UK HEP Forum 2023

21 November 2023

# Precision Calculations of Higgs Production and Decay

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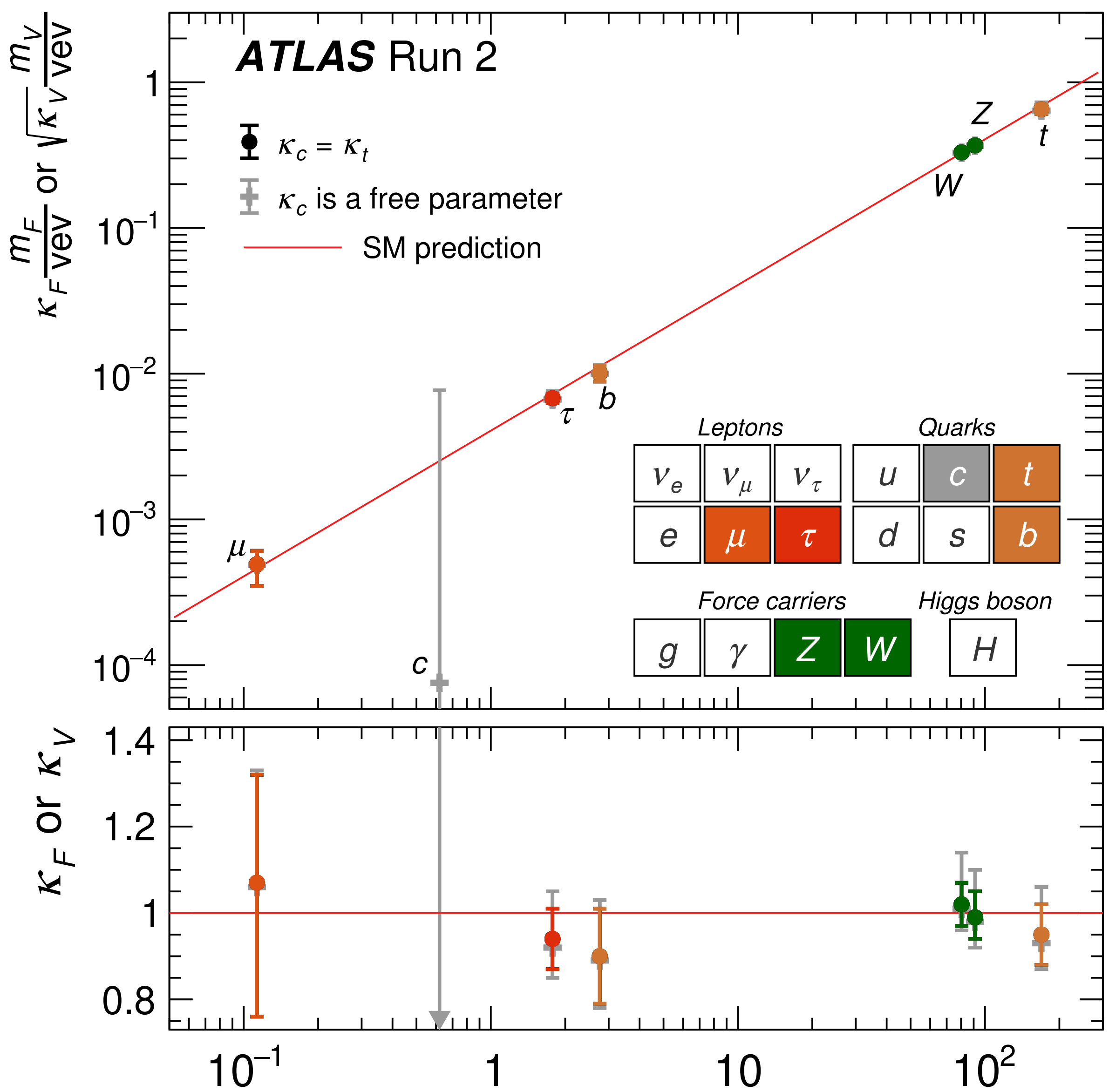
THE ROYAL SOCIETY

# Higgs Discoveries

The Higgs sector continues to yield impressive fundamental discoveries




CERN-EP-2022-057

**2018 ($t\bar{t}H$):** First direct observation of top-quark Yukawa coupling

CMS 1804.02610/ ATLAS 1806.00425

**2020 ($H \to \mu\mu$):** First direct evidence that Higgs field is responsible for mass of 2nd gen. leptons

CMS 2009.04363 / ATLAS 2007.07830

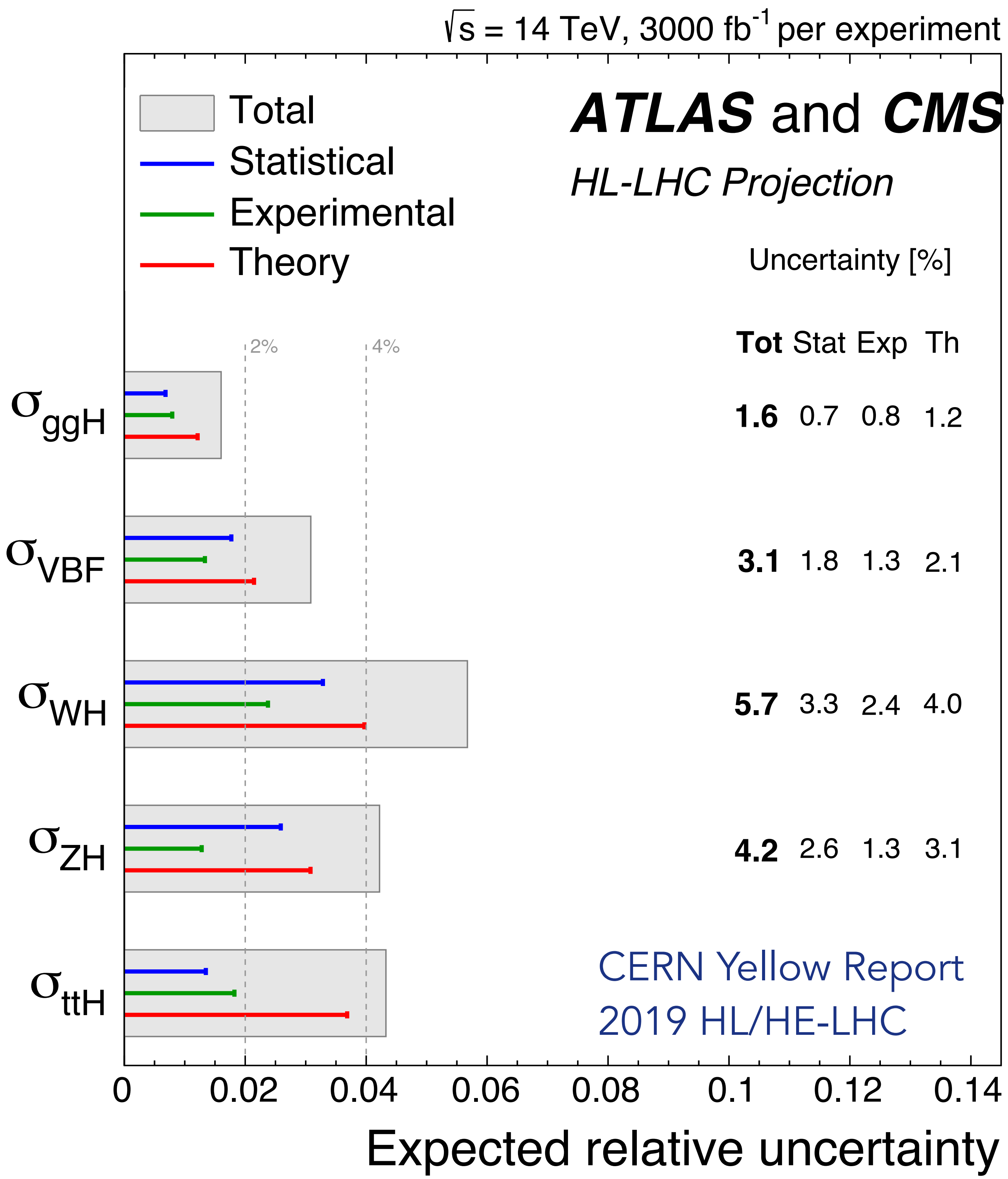
**2022** ($H \to c\bar{c}$): First hints that Higgs field is responsible for mass of 2nd gen. quarks CMS

2205.05550 / ATLAS 2201.11428

**2023** ($H \to Z\gamma$): First evidence of rare decay, sensitive to BSM loops

CERN-EP-2023-157

# Upcoming Experiments




HL-LHC construction underway

~10x integrated luminosity of LHC (LHC 0.3 ab$^{-1}$, HL-LHC: 3 ab$^{-1}$)

**1)** Experimental projection is pessimistic considering current performance

**2)** Plot shown assumes reduction by factor 2 of today's theory uncertainties

**Theory uncertainty is expected to dominate HL-LHC Higgs physics**

# Outline

**Gluon Fusion**

PDF theory uncertainties

Signal-background interference

**Vector Boson Fusion**

Non-factorisable corrections

Parton shower uncertainties

**ZH Production**

Gluon induced contribution

**ttH Production**

Approximate NNLO

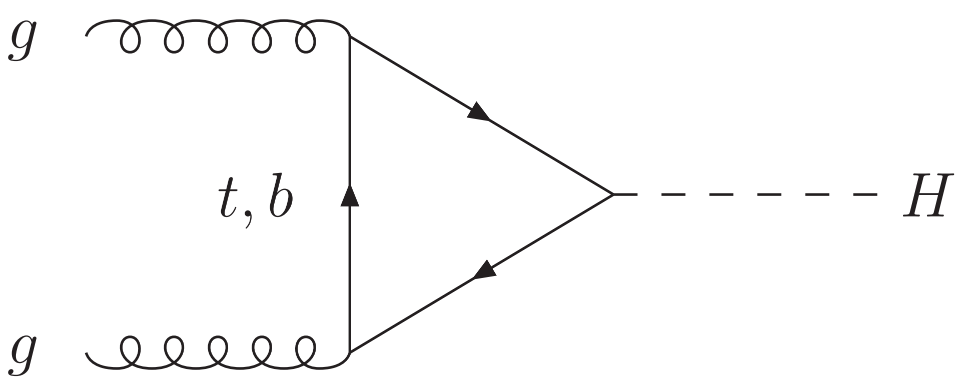
**HH Production**

Background processes ($b\bar{b}H$)

Electroweak corrections

*Too many interesting recent results, apologies for the very biased topic selection

# Gluon Fusion



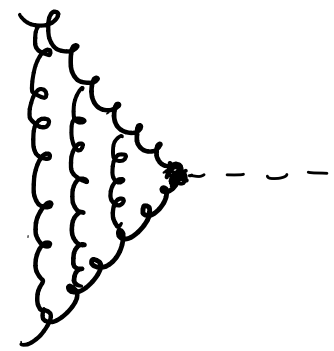
# Theory Uncertainties

Perturbative theoretical predictions for gluon fusion are **remarkably advanced**

$$\delta\sigma_{PP\to H+X} = \left[\delta(\text{scale}) + \delta(\text{EWK}) + \delta(t,b,c) + \delta(m_t)\right] + \left[\delta(\text{PDF}+\alpha_s)\right] + \left[\delta(\text{PDF}-\text{TH})\right]$$

## N$^3$LO Heavy Top Limit

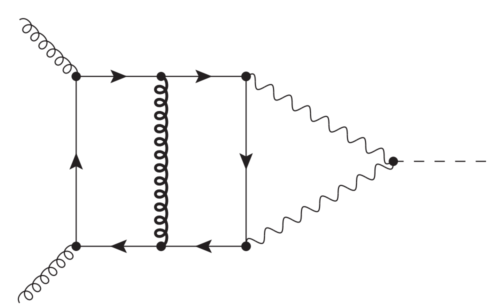
Anastasiou, Duhr, Dulat, (Furlan), (Gehrmann), Herzog, Mistlberger 16; Mistlberger 18;



$$\delta(\text{scale}) = ^{+0.3\%}_{-2.5\%}$$

## Mixed QCD-EW Corrections

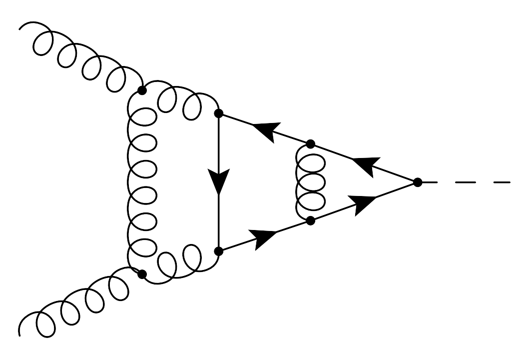
Bonetti, Melnikov, Tancredi 17
Bonetti, Panzer, Smirnov, Tancredi 20
Becchetti, Bonciani, Del Duca, Hirschi, Moriello, Schweitzer 20



$$\delta(\text{EWK}) = \pm 1\,\% \;\to 0.5\%?$$

## NNLO Top Quark Corrections

Czakon, Niggetiedt 20;
Czakon, Harlander, Klappert, Niggetiedt 21



$$\delta(t,b,c) = \pm 0.85\,\%$$
$$\delta(1/m_t) = \pm 1\,\% \;\to \pm 0.15\,\%$$

# PDF Uncertainties

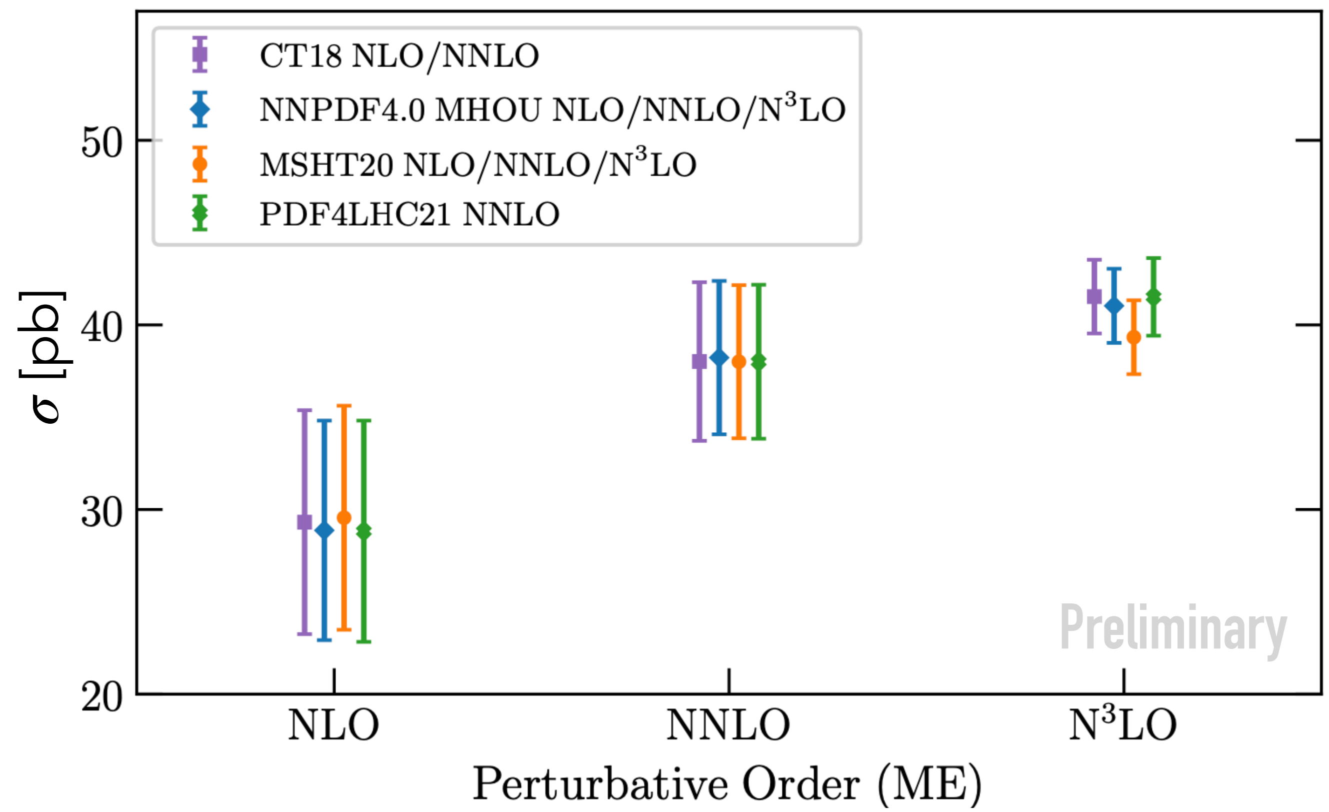
**Many sources of uncertainty** relevant, including PDF/Missing Higher Orders

| $\sqrt{s}$ [TeV] | $M_{\mathrm{H}}$ [GeV] | $\sigma$ [pb] | $\delta$(theory) | $\delta$(scale) | $\delta$(EWK) | $\delta$(t, b, c) | $\delta(1/m_{\mathrm{t}})$ | $\delta(\mathrm{PDF}+\alpha_s)$ | $\delta$(PDF) | $\delta(\alpha_s)$ | $\delta$(PDF − TH) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 13.6 | 125.09 | 52.07 | $^{+3.11}_{-5.31}\%$ | $^{+0.28}_{-2.48}\%$ | $\pm1.00\%$ | $\pm0.83\%$ | $\pm1.00\%$ | $^{+2.67}_{-2.25}\%$ | $^{+1.64}_{+1.64}\%$ | $^{+2.10}_{-1.54}\%$ | $\pm1.18\%$ |

