Precision HH predictions: QCD and EW corrections

IPPP Seminar, Durham

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18th May, 2023

Higgs self coupling

Standard Model Higgs potential:

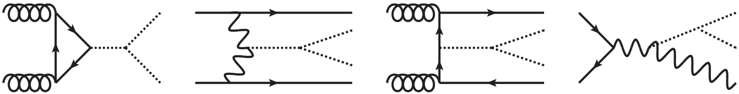
$$V(H) = \frac{1}{2} m_H^2 H^2 + \lambda v H^3 + \frac{\lambda}{4} H^4,$$

where $\lambda = m_H^2/(2v^2) \approx 0.13$.

Want to measure λ, to determine if $V(H)$ is consistent with nature.

- Challenging! Cross-section $\approx 10^{-3} \times H$ prod.
- $-3.3 < \lambda/\lambda_{SM} < 8.5$ [CMS '21]

λ appears in various production channels:



- Gluon fusion – dominant, 10x
- VBF
- $t\bar{t}$ associated production
- H-strahlung

Gluon Fusion

Leading order (1 loop) partonic amplitude:

$$\mathcal{M}^{\mu\nu} \sim \mathcal{A}_1^{\mu\nu}(\mathcal{F}_{tri} + \mathcal{F}_{box1}) + \mathcal{A}_2^{\mu\nu}(\mathcal{F}_{box2})$$

- $\mathcal{F}_{tri}$ contains the dependence on λ at LO

Form factors:

- LO: known exactly [Glover, van der Bij '88]
- Beyond LO... no fully-exact (analytic) results to date
  - QCD: numerical evaluation, expansion in various kinematic limits
  - EW: first steps: HE expansion [Davies, Mishima, Schönwald, Steinhauser, Zhang '22]
    - (see also HTL considerations) [Mühlleitner,Schlenk,Spira '22]

$gg \to HH$ Beyond LO QCD

NLO QCD:

- large-m_t [Dawson,Dittmaier,Spira '98] [Grigo,Hoff,Melnikov,Steinhauser '13]
- numeric [Borowka,Greiner,Heinrich,Jones,Kerner,Schlenk,Schubert,Zirke '16] [Baglio,Campanario,Glaus,Mühlleitner,Spira,Streicher '19]
- large-m_t + threshold exp. Padé [Gröber, Maier, Rauh '17]
- high-energy expansion [Davies, Mishima, Steinhauser, Wellmann '18,'19]
- small-p_T expansion [Bonciani, Degrassi, Giardino, Gröber '18]
- small-t expansion [Davies, Mishima, Schönwald, Steinhauser '23]

NNLO QCD:

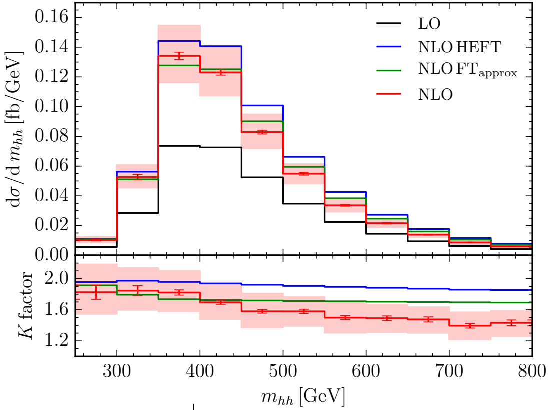
- large-m_t virtuals [de Florian, Mazzitelli '13] [Grigo, Hoff, Steinhauser '15][Davies, Steinhauser '19]
- HTL+numeric real ("FTapprox") [Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli '18]
- large-m_t reals [Davies, Herren, Mishima, Steinhauser '19 '21]
- small-t expansion ? [work in progress: Davies, Schönwald, Steinhauser]

N3LO QCD:

- Wilson coefficient C_{HH} [Spira '16][Gerlach, Herren, Steinhauser '18]
- HTL [Chen, Li, Shao, Wang '19]

$gg \to HH$ Beyond LO QCD

[Borowka, Greiner, Heinrich, Jones, Kerner '16]

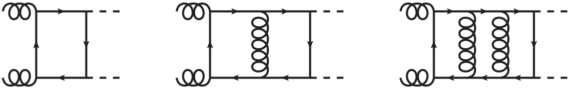


Total XS (14TeV):

