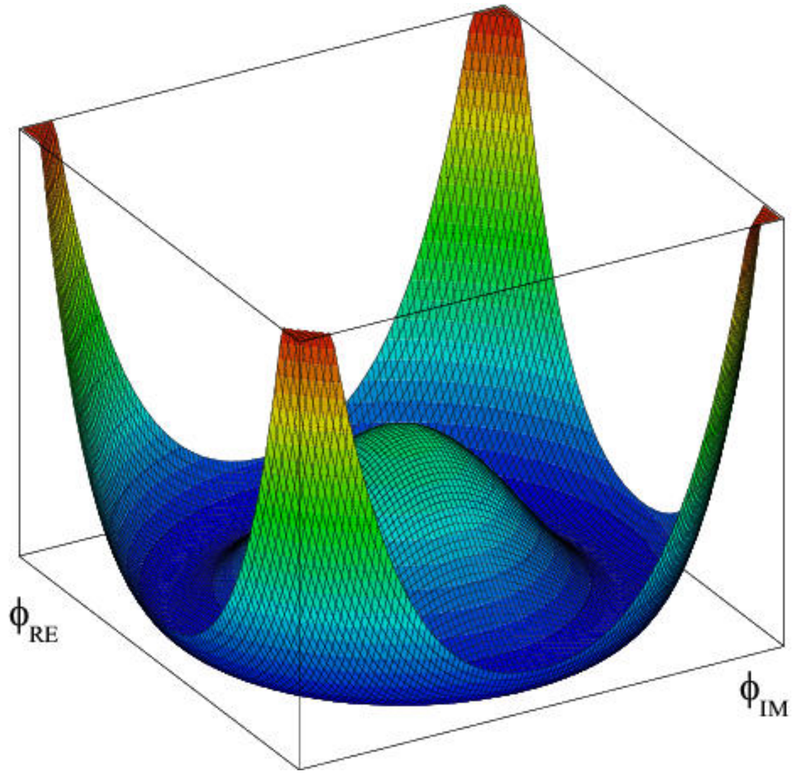
Higgs Field: a Superconnection?



Source: Wikipedia

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Hints

$$\mathcal{L}_{SM} = -\frac{1}{4}\mathrm{Tr}\left(G_{\mu\nu}G^{\mu\nu}\right) - \frac{1}{4}\mathrm{Tr}\left(W_{\mu\nu}W^{\mu\nu}\right) - \frac{1}{4}\mathrm{Tr}\left(B_{\mu\nu}B^{\mu\nu}\right)$$

$$i\left[\bar{q}_L \not{D} q_L + \bar{u}_R \not{D} u_R + \bar{d}_R \not{D} d_R + \bar{l}_L \not{D} l_L + \bar{\nu}_R \not{D} \nu_R + \bar{e}_R \not{D} e_R\right]$$

$$-(D_\mu h)^\dagger (D^\mu h) + m^2 h^\dagger h - \frac{\lambda}{4}(h^\dagger h)^2$$

$$-\left[\bar{q}_L^i Y_u^{ij} \tilde{h}\, u_R + \bar{q}_L^i Y_d^{ij} h\, d_R + \bar{l}_L^i Y_\nu^{ij} \tilde{h}\, \nu_R + \bar{l}_L^{ij} Y_e^{ij} h\, e_R + \text{h.c.}\right]$$

- The Standard Model Lagrangian is conformally invariant except for the terms involving the Higgs Boson.
- The Standard Model Lagrangian can be broken into self-dual and anti-self dual parts except for the terms involving the Higgs.

Superspace and Superalgebra

- A vector space V is called a superspace if it has a $\mathbb{Z}_2$ grading.

