

Recap

Symmetries as groups, fields as representations
 continuous symmetries $\xrightarrow{\text{Noether}}$ conserved currents
 eq. Dirac $\mathcal{L} = \bar{\psi} \gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi$ inv.
 $\psi \rightarrow e^{i\theta} \psi \rightarrow j^\mu = \bar{\psi} \gamma^\mu \psi$
 properties of LSM.

Fermi theory of weak interaction

Although weak interactions are v. weak compared to strong & EM interactions, they are very important - give rise to many kinds of physical phenomena which strong & EM interactions cannot eg. parity violation.

Consider nuclear β decay - nucleus of atomic # Z xfrm into another nucleus of $Z+1$, emits e^- & $\bar{\nu}_e$:

$$n \rightarrow p + e^- + \bar{\nu}_e$$

Fermi supposed 2 fermions interact at a point, via a current-current interaction:

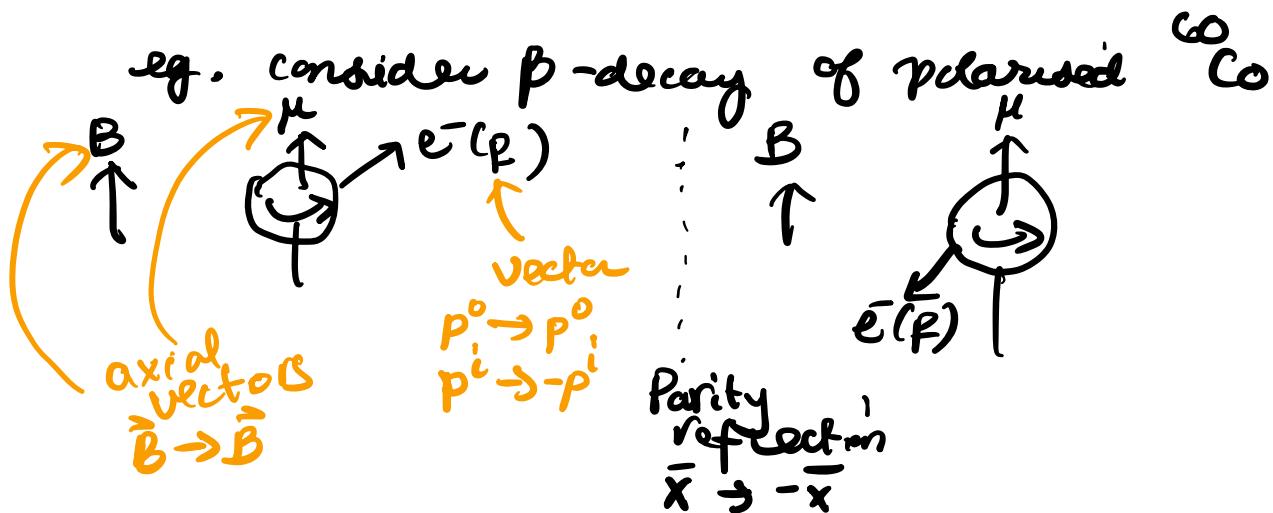
$$\mathcal{L}_{\text{int}} = -G_F [\bar{\psi}_{(p)} \gamma^\mu \psi_{(n)}] [\bar{\psi}_{(e)} \gamma_\mu \psi_{(\bar{\nu}_e)}]$$

creates proton annihilates neutron e^- creates $\bar{\nu}_e$ $\bar{\nu}_e$ creates e^-
 Vectors Vectors Vectors Vectors
 contracted to give scalar

Many other phenomena that may be described by $\bar{\ell}$ -fermion current-current interaction:

- eg. $n + \bar{\nu}_e \rightarrow p + \bar{e}$
- $\mu^- \rightarrow \bar{e}^- + \bar{\nu}_\mu + \bar{\nu}_e$
- $e^- + \bar{\nu}_e \rightarrow e^- + \bar{\nu}_e$

In fact, these processes can involve parity violation:



expt \Rightarrow e^- 's emitted mostly in direction opposite to B
 \Rightarrow violates parity.

Parity violation cannot arise from [VVJ] type-interaction, nor from [AAJ] interaction
- need [VAJ] or [AVJ] type interaction

$$\Rightarrow L_{int} = \frac{G_F}{\sqrt{2}} [\bar{t}_1 \gamma^\mu (c_V \pm c_A \gamma_5) t_2] [\bar{t}_3 \gamma_\mu (1 \mp \gamma_5) t_4]$$

"Fermi const." $G_F \sim 1.2 \times 10^{-11}$ GeV⁻² vector current \pm axial current

t_1 shows this!

\hookrightarrow "V+A" or "V-A" interactions
exptally, $\bar{\ell}$ -fermi interaction is (V-A)

Note: we can use Fermi identities to express Lint w/ different groupings.