	σ_{LO}	σ_{NLO}	σ_{NNLO}
B-i HTL	–	$38.32^{+18.1\%}_{-14.9\%}$	$39.58^{+1.4\%}_{-4.7\%}$
FTapprox	–	$34.25^{+14.7\%}_{-13.2\%}$	$36.69^{+2.1\%}_{-4.9\%}$
Full	$19.85^{+27.6\%}_{-20.5\%}$	$32.88^{+13.5\%}_{-12.5\%}$	–

QCD Corrections

Example diagrams at LO, NLO, NNLO:



Diagrams depend on ϵ, s, t, m_t, m_H:

- analytic result very complicated
- simplify: expand in certain kinematic limits

Here we consider two expansions:

- high-energy: description for larger p_T values $\quad s, |t| > m_t^2 > m_H^2$
- small-t: description for smaller p_T values $\quad s, m_t^2 > |t|, m_H^2$

High-energy expansion

Seek an expansion where $s, |t| > m_t^2 > m_H^2$: [Davies,Mishima,Steinhauser,Wellmann '18,'19]

1. FFs in terms of Feynman integrals: $I(m_H^2, m_t^2, s, t, \epsilon)$,
2. Taylor expand for $m_H^2 \to 0$ (with `LiteRed`): [Lee '14]

$$I(m_H^2, \ldots) = I(0, \ldots) + m_H^2 I'(0, \ldots) + \cdots ,$$

3. IBP reduce to Master Integrals: $J(0, m_t^2, s, t, \epsilon)$ (`FIRE`, `Kira`),
4. Determine MIs as an expansion around $m_t^2 \to 0$:

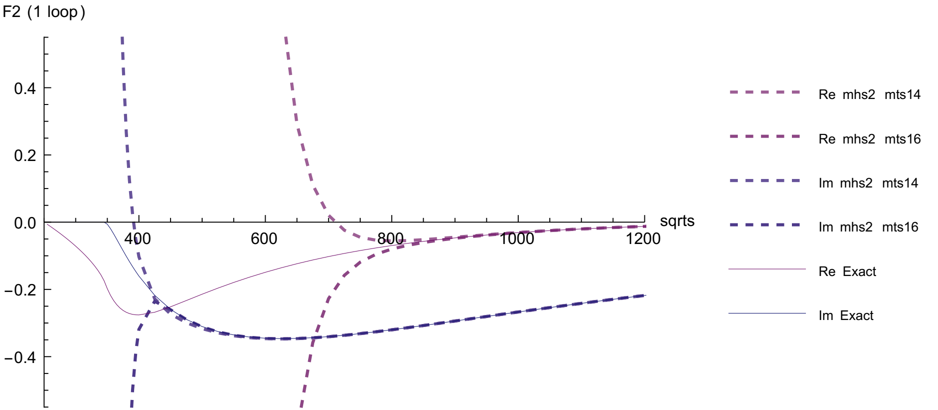
$$J(0, m_t^2, s, t, \epsilon) = \sum_{i,j,k} C_{ijk}(s,t)\epsilon^i (m_t^2)^j \log(m_t^2)^k$$

   - Diff. eq. for MIs w.r.t $m_t^2 \to$ linear equations for C_{ijk}.
   - Boundary conditions: leading exp. terms from Expansion-by-Regions

Result: power series in m_t^2 and $\log(m_t^2)$.

- coefficients: func. of s, t written in terms of Harmonic Polylogarithms

High-energy expansion



High-energy expansion: Padé approximants

The expansion diverges for $\sqrt{s}$ below $\sim$ 750GeV.

The convergence can be improved by making use of Padé approximants:

- approximate a function using a rational polynomial,

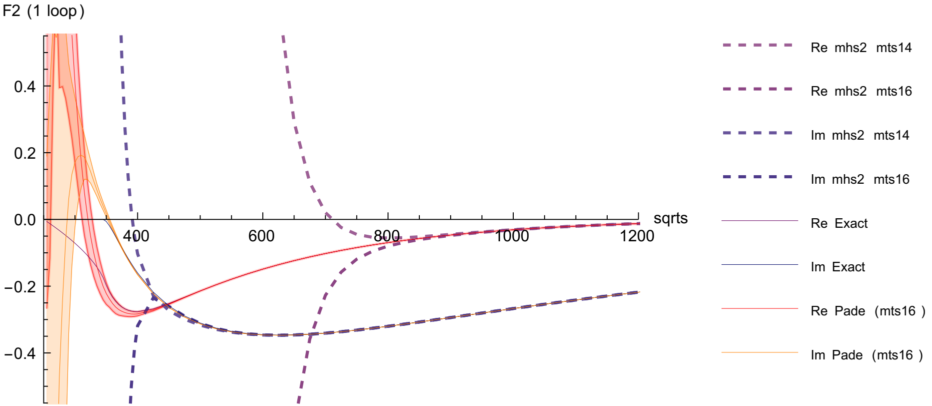
$$f(x) \approx [n/m](x) = \frac{a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n}{1 + b_1 x + b_2 x^2 + \cdots + b_m x^m},$$

