Electroweak logarithms at High Energies

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Artwork by C.Z.

Introduction

- In the energy range above the **EW** scale ($\sqrt{s} \gg M_W$), *Sudakov logarithms* represent the leading contribution of **EW** radiative corrections

- *Sudakov logarithms* from N^{n}LO **EW** corrections

$$\alpha^n \log^k \frac{s}{M_W^2}, \qquad 1 \leq k \leq 2n$$

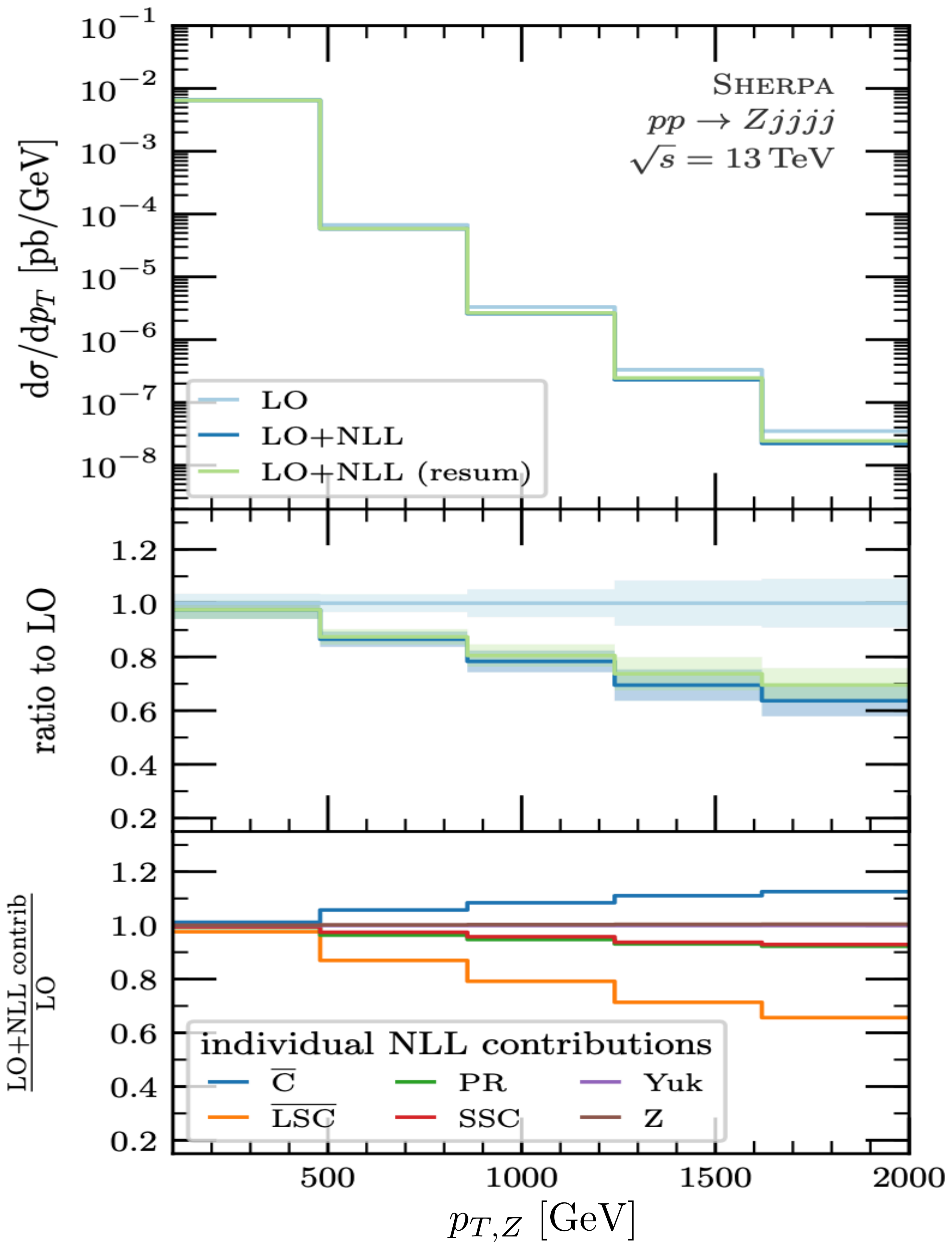
- At **NLO**

Double logs: $$L(s) = \frac{\alpha}{4\pi} \log^2 \frac{s}{M_W^2},$$

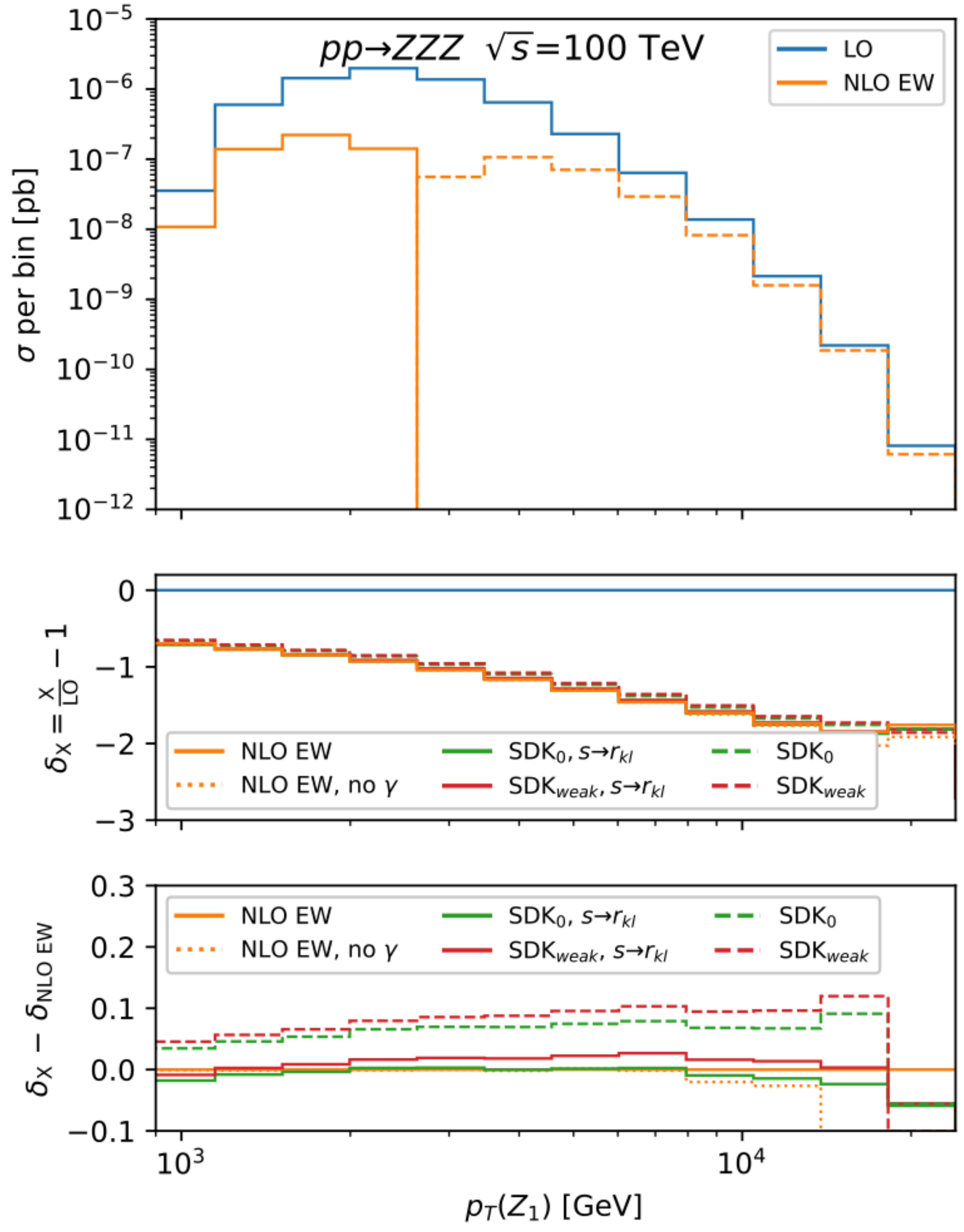
Single logs: $$l(s) = \frac{\alpha}{4\pi} \log \frac{s}{M_W^2}$$

Introduction

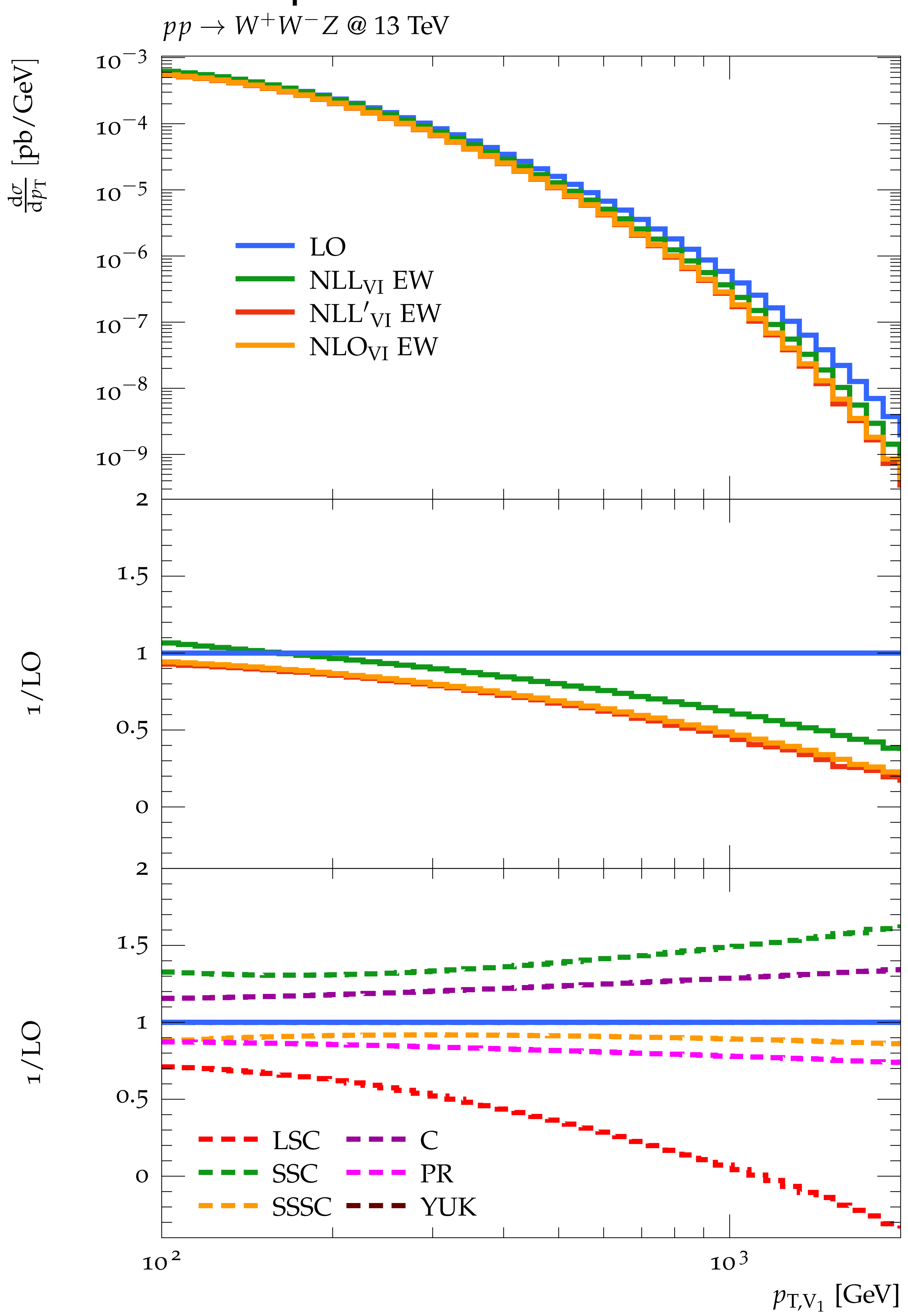
- Significant enhancement of tails of kinematic distributions up to several tens percent



[Bothmann, Napoletano 2006.14635; *2020*]



[Pagani, Zaro 2110.03714; *2021*]

