + x_3 = 2$$
$$0 \leq x_1, x_2 \leq 1, \quad \text{and} \quad x_1 + x_2 \geq 1$$



Why is $x_1 = x_2 = 1$ the soft case?

$$\sigma^{q\bar{q}g} = \frac{4\pi^2}{3s} f_q^2 C_F \frac{\alpha_s}{2\pi} \int\int dx_1 dx_2 \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

Integral diverges if $x_1 \to 1$ or $x_2 \to 1$ or $x_1, x_2 \to 1$!

What happens now?

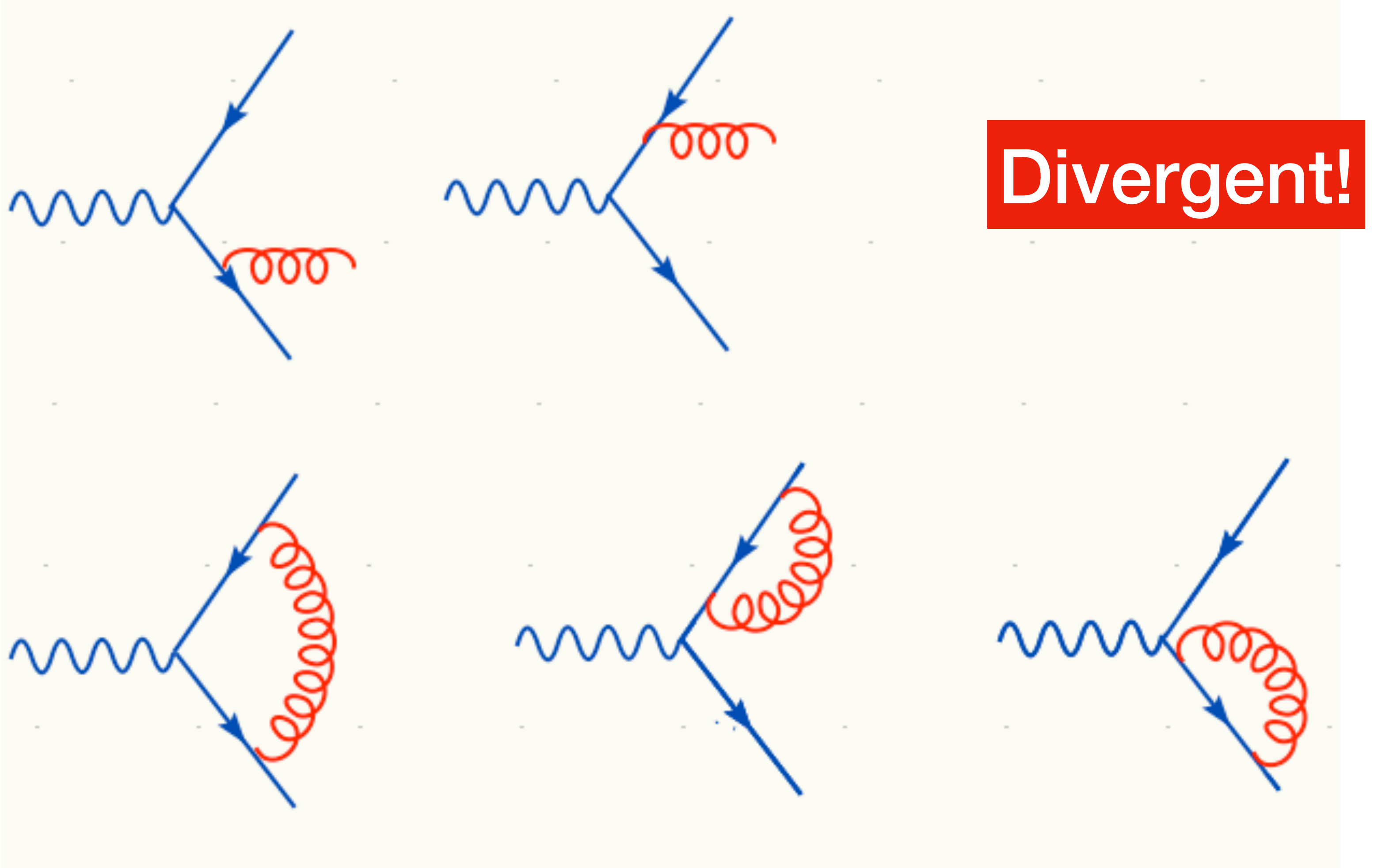
IR singularities

IR singularities arise when a parton is too soft or if two partons are collinear

- Infrared divergences arise from interactions that happen a long time after the creation of the quark/antiquark pair.
- When distances become comparable to the hadron size of ~1 Fermi, quasi-free partons of the perturbative calculation are confined/hadronized non-perturbatively.

How do we proceed with our calculation?

Cancellation of divergences



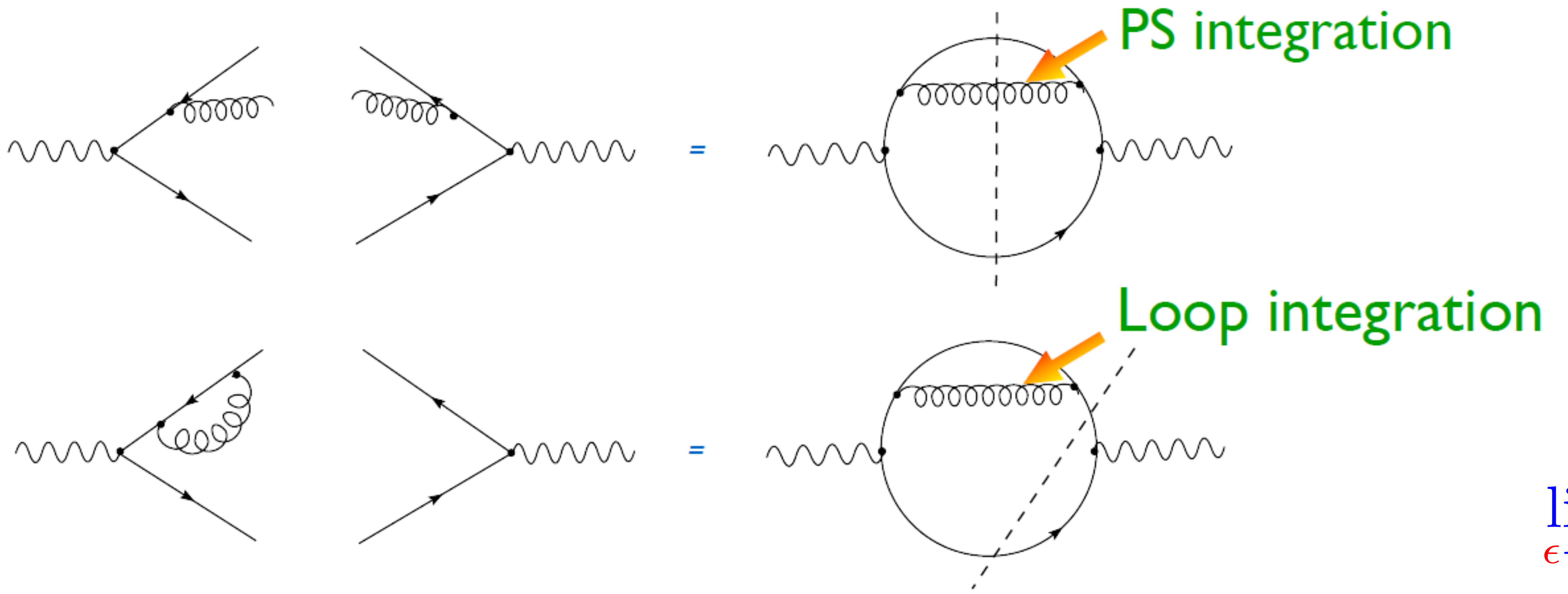
Divergent!

Real

Virtual

Also divergent!

In practice: regularise both divergences (with either dimensional regularisation or mass regulator)



$$\sigma^{\mathrm{REAL}} = \sigma^{\mathrm{Born}} C_F \frac{\alpha_S}{2\pi}\left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2\right)$$

$$\sigma^{\mathrm{VIRT}} = \sigma^{\mathrm{Born}} C_F \frac{\alpha_S}{2\pi}\left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2\right)$$

$$\lim_{\epsilon\to 0}(\sigma^{\mathrm{REAL}} + \sigma^{\mathrm{VIRT}}) = C_F \frac{3}{4}\frac{\alpha_S}{\pi}\sigma^{\mathrm{Born}}$$

$$R_1 = R_0\left(1 + \frac{\alpha_S}{\pi}\right)$$ **Finite!**

KLN Theorem

Why does this work?

Kinoshita-Lee-Nauenberg theorem: Infrared singularities in a massless theory cancel out after summing over degenerate (initial and final) states

hard   hard + soft gluon   2 collinear partons

Physically a hard parton can not be distinguished from a hard parton plus a soft gluon or from two collinear partons with the same energy. They are degenerate states. A final state with a soft gluon is nearly degenerate with a final state with no gluon at all (virtual)

Hence, one needs to add all degenerate states to get a physically sound observable

Infrared safety

How can we make sure IR divergences cancel?

We need to pick observables which are insensitive to soft and collinear radiation. These observables are determined by hard, short-distance physics, with long distance effects suppressed by an inverse power of a large momentum scale.

Schematically for an IR safe observable:

$$\mathcal{O}_{n+1}(k_1, k_2, \ldots, k_i, k_j, \ldots k_n) \to \mathcal{O}_n(k_1, k_2, \ldots k_i + k_j, \ldots k_n)$$

whenever one of the k_i/k_j becomes soft or k_i and k_j are collinear

Which observables are infrared safe?

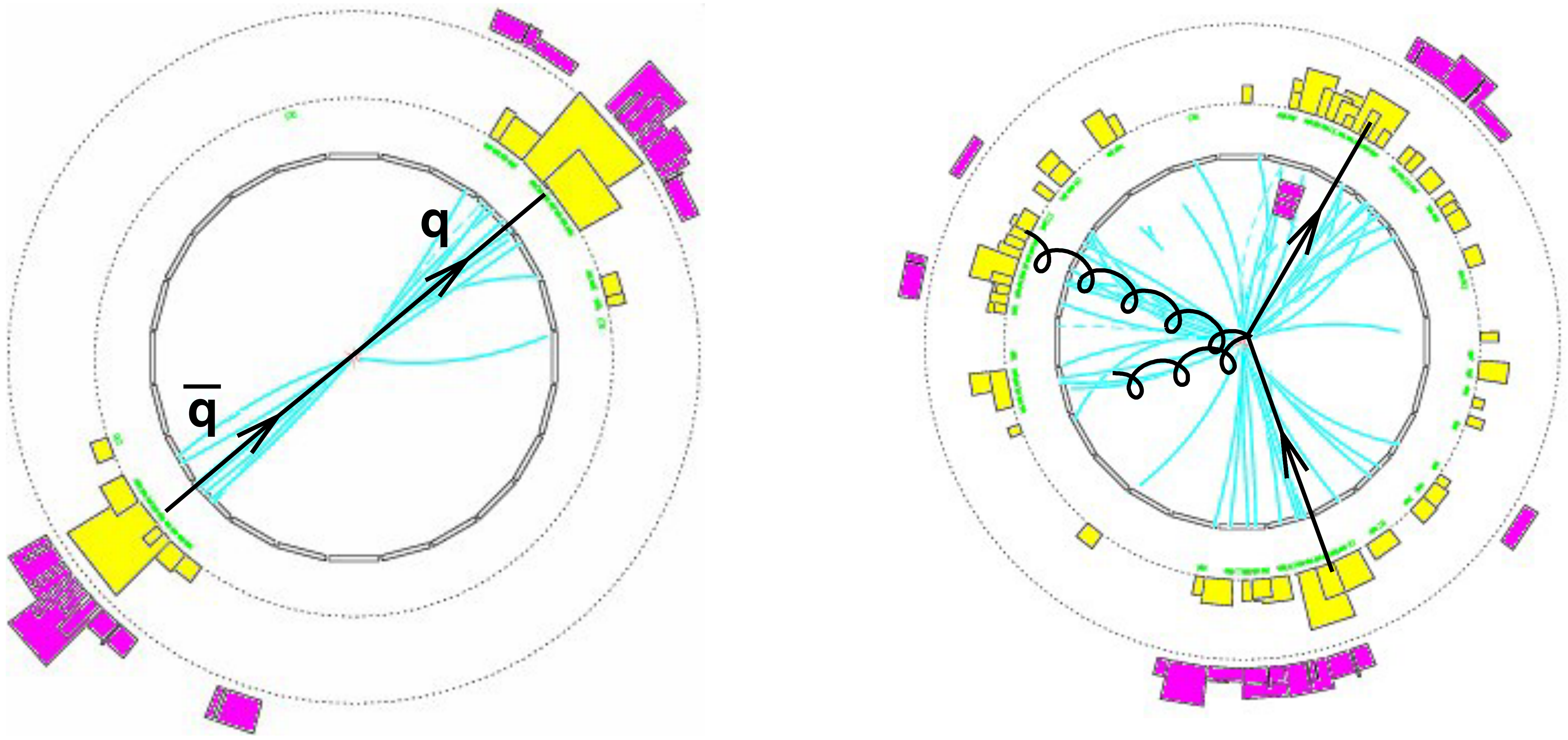
- energy of the hardest particle in the event — NO
- multiplicity of gluons — NO
- momentum flow into a cone in rapidity and angle — YES
- jet cross-sections — DEPENDS

See exercises!

Event shapes

Event shapes: describe the shape of the event, but are largely insensitive to soft and collinear branching

- widely used to measure αs
- measure colour factors
- test QCD
- learn about non-perturbative physics



pencil-like spherical

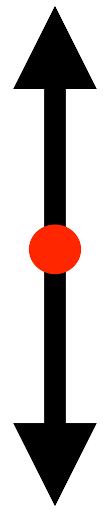
Thrust

Event-shape example

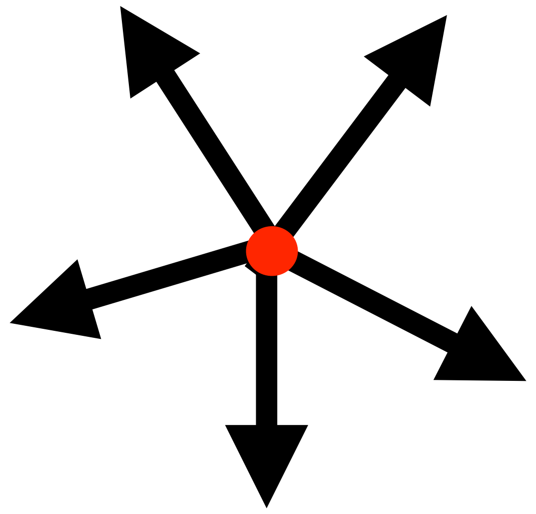
$$T = \max_{\vec{\hat{n}}} \frac{\sum_i |\vec{p_i} \cdot \vec{\hat{n}}|}{\sum_i |\vec{p_i}|}$$

Sum over all final state particles

Find axis n which maximises this projection



$T = 1$



$T = 1/2$

Noteby: if one of the partons emits a soft or collinear gluon the value of thrust is not changing. **IRC safe**

What happens in an $e^+e^- \to q\bar{q}g$ event?

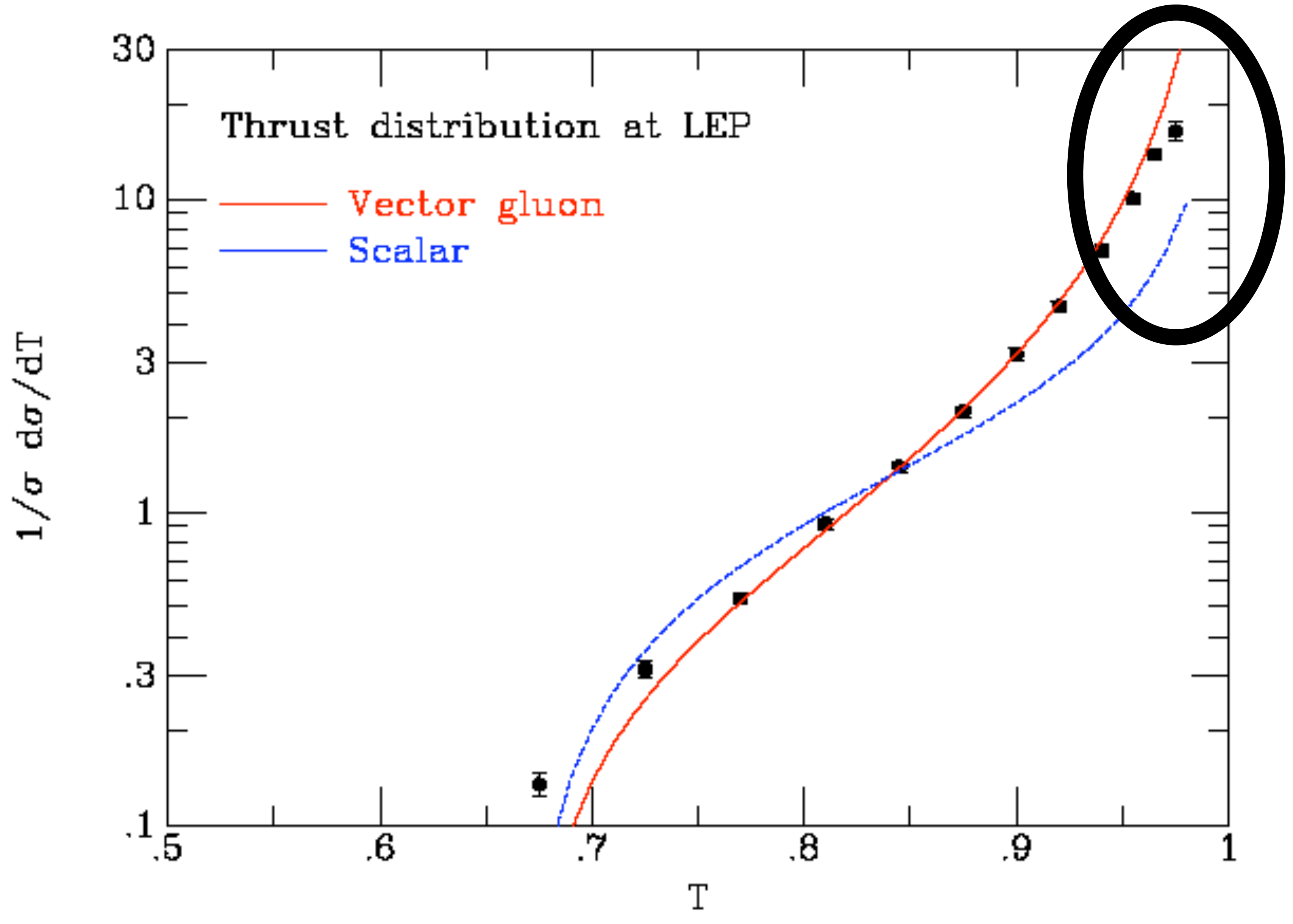
Thrust

What happens in an $e^+e^- \to q\bar{q}g$ event?

$$T = \max_{\vec{\hat{n}}} \frac{\sum_i |\vec{p_i} \cdot \vec{\hat{n}}|}{\sum_i |\vec{p_i}|}$$

$$\frac{1}{\sigma}\frac{d\sigma}{dT} = C_F \frac{\alpha_S}{2\pi}\left[\frac{2(3T^2-3T+2)}{T(1-T)}\log\left(\frac{2T-1}{1-T}\right) - \frac{3(3T-2)(2-T)}{1-T}\right]$$

Divergent for T=1 Why?



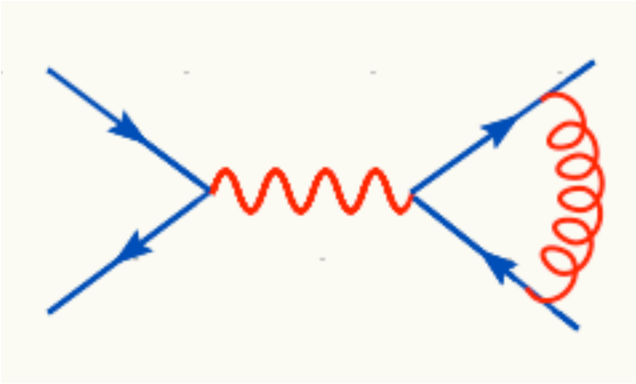
$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}T} \xrightarrow{T\to 1} -C_F\frac{\alpha_S}{2\pi}\left[\frac{4}{(1-T)}\ln(1-T) + \frac{3}{1-T}\right]$$

Large higher order terms of the form $\alpha_S^N \frac{\mathrm{Log}^{2N-1}(1-T)}{1-T}$ need to be resummed.

Use either analytic resummation or the parton shower! See later!

Asymptotic freedom

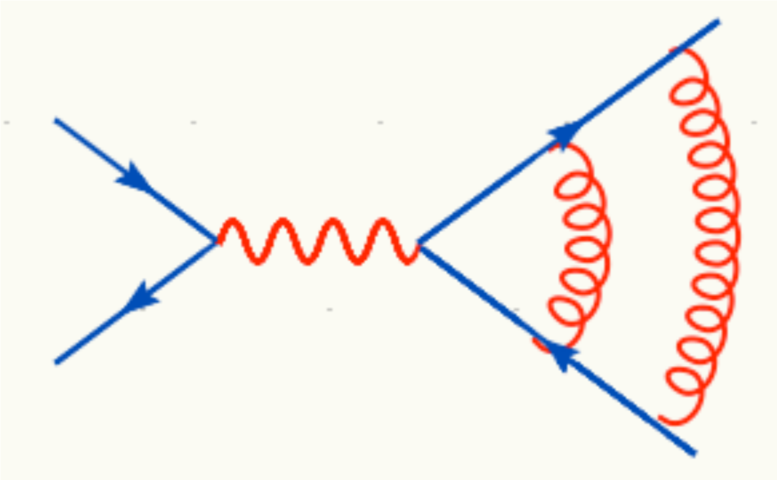
How about the UV?



$$R_1 = R_0\left(1+\frac{\alpha_S}{\pi}\right)$$

No divergences!