PDFs are only fully known at NNLO

↪ Mismatch between coefficient function $\hat{\sigma}^{(3)}(x_a, x_b)$ and PDFs $f^{(2)}(x, Q^2)$

↪ Recently, approximate N3LO PDFs became available McGowan, Cridge, Harland-Lang, Thorne 22
NNPDF Collaboration (work in progress)

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The aN3LO PDFs lead to a decrease in the total cross section prediction (current estimate ~2-4%)

Active benchmarking study ongoing

# Gluon Fusion: PDF Uncertainties

## Higher order theory input can help us...

- Several ingredients required to perform N3LO PDF fits, many available some missing

➡ 4-loop DGLAP Splitting Functions to evolve PDFs

$$Q^2 \frac{df_i}{dQ^2} = P_{ij}(x, \alpha_s) \otimes f_i(x, Q^2)$$

**non-singlet** - large $n_F$ limit [NPB 915 (2017) 335; arXiv:2308.07958]
- small-x [JHEP 08 (2022) 135] and large-x [JHEP 10 (2017) 041] limits
- lowest 8 Mellin moments [JHEP 06 (2018) 073]

**singlet** - large $n_F$ limit [NPB 915 (2017) 335; arXiv:2308.07958, arXiv:2310.01245]
- small-x [JHEP 06 (2018) 145] and large-x [NPB 832 (2010) 152; JHEP 04 (2020) 018; JHEP 09 (2022) 155] limits
- lowest 5 (10) Mellin moments [PLB 825 (2022) 136853; ibid. 842 (2023) 137944; ibid. 846 (2023) 138215]

➡ PDF matching conditions to change number of PDF flavours at heavy-quark matching scales
- all known [NPB 820 (2009) 417; NPB 886 (2014) 733; JHEP 12 (2022) 134] except for complete $A^{(3)}_{H,g}$ [arXiv::2311.00644]

$$f_i^{(n_f+1)}(x, Q^2) = A_{ij}(x, \alpha_s) f_j^{(n_f)}(x, Q^2)$$

➡ DIS Structure Functions
- DIS NC (massless) [NPB 492 (1997) 338; PLB 606 (2005) 123; NPB 724 (2005) 3]
- DIS CC (massless) [NPB 813 (2009) 220]
- Massive from param. combining known limits and damping functions [NPB 864 (2012) 399]

$$F_k = x \sum_{i=-n_f}^{n_f} C_{k,i}(x, \alpha_s) \otimes f_i(x, Q^2), \quad k = \{1,2,3\}$$

➡ Hadronic cross section
- Drell-Yan (inclusive) [JHEP 11 (2020) 143]
- Drell-Yan (differential in $p_T$) [PLB 845 (2023) 138125]
- Drell-Yan (differential in rapidity) [PRL 128 (2022) 052001]

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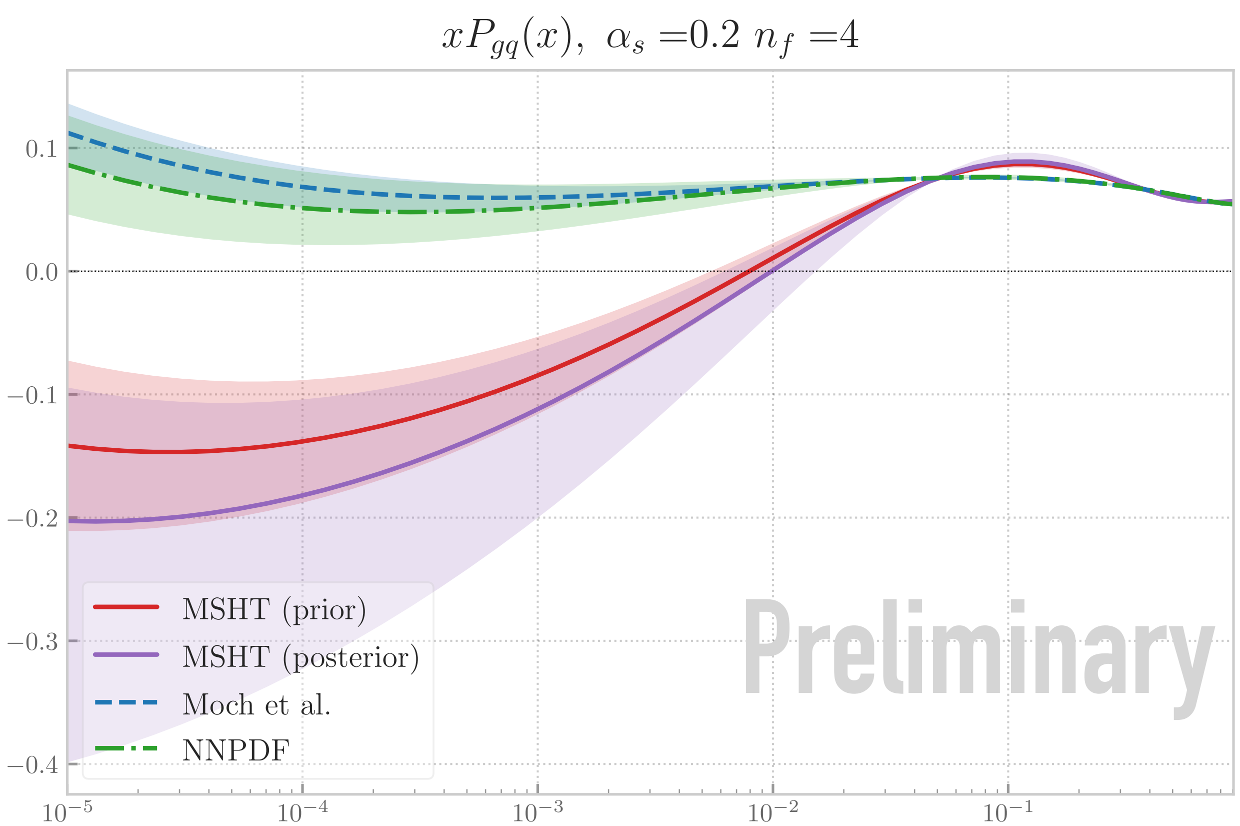
But differences also depend on the choice of $Q_0^2$ scale and methodology

# New/Anticipated Theory Results

## Splitting functions for N3LO PDFs

Singlet (moments): Falcioni, Herzog, Moch, Vogt 23
Singlet ($N_f^2$, $N_f C_F^3$ ): Gehrmann, von Manteuffel, Sotnikov, Yang 23, 23;

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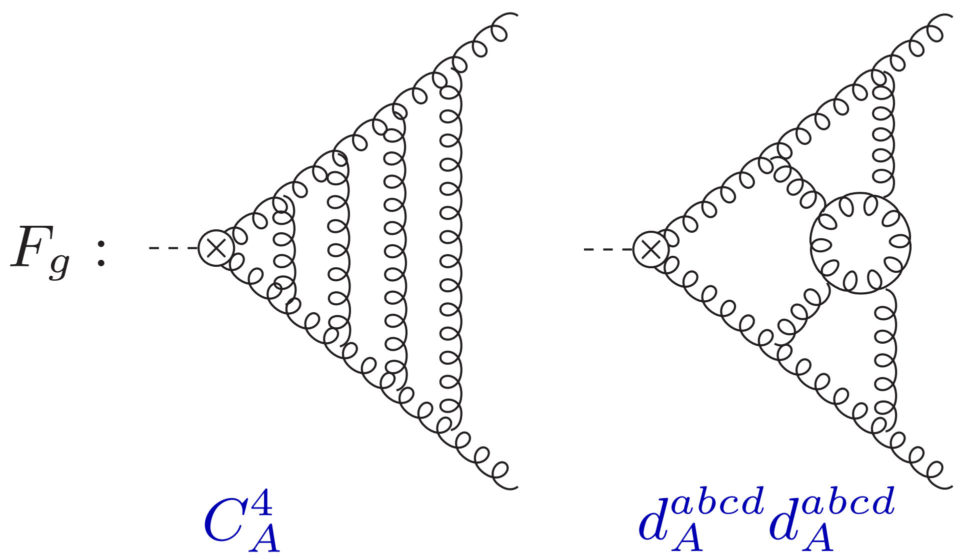
## Towards N4LO Higgs Production

Das, Moch, Vogt 20; Lee, von Manteuffel, Schabinger, Smirnov, Smirnov, Steinhauser 22, 23;




Virtuals known @ $\mathrm{N^4LO}$

RVs still considerable work for full XS

Soft-Virtual Approx: $+0.2 - 2.7\,\%$

# Signal-background Interference

Signal-background interference can be used in $H \to \gamma\gamma$ to place model-dependent bounds on $\Gamma_H$ by lifting degeneracy on couplings and width

$$\left|\mathcal{M}_{gg\to\gamma\gamma}\right|^2 = |S|^2 \quad + \quad |B|^2 \quad + \quad \frac{2m_{\gamma\gamma}^2}{(m_{\gamma\gamma}^2 - m_H^2)^2 + \Gamma_H^2 m_H^2}\left[\boxed{(m_{\gamma\gamma}^2 - m_H^2)\mathrm{Re}\, I} + \boxed{\Gamma_H m_H \mathrm{Im}\, I}\right]$$

## Mass Shift ($\mathrm{Re}\, I$)

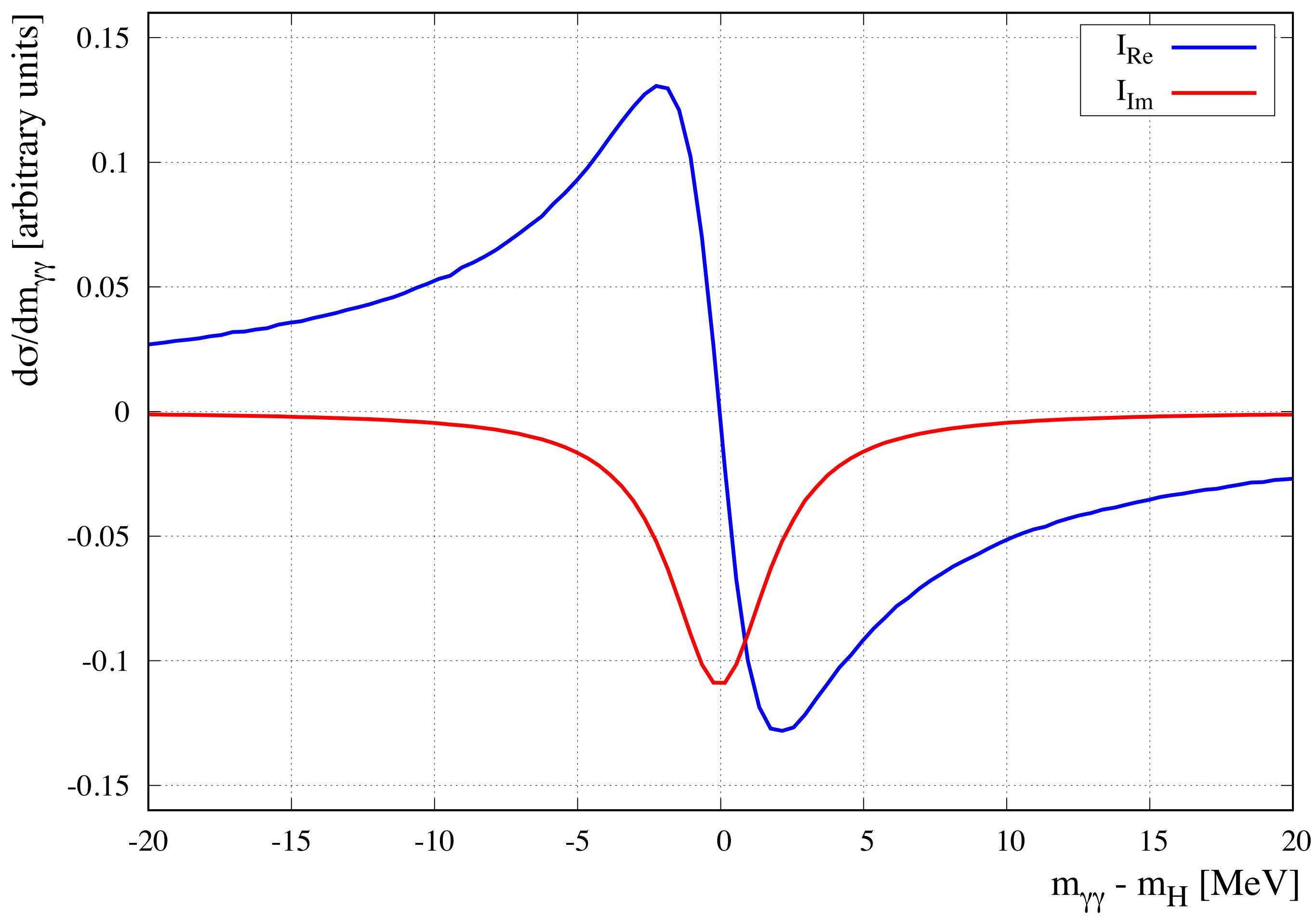
$$\Delta M_{\gamma\gamma} \propto \sqrt{\frac{\Gamma_H}{\Gamma_H^{\rm SM}}}$$

Martin 12; Dixon, Li 13;

## Destructive Interference ($\mathrm{Im}\, I$)

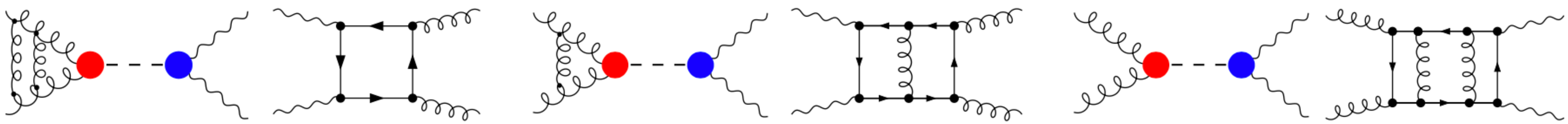
$$\sigma = \sigma_{\rm sig}\left(1 + \sqrt{\frac{\Gamma_H}{\Gamma_H^{\rm SM}}}\frac{\sigma_{\rm int}^{\rm SM}}{\sigma_{\rm sig}^{\rm SM}}\right)$$

Campbell, Carena, Harnik, Liu 17



# Signal-background Interference

Thanks to a recent calculation of $gg \to \gamma\gamma$ @ NNLO (3-loops) now possible to compute interference @ NNLO Bargiela, Caola, von Manteuffel, Tancredi 21 + Buccioni, Devoto 22