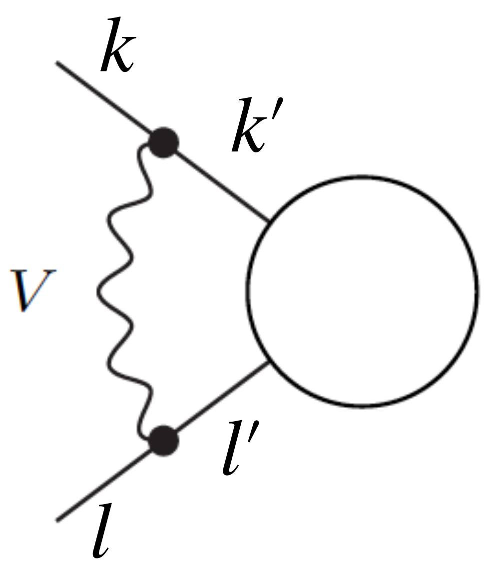


[Lindert, L.M.; *to appear soon*]

Origin of DL and SL

- DL originate from triangle diagrams where two external legs exchange a ***soft and collinear* (SC)** gauge boson V

$$\delta^{\mathrm{DL}}\mathcal{M} \sim \frac{\alpha}{4\pi}\, I^{V}_{kk'} I^{\bar{V}}_{ll'}\, \log^2 \frac{|r_{kl}|}{M_V^2}\, \mathcal{M}_0$$



$r_{kl} = (p_k + p_l)^2$

- DL can be split into:

  ➔ **LSC** (angular independent): $\sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \log^2\left(\frac{s}{M_V^2}\right)$

  ➔ **SSC** (angular dependent): $\sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \log\left(\frac{s}{M_V^2}\right) \log\left(\frac{|r_{kl}|}{s}\right)$

  ➔ **Sub-SSC** (angular dependent): $\sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \log^2\left(\frac{|r_{kl}|}{s}\right)$

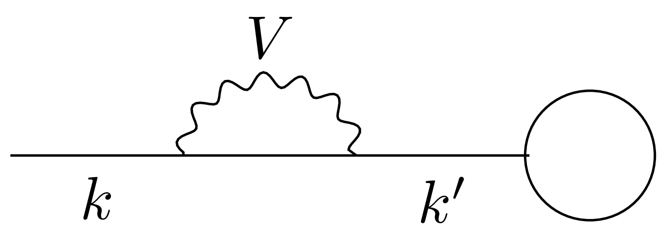
- SL have a triple origin:

  ➔ **PR**: UV renormalisation of **EW** dimensionless parameters

$$g_{i,0} = g_i + \delta g_i = (1 + \delta Z_{g_i})\, g_i$$

  ➔ **WF**: wave-function renormalisation of external fields

$$\delta^{\mathrm{WF}}\mathcal{M} \sim \frac{\alpha}{4\pi}\, I^V I^{\bar{V}} \log\left(\frac{s}{M_V^2}\right) \mathcal{M}_0$$



  ➔ **Coll**: external leg emission of a collinear gauge boson

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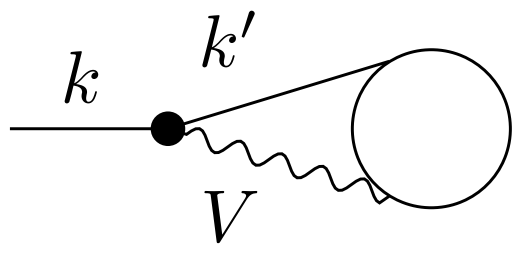

- **C**=**WF**+**Coll**: Full gauge-invariant SL associated to external fields

Implementation in OpenLoops: why

- **NLO EW** corrections have been automated nowadays

- **EW** *Sudakov logarithms* at one-loop already implemented in
    - ALPGEN: Chiesa *et al*, 1305.6837; ***2013***
    - Sherpa: Bothmann, Napoletano 2006.14635; ***2020***
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However:

- Even if automated, one-loop computations can be very complicated (e.g. high multiplicity processes)
- No **NNLO/two-loop** level automation available
- **EW** *Sudakov logs* have nice properties: ***factorisation***, being the leading contribution of radiative corrections

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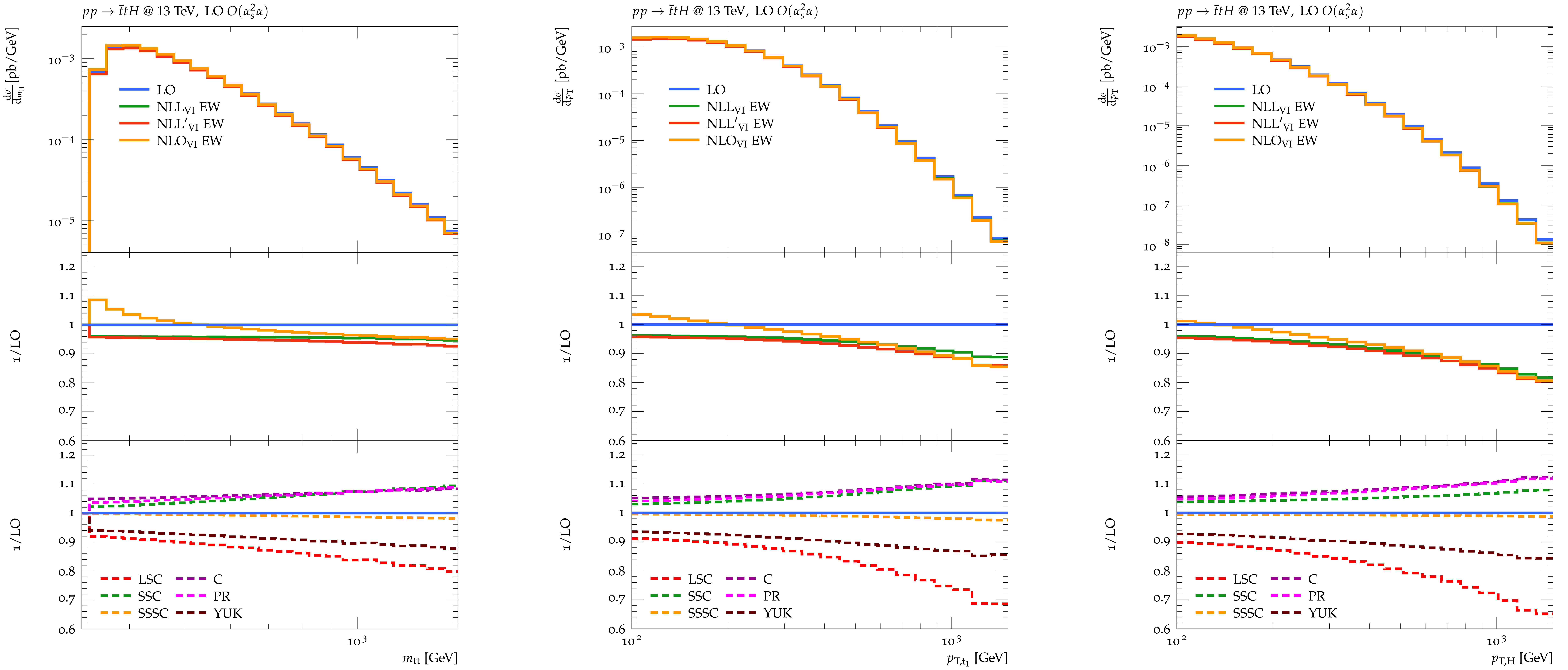
- OpenLoops (OL): automated tool for the calculation of tree and one-loop amplitudes [Buccioni *et al*, 1907.13071; ***2019***]

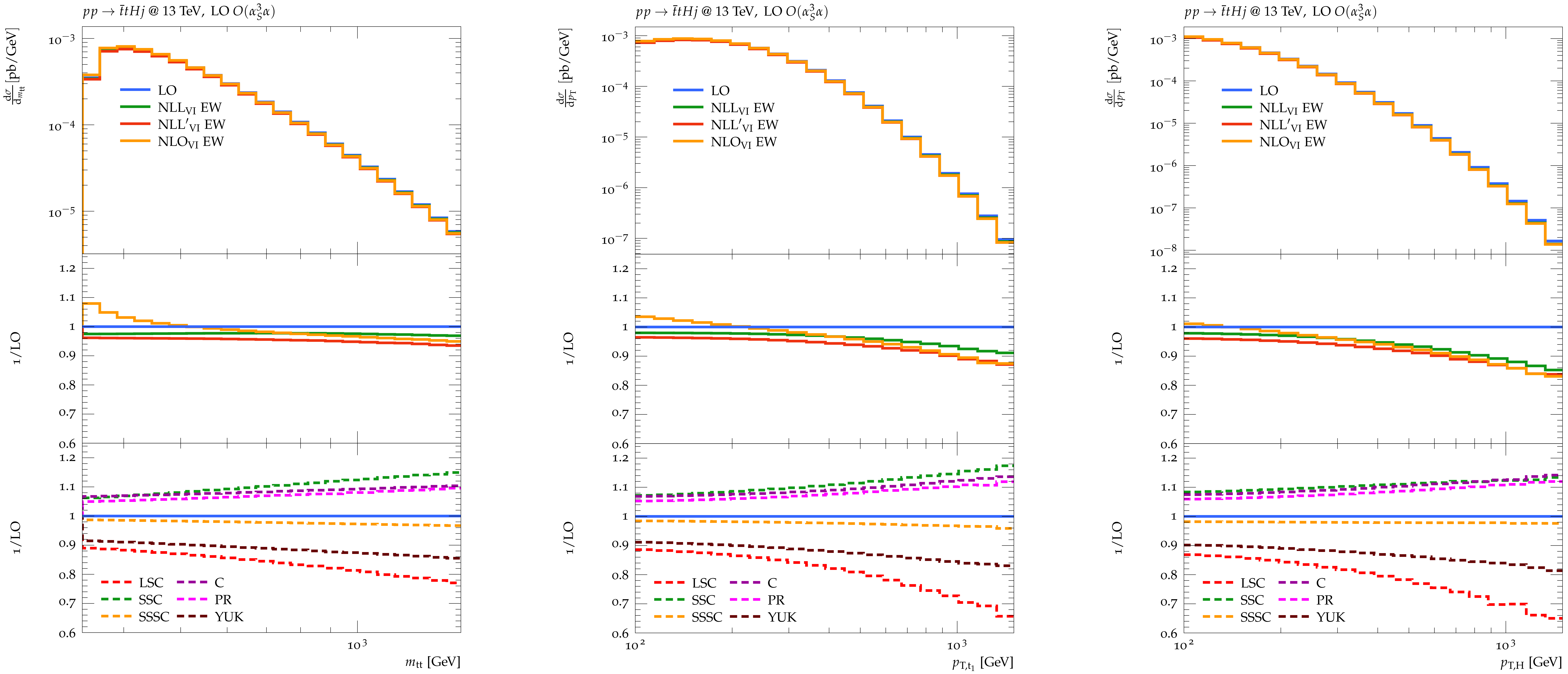
- Goal of the implementation: evaluate **NLO EW** *Sudakov* corrections via tree amplitudes (w/o loop computations) $\Longrightarrow$ ~30 times faster than computing full **NLO EW**!!

