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EFT FOR $b \to c \ell \nu$

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1. QFTs and EFTs
2. EFT BELOW THE EW SCALE
3. EFT FOR $b \to c \ell \nu$

- EXTENDED NOTES:

A. THE SM and FLAVOUR SYMMETRY

B. THE SM AS AN EFT

C. FLAVOUR IN THE SMEFT

D. BELOW THE EW SCALE

E. WEAK DECAYS AS PROBES OF BSM vs EW

F. EFT FOR $b \to c \ell \nu$

G. SCALE DEPENDENCE AND RGE

1. "All" QFTs are EFTs

- What is an EFT?
- What is a QFT?
- What is the Standard Model (SM)?

  - Relativity + QM ⇒ QFT framework
  - Gauge symmetry G
  - Field content in irreps of Lorentz + G
  - SSB pattern
  - Renormalitzability

    Why? → otherwise we can't calculate

    ↳ REALLY???

⇒ Put all this into a Lagrangian

→ read off vertices (Feynman rules)

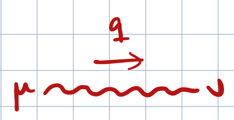
→ calculate S-matrix elements (amplitudes)

$$\mathcal{L} = \sum_i g_i \, O_i$$

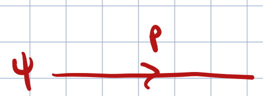
operators of dimension ≤ 4 respecting these symmetries.

EXAMPLE: QED and μ-decay

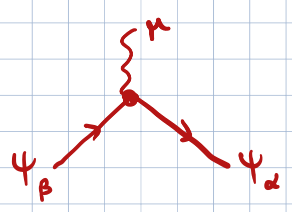
$$\mathcal{L}_{QED} = -\frac{1}{4}\underbrace{F_{\mu\nu}F^{\mu\nu}}_{d=4} + \sum_{\psi=\{\mu,e,\nu_\mu,\nu_e\}} \bar{\psi}\,(i\underbrace{\not{D}}_{d=4} - \underbrace{m}_{d=3})\,\psi$$



$$= \frac{-i g_{\mu\nu}}{q^2}$$

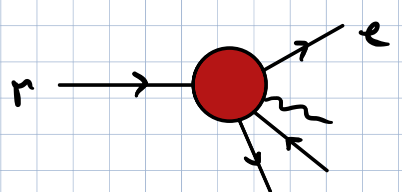


$$= \frac{i(\not{p}+m)}{p^2-m^2}$$



$$= i e Q_\psi (\gamma_\mu)_{\alpha\beta}$$

In this theory μ is stable:



DOES NOT EXIST.

This is because lepton flavor is an ACCIDENTAL SYMMETRY of QED.

Let's add a new "non-renormalizable" operator to the Lagrangian:

$$\delta\mathcal{L} = \frac{c_6}{\Lambda^2}\left(\bar{\nu}_\mu \gamma^\alpha P_L \mu\right)\left(\bar{e}\gamma_\alpha P_L \nu_e\right) \qquad (1)$$

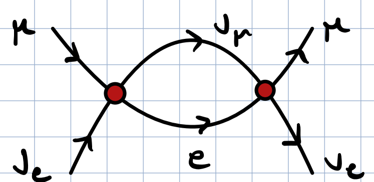
The operator is $d=6$, and therefore the coupling (or "Wilson coefficient") must have dimension of m^{-2}. Thus a new mass scale Λ appears.

When calculating an observable related to a process with energy E, we will find:

$$\text{Observable} = A^{(D=4)} + B\cdot c_5 \frac{E}{\Lambda} + \left(C\, c_5^2 + D c_6\right)\frac{E^2}{\Lambda^2} + \cdots$$

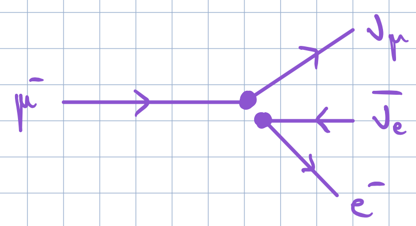
and if $E \ll \Lambda$, $A^{(D=4)}$ might be an excellent approximation $\Rightarrow$ Explains success of SM

In addition, order-by-order in E/Λ, renorma-litzability is not an issue:



needs counterterm of $d=8$ op, negligible at $\mathcal{O}(E/\Lambda)$.

Our $d=6$ operator (1) breaks the accidental symmetry of QED, and mediates μ-decay :



At tree level (EXERCISE) : $\Gamma_\mu \simeq \dfrac{m_\mu^5 c^2}{1536\,\pi^3 \Lambda^4}$ (2)

Experimentally: $\tau_\mu^{exp} = 2.2 \times 10^{-6}\,s$

$\Rightarrow \Gamma_\mu^{exp} = 3 \cdot 10^{-19}$ GeV (3)

Comparing (2) and (3), with $c \sim 1$:

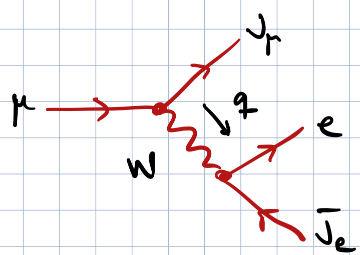
$\Lambda \simeq 172$ GeV $\simeq v/\sqrt{2}$ $\Rightarrow$ $\Lambda \simeq \Lambda_{EW}$! ✓

$\Rightarrow$ We "discover" the EW scale, and measure it !

(EW physics is "new physics" for QED)

The "cut-off" scale Λ usually related to the mass of a heavy particle beyond the EFT.
In this example the EW SM is a UV-completion of QED:
(one of many)

$$\mathcal{M}(\mu \to e \nu \bar{\nu}) =$$



$$= \underbrace{\frac{g^2}{2(q^2 - M_W^2)}} \bar{u}_{\nu_\mu} \gamma_\alpha P_L u_\mu \cdot \bar{u}_e \gamma^\alpha P_L v_{\nu_e}$$

$$= -\frac{g^2}{2M_W^2} \cdot \frac{1}{1 - q^2/M_W^2} = -\frac{g^2}{2M_W^2}\left[1 + \frac{q^2}{M_W^2} + \cdots\right]$$

$\Rightarrow$ Equivalent to $\delta\mathcal{L}$ with

$$\frac{c_6}{\Lambda^2} = -\frac{g^2}{2M_W^2} = -\frac{4G_F}{\sqrt{2}}$$

This calculation of a Wilson coefficient of an EFT in terms of the parameters of a UV-completion is called a <u>MATCHING CALCULATION</u>.

EXAMPLE: The SM as an EFT

The SMEFT Lagrangian is

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \sum_i C_i^{(5)} Q_i^{(5)} + \sum_i C_i^{(6)} Q_i^{(6)} + \cdots$$

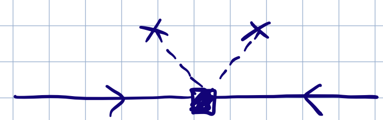
where

$$C_i^{(d)} \sim c/\Lambda^{d-4} \,.$$

The renormalizable SM has $C_i^{(d)} = 0$. Thus in order to probe this idea we need to measure non-zero values for some $C_i^{(d)}$.

In a sense we already have evidence of $C_i^{(5)}$:

$$Q_{\ell\ell HH} = (\tilde{H}^{+}\ell)^{T} C (\tilde{H}^{+}\ell) \xrightarrow[\langle H\rangle = v/\sqrt{2}]{} \frac{1}{2} v^{2} \nu^{T} C \nu + \cdots$$



which is a Majorana mass for the neutrino:

$$M_\nu \sim c \frac{v^2}{\Lambda}$$

The smallness of M_ν implies $\Lambda \sim 10^{14}\,\mathrm{GeV}$.

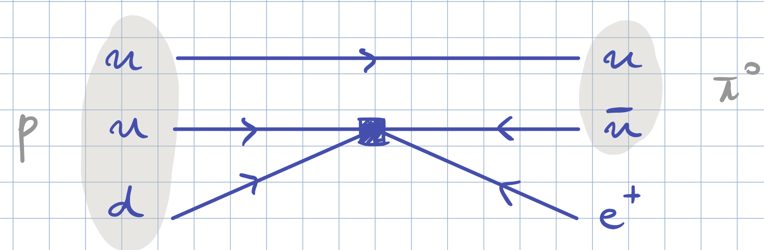
(with assumptions)

Note that Q_{eeHH} breaks Lepton number, which is an ACCIDENTAL SYMMETRY of $\mathcal{L}(D\leq 4)$.

Thus it seems that in order to test $\mathcal{L}_{D>4}$ it is a good idea to look at lepton number violating observables. These are obvious null tests of the SM.

Another example are BARYON NUMBER VIOLATING observables, such as proton decay. This can be mediated by e.g. the previous dim=6 op:

$$Q_{duue} = \varepsilon_{\alpha\beta\sigma}\,(d_\alpha^T C u_\beta)(u_\sigma^T C e)$$



Currently $\tau_p \gtrsim 10^{30}$ yrs $\Rightarrow$ $\Lambda \gtrsim 10^{15}$ GeV

Note that the scale Λ related to neutrino mass need not be the same as the scale Λ related to p-decay.

2. EFT Below the EW SCALE

[E.g. hep-ph/9512380 ; 1704.05672 ; 1709.04486]

Write the most general Lagrangian built out of fields lighter than M_W, and compatible with $SU(3)_c \times U(1)_{em}$ gauge invariance:

$$\boxed{\mathcal{L}_{WET/LEFT} = \mathcal{L}_{QCD} + \mathcal{L}_{QED} + \sum_i C_i \, \mathcal{O}_i}$$

without top quark

"Wilson" coefficients

Operators build from light fields and invariant under $SU(3)_c \times U(1)_{em}$.

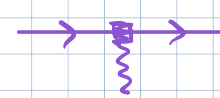
Some WET/LEFT operators:

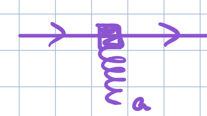
dim-5 :

$$\mathcal{O}_{e\gamma}^{11} = \bar{e}_L \sigma^{\mu\nu} e_R F_{\mu\nu}$$

$$\mathcal{O}_{d\gamma}^{23} = \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}$$

$$\mathcal{O}_{dG}^{13} = \bar{d}_L \sigma^{\mu\nu} T^a b_R G^a_{\mu\nu}$$

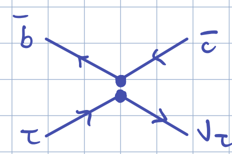


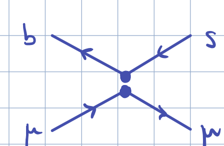


dim - 6 :

$$\left[\mathcal{O}^{V,LL}_{\nu edu}\right]_{3332} = (\bar{\nu}_\tau \gamma^\alpha \tau)(\bar{b}_L \gamma_\alpha c_L)$$

$$\left[\mathcal{O}^{V,RR}_{ed}\right]_{2232} = (\bar{\mu}_R \gamma^\alpha \mu_R)(\bar{b}_R \gamma_\alpha s_R)$$





Some important comments:

- → QCD + QED is the Low-E EFT of the SM (EFT)
- → the cut-off is $\Lambda \sim \Lambda_{EW} \sim v$ (EW is "NP")
- → The full FLAVOUR Group is ACCIDENTAL SYM.

The p-decay and μ-decay phylosophy applies to all flavor transitions:

$$\boxed{\text{LOW-LYING HADRONS are STABLE at } D \leq 4}$$

$K^0 \sim \bar{s} d$; $K^+ \sim \bar{s} u$; $D^0 \sim \bar{u} c$; $D^+ \sim \bar{d} c$

$B^0 \sim \bar{b} d$; $B^+ \sim \bar{b} u$; $B_s^0 \sim \bar{b} s$; $\Lambda_b \sim udb$; ...

Exercise: Look at the PDG (Particle data group) particle listings and check that:

$$\tau_{\text{mesons}} \sim \tau_{\text{baryons}} \gtrsim 10^{-12} s \sim 10^{12}\, GeV^{-1}$$

$$\tau_{\text{Resonances}} \lesssim 10^{-23} s \sim 10\, GeV^{-1}$$

This fact has two consequences:

1. Definition of Hadrons

$$H = \underbrace{H_0}_{QCD\ (+QED)} + \underbrace{H_{int}}_{D>4} \; ; \qquad \text{e.g. } H_0|B\rangle = m_B|B\rangle$$

2. Weak decays as probes of BSM vs EW.

Decay rates are linear in C_i^{EW}, C_i^{BSM}

Example: $b \to c \ell \nu$ decay

Consider the transition $b \to c \ell \nu$ mediated by the term in the eff. Lagrangian:

$$\mathcal{L}_{eff}^{(6)} = C_1 (\bar{c} \gamma_\mu P_L b)(\bar{\ell} \gamma^\mu P_L \nu)$$

(there are actually 5 independent $d=6$ operators of the $b \to c \ell \nu$ type. For this example one is enough.)