What happens at higher orders?



$$R^{(2)} = R^{(0)}\left(1+\frac{\alpha_S}{\pi}+\left(\frac{\alpha_S}{\pi}\right)^2\left(c+\pi b_0\log\left(\frac{M_{\rm UV}^2}{Q^2}\right)\right)\right) \qquad b_0 = \frac{11N_c-4n_fT_R}{12\pi}$$

UV divergences don't cancel! We need renormalisation!

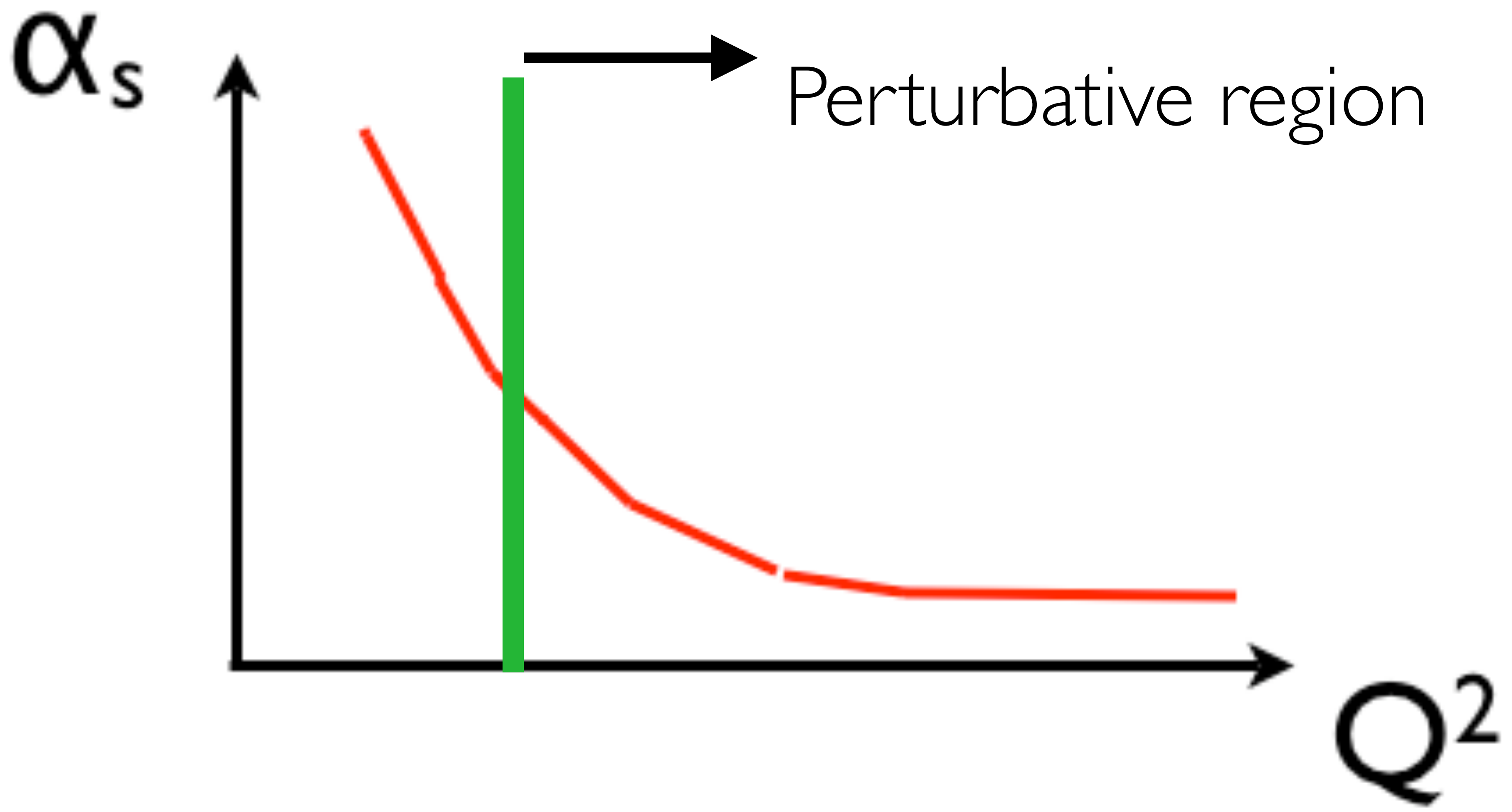
Renormalising the bare coupling we have:

$$\alpha_S(\mu) = \alpha_S^{\rm bare} + b_0\log\left(\frac{M_{\rm UV}^2}{\mu^2}\right)(\alpha_S^{\rm bare})^2$$

$$R_2^{\rm ren}\left(\alpha_S(\mu),\frac{\mu^2}{Q^2}\right) = R_0\left(1+\frac{\alpha_S(\mu)}{\pi}+\left[c+\pi b_0\log\frac{\mu^2}{Q^2}\right]\left(\frac{\alpha_S(\mu)}{\pi}\right)^2\right)$$

Finite but scale dependent!

Asymptotic freedom



QCD $b_0 = \frac{11N_c - 2n_f}{12\pi}$ $\Rightarrow$ $\beta(\alpha_S) < 0$

QED $b_0 = -\frac{n_f}{3\pi}$ $\Rightarrow$ $\beta(\alpha_{\rm EM}) > 0$

$$\mu^2 \frac{d\alpha}{d\mu^2} = \beta(\alpha) = -(b_0\alpha^2 + b_1\alpha^3 + b_2\alpha^4 + \cdots)$$

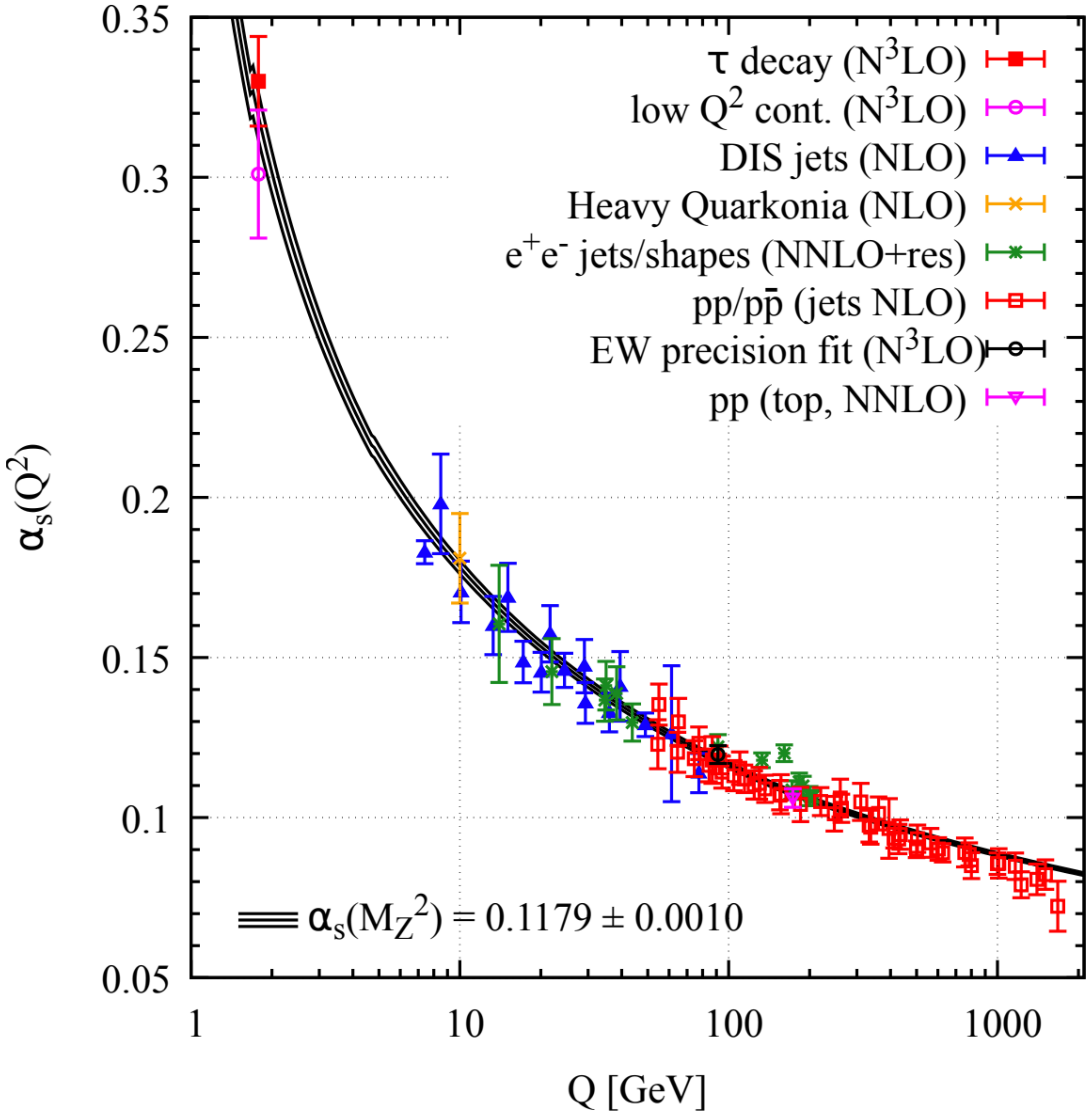
1-loop

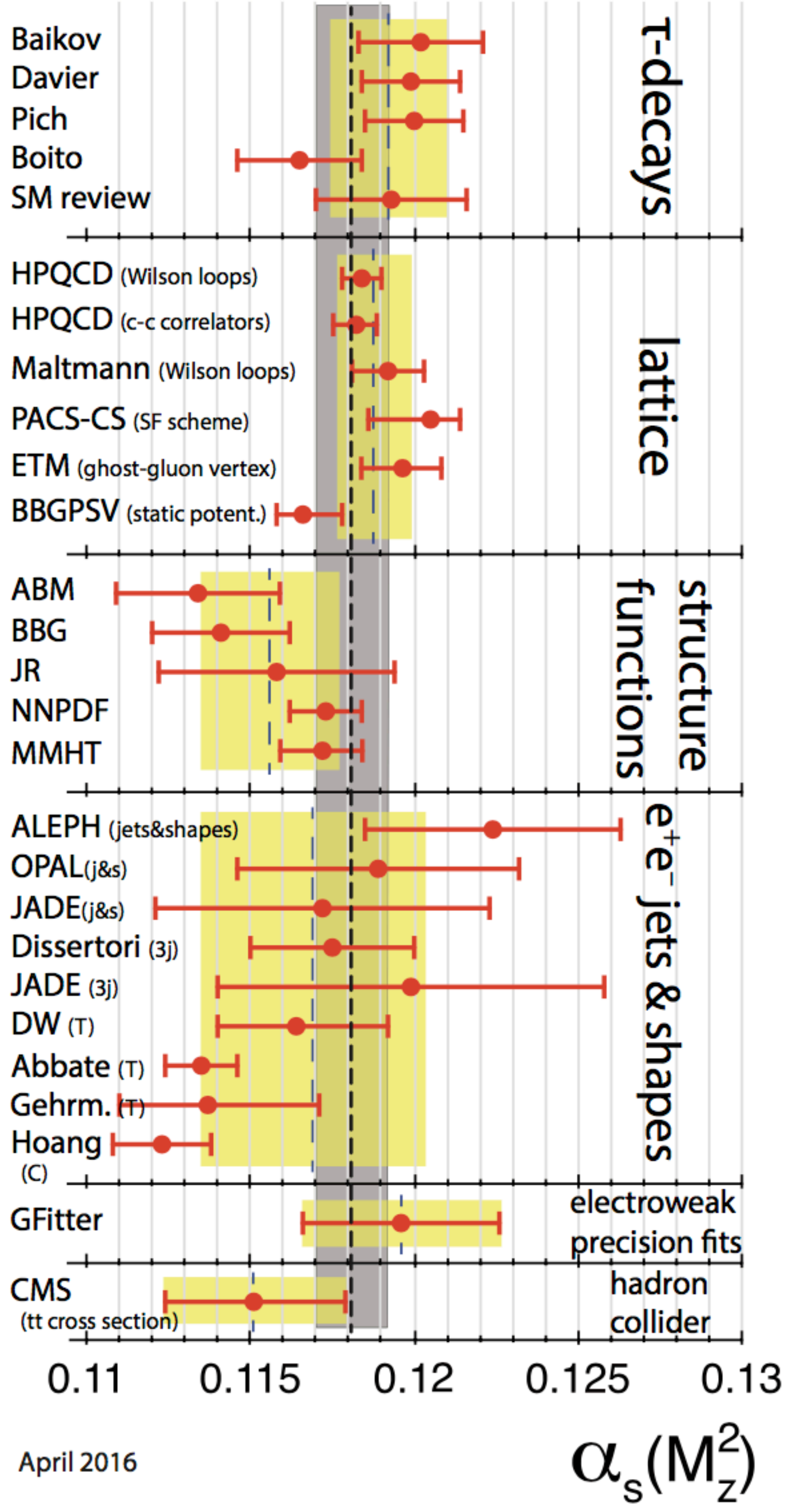
$$\beta(\alpha_S) \equiv \mu^2 \frac{\partial \alpha_S}{\partial \mu^2} = -b_0 \alpha_S^2 \quad \Rightarrow \quad \alpha_S(\mu) = \frac{1}{b_0 \log \frac{\mu^2}{\Lambda^2}}$$

2-loop

$$\alpha_S(\mu) = \frac{1}{b_0 \log \frac{\mu^2}{\Lambda^2}} \left[1 - \frac{b_1}{b_0^2} \frac{\log \log \mu^2/\Lambda^2}{\log \mu^2/\Lambda^2} \right]$$

Running of α_s





Many measurements at different scales all leading to very consistent results once evolved to the same reference scale, M_Z.

Going back to the Master formula

$$\sum_{a,b} \int dx_1 dx_2 \, d\Phi_{PS} \, f_a(x_1) f_b(x) \, \hat{\sigma}(\hat{s})$$

$$\Big\downarrow$$

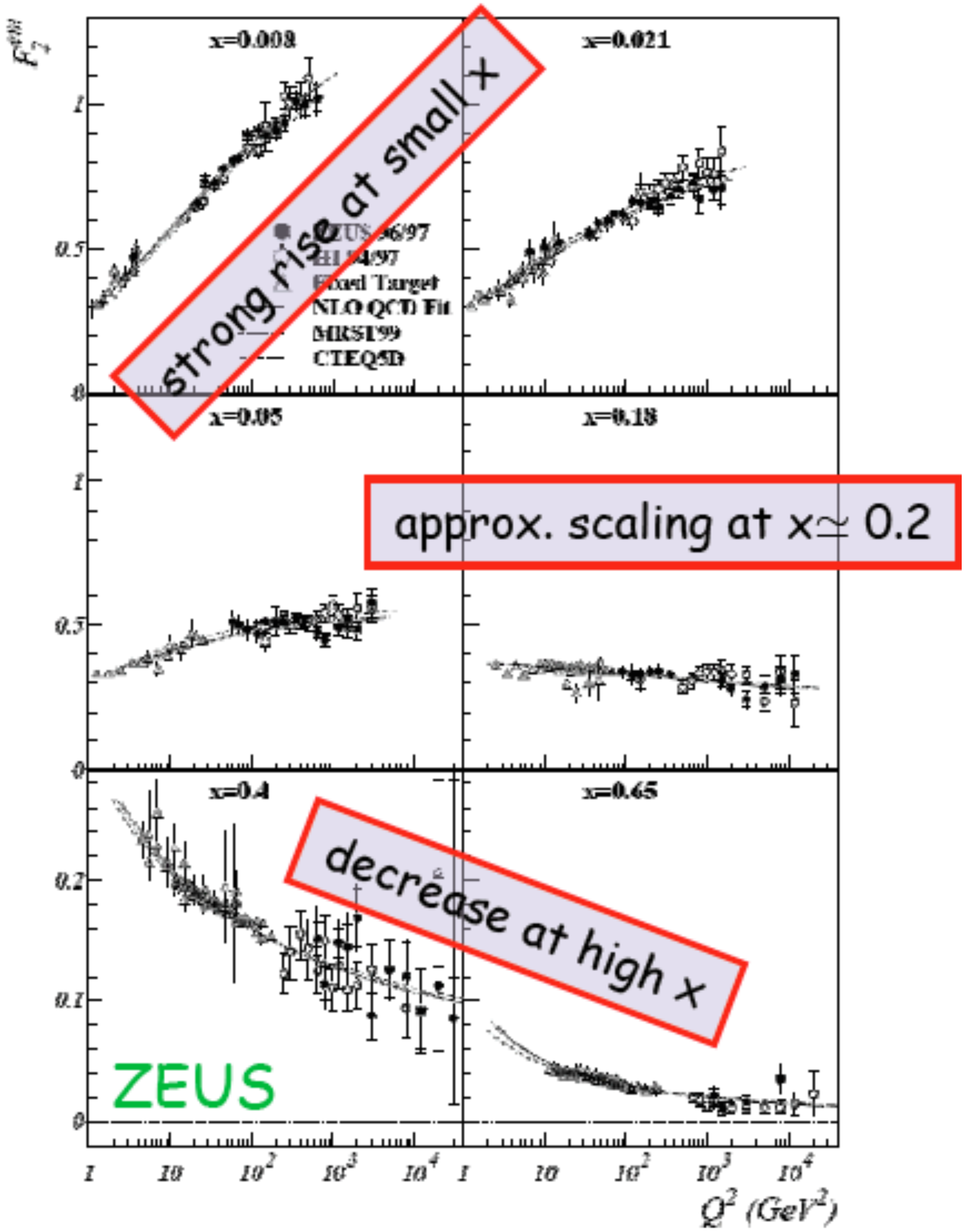
$$\sum_{a,b} \int dx_1 dx_2 \, d\Phi_{PS} \, f_a(x_1) f_b(x) \, \hat{\sigma}(\hat{s}, \mu_R)$$

$$\Big\downarrow \; \textbf{???}$$

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\mathrm{FS}} \, f_a(x_1, \mu_F) f_b(x_2, \mu_F) \, \hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$$

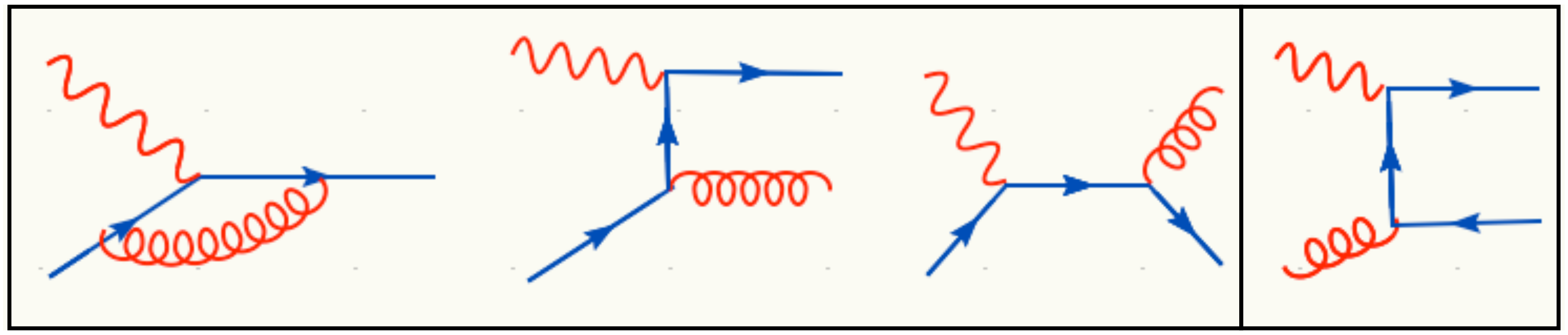
QCD improved parton model

The parton model predicts scaling. Experiment shows:



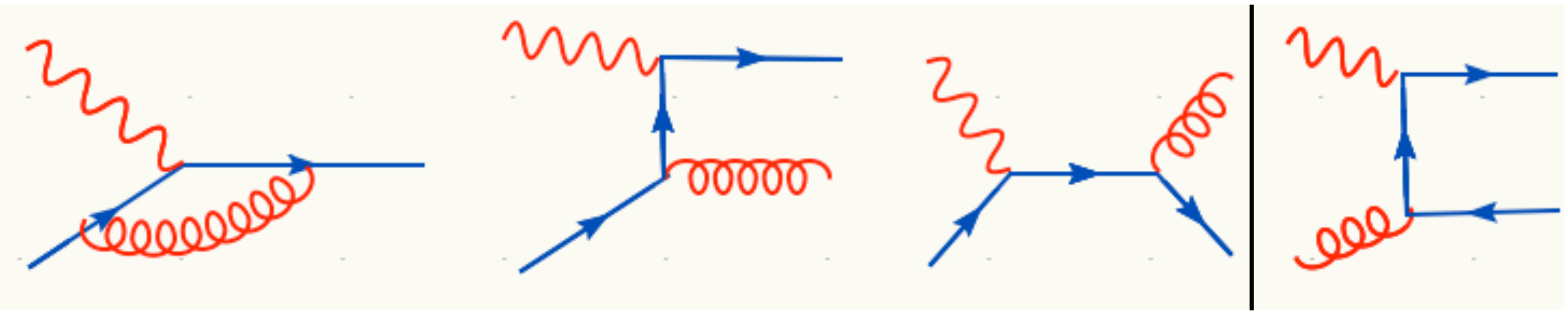
Scaling violation

What are we missing?