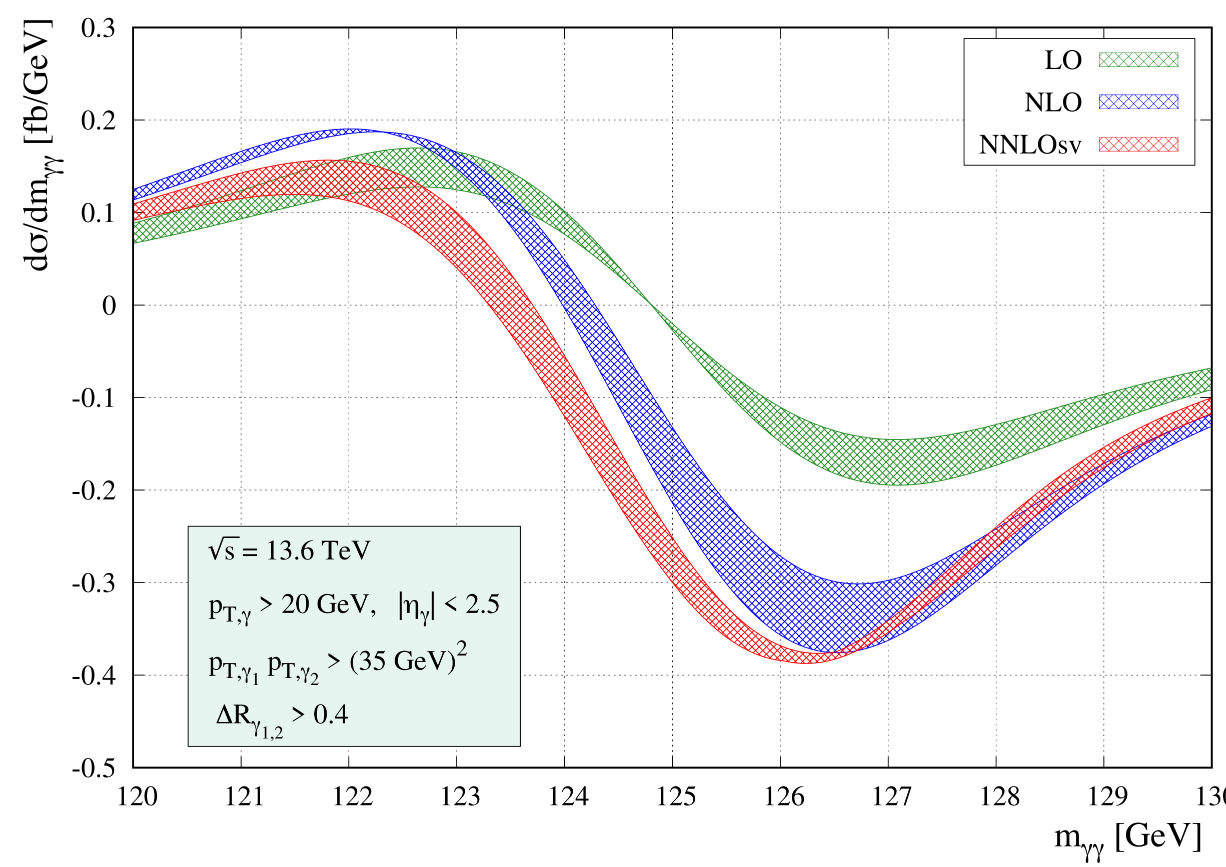


**NNLO corrections sizeable**

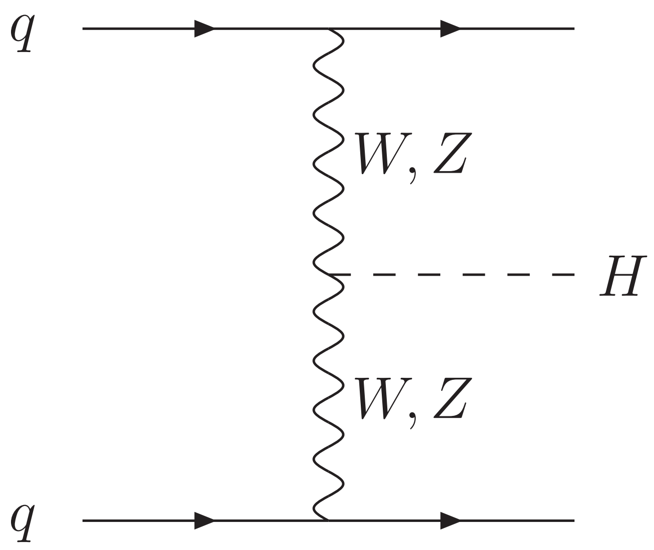
↪ Mass shift less pronounced

↪ Destructive interference enhanced, -1.7% decrease of total cross section

| $\Delta m_{\gamma\gamma}$ [MeV] | 7 TeV | 8 TeV | 13.6 TeV |
|---|---|---|---|
| LO | $-77.2^{+0.8\%}_{-1.0\%}$ | $-79.5^{+0.6\%}_{-0.8\%}$ | $-83.1^{+0\%}_{-0.3\%}$ |
| NLO | $-56.2^{+13\%}_{-15\%}$ | $-56.8^{+13\%}_{-14\%}$ | $-55.2^{+12\%}_{-12\%}$ |
| NNLOsv | $-46.3^{+15\%}_{-17\%}$ | $-47.0^{+14\%}_{-16\%}$ | $-46.0^{+11\%}_{-12\%}$ |
| NNLOsv′ | $-39.5^{+20\%}_{-24\%}$ | $-39.7^{+19\%}_{-22\%}$ | $-39.4^{+16\%}_{-17\%}$ |



# Vector Boson Fusion (VBF)

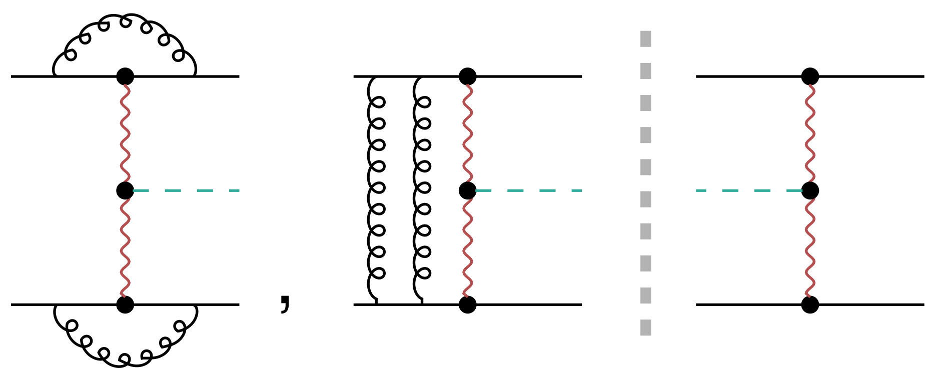


# Non-Factorisable Corrections (II)

Factorisable contributions are known to N3LO inclusive/ NNLO differential

Dreyer, Karlberg 16; Bolzoni, Maltoni, Moch, Zaro 10, 12; Cacciari, Dreyer, Karlberg, Salam, Zanderighi 15; Cruz-Martinez, Gehrmann, Glover, Huss 18; Asteriadis, Caola, Melnikov, Röntsch 22, 23

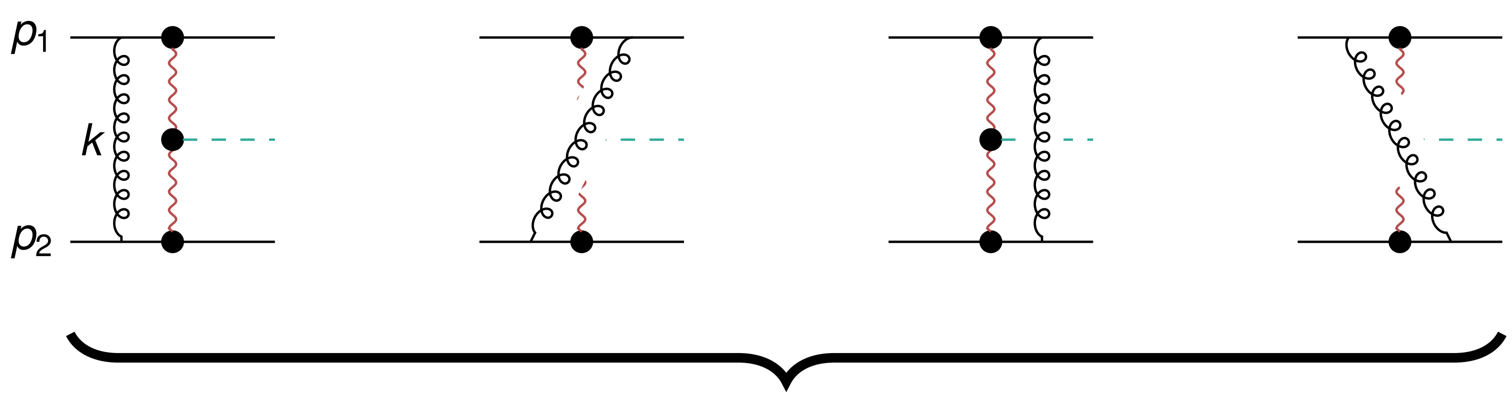
Non-factorisable contributions are **colour suppressed**



$$\Longrightarrow \frac{(N_c^2-1)^2}{4}, \quad \frac{(N_c^2-1)}{4}$$

However, (soft/eikonal approximation) it was found they are $\pi^2$ **enhanced**

Liu, Melnikov, Penin 19




$$\left[\frac{1}{2p_1k+i0}+\frac{1}{-2p_1k+i0}\right]\left[\frac{1}{-2p_2k+i0}+\frac{1}{2p_2k+i0}\right] \Longrightarrow \pi^2$$

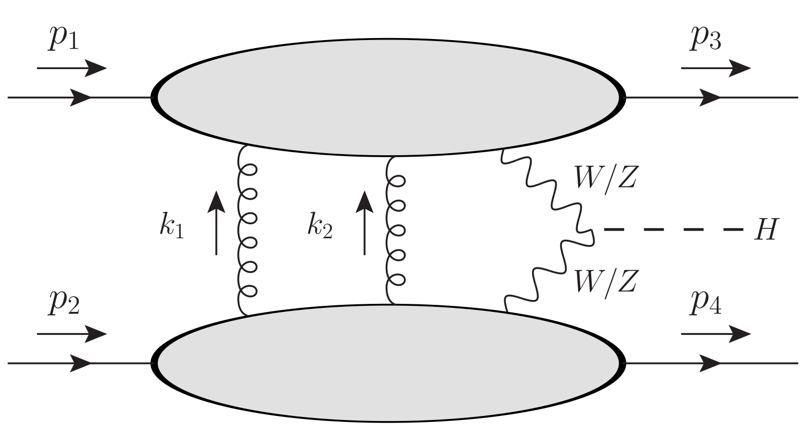
Figures: Ming-Ming Long (20th LHCHWG Meeting)

# Non-Factorisable Corrections (III)

NNLO non-factorisable contribution computed beyond the eikonal approximation

Brønnum-Hansen, Long, Melnikov, Juvin-Quarroz 23, 23;

Used expansion-by-regions in forward limit, $\mathbf{p}_{3,\perp}^2/s \sim \mathbf{p}_{4,\perp}^2/s \sim \lambda$




$$k_1 = \alpha_1 p_1 + \beta_1 p_2 + k_{1,\perp}, \quad \frac{\mathrm{d}^d k_1}{(2\pi)^d} = -\frac{s}{2}\frac{\mathrm{d}\alpha_1}{2\pi i}\frac{\mathrm{d}\beta_1}{2\pi i}\frac{\mathrm{d}^{d-2}\mathbf{k}_{1,\perp}}{(2\pi)^{d-2}}$$

| | $\alpha_1$ | $\beta_1$ | $\mathbf{k}_{1,\perp}$ | $\mathcal{M}_1$ |
|---|---|---|---|---|
| G | $\lambda$ | $\lambda$ | $\sqrt{\lambda}$ | -2 |
| G-S | $\lambda$ | $\sqrt{\lambda}$ | $\sqrt{\lambda}$ | -3/2 |
| S | $\sqrt{\lambda}$ | $\sqrt{\lambda}$ | $\sqrt{\lambda}$ | -1 |
| C | 1 | $\lambda$ | $\sqrt{\lambda}$ | 0 |
| H | 1 | 1 | 1 | 0 |

**Only Glauber/ Mixed @ next-to-leading power**

Sub-eikonal correction about 20% of eikonal correction

**Result:** NNLO non-factorisable contribution similar in size to N3LO correction

| | $\sigma^{(13\ \mathrm{TeV})}$ [pb] | $\sigma^{(14\ \mathrm{TeV})}$ [pb] | $\sigma^{(100\ \mathrm{TeV})}$ [pb] |
|---|---|---|---|
| LO | $4.099\,^{+0.051}_{-0.067}$ | $4.647\,^{+0.037}_{-0.058}$ | $77.17\,^{+6.45}_{-7.29}$ |
| NLO | $3.970\,^{+0.025}_{-0.023}$ | $4.497\,^{+0.032}_{-0.027}$ | $73.90\,^{+1.73}_{-1.94}$ |
| NNLO | $3.932\,^{+0.015}_{-0.010}$ | $4.452\,^{+0.018}_{-0.012}$ | $72.44\,^{+0.53}_{-0.40}$ |
| N3LO | $3.928\,^{+0.005}_{-0.001}$ | $4.448\,^{+0.006}_{-0.001}$ | $72.34\,^{+0.11}_{-0.02}$ |

+

$$\sigma_{\mathrm{LO}}^{\mathrm{non-fac}} = -\,2.97^{+0.52}_{-0.69}\ \mathrm{fb}$$

$$\sigma_{\mathrm{NLO}}^{\mathrm{non-fac}} = -\,3.20^{+0.14}_{-0.01}\ \mathrm{fb}$$

Figures/Tables: Ming-Ming Long (20th LHCHWG Meeting)

# VBF Parton Shower Uncertainties

Parton shower uncertainties dominate the theory uncertainty for VBF

↪ Currently ±15% on inclusive measurement, will limit interpretation in Run 3

↪ Not clear if Pythia dipole vs Herwig 7 captures true uncertainty

## Several studies completed & ongoing:

**NNLO QCD vs NLO+PS**

Good agreement theoretically

Less so for experimental PS studies (underlying event? hadronisation? tuning vs recoil scheme?)

Buckley et al. 21

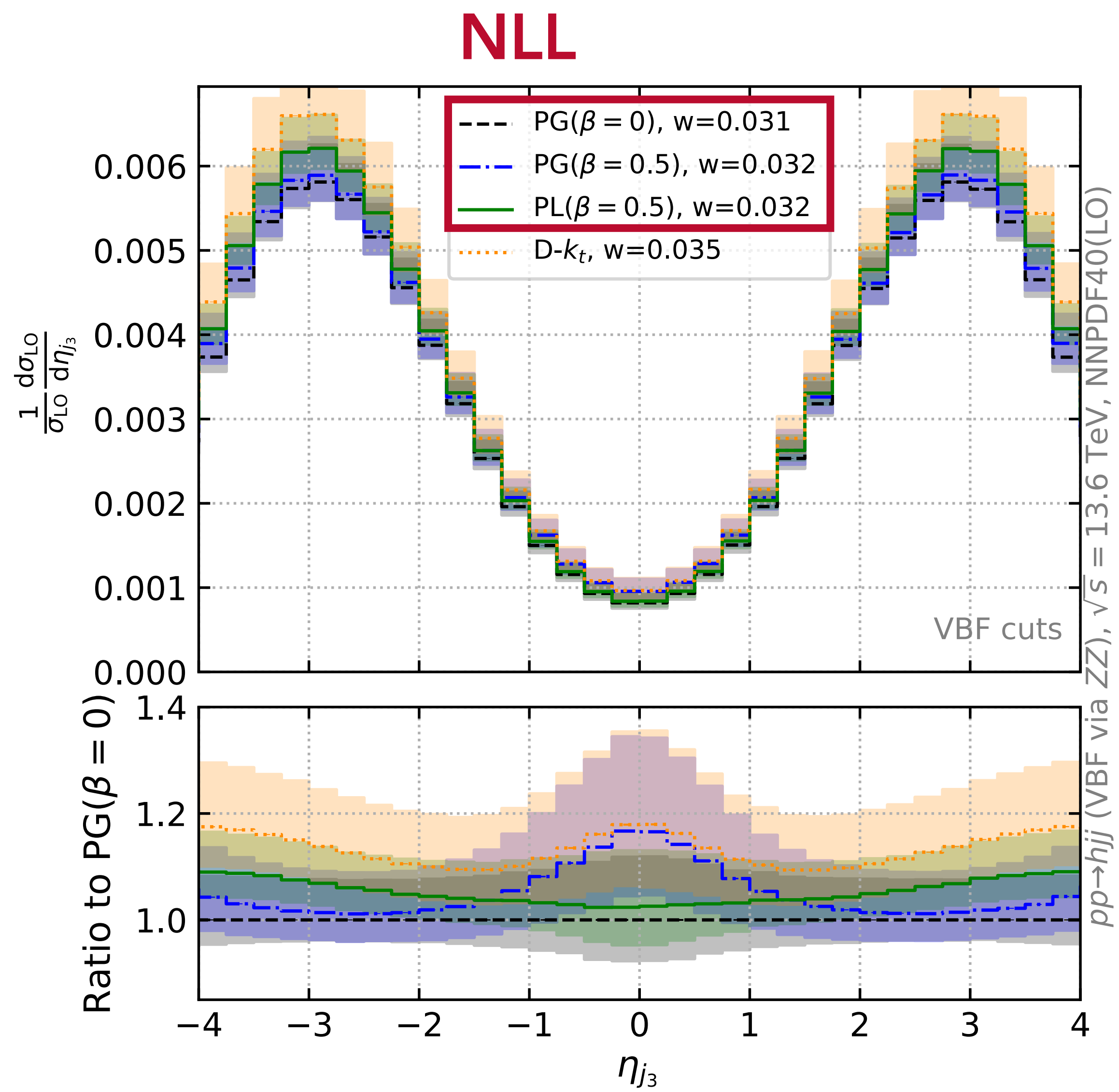
**NLL PanScales showers**

LL vs NLL differ by ~15% for 3rd jet (within scale var.)