This quark-level transition is realized in decay observables such as

$$B \to D\, l \bar{\nu}\,, \quad B \to D^{*} l \bar{\nu}\,, \quad B_s \to D_s\, l \bar{\nu}\,, \quad \Lambda_b \to \Lambda_c\, l \bar{\nu}\,, \; \dots$$

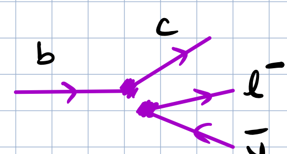
$$B \to X_c\, l \bar{\nu}\,, \quad \dots$$

The theory predictions for all these observables will depend on the Wilson coefficient C_1.

What is C_1 in the SM and in BSM models?
To answer this question we need to do a <u>matching calculation</u>.

We consider the partonic amplitude $\mathcal{A}(b \to c\, l \bar{\nu})$, and require the EFT amplitude to be equal to the (expanded) "full theory" amplitude.

We have (up to higher perturbative orders)

$i \mathcal{A}_{EFT} =$ 

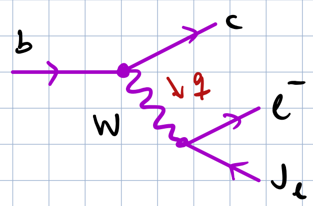
$= i\, C_1 \left(\bar{u}_c \gamma_\mu P_L u_b \right) \left(\bar{u}_l \gamma^\mu P_L v_\nu \right)$

$$\Rightarrow \quad \mathcal{A}_{EFT} = C_1\, \bar{u}_c \gamma_\mu P_L u_b \cdot \bar{u}_l \gamma^\mu P_L v_\nu$$

In the SM, the process $b \to c \ell \nu$ is mediated by W exchange:

$q^2 = (p_b - p_c)^2 \leq (m_b - m_c)^2 \ll M_W^2$

$$i\mathcal{A}_{SM} = $$



$$+ \dots$$

$$= -\frac{g^2}{2} V_{cb}\, \bar{u}_c \gamma_\mu P_L u_b \cdot \bar{u}_e \gamma_\nu P_L v_\nu \cdot \overbrace{\frac{-i}{q^2 - M_W^2}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{M_W^2}\right)}^{\frac{i}{M_W^2} g^{\mu\nu} + \mathcal{O}\left(q^2/M_W^2\right)}$$

$$+ \text{ higher perturbative orders}$$

$$= -i \frac{g^2}{2M_W^2} V_{cb}\, \bar{u}_c \gamma_\mu P_L u_b \cdot \bar{u}_e \gamma^\mu P_L v_\nu + \cdots$$

$$\Rightarrow \quad \mathcal{A}_{SM} = -\frac{g^2}{2M_W^2} V_{cb}\, \bar{u}_c \gamma_\mu P_L u_b \cdot \bar{u}_e \gamma^\mu P_L v_\nu + \cdots$$

The LO SM matching condition is thus:

$$C_1^{SM} = -\frac{g^2 V_{cb}}{2M_W^2}$$

Flavor suppression $\sim \lambda^2$

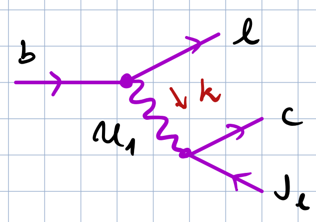
Let's consider now a new "vector leptoquark" U_1:

$$\mathcal{L}_{LQ} = g_b\, \bar{e}\, \not{U}_1 P_L b + g_c\, \bar{\nu}_e \not{U}_1 P_L c + h.c.$$

$$+ \frac{1}{2} M_{U_1}^2 U_1^\mu U_{1\mu}$$

The LO contribution to the $b \to c \ell \nu$ amplitude from U_1 exchange is given by:

$$i A_{U_1} =$$



$$+ \dots$$

$$= - g_b g_c \cdot \left[\frac{i}{M_{U_1}^2} + \mathcal{O}\left(k^2/M_{U_1}^2\right) \right] \bar{u}_\ell \gamma_\mu P_L u_b \cdot \bar{u}_c \gamma^\mu P_L v_\nu + \dots$$

$$= -i \frac{g_b g_c}{M_{U_1}^2} \underbrace{\bar{u}_\ell \gamma_\mu P_L u_b \cdot \bar{u}_c \gamma^\mu P_L v_\nu}_{-\bar{u}_c \gamma_\mu P_L u_b \cdot \bar{u}_\ell \gamma^\mu P_L v_\nu \quad \text{(Fierz)}} + \dots$$

$$\Rightarrow \quad A_{U_1} = \frac{g_b g_c}{M_{U_1}^2} \bar{u}_c \gamma_\mu P_L u_b \cdot \bar{u}_\ell \gamma^\mu P_L v_\nu$$

Thus, the LO contribution to the matching condition from U_1 exchange is:

$$C_1^{U_1} = \frac{g_b g_c}{M_{U_1}^2}$$

Combining everything:

$$\boxed{C_1 = -\frac{g^2 V_{cb}}{2 M_W^2} + \frac{g_b g_c}{M_{U_1}^2} + \dots = C_1^{SM} \left[1 - 2 \frac{g_b g_c}{g^2 V_{cb}} \cdot \frac{M_W^2}{M_{U_1}^2} + \dots \right]}$$

What does this mean?

Imagine we measure some $b \to c l \nu$ observable and extract an experimental value for C_1 with a relative uncertainty σ, and consistent with the SM prediction:

$$C_1^{exp} = C_1^{SM} (1 \pm \sigma)$$

Then we conclude that

$$2 \frac{g_b g_c}{g^2 V_{cb}} \cdot \frac{M_W^2}{M_{U_1}^2} < \sigma \qquad (\text{e.g. @ } 68\% \text{ C.L.})$$

and therefore,

$$M_{U_1} > \sqrt{\frac{2 g_b g_c}{g^2 \sigma V_{cb}}} \, M_W .$$

Setting $g_b \sim g_c \sim g$

$V_{cb} \sim \lambda^2 \sim 0.04$

$\sigma \sim 10\%$

we find that $M_{U_1} \gtrsim 20\, M_W \sim 2\,\text{TeV}.$

We probe the TeV scale!

3. EFT for $b \to c \ell \nu$

[E.g. 1704.06639]

Write all local operators built out of

$u, d, s, c, b, e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau, F_{\mu\nu}, G^a_{\mu\nu}, D_\mu$

which are:

- Lorentz-invariant
- Local $SU(3)_c \times U(1)_{em}$ invariant
- Canonical dimension $d \leq 6$
- Not related by EOMs or Dirac Identities.

The result is:

$$\mathcal{L}_{EFT} = \mathcal{L}_{QCD+QED} + \sum_{i=1}^{5} C_i O_i$$

with

$$O_1 = (\bar{c} P_R \gamma^\mu b)(\bar{\ell} \gamma_\mu \nu_\ell); \quad O_4 = (\bar{c} P_L \gamma^\mu b)(\bar{\ell} \gamma_\mu \nu_\ell);$$

$$O_2 = (\bar{c} P_R b)(\bar{\ell} \nu_\ell); \quad O_5 = (\bar{c} P_L b)(\bar{\ell} \nu_\ell)$$

$$O_3 = (\bar{c} P_R \sigma^{\mu\nu} b)(\bar{\ell} \sigma_{\mu\nu} \nu_{\ell'})$$

– See App. F for specific details. –

As in any QFT, this Lagrangian must be renormalized. In particular the couplings (the WC's C_i) must be define according to some renormalization conditions, and are scale dependent:

$$C_i = C_i(\mu)$$

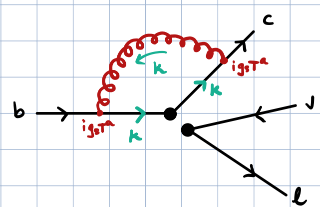
This scale dependence is given by the beta functions (or anomalous dimensions):

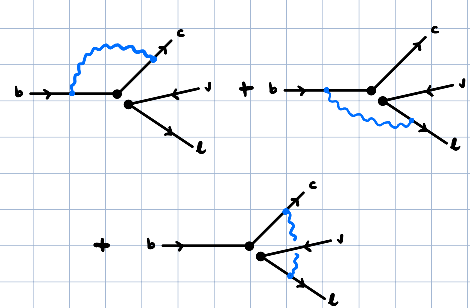
$$\boxed{\frac{d\, C_i(\mu)}{d \log \mu} = \gamma_{ji} \; C_j(\mu)} \qquad \text{(RGE)}$$

with

$$\gamma = \hat{\alpha}_s \gamma^{(0)} + \hat{\alpha} \gamma_e^{(0)} + \hat{\alpha}_s^2 \gamma^{(1)} + \cdots$$

The ADMs $\gamma^{(n)}, \gamma_e^{(n)}$ are obtained by renormalizing the $b \to c l \nu$ amplitudes at $n+1$ loops. E.g.

$\gamma^{(0)}$: 

; $\gamma_e^{(0)}$: 

leading to (see 1704.06639)

$$\gamma^{(0)} = \begin{pmatrix} 0 & & & & \\ & -8 & & & \\ & & 8/3 & & \\ & & & 0 & \\ & & & & -8 \end{pmatrix} \; ; \quad \gamma_e^{(0)} = \begin{pmatrix} -2 & & & & \\ & 4/3 & 1/6 & & \\ & 8 & -40/9 & & \\ & & & -4 & \\ & & & & 4/3 \end{pmatrix}$$

The matrix $\gamma^{(0)}$ is explicitly derived in App. G.

The functions $C_i(\mu)$ are obtained solving the RGE with an initial condition $C_i(M)$:

"Evolution matrix"

$$\boxed{C_i(\mu) = \mathcal{U}_{ij}(\mu, M) \cdot C_j(M)}$$

taking into account that $\hat{\alpha}_s \equiv \hat{\alpha}_s(\mu)$ and $\hat{\alpha} \equiv \hat{\alpha}(\mu)$ are given by the QCD and QED β-functions.

(See e.g. A.Buras hep-ph/9806471)

SUMMARY OF A TYPICAL ANALYSIS:

1. Measure a number of $b \to c \ell \nu$ observables (e.g. $BR(B \to D \ell \nu)$, $BR(B \to D^* \ell \nu)$, AOs, ...)

2. Calculate these observables within the EFT.

   E.g.

$$\mathcal{A}(B \to D \ell \nu) = C_i(\mu_b) \langle D \ell \nu | \underbrace{O_i(\mu_b)}_{\text{calculated @ } \mu \sim m_b} | B \rangle$$

$$\Rightarrow \quad BR(B \to D \ell \nu) = F(C_i(\mu_b))$$

3. Compare theory and experiment to perform a fit to the 5 coefficients:

$$\{ C_1(\mu_b), \ldots, C_5(\mu_b) \}$$

4. Use the RGE to obtain $C_i(M_W)$ and compare to

$$C_1(M_W) = -\frac{g^2 V_{cb}}{2 M_W^2} + \cdots + C_1^{BSM}(M_W) \, ; \qquad C_{i \neq 1}(M_W) = C_i^{BSM}(M_W)$$

A. BRIEF REMINDER: SM & FLAVOR SYMMETRY

The Standard Model (SM) is defined by:

- RENORMALIZABLE(*) QFT with
- GAUGE SYMMETRY: $SU(3) \times SU(2) \times U(1)$
- FIELD CONTENT (+REPS): $(Q, U_R, D_R, L, E_R) \times 3 + H$
- SSB: $\langle H^\dagger H \rangle = v^2/2 \neq 0$.

with the following reps:

	Q	U_R	D_R	L	E_R	H
$SU(3)_c$	3	3	3	1	1	1
$SU(2)_L$	2	1	1	2	1	2
$U(1)_Y$	$1/6$	$2/3$	$-1/3$	$-1/2$	-1	$1/2$

Note: Remembering hypercharges:

- $Y_H = 1/2$
- $Y_{\psi_R} = Q_{\psi_R}$
- For ψ_L do $\bar{Q}\, \underset{1/2}{H}\, \underset{-1/3}{D_R}$ and $\bar{L}\, \underset{1/2}{H}\, \underset{-1}{E_R}$

The procedure is the following:

1) Write a Lorentz-invariant, Gauge-invariant Lagrangian with all available fields, with all possible terms up to canonical dim ≤ 4, and give vev to the Higgs field

$$\Rightarrow \quad \mathcal{L}_{SM} = \mathcal{L}_{D\leq 4}. \quad (*)$$

2) Renormalize this Lagrangian and fix all renormalized couplings (Lagrangian parameters) from a set of experimental measurements according to some renormalization scheme.

3) Use the theory!

- Calculate observables ("predictions")
- Test experimental measurement
- Learn about the fundamental principles.

The level of success of this program up to now is the LEGACY OF THE 20th CENTURY.