α_S corrections to the LO process photon-gluon fusion

QCD improved parton model

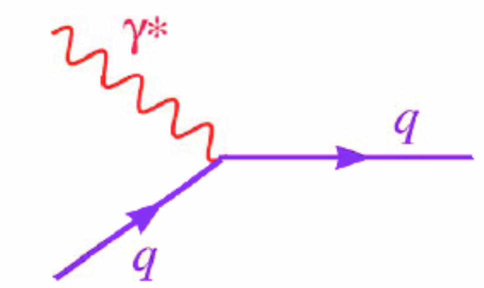


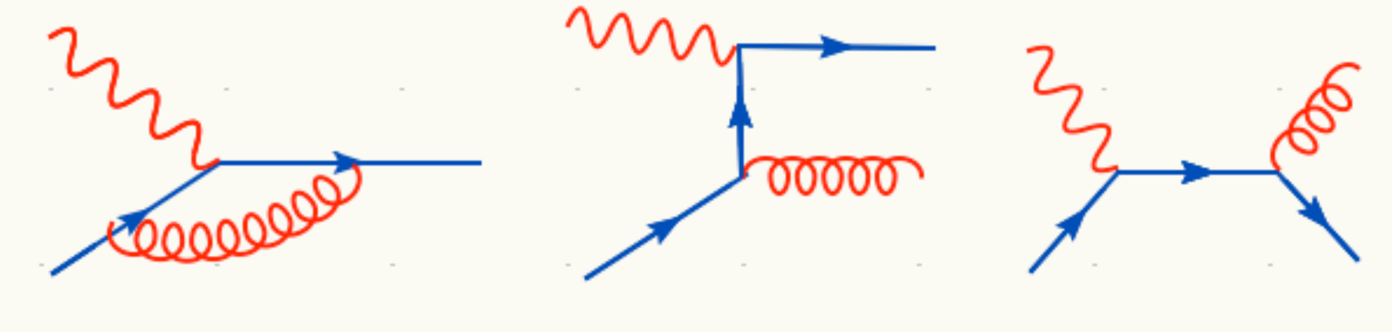
What do we expect?

Given the computation of R at NLO, we expect IR divergences

We need to regulate these, and hope that they cancel!

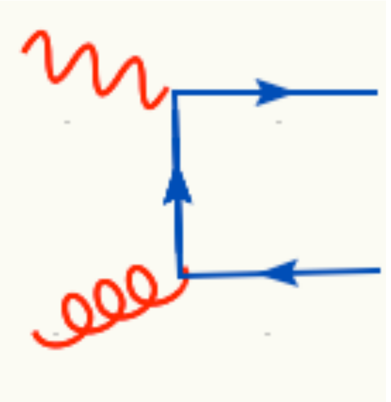
$$\frac{d^2\hat{\sigma}}{dx dQ^2}\Big|_{F_2} \equiv \hat{F}_2^q$$



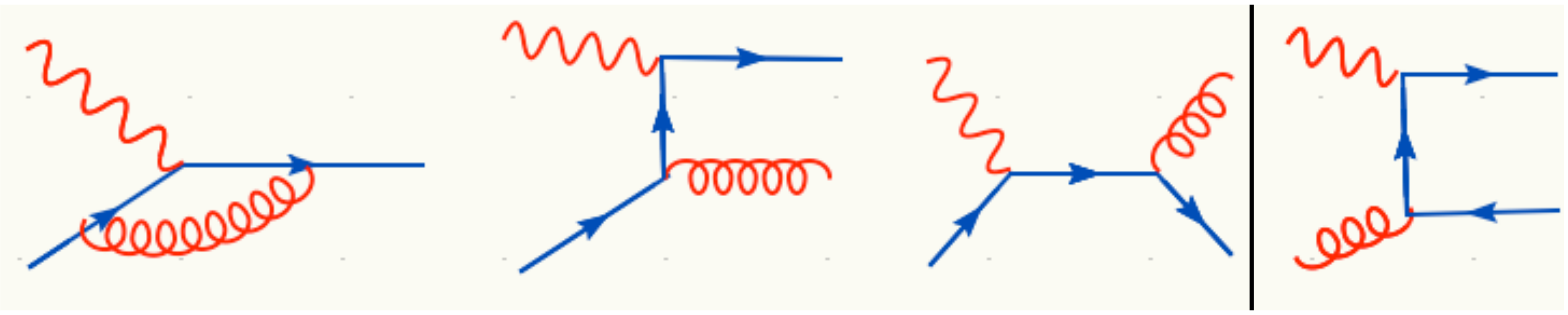


Soft and UV divergences cancel but a collinear divergence arises:

$$\hat{F}_2^q = e_q^2 x[\delta(1-x) + \frac{\alpha_s}{4\pi} P_{qq} \log\frac{Q^2}{m_g^2} + C_2^q(x)] \qquad \hat{F}_2^g = e_q^2 x[0 + \frac{\alpha_s}{4\pi} P_{qg} \log\frac{Q^2}{m_g^2} + C_2^g(x)]$$



QCD improved parton model



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IR cut-off

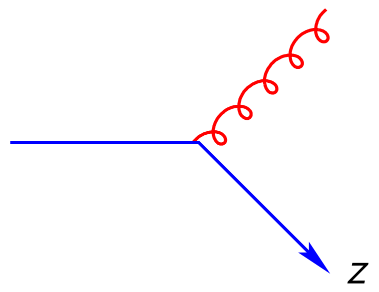
What are functions P_{qq} and P_{qg}?

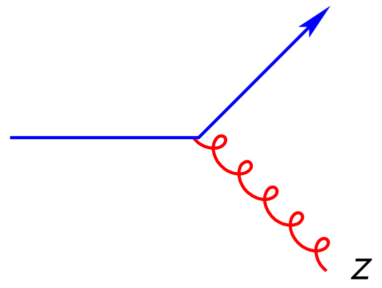
Splitting functions $P_{ij}(x)$: they give the probability of parton j splitting into parton i which carries momentum fraction x of the original parton

Altarelli-Parisi Splitting functions

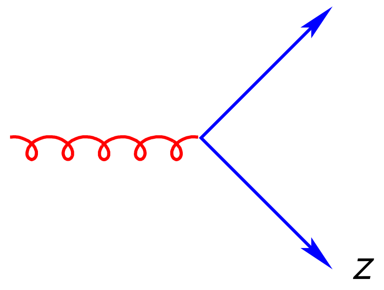
Branching has a universal form given by the Altarelli-Parisi splitting functions

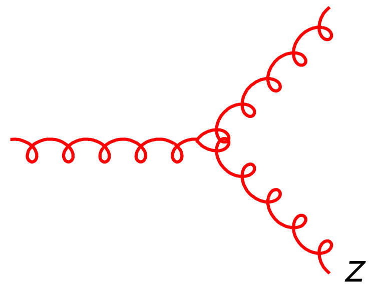
$$P_{q\to qg}(z) = C_F\left[\frac{1+z^2}{1-z}\right], \qquad P_{q\to gq}(z) = C_F\left[\frac{1+(1-z)^2}{z}\right].$$





$$P_{g\to qq}(z) = T_R\left[z^2+(1-z)^2\right], \qquad P_{g\to gg}(z) = C_A\left[z(1-z)+\frac{z}{1-z}+\frac{1-z}{z}\right]$$





These functions are universal for each type of splitting

What does this collinear divergence mean?

Residual long-distance physics, not disappearing once real and virtual corrections are added. These appear along with the universal splitting functions.

Can a physical observable be divergent?

No, as the physical observable is the hadronic structure function:

$$F_2^q(x,Q^2) = x \sum_{i=q,\bar{q}} e_q^2 \left[f_{i,0}(x) + \frac{\alpha_S}{2\pi} \int_x^1 \frac{d\xi}{\xi} f_{i,0}(\xi) \left[P_{qq}(\frac{x}{\xi}) \log \frac{Q^2}{m_g^2} + C_2^q(\frac{x}{\xi}) \right] \right]$$

We can absorb the dependence on the IR cutoff into the PDF:

$$f_q(x,\mu_f) \equiv f_{q,0}(x) + \frac{\alpha_S}{2\pi} \int_x^1 \frac{d\xi}{\xi} f_{q,0}(\xi) P_{qq}(\frac{x}{\xi}) \log \frac{\mu_f^2}{m_g^2} + z_{qq}$$

Renormalised PDFs!

Factorisation

Structure function is a measurable object and cannot depend on scale at all orders (renormalisation group invariance)

$$F_2^q(x,Q^2) = x \sum_{i=q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} f_i(\xi,\mu_f^2) \left[\delta(1-\frac{x}{\xi}) + \frac{\alpha_S(\mu_r)}{2\pi}\left[P_{qq}(\frac{x}{\xi})\log\frac{Q^2}{\mu_f^2} + (C_2^q - z_{qq})(\frac{x}{\xi})\right]\right]$$

Long distance physics is universally factorised into the PDFs, which now depend on μ_f. PDFs are not calculable in perturbation theory. PDFs are universal, they don't depend on the process.

Factorisation scale μ_f acts as a cut-off, emissions below μ_f are included in the PDFs.

DGLAP

We can't compute PDFs in perturbation theory but we can predict their evolution in scale:

$$\boxed{\mu^2 \frac{\partial f(x, \mu^2)}{\partial \mu^2} = \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P(z) f\left(\frac{x}{z}, \mu^2\right)}$$

Altarelli, Parisi; Gribov-Lipatov; Dokshitzer '77

Universality of splitting functions: we can measure pdfs in one process and use them as an input for another process

$$P_{ab}(\alpha_S, z) = \frac{\alpha_S}{2\pi} P_{ab}^{(0)}(z) + \left(\frac{\alpha_S}{2\pi}\right)^2 P_{ab}^{(1)}(z) + \left(\frac{\alpha_S}{2\pi}\right)^3 P_{ab}^{(2)}(z) + \ldots..$$

LO (1974) NLO (1980) NNLO (2004)

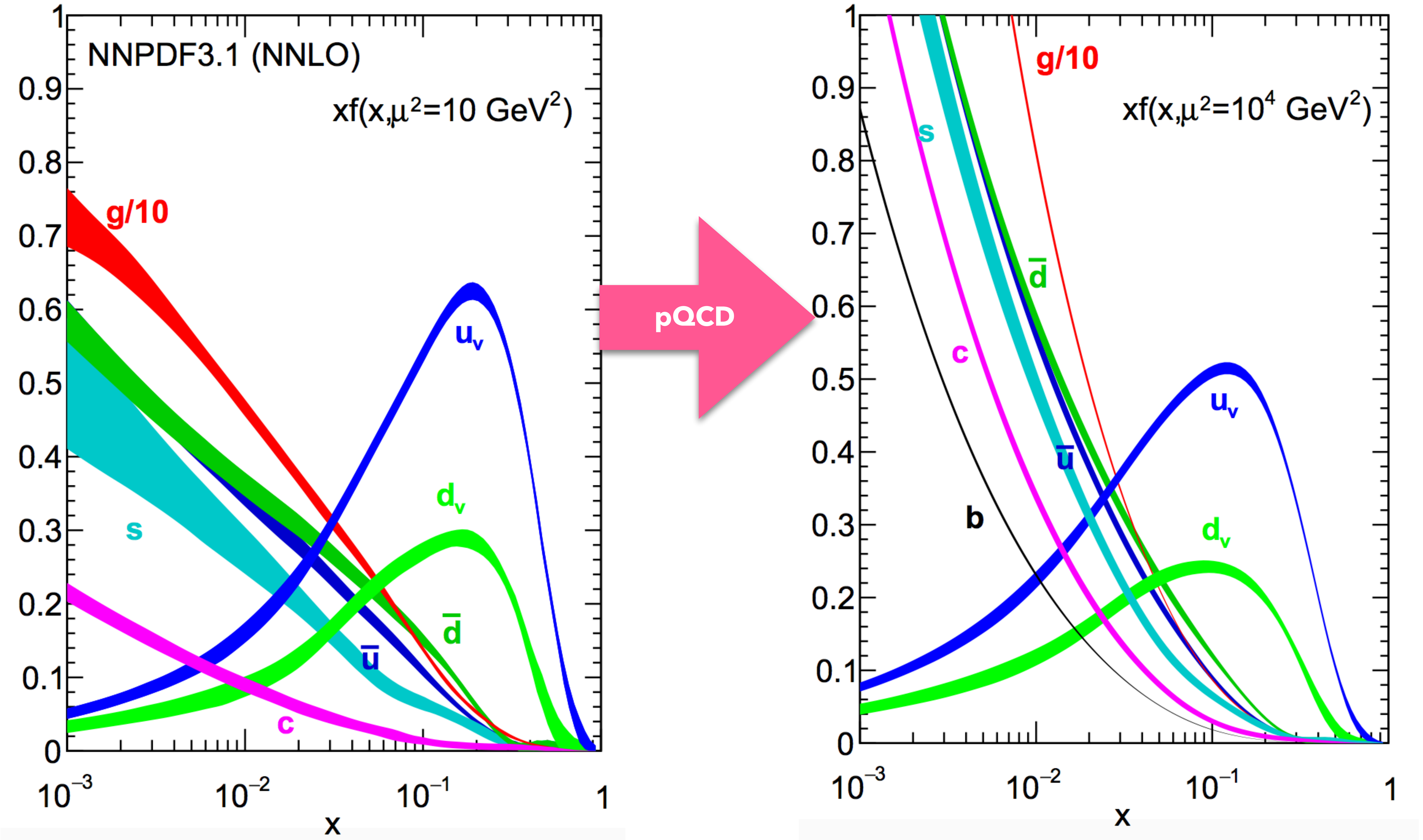
Splitting functions improved in perturbation theory!

LO Dokshitzer; Gribov, Lipatov; Altarelli, Parisi (1977)

NLO Floratos,Ross,Sachrajda; Floratos, Lacaze, Kounnas Gonzalez-Arroyo,Lopez,Yndurain; Curci,Furmanski Petronzio, (1981)

NNLO - Moch, Vermaseren, Vogt, 2004

PDF evolution



Collider Phenomenology (3)

Eleni Vryonidou

STFC school, Oxford
11-15/9/23

Plan for the lectures

- Basics of collider physics
- Basics of QCD
  - DIS and the Parton Model
  - Higher order corrections
  - Asymptotic freedom
  - QCD improved parton model
- State-of-the-art computations for the LHC
- Monte Carlo generators
- Higgs phenomenology
- Top phenomenology
- Searching for New Physics: EFT

LHC Master Formula

$$\sum_{a,b} \int dx_1 dx_2 \, d\Phi_{PS} \, f_a(x_1) f_b(x) \, \hat{\sigma}(\hat{s})$$

Parton model

$$\Big\downarrow \text{ Renormalisation}$$

$$\sum_{a,b} \int dx_1 dx_2 \, d\Phi_{PS} \, f_a(x_1) f_b(x) \, \hat{\sigma}(\hat{s}, \mu_R)$$

$$\Big\downarrow \text{ QCD improved parton model}$$

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\mathrm{FS}} \, f_a(x_1, \mu_F) f_b(x_2, \mu_F) \, \hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$$

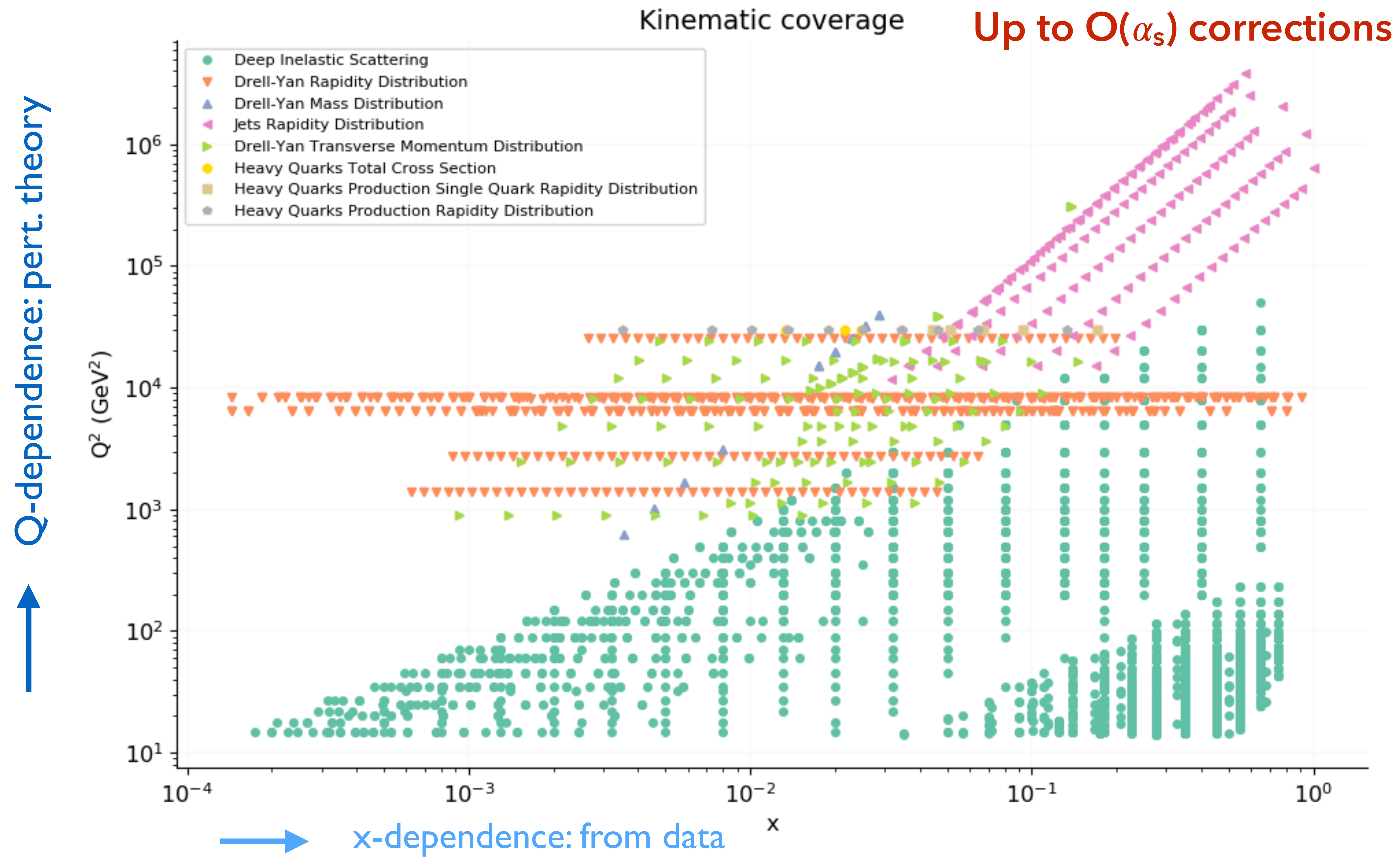
PDF extraction

We can't compute PDFs in perturbation theory but we can extract them from data, and use DGLAP equations to evolve them to different scales.

- Choose **experimental data** to fit and include all info on correlations

  Theory settings: perturbative order, EW corrections, intrinsic heavy quarks, α_s, quark masses value and scheme
- Choose a starting scale Q_0 where pQCD applies
- **Parametrise** independent quarks and gluon distributions at the starting scale
- Solve **DGLAP equations** from initial scale to scales of experimental data and build up observables
- **Fit** PDFs to data
- Provide PDF **error sets** to compute PDF uncertainties

Data for PDF determination



LHC kinematics

How can we tell which x data probes?