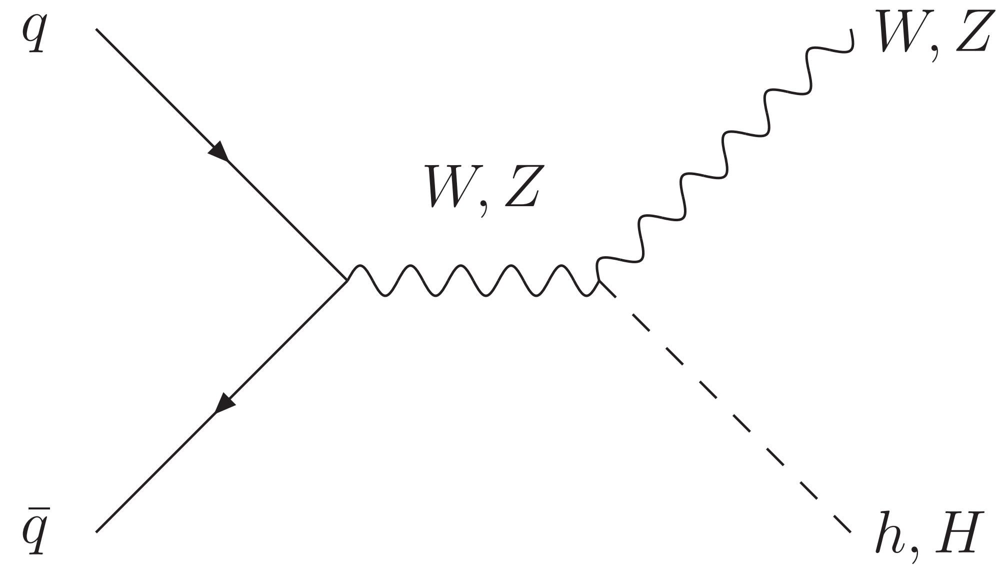
van Beekveld, Ferrario Ravasio 23

**NLO EW+PS** recently available

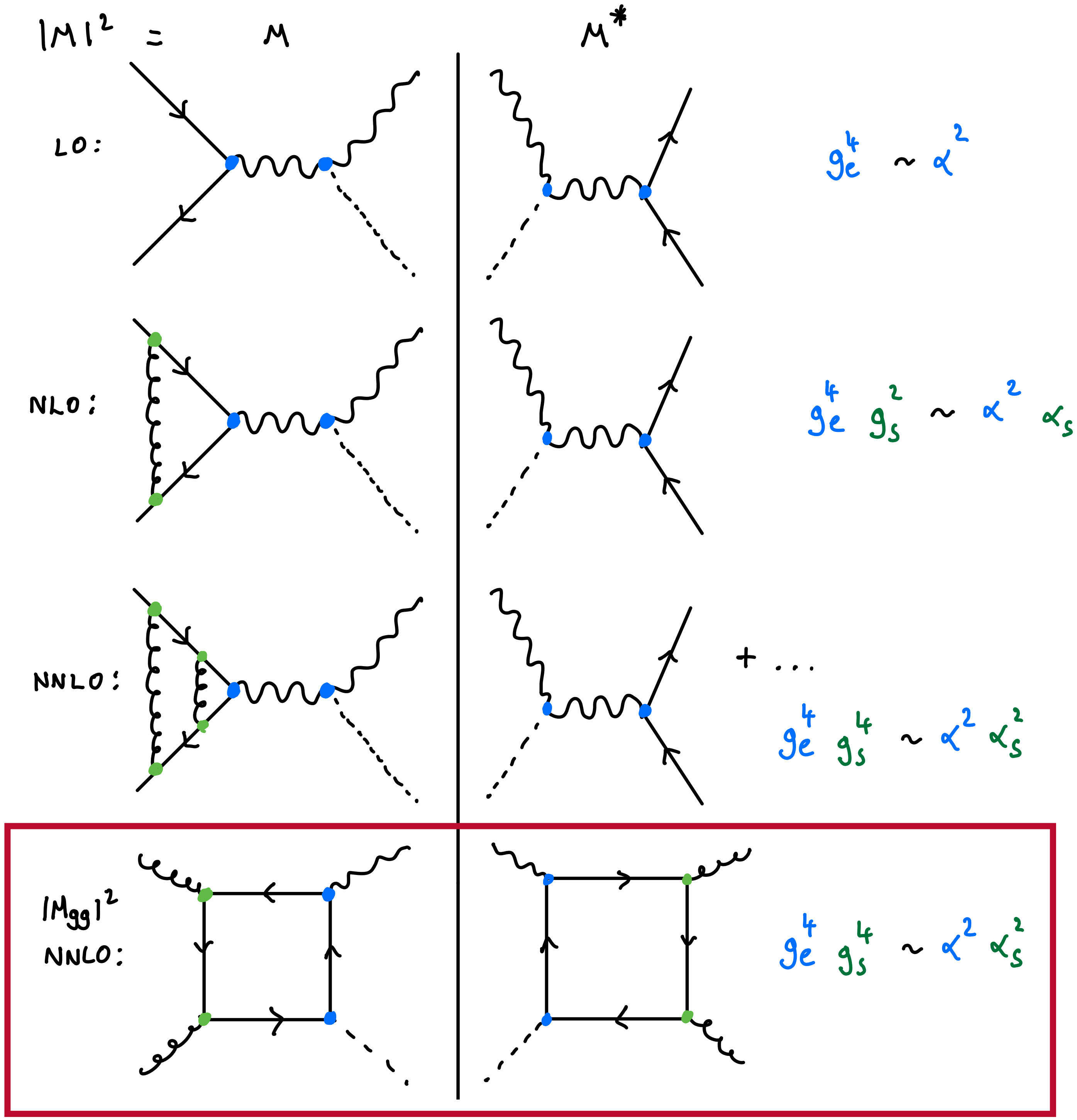
Jäger, Scheller 22



# ZH Production



# $pp \to ZH$: Role of $gg$ Channel




Drell-Yan-like contribution recently computed @ N$^3$LO

Baglio, Duhr, Mistlberger, Szafron 22

Gluon channel contributes to $pp \to ZH$ @ NNLO

1) ~10% of the total cross section at LHC (due to large gluon luminosity)
2) Has large uncertainty >100%
3) A dominant TH uncertainty on ZH analyses

**This motivates calculating $ggZH$ @ NLO (2-loop)**

# $gg \to ZH$: Results

Putting all pieces together (Born + reals + virtual) can obtain full NLO results

Wang, Xu, Xu, Yang 21; Chen, Davies, Heinrich, SPJ, Kerner, Mishima, Schlenk, Steinhauser 22; Degrassi, Gröber, Vitti, Zhao 22

## Total Cross-section

| $\sqrt{s}$ | LO [fb] | NLO [fb] |
|---|---|---|
| 13 TeV | $52.42^{+25.5\%}_{-19.3\%}$ | $103.8(3)^{+16.4\%}_{-13.9\%}$ |
| 13.6 TeV | $58.06^{+25.1\%}_{-19.0\%}$ | $114.7(3)^{+16.2\%}_{-13.7\%}$ |
| 14 TeV | $61.96^{+24.9\%}_{-18.9\%}$ | $122.2(3)^{+16.1\%}_{-13.6\%}$ |

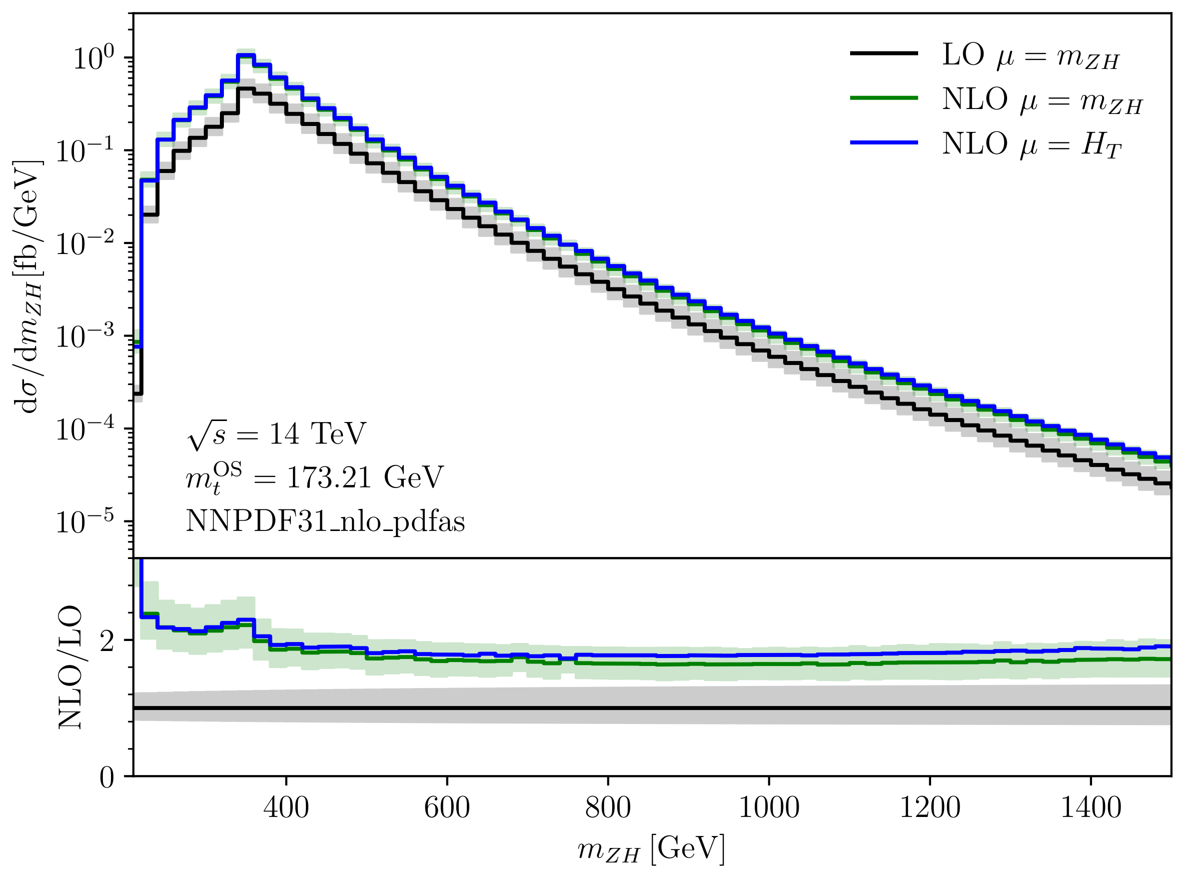
NNPDF31_nlo_pdfas

$m_t^{\mathrm{OS}} = 173.21$ GeV

$\mu = m_{ZH}$

$\mu_{R,F} \in \left[\frac{\mu}{2}, 2\mu\right]$ (7 – point)

## Invariant Mass



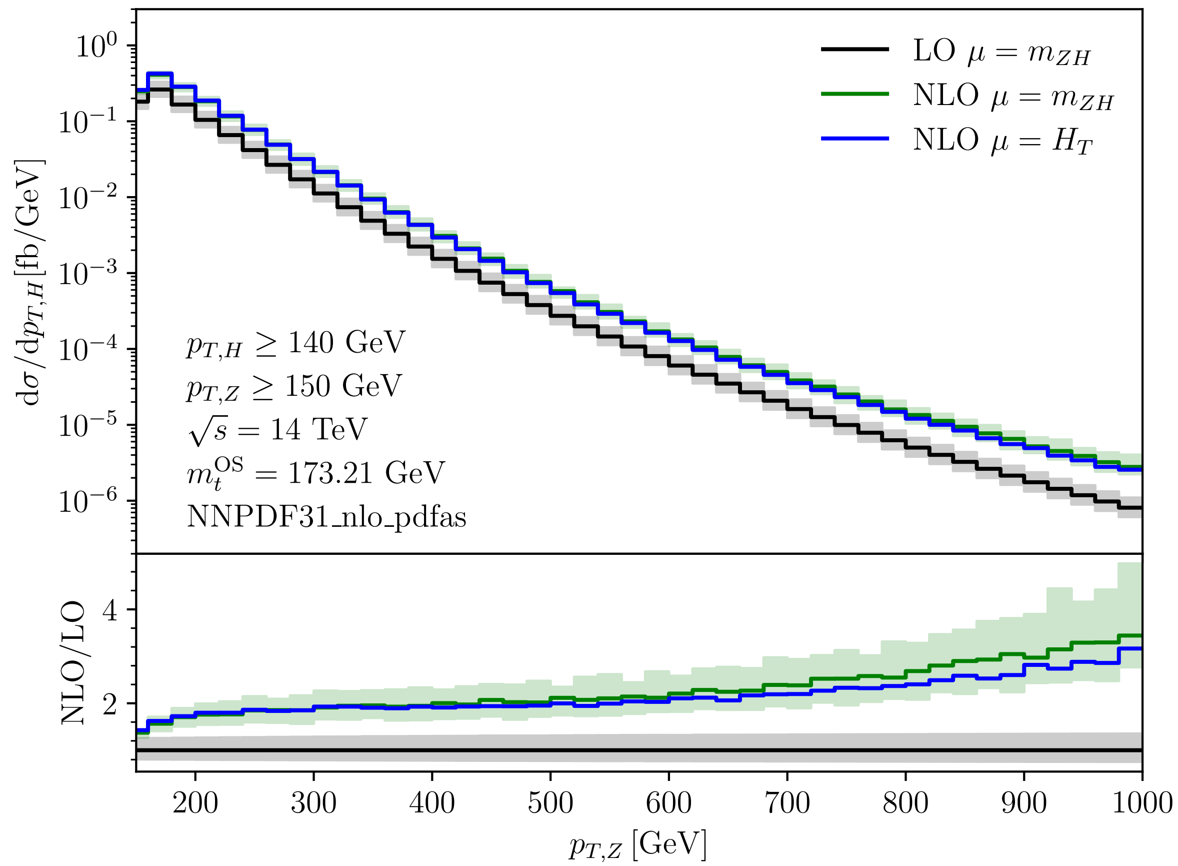

NLO corrections are large and lie outside the usual LO scale uncertainties

NLO/LO somewhat* flat except at production & top thresholds

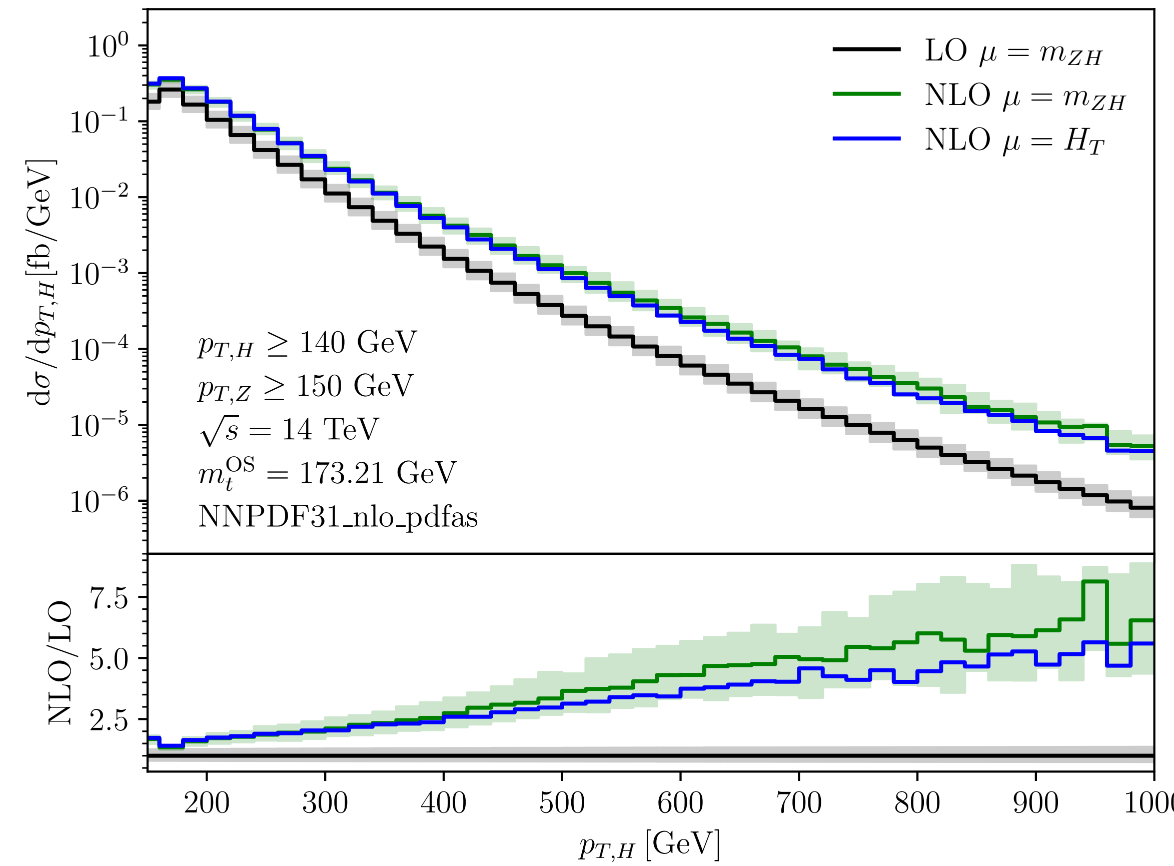
*Starts to rise above ~1 TeV (also depends on what real diagrams are included)