Step 1 gives:

$$\mathcal{L}_{SM} = \mathcal{L}_G + \mathcal{L}_S + \mathcal{L}_F + \mathcal{L}_Y$$

with:

$$\mathcal{L}_G = -\frac{1}{4} \sum_{F=G,W,B} F^a_{\mu\nu} F^{a,\mu\nu}$$

$$\mathcal{L}_S = (D_\mu H)^\dagger (D^\mu H)$$

$$\mathcal{L}_F = \sum_{\psi,ij} \delta_{ij} \bar{\psi}_i (i \not{D}) \psi_j = \sum_{\psi,i} \bar{\psi}_i (i\not{D}) \psi_i$$

Always possible

where: $\psi_i = \{ L^i, E^i_R, Q^i, U^i_R, D^i_R \}$; $L = \begin{pmatrix} N_L \\ E_L \end{pmatrix}$; $Q = \begin{pmatrix} U_L \\ D_L \end{pmatrix}$

$i = \{1,2,3\}$ (generation)

The blue part of the SM Lagrangian has a $U(3)^5$ symmetry under which:

$$\psi^i \longrightarrow U^{ij}_\psi \psi^j \quad , \quad U_\psi \in U(3)$$

This $U(3)^5$ symmetry is called the

FLAVOR SYMMETRY.

Note: $\mathcal{U}(3)^5 = \mathcal{U}(1)^5 \times SU(3)^5$

Separate Global phase transformations for $\{L, E_R, Q, U_R, D_R\}$

The $\mathcal{U}(3)^5$ Flavor symmetry is broken (partially) by $\mathcal{L}_Y$:

$$\mathcal{L}_Y = - y_d^{ij} \, \bar{Q}^i \cdot H \, D_R^j - y_u^{ij} \, \bar{Q}^i \cdot \underbrace{\tilde{H}}_{i\sigma^2 H^*} \, U_R^j$$
$$- y_e^{ij} \, \bar{L}^i \cdot H \, E^j + h.c.$$

Using the $\mathcal{U}(3)^5$ symmetry of $\mathcal{L}_G + \mathcal{L}_S + \mathcal{L}_F$ one can always write (EXERCISE)

$$y_e^{ij} = \mathrm{diag}\,(y_e, y_\mu, y_\tau)$$

$$y_u^{ij} = \mathrm{diag}\,(y_u, y_c, y_t)$$

$$y_d^{ij} = \underbrace{V_{CKM}}_{\text{"CKM matrix"}} \cdot \mathrm{diag}\,(y_d, y_s, y_b)$$

V_{CKM} is a unitary 3x3 matrix parametrized by 3 angles + 1 phase.

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

A convenient parametrization of the CKM Matrix is the Wolfenstain Parametrization:

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2+ & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)\,.$$

in terms of 4 (real) Wolfenstein Parameters,

$$\{\lambda, A, \rho, \eta\}$$

where $\lambda \simeq 0.22$ is the Cabibbo parameter, acting as an expansion parameter.

We'll extract the values of λ, A, ρ, η from experiment in the exercise session.

Once the Higgs acquires a vev:

$$H(x) = \frac{1}{\sqrt{2}}\, \mathcal{U}(x) \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

then $\mathcal{L}_Y$ contains quadratic terms in fermion fields:

$$\mathcal{L}_{m_\psi} = - \sum_{\psi = E,D,U} M_\psi^{ij}\, \bar{\psi}_L^i\, \psi_R^j \;+\; h.c.$$

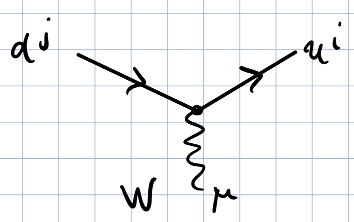
with

$$\boxed{M_\psi = \frac{v}{\sqrt{2}}\, y_\psi} \qquad (\#)$$

i.e.:

$$M_e = M_E = \mathrm{diag}\,(m_e, m_\mu, m_\tau)$$
$$M_u = M_U = \mathrm{diag}\,(m_u, m_c, m_t)$$
$$M_D = V_{CKM} \times \mathrm{diag}\,(m_d, m_s, m_b) \equiv V_{CKM} \cdot M_d .$$

Rotating now the d_L^i fields separately from the u_L^i fields the matrix V_{CKM} can be removed from M_D, but then V_{CKM} appears in the W couplings:



$$= i\, \frac{g}{\sqrt{2}}\, V_{ij}\, \gamma_\mu P_L$$

Note: The relation (#) between the masses and the yukawa couplings is modified in the presence of BSM physics, as we will see in Chapter 3.

$\mathcal{L}_Y$ respects the subgroup (EXERCISE)

$$\mathcal{U}(1)^5_{SM} = \mathcal{U}(1)_B \times \mathcal{U}(1)_e \times \mathcal{U}(1)_\mu \times \mathcal{U}(1)_\tau \times \mathcal{U}(1)_Y$$

where:

$\mathcal{U}(1)_B$:
$$Q \to e^{i\alpha} Q$$
$$U_R \to e^{i\alpha} U_R$$
$$D_R \to e^{i\alpha} D_R$$

$\mathcal{U}(1)_e$:
$$e_L \to e^{i\alpha} e_L$$
$$e_R \to e^{i\alpha} e_R$$

$\mathcal{U}(1)_{\mu,\tau}$: analogous to $\mathcal{U}(1)_e$

$\mathcal{U}(1)_Y$: Hypercharge transformation (Gauged)

Comments:

1) the unbroken $\mathcal{U}(1)^5_{SM}$ symmetry is an ACCIDENTAL SYMMETRY of the SM.

2) If $y_f^{ij} \ll 1$, then the broken flavor symmetry is an APPROXIMATE SYMMETRY. (E.g. $\mathcal{U}(2)^5$)

B. THE STANDARD MODEL AS AN EFT

There is one axiom of the SM which was intro-duced only for the purpose of calculation: the axiom of RENORMALITZABILITY.

Imagine adding to the SM the dim-6 operator

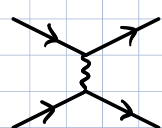
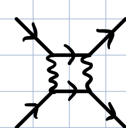
$$Q^{VLL}_{1122} = (\bar{e}_L \gamma_\alpha e_L)(\bar{\mu}_L \gamma^\alpha \mu_L)$$

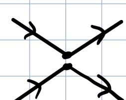
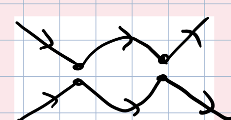
such that

has dim $= 1/M^2$

$$\mathcal{L} = \mathcal{L}_{SM} + C^{VLL}_{1122}\, Q^{VLL}_{1122}$$

and calculate the amplitude for $e^-\mu^- \to e^-\mu^-$

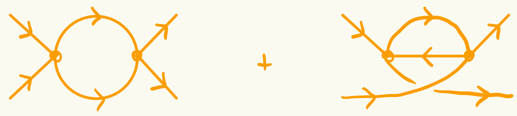
$i\mathcal{A}(e^-\mu^- \to e^-\mu^-) =$  $+$  $+ \cdots$

$+$  $+$  $+ \cdots$

Needs counterterm of order $1/M^4$

$\Rightarrow$ Needs counterterm of dim=8 operator!

$$\left(\text{e.g. } C^{(8)} \cdot [\bar{e}_L \gamma_\alpha \partial_\beta e_L][\bar{\mu}_L \gamma^\alpha \partial^\beta \mu_L]\right) \quad (*)$$

(*) 

$$= \frac{1}{\varepsilon} \cdot \frac{2}{3}(3s-u)\; \bar{u}(p_1')\gamma_\mu P_L u(p_1) \cdot \bar{u}(p_2')\gamma^\mu P_L u(p_2) \;+\; \text{finite}$$

This divergence can be cancelled by a counterterm of the dim = 8 operator:

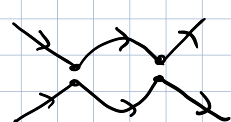
$$O^{(8)} = 3\left(\bar{e}_L \gamma_\mu P_L \partial_\nu e_L\right)\left(\bar{\mu}_L \gamma^\mu P_L \partial^\nu \mu_L\right) - \left(\bar{e}_L \gamma_\mu P_L \partial_\nu e_L\right)\left(\partial^\nu \bar{\mu}_L \gamma^\mu P_L \mu_L\right)$$

The solution is to adopt a POWER COUNTING :

$$C^{VLL}_{1122} \sim 1/\Lambda^2 \quad ; \quad C^{(8)} \sim 1/\Lambda^4$$

for $\Lambda \gg E$, $\mathcal{A} = 1 + E^2/\Lambda^2 + E^4/\Lambda^4 + \dots$

$\Rightarrow$ We work to fixed order in $E/\Lambda \ll 1$.

To $O(E^2/\Lambda^2)$ the contribution  is of higher order and can be dropped. Then it is consistent to add only dim-6 operators.

$\Rightarrow$ This EFT can be renormalized.

Thus, removing the renormalitzability axiom:

$$\mathcal{L} = \underbrace{\mathcal{L}^{(D\leq 4)}}_{\mathcal{L}_{SM}} + \frac{1}{\Lambda^{D-4}} \mathcal{L}^{(D>4)}$$

"cut-off" scale

$\leftrightarrow$ Power counting parameter

At energies $E \ll \Lambda$:

$$\text{Observable} = \text{Observable}(SM) + c_5 \, E/\Lambda + c_6 \, E^2/\Lambda^2 + \cdots$$

The fact that $E \ll \Lambda$ for all energies E probed so far explains why the SM works so well !!!

The current consensus is that

$$\boxed{SM = EFT = SMEFT}$$

The SMEFT contains: (e.g. 1008.4884)

$$\left.\begin{array}{l} 1 \ \text{dim}=5 \ \text{operator} \ (\text{Weinberg's op.}) \\ 59+4 \ \text{dim}=6 \ \text{operators.} \end{array}\right\} \times \text{Flavor}$$

$$= \left| \begin{array}{l} 12 \ \text{dim}=5 \ \text{ops} \\ 2499 + 546 \ \ \text{dim}=6 \ \ \text{ops} \end{array}\right.$$

B-conserving B-violating

Examples:

1) Dim-5 operators:

$$Q_{\ell\ell HH} = (\tilde{H}^\dagger \ell)^T C (\tilde{H}^\dagger \ell) \qquad \left(\text{and} \quad Q_{\ell\ell HH}^\dagger\right)$$

× Flavor:

$$Q_{\ell\ell HH}^{ij} = (\tilde{H}^\dagger \ell_i)^T C (\tilde{H}^\dagger \ell_j)$$

$$i,j = \{1,2,3\}$$

But: $Q_{\ell\ell HH}^{ij} = Q_{\ell\ell HH}^{ji}$

$\Rightarrow$ 6 indep. ops $(+ 6 \text{ h.c.}) = \underline{12}$

2) Dim-6 B-violating op:

$$Q_{duue} = \varepsilon_{\alpha\beta\sigma} (d_\alpha^T C u_\beta)(u_\sigma^T C e) \qquad [+ \text{h.c.}]$$

× Flavor:

$$Q_{duue}^{ijkl} = \varepsilon_{\alpha\beta\sigma} (d_{\alpha i}^T C u_{\beta j})(u_{\sigma k}^T C e_l)$$

$$i,j,k,l = \{1,2,3\}$$

$\Rightarrow$ 81 indep ops $(+ 81 \text{ h.c.}) = \underline{162}$

The SMEFT Lagrangian is

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \sum_i C_i^{(5)} Q_i^{(5)} + \sum_i C_i^{(6)} Q_i^{(6)} + \cdots$$

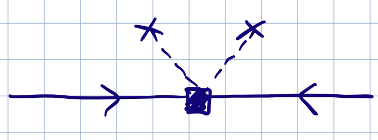
where

$$C_i^{(d)} \sim c/\Lambda^{d-4} .$$

The renormalizable SM has $C_i^{(d)} = 0$. Thus in order to probe this idea we need to measure non-zero values for some $C_i^{(d)}$.

In a sense we already have evidence of $C_i^{(5)}$:

$$Q_{\ell\ell HH} = (\tilde{H}^+ \ell)^T C (\tilde{H}^+ \ell) \xrightarrow[\langle H \rangle = v/\sqrt{2}]{} \frac{1}{2} v^2 \nu^T C \nu + \cdots$$



which is a majorana mass for the neutrino:

$$M_\nu \sim c \frac{v^2}{\Lambda}$$

The smallness of M_ν implies $\Lambda \sim 10^{14}\,\text{GeV}$.

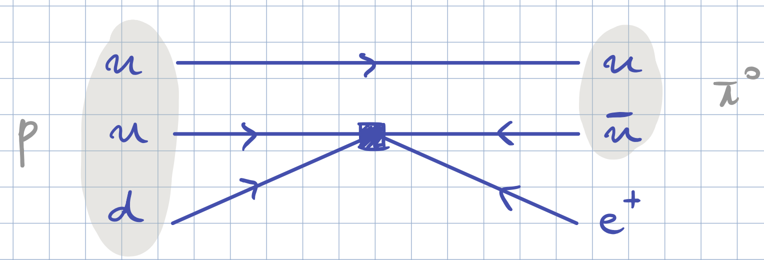
(with assumptions)

Note that Q_{eeHH} breaks Lepton number, which is an ACCIDENTAL SYMMETRY of $\mathcal{L}(D\leq 4)$.

Thus it seems that in order to test $\mathcal{L}_{D>4}$ it is a good idea to look at lepton number violating observables. These are obvious null tests of the SM.