Results: ttH



pp → t̄tH @ 13 TeV, LO O(α_s²α)
dσ/dm_tt [pb/GeV]
dσ/dp_T [pb/GeV]
LO
NLL_VI EW
NLL'_VI EW
NLO_VI EW
1/LO
LSC
SSC
SSSC
C
PR
YUK
m_tt [GeV]
p_T,t1 [GeV]
p_T,H [GeV]

Results: $ttHj$



Conclusions and outlook

- In the **EW** sector, radiative corrections at high energies are dominated by *Sudakov logarithms* which significantly enhance tails of kinematic distributions ($>10\%$)

- Exploiting their ***universality***, we develop and implemented in OpenLoops an approach to compute one-loop **EW** *Sudakov logs* via tree amplitudes only

Reduction from loop computations to a tree-level problem with $\sim \mathcal{O}(1\%)$ of accuracy

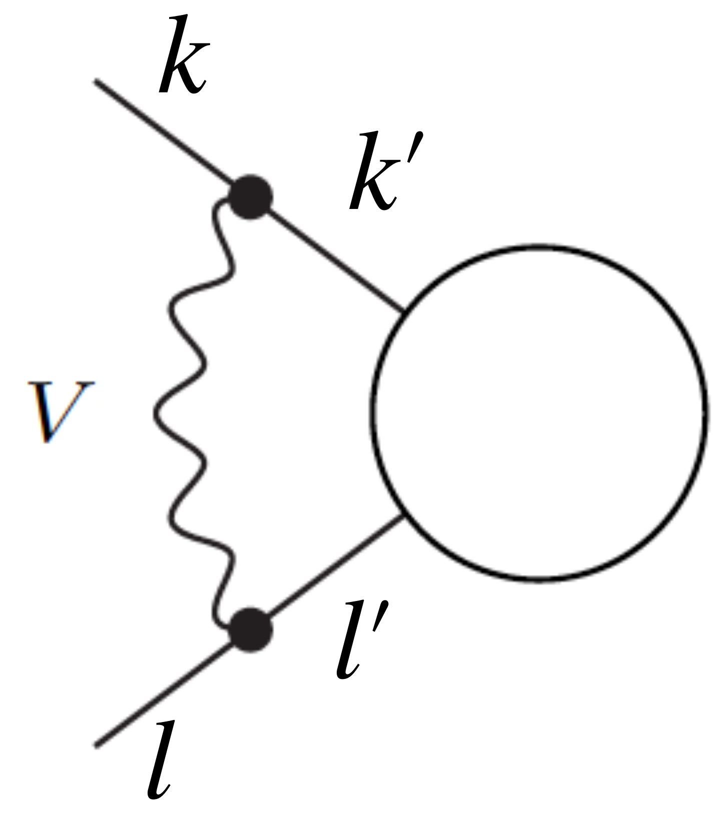
- Additional aspects of the implementation:
  - ▶ Model independent (applicable to both **SM** and **BSM** scenarios)
  - ▶ Direct employment in PS Event Generators with OL interface
  - ▶ Can be used together with differential QED radiation at **NLO** (both mass and dim reg are available)
  - ▶ Support **EW** corrections for resonant processes

- Outlook:
  - ▶ Dressing **NLL** **EW** *Sudakov logs* with **QCD** loops, i.e. **mixed** **QCD-EW** corrections
  - ▶ **NNLO/two-loop** extension

Backup

Double Logs (DL)

- DL originate from triangle diagrams where two external legs exchange a ***soft and collinear*** **(SC)** gauge boson V



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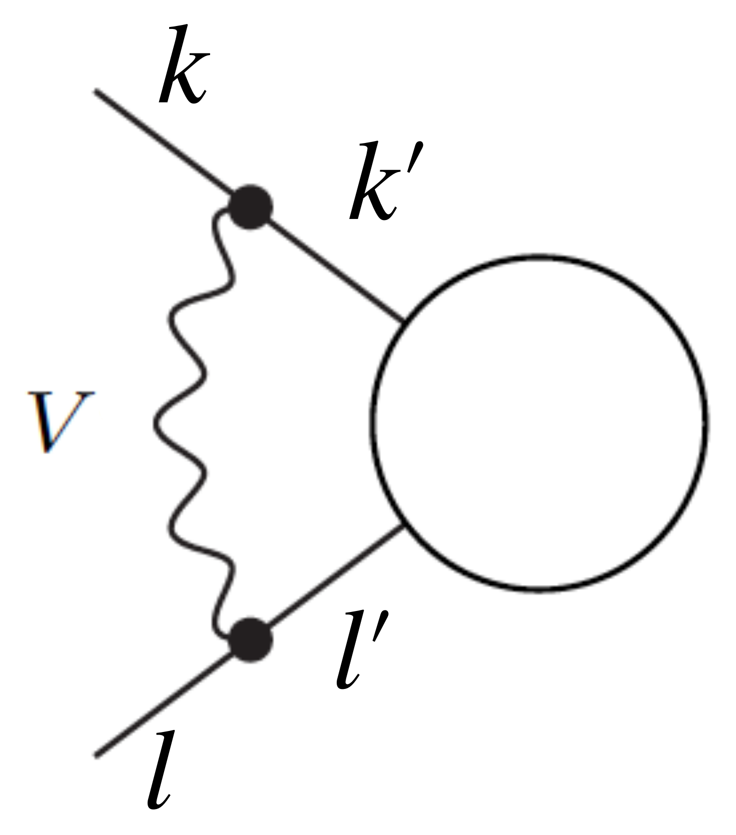

- DL originate from triangle diagrams where two external legs exchange a ***soft and collinear* (SC)** gauge boson V

- In the *Eikonal approximation*, the loop integral reduces to the scalar three-point function C_0, which ***factorises***

$$\delta^{\mathrm{DL}}\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}} \overset{\mathrm{LA}}{=} \sum_{k}\sum_{l<k}\sum_{V}\sum_{k',l'} \frac{\alpha}{4\pi} I^{V}_{kk'} I^{\bar{V}}_{ll'} \underbrace{\left[\log^2\frac{|r_{kl}|}{M_V^2} - 2i\pi\Theta(r_{kl})\log\frac{|r_{kl}|}{M_V^2}\right]}_{\propto\, C_0|_{\mathrm{LA}}} \mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i'_k}\cdots\varphi_{i'_l}\cdots\varphi_{i_n}}$$

with $r_{kl} = (p_k + p_l)^2$

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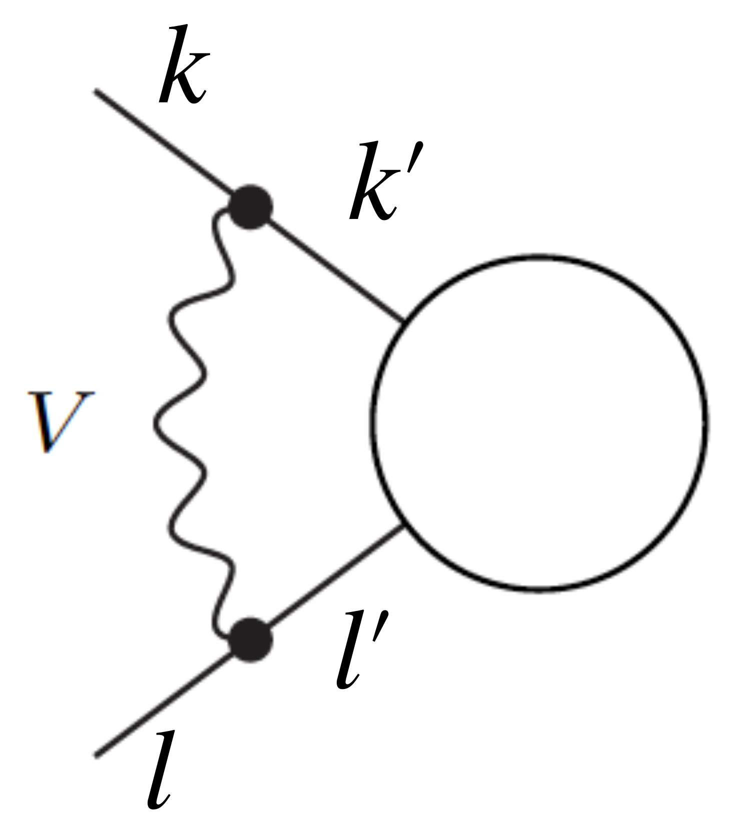

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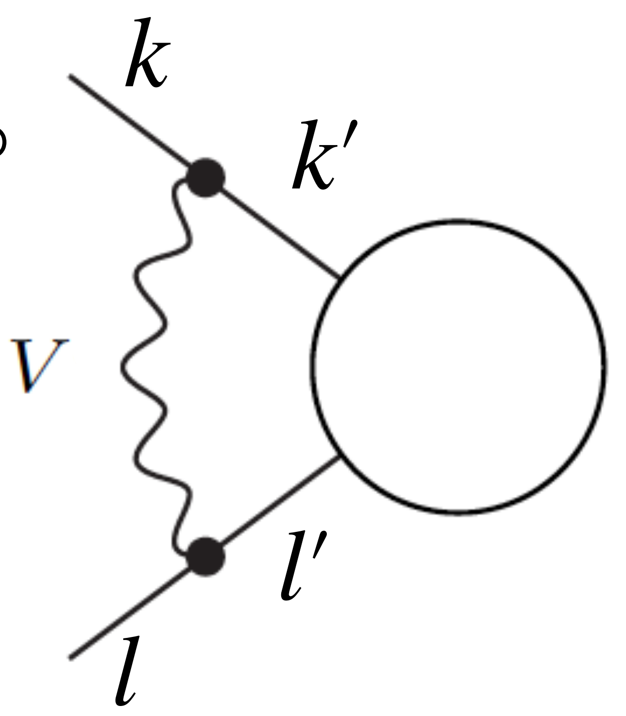
with $r_{kl} = (p_k + p_l)^2$

- Consequence of C_0 ***factorisation***: DL are ***universal***, i.e. process independent

Double Logs: LSC, SSC, S-SSC

DL originate when two external legs exchange a ***soft and collinear* (SC)** gauge boson V



- DL can be split into

→ **Leading *Soft-Collinear* (LSC)**: angular independent, single sum over external legs

$$\delta^{\mathrm{LSC}}\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}} = \sum_k \sum_{k'} \sum_V \delta^{\mathrm{LSC,V}}_{kk'} \mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i'_k}\cdots\varphi_{i_n}}, \qquad \delta^{\mathrm{LSC,\ V}}_{kk'} \sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \boxed{\log^2\left(\frac{s}{M_V^2}\right)}$$

→ **Subleading *Soft-Collinear* (SSC)** and **Sub-SSC**: angular dependent, double sum over external legs

$$\delta^{(\mathrm{S-})\mathrm{SSC}}\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}} = \sum_k \sum_{l<k} \sum_{k',l'} \sum_V \delta^{(\mathrm{S-})\mathrm{SSC,V}}_{kk'll'} \mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i'_k}\cdots\varphi_{i'_l}\cdots\varphi_{i_n}}$$

$$\delta^{\mathrm{SSC,\ V}}_{kk'll'} \sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \boxed{\log\left(\frac{s}{M_V^2}\right) \log\left(\frac{|r_{kl}|}{s}\right)}, \qquad \delta^{\mathrm{S\text{-}SSC,\ V}}_{kk'll'} \sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \boxed{\log^2\left(\frac{|r_{kl}|}{s}\right)} \qquad r_{kl} = (p_k + p_l)^2$$

[Denner and Pozzorini 0010201; *2001*]

Single Logs (SL)

- SL have a triple origin

Single Logs (SL): PR

$$\mu_{i,0}^2 = \mu_i^2 + \delta\mu_i^2$$

$$\varphi_{i,0} = \left(1 + \frac{1}{2}\delta Z_{\varphi_i\varphi_j}\right)\varphi_j$$

$$g_{i,0} = g_i + \delta g_i = (1 + \delta Z_{g_i})\, g_i$$

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→ **PR**: UV renormalisation of EW dimensionless parameters

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Single Logs (SL): PR & WFR

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→ WF: wave-function renormalisation of external fields

$$\varphi_{i,0} = \left(1 + \frac{1}{2}\delta Z_{\varphi_i\varphi_j}\right)\varphi_j$$

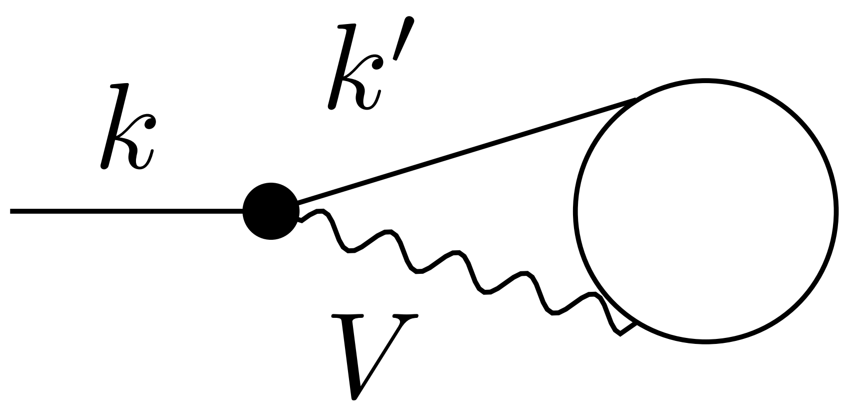
yields to the ***factorised*** correction

$$\delta^{\mathrm{WF}}\mathcal{M}^{\varphi_{i_1}\ldots\varphi_{i_n}} \stackrel{\mathrm{LA}}{=} \sum_k \sum_{k'} \delta^{\mathrm{WF}}_{kk'}\mathcal{M}_0^{\varphi_{i_1}\ldots\varphi_{i'_k}\ldots\varphi_{i_n}}, \qquad \delta^{\mathrm{WF}}_{kk'} = \frac{1}{2}\delta Z_{kk'}$$