# $gg \to ZH$: Transverse Momentum

## Z Transverse Momentum




## H Transverse Momentum



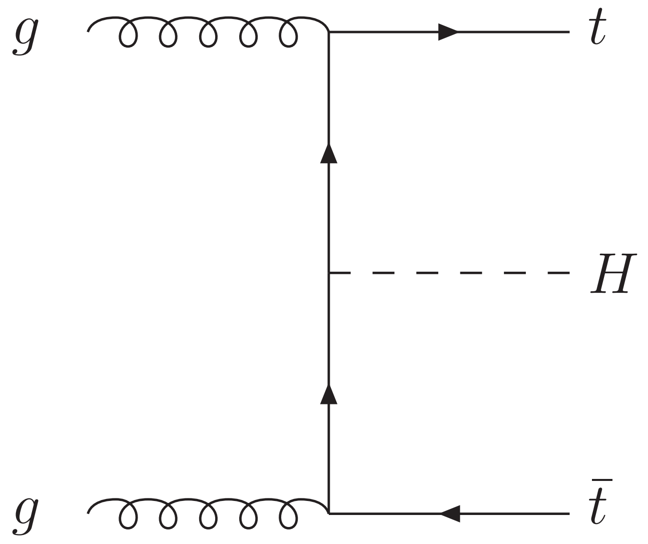

Z $p_T$ : **Large** NLO corrections, rising sharply at large $p_{T,Z}$

H $p_T$ : **Extremely** large NLO corrections, rising very sharply at large $p_{T,H}$

Placing cuts on soft $Z$ or $H$ emission slightly tames growth

**Radiating an additional jet opens up an important new region of phase-space**
**Very important to include higher order corrections in this region**

# $t\bar{t}H$ Production



# Soft-Higgs Approximation

The 2-loop virtual matrix elements for $t\bar{t}H$ are extremely challenging to compute:
↪ $2 \to 3$ process involving two additional scales ($m_t$, $m_h$ )

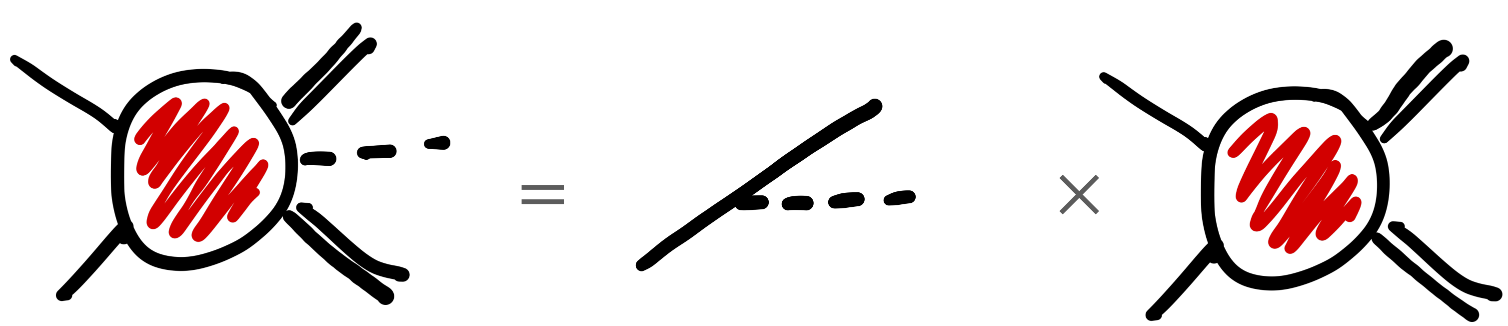
**Idea:**
Soft-Higgs boson emission from on-shell top quarks gives soft singularity

$$\lim_{k\to 0}\,[(p+k)^2 - m_t^2]^{-1} \to [(p^2 - m_t^2)]^{-1}, \quad p^2 = m_t^2$$

Can derive **factorisation formula** from eikonal approx/low energy theorem (emission from highly off-shell propagators not captured)

Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Savoini 22

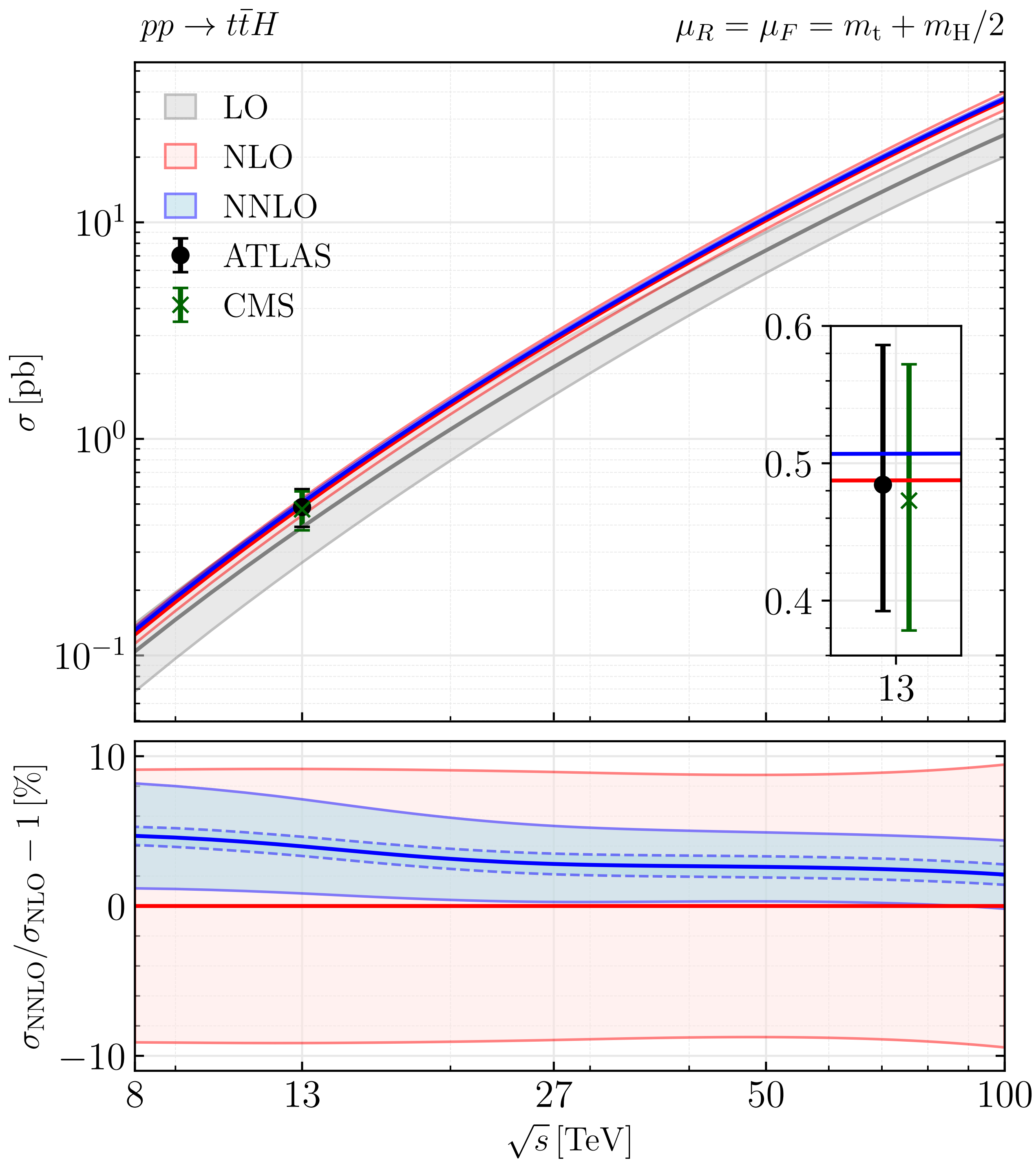
Figure: Simone Devoto (QCD@LHC 2023)



$$\mathcal{M}_{q\bar{q}'\to t\bar{t}H}\big(\{p_i\},k\big) \simeq F\big(\alpha_S(\mu_R); m_t/\mu_R\big)\,\frac{m_t}{v}\sum_{i=3,4}\frac{m_t}{p_i\cdot k}\mathcal{M}_{q\bar{q}'\to t\bar{t}}\big(\{p_i\}\big)$$

# $t\bar{t}H$ @ Approximate NNLO

Use $c\bar{c} \to t\bar{t}$ $(c = q, g)$ amplitudes + $Q\overline{Q}F$ generalisation of $q_T$ subtraction




| $\sigma$ [pb] | $\sqrt{s} = 13$ TeV | $\sqrt{s} = 100$ TeV |
|---|---|---|
| $\sigma_{LO}$ | $0.3910^{+31.3\%}_{-22.2\%}$ | $25.38^{+21.1\%}_{-16.0\%}$ |
| $\sigma_{NLO}$ | $0.4875^{+5.6\%}_{-9.1\%}$ | $36.43^{+9.4\%}_{-8.7\%}$ |
| $\sigma_{NNLO}$ | $0.5070\,(31)^{+0.9\%}_{-3.0\%}$ | $37.20(25)^{+0.1\%}_{-2.2\%}$ |

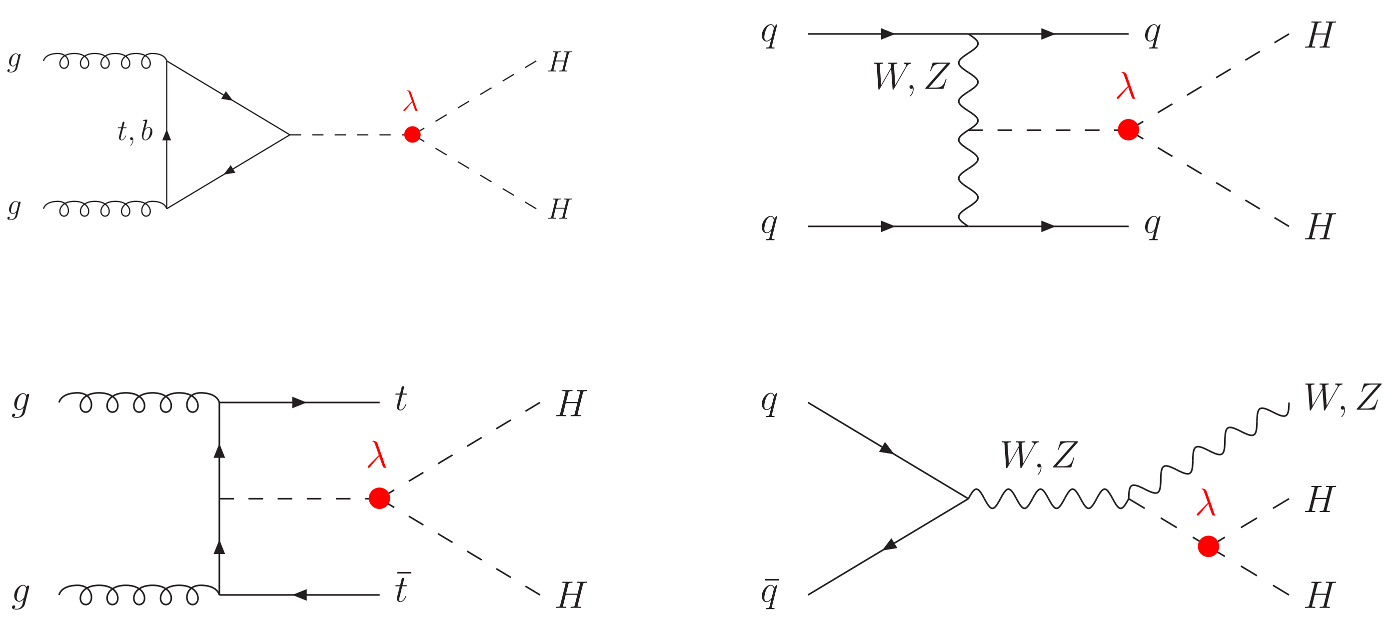
Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Savoini 22

NNLO +4% @ 13 TeV

Significant reduction of scale uncertainties

Soft approximation uncertainty estimated to be significantly smaller than scale uncertainty (using NLO)

# HH Production




$$\sigma(pp \to HH) \sim \frac{\sigma(pp \to H)}{1000}$$

# $b\bar{b}H$ Background

HH production known @ NLO (full), N3LO (HTL)

Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert Zirke 16, 16; Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira, Streicher 18, 20; Chen, Li, Shao, Wang 19;

Small signal means we need excellent control of the backgrounds

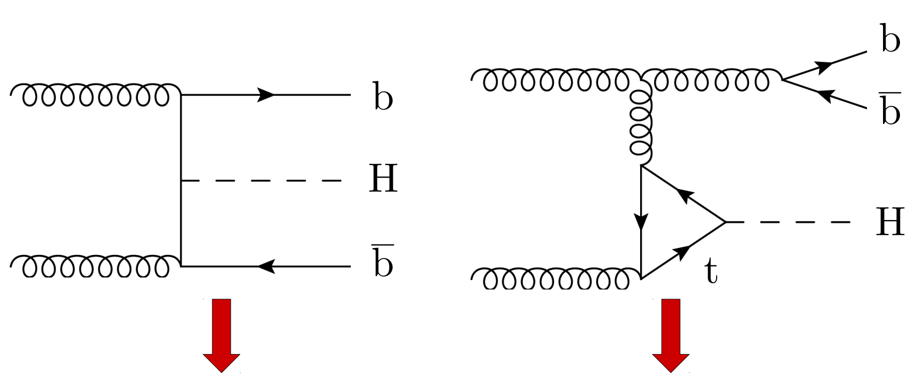
Higgs production in association with $b\bar{b}$ is a **background** to $H(\to b\bar{b})H(\to X)$

**NLO corrections** known for some time

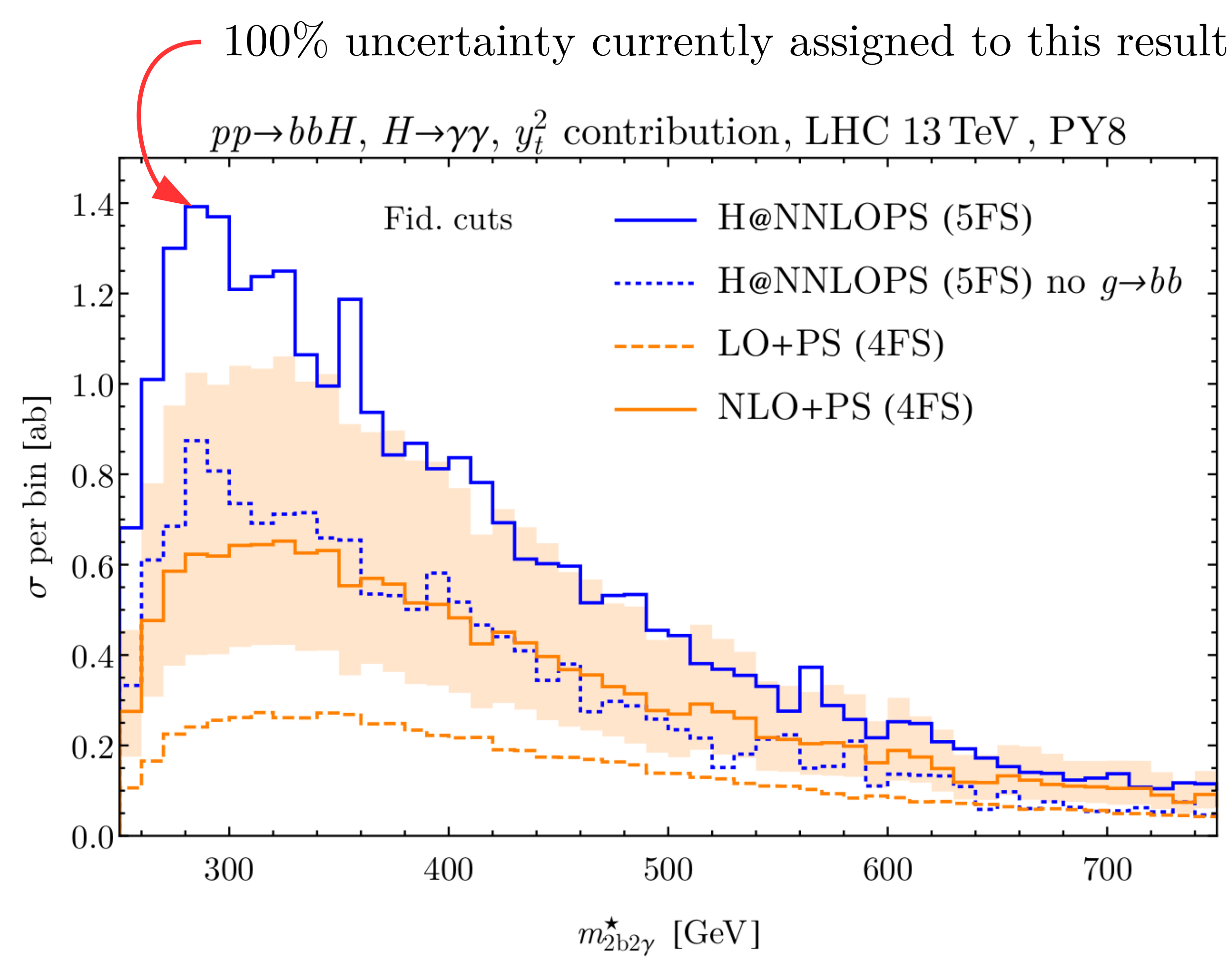
Deutschmann, Maltoni, Wiesemann, Zaro 18