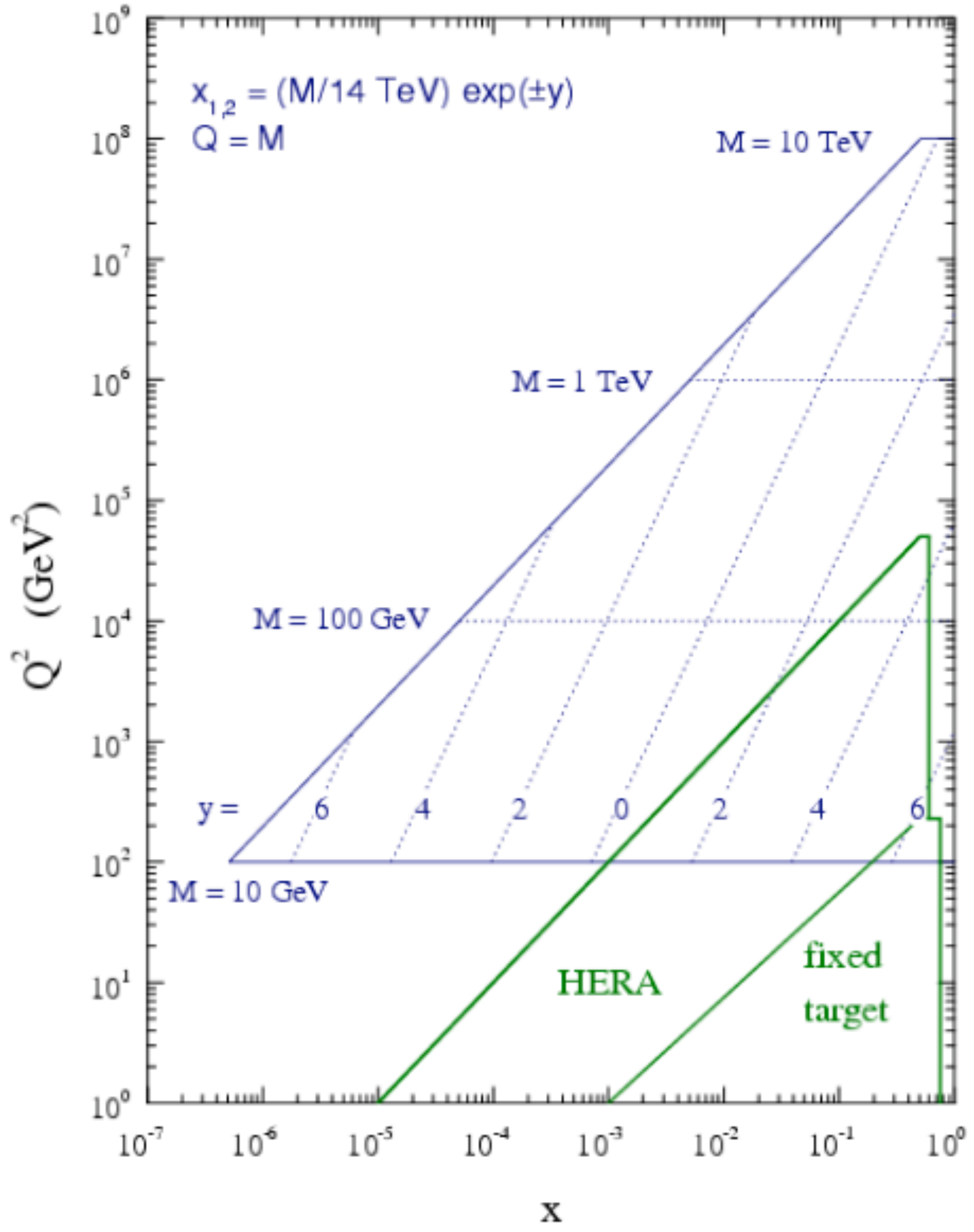
For the production of a particle of mass M:

$$M^2 = x_1 x_2 S = x_1 x_2 4E_{\text{beam}}^2$$

$$y = \frac{1}{2}\log\frac{x_1}{x_2}$$

$$x_1 = \frac{M}{\sqrt{S}}e^{y} \quad x_2 = \frac{M}{\sqrt{S}}e^{-y}$$

See exercises!



Data complementarity

GLUON

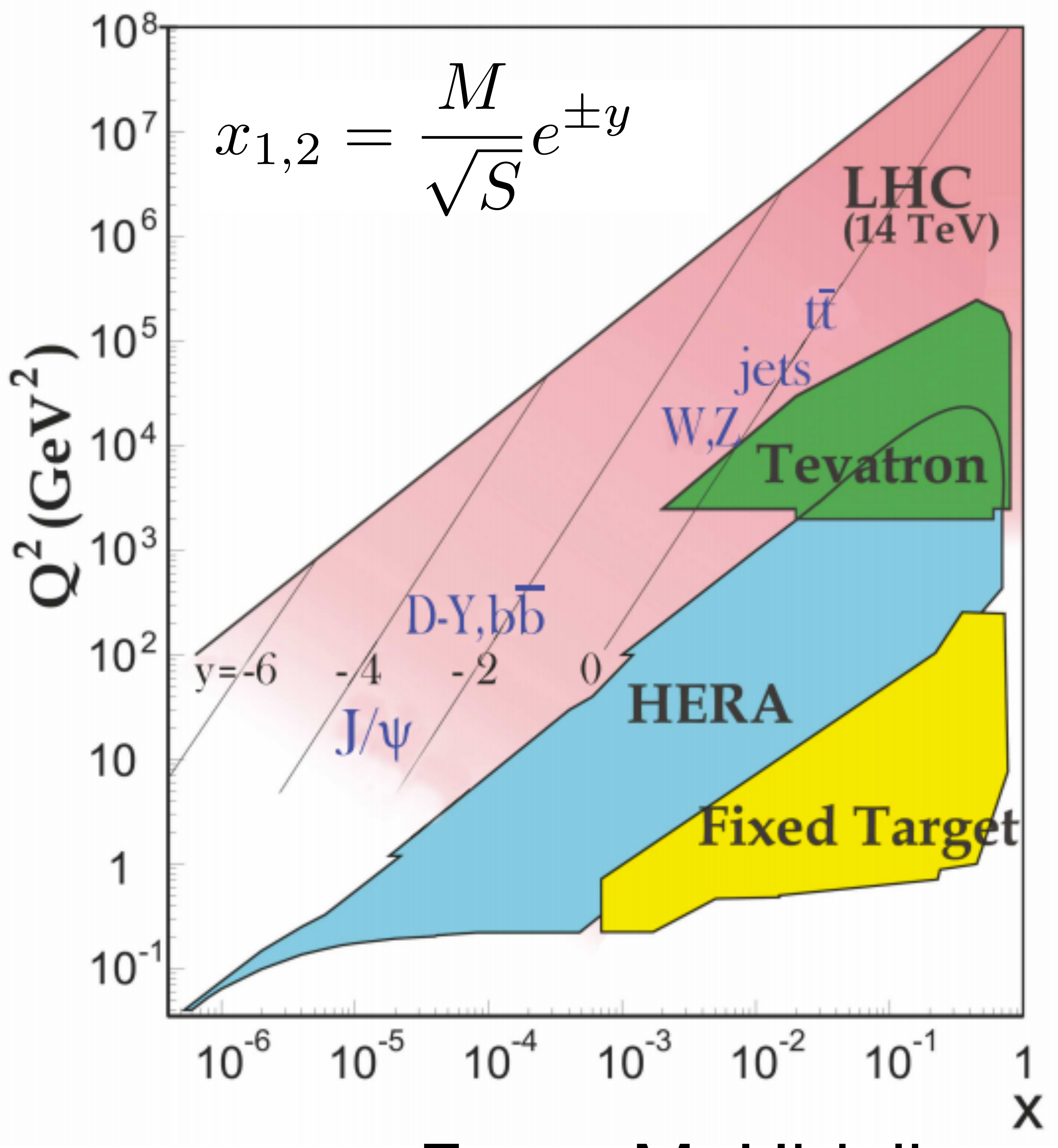
- Inclusive jets and dijets **(medium/large x)**
- Isolated photon and γ+jets **(medium/large x)**
- Top pair production **(large x)**
- High p_T V(+jets) distribution **(small/medium x)**

QUARKS

- High p_T W(+jets) ratios **(medium/large x)**
- W and Z production **(medium x)**
- Low and high mass Drell-Yan **(small and large x)**
- Wc **(strangeness at medium x)**

PHOTON

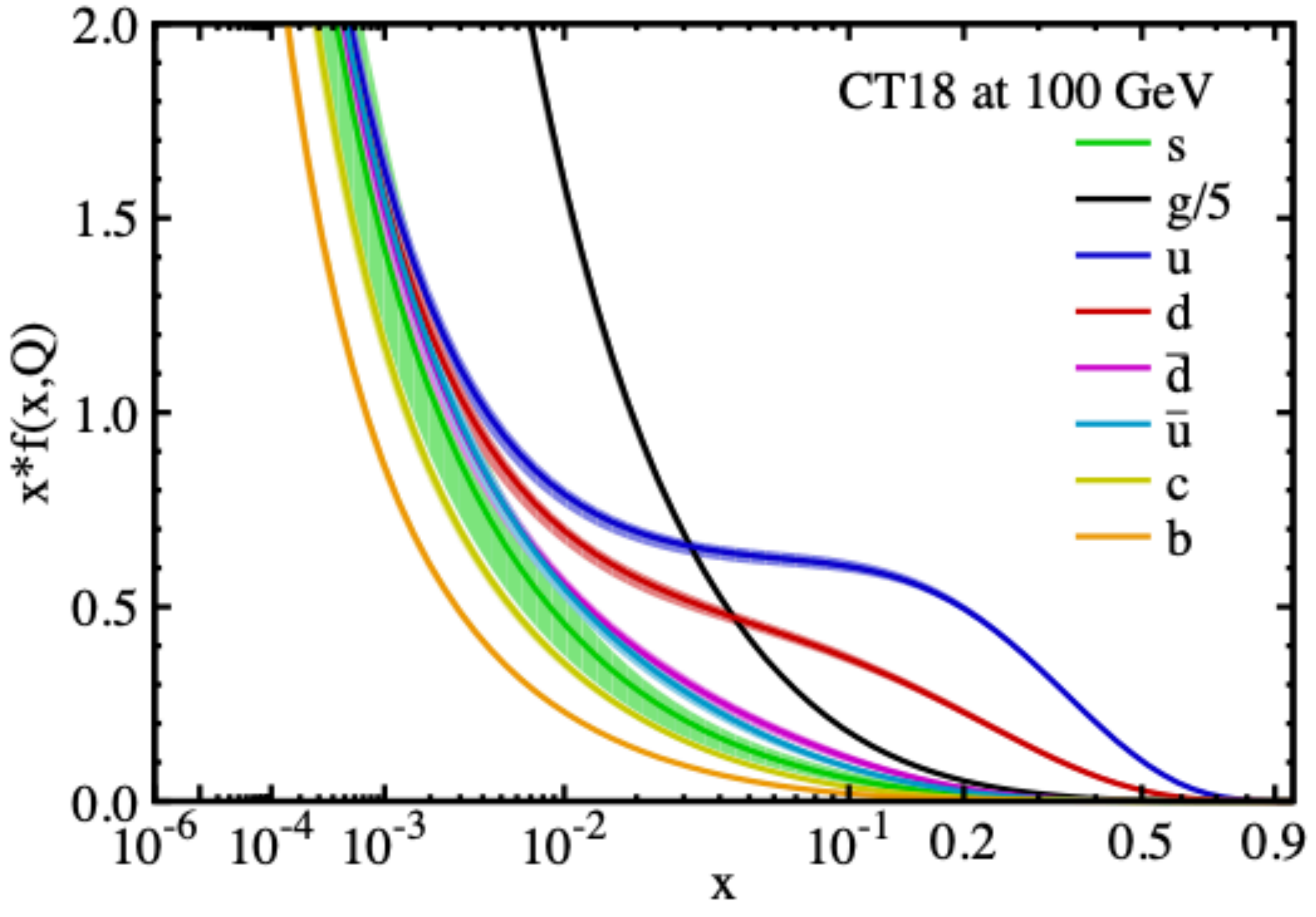
- Low and high mass Drell-Yan
- WW production