Another example are BARYON NUMBER VIOLATING observables, such as proton decay. This can be mediated by e.g. the previous dim=6 op:

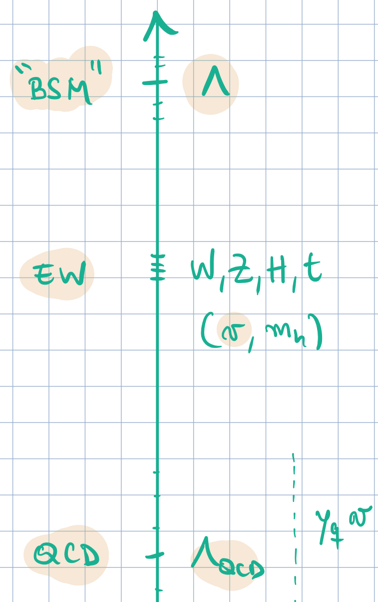
$$Q_{duue} = \varepsilon_{\alpha\beta\sigma}\,(d_\alpha^T\, C\, u_\beta)(u_\sigma^T\, C\, e)$$



Currently $\tau_p \gtrsim 10^{30}$ yrs $\Rightarrow$ $\Lambda \gtrsim 10^{15}$ GeV

Note that the scale Λ related to neutrino mass need not be the same as the scale Λ related to p-decay.

The "cut-off" scale Λ is some scale related to some new physics at high energy:



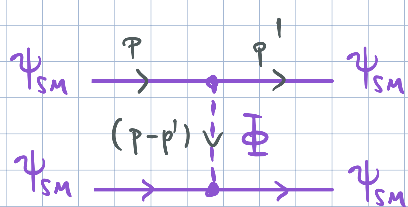
$$\mathcal{L} = \overbrace{\mathcal{L}_{QCD} + \mathcal{L}_{QED} + \mathcal{L}_{EW}}^{\mathcal{L}_{SM}} + \mathcal{L}_{\Lambda}$$

renorm.

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \mathcal{L}^{D>4}$$

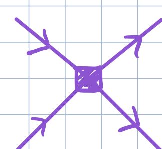
$$\mathcal{L}_{WET} = \mathcal{L}_{LEFT}$$

E.g.



$$= \left(\bar{\psi}_{SM}\psi_{SM}\right) \cdot \frac{i\, g^2}{t - M_\Phi^2} \cdot \left(\bar{\psi}_{SM}\psi_{SM}\right)$$

$$= -i\, \frac{g^2}{m_\phi^2}\left[1 + \frac{t}{m_\phi^2} + \frac{t^2}{m_\phi^4} + \dots\right] =$$



$$\mathcal{L}_{eff} = -\frac{g^2}{m_\phi^2}\left(\bar{\psi}_{SM}\psi_{SM}\right)^2 + \dots$$

$$\Rightarrow \quad \mathcal{L}^{D>4} = \sum_i C_i(\mu)\, Q_i(\mu)$$

Depends on IR scales

Depends on UV scales.

The "Wilson coefficients" $C_i \sim c_i/\Lambda^{d-4}$ are just some couplings that can be calculated if we know the UV theory

Note: The SMEFT has to be renormalized at the given order in the $1/\Lambda$ expansion. This means the Wilson coefficients $C_i(\mu)$ and the operators $O_i(\mu)$ depend on the renormalization scale μ. The dependence of $C_i(\mu)$ (or $O_i(\mu)$) with μ is described by the BETA FUNCTIONS or ANOMALOUS DIMENSIONS of the SMEFT operators, in the form of an RGE, e.g:

$$\boxed{\frac{d}{d\log\mu} C_i(\mu) = \beta_i(\vec{C})}$$

and leads also to operator mixing.

(See: 1308.2627; 1310.4838; 1312.2014)

DIMENSION-6 OPS IN THE SMEFT

$(\varphi \equiv H)$ [From 1704.04504. Original: 1008.4884]

dim	class	# operators	quantum numbers
5	Dimension-five	1	$\Delta L = 2$
6	X^3	4	
6	φ^6	1	
6	$\varphi^4 D^2$	2	
6	$X^2\varphi^2$	8	
6	$\psi^2\varphi^3$	3	
6	$\psi^2 X\varphi$	8	
6	$\psi^2\varphi^2 D$	8	
6	$(\bar{L}L)(\bar{L}L)$	5	
6	$(\bar{R}R)(\bar{R}R)$	7	
6	$(\bar{L}L)(\bar{R}R)$	8	
6	$(\bar{L}R)(\bar{L}R)$	4	
6	$(\bar{L}R)(\bar{R}L)$	1	
6	Baryon-number-violating	4	$\Delta B = \Delta L = 1$

× Flavor

PURELY BOSONIC

X^3		$X^2\varphi^2$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi G}$	$\varphi^\dagger \varphi G^A_{\mu\nu} G^{A\mu\nu}$
$Q_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi B}$	$\varphi^\dagger \varphi B_{\mu\nu} B^{\mu\nu}$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi W}$	$\varphi^\dagger \varphi W^I_{\mu\nu} W^{I\mu\nu}$
$Q_{\widetilde{W}}$	$\epsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi WB}$	$\varphi^\dagger \tau^I \varphi W^I_{\mu\nu} B^{\mu\nu}$
φ^6		$Q_{\varphi\widetilde{G}}$	$\varphi^\dagger \varphi \widetilde{G}^A_{\mu\nu} G^{A\mu\nu}$
Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{\varphi\widetilde{B}}$	$\varphi^\dagger \varphi \widetilde{B}_{\mu\nu} B^{\mu\nu}$
$\varphi^4 D^2$		$Q_{\varphi\widetilde{W}}$	$\varphi^\dagger \varphi \widetilde{W}^I_{\mu\nu} W^{I\mu\nu}$
$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\,\Box\,(\varphi^\dagger\varphi)$	$Q_{\varphi\widetilde{W}B}$	$\varphi^\dagger \tau^I \varphi \widetilde{W}^I_{\mu\nu} B^{\mu\nu}$
$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu \varphi\right)^* \left(\varphi^\dagger D_\mu \varphi\right)$		

2-FERMION

$\psi^2\varphi^3$		$\psi^2\varphi^2 D$	
$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q u \widetilde{\varphi})$	$Q^{(1)}_{\varphi\ell}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar\ell \gamma^\mu \ell)$
$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q d \varphi)$	$Q^{(3)}_{\varphi\ell}$	$(\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar\ell \tau^I \gamma^\mu \ell)$
$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar \ell e \varphi)$	$Q_{\varphi e}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar e \gamma^\mu e)$
$\psi^2 X\varphi$		$Q^{(1)}_{\varphi q}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar q \gamma^\mu q)$
Q_{eW}	$(\bar\ell \sigma^{\mu\nu} e)\, \tau^I \varphi W^I_{\mu\nu}$	$Q^{(3)}_{\varphi q}$	$(\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar q \tau^I \gamma^\mu q)$
Q_{eB}	$(\bar\ell \sigma^{\mu\nu} e)\, \varphi B_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar u \gamma^\mu u)$
Q_{uG}	$(\bar q \sigma^{\mu\nu} T^A u)\, \widetilde{\varphi} G^A_{\mu\nu}$	$Q_{\varphi d}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar d \gamma^\mu d)$
Q_{uW}	$(\bar q \sigma^{\mu\nu} u)\, \tau^I \widetilde{\varphi} W^I_{\mu\nu}$	$Q_{\varphi ud}$	$(\widetilde{\varphi}^\dagger i D_\mu \varphi)(\bar u \gamma^\mu d)$
Q_{uB}	$(\bar q \sigma^{\mu\nu} u)\, \widetilde{\varphi} B_{\mu\nu}$		
Q_{dG}	$(\bar q \sigma^{\mu\nu} T^A d)\, \varphi G^A_{\mu\nu}$		
Q_{dW}	$(\bar q \sigma^{\mu\nu} d)\, \tau^I \varphi W^I_{\mu\nu}$		
Q_{dB}	$(\bar q \sigma^{\mu\nu} d)\, \varphi B_{\mu\nu}$		

4- FERMION

$(\bar{L}L)(\bar{L}L)$	
$Q_{\ell\ell}$	$(\bar{\ell}\gamma_\mu \ell)(\bar{\ell}\gamma^\mu \ell)$
$Q_{qq}^{(1)}$	$(\bar{q}\gamma_\mu q)(\bar{q}\gamma^\mu q)$
$Q_{qq}^{(3)}$	$(\bar{q}\gamma_\mu \tau^I q)(\bar{q}\gamma^\mu \tau^I q)$
$Q_{\ell q}^{(1)}$	$(\bar{\ell}\gamma_\mu \ell)(\bar{q}\gamma^\mu q)$
$Q_{\ell q}^{(3)}$	$(\bar{\ell}\gamma_\mu \tau^I \ell)(\bar{q}\gamma^\mu \tau^I q)$

$(\bar{R}R)(\bar{R}R)$	
Q_{ee}	$(\bar{e}\gamma_\mu e)(\bar{e}\gamma^\mu e)$
Q_{uu}	$(\bar{u}\gamma_\mu u)(\bar{u}\gamma^\mu u)$
Q_{dd}	$(\bar{d}\gamma_\mu d)(\bar{d}\gamma^\mu d)$
Q_{eu}	$(\bar{e}\gamma_\mu e)(\bar{u}\gamma^\mu u)$
Q_{ed}	$(\bar{e}\gamma_\mu e)(\bar{d}\gamma^\mu d)$
$Q_{ud}^{(1)}$	$(\bar{u}\gamma_\mu u)(\bar{d}\gamma^\mu d)$
$Q_{ud}^{(8)}$	$(\bar{u}\gamma_\mu T^A u)(\bar{d}\gamma^\mu T^A d)$

$(\bar{L}L)(\bar{R}R)$	
$Q_{\ell e}$	$(\bar{\ell}\gamma_\mu \ell)(\bar{e}\gamma^\mu e)$
$Q_{\ell u}$	$(\bar{\ell}\gamma_\mu \ell)(\bar{u}\gamma^\mu u)$
$Q_{\ell d}$	$(\bar{\ell}\gamma_\mu \ell)(\bar{d}\gamma^\mu d)$
Q_{qe}	$(\bar{q}\gamma_\mu q)(\bar{e}\gamma^\mu e)$
$Q_{qu}^{(1)}$	$(\bar{q}\gamma_\mu q)(\bar{u}\gamma^\mu u)$
$Q_{qu}^{(8)}$	$(\bar{q}\gamma_\mu T^A q)(\bar{u}\gamma^\mu T^A u)$
$Q_{qd}^{(1)}$	$(\bar{q}\gamma_\mu q)(\bar{d}\gamma^\mu d)$
$Q_{qd}^{(8)}$	$(\bar{q}\gamma_\mu T^A q)(\bar{d}\gamma^\mu T^A d)$

$(\bar{L}R)(\bar{R}L)$	
$Q_{\ell edq}$	$(\bar{\ell}^j e)(\bar{d} q^j)$

$(\bar{L}R)(\bar{L}R)$	
$Q_{quqd}^{(1)}$	$(\bar{q}^j u)\,\epsilon_{jk}\,(\bar{q}^k d)$
$Q_{quqd}^{(8)}$	$(\bar{q}^j T^A u)\,\epsilon_{jk}\,(\bar{q}^k T^A d)$
$Q_{\ell equ}^{(1)}$	$(\bar{\ell}^j e)\,\epsilon_{jk}\,(\bar{q}^k u)$
$Q_{\ell equ}^{(3)}$	$(\bar{\ell}^j \sigma_{\mu\nu} e)\,\epsilon_{jk}\,(\bar{q}^k \sigma^{\mu\nu} u)$

Baryon-number-violating	
$Q_{duq\ell}$	$(d^T C u)(q^T C \ell)$
Q_{qque}	$(q^T C q)(u^T C e)$
$Q_{qqq\ell}$	$\epsilon_{il}\epsilon_{jk}\,(q_i^T C q_j)(q_k^T C \ell_l)$
Q_{duue}	$(d^T C u)(u^T C e)$

C. FLAVOR IN THE SMEFT

Let's start by looking at fermion masses and the CKM matrix within the setting of the SMEFT.

The relevant terms in the Lagrangian that will lead to fermion mass terms after SSB include of course the Yukawas, but now there are additional contributions. At dim = 6 we have:

$$Q_{uH}^{ij} = (H^\dagger H)(\bar{q}^i u^j \tilde{H}) \longrightarrow \frac{v^3}{2\sqrt{2}} \bar{u}_L^i u_R^j$$

$$Q_{dH}^{ij} = (H^\dagger H)(\bar{q}^i d^j H) \longrightarrow \frac{v^3}{2\sqrt{2}} \bar{d}_L^i d_R^j$$

$$Q_{eH}^{ij} = (H^\dagger H)(\bar{\ell}^i e^j H) \longrightarrow \frac{v^3}{2\sqrt{2}} \bar{e}_L^i e_R^j$$

Exercise: Check this and convince yourself that no other $d=6$ SMEFT ops contribute to $\mathcal{L}_{m_\psi}$.

Thus in the SMEFT at $d=6$ we have:

$$\mathcal{L}_{m_\psi} = - \sum_{\psi = E,D,U} M_\psi^{ij} \bar{\psi}_L^i \psi_R^j + h.c.$$

with now:

$$\boxed{M_\psi = \frac{v}{\sqrt{2}}\left[y_\psi - \frac{v^2}{2}\, C_{\psi H}\right]} \qquad (*)$$

and thus the relationship among fermion masses and Yukawas is modified.