  where a_i, b_j coefficients are fixed by the series coefficients of $f(x)$.

Compute a set of approximants (various choices of n, m):

- combine to give central value and error estimate
- deeper expansion $\to$ larger $n+m$ $\to$ smaller error
- expansion to m_t^{120} allows for very high-order approximants

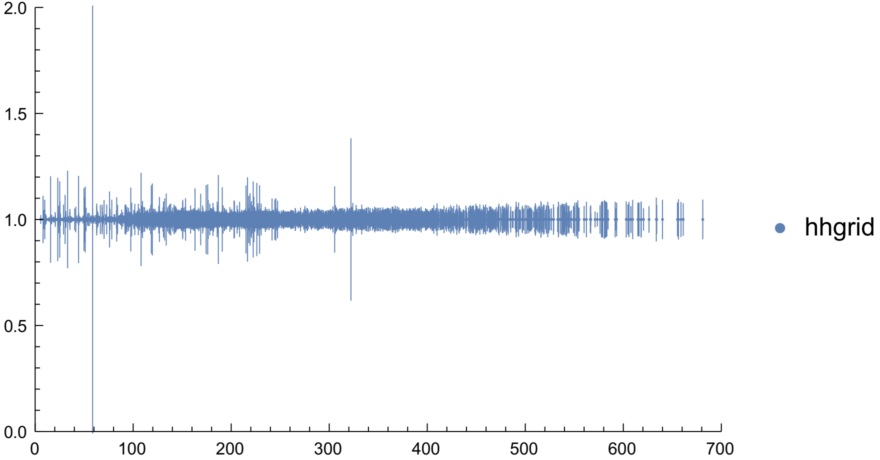
High-energy expansion: Padé approximants



High-energy expansion: "V_{fin}"

Comparison with `hhgrid`: [https://github.com/mppmu/hhgrid]

- interpolation grid of 6320 points evaluated by `pySecDec`
- grid points normalized to `hhgrid` vals. , function of p_t:



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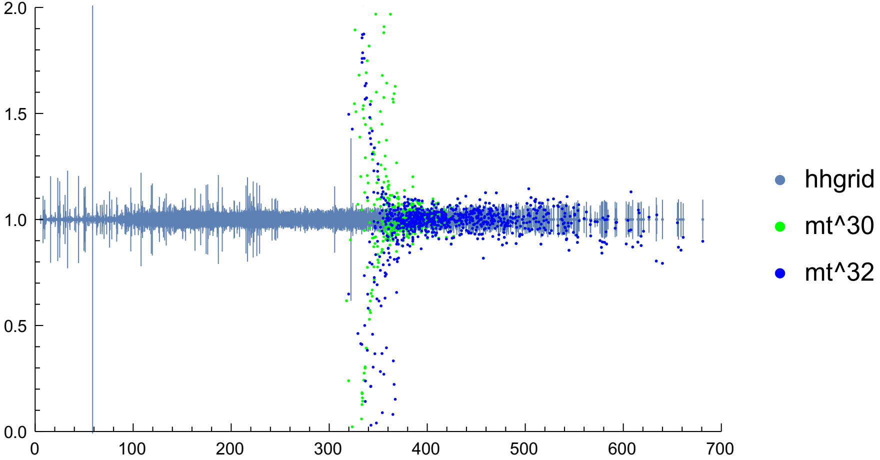