**New NLO+PS** analysis targeting HH signal region, allowing PS $g \to b\bar{b}$ vastly overestimates background

Manzoni, Mazzeo, Mazzitelli, Wiesemann, Zaro 23




$\mathcal{O}(\alpha_s^2 y_b^2)$ $\mathcal{O}(\alpha_s^4 y_t^2)$




**Improves cross-section limits by 2-20%** (depending on channel LHC/HL-LHC)

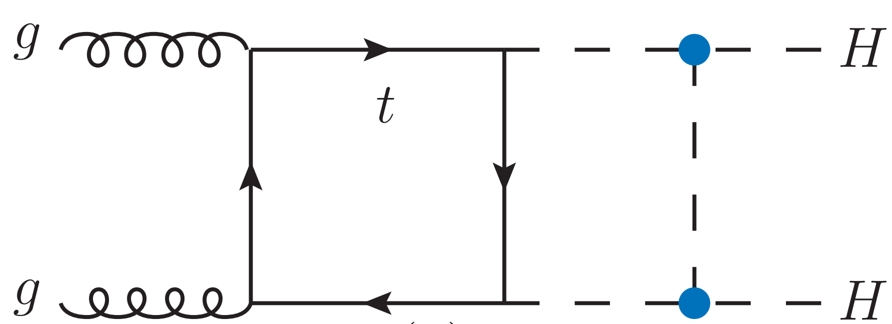
# Electroweak Corrections

It is also interesting to explore the impact of EW corrections (in single Higgs for off-shell Higgs have $\pm 5\,\%$ impact) Actis, Passarino, Sturm, Uccirati 08

Richer structure in the SM and much richer structure in the context of EFT

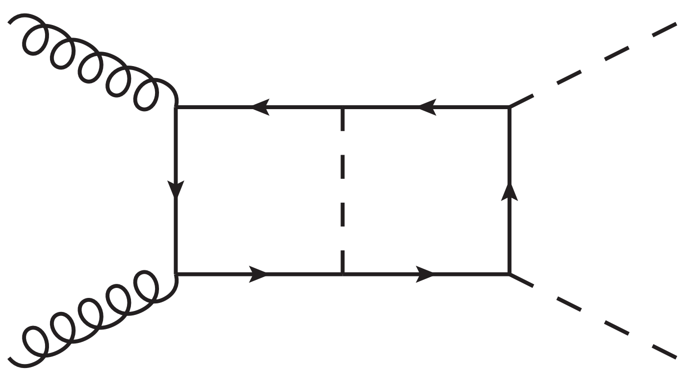
**Partial 2-loop EW corrections known:**

Involving $\lambda_3$ and $\lambda_4$




Borowka, Duhr, Maltoni, Pagani, Shivaji, Zhao 18

Leading top-Yukawa contributions



(Small $m_T$) Davies, Mishima, Schönwald, Steinhauser, Zhang 22
+ (EFT approach) Mühlleitner, Schlenk, Spira 22

**Complete EW corrections will modify distributions and bounds in the SM & EFT frameworks**

# Electroweak Corrections (Partial)

Would be desirable to have complete EW corrections

↪ First step: both self-coupling/Yukawa corrections with exact mass dependence

↪ 2-loop 4-point fully massive amplitudes w/ 4 scales $(s, t, m_h, m_t + d = 4 - 2\epsilon)$

WIP: Gudrun Heinrich, SJ, Matthias Kerner, Tom Stone, Augustin Vestner

## Workflow:

1. Reduce amplitude to "master integrals" $\mathcal{O}(1k) \to 494$ ✓
   (8.5 GB symbolic expression!)

2. Evaluate master integrals
   + Numerically (pySecDec) ✓
   + Series solution (DiffExp) (✓)

3. Compute counter-term amplitude/renormalization (✓)

→ **See Talk: Tom Stone**

PRELIMINARY


```
{"s": 6392./1000., 't': -1038./1000.}
AMPgs2ghh3ght@1=
  +eps^-1*(-8.0501761760992874e-01-3.8847967242213449e-01j)
  ±eps^-1*(+6.4683122144165918e-18+6.0907618364907079e-18j)
  +eps^0*(-2.8972433508074156e+00+1.0897854394801396e+01j)
  ±eps^0*(+3.6435347347217201e-07+3.4598017999757325e-07j)
AMPgs2ghhghhhght@1=
  +eps^-1*(-1.4596999451695023e+01-7.0441161054046244e+00j)
  ±eps^-1*(+1.1728681184339178e-16+1.1044087140803378e-16j)
  +eps^0*(+8.5908827457094041e+00-2.3032424593432740e+01j)
  ±eps^0*(+2.1379793495768672e-06+2.0307809283694008e-06j)
AMPgs2ghhhght2@1=
  +eps^0*(-2.9328689251200672e+01+6.2777695560284613e+01j)
  ±eps^0*(+4.2325534773895372e-05+4.7059980270574143e-05j)
AMPgs2ght4@2=
  +eps^-1*(+1.0795435677340720e+01-1.5627850302093538e+01j)
  ±eps^-1*(+6.8356820307489631e-07+8.2071745646177096e-07j)
  +eps^0*(+1.2494307071197568e+01-1.3334138655224328e+01j)
  ±eps^0*(+5.5369904028054630e-05+5.7556899298774435e-05j)
AMPgs2ght4@1=
  +eps^-1*(+8.9965909545280738e+02+2.0129450721915367e+02j)
  ±eps^-1*(+3.1141662895359273e-07+3.9765520889843626e-07j)
  +eps^0*(-3.2569582999240259e+02+7.4212291343799063e+02j)
  ±eps^0*(+1.2062257017861179e-05+1.3260886643234515e-05j)
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  ±eps^0*(+6.4890537787570324e-06+7.3624383819487271e-06j)
AMPgs2ghh2ght2@1=
  +eps^0*(+5.9931726494394134e+01-2.7172796094458263e+01j)
  ±eps^0*(+7.2171240278777598e-06+8.0229678514775212e-06j)
AMPgs2ghhght3@2=
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  ±eps^0*(+5.2242846406267860e-05+5.2540153789084503e-05j)
AMPgs2ghhght3@1=
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  ±eps^-1*(+1.2444914353984587e-11+1.1939919132080516e-11j)
  +eps^0*(-5.6920823777757072e+01-1.4139018713494380e+02j)
  ±eps^0*(+2.3643191708284244e-05+1.9217507136723785e-05j)
```

# Electroweak Corrections (HTL)

Very recently, complete EW results were obtained in the large-$m_t$ limit (HTL)
Davies, Steinhauser, Schönwald, Zhang 23

Complete EW corrections are **somewhat complicated**…

- SM fields: $\{t,\, b,\, H,\, \gamma,\, Z,\, W^{\pm},\, \chi,\, \phi^{\pm}\}$
- ghosts: $\{u^{\gamma},\, u^{Z},\, u^{\pm}\}$

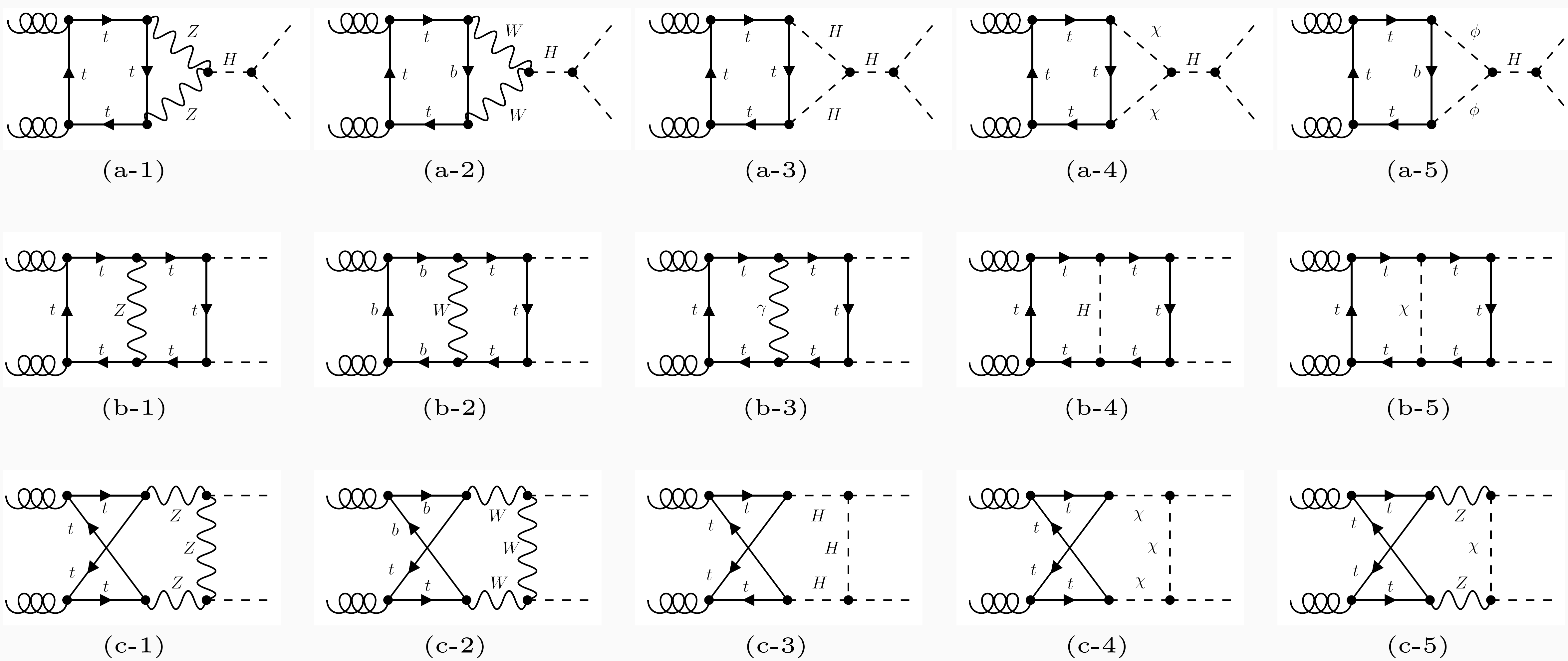



Figure: Kai Schönwald (20th LHCHWG Meeting)

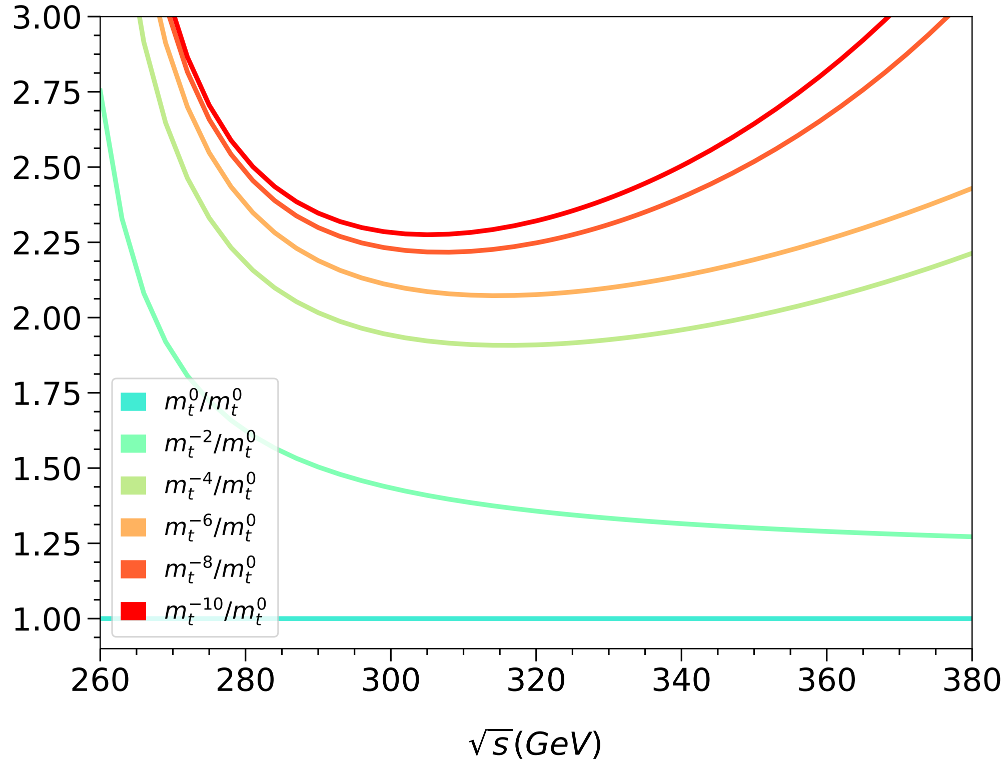
# Electroweak Corrections (HTL)

In the large-$m_t$ limit ( $m_t^2 \gg \xi_W m_W^2, \xi_Z m_Z^2 \gg s, t, m_H^2, m_W^2, m_Z^2$ )

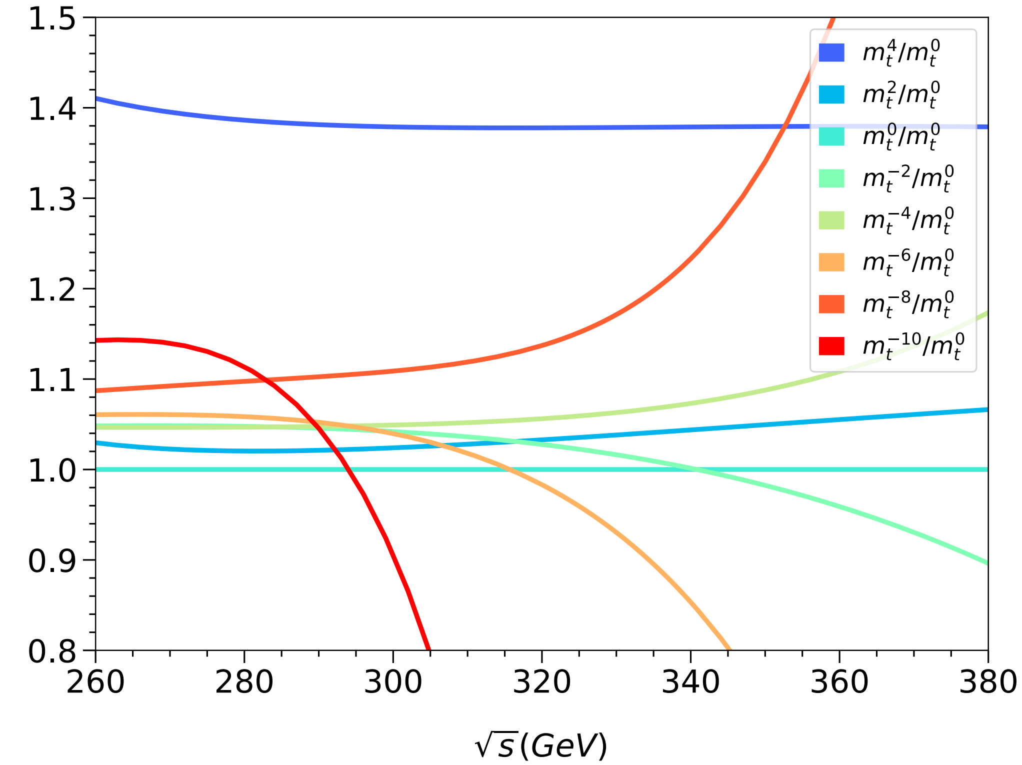
↪ LO expansion converges up to $\sqrt{s} \sim 2m_t$