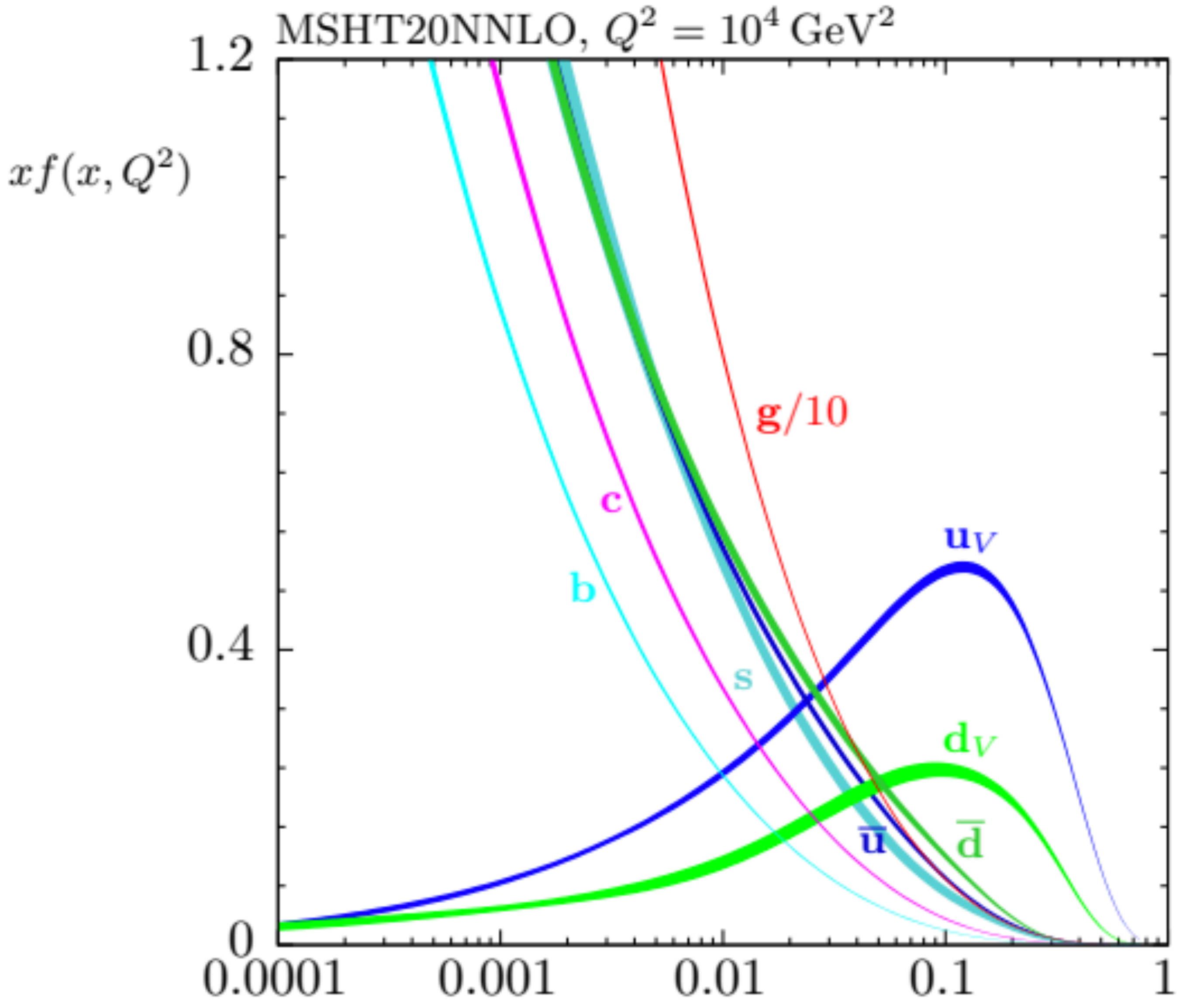
From. M. Ubiali

Modern PDFs

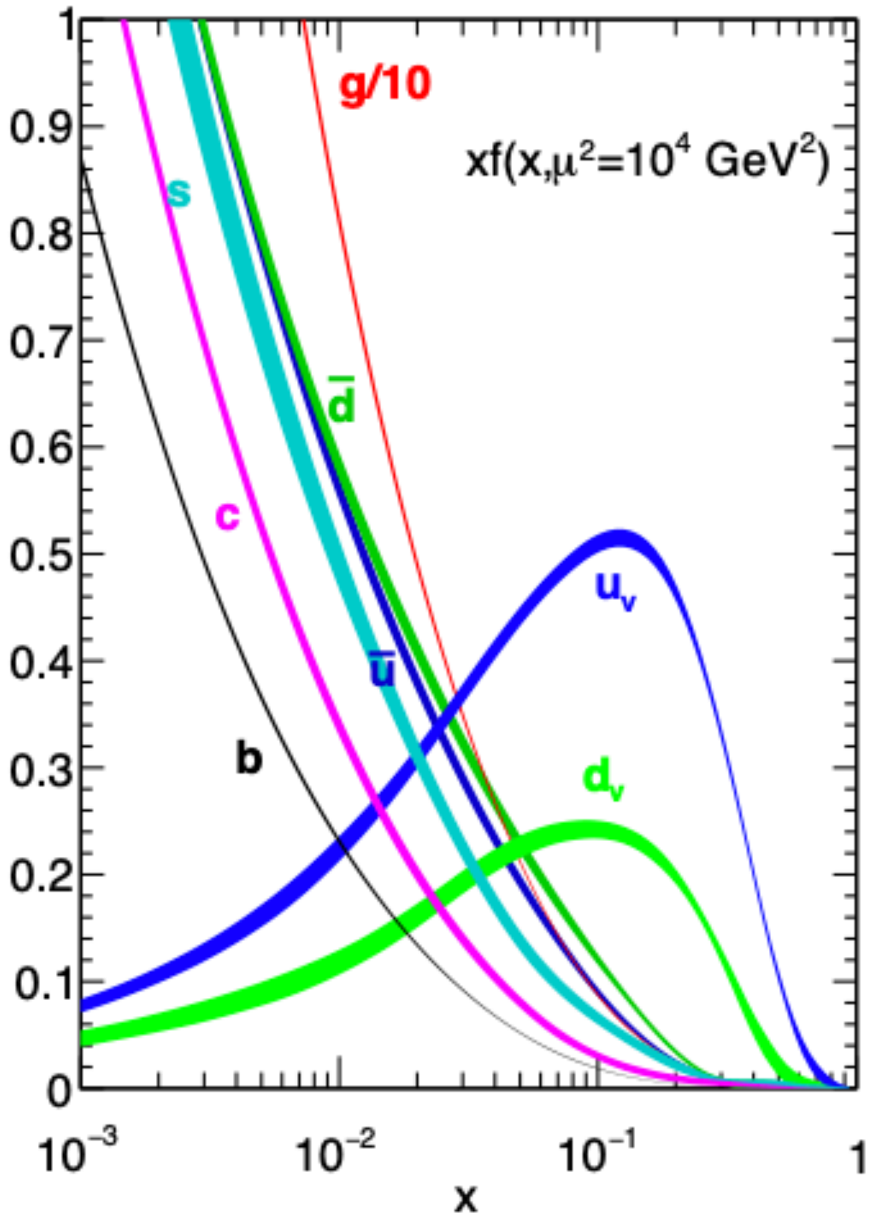
CT18



MSTH20

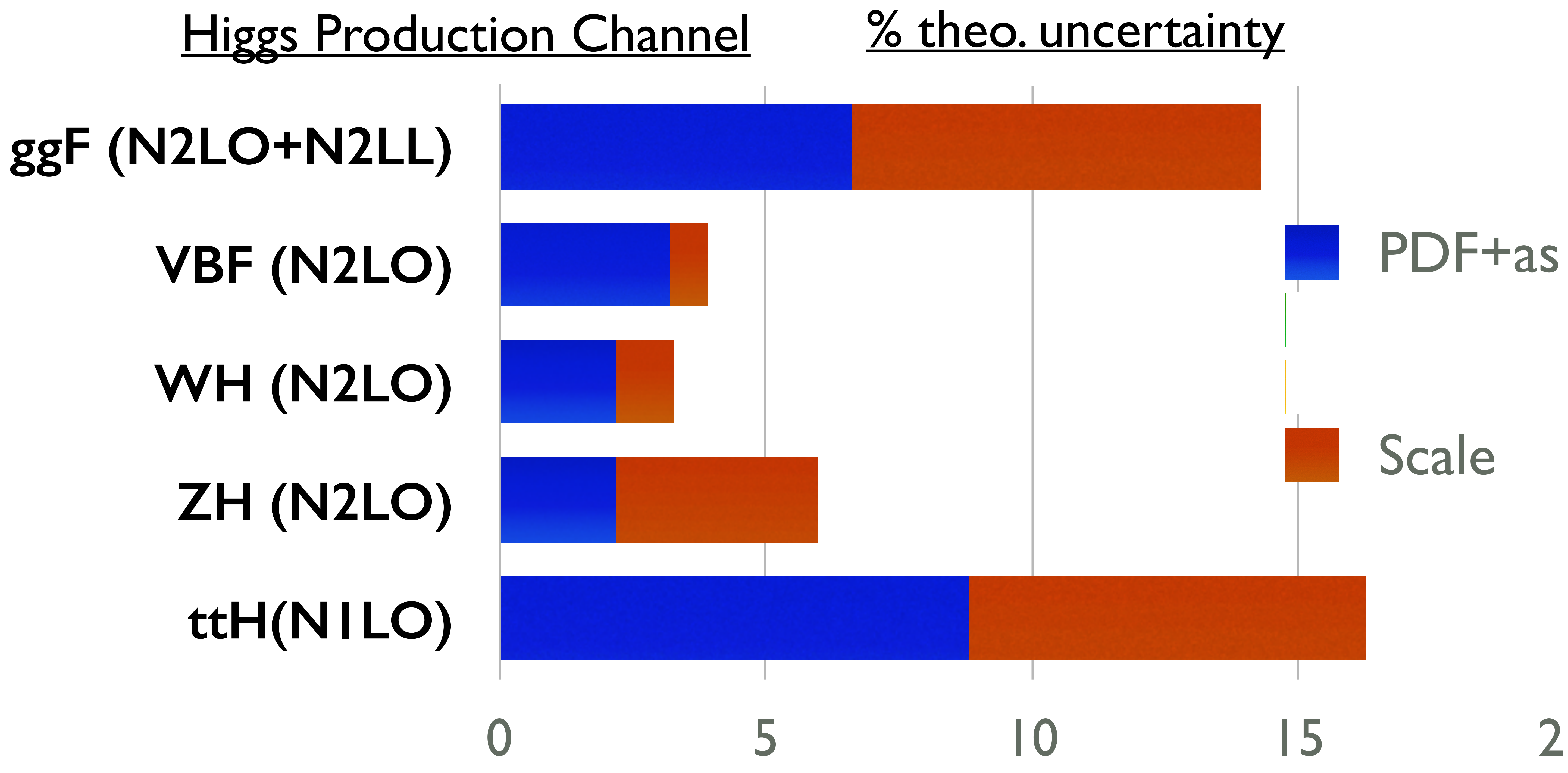


NNPDF3.1

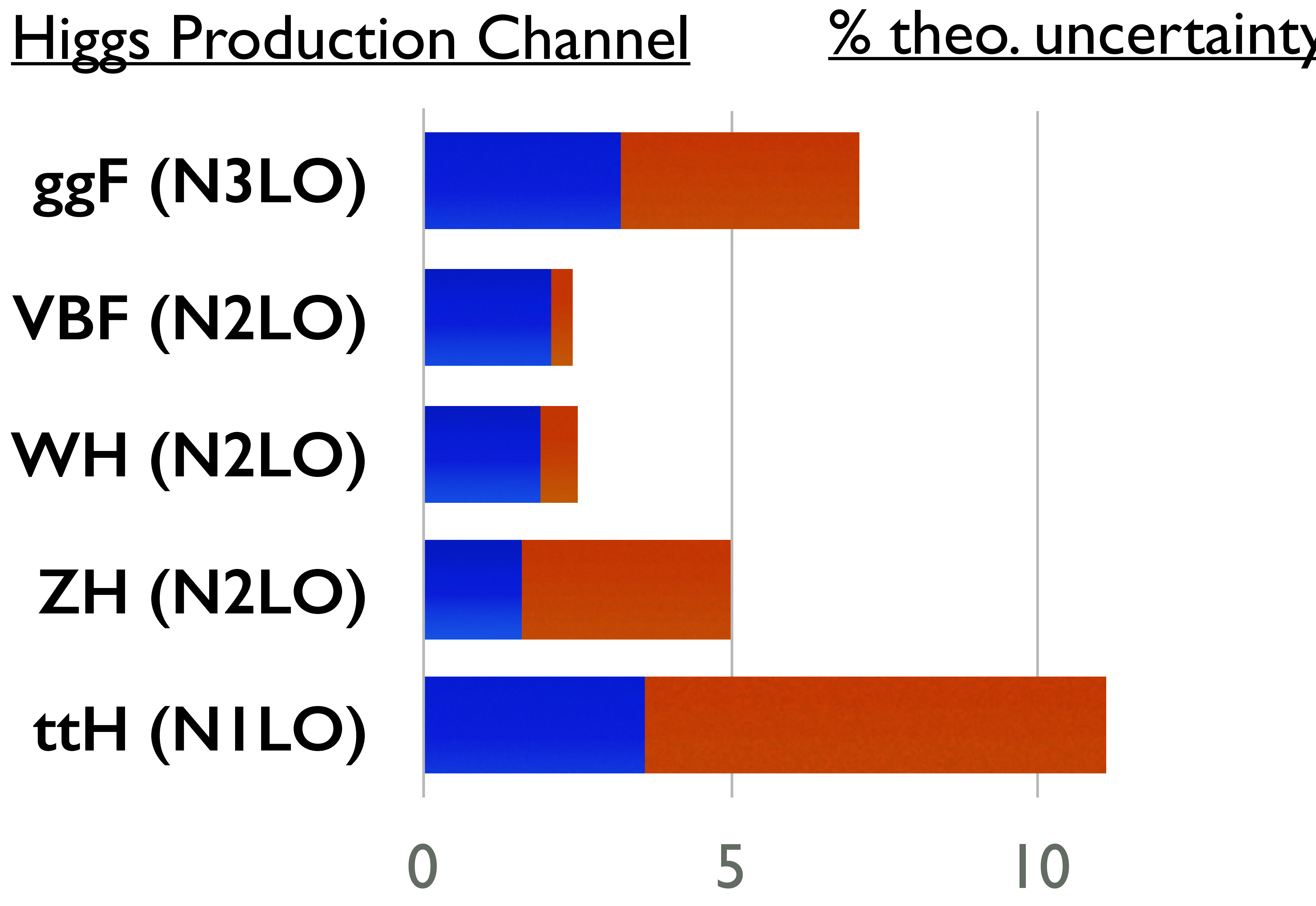


Different collaborations, predictions usually computed with different PDFs to extract an uncertainty envelope.

Impact of PDF uncertainties



Yellow Report 3 (2013)

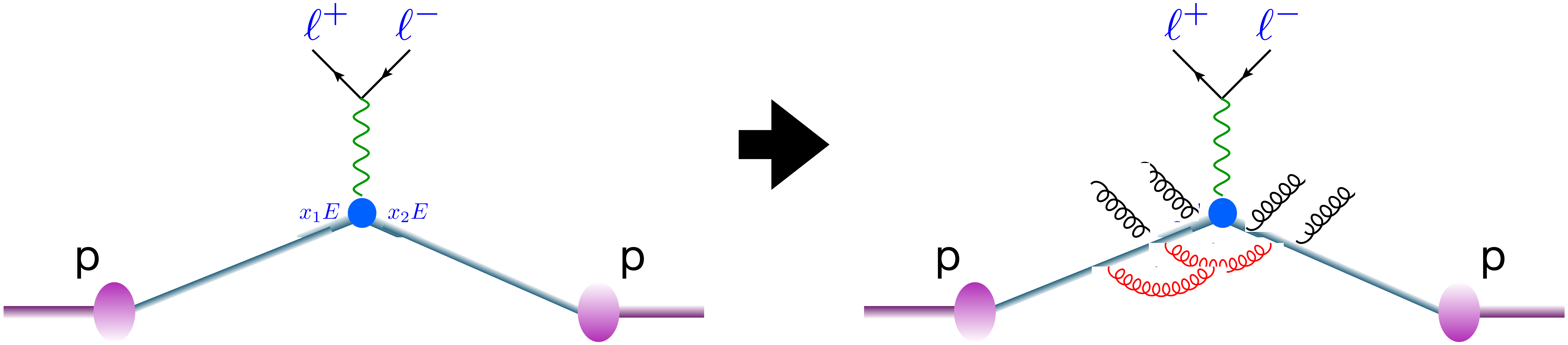


Yellow Report 4 (2016)

Progress in PDFs!

Fixed order computations

Going to higher orders



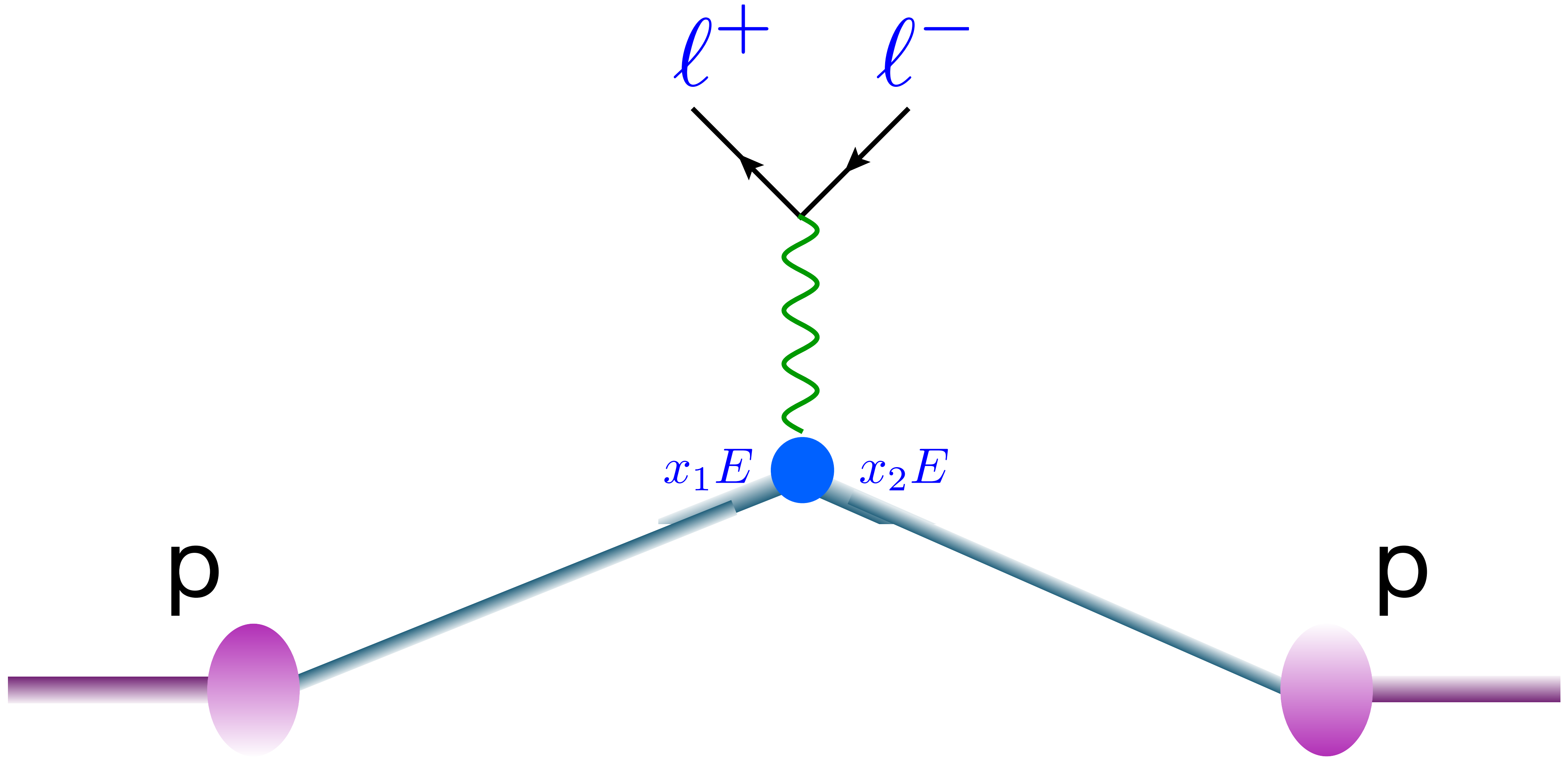
$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\mathrm{FS}}\, f_a(x_1, \mu_F) f_b(x_2, \mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$$

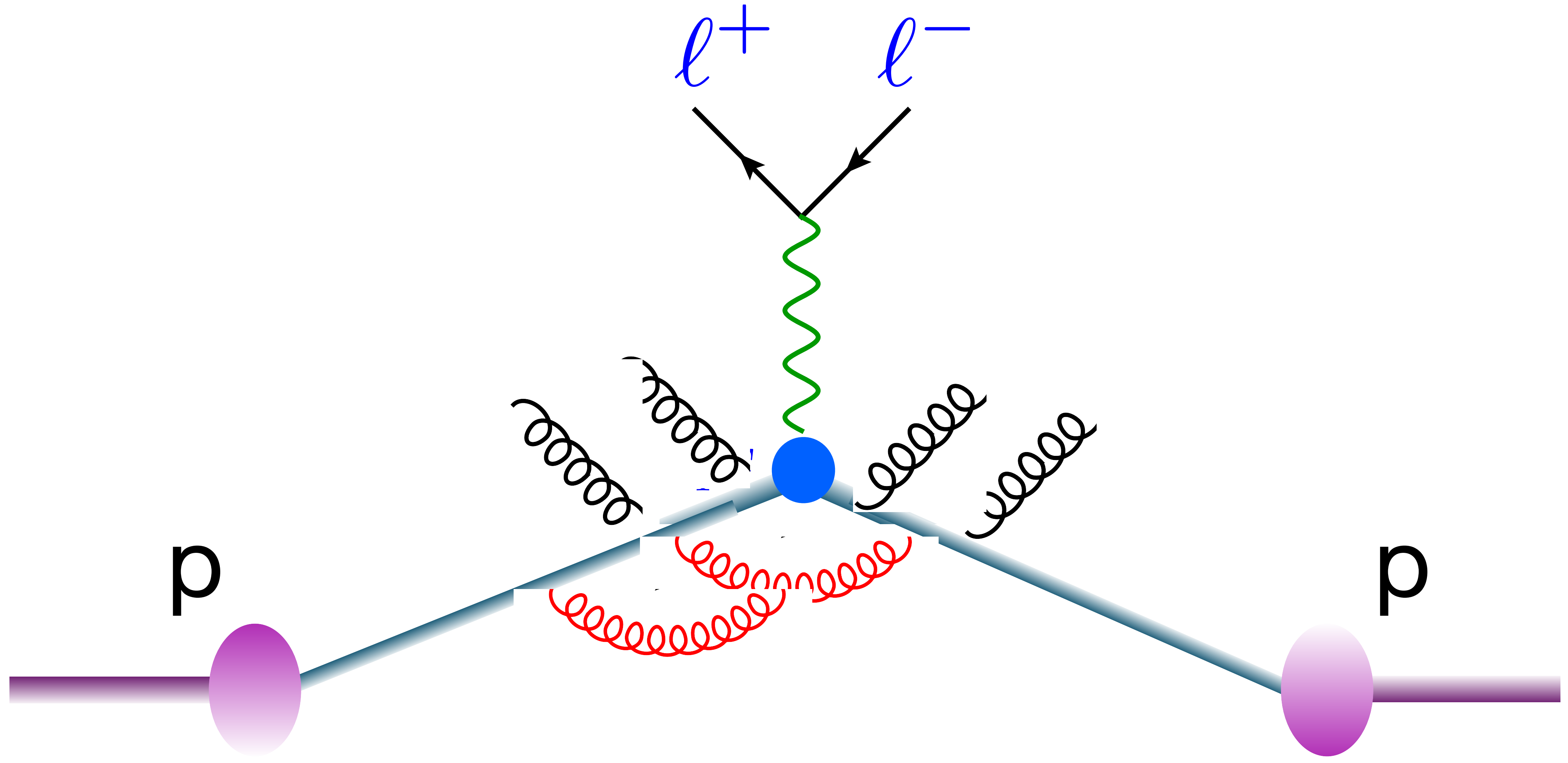
$$\hat{\sigma} = \sigma^{\mathrm{Born}} \left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3 \sigma^{(3)} + \ldots \right)$$

LO NLO NNLO N3LO

Fixed order computations

Going to higher orders

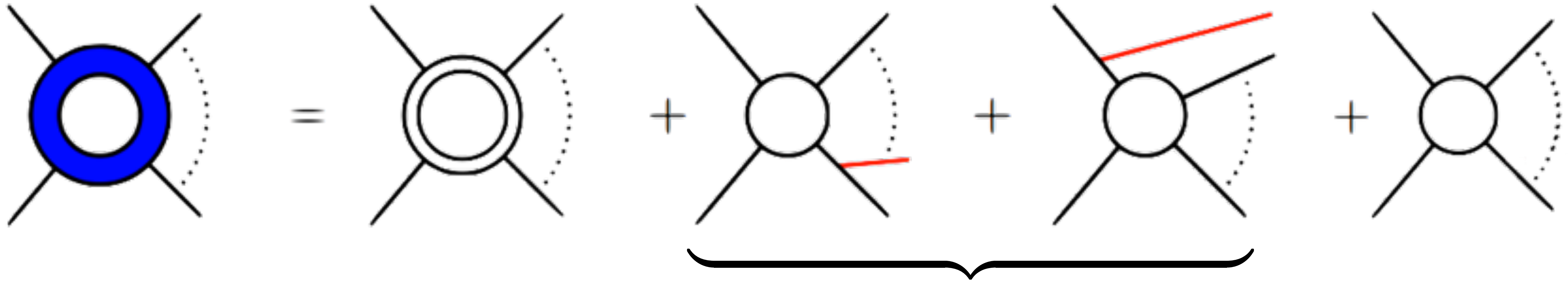




We need to add real and virtual corrections to the hard scattering dealing with singularities

Relatively straightforward at NLO (automated), complicated at NNLO (tens of processes), extremely hard at NNNLO (handful of processes known)

Structure of an NLO calculation



$$\sigma^{\text{NLO}} = \int_m d^{(d)}\sigma^V + \int_{m+1} d^{(d)}\sigma^R + \int_m d^{(4)}\sigma^B$$

Virtual part | Real emission part | Born

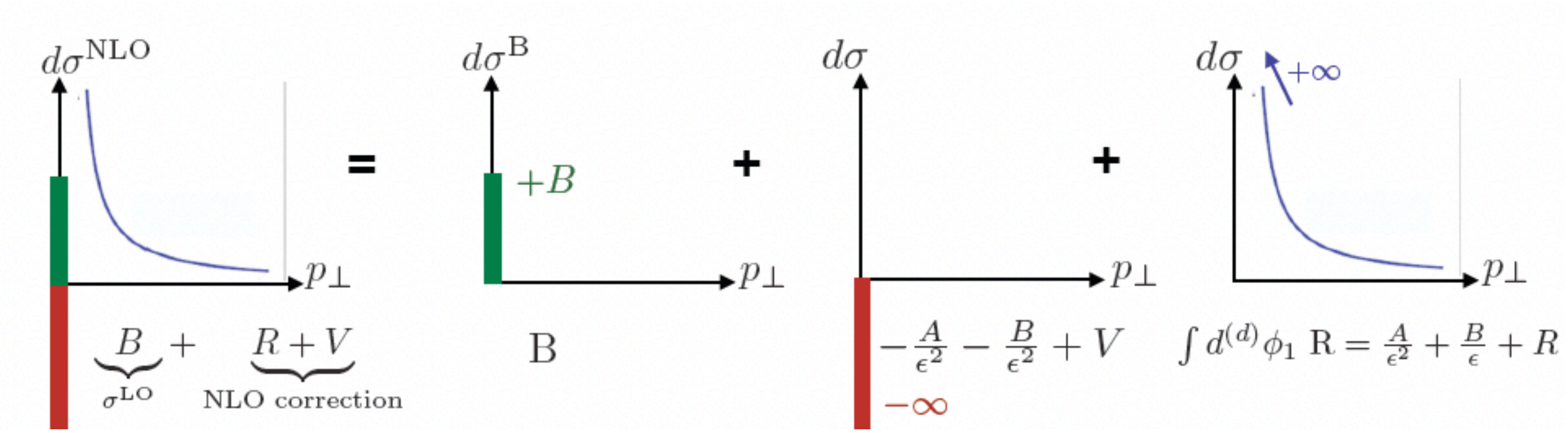
Difficulties:

- Loop calculations: tough and time consuming
- Divergences: Both real and virtual corrections are divergent
- More channels, more phase space integrations

Difficulty

How to deal with NLO in practice?

NLO corrections involve divergences: Divergences are bad for numerical computations



$$\sigma_{\mathrm{NLO}} = \int d\Phi^{(n)}\mathcal{B} + \int d\Phi^{(n)}\mathcal{V} + \int d\Phi^{(n+1)}\mathcal{R}$$

Subtraction:

$$= \int d\Phi^{(n)}\mathcal{B} + \int d\Phi^{(n)}\underbrace{\left[\mathcal{V} + \int d\Phi^{(1)} S\right]}_{\text{finite}} + \int d\Phi^{(n+1)}\underbrace{\left[\mathcal{R} - S\right]}_{\text{finite}}$$

Subtraction techniques at NLO

Dipole subtraction

- Catani, Seymour hep-ph/9602277
- Automated in MadDipole, Sherpa, HELAC-NLO

FKS subtraction

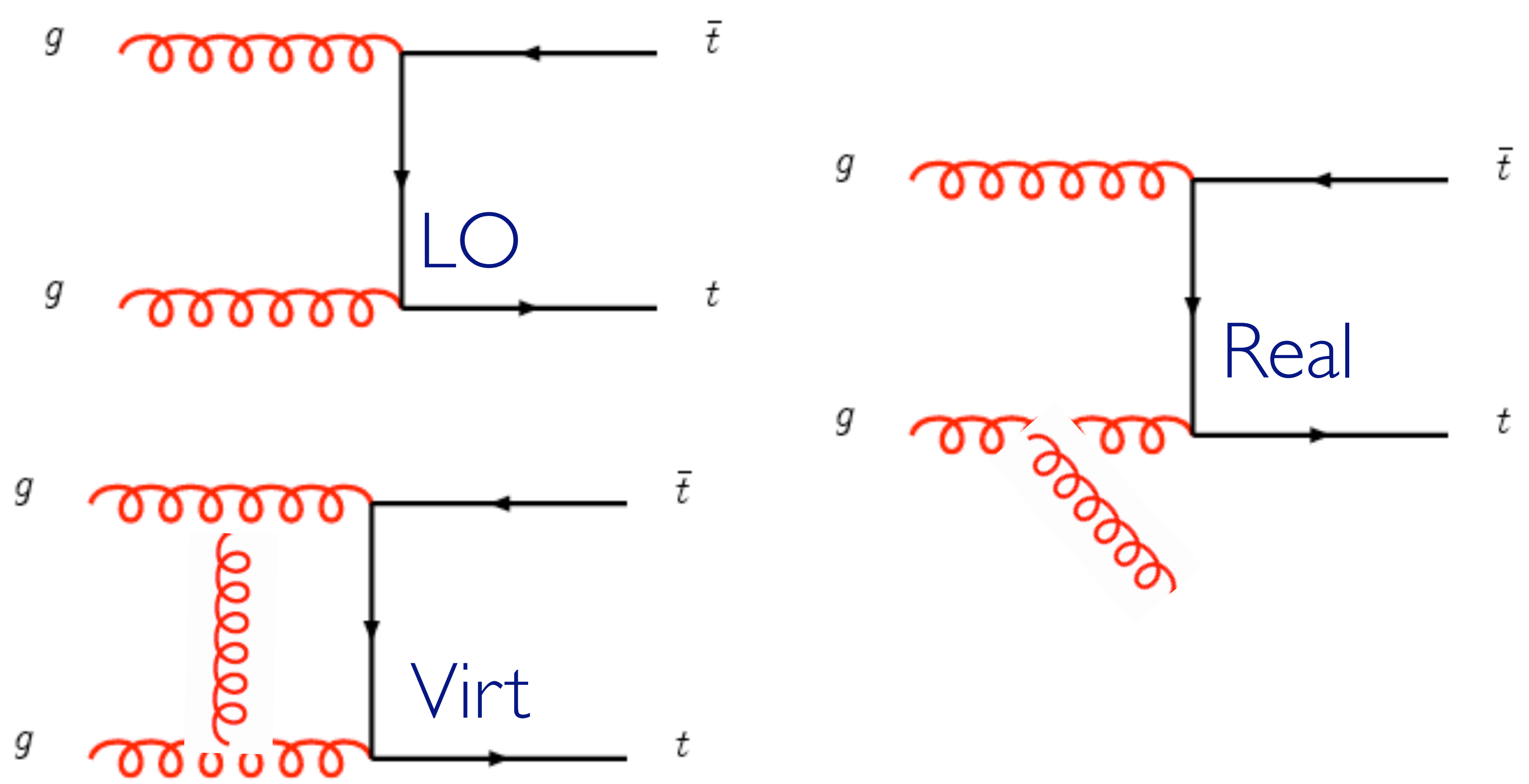
- Frixione, Kunszt, Signer hep-ph/9512328
- Automated in MadGraph5_aMC@NLO and Powheg/Powhel

Detailed discussion of these could be another lecture course!