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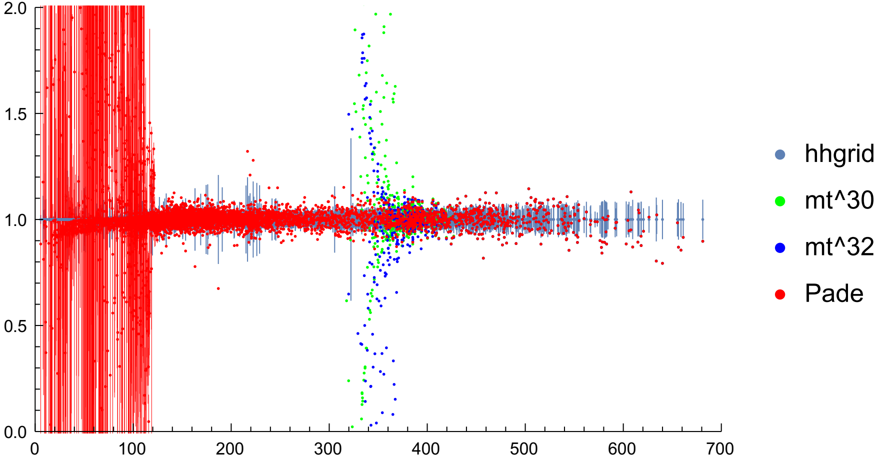
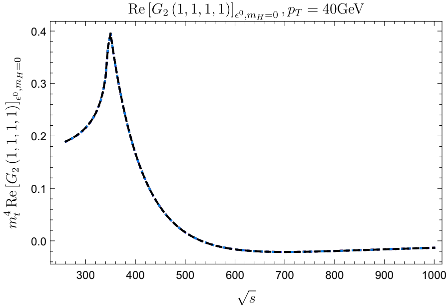
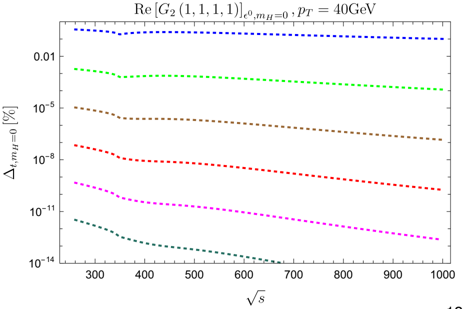


Small-t expansion

As for HE, first expand around $m_H \to 0$.
Then, two approaches (which give the same result):

- take the IBP-reduced amplitude of the HE expansion
  - expand around $t \to 0$ instead of $m_t \to 0$
- expand un-reduced amplitude around $q_3 \to -q_1$ ($t \to 0$)
  - IBP reduce integrals which depend only on ϵ, s, m_t
  - can be applied at NNLO





Small-t expansion: "expand and match" MIs

"Semi-analytic" determination of the $t \to 0$ MIs: [Fael,Lange,Schönwald,Steinhauser '21]

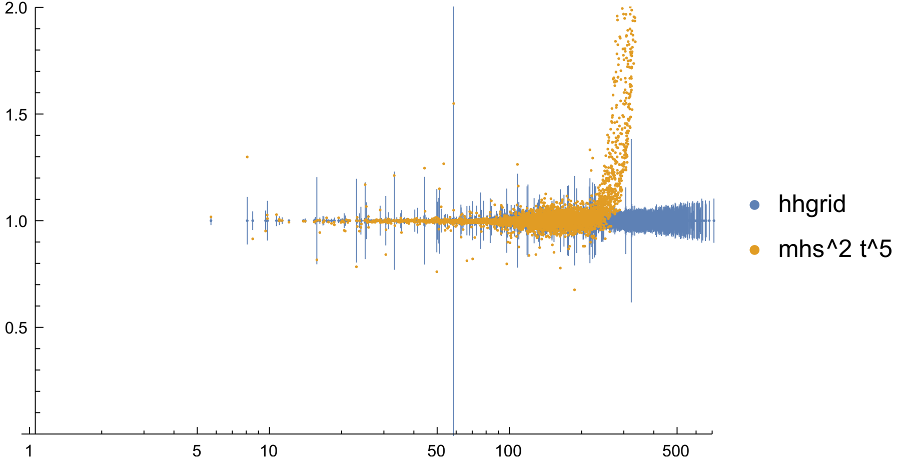
1. establish system of DEs for the MIs, w.r.t. $\hat{s} = s/m_t^2$
2. expand around $\hat{s} = 0$:
   - insert ansatz into DE: $J(\epsilon, \hat{s} = 0) = \sum_{i,j} c_{ij}\, \epsilon^i\, \hat{s}^j$
   - determine minimal set of c_{ij} (`Kira+FireFly`)
   - evaluate minimal boundary constants analytically (LME)
3. expand around a new point $\hat{s} = \hat{s}_0$ (repeat the above, modify ansatz)
4. match expansions (numerically) at a point where they both converge