One can still use the flavor symmetry to write:

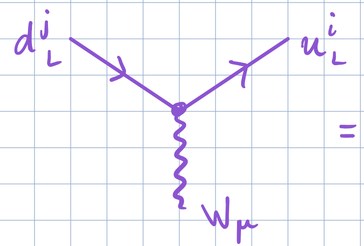
$$M_e = M_E = \mathrm{diag}\,(m_e, m_\mu, m_\tau)$$
$$M_u = M_U = \mathrm{diag}\,(m_u, m_c, m_t)$$
$$M_D = V_{CKM} \times \mathrm{diag}\,(m_d, m_s, m_b) \equiv V_{CKM} \cdot M_d .$$

but now the Yukawa couplings do not follow this pattern anymore.

The matrix V_{CKM} is still called the CKM matrix, although it is affected by $d=6$ operators. It is still a unitary 3×3 matrix, and still parametrized by 4 real parameters (including one phase). We can still use the Wolfenstein parametrization.

But contrary to the SM, the structure of charged currents is not uniquely determined by the

CKM matrix, but it is also affected by the presence of dim=6 operators, e.g.:



$$= -i \frac{g}{\sqrt{2}} \left[V_{ij} + v^2 \left[C_{Hq}^{(3)} \right]_{ij} \right] \gamma_\mu \qquad (\#)$$

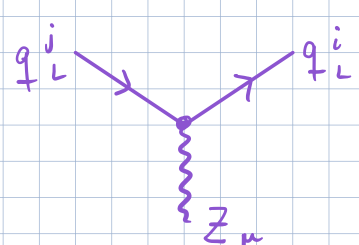
Exercise: Prove this and check if there are further contributions from other dim-6 ops in the SMEFT.

In eqs. (*) and (#) the quantity v refers to the Higgs vev in the presence of dim-6 ops:

$$v = \left(1 + \frac{3 C_H v^2}{8 \lambda} \right) v_{SM}$$

where C_H is the coefficient of $O_H = (H^\dagger H)^3$.

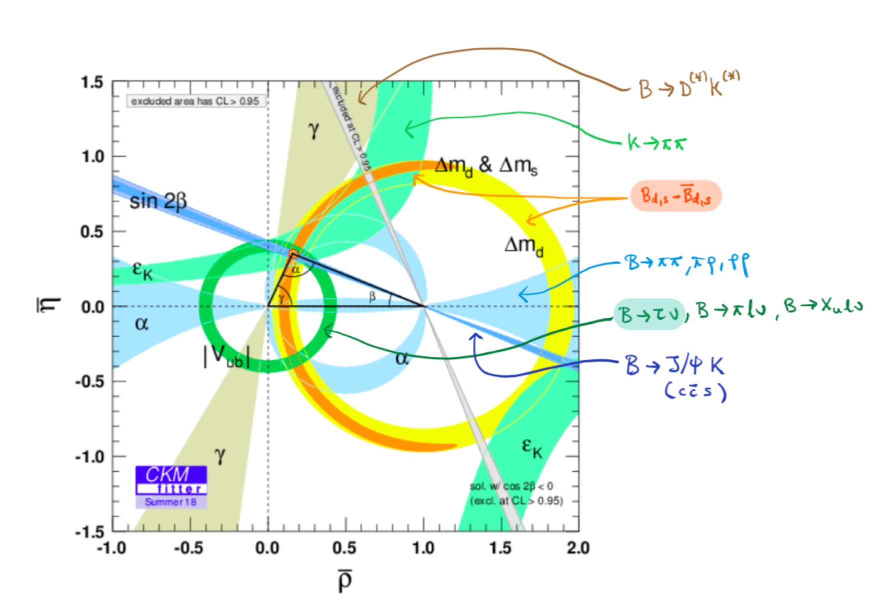
In the presence of dim-6 ops there are many new contributions that lead to ΔF transitions, e.g.



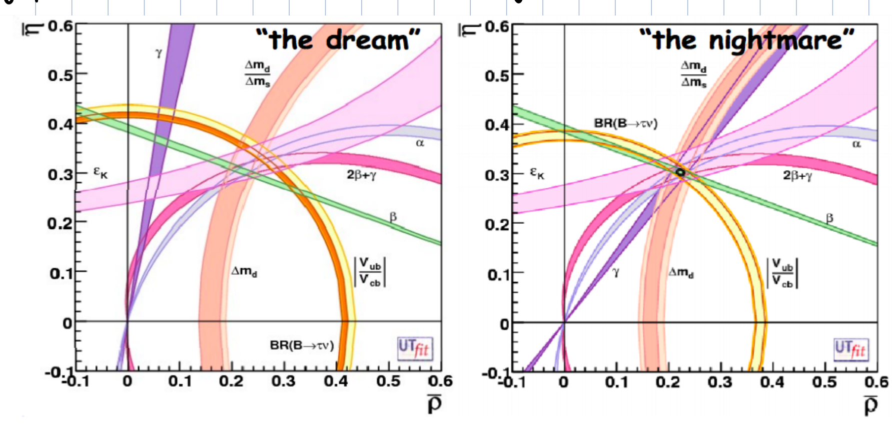
$$\propto \left[C_{Hq}^{(1)} \right]_{ij}, \left[C_{Hq}^{(3)} \right]_{ij}$$

($\equiv$ anomalous Z couplings)

All this has many implications. One of them is that the global (SM) determination of the CKM parameters will not work if some SMEFT dim = 6 coefficients are $\neq 0$.



Two typical situations may be the following:



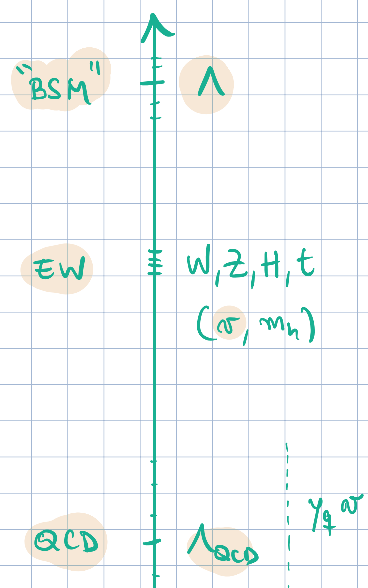
We'll see an example in the Exercise session.

[The general case is discussed in 1812.08163]

D. BELOW THE EW SCALE

[E.g. hep-ph/9512380 ; 1704.05672 ; 1709.04486]

In experiments with $E \ll \Lambda_{EW} \sim v$ we can perform the same type of EFT expansion as before, but with EW-scale particles integrated out:

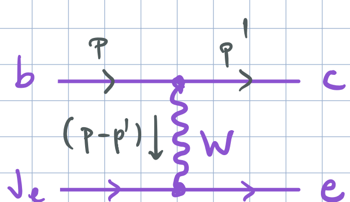


$$\mathcal{L}_{SMEFT} = \overbrace{\mathcal{L}_{QCD} + \mathcal{L}_{QED} + \mathcal{L}_{EW}}^{\mathcal{L}_{SM}} + \mathcal{L}^{D>4}$$

$$\boxed{\mathcal{L}_{WET\,(LEFT)} = \mathcal{L}_{QCD} + \mathcal{L}_{QED} + \mathcal{L}^{D>4}_{WET}}$$

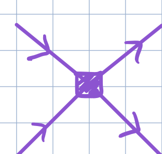
[WET ≡ WEAK EFFECTIVE THEORY
LEFT ≡ LOW ENERGY EFT]

E.g.



$$= \left(\bar{c}\,\gamma_\mu P_L b\right)\cdot \frac{i\,g^2/2}{t - M_W^2}\cdot\left(\bar{e}\,\gamma^\mu P_L \nu_e\right)$$

$$= -i\,\frac{g^2}{2\,m_W^2}\left[1 + \frac{t}{m_W^2} + \frac{t^2}{m_W^4} + \cdots\right]\langle O_{eff}\rangle =$$



$$\mathcal{L}_{eff} = -\frac{4G_F}{\sqrt{2}}\left(\bar{d}\,\gamma_\mu P_L c\right)\left(\bar{\nu}_e \gamma^\mu e\right) + \cdots$$

where $G_F \equiv \dfrac{g^2}{4\sqrt{2}\, m_W^2}$ is FERMI'S CONSTANT.

Thus:

$$\boxed{\mathcal{L}_{WET/LEFT} = \mathcal{L}_{QCD} + \mathcal{L}_{QED} + \sum_i L_i\, \mathcal{O}_i}$$

without top quark

"Wilson" coefficients

Operators build from light fields and invariant under $SU(3)_c \times U(1)_{em}$.

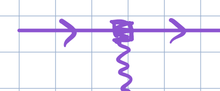
Some WET/LEFT operators:

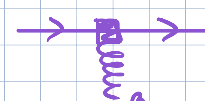
dim-5:

$$\mathcal{O}_{e\gamma}^{11} = \bar{e}_L\, \sigma^{\mu\nu} e_R\, F_{\mu\nu}$$

$$\mathcal{O}_{d\gamma}^{23} = \bar{s}_L\, \sigma^{\mu\nu} b_R\, F_{\mu\nu}$$

$$\mathcal{O}_{dG}^{13} = \bar{d}_L\, \sigma^{\mu\nu} T^a b_R\, G^a_{\mu\nu}$$

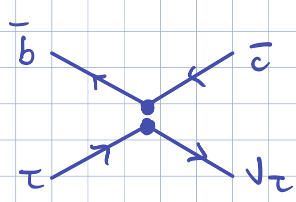


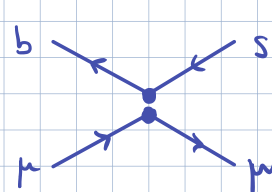


dim-6:

$$\left[\mathcal{O}^{V,LL}_{\nu edu}\right]_{3332} = (\bar{\nu}_\mu \gamma^\alpha \mu)(\bar{b}_L \gamma_\alpha c_L)$$

$$\left[\mathcal{O}^{V,RR}_{ed}\right]_{2232} = (\bar{\mu}_R \gamma^\alpha \mu_R)(\bar{b}_R \gamma_\alpha s_R)$$





The number of independent operators is even larger than the case of SMEFT (e.g. 1709.04486)

$$\left.\begin{array}{l} 6 \text{ dim-5 ops (dipoles)} \\ 73+16 \text{ dim-6 ops} \end{array}\right\} \times \text{ Flavor}$$

$$= \left| \begin{array}{l} 75 \text{ dim-5 ops} \\ \underset{\text{B-conserving}}{6115} + \underset{\text{B-violating}}{1032} \text{ dim-6 ops} \end{array}\right.$$

(see notebook)

We can also calculate the Wilson coefficients if we know the UV theory. In this case, the WCs are non-zero in the pure SM case. E.g.:

$$[L_{\nu edu}]_{3332} = -\frac{2}{v^2} V_{cb}^* + 2 V_{ib}^* \left[C_{\ell q}^{(3)}\right]_{33i2} - 2 V_{ib}^* \left[C_{Hq}^{(3)}\right]^*_{2i} - 2 V_{cb}^* \left[C_{He}^{(3)}\right]_{33}$$

Exercise: Prove this and show that there are no further contributions from other dim-6 ops in the SMEFT.

Some important coments:

→ QCD + QED is the Low-E EFT of the SM (EFT)

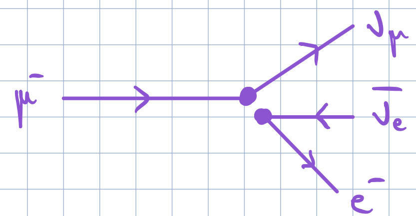
→ the cut-off is $\Lambda \sim \Lambda_{EW} \sim v$ (EW is "NP")

→ The full FLAVOUR Group is ACCIDENTAL SYM.

Example: Muon decay

The muon is stable in QCD + QED. Its decay amplitude arises from dim-6 ops in the LEFT, e.g.

$$\delta \mathcal{L} = \frac{c}{\Lambda^2} (\bar{\nu}_\mu \gamma^\alpha P_L \mu)(\bar{e} \gamma_\alpha P_L \nu_e)$$



At tree level (EXERCISE): $\Gamma_\mu \simeq \dfrac{m_\mu^5 \, c^2}{1536 \, \pi^3 \Lambda^4}$ (1)

Experimentally: $\tau_\mu^{exp} = 2.2 \times 10^{-6}$ s

$\Rightarrow \Gamma_\mu^{exp} = 3 \cdot 10^{-19}$ GeV (2)

Comparing (1) and (2), with $c \sim 1$:

$$\Lambda \simeq 172 \text{ GeV} \simeq v/\sqrt{2} \quad \Rightarrow \quad \Lambda \simeq \Lambda_{EW} \,!$$ ✓

⇒ We "discover" the EW scale, and measure it!