A note about NLO

NLO is relative

Example: top pair production



NLO

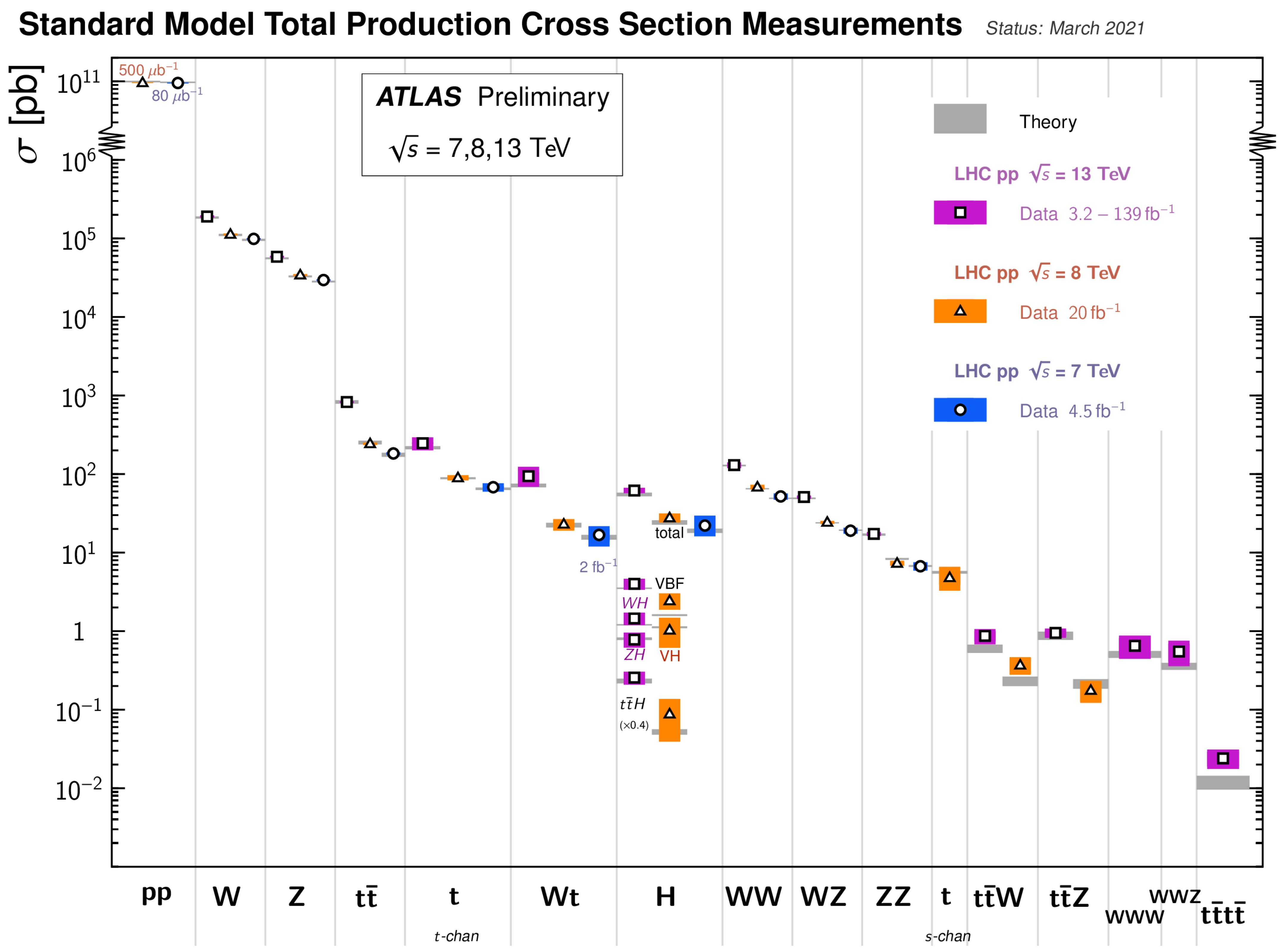
Which observables do we compute at NLO?

- Total cross-section ✓
- pT of a top quark ✓
- pT of top pair ✗
- pT of hardest jet ✗
- tt invariant mass ✓

It is certain observables which are computed at NLO

Need for higher-orders

Why is this so important?

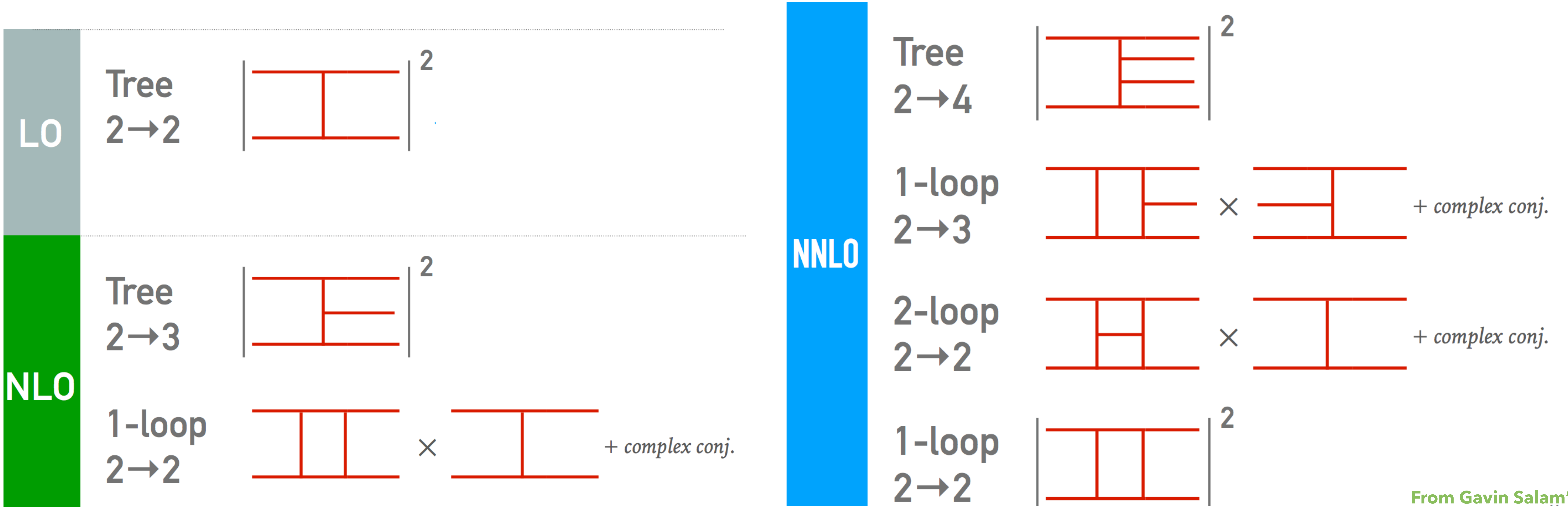


Reminder:

Level of experimental precision demands precise theoretical predictions

Theorists are not simply having fun!!!

Higher order computations



From Gavin Salam's lectures
Quy Nhon Vietnam 2018

Complexity rises a lot with each N!

Status of hard scattering cross-sections

LO automated

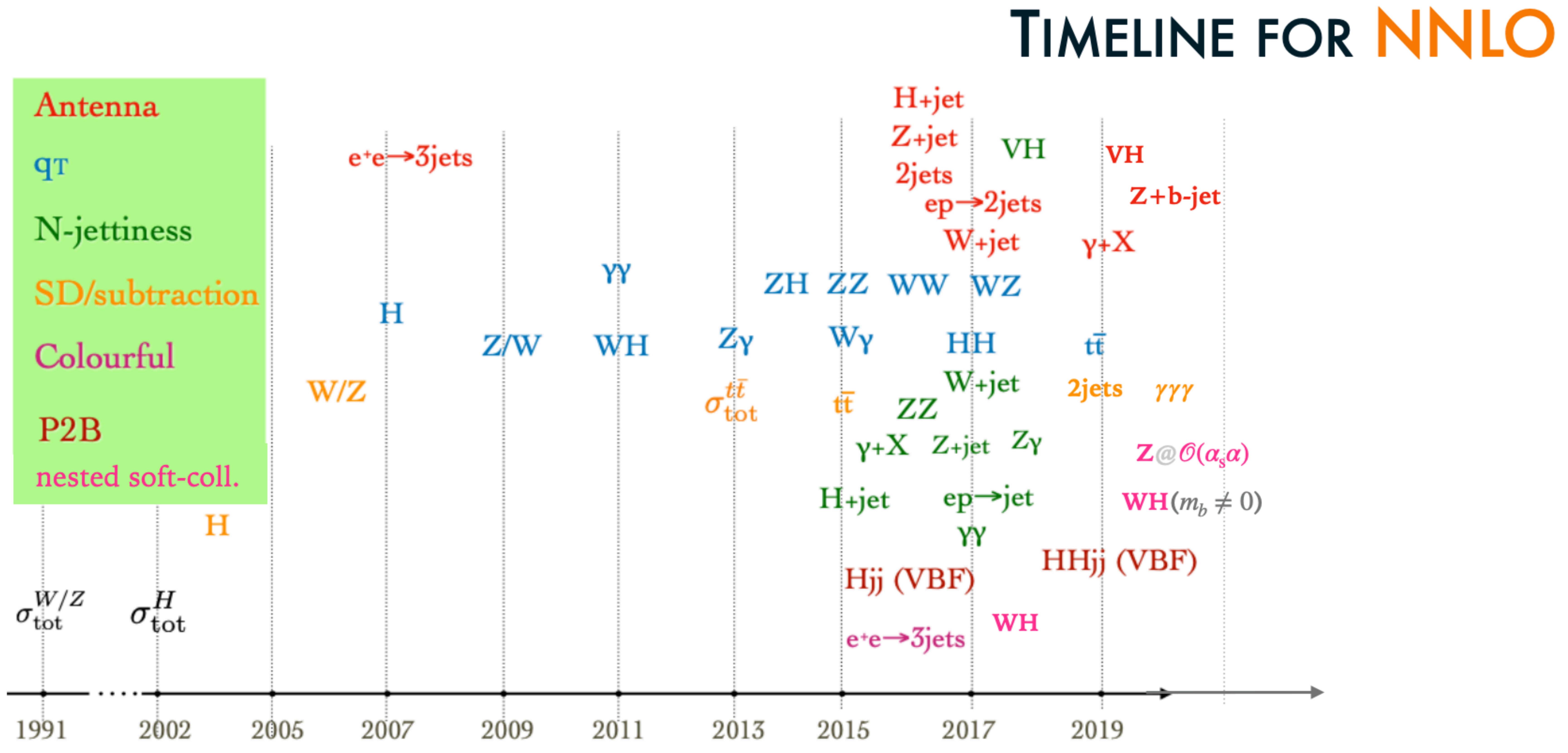
NLO automated

NNLO: Several processes known (VV production, top pair production, all $2 \to 1$ processes)

NNNLO: only a handful of processes!

- Higgs in gluon fusion (Anastasiou et al, arXiv:1602.00695)
- Higgs in VBF (Dreyer et al, arXiv:1811.07906)
- Higgs in bottom annihilation (Duhr et al, arXiv:1904.09990)
- Drell-Yan (Duhr et al, arXiv:2001.07717, 2007.13313)

Progress in higher-order computations



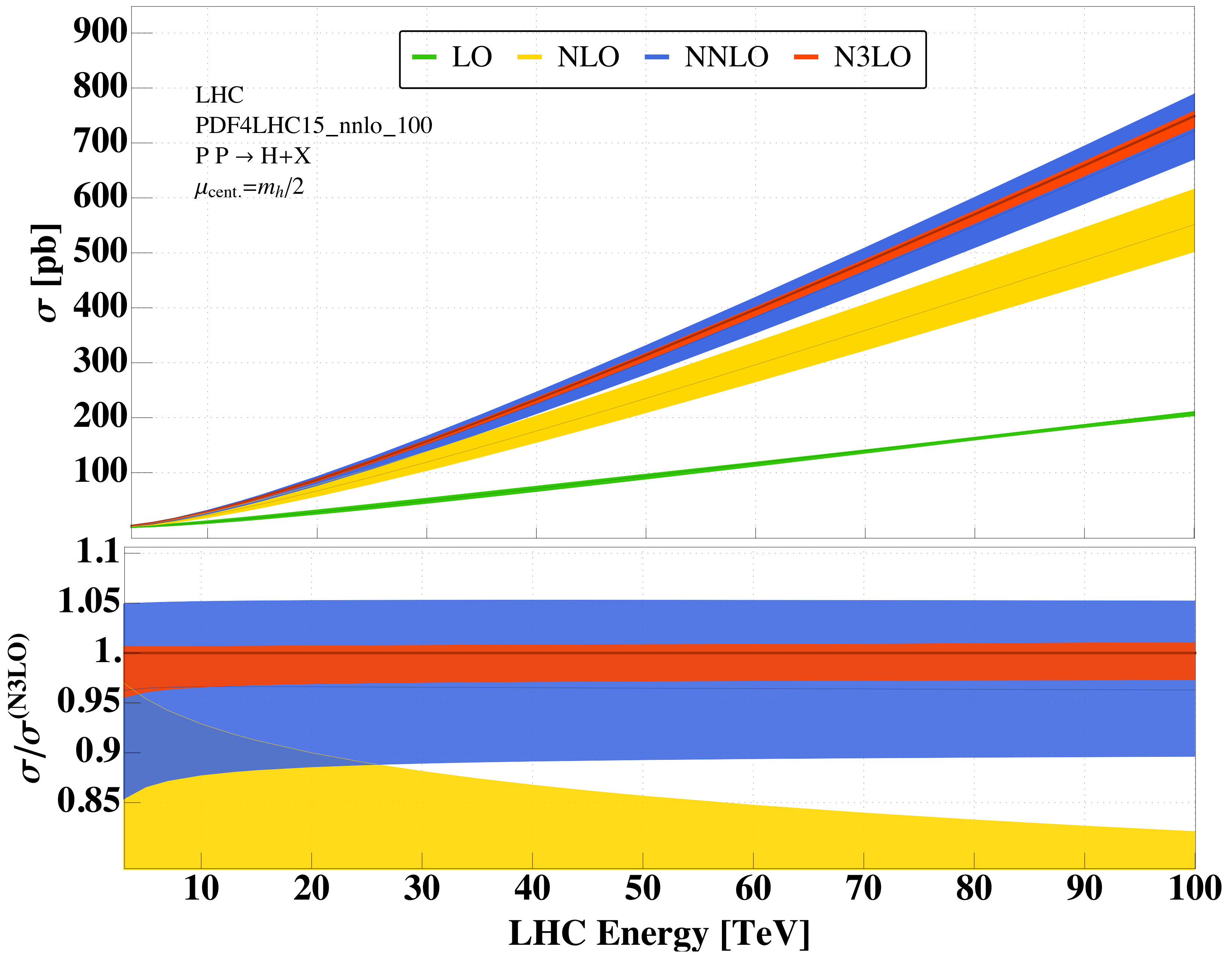
A. Huss, QCD@LHC-X 2020

Hard scattering cross-section

Perturbative expansion

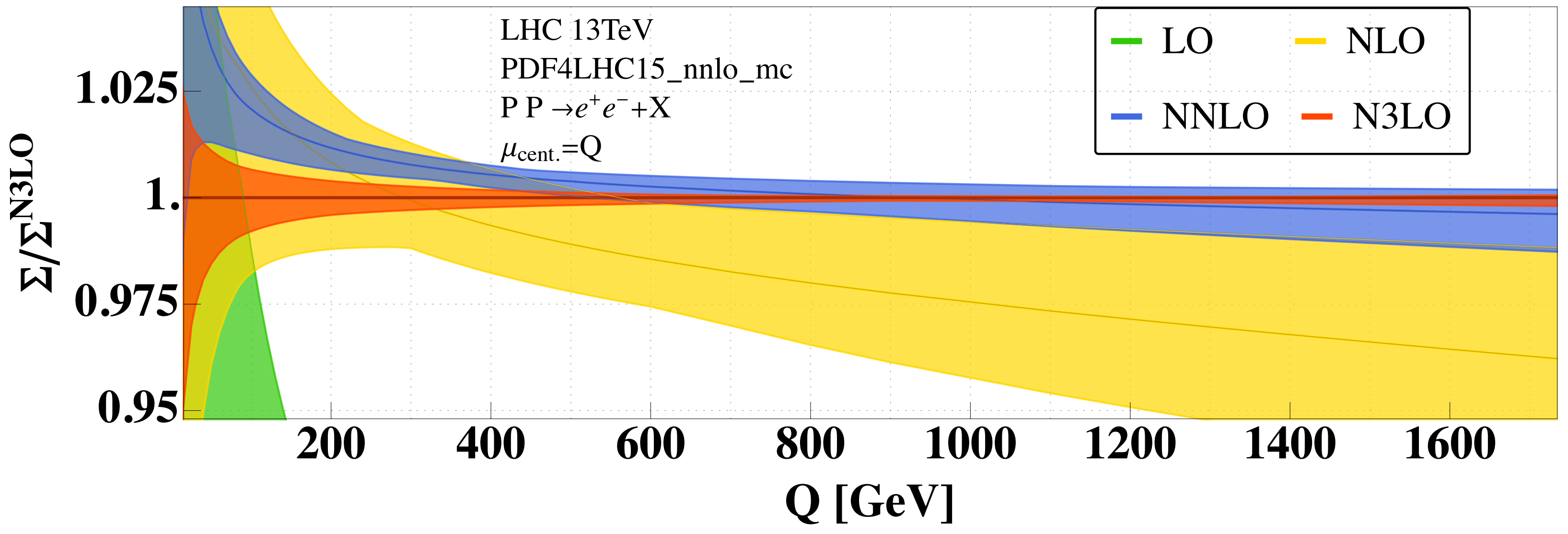
$$\hat{\sigma} = \sigma^{\text{Born}} \left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3 \sigma^{(3)} + \ldots \right)$$