Here we have such "semi-analytic" expansions for the MIs at:

$$\hat{s} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 16, 20, 25, 30, 40, 50, \infty\}$$

Small-t expansion: "V_{fin}"

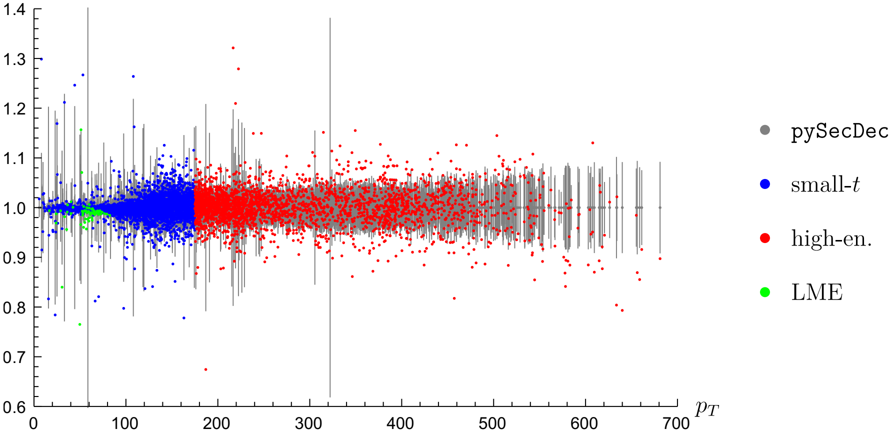
Comparison with `hhgrid`: [https://github.com/mppmu/hhgrid]



HE and $t \to 0$ combination: "V_{fin}"

Comparison with `hhgrid`: [`https://github.com/mppmu/hhgrid`]

- merge both results, switch at $p_T = 175$ GeV.



"ggxy"

`C++` library implementing the $gg \to HH$ form factors, currently including:

- exact leading-order expressions
- $t \to 0$ expansion
- high-energy expansion, incl. Padé approximants

Computes the 6320 phase-space points of `hhgrid` at avg 0.002s/point.

TODO:

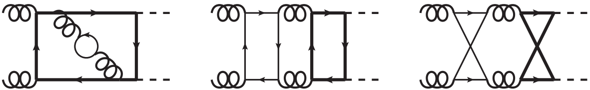
- m_t scheme and scale variation
- include known results for $gg \to ZH$, $gg \to ZZ$, $gg \to \gamma\gamma$
- make public!

Small-t expansion: NNLO?

We would like to understand $gg \to HH$ at NNLO, due to the large NLO, and uncertainty from the top quark mass scheme and scale choice.

First steps: diagrams with light fermion loop, expand to $m_H^0 t^0$

[work in progress: Davies, Schönwald, Steinhauser]



- done: IBP reduction: 176 MIs. Compute with "expand-and-match".

Next steps:

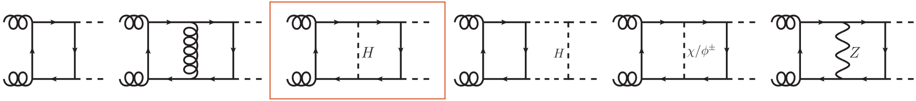
- diagrams with light-fermion loop: attempt $m_H^2 t^0$?
- all remaining diagrams, first to $m_H^0 t^0$

EW Corrections

Since we look at NNLO QCD, we should also look at 2L EW corrections.

This is a much more difficult computation:

- 2L QCD: 118 Feynman diagrams
- 2L EW: 3810 Feynman diagrams



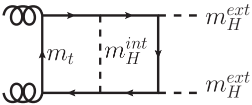
There are also more scales to deal with, compared to the QCD.