↪ However, NLO expansion does not converge well above $HH$ threshold

**LO**

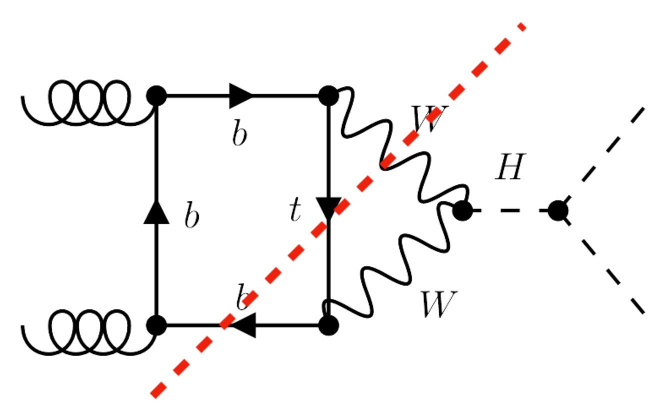



**NLO**




Traced to configurations with cut through W-t-b line

( $m_T + m_W + m_b \sim 250\text{GeV}$ )



# Summary

The last few years has seen impressive theory progress

Uncertainties beyond scale variations are becoming increasingly relevant, e.g. full understanding of massive corrections are important (SCET, beyond LP, fully massive results)

A lot of pQCD knowledge can be transferred to the computation of EW processes, starting to happen in a significant way (mixed QCD-EW)

**Still plenty to do!** →

Les Houches Wishlist 21

**Thank you for listening**

| process | known | desired |
|---|---|---|
| $pp \to H$ | $\mathrm{N^3LO_{HTL}}$<br>$\mathrm{NNLO^{(t)}_{QCD}}$<br>$\mathrm{N^{(1,1)}LO^{(HTL)}_{QCD\otimes EW}}$<br>$\mathrm{NLO_{QCD}}$ | $\mathrm{N^4LO_{HTL}}$ (incl.)<br>$\mathrm{NNLO^{(b,c)}_{QCD}}$ |
| $pp \to H+j$ | $\mathrm{NNLO_{HTL}}$<br>$\mathrm{NLO_{QCD}}$<br>$\mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$ | $\mathrm{NNLO_{HTL} \otimes NLO_{QCD} + NLO_{EW}}$ |
| $pp \to H+2j$ | $\mathrm{NLO_{HTL} \otimes LO_{QCD}}$<br>$\mathrm{N^3LO^{(VBF^*)}_{QCD}}$ (incl.)<br>$\mathrm{NNLO^{(VBF^*)}_{QCD}}$<br>$\mathrm{NLO^{(VBF)}_{EW}}$ | $\mathrm{NNLO_{HTL} \otimes NLO_{QCD} + NLO_{EW}}$<br>$\mathrm{N^3LO^{(VBF^*)}_{QCD}}$<br>$\mathrm{NNLO^{(VBF)}_{QCD}}$ |
| $pp \to H+3j$ | $\mathrm{NLO_{HTL}}$<br>$\mathrm{NLO^{(VBF)}_{QCD}}$ | $\mathrm{NLO_{QCD} + NLO_{EW}}$ |
| $pp \to VH$ | $\mathrm{NNLO_{QCD} + NLO_{EW}}$<br>$\mathrm{NLO^{(t,b)}_{gg\to HZ}}$ | |
| $pp \to VH+j$ | $\mathrm{NNLO_{QCD}}$<br>$\mathrm{NLO_{QCD} + NLO_{EW}}$ | $\mathrm{NNLO_{QCD} + NLO_{EW}}$ |
| $pp \to HH$ | $\mathrm{N^3LO_{HTL} \otimes NLO_{QCD}}$ | $\mathrm{NLO_{EW}}$ |
| $pp \to HH+2j$ | $\mathrm{N^3LO^{(VBF^*)}_{QCD}}$ (incl.)<br>$\mathrm{NNLO^{(VBF^*)}_{QCD}}$<br>$\mathrm{NLO^{(VBF)}_{EW}}$ | |
| $pp \to HHH$ | $\mathrm{NNLO_{HTL}}$ | |
| $pp \to H+t\bar{t}$ | $\mathrm{NLO_{QCD} + NLO_{EW}}$<br>$\mathrm{NNLO_{QCD}}$ (off-diag.) | $\mathrm{NNLO_{QCD}}$ |
| $pp \to H+t/\bar{t}$ | $\mathrm{NLO_{QCD} + NLO_{EW}}$ | $\mathrm{NNLO_{QCD}}$ |

# Backup

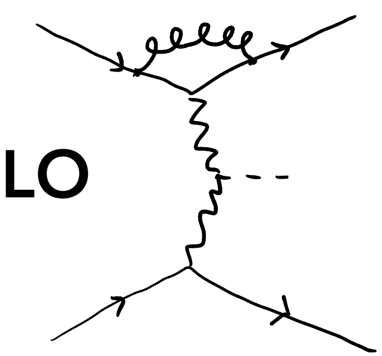
# Non-Factorisable Corrections

VBF is usually computed in the **structure function approach**

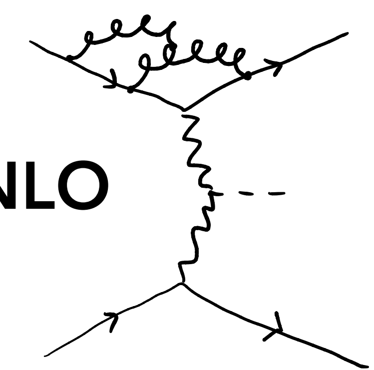
## Approximations & Precision

Known to N3LO in the **structure function / DIS approximation**

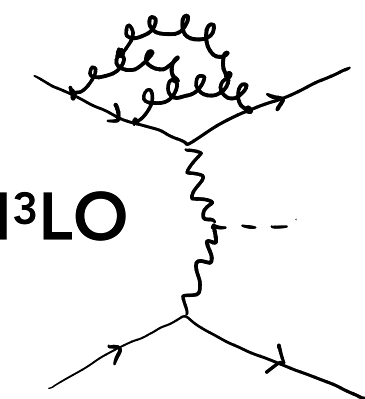
**NLO**



**NNLO**

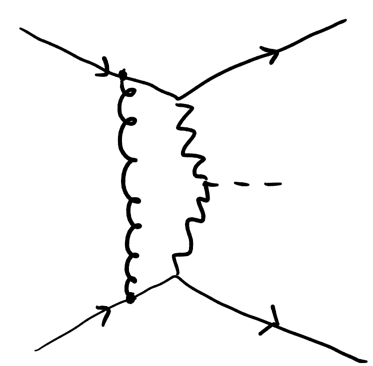


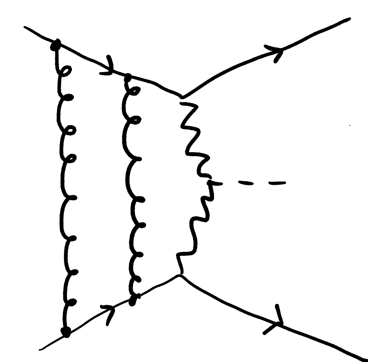
**N$^3$LO**



NLO: **non-factorisable** virtual contribution vanishes due to colour conservation

NNLO: get colour suppressed **non-factorisable** contribution





# Inclusion of NNLO $m_t$

**iHixs:**

$$\begin{aligned}\hat{\sigma}_{ij} &= \mathrm{R}_{\mathrm{LO}} C^2 \left[\sigma_{ij}^{\mathrm{LO,\,EFT}} + \sigma_{ij}^{\mathrm{NLO,\,EFT}} + \sigma_{ij}^{\mathrm{NNLO,\,EFT}} + \sigma_{ij}^{\mathrm{N^3LO,\,EFT}}\right] \\ &+ \delta\sigma_{ij}^{\mathrm{LO,\,(t,b,c)}} + \delta\sigma_{ij}^{\mathrm{NLO,\,(t,b,c)}} + \delta\sigma_{ij}^{\mathrm{NNLO,\,(t)}} + \mathrm{R}_{\mathrm{LO}} C^2\, \delta\sigma_{ij}^{\mathrm{Res}}\,.\end{aligned}$$

- start with iHixs prediction and systematically incorporate new results
- exact top mass at NNLO Czakon, Harlander, Klappert, Niggetiedt 21

$$\delta\sigma_{ij}^{\mathrm{NNLO,\,(t)}} = \boxed{\sigma_{ij}^{\mathrm{NNLO,\,approx.}}} - \left[C_{\mathrm{QCD}}^2 \mathrm{R}_{\mathrm{LO}} \sigma_{ij}^{\mathrm{EFT}}\right]_{\alpha_S^4} \quad \text{for} \quad (ij) \in \{(gg),(gq)\}$$

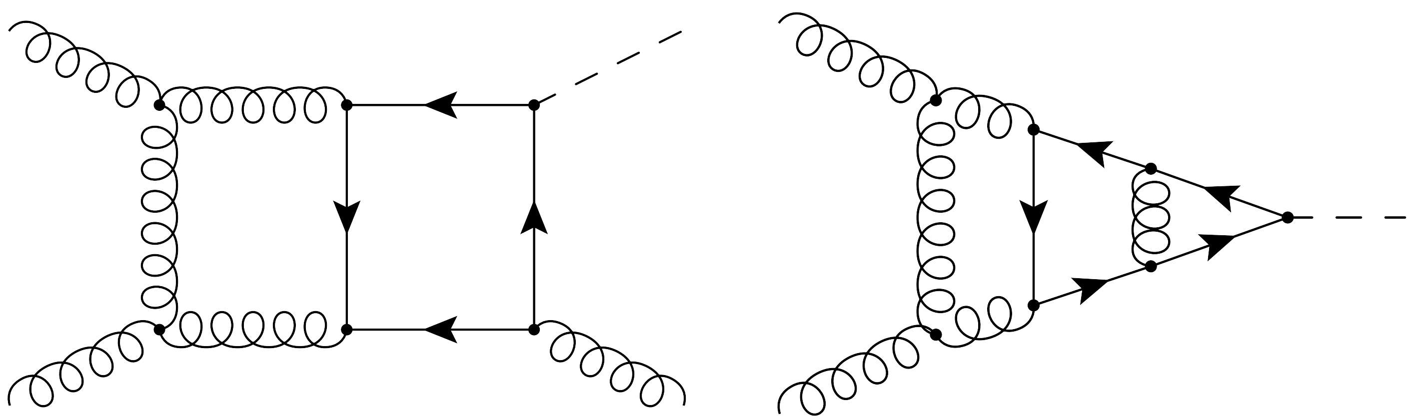
iHixs gives access to each part:

$\hookrightarrow$ substitution $\sigma_{ij}^{\mathrm{NNLO,\,approx}} \to \sigma_{ij}^{\mathrm{NNLO,\,exact}}$ straightforward

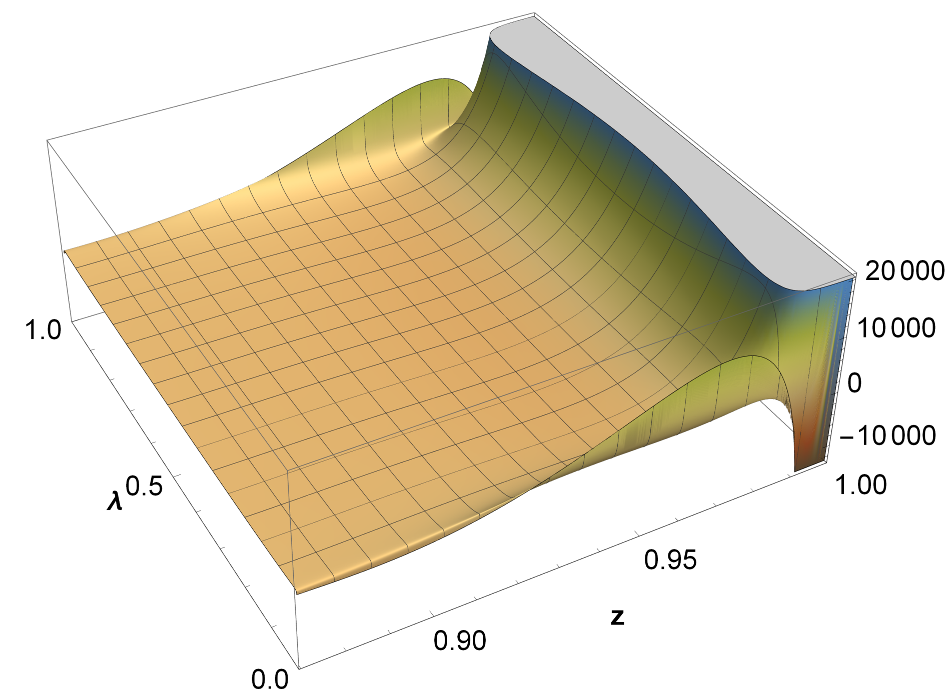
(computation of "exact" already as a difference to EFT $\rightsquigarrow$ compatibility checks)

# Gluon Fusion: NNLO with Full top-quark Mass

Returning to the $\delta(1/m_T)$ uncertainty in $gg \to H$



$gg \to Hg$




$2\mathrm{Re}\langle M^{(1)}_{\mathrm{exact}} | M^{(2)}_{\mathrm{exact}} \rangle |_{\mathrm{regulated}}$

H+j @ 2-loop & H @ 3-loop with $m_T$ using numerical solution of differential equations

Czakon, Niggetiedt 20;
Czakon, Harlander, Klappert, Niggetiedt 21

Decreases $\sigma_{\mathrm{tot}}$ by $-0.26\,\%$ @ 13 TeV compared to heavy top limit (HTL)

Intricate interplay between mass effects $gg\ (+0.62\%), qg\ (-16\%), qq\ (-15\%)$
Complete NNLO results obtained using STRIPPER framework

# Inclusion of mixed QCD-EW

**iHixs:**

$$\hat{\sigma}_{ij} = \mathrm{R_{LO}}C^2\left[\sigma_{ij}^{\text{LO, EFT}} + \sigma_{ij}^{\text{NLO, EFT}} + \sigma_{ij}^{\text{NNLO, EFT}} + \sigma_{ij}^{\text{N}^3\text{LO, EFT}}\right] + \delta\sigma_{ij}^{\text{LO, (t,b,c)}} + \delta\sigma_{ij}^{\text{NLO, (t,b,c)}} + \delta\sigma_{ij}^{\text{NNLO, (t)}} + \mathrm{R_{LO}}C^2\,\delta\sigma_{ij}^{\text{Res}}\,.$$

- start with iHixs prediction and systematically incorporate new results
- inclusion of EW corrections by Becchetti, Bonciani, Del Duca, Hirschi, Moriello, Schweitzer 20
  iHixs formula based on *factorization hypothesis*:

$$C = C_{\text{QCD}} + \lambda_{\text{EWK}}(1 + \frac{\alpha_S}{\pi}C_{1w} + \ldots).$$

↪ iHixs uses $C_{1w} = 7/6$ as estimated from the $M_V \to \infty$ limit

↪ full result gives: $C_{1w} = -1.7$ $(\mu_R = M_{\text{H}}/2)$ $C_{1w} = -2.1$ $(\mu_R = M_{\text{H}})$

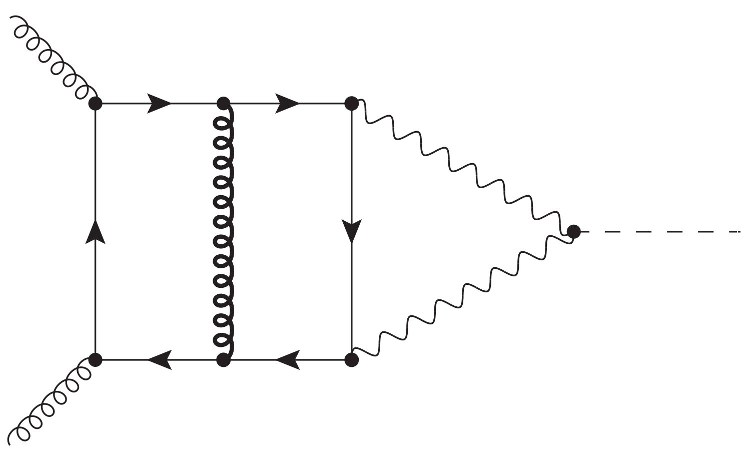
**but note:** $\delta(\text{EW}) \sim \pm 1\,\%$ ⇝ vary $C_{1w}$ by factor in range $[-3, 6]$

**Initial proposal:** incorporate new result with an additional correction term (1st step)

$$\delta\sigma_{ij}^{\text{EW}} = \sigma_{ij}^{\text{EW}} - \left[C^2 R_{\text{LO}}\sigma_{ij}^{\text{EFT}}\right]_{\alpha_s^3\alpha^2}$$