LO NLO NNLO N3LO



Higgs production

Improved accuracy and precision



Dilepton production

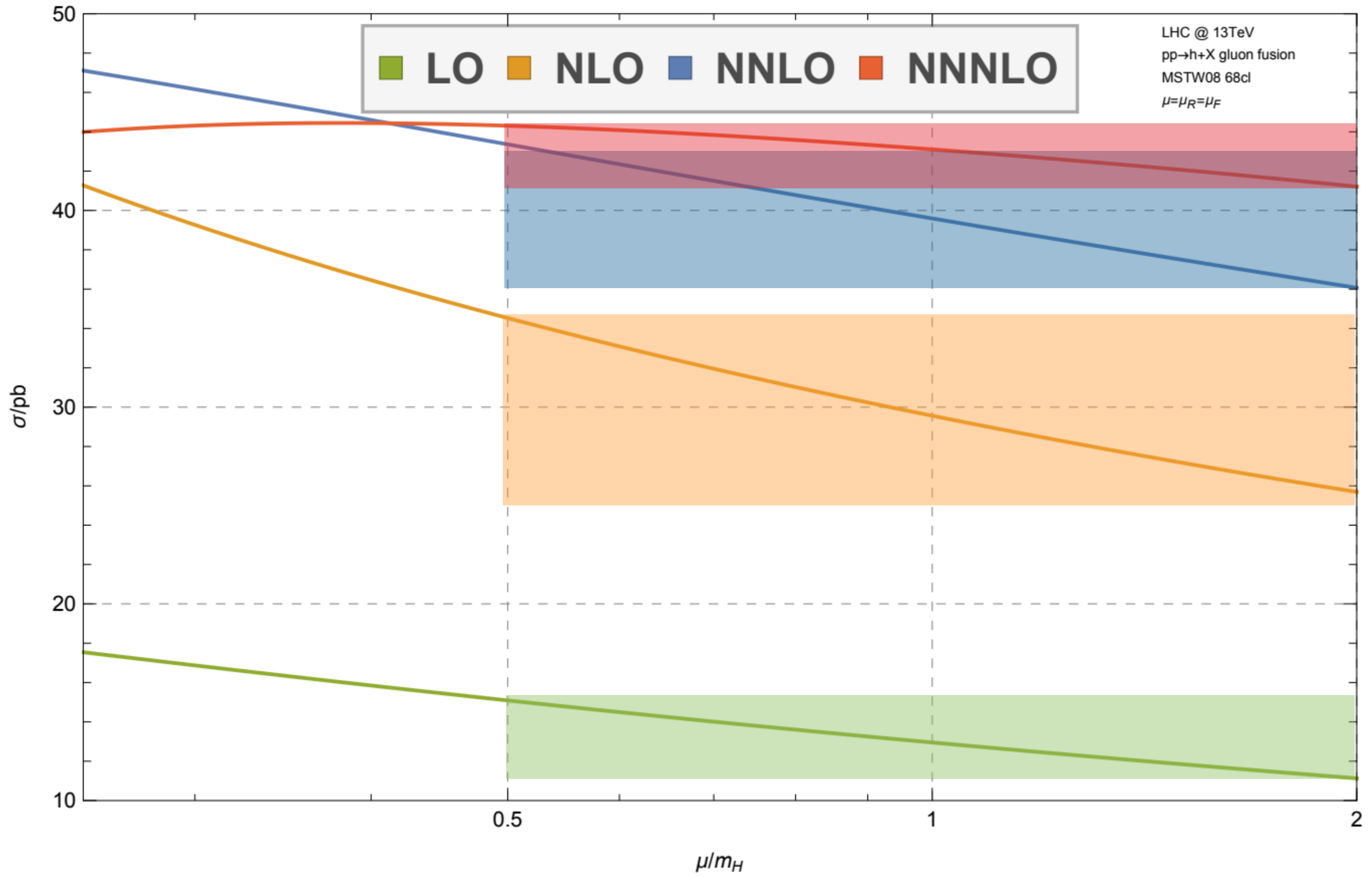
arXiv:2203.06730

Uncertainties in theory predictions

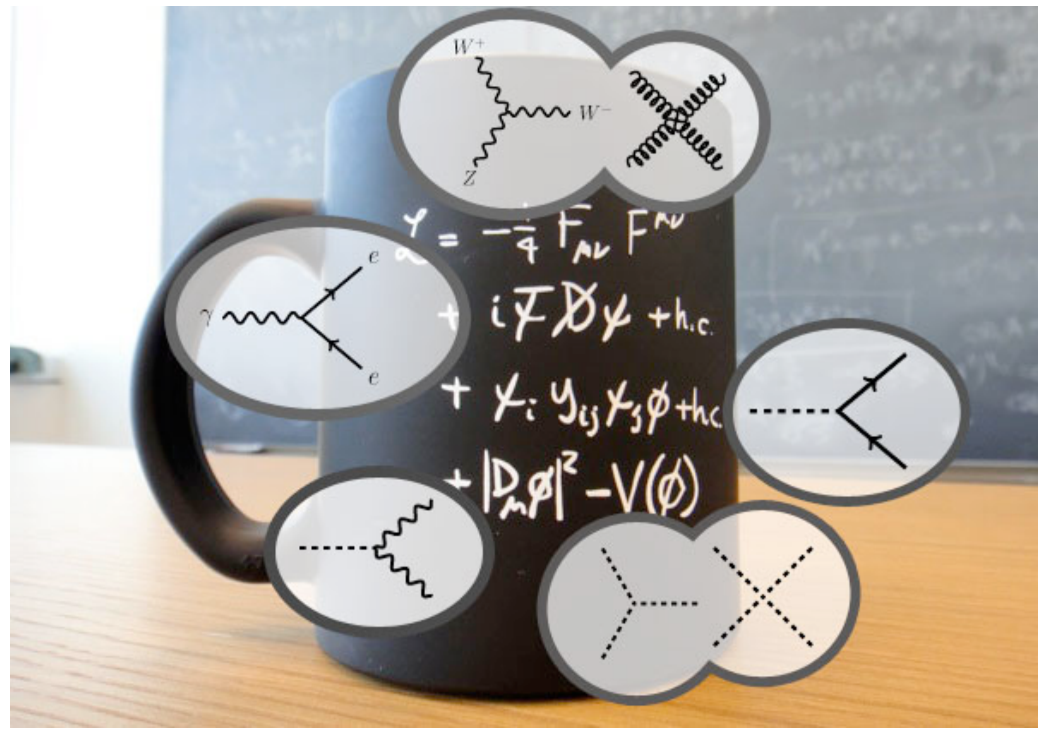
How do we estimate uncertainties?

Vary the renormalisation and factorisation scale

Typically pick some central scale μ_0 and vary the scale up and down by a factor of 2



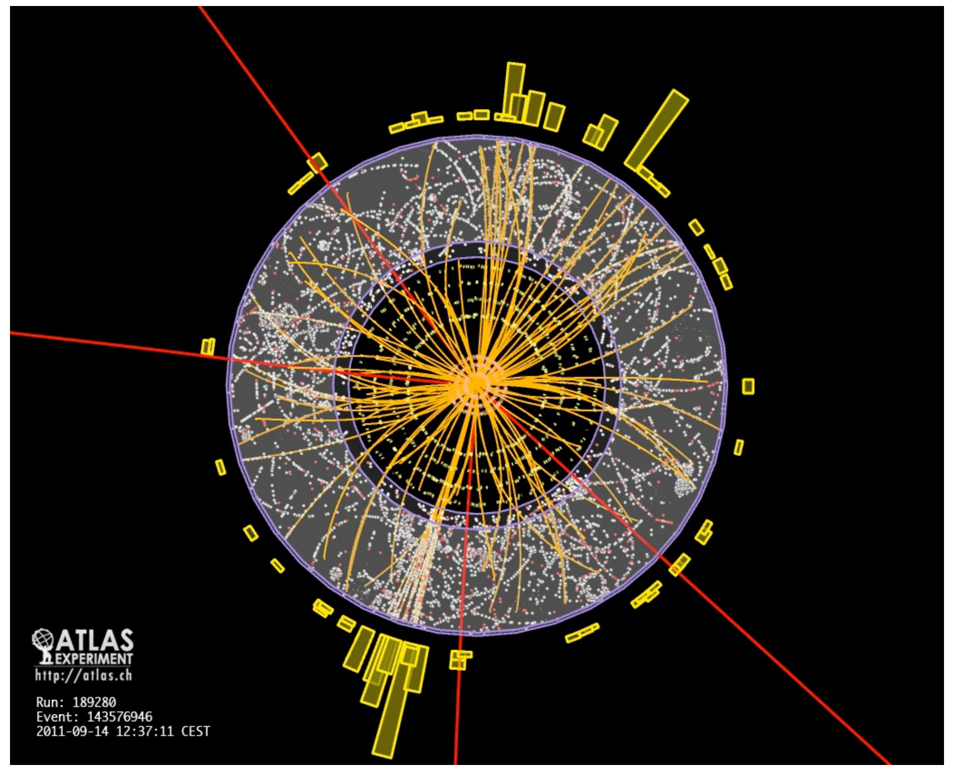
How do we actually compute all of these?



Theory

Monte Carlo

Experiment



Focusing on LO

How to compute a LO cross-section

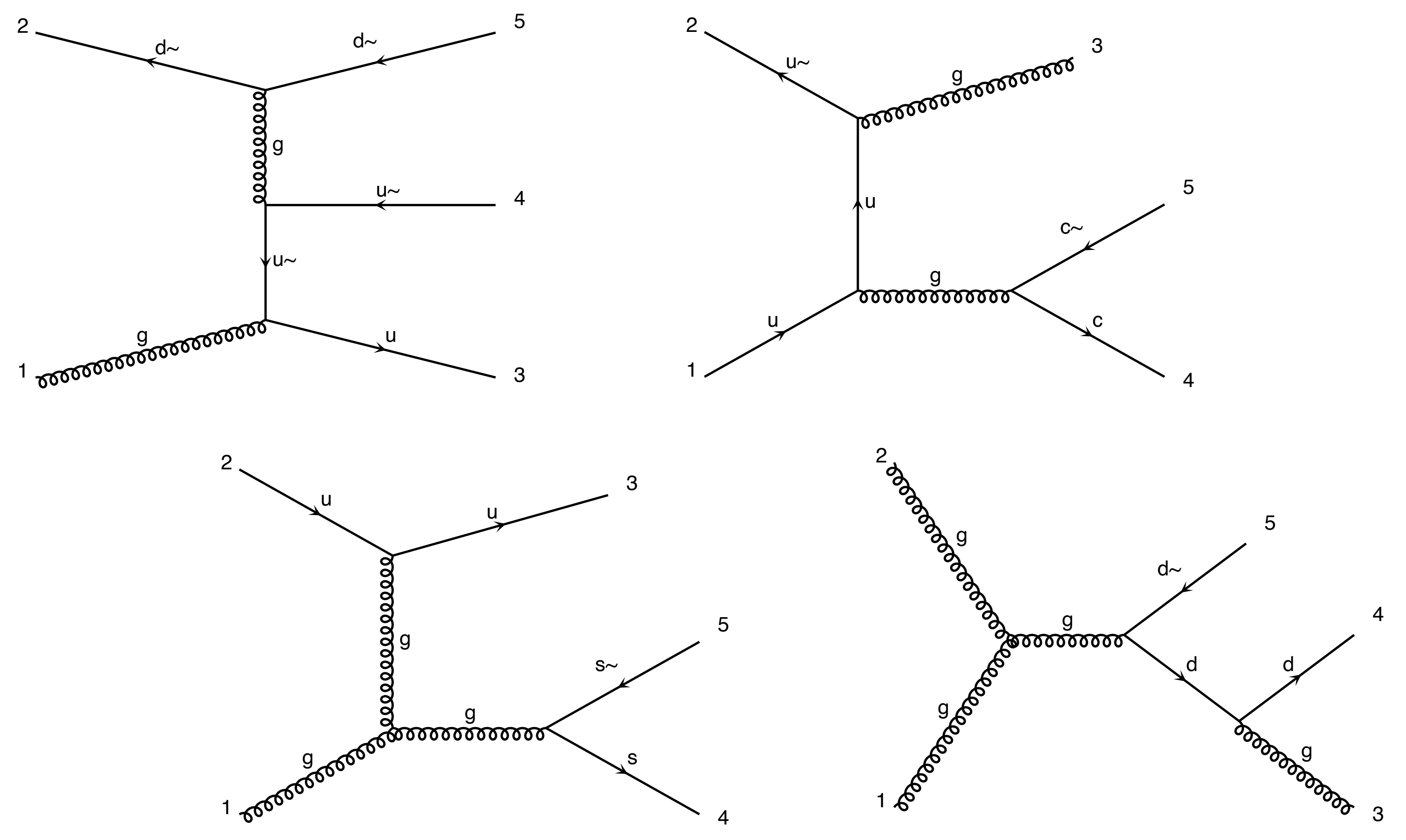
Example: 3 jet production in pp collisions

1. Know the Feynman rules (SM or BSM)
2. Find all possible Subprocesses

97 processes with 781 diagrams generated in 2.994 s

Total: 97 processes with 781 diagrams

3. Compute the amplitude
4. Compute $|M|^2$ for each subprocess, sum over spin and colour
5. Integrate over the phase space



$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

LO calculation of a cross-section

How many subprocesses?

Amplitude computation (Feynman diagrams)

Square the amplitude, sum over spin and colour

Integrate over the phase space

Difficulty

Complexity increases with

- number of particles in the final state
- number of Feynman diagrams for the process (typically organise these in terms of leading couplings: see tutorial)

Structure of an automated MC generator

I. Input Feynman rules
II. Define initial and final state
III. Automatically find all subprocesses
IV. Compute matrix element (including tricks like helicity amplitudes)
V. Integrate over the phase space by optimising the PS parametrisation and sampling
VI. Unweight and write events in the Les Houches format

Next: Shower+Hadronisation
Detector simulation and reconstruction

Output of LO MC generators

Les houches events

Example: gg>ZZ

```
<event>
 4      0 +1.1211000e+00 1.89058500e+02 7.81859000e-03 1.15931300e-01
       21 -1    0    0  502  501 +0.0000000000e+00 +0.0000000000e+00 +4.6570159241e+01 4.6570159241e+01 0.0000000000e+00 0.0000e+00 1.0000e+00
       21 -1    0    0  501  502 -0.0000000000e+00 -0.0000000000e+00 -1.9187776299e+02 1.9187776299e+02 0.0000000000e+00 0.0000e+00 1.0000e+00
       23  1    1    2    0    0 +1.3441082214e+01 +1.3065682927e+01 -5.1959303141e+01 1.0661295577e+02 9.1187600000e+01 0.0000e+00 1.0000e+00
       23  1    1    2    0    0 -1.3441082214e+01 -1.3065682927e+01 -9.3348300610e+01 1.3183496646e+02 9.1187600000e+01 0.0000e+00 1.0000e+00
</event>
```

PDG Momenta Mass

All Information needed to pass to parton shower is included in the event

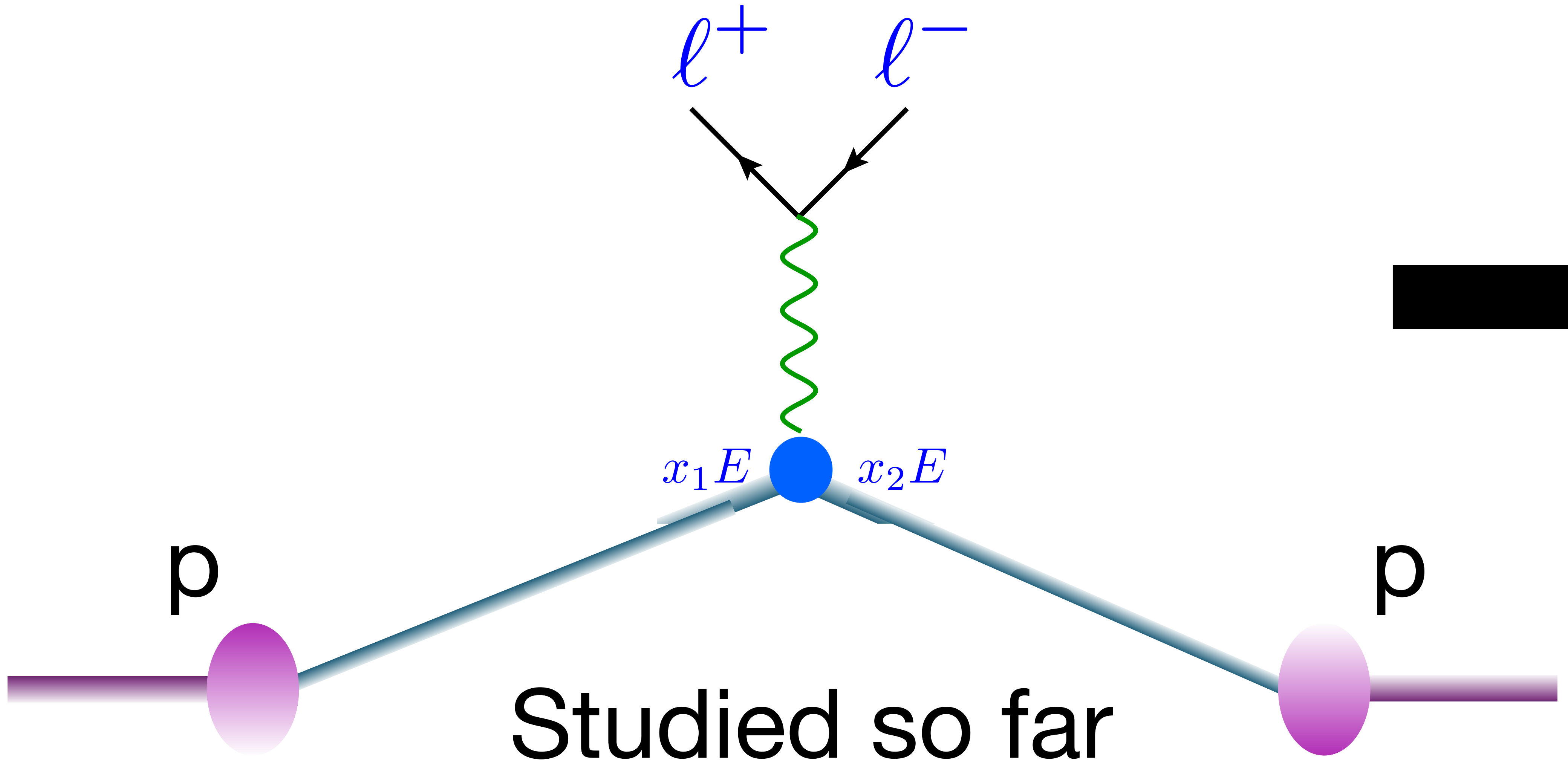
Available public MC generators

Matrix element generators (and integrators):

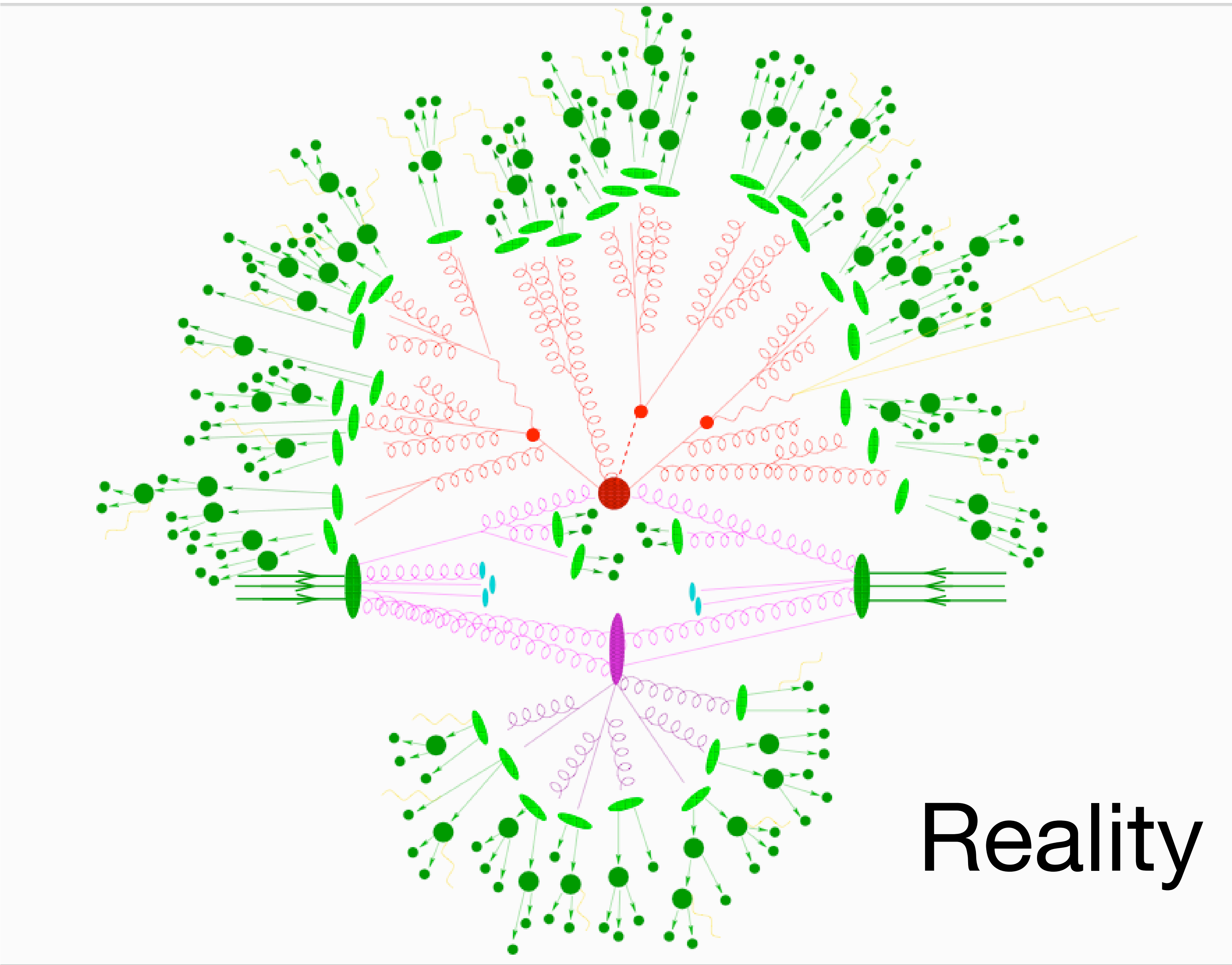
- MadGraph/MadEvent
- Comix/AMEGIC (part of Sherpa)
- HELAC/PHEGAS
- Whizard
- CalcHEP/CompHEP

Is Fixed Order enough?