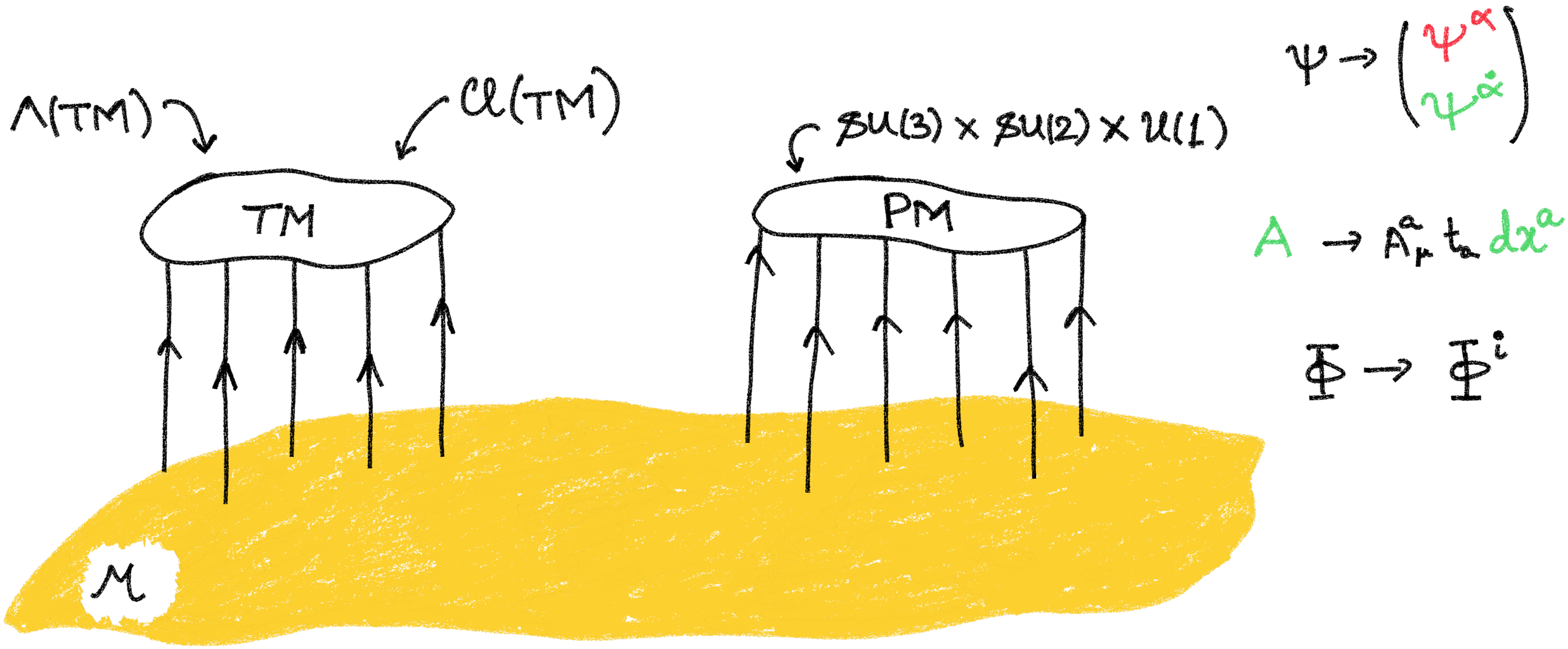
$$V = V_0 \oplus V_1$$

- A superalgebra is an algebra over a super space such that:

$$A^{\pm} \circ A^{\pm} = A^{+} \quad ; \quad A^{\pm} \circ A^{\mp} = A^{-}$$

- Examples:
- Exterior algebra of forms over a vector space. $\wedge(V) = \wedge^{+}(V) \oplus \wedge^{-}(V)$
- Clifford algebra over a vector space. $\mathcal{C}(V) = \mathcal{C}^{+}(V) \oplus \mathcal{C}^{-}(V)$

Standard Model using Bundles over Spacetime



Dirac Lagrangian: Higgs as a Superconnection ??

- A derivation on the Superbundle

$$D : \mathcal{A}(TM^{\pm}) \longrightarrow \mathcal{A}(TM^{\mp})$$

- Modified Dirac Lagrangian $\overline{\Psi}\not{\partial}\Psi + \overline{\Psi}\not{\Phi}\Psi$

$$D = \not{\partial} + \not{\Phi} = \gamma_a\left(e^a \, \lrcorner \, A\right) + \not{\Phi} \quad \xrightarrow{??}$$

- Should the Higgs be a superconnection?

A Possible Resolution : SU(2|1) Lie Supergroup

- Lie Superalgebras:

$$[t^a, t^b]_{\pm} = t^a t^b - (-1)^{|a||b|} t^b t^a = f^{ab}{}_c \, t^c$$

- $$\mathfrak{su}(2|1) \longrightarrow \left(\begin{array}{c|c} \mathfrak{su}(2) & \Phi \\ \hline \Phi^+ & u(1)_r \end{array} \right)$$

Why Superconnections?

- Calculation of the Weinberg angle and the Higgs-Weak mass ratio.
  (D. Fairlie, T. Mieg, Y. Ne'eman, S. Sternberg) (I. Todorov).

- Better understanding of Electroweak symmetry breaking.

- Might provide insights into conformal symmetry breaking and early universe cosmology.

Thank You!