* Problems of Fermi theory

Fermi theory very successful in describing low energy weak interactions but

- Fermi const G_F has mass dim = -2
=> theory is non-renormalizable
- even sticking to tree-level diagrams, cross-sections

$$\sigma \propto G_F^2 S$$

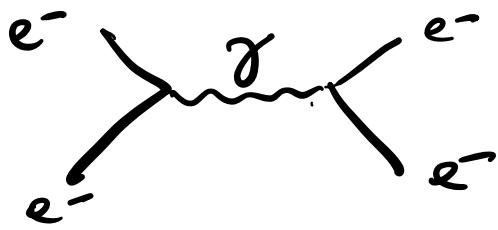
\propto Mandelstam variable - c.p.m. energy available from process

growing linearly w/ $S \Rightarrow$ break down of unitarity at energies $\propto G_F^{-1/2}$

=> a low energy approximation to some more complete theory, which should be renormalizable and unitary.

Intermediate Vector Bosons

e.g. QED $e^- + e^- \rightarrow e^- + e^-$



interaction involves four fermions but via basic interaction $e^- \gamma \rightarrow e^-$

Rather than current-current interaction, consider current-vector boson-current interaction



Write vector boson interaction of fermions as:

$$L_{\text{int}} = -g J^\mu W_\mu^\dagger + \text{h.c.}$$

\uparrow
coupling
const

\uparrow
superposition
of fermion lines

\Rightarrow Amplitude for :

$$iM = (-ig)^2 J^\mu \Gamma i D_{\mu\nu} J^\nu$$

w/ massive vector field:

$$D_{\mu\nu}(q) = \frac{1}{q^2 - M_W^2} \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_W^2} \right)$$

\uparrow propagator for intermediate vector boson:

Then in regime of low energies $q^\mu \ll M_W$

$$D_{\mu\nu} \rightarrow \frac{g_{\mu\nu}}{M_W^2} \quad \text{and}$$

$$\Rightarrow iM \sim g^2 \frac{1}{M_W^2} J^\mu \Gamma J_\mu$$

Exactly how Fermi interaction looks!

$$\text{w/ } G_F \sim \frac{g^2}{M_W^2} \quad \begin{matrix} \text{dimensionless} \\ \text{combination of coupling} \\ \text{const and vector boson} \\ \text{mass} \end{matrix}$$

\Rightarrow Fermi theory emerges as an approx' for momentum transfers small compared to the vector boson mass.

BUT rules for renormalisability:

- =
1. no coupling const w/ -ve mass dim.
 2. $D_{\text{boson}} \sim 1/q^2$ $D_{\text{fermion}} \sim 1/q$ for large q .

Our propagator for massive vector boson:

$$D_{\mu\nu} \sim \frac{1}{q^2 - M_W^2} \left(g_{\mu\nu} + \frac{q_\mu q_\nu}{M_W^2} \right) \rightarrow \frac{1}{q^2} \left(\frac{q_\mu q_\nu}{M_W^2} \right)$$

as $q \gg M_W$

so does not fall off as required.

\Rightarrow UV divergences.

Need to invent theory for massive vector bosons.

1. More symmetries! $\xrightarrow{\text{Global \& gauged}}$ and Abelian and Non-Abelian.

* Global Abelian U(1)

$$\phi = \phi_1(x) + i\phi_2(x)$$

Consider e.g. a complex scalar field, transforming under global U(1) symmetry:

$$\phi(x) \rightarrow \phi'(x) = e^{i\theta} \phi(x)$$

w/ invariant Lagrangian (up to dim 4 operators)

$$\mathcal{L}[\phi(x), \partial_\mu(\phi(x))] = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi - \frac{\lambda}{2} (\phi^* \phi)^2$$

Symmetry \Rightarrow masses of $\phi_{1,2}(x)$ are equal.
 $(\mathcal{L} > -m^2(\phi_1^2 + \phi_2^2))$

Symmetry \Rightarrow conserved current $j^\mu = \frac{\partial \mathcal{L}}{\partial \phi}, \frac{\delta \mathcal{L}}{\delta \dot{\phi}}$

$$j^\mu = -i\phi^* \partial_\mu \phi + i\phi \partial_\mu \phi^*$$

as conserved current $Q \equiv (d^3x) j_0$.