Fixed order computations can't give us the full picture of what we see at the LHC

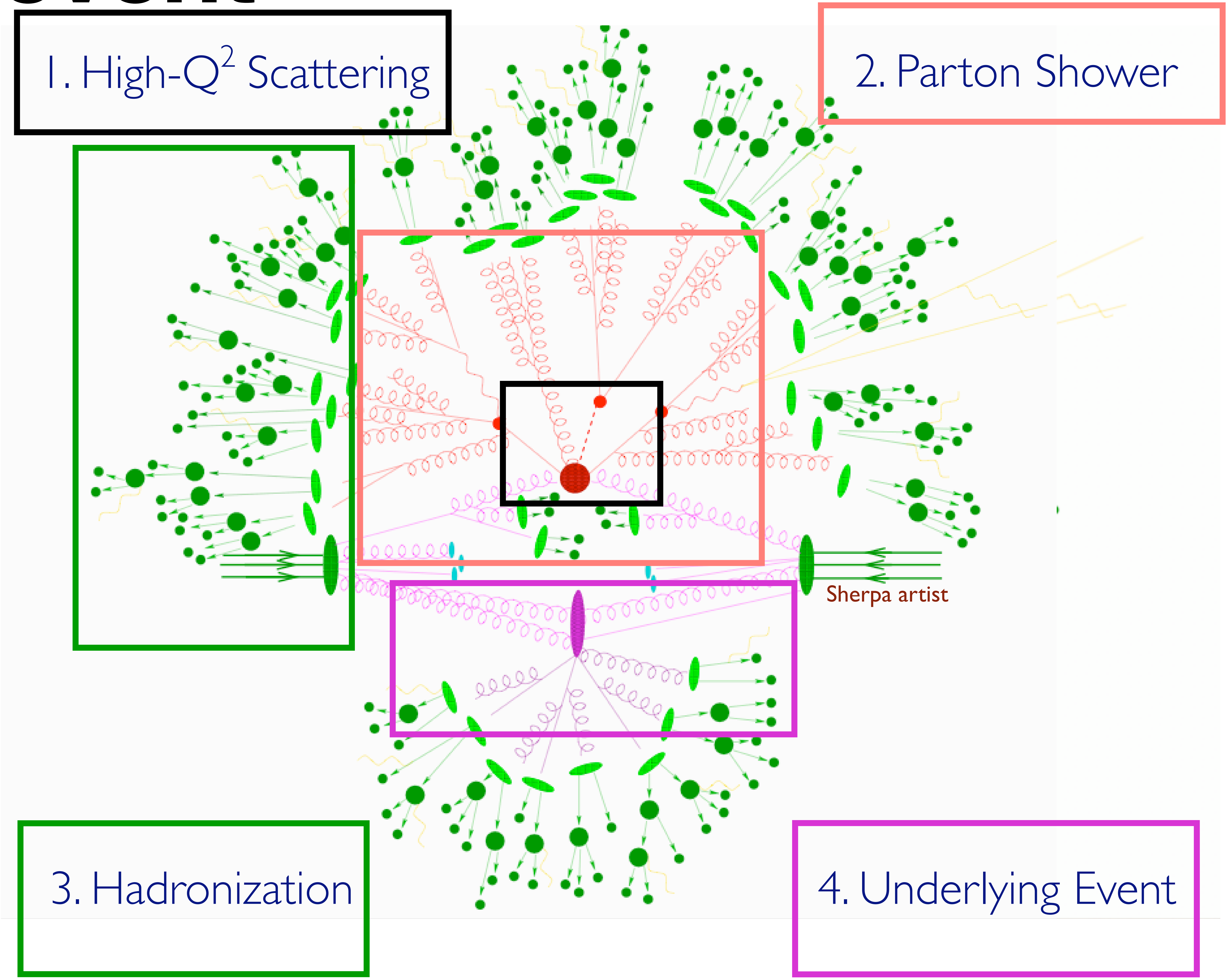


Studied so far

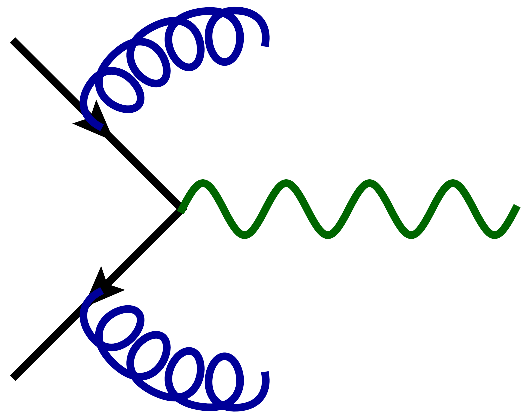


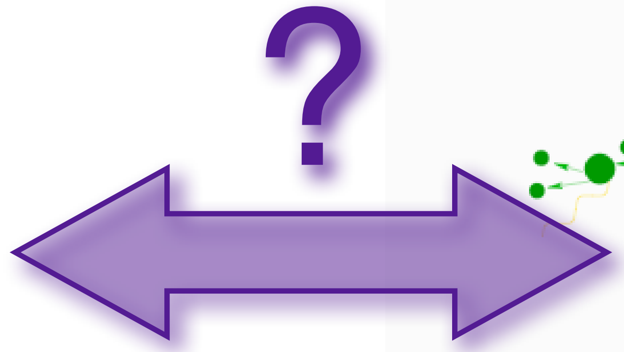
Reality

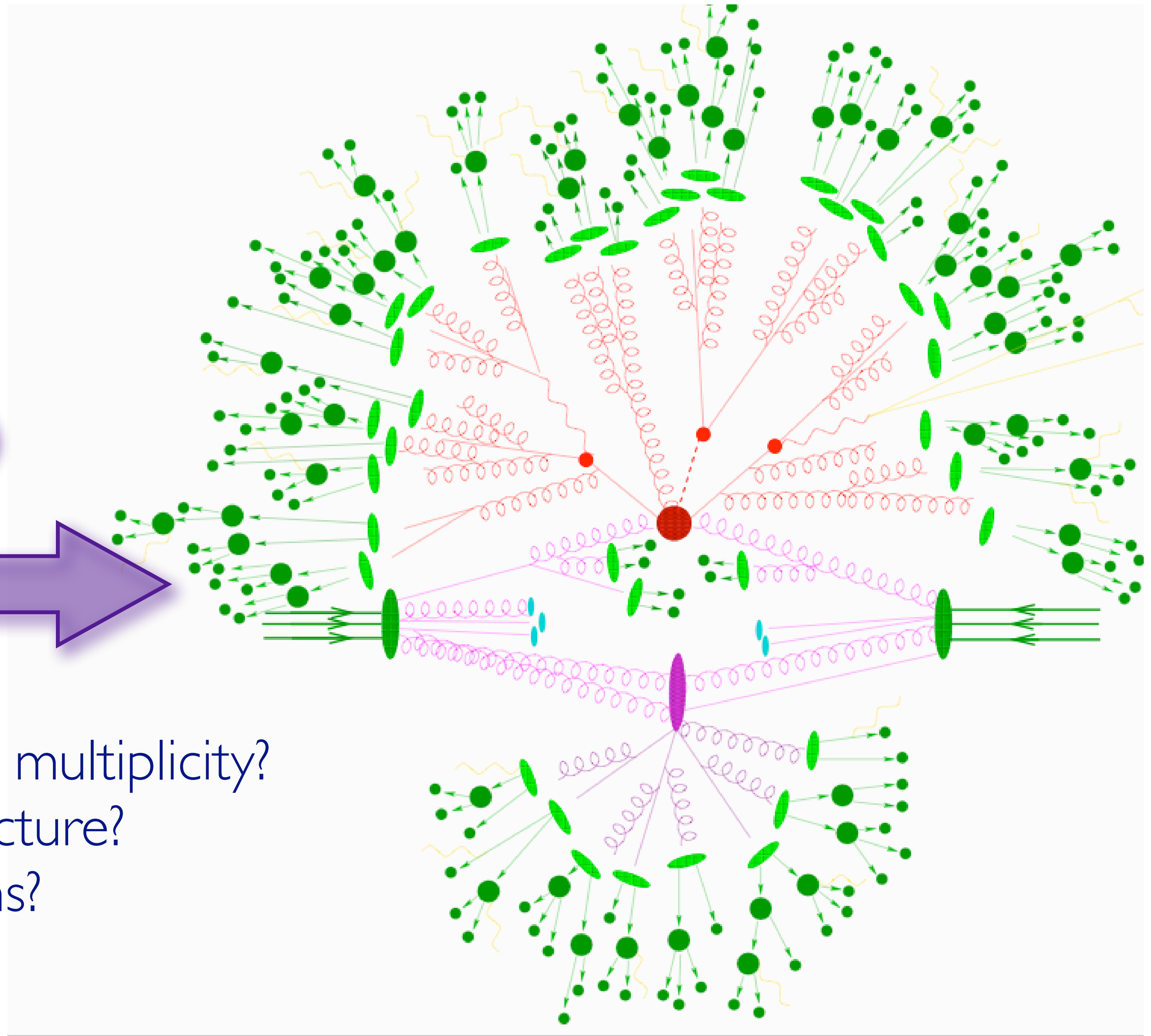
An LHC event


1. High-Q^2 Scattering
2. Parton Shower
3. Hadronization
4. Underlying Event
Sherpa artist

Is fixed order enough?







- Particle multiplicity?
- Jet structure?
- Hadrons?

- Fixed order calculations involve only a few partons
- Not what we see in detectors
- Need Shower and Hadronisation

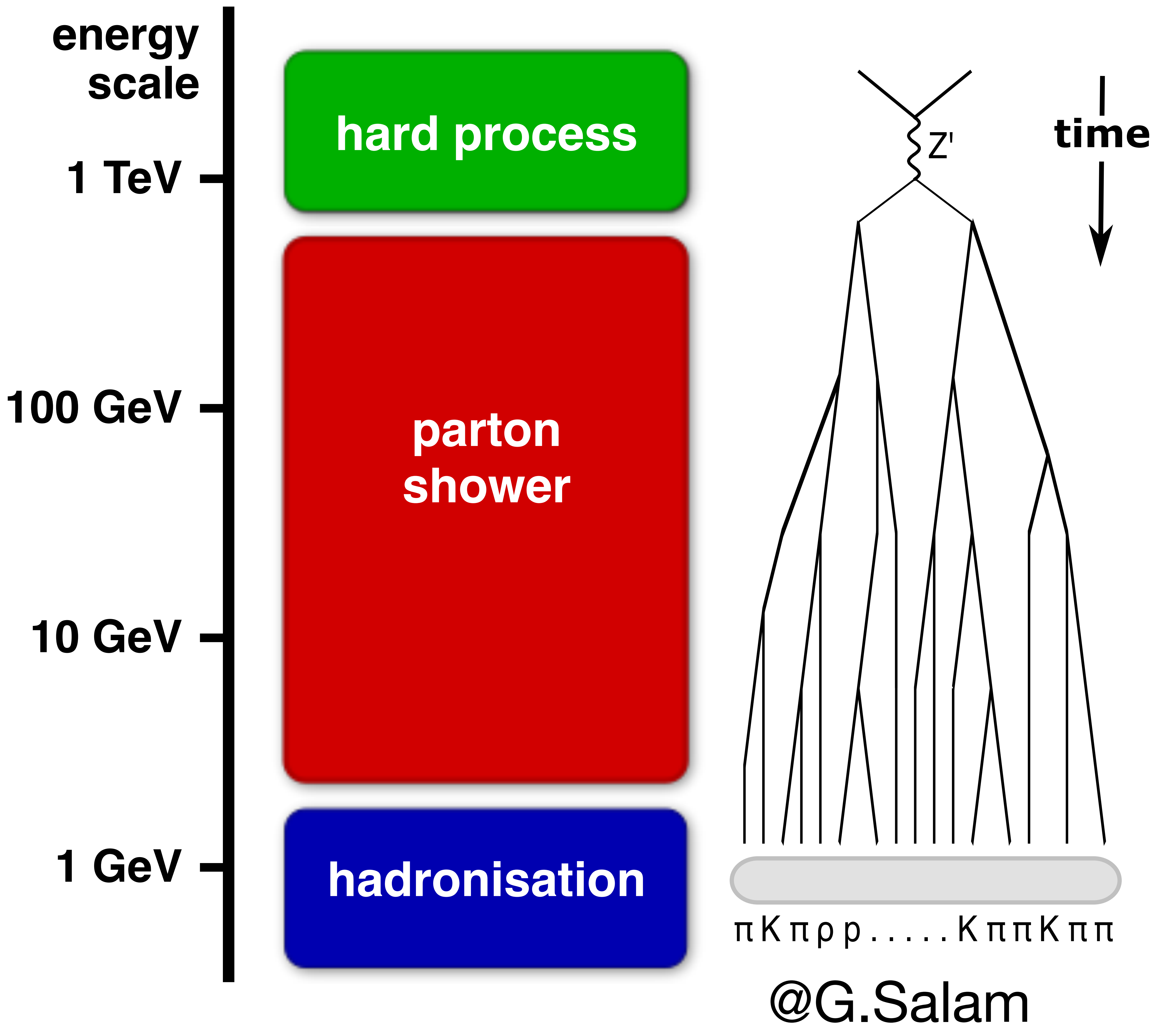
A multiscale story

High-Q^2 scattering: process dependent, systematically improvable with higher order corrections, where we expect new physics

Parton Shower: QCD, universal, soft and collinear physics

Hadronisation: low Q^2, universal, based on different models

Underlying event: low Q^2, involves multiple interactions



Parton Shower

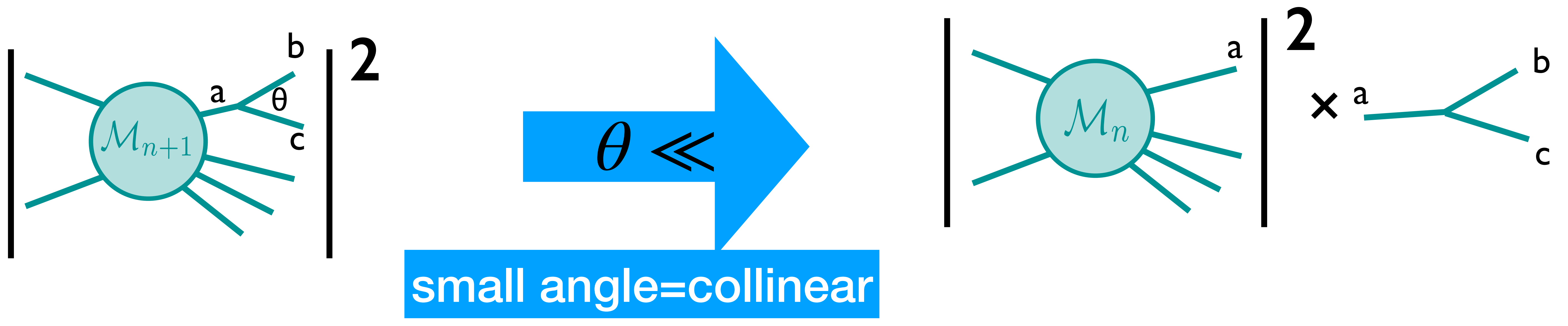
What does the parton shower do/should do?

- Dress partons with radiation with an arbitrary number of branchings
- Preserve the inclusive cross-section: unitary
- Needs to evolve in scale from Q~1TeV (hard scattering) down to ~GeV

Basics of parton shower

Collinear factorisation

Starting with one splitting



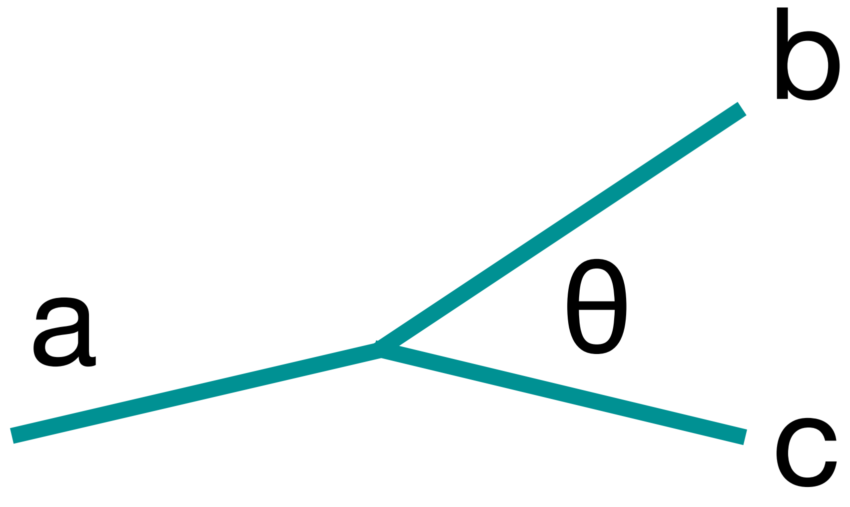
- Time scale associated with splitting much longer than the one of the hard scattering
- This kind of splitting should be described by a branching probability
- The parton shower will quantify the probability of emission

Collinear factorisation:

$$\boxed{|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)}$$

Collinear factorisation and splitting functions

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$



- t is the evolution variable
  - t tends to zero in the collinear limit (this factor is singular)
- z energy fraction transferred from parton a to parton b in splitting ($z \to 1$ in the soft limit)
- ϕ azimuthal angle

The branching probability has the same form for all quantities $\propto \theta^2$

- transverse momentum $k_\perp \sim z^2(1-z)^2\theta^2E^2$
- invariant mass $Q^2 \sim z(1-z)\theta^2E^2$

$$\frac{d\theta^2}{\theta^2} = \frac{dk_\perp^2}{k_\perp^2} = \frac{dQ^2}{Q^2}$$

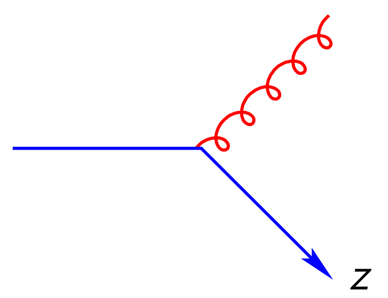
$$t \in \{\theta^2, k_\perp^2, Q^2\}$$

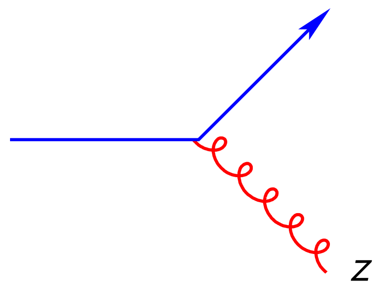
Altarelli-Parisi Splitting functions

Branching has a universal form given by the Altarelli-Parisi splitting functions (as we saw in DIS)

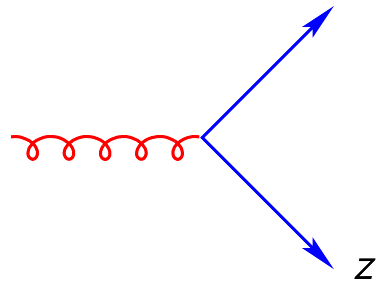
$$\frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

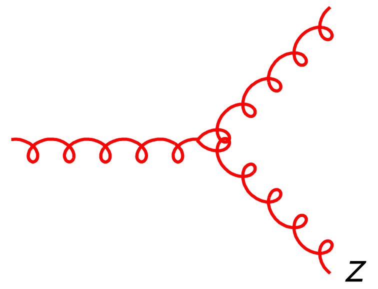
$$P_{q\to qg}(z) = C_F \left[\frac{1+z^2}{1-z}\right], \qquad P_{q\to gq}(z) = C_F \left[\frac{1+(1-z)^2}{z}\right].$$





$$P_{g\to qq}(z) = T_R \left[z^2 + (1-z)^2\right], \qquad P_{g\to gg}(z) = C_A \left[z(1-z) + \frac{z}{1-z} + \frac{1-z}{z}\right]$$

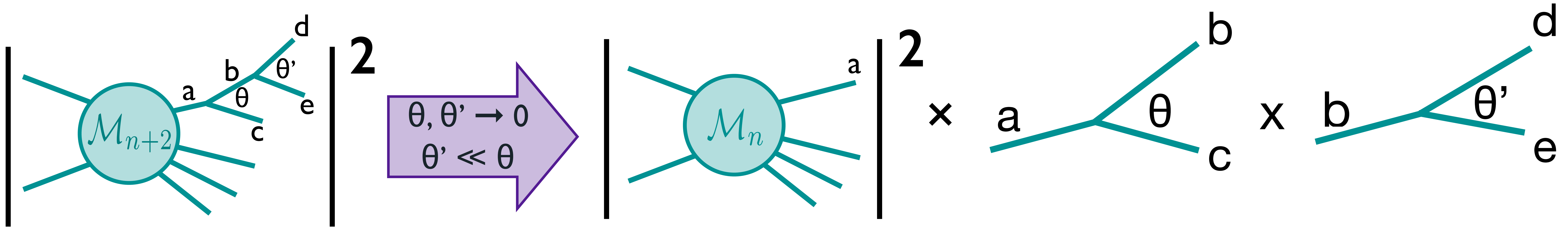




These functions are universal for each type of splitting

Multiple emissions

How does this change with multiple emissions?



$$|\mathcal{M}_{n+2}|^2 d\Phi_{n+2} \;\simeq\; |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z) \;\times \frac{dt'}{t'} dz' \frac{d\phi'}{2\pi} \frac{\alpha_S}{2\pi} P_{b\to de}(z')$$

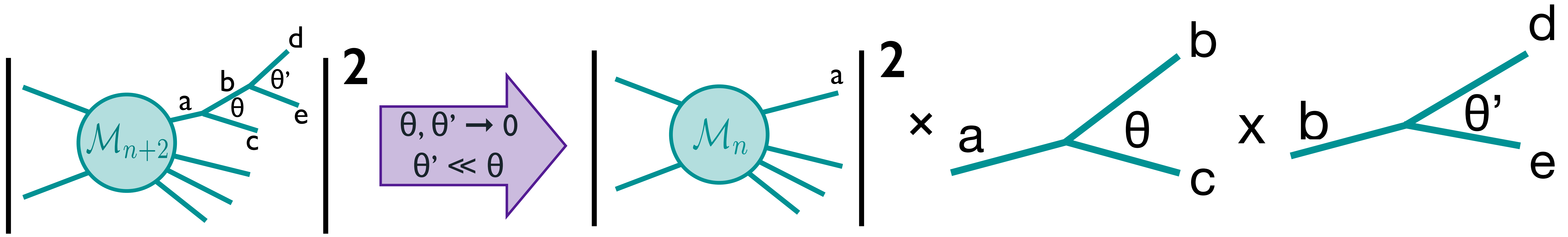
We can generalise this for an arbitrary number of emissions

Iterative sequence of emissions which does not depend on the history (Markov Chain)

➡ **No interference: Classical**

Multiple emissions

How does this change with multiple emissions?



Dominant contribution comes from subsequent emissions which satisfy strong ordering $\theta \gg \theta' \gg \theta''$

For k emissions the rate takes the form:

$$\sigma_{n+k} \propto \alpha_S^k \int_{Q_0^2}^{Q^2} \frac{dt}{t} \int_{Q_0^2}^{t} \frac{dt'}{t'} \dots \int_{Q_0^2}^{t^{(k-2)}} \frac{dt^{(k-1)}}{t^{(k-1)}} \propto \sigma_n \left(\frac{\alpha_S}{2\pi}\right)^k \log^k(Q^2/Q_0^2)$$

- Q is the hard scale and Q_0 is an infrared cut off (separating non-perturbative regime)
- Each power of α_s comes with a logarithm (breakdown of perturbation theory when large)