Single Logs (SL): Coll

- SL have a triple origin

→ **Coll**: external leg emission of a collinear gauge boson



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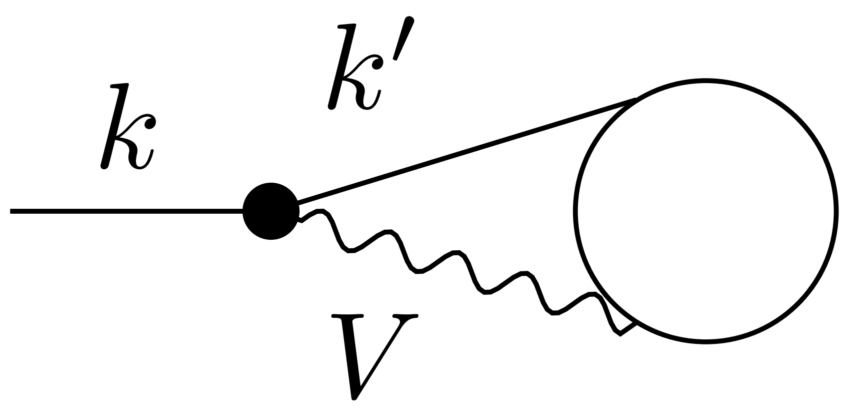

Its evaluation in *Collinear approximation* leads to the ***factorised*** contribution

$$\delta^{\mathrm{coll}} \mathcal{M}^{\varphi_{i_1} \ldots \varphi_{i_n}} \stackrel{\mathrm{LA}}{=} \sum_k \sum_{k'} \delta^{\mathrm{coll}}_{kk'} \mathcal{M}_0^{\varphi_{i_1} \ldots \varphi_{i'_k} \ldots \varphi_{i_n}} , \qquad \delta^{\mathrm{coll}}_{kk'} \sim \frac{\alpha}{4\pi} \sum_V I^V I^{\bar{V}} \log\left(\frac{s}{M_V^2}\right)$$

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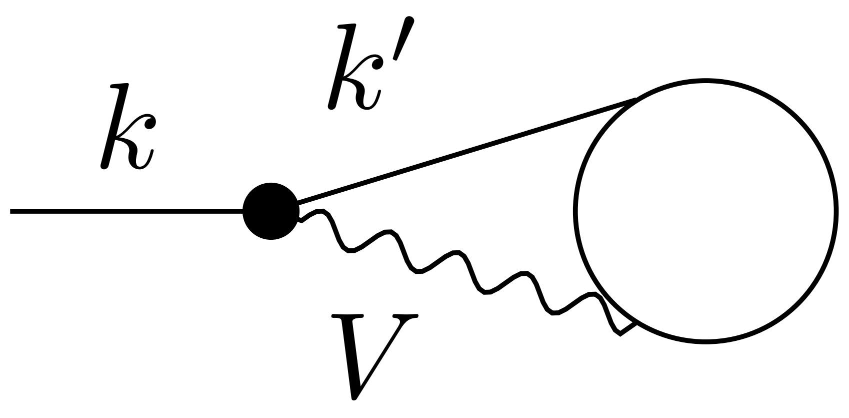

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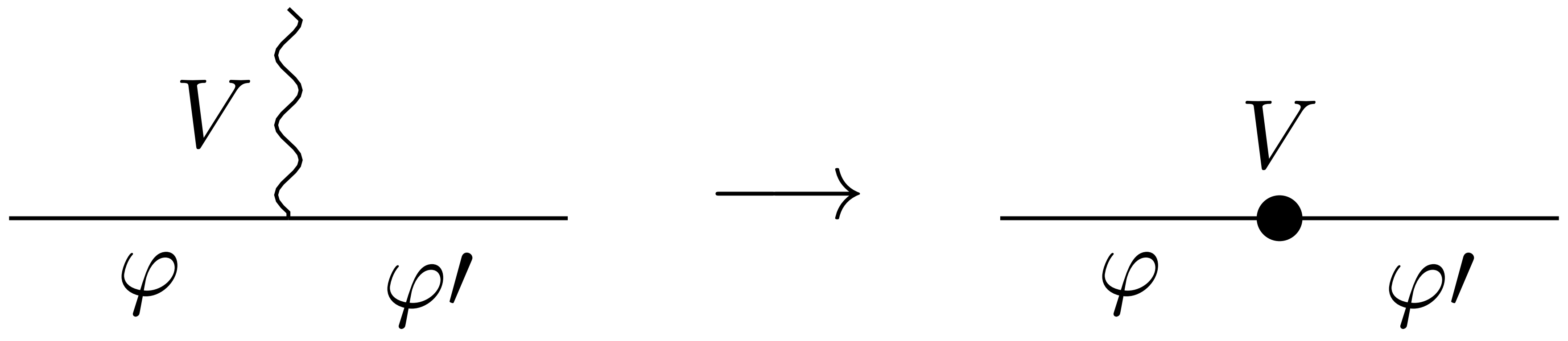
→ **C**: Full gauge-invariant SL correction associated to external fields:

$$\delta^{\text{C}} \mathcal{M}^{\varphi_{i_1} \dots \varphi_{i_n}} \overset{\text{LA}}{=} \sum_{k} \sum_{k'} \delta^{\text{C}}_{kk'} \mathcal{M}_0^{\varphi_{i_1} \dots \varphi_{i'_k} \dots \varphi_{i_n}} , \qquad \delta^{\text{C}}_{kk'} = \left(\delta^{\text{coll}}_{kk'} + \delta^{\text{WF}}_{kk'}\right)\Big|_{\mu^2 = s}$$

Implementation in OpenLoops: how

- Representation of Denner-Pozzorini algorithm via effective CT vertices



$$= ieI^{V}_{\varphi\varphi'}K^{V}_{\mathrm{ew}}$$

reducing one-loop amplitudes to tree-level ones via double CT insertions

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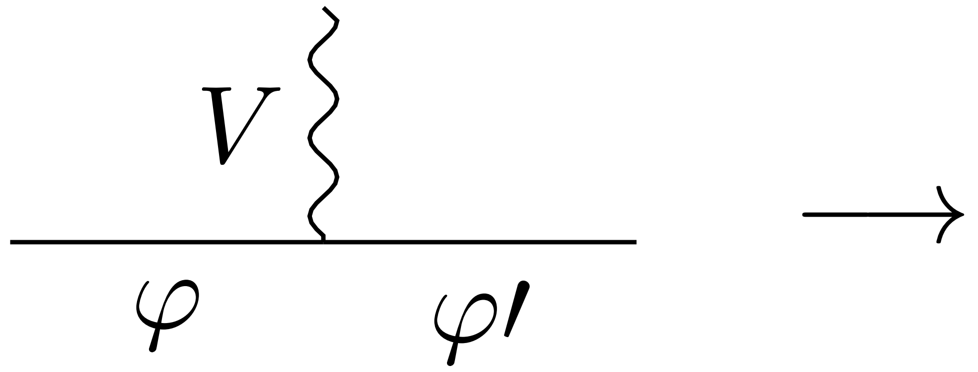

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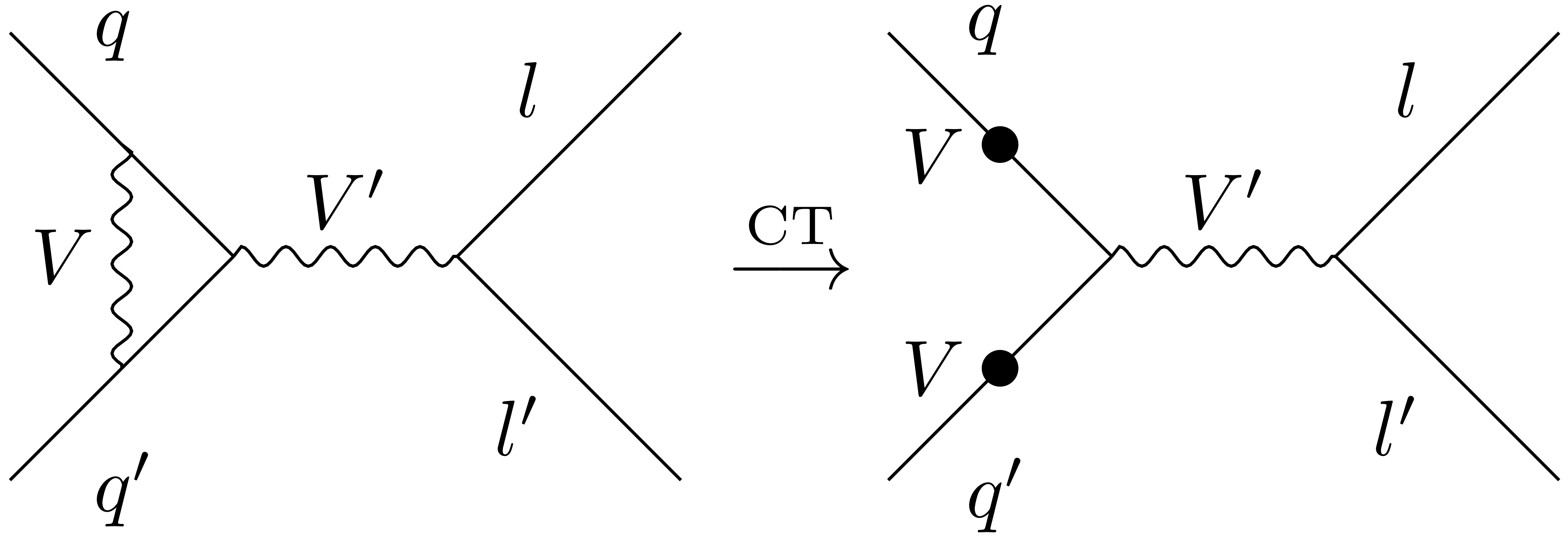
reducing one-loop amplitudes to tree-level ones via double CT insertions

Eg.: Drell-Yann



$$\Rightarrow \mathcal{M} \sim e^2 \sum_{V=A,Z,W^{\pm}} I^{V}_{q} I^{V}_{q'} \left(K^{V}_{\mathrm{ew}}\right)^2 \mathcal{M}_0$$

$$\sim \delta^{\mathrm{DL,\,SL}}$$

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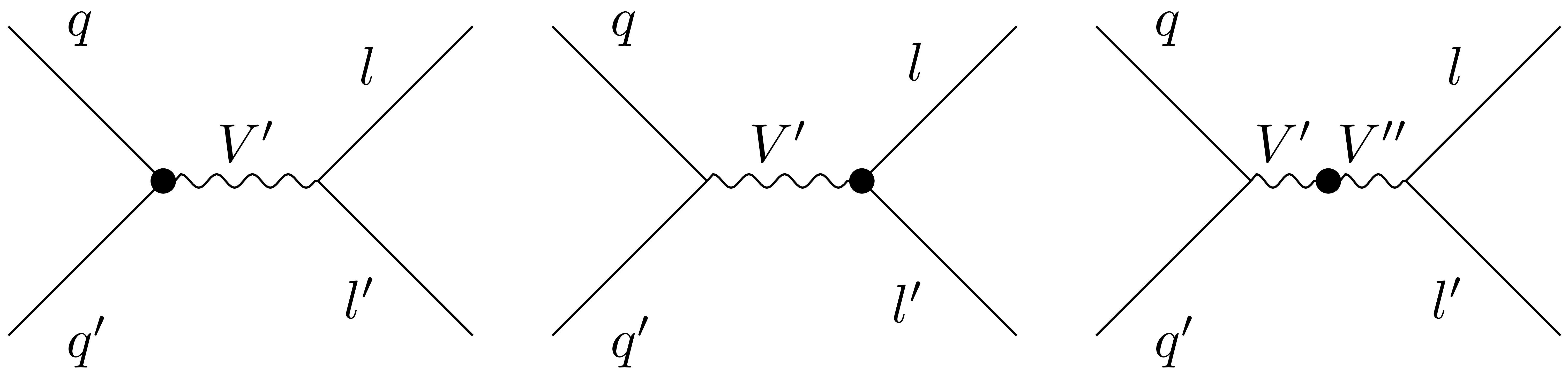
- Effective CT vertices are suitable for evaluation of ***soft-collinear*** and ***collinear*** *Sudakov* corrections