Note: the tree-level SM matching calculation gives:

$$c/\Lambda^2 = -g^2/2M_W^2 = -2/v^2 = -4G_F/\sqrt{2}$$

τ_μ is used to determine v (or G_F):

$$v = 246.21965(6) \text{ GeV}$$
$$G_F = 1.1663787(5) \text{ GeV}^{-2}$$

(SM)

But this determination is modified if there is BSM physics:

$$v = 246.21965(6)\,(1 + \delta_v)$$

with

$$\delta_v = v^2 \left([C_{He}^{(3)}]_{\mu\mu} + [C_{He}^{(3)}]_{ee} - \frac{1}{2}[C_{ll}]_{\mu ee\mu} - \frac{1}{2}[C_{ll}]_{e\mu\mu e} \right)$$

(See e.g. 1812.08163)

The p-decay and μ-decay philosophy applies to all flavor transitions:

$$\boxed{\text{LOW-LYING HADRONS are STABLE at } D \leq 4}$$

$K^0 \sim \bar{s}d$; $K^+ \sim \bar{s}u$; $D^0 \sim \bar{u}c$; $D^+ \sim \bar{d}c$

$B^0 \sim \bar{b}d$; $B^+ \sim \bar{b}u$; $B_s^0 \sim \bar{b}s$; $\Lambda_b \sim udb$; ...

<u>Exercise</u>: Look at the PDG (Particle data group) particle listings and check that:

$$\tau_{\text{mesons}} \sim \tau_{\text{baryons}} \gtrsim 10^{-12}\,s \sim 10^{12}\,\text{GeV}^{-1}$$

$$\tau_{\text{Resonances}} \lesssim 10^{-23}\,s \sim 10\,\text{GeV}^{-1}$$

This fact has two consequences:

1. Definition of Hadrons

$$H = \underset{\uparrow \atop \text{QCD (+QED)}}{H_0} + \underset{\nwarrow \atop D>4}{H_{int}} \quad ; \quad \text{e.g. } H_0|B\rangle = m_B|B\rangle$$

2. Weak decays as probes of BSM vs EW. (Chapter 5).

E. WEAK DECAYS AS PROBES OF BSM vs EW

We have seen that studying transitions in hadrons that are stable under QCD we are directly probing flavor transitions, which can happen through either EW physics or BSM physics. And therefore weak decays of hadrons are excellent tests of EW physics and probes of New Physics.

We will go through 3 simple examples using the formalism that we have described in previous sections.

EXAMPLE 1 : $b \to c \, l \nu$

Consider the transition $b \to c l \nu$ mediated by the term in the eff. Lagrangian:

$$\mathcal{L}_{eff}^{(6)} = C_1 \, (\bar{c} \gamma_\mu P_L b)(\bar{l} \gamma^\mu P_L \nu)$$

(there are actually 5 independent $d=6$ operators of the $b \to c l \nu$ type. For this example one is enough.)

This quark-level transition is realized in decay observables such as

$$B \to D\, l \bar{\nu}\,,\quad B \to D^* l \bar{\nu}\,,\quad B_s \to D_s\, l\bar{\nu}\,,\quad \Lambda_b \to \Lambda_c\, l \bar{\nu}\,, \cdots$$

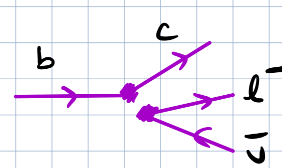
$$B \to X_c\, l \bar{\nu}\,,\quad \cdots$$

The theory prediction for all these observables will depend on the Wilson coefficient C_1.

What is C_1 in the SM and in BSM models?
To answer this question we need to do a matching calculation.

We consider the partonic amplitude $\mathcal{A}(b \to c\, l \bar{\nu})$, and require the EFT amplitude to be equal to the (expanded) "full theory" amplitude.

We have (up to higher perturbative orders)

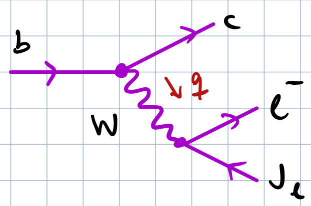
$i\mathcal{A}_{EFT}$ = 

$= i\, C_1 \left(\bar{u}_c \gamma_\mu P_L u_b\right)\left(\bar{u}_e \gamma^\mu P_L v_\nu\right)$

$$\Rightarrow \quad \mathcal{A}_{EFT} = C_1\, \bar{u}_c \gamma_\mu P_L u_b \cdot \bar{u}_l \gamma^\mu P_L v_\nu$$

In the SM, the process $b \to c \ell \nu$ is mediated by W exchange:

$$q^2 = (p_b - p_c)^2 \leq (m_b - m_c)^2 \ll M_W^2$$

$i\mathcal{A}_{SM} =$  $+ \dots$

$$= -\frac{g^2}{2} V_{cb}\, \bar{u}_c \gamma_\mu P_L u_b \cdot \bar{u}_e \gamma_\nu P_L v_\nu \cdot \overbrace{\frac{-i}{q^2 - M_W^2}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{M_W^2}\right)}^{\frac{i}{M_W^2} g^{\mu\nu} + \mathcal{O}\left(q^2/M_W^2\right)}$$

$+$ higher perturbative orders

$$= -i \frac{g^2}{2M_W^2} V_{cb}\, \bar{u}_c \gamma_\mu P_L u_b \cdot \bar{u}_e \gamma^\mu P_L v_\nu + \cdots$$

$$\Rightarrow \quad i\mathcal{A}_{SM} = -\frac{g^2}{2M_W^2} V_{cb}\, \bar{u}_c \gamma_\mu P_L u_b \cdot \bar{u}_e \gamma^\mu P_L v_\nu + \cdots$$

The LO SM matching condition is thus:

$$C_1^{SM} = -\frac{g^2 V_{cb}}{2M_W^2}$$

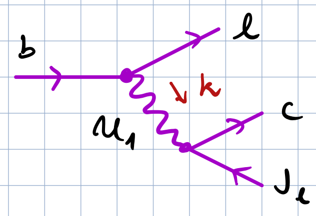
Flavor suppression $\sim \lambda^2$

Let's consider now a new "vector leptoquark" U_1:

$$\mathcal{L}_{LQ} = g_b\, \bar{e}\, \not{U}_1 P_L b + g_c\, \bar{\nu}_e \not{U}_1 P_L c + h.c.$$

$$+ \frac{1}{2} M_{U_1}^2 U_1^\mu U_{1\mu}$$

The LO contribution to the $b \to c \ell \nu$ amplitude from U_1 exchange is given by:

$i\mathcal{A}_{U_1} =$  $+ \dots$

$$= - g_b g_c \cdot \left[\frac{i}{M_{U_1}^2} + \mathcal{O}\left(k^2 / M_{U_1}^2 \right) \right] \bar{u}_\ell \gamma_\mu P_L u_b \cdot \bar{u}_c \gamma^\mu P_L v_\nu + \dots$$

$$= -i \frac{g_b g_c}{M_{U_1}^2} \underbrace{\bar{u}_\ell \gamma_\mu P_L u_b \cdot \bar{u}_c \gamma^\mu P_L v_\nu}_{- \bar{u}_c \gamma_\mu P_L u_b \cdot \bar{u}_\ell \gamma^\mu P_L v_\nu \quad \text{(Fierz)}} + \dots$$

$$\Rightarrow \quad \mathcal{A}_{U_1} = \frac{g_b g_c}{M_{U_1}^2} \bar{u}_c \gamma_\mu P_L u_b \cdot \bar{u}_\ell \gamma^\mu P_L v_\nu$$

Thus, the LO contribution to the matching condition from U_1 exchange is:

$$C_1^{U_1} = \frac{g_b g_c}{M_{U_1}^2}$$

Combining everything:

$$C_1 = - \frac{g^2 V_{cb}}{2 M_W^2} + \frac{g_b g_c}{M_{U_1}^2} + \dots = C_1^{SM} \left[1 - 2 \frac{g_b g_c}{g^2 V_{cb}} \cdot \frac{M_W^2}{M_{U_1}^2} + \dots \right]$$

What does this mean?

Imagine we measure some $b \to c l \nu$ observable and extract an experimental value for C_1 with a relative uncertainty σ, and consistent with the SM prediction:

$$C_1^{exp} = C_1^{SM} (1 \pm \sigma)$$

Then we conclude that

$$2 \frac{g_b g_c}{g^2 V_{cb}} \cdot \frac{M_W^2}{M_{U_1}^2} < \sigma \qquad (\text{e.g. @ } 68\% \text{ C.L.})$$

and therefore,

$$M_{U_1} > \sqrt{\frac{2 g_b g_c}{g^2 \sigma V_{cb}}} \, M_W .$$

Setting $g_b \sim g_c \sim g$

$V_{cb} \sim \lambda^2 \sim 0.04$

$\sigma \sim 10\%$

we find that $M_{U_1} \gtrsim 20 \, M_W \sim 2 \text{ TeV}.$

We probe the TeV scale!

EXAMPLE 2 : $b \to s \ell^+ \ell^-$ (FCNCs)

We consider now the FCNC transition $b \to s \ell^+ \ell^-$ which is mediated in the EFT by operators of the form (barring dipole ops)

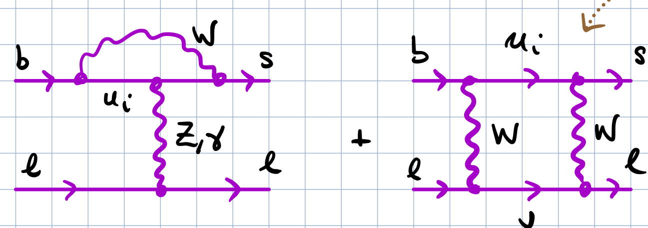
$$C_i \, (\bar{s}\, \Gamma_1^{(i)} b)(\bar{\ell}\, \Gamma_2^{(i)} \ell)$$

for different Dirac structures $\Gamma_j^{(i)}$.

The matching coefficients C_i in the SM arise only at one loop :

(c.f. Ch. 6.)

$$C_i^{SM} \sim$$



$+ \dots$

$$\sim \frac{g^4}{M_W^2} \times \frac{1}{(4\pi)^2} \times V_{is}^* V_{ib}$$

Loop   CKM

Notes:

1. It happens that $V_{us}^* V_{ub} \sim \lambda^4 \ll V_{cs}^* V_{cb} \sim V_{ts}^* V_{tb} \sim \lambda^2$
2. In the literature you will find:

$[\bar{s}\gamma_\mu P_L b][\bar{\ell}\gamma^\mu(\gamma_5)\ell]$

$$\mathcal{L}_{eff} = \underbrace{\frac{4G_F}{\sqrt{2}}}_{g^2/2M_W^2} V_{ts}^* V_{tb} \underbrace{\frac{\alpha}{4\pi}}_{e^2/(4\pi)^2} \left(C_9 O_9 + C_{10} O_{10} \right)$$

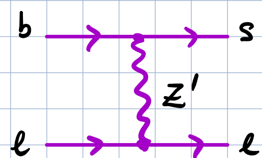
$\simeq 4$ $\simeq -4$

Let us consider two possible tree-level models:

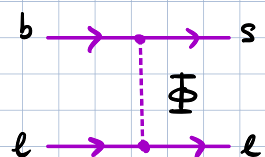
$$\mathcal{L}_{Z'} = g_{bs}\, \bar{s}\, \not{Z}'\, P_L\, b + g_\ell\, \bar{\ell}\, \not{Z}'\, \ell + h.c. + \frac{1}{2} M_{Z'}^2\, Z'_\mu Z'^\mu$$

$$\mathcal{L}_\Phi = g_b\, \Phi\, \bar{\ell}\, P_R\, b + g_s\, \Phi\, \bar{\ell}\, P_R\, s + h.c. + M_\phi^2\, \Phi^\dagger \Phi$$

We have:

$C_i^{Z'} \sim$ 

$\sim \dfrac{g_{bs}\, g_\ell}{M_{Z'}^2}$

$C_i^{\Phi} \sim$ 

$\sim \dfrac{g_b\, g_s}{M_\Phi^2}$

Thus, assuming as before that C_i is measured compatible with the SM with a relative accuracy of $\sigma \sim 10\%$, and assuming

$$g_s \sim g_b \sim g_{sb} \sim g_\ell \sim g$$

we find that

Tens of TeV!

$$M_{Z',\Phi} \gtrsim \frac{4\pi}{g \lambda \sqrt{\sigma}} M_W \simeq 20 \text{ TeV}$$

[For numerics note that $G_F = \dfrac{g^2}{4\sqrt{2} M_W^2} = 1.17 \cdot 10^{-5}\, \text{GeV}^{-2}$]

We will consider specific bounds on C_{10} and to $M_{Z'}$ and M_ϕ from the measurement of the $B_s \to \mu^+\mu^-$ branching ratio in the Exercise session.

Comment: There are currently some tensions between experimental determination and SM prediction for C_9, C_{10} from $b \to s\mu\mu$ observables. These tension are sometimes called the "Neutral-current B anomalies".

We can try to explain the anomalies by introducing either a Z' or a LQ ϕ.