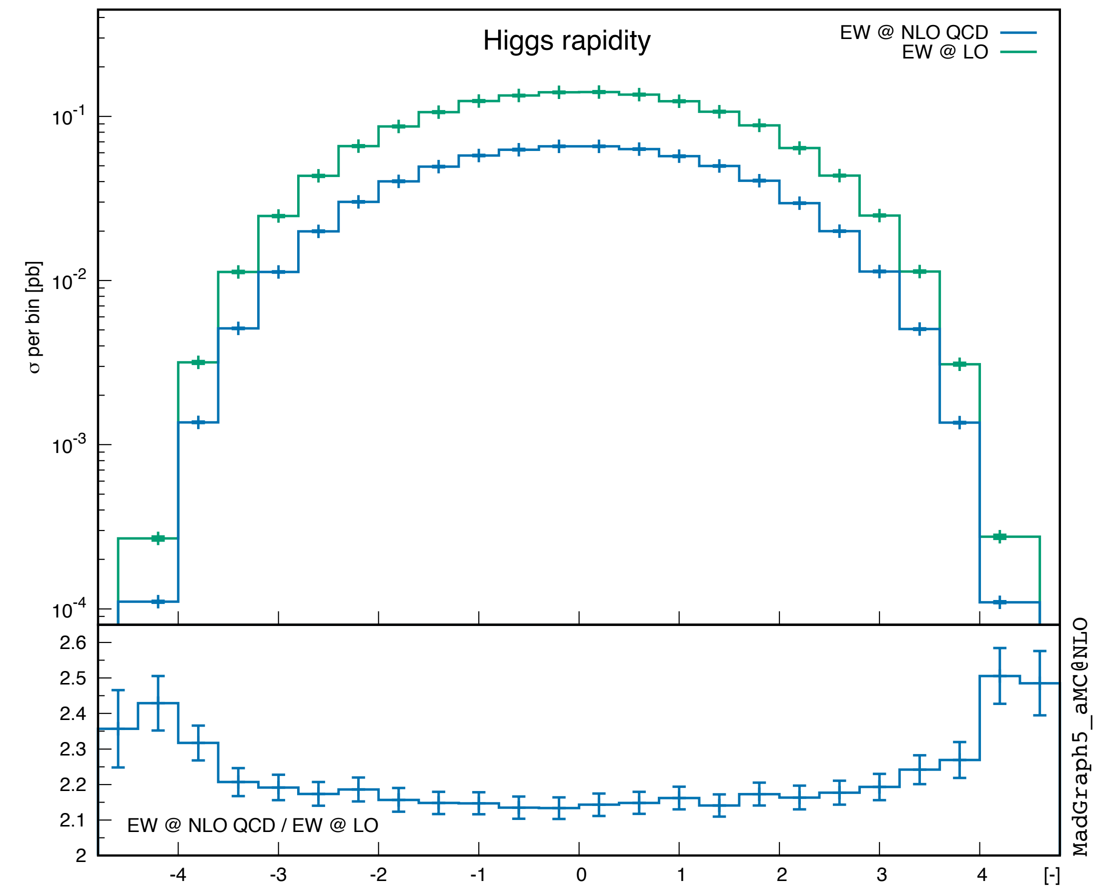
and define error estimates on correction factor (beyond light quarks, $gg$ channel, …)

# Gluon Fusion: Mixed QCD-EW Corrections



Challenging calculations

Bonetti, Melnikov, Tancredi 17
Bonetti, Panzer, Smirnov, Tancredi 20




Becchetti, Bonciani, Del Duca, Hirschi, Moriello, Schweitzer 20

Increases $\sigma_{\rm tot}$ by $+5.1\,\%$ @ 13 TeV, reduces residual uncertainty $\delta(\mathrm{EW}) \sim 0.6\,\%$

Favouring factorisation of EW corrections: $\sigma = \sigma_{\rm LO}\,(1+\delta_{\rm QCD}) \times (1+\delta_{\rm EWK})$

Compatible with previous estimates:

Soft approx: $+5.4\,\%$ ,
Bonetti, Melnikov, Tancredi 18;

$M_H \ll M_V: +5.2\,\%$ ,
Anastasiou, Boughezal, Petriello 09;

$M_H \gg M_V: +5.4\,\%$
Anastasiou, Del Duca, Furlan, Mistlberger, Moriello, Schweitzer, Specchia 19

# HH: Mass Scheme Uncertainty

Combination of scale ($\mu_R, \mu_F$) and top mass scheme ($\mathrm{OS}$ / $\overline{\mathrm{MS}}$) studied
Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira 20

If we wish to take the **envelope** of the predictions as the uncertainty, then the two uncertainties should be added **linearly** (validated at NLO)

**Scale ($\mu_R, \mu_F$)**

$$\begin{aligned}
\kappa_\lambda = -10: &\quad \sigma_{tot} = 1680^{+3.0\%}_{-7.7\%}\ \mathrm{fb},\\
\kappa_\lambda = -5: &\quad \sigma_{tot} = 598.9^{+2.7\%}_{-7.5\%}\ \mathrm{fb},\\
\kappa_\lambda = -1: &\quad \sigma_{tot} = 131.9^{+2.5\%}_{-6.7\%}\ \mathrm{fb},\\
\kappa_\lambda = 0: &\quad \sigma_{tot} = 70.38^{+2.4\%}_{-6.1\%}\ \mathrm{fb},\\
\kappa_\lambda = 1: &\quad \sigma_{tot} = 31.05^{+2.2\%}_{-5.0\%}\ \mathrm{fb},\\
\kappa_\lambda = 2: &\quad \sigma_{tot} = 13.81^{+2.1\%}_{-4.9\%}\ \mathrm{fb},\\
\kappa_\lambda = 2.4: &\quad \sigma_{tot} = 13.10^{+2.3\%}_{-5.1\%}\ \mathrm{fb},\\
\kappa_\lambda = 3: &\quad \sigma_{tot} = 18.67^{+2.7\%}_{-7.3\%}\ \mathrm{fb},\\
\kappa_\lambda = 5: &\quad \sigma_{tot} = 94.82^{+4.9\%}_{-8.8\%}\ \mathrm{fb},\\
\kappa_\lambda = 10: &\quad \sigma_{tot} = 672.2^{+4.2\%}_{-8.5\%}\ \mathrm{fb}
\end{aligned}$$

+

**NLO Mass Scheme Unc.**

$$\begin{aligned}
\kappa_\lambda = -10: &\quad \sigma_{tot} = 1438(1)^{+10\%}_{-6\%}\ \mathrm{fb},\\
\kappa_\lambda = -5: &\quad \sigma_{tot} = 512.8(3)^{+10\%}_{-7\%}\ \mathrm{fb},\\
\kappa_\lambda = -1: &\quad \sigma_{tot} = 113.66(7)^{+8\%}_{-9\%}\ \mathrm{fb},\\
\kappa_\lambda = 0: &\quad \sigma_{tot} = 61.22(6)^{+6\%}_{-12\%}\ \mathrm{fb},\\
\kappa_\lambda = 1: &\quad \sigma_{tot} = 27.73(7)^{+4\%}_{-18\%}\ \mathrm{fb},\\
\kappa_\lambda = 2: &\quad \sigma_{tot} = 13.2(1)^{+1\%}_{-23\%}\ \mathrm{fb},\\
\kappa_\lambda = 2.4: &\quad \sigma_{tot} = 12.7(1)^{+4\%}_{-22\%}\ \mathrm{fb},\\
\kappa_\lambda = 3: &\quad \sigma_{tot} = 17.6(1)^{+9\%}_{-15\%}\ \mathrm{fb},\\
\kappa_\lambda = 5: &\quad \sigma_{tot} = 83.2(3)^{+13\%}_{-4\%}\ \mathrm{fb},\\
\kappa_\lambda = 10: &\quad \sigma_{tot} = 579(1)^{+12\%}_{-4\%}\ \mathrm{fb}
\end{aligned}$$

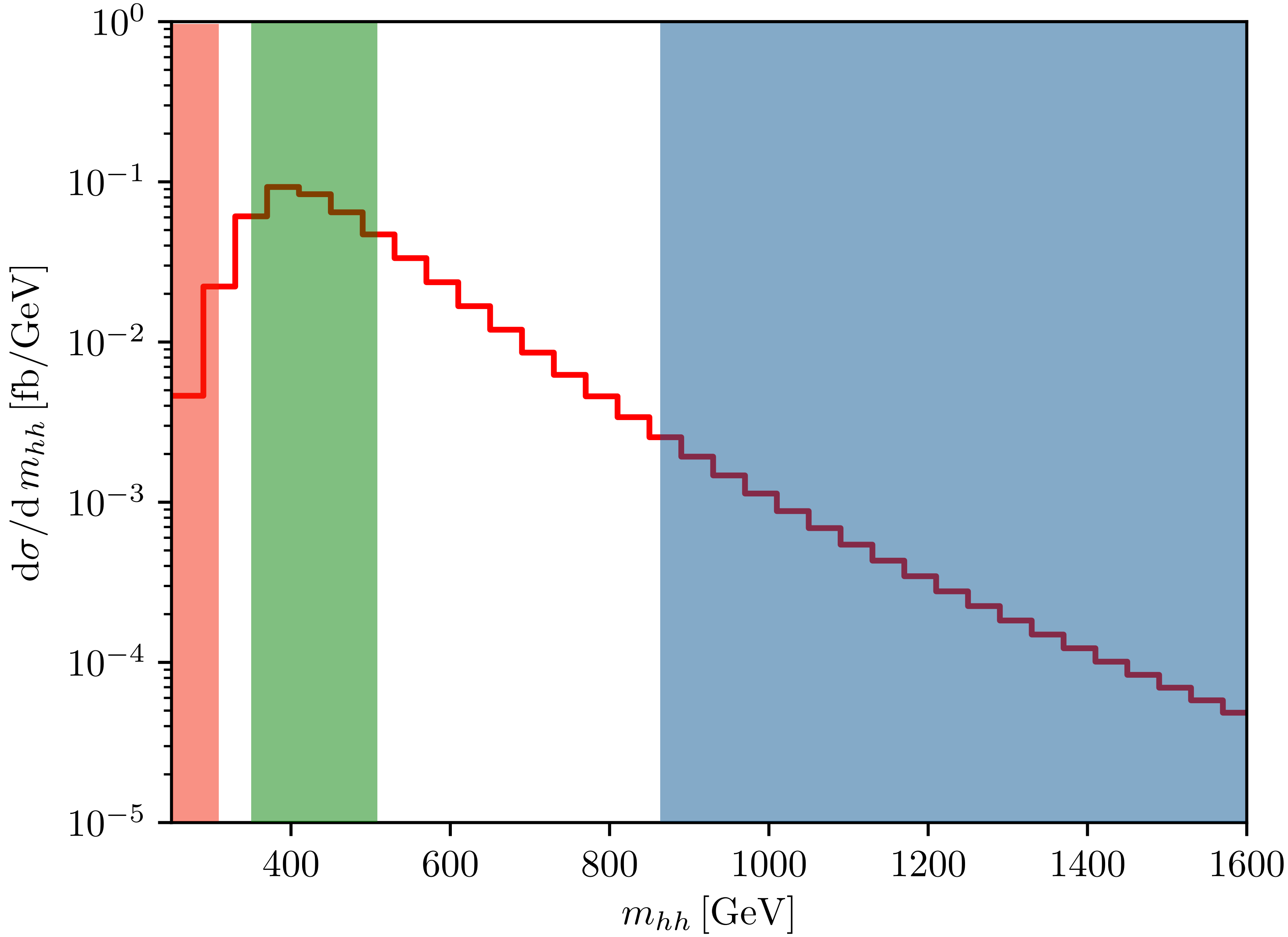
=

**Proposed Combination**

$$\boxed{\begin{aligned}
\kappa_\lambda = -10: &\quad \sigma_{tot} = 1680^{+13\%}_{-14\%}\ \mathrm{fb},\\
\kappa_\lambda = -5: &\quad \sigma_{tot} = 598.9^{+13\%}_{-15\%}\ \mathrm{fb},\\
\kappa_\lambda = -1: &\quad \sigma_{tot} = 131.9^{+11\%}_{-16\%}\ \mathrm{fb},\\
\kappa_\lambda = 0: &\quad \sigma_{tot} = 70.38^{+8\%}_{-18\%}\ \mathrm{fb},\\
\kappa_\lambda = 1: &\quad \sigma_{tot} = 31.05^{+6\%}_{-23\%}\ \mathrm{fb},\\
\kappa_\lambda = 2: &\quad \sigma_{tot} = 13.81^{+3\%}_{-28\%}\ \mathrm{fb},\\
\kappa_\lambda = 2.4: &\quad \sigma_{tot} = 13.10^{+6\%}_{-27\%}\ \mathrm{fb},\\
\kappa_\lambda = 3: &\quad \sigma_{tot} = 18.67^{+12\%}_{-22\%}\ \mathrm{fb},\\
\kappa_\lambda = 5: &\quad \sigma_{tot} = 94.82^{+18\%}_{-13\%}\ \mathrm{fb},\\
\kappa_\lambda = 10: &\quad \sigma_{tot} = 672.2^{+16\%}_{-13\%}\ \mathrm{fb}
\end{aligned}}$$

@13 TeV

# Tackling Mass Scheme Uncertainties




**Low invariant mass:**
expand in $1/m_t^2$
known to NNLO
Grigo, Hoff, Steinhauser 15;

**Around Peak:**
threshold expansion
Gröber, Maier, Rauh 17

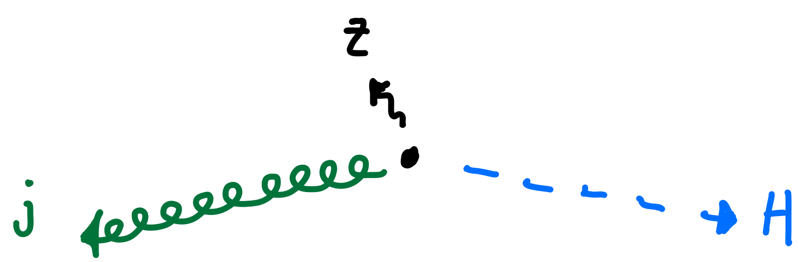
**High energy:**
small-$m_t$ expansion
known at NLO
Davies, Mishima, Steinhauser, Wellmann 18, 19

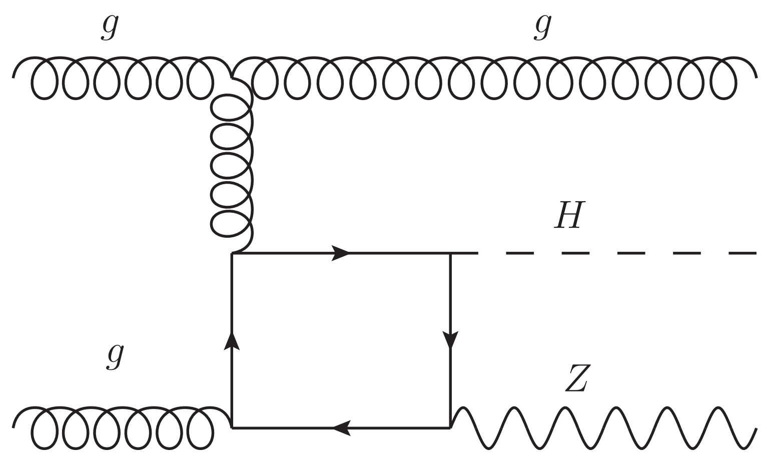
**Options:**
1) Try to understand structure of mass logarithms
2) Keep calculating
3) Other ideas (?)

# $gg \to ZH$: Z vs H

The different behaviour of $p_{T,Z}$ and $p_{T,H}$ was observed previously in $gg \to ZH + j$

Hespel, Maltoni, Vryonidou 15; Les Houches 19




Traced to configurations where Higgs recoils against a hard jet, with a soft $Z$

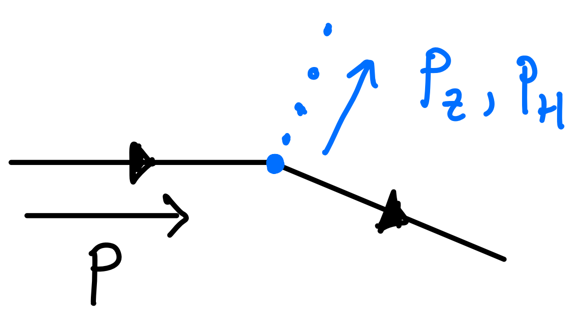
Maltoni et al. attributed this to $t$-channel gluon exchange

## One observation

If we apply an eikonal approximation to such diagrams, the enhancement of soft $Z$ bosons can be understood

$$\text{(Soft } Z \text{ emission)} : \frac{p^\mu}{p \cdot p_Z}$$

$$\text{(Soft } H \text{ emission)} : \frac{m_t}{p \cdot p_H}$$




Ratio for large radiator (transverse) momentum $\sim p_T/m_t \gg 1$