Implementation in OpenLoops: how

- Effective CT vertices are suitable for evaluation of ***soft-collinear*** and ***collinear*** *Sudakov* corrections

- Single logs coming from **PR** contributions can be evaluated via generation of standard UV counterterms, e.g.



setting all the **WFRCs** to zero

Implementation in OpenLoops: how

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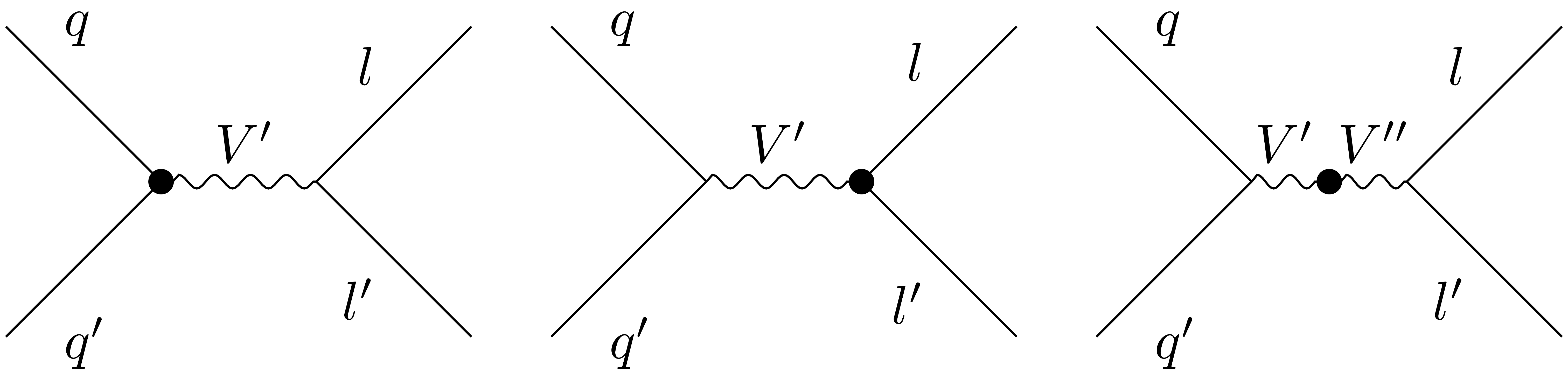
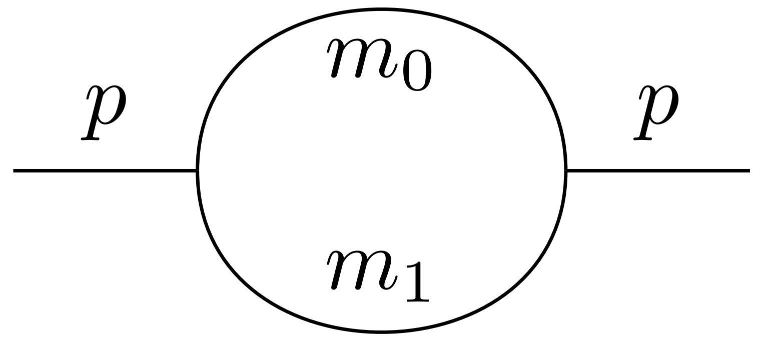


setting all the **WFRCs** to zero

- Alternative way: set $\delta^{\mathrm{WF}}_{kk'}$ to zero and evaluate **WF** + **PR** via standard UV counterterms

Single Logs: PR

- Generic two-point function



$$\frac{\mathrm{i}}{(4\pi)^2} B_{\{0,\mu,\mu\nu\}}\left(p, m_0, m_1\right) := \mu^{4-D} \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{\{1, q_\mu, q_\mu q_\nu\}}{\left(q^2 - m_0^2 + \mathrm{i}\varepsilon\right)\left[(q+p)^2 - m_1^2 + \mathrm{i}\varepsilon\right]}$$

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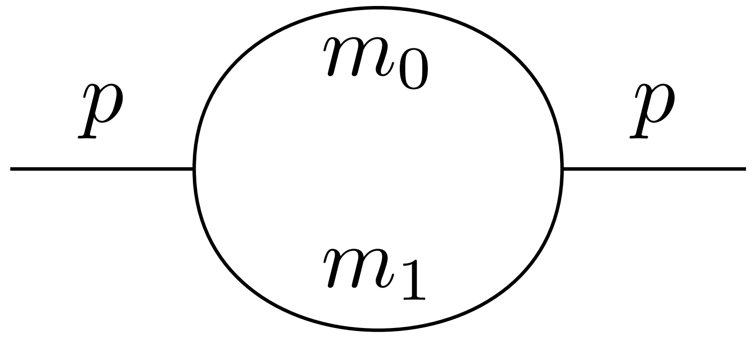

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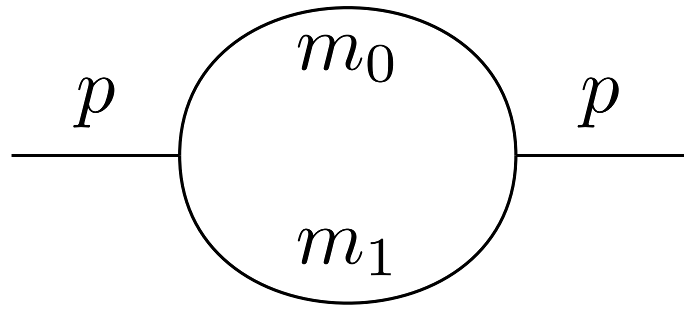
- In LA $\mu^2 = s \gg p^2, m_0^2, m_1^2 \Rightarrow$ four possible hierarchy of masses

$$\begin{aligned}
&(a) \quad m_i^2 \ll p^2 \text{ and } p^2 - m_{1-i}^2 \ll p^2 \quad \text{for } i = 0 \text{ or } \quad i = 1, \\
&(b) \quad \text{not} \quad (a) \text{ and } m_i^2 \not\gg p^2 \text{ for } i = 0, 1, \\
&(c) \quad m_0^2 = m_1^2 \gg p^2 \\
&(d) \quad m_i^2 \gg p^2 \not\gg m_{1-i}^2 \text{ for } i = 0 \text{ or } i = 1
\end{aligned}$$

Single Logs: PR



- Generic two-point function

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- Results for two point functions and their derivatives

$$B_0(p, m_0, m_1) \overset{\text{LA}}{=} \log \frac{\mu^2}{M^2},$$

$$B_1\left(p^2, m_0, m_1\right) \overset{\text{LA}}{=} -\frac{1}{2} \log \frac{\mu^2}{M^2},$$

$$\frac{1}{p^2} B_{00}\left(p^2, m_0, m_1\right) \overset{\text{LA}}{=} \frac{3m_0^2 + 3m_1^2 - p^2}{12p^2} \log \frac{\mu^2}{M^2},$$

$$\frac{1}{p^2} g^{\mu\nu} B_{\mu\nu}\left(p^2, m_0, m_1\right) \overset{\text{LA}}{=} \frac{m_0^2 + m_1^2}{p^2} \log \frac{\mu^2}{M^2}$$

$$p^2 B_0'(p, m_0, m_1) \overset{\text{LA}}{=} \frac{1}{2} \log \frac{m_{1-i}^2}{m_i^2} = \frac{1}{2} \log \frac{p^2}{\lambda^2},$$

$$p^2 B_1'\left(p^2, m_0, m_1\right) + \frac{1}{2} p^2 B_0'(p, m_0, m_1) \overset{\text{LA}}{=} -\frac{1}{4} \log \frac{m_0^2}{m_1^2}$$