This provides a measurement of the combinations (respectively)

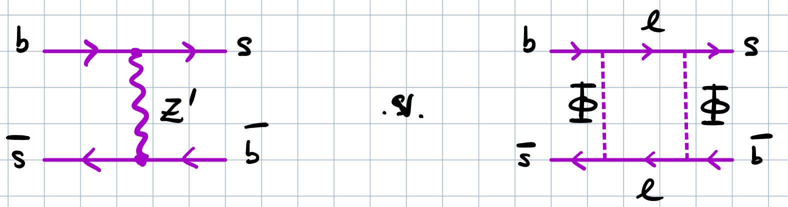
$$\frac{g_{bs}\, g_\ell}{M_{Z'}^2} \, , \quad \frac{g_b\, g_s}{M_\Phi^2} \neq 0$$

which need to be different from zero in order to address the anomalies.

However, these new particles contribute also to other processes that do <u>not</u> deviate from the SM.

An example is given by ΔM_s, which is related to the $b\bar{s} \to s\bar{b}$ amplitude.

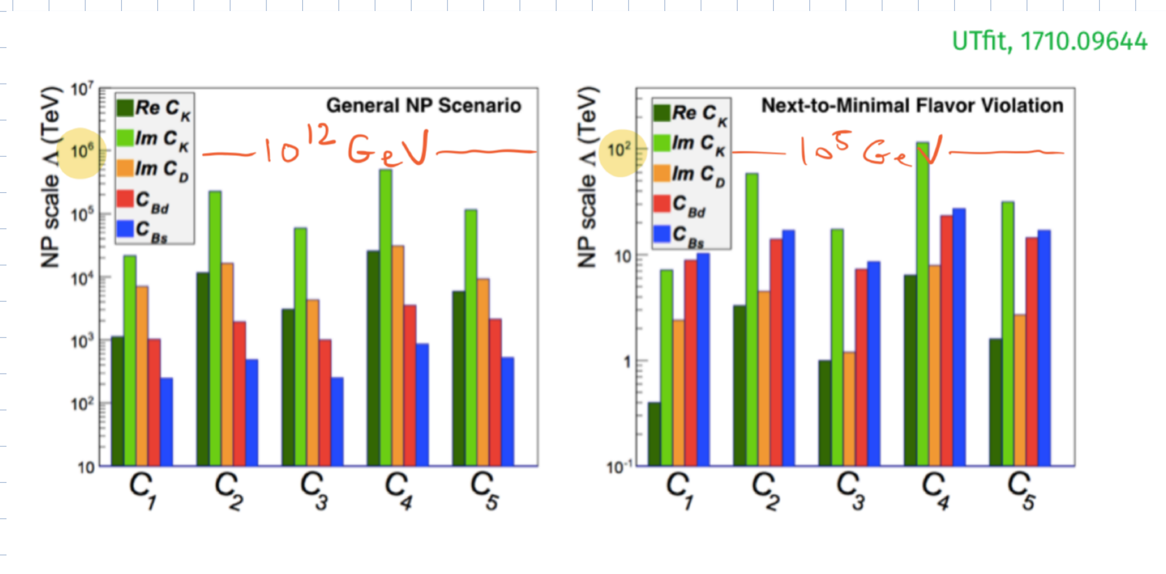
Note that this constraint is much more important for Z' than for ϕ, because:



This is one reason for which LQ models are so popular when addressing the B anomalies.

EXAMPLE 3 : Neutral meson mixing

(e.g. 0707.0636)



SUMMARY SO FAR

- SM = EFT (one less axiom)

  No problem (power counting)

  → $\Lambda \gg \Lambda_{EW}$ explains success of SM

  → Best tests : Accidental symmetries (p-decay , ν-masses)

$$\boxed{\mathcal{L} = \mathcal{L}^{D\leq 4} + \sum_i \frac{c_i}{\Lambda^{d-4}} \mathcal{O}_i^{(d)}}$$

Measure to learn about UV

- Flavor = Accidental symmetry at $E \ll \Lambda_{EW}$

  ↳ Look at weak decay & mixing of "stable" hadrons (e.g. K, $D_{(s)}$, $B_{(s)}$, Λ_b ...)
  (Leptons, e.g. τ, equally interesting)

  ↳ FCNC suppression of EW physics allows to reach $\underbrace{\Lambda \gg \Lambda_{EW}}$.

  Beyond the reach of direct production.

F. EFT for $b \to c \ell \bar{\nu}$

Write all local operators built out of

$u, d, s, c, b, e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau, F_{\mu\nu}, G^a_{\mu\nu}, D_\mu$

which are:

- Lorentz-invariant
- Local $SU(3)_c \times U(1)_{em}$ invariant
- Canonical dimension $d \leq 6$
- Not related by EOMs or Dirac Identities.

We have seen that, up to $d=4$

$$\mathcal{L}^{d\leq 4}_{LEFT} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} + \sum_{\substack{\psi = u,d,s,c,b \\ e,\mu,\tau,\nu_e,\nu_\mu,\nu_\tau}} \bar{\psi} (i\not{D} - m_\psi) \psi$$

Diagonal order-by-order

All operators in this Lagrangian satisfy

$$\Delta B = \Delta C = \Delta S = 0$$

and therefore cannot mediate FC transitions.

At $d=5$ we can only build "dipole operators":

$$\bar{\psi}_i \, \Gamma^{\mu\nu} \psi_j \, F_{\mu\nu} \qquad \text{and} \qquad \bar{q}_i \, \Gamma^{\mu\nu} T^a q_j \, G^a_{\mu\nu}$$

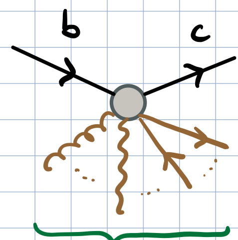
Dirac structure ($\sigma^{\mu\nu}$ or $\sigma^{\mu\nu}\gamma_5$)

These operators only mediate FCNCs.

We conclude that

$$\boxed{\text{FCCCs are mediated by } d \geq 6 \text{ operators in the WET / LEFT}}$$

Let's find all such operators of the type $\Delta B = -\Delta C = 1$:



Must be :
- $d=3$
- $Q=1$
- $\Delta B = \Delta C = 0$

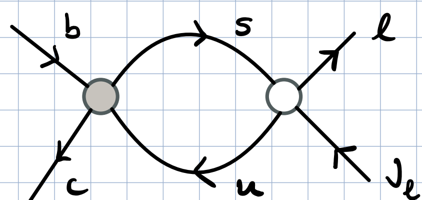
$\Rightarrow$ $\bar{d}u$, $\bar{s}u$, $\bar{\ell}\nu_{\ell'}$ $\qquad (\ell, \ell' = e, \mu, \tau)$

We are thus led to operators with field content:

$$\bar{c}\, b\, \bar{d}\, u\,, \quad \bar{c}\, b\, \bar{s}\, u\,, \quad \bar{c}\, b\, \bar{\ell}\, \nu_e\,,$$

Due to Flavour conservation of the $d \leq 4$ Lagrangian, up to dimension-6 we can consider the three types of operators separately. The technical statement is that they do not mix under renormalization.

Simply: $A(b \to c \ell \nu)$ will not receive contributions (up to $1/\Lambda^2$) from any ops other that $\bar{c} b \bar{\ell} \nu$ to any order in the coupling expansion.



$\sim \quad {}^{1}/_{\Lambda^4}$

The same is true for e.g. $\bar{c} b \bar{e} \nu_e$ vs $\bar{c} b \bar{\mu} \nu_\mu$.

Thus we need to construct a basis of operators with the field content

$$\bar{c}\, b\, \bar{\ell}\, \nu_e \qquad (\ell \text{ fixed})$$

Lorentz invariance requires these operators to be a product of two currents:

$$\bar{c}\,\Gamma\, b \cdot \bar{\ell}\,\Gamma'\,\nu_{\ell'} \qquad \text{or} \qquad \bar{c}\,\Gamma\,\nu_{\ell'} \cdot \bar{\ell}\,\Gamma'\, b$$

with Γ, Γ' arbitrary Dirac matrices, provided all Lorentz indices are contracted. E.g:

$$\Gamma \otimes \Gamma' = 1 \otimes 1; \; 1 \otimes \gamma_5 \; ; \; \gamma_\mu \otimes \gamma^\mu ; \; \gamma_\mu \otimes \gamma^\mu \gamma_5 \; ;$$

$$\gamma_\mu \gamma_\nu \otimes \gamma^\nu \gamma^\mu \; ; \; \gamma_\mu \gamma_\nu \gamma_5 \otimes \gamma^\mu \gamma^\nu \gamma_5 \, ; \; \cdots$$

in principle containing strings of arbitrary number of γ's and γ_5's.

In 4-dimensional spacetime, Γ, Γ' are 4×4 matrices in Dirac space:

$$\Gamma_{\alpha\beta} \quad , \quad \alpha, \beta = 1, \cdots, 4$$

and have 16 independent components. One can always write:

$$\boxed{\begin{aligned} \Gamma_{\alpha\beta} = \; & a\, \delta_{\alpha\beta} + a_5 (\gamma_5)_{\alpha\beta} + a_\mu (\gamma^\mu)_{\alpha\beta} \\ & + a_{5\mu} (\gamma^\mu \gamma_5)_{\alpha\beta} + a_{\mu\nu} (\sigma^{\mu\nu})_{\alpha\beta} \end{aligned}} \qquad (\Gamma)$$

where $\boxed{\sigma^{\mu\nu} \equiv \frac{i}{2}[\gamma^\mu,\gamma^\nu]}$ is an antisymmetric tensor, and thus $a_{\mu\nu}$ contains 6 independent components. In total

$$\{a, a_5, a_\mu, a_{\mu 5}, a_{\mu\nu}\} \rightarrow 16 \text{ degrees of freedom for } \Gamma_{\alpha\beta}.$$

Dirac algebra can be carried out by means of a few 4-dimensional relations:

- $\{\gamma_\mu,\gamma_\nu\} = 2g_{\mu\nu}$
- $\{\gamma_\mu,\gamma_5\} = 0$
- $Tr(\gamma_\mu) = Tr(\gamma_5) = 0$; $Tr(\gamma_\mu\gamma_\nu) = 4g_{\mu\nu}$
- $Tr(\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\lambda\gamma_5) = -4i\,\varepsilon_{\mu\nu\rho\lambda}$

Exercise:

(a) Using these relations, show that the coefficients in eq. (Γ) are given by:

$$a = \frac{1}{4}Tr(\Gamma)\ ;\quad a_5 = \frac{1}{4}Tr(\Gamma\gamma_5)\ ;\quad a_\mu = \frac{1}{4}Tr(\Gamma\gamma_\mu)\ ;$$

$$a_{5\mu} = -\frac{1}{4}Tr(\Gamma\gamma_\mu\gamma_5)\ ;\quad a_{\mu\nu} = \frac{1}{8}Tr(\Gamma\sigma_{\mu\nu})$$

(b) Express $\gamma_\mu \gamma_\nu \gamma_\rho$ and $\gamma_\mu \gamma_\nu \gamma_\rho \gamma_\lambda$ in terms of $\{1, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}\}$.

A last trick to reduce the number of independent operators is provided by Fierz Identities. Consider the operator

$$O_{VV} \equiv \bar{c}\gamma_\mu \nu_e \cdot \bar{\ell}\gamma^\mu b$$

Making Dirac indices explicit, we write:

$$O_{VV} = (\bar{c})_\alpha (\gamma_\mu)_{\alpha\beta} (\nu_e)_\beta \, (\bar{\ell})_\sigma (\gamma^\mu)_{\sigma\lambda} (b)_\lambda$$

$$= -\underbrace{(b)_\lambda (\bar{c}_\alpha)}_{M_{\lambda\alpha}} (\gamma_\mu)_{\alpha\beta} \underbrace{(\nu_e)_\beta (\bar{\ell})_\sigma}_{M'_{\beta\sigma}} (\gamma^\mu)_{\sigma\lambda}$$

anticommute b field 3 times

$$= - \mathrm{Tr}\left(M \gamma_\mu M' \gamma^\mu\right)$$

We can now decompose M, M' using eq.(Γ):

$$M = b\bar{c} = \frac{1}{4}\mathrm{Tr}(b\bar{c})\,\mathbb{1} + \frac{1}{4}\mathrm{Tr}(b\bar{c}\gamma_5)\,\gamma_5$$

$$+ \frac{1}{4}\mathrm{Tr}(b\bar{c}\gamma_\mu)\,\gamma^\mu - \frac{1}{4}\mathrm{Tr}(b\bar{c}\gamma_\mu\gamma_5)\,\gamma^\mu\gamma_5 + \frac{1}{8}\mathrm{Tr}(b\bar{c}\sigma_{\mu\nu})\,\sigma^{\mu\nu}$$

$$= -\frac{1}{4}[\bar{c}b]\,\mathbb{1} - \frac{1}{4}[\bar{c}\gamma_5 b]\gamma_5 - \frac{1}{4}[\bar{c}\gamma_\mu b]\gamma^\mu$$

$$+ \frac{1}{4}[\bar{c}\gamma_\mu\gamma_5 b]\gamma^\mu\gamma_5 - \frac{1}{8}[\bar{c}\sigma_{\mu\nu}b]\sigma^{\mu\nu}$$

and similarly for M'.