- start with $\alpha_s \alpha_t^2$ "y_t^4" diagrams with internally propagating Higgs
  - expansion parameter $\alpha_t = \alpha m_t^2/(2 s_W^2 m_W^2) \sim \alpha_s/2$
  - only planar integrals in this subset [Davies, Mishima, Schönwald, Steinhauser, Zhang '22]

EW Corrections: High-energy expansion

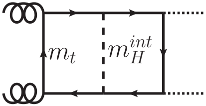
Again, full diagrams depend on many variables:

- ϵ, s, t, m_t, m_H



As for QCD, Taylor expand around $m_H^{ext} = 0$:

- expand integrals with `LiteRed`



Unlike the QCD case, m_H^{int} remains in the propagator:

- complicates IBP reduction, MIs with extra scale are difficult

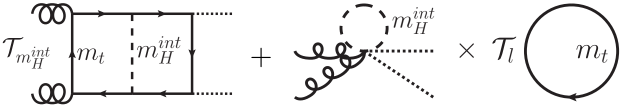
Expand in m_H^{int} also, propose two ways to do it:

- “A”: $s, |t| \gg m_t^2 \gg {m_H^{int}}^2 \sim {m_H^{ext}}^2$,
- “B”: $s, |t| \gg m_t^2 \sim {m_H^{int}}^2 \gg {m_H^{ext}}^2$.

High-Energy Expansion "A"

Option A: asymptotic expansion around $m_H^{int} = 0$. Exp.-by-subgraph:

- two sub-graphs:



The two-loop subgraph is a Taylor expansion of the Higgs propagator:

- results in integrals with a massless internal line, scales s, t, m_t.
- IBP reduce with `FIRE` and `Kira` [Smirnov '15] [Klappert,Lange,Maierhöfer,Usovitsch '21]
- these coincide with the QCD Master Integrals – reuse the old results [Davies,Mishima,Steinhauser,Wellmann '18,'19]

The massive tadpoles are easily computed by `MATAD`. [Steinhauser '00]

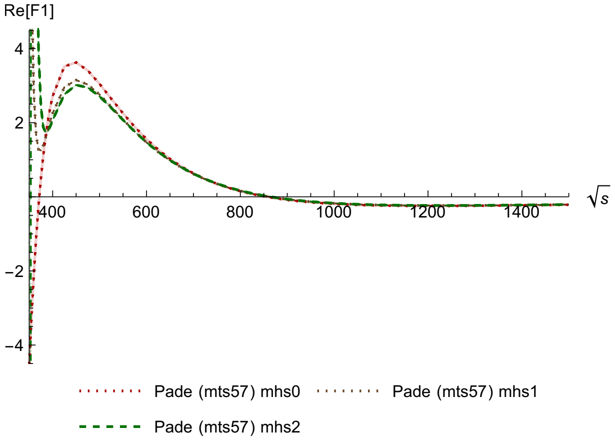
The asymp. expansion procedure is done by `exp` and `FORM` [Harlander,Seidelsticker,Steinhauser '97] [Ruijl,Ueda,Vermaseren '17]

We expand to quartic order: $(m_H^{int})^a\,(m_H^{ext})^b,\ \ 0 \le (a+b) \le 4$.

High-Energy Expansion “A”: convergence

Re(F_{box1}), fixed $\cos\theta = 0$, expansion “A” Padé (to $(m_H^2)^{\{0,1,2\}}$):

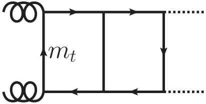
- $(m_H^2)^1$ and $(m_H^2)^2$ terms differ by at most 5% for $\sqrt{s} \geq 400$GeV



High-energy Expansion “B”

Option B: expand around $m_H^{int} \approx m_t$,

- simple Taylor expansion, easy to implement



Write Higgs propagator as: $\frac{1}{p^2 - m_H^2} = \frac{1}{p^2 - m_t^2(1 - [2-\delta]\delta)}$

- expand around $\delta \to 0$ where $\delta = 1 - m_H/m_t \approx 0.28$.

This yields new integral families compared to the QCD computation:

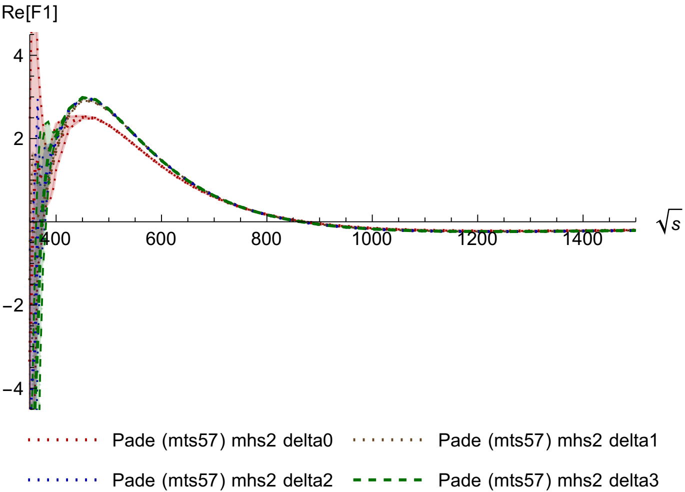
- all lines have the mass m_t,
- IBP reduce and compute the MIs in the high-energy limit

Expand to $(m_H^{ext})^4$ and δ^3.