Additional results

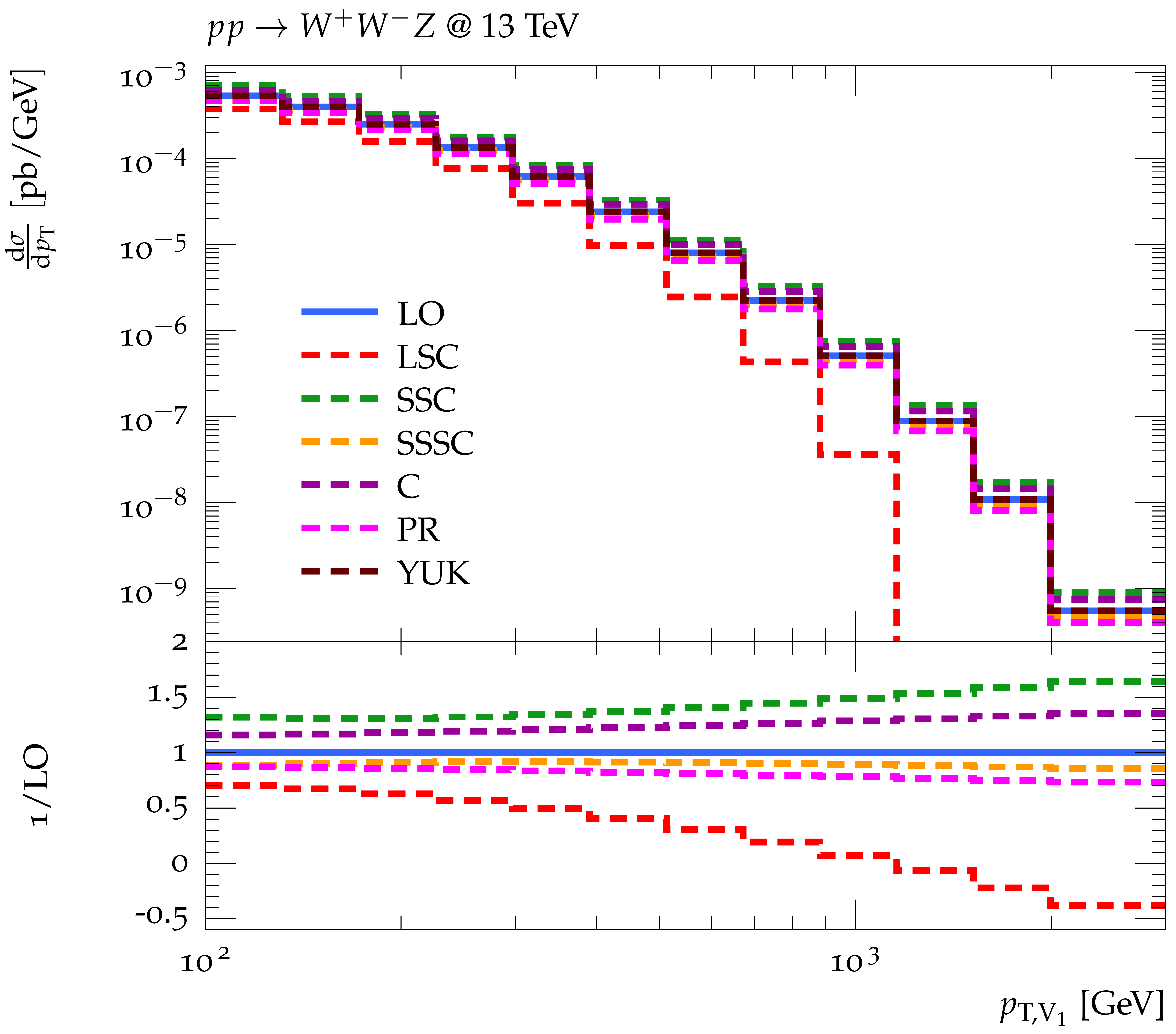
Validation: Energy scan

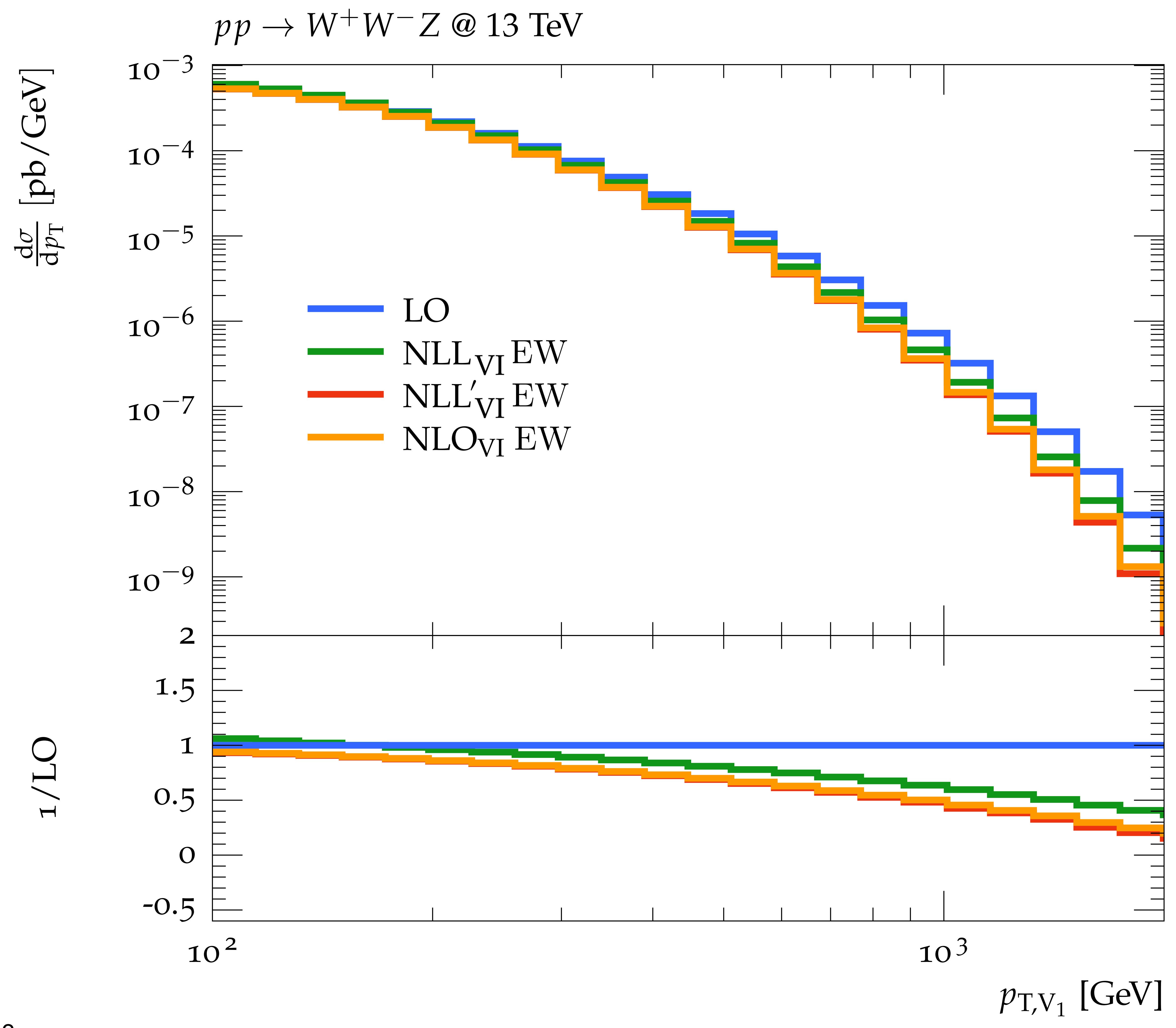
- In the high energy limit, for not mass-suppressed matrix elements we expect $\mathrm{NLO}_{\mathrm{VI}} - \mathrm{NLL'}_{\mathrm{VI}} \propto$ const
- Inclusion of the phase from a LA of C_0 in DL, i.e.

$$C_0|_{\mathrm{LA}} \propto \left[\log^2 \frac{|r_{kl}|}{M_V^2} - 2i\pi\Theta(r_{kl}) \log \frac{|r_{kl}|}{M_V^2}\right]$$

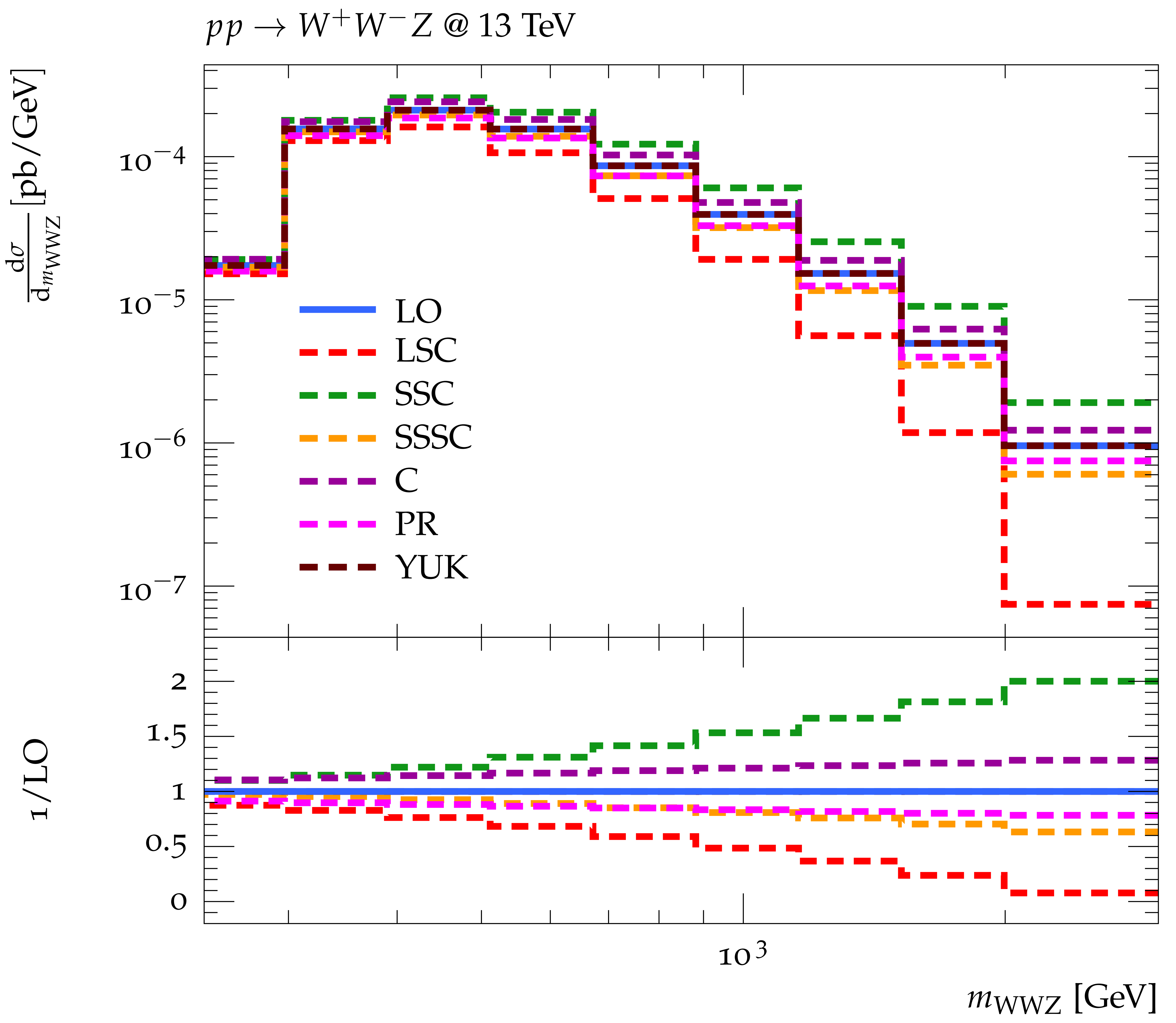
is crucial in $2 \to n$ processes with $n \geq 3$: without phase $\mathrm{NLO}_{\mathrm{VI}} - \mathrm{NLL'}_{\mathrm{VI}}$ shows a logarithmic dependence