Exercise:

Show that :

$$\mathrm{Tr}(M\gamma_\mu M'\gamma^\mu) = [\bar{c}b\cdot\bar{\ell}\nu_e] - [\bar{c}\gamma_5 b\cdot\bar{\ell}\gamma_5\nu_e]$$

$$+ [\bar{c}\gamma_\mu b\cdot\bar{\ell}\gamma^\mu\nu_e] + [\bar{c}\gamma_\mu\gamma_5 b\cdot\bar{e}\gamma^\mu\gamma_5\nu_e]$$

Thus,

$$O_{VV} \equiv \bar{c}\gamma_\mu\nu_e\cdot\bar{\ell}\gamma^\mu b = -\bar{c}b\,\bar{\ell}\nu_e + \bar{c}\gamma_5 b\,\bar{\ell}\gamma_5\nu_e$$

$$-\bar{c}\gamma_\mu b\,\bar{\ell}\gamma^\mu\nu_e - \bar{c}\gamma_\mu\gamma_5 b\,\bar{\ell}\gamma^\mu\gamma_5\nu_e .$$

In general, $\boxed{\bar{c}\Gamma\nu_e\,\bar{\ell}\Gamma' b = \sum_i a_i\,\bar{c}\Gamma_i b\,\bar{\ell}\Gamma_i'\nu_e}$

for any Γ, Γ. This is called a FIERZ IDENTITY and allows to write (in 4D) any operator of the form $\bar{c}\nu_e\,\bar{\ell}b$ in terms of operators of the form $\bar{c}b\,\bar{\ell}\nu_e$.

Exercise: (Here, $P_L \equiv \frac{1-\gamma_5}{2}$, $P_R \equiv \frac{1+\gamma_5}{2}$)

(a) Write $\bar{c}\gamma_\mu P_L \nu_e \, \bar{\ell}\gamma^\mu P_L b$ in Fierzed form.

(b) Write $\bar{c}\gamma_\mu P_L \nu_e \, \bar{\ell}\gamma^\mu P_R b$ in Fierzed form.

Concluding: In 4 dimensions, our set of independent $b \to c\ell\bar{\nu}$ operators up to $d \leq 6$ is given by:

$$(\bar{c}\Gamma b)(\bar{\ell}\Gamma'\nu_e) \quad \text{with} \quad \Gamma, \Gamma' = \{1, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}\}.$$

with Lorentz indices contracted.

Taking into account only LH neutrinos, we have that:

$$\nu_e = P_L \nu_e \quad ; \quad P_R \nu_e = 0$$

and thus a "chiral basis" with P_L, P_R instead of γ_5 seems adequate. One such basis is:

$$O_1 = (\bar{c}\, P_R \gamma^\mu b)(\bar{\ell}\gamma_\mu \nu_e)\,; \qquad O_4 = (\bar{c}\, P_L \gamma^\mu b)(\bar{\ell}\gamma_\mu \nu_e)\,;$$

$$O_2 = (\bar{c}\, P_R b)(\bar{\ell}\nu_e)\,; \qquad O_5 = (\bar{c} P_L b)(\bar{\ell}\nu_e)$$

$$O_3 = (\bar{c}\, P_R \sigma^{\mu\nu} b)(\bar{\ell}\sigma_{\mu\nu}\nu_{e'})$$

G. RENORMALIZATION AND SCALE DEPENDENCE

Our EFT for $b \to c \ell \bar{\nu}$ processes is given by

$$\mathcal{L}_{EFT} = \mathcal{L}_{QCD+QED} + \sum_{i=1}^{5} C_i^{(0)} O_i^{(0)}$$

As usual, the theory must be renormalized. We assume QED and QCD are renormalized in the standard way, and in the $d=6$ part we have indicated that the Wilson coeffs. $C_i^{(0)}$ and the $d=6$ operators $O_i^{(0)}$ are BARE QUANTITIES.

The relation to renormalized quantities is:

$$C_i^{(0)} = Z_{ij}^{c}\, C_j \quad ; \quad O_i^{(0)} = Z_{ki}^{0}\, O_k \qquad (\text{sum over } j, k)$$

where Z_{ij}^{c} renormalize the WCs and Z_{ki}^{0} takes care of field renormalization. These are expanded as:

$$Z_{ij}^{X} = \delta_{ij} + \frac{1}{\hat{\epsilon}}\left\{ \hat{\alpha}_s\, \delta Z_{ij}^{X,(1,0)} + \hat{\alpha}\, \delta Z_{ij}^{X,(0,1)} \right\} + \mathcal{O}(\alpha_s^2, \alpha^2, \alpha\alpha_s)$$

with the $\overline{MS}$ prescription (in $d = 4 - 2\epsilon$):

$$\frac{1}{\hat{\epsilon}} = \frac{1}{\epsilon} - \gamma_E + \log 4\pi - \log \mu^2$$

In terms of counterterms we thus have:

$$C_i^{(0)} O_i^{(0)} = Z_{ij}^{c} \cdot Z_{ki}^{O} \; C_j \, O_k$$

$$= C_i O_i + \frac{1}{\hat\epsilon} \hat\alpha_s \left[\delta Z_{ki}^{c\,(1,0)} + \delta Z_{ki}^{O\,(1,0)} \right] C_i O_k$$

$$+ \frac{1}{\hat\epsilon} \hat\alpha \left[\delta Z_{ki}^{c\,(0,1)} + \delta Z_{ki}^{O\,(0,1)} \right] C_i O_k$$

$$+ \mathcal{O}(\alpha_s^2, \alpha^2, \alpha_s \alpha)$$

Exercise: Show that, for $\bar c b \bar e \nu$ ops:

$$\delta Z_{ij}^{O,(1,0)} = - C_F \, \delta_{ij}$$

Let's calculate the $b \to c \ell \bar\nu$ amplitude order-by-order in perturbation theory:

$$\mathcal{A}(b \to c \ell \bar\nu) = \mathcal{A}^{(0,0)} + \hat\alpha_s \mathcal{A}^{(1,0)} + \hat\alpha \mathcal{A}^{(0,1)} + \cdots$$

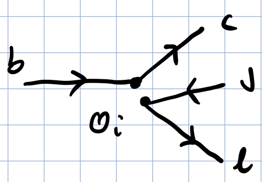
with $\hat\alpha_s \equiv \alpha_s/4\pi = (g_s/4\pi)^2$ and $\hat\alpha \equiv \alpha/4\pi = (e/4\pi)^2$.

We also ignore $1/\Lambda^4$ corrections, and thus $\mathcal{A}$ is linear in C_i:

$$\mathcal{A}^{(k,l)} = \sum_{i=1}^{5} C_i \, \mathcal{A}_i^{(k,l)}$$

At tree-level:

$$\mathcal{A}_i^{(0,0)} =$$



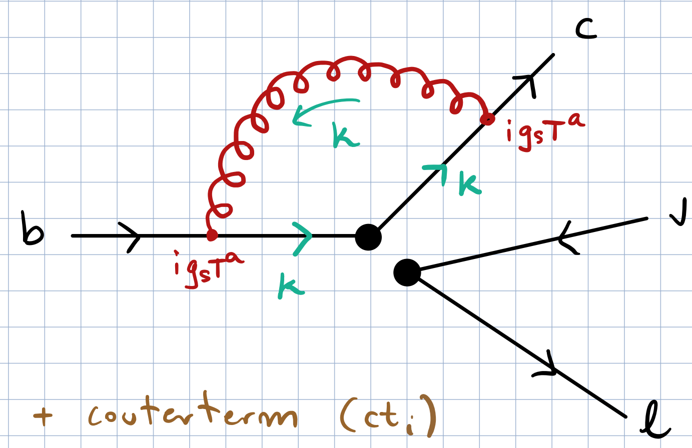
$$= \bar{u}_c \Gamma_i u_b \cdot \bar{u}_\ell \Gamma_i' v_\nu \equiv \langle O_i \rangle^{(tree)}$$

with $\Gamma_{1,4} = P_{R,L}\gamma_\mu$; $\Gamma_{2,5} = P_{R,L}$; $\Gamma_3 = P_R \sigma_{\mu\nu}$

$\Gamma'_{1,4} = \gamma^\mu$; $\Gamma'_{2,5} = 1$; $\Gamma'_3 = \sigma^{\mu\nu}$

At order α_s:

$$\mathcal{A}_i^{(1,0)} =$$



$+$ couterterm (ct_i)

$$= -i\, g_s^2 \overbrace{C_F}^{T^a T^a} \, \bar{u}_\ell \Gamma_i' v_\nu \cdot \bar{u}_c \int \frac{đk \; \gamma_\alpha \not{k} \Gamma_i \not{k} \gamma^\alpha}{k^4 (k^2 - \lambda^2)} u_b + ct_i$$

with the notation $đk \equiv d^d k / (2\pi)^d$.

We use dimensional regularization in $d=4-2\varepsilon$ dimensions, and consider zero external momenta and massless quarks. This introduces an IR divergence that we regularize by a fictitious gluon mass λ.

We need the loop integral:

$$I_{\mu\nu} \equiv \int \frac{d^dk \; k_\mu k_\nu}{k^4 \, (k^2-\lambda^2)} = \frac{i \, g_{\mu\nu}}{4(4\pi)^2} \left[\frac{1}{\hat{\epsilon}} + \frac{3}{2} + \log \mu^2/\lambda^2 + O(\epsilon) \right]$$

With this at hand, we get:

$$\mathcal{A}_i^{(1,0)} = \frac{g_s^2 C_F}{4(4\pi)^2} \left[\frac{1}{\hat{\epsilon}} + \frac{3}{2} + \log \frac{\mu^2}{\lambda^2} \right] \bar{u}_c \gamma_\alpha \gamma_\rho \Gamma_i \gamma^\rho \gamma^\alpha u_b \cdot \bar{u}_\ell \Gamma_i' v_\nu$$
$$+ \, ct_i$$

Exercise:

Show that

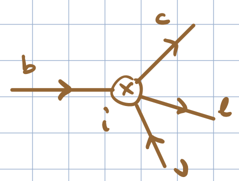
$$\mathcal{A}_{1,4}^{(1,0)} = \hat{\alpha}_s C_F \left[\frac{1}{\hat{\epsilon}} - \frac{1}{2} + \log \frac{\mu^2}{\lambda^2} \right] \cdot \langle O_{1,4} \rangle^{(\text{tree})} + ct_{1,4}$$

$$\mathcal{A}_{2,5}^{(1,0)} = \hat{\alpha}_s C_F \left[\frac{4}{\hat{\epsilon}} + 2 + 4 \log \frac{\mu^2}{\lambda^2} \right] \cdot \langle O_{2,5} \rangle^{(\text{tree})} + ct_{2,5}$$

$$\mathcal{A}_3^{(1,0)} = \mathcal{O}(\epsilon) + ct_3$$

The counterterms are given by:

$$ct_i = $$



$$= \frac{1}{\hat{\epsilon}}\, \hat{\alpha}_s \left(\delta Z^{c,(1,0)}_{ki} + \underbrace{\delta Z^{O,(1,0)}_{ki}}_{-C_F\,\delta_{ki}} \right) \langle O_k \rangle^{(tree)}$$

leading to:

$$\delta Z^{c,(1,0)}_{ki} = 0 \qquad \text{for } k = 1,4$$

$$\delta Z^{c,(1,0)}_{ki} = -3C_F\,\delta_{ki} = -4\,\delta_{ki} \qquad \text{for } k = 2,5$$

$$\delta Z^{c,(1,0)}_{ki} = C_F\,\delta_{ki} = 4/3\;\delta_{ki} \qquad \text{for } k = 3$$

With these results at hand, we come back to the relation between bare and renormalized coefficients:

$$C_i^{(0)} = Z^c_{ij}\, C_j$$

and recognizing that $C_i^{(0)}$ is independent of μ:

$$0 = \frac{d\,C_i^{(0)}}{d\log\mu} = \frac{d Z^c_{ij}}{d\log\mu}\cdot C_j + Z^c_{ij}\,\frac{d\,C_j}{d\log\mu}$$

which leads to:

$$\frac{d C_i}{d \log\mu} = -\left(Z^c\right)^{-1}_{ij} \frac{d Z^c_{jk}}{d \log\mu} C_k .$$

We define the ANOMALOUS DIMENSION MATRIX γ by:

$$\boxed{\frac{d C_i}{d \log\mu} = \gamma_{ji} C_j}$$

so that:

$$\gamma^T = - Z^{c^{-1}} \frac{d Z^c}{d \log\mu}$$

we have:

$$Z^{c^{-1}} = 1 + O(\hat{\alpha}_s, \alpha)$$

$$\frac{d Z^c}{d \log\mu} = - 2 \hat{\alpha}_s \delta Z^{c,(1,0)} + O(\hat{\alpha}_s^2, \alpha)$$

such that $\gamma^T = \hat{\alpha}_s \left(2 \delta Z^{c,(1,0)}\right) + O(\hat{\alpha}_s^2, \alpha)$

$$\Rightarrow \quad \boxed{\gamma = \hat{\alpha}_s \begin{pmatrix} 0 & 8/3 & & \\ -8 & & & \\ & & 0 & \\ & & -8 & \end{pmatrix} + O(\hat{\alpha}_s^2, \alpha)}$$