Quantum integration of elementary particle processes

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→ Based on **arXiv:2201.01547, Phys.Lett.B 832 (2022) 137228**

In collaboration with:
Gabriele Agliardi, **Michele Grossi**, **Enrico Prati**

Quantum computing for high-energy physics

Durham, United Kingdom

19th of September 2023


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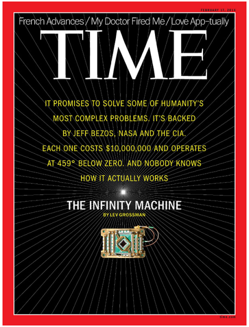
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TIME
IT PROMISES TO SOLVE SOME OF HUMANITY'S
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