Results: $pp \to W^+W^-Z$

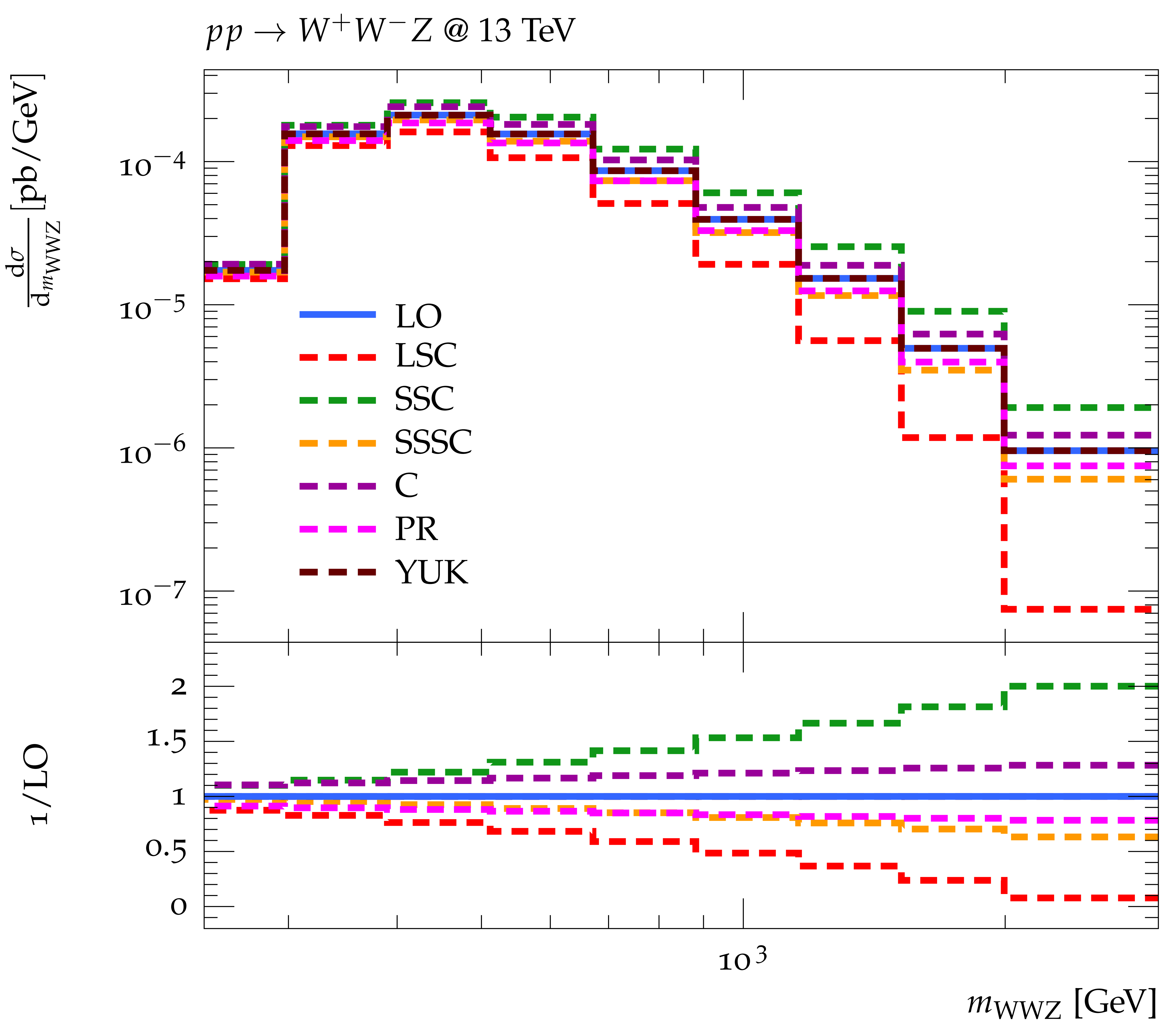




Results: $pp \to W^+W^-Z$



Results: $pp \to W^+W^-Z$



SSC and S-SSC become very sizeable for PS regions where Sudakov condition

$$s \sim (p_k + p_l)^2 \gg M^2_{Z,W} \quad \forall\, k,l$$

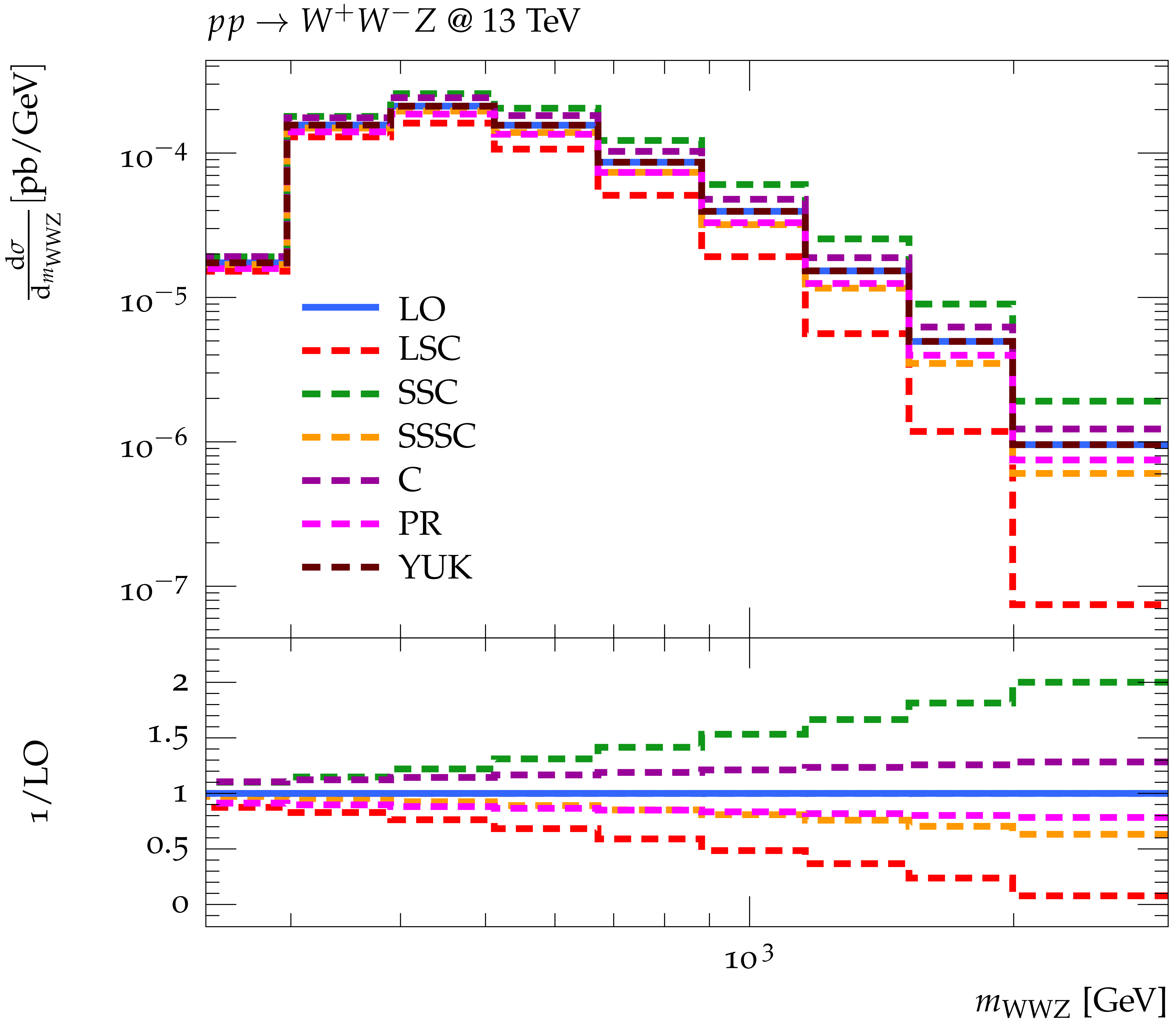
is violated, with hierarchy among invariants

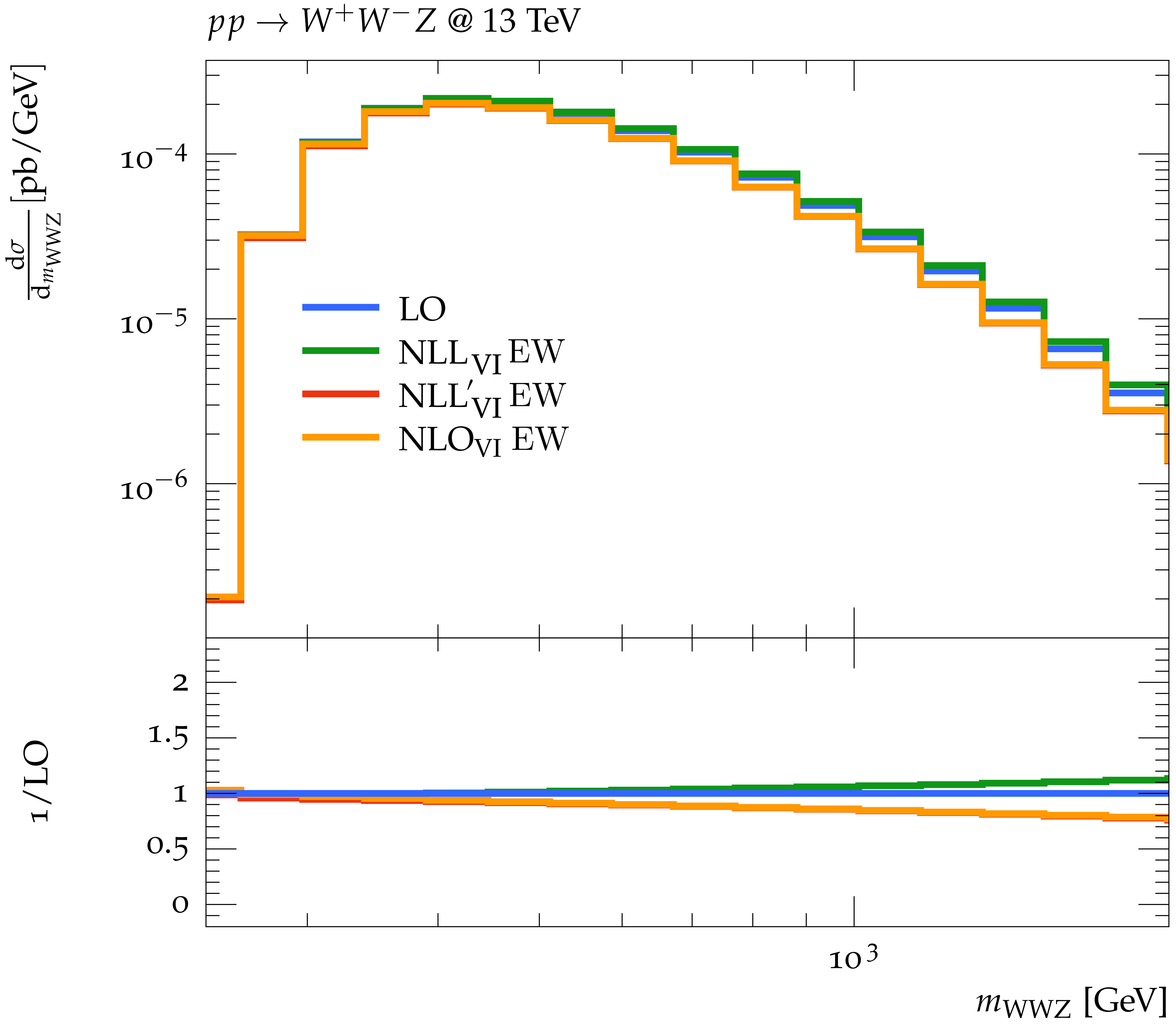
$$s \sim (p_k + p_l)^2 \gg (p_{k'} + p_{l'})^2 \gg M^2_{Z,W}$$

$$\delta^{\text{SSC, V}}_{kk'll'} \sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \log\left(\frac{s}{M_V^2}\right) \log\left(\frac{|r_{kl}|}{s}\right)$$

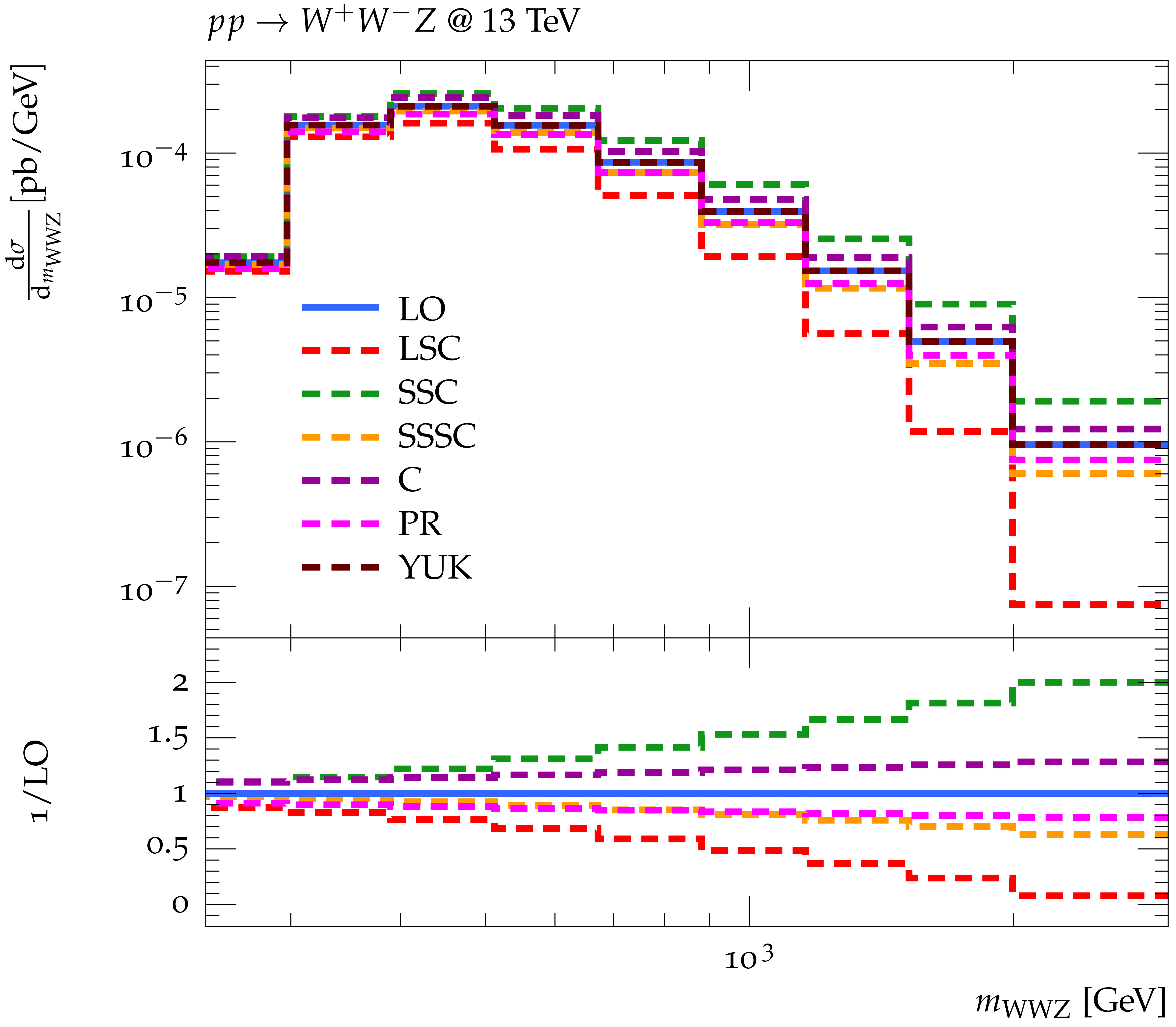
$$\delta^{\text{S-SSC, V}}_{kk'll'} \sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \log^2\left(\frac{|r_{kl}|}{s}\right)$$