*Gauge (local) U(1)

With a local symmetry transformation:

$$\phi(x) \rightarrow \phi'(x) = e^{i\Theta(x)} \phi(x)$$

Lagrangian is no longer invariant.

$$\nabla^\mu \phi \nabla^\nu \phi^*$$

Promote ∂_μ to covariant derivative:

$$\partial_\mu \phi = (\partial_\mu - ieA_\mu) \phi$$

"gauge field"

with the $\partial_\mu \phi$ transform covariantly:

$$\partial_\mu \phi \rightarrow e^{i\Theta(x)} \partial_\mu \phi$$



$$\text{then } \partial_\mu \phi \rightarrow (\partial_\mu \phi)' = (\partial_\mu - ieA'_\mu) e^{i\Theta(x)} \phi$$

$$= e^{i\Theta(x)} (\partial_\mu \phi + i\phi \partial_\mu \Theta - ieA'_\mu \phi)$$

$$\stackrel{!}{=} e^{i\Theta(x)} (\partial_\mu \phi - ieA_\mu \phi)$$

$D_\mu \phi$

provided that gauge field transforms as:

$$A_\mu \rightarrow A'_\mu = A_\mu + \frac{1}{e} \partial_\mu \Theta$$

Having now massless vector boson, introduce its field strength tensor:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \\ = \frac{i}{e} [\partial_\mu, \partial_\nu] \text{ covariant form}$$

\Rightarrow invariant Lagrangian (scalar electrodynamics)

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (\partial_\mu \phi)^* (\partial^\mu \phi) - V(\phi^* \phi)$$

Note * we still have $m_1 = m_2 = m$ for scalar fields,

* two polarizations for the massless vector field

* conserved charge.

(see QED lectures for fermion matter field)

* global non-abelian

Consider now e.g. N scalar fields ($N=3$ for QCD, $N=2$ for EW case)

$$\Phi = \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_N \end{pmatrix} \quad \Phi' = U \Phi$$

$$\text{w/ } x^{\mu} \phi_i(x) \rightarrow \phi'_i(x) = U_{ij} \phi_j(x)$$

where $U \in \text{SU}(N)$ ($N \times N$ matrices, $U^\dagger U = I$, $\det U = 1$ w/ components U_{ij})

parameterized by $N^2 - 1 \equiv r$ parameters θ^α ad:

$$U = \exp \left(i \sum_{a=1}^r \theta^\alpha t^\alpha \right) \quad \text{eg } t^\alpha = \sigma^\alpha / 2 \text{ for SU(2), generators}$$

Unitary x^{μ} leave $\Phi^\dagger \Phi = \phi_i^* \phi_i$ invariant.

$$\text{so } \mathcal{L} = \partial_\mu \phi_i^* \partial^\mu \phi_i - m^2 \phi_i^* \phi_i - \lambda (\phi_i^* \phi_i)^2 \quad (7)$$

is invariant under global $SU(N)$ transformation.

Symmetry \Rightarrow all fields ϕ_i have same mass and same coupling const.

* set of conserved charges.

* Gauge (local) non-Abelian

\mathcal{L} in (7) no longer invariant if ϕ_i form a coordinate dependent:

$$\phi_i(x) \rightarrow \phi_i^U(x) = U_{ij}(x) \phi_j(x) \quad \text{where } U(x) \in SU(N)$$

$$\Phi(x) \rightarrow \Phi'(x) = U(x) \Phi(x)$$

check $\partial_\mu \Phi'(x) = U \partial_\mu \Phi(x) + \partial_\mu U(x) \cdot \Phi(x)$

renders kinetic term not invariant.

As for Abelian case, introduce gauge field and promote derivatives to covariant derivatives:

$$\partial_\mu \phi_i = \partial_\mu \phi_i - ig \underbrace{A_\mu^a T_{ij}^a}_{\substack{\text{new vector} \\ \text{field} \\ \text{cf. photon}}} \phi_j$$

$A_\mu^a T_{ij}^a = A_\mu^{ij}$ generators of $SU(N)$ in rep. of ϕ_j

where $A_{\mu ij}$ formed s.t. $(A_\mu \phi_i)' \rightarrow U_{ij} A_\mu \phi_j$:

$$\begin{aligned}\partial_\mu \bar{\Psi}' &= \partial_\mu \bar{\Psi}' - ig A_\mu \bar{\Psi}' \quad \bar{\Psi}' = U \bar{\Psi} \\ &= U \partial_\mu \bar{\Psi} + \partial_\mu U \cdot \bar{\Psi} - ig \sqrt{g} U \bar{\Psi}\end{aligned}$$

$$\Rightarrow A_\mu \rightarrow A'_\mu = U A_\mu U^{-1} - \frac{i}{g} (\partial_\mu U) U^{-1}$$

Writing $A_{\mu i j} = A_\mu^a T_{i j}^a$ and $U = \exp(i g T_a \theta^a)$

$$\begin{aligned}A'_\mu^a &= A_\mu^a + g f_{bc a} \theta^b A_\mu^c + \partial_\mu \theta^a \\ &= A_\mu^a - i g \theta^b (t_b^{adj})_{ac} A_\mu^c \text{ (global)}$$

$\Rightarrow A_\mu^a$ ^{x fm in adjoint of the symmetry}
^{gp.} self interaction ✓

Introduce $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f_{bc a} A_\mu^b A_\nu^c$
st $F_{\mu\nu}(x) \rightarrow F'_{\mu\nu}(x) = U(x) F_{\mu\nu}(x) U^{-1}(x)$