High-energy Expansion "B": convergence

Re(F_{box1}), fixed $\cos\theta = 0$, expansion "B" Padé (to $(m_H^2)^2\delta^{\{0,1,2,3\}}$):

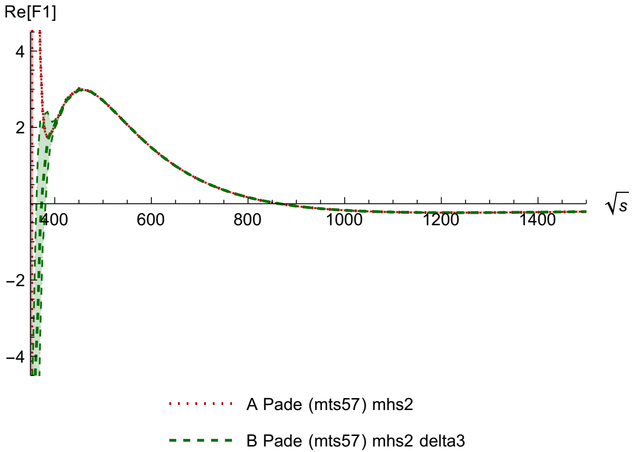
- δ^2 and δ^3 terms differ by at most 0.5% for $\sqrt{s} \geq 400$GeV



High-energy Expansion: “A”, “B” comparison

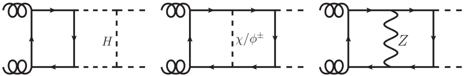
Re(F_{box1}), fixed $\cos\theta = 0$, best “A” and “B” Padé

- “A”, “B” differ by at most 2% for $\sqrt{s} \geq 400$GeV,
- 0.1% for $\sqrt{s} \geq 500$GeV



EW Corrections: next steps

High-energy: same idea, for remaining diagram classes:



- Diagrams with more internal Higgs:
  - "B" expansion and IBP reduction done
  - new topologies: non-planar with all lines massive
  - in progress: compute master integrals
- Diagrams with charged goldstone exchange:
  - new topologies: massive loop doesn't close
  - in progress: compute master integrals

Large mass expansion of all dias: done, to understand renormalization

- not very useful: diagrams with tWb cuts: $m_t + m_W \approx 2m_H$

Small-t expansion: not yet studied.

Conclusion

Multi-scale multi-loop integrals are hard.

- expand!

Expansions give a good description for HH at NLO QCD:

- high-energy + small-t covers whole phase space
- implemented in `C++`: to be made public

First steps:

- NNLO QCD: small-t expansion of light-fermion diagrams
  - to come: remaining diagrams
  - deeper expansion? Challenging IBP reduction.
- EW: high-energy expansion of y_t^4 diagrams
  - to come: remaining diagrams
  - small-t expansion

Padé Approximants in C++

Determine a_i, b_i s.t. upon series expansion,

$$\frac{a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n}{1 + b_1 x + b_2 x^2 + \cdots + b_m x^m} = \sum_{i=0}^{n+m} t_i x^i \, .$$

Solve (explicit $n = 4$, $m = 3$ example):

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -t_0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -t_1 & -t_0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -t_2 & -t_1 & -t_0 \\ 0 & 0 & 0 & 0 & 1 & -t_3 & -t_2 & -t_1 \\ 0 & 0 & 0 & 0 & 0 & -t_4 & -t_3 & -t_2 \\ 0 & 0 & 0 & 0 & 0 & -t_5 & -t_4 & -t_3 \\ 0 & 0 & 0 & 0 & 0 & -t_6 & -t_5 & -t_4 \end{pmatrix} \cdot \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} t_0 \\ t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \\ t_6 \\ t_7 \end{pmatrix}$$

Easy to solve for numerical values of t_i using, e.g. `Eigen` library.
Also in quad precision using `boost::multiprecision`.