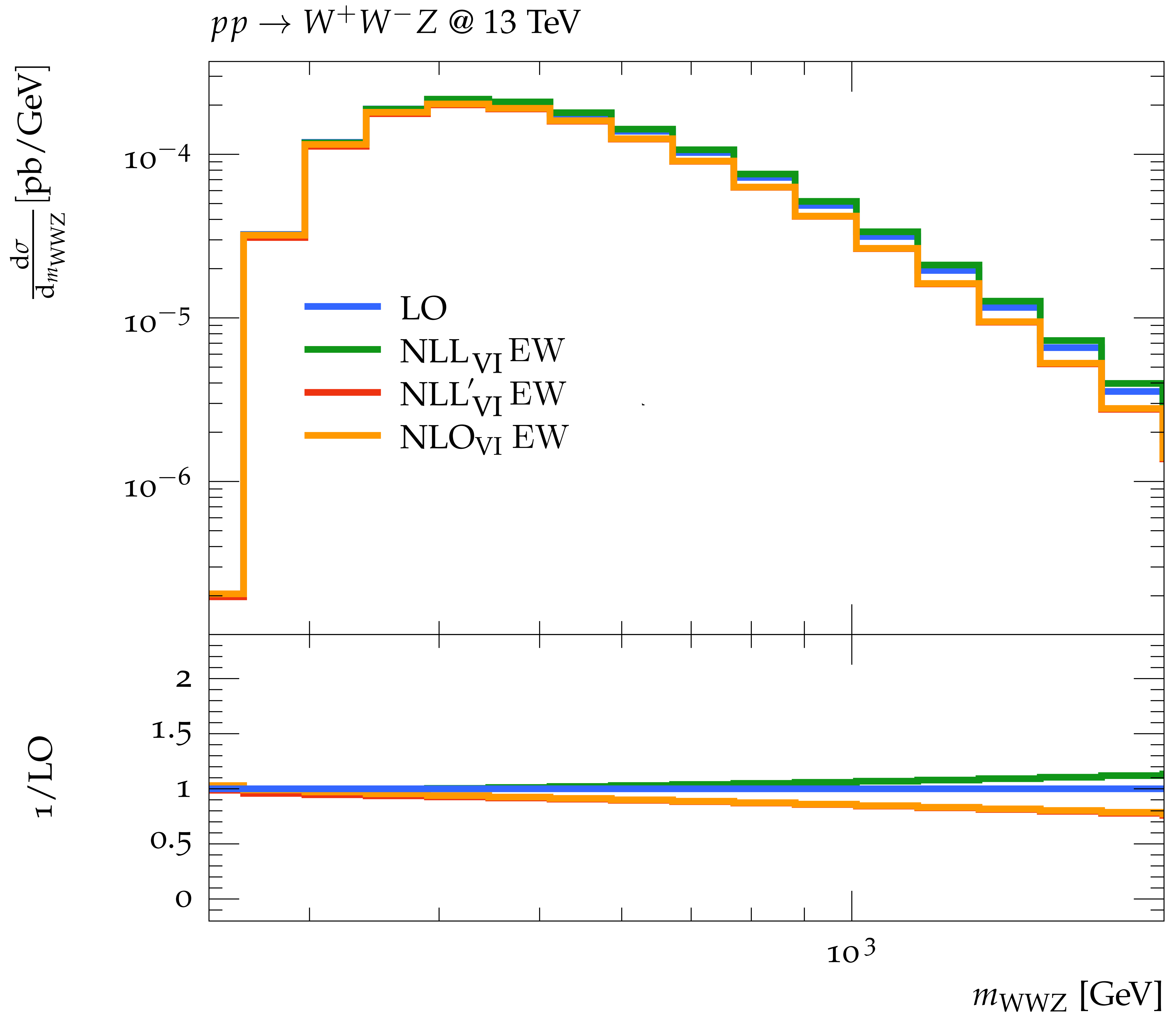
Results: $pp \to W^+W^-Z$





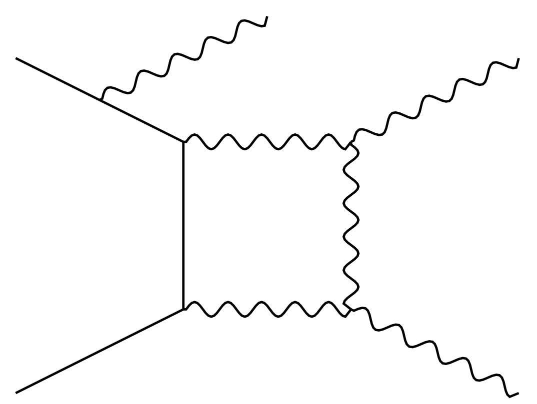
Results: $pp \to W^+W^-Z$





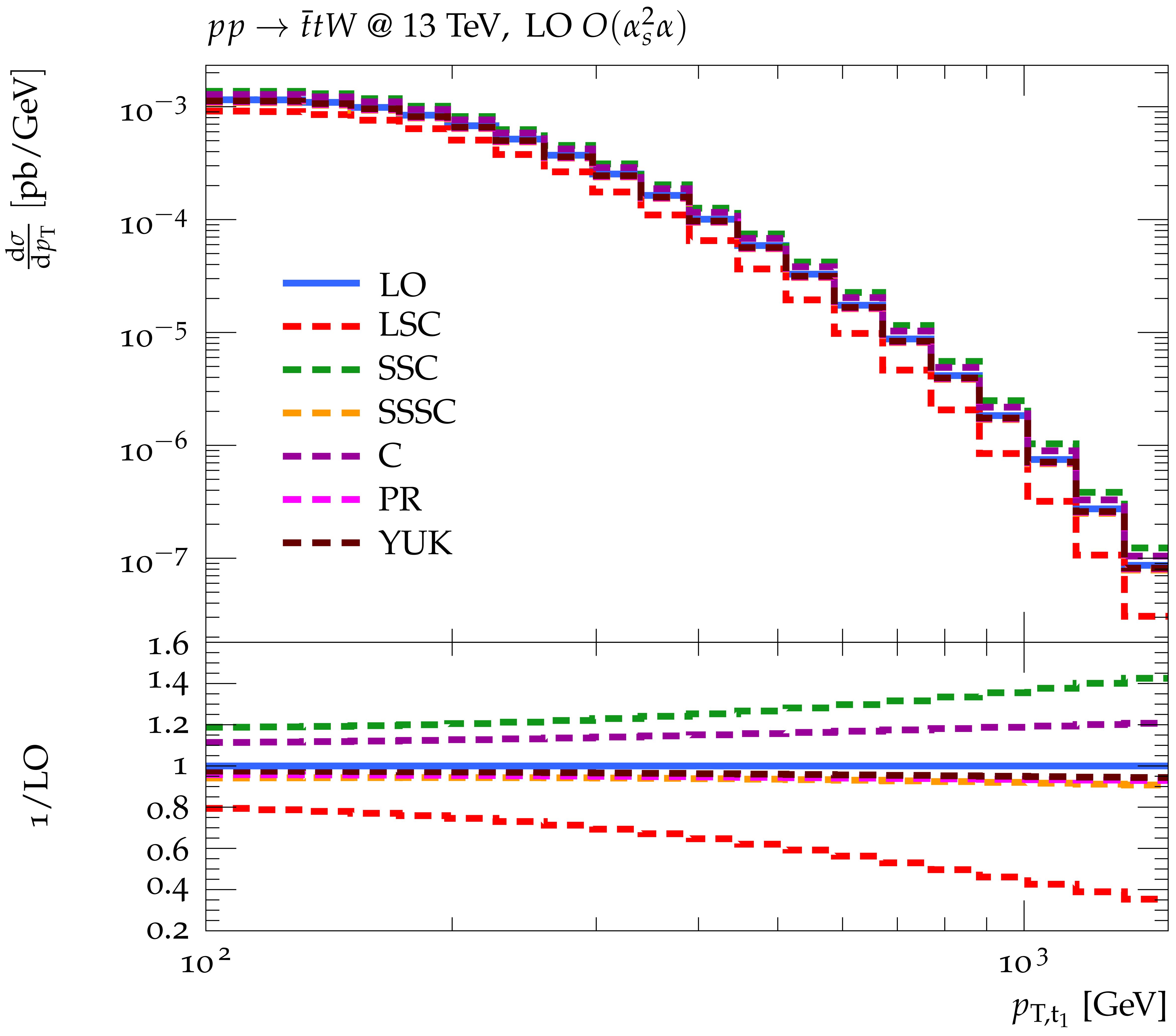
However, no full control on **S-SSC** term!

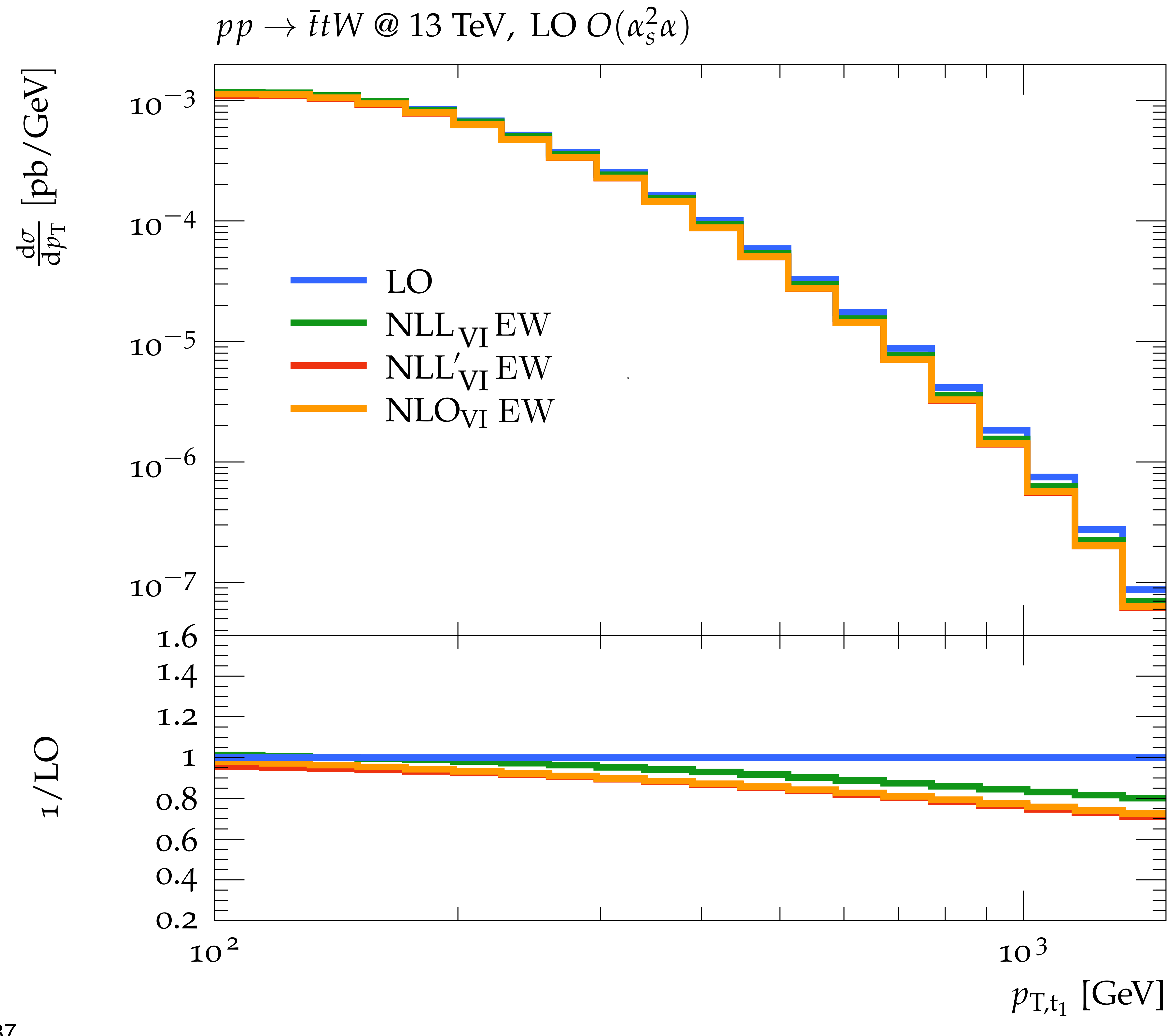
S-SSC-like terms arise also from box diagrams



$\sim D_0 \sim \log\left(\frac{|r_{kl}|}{s}\right)$

Results: $pp \to \bar{t}tW$





Results: $pp \to \bar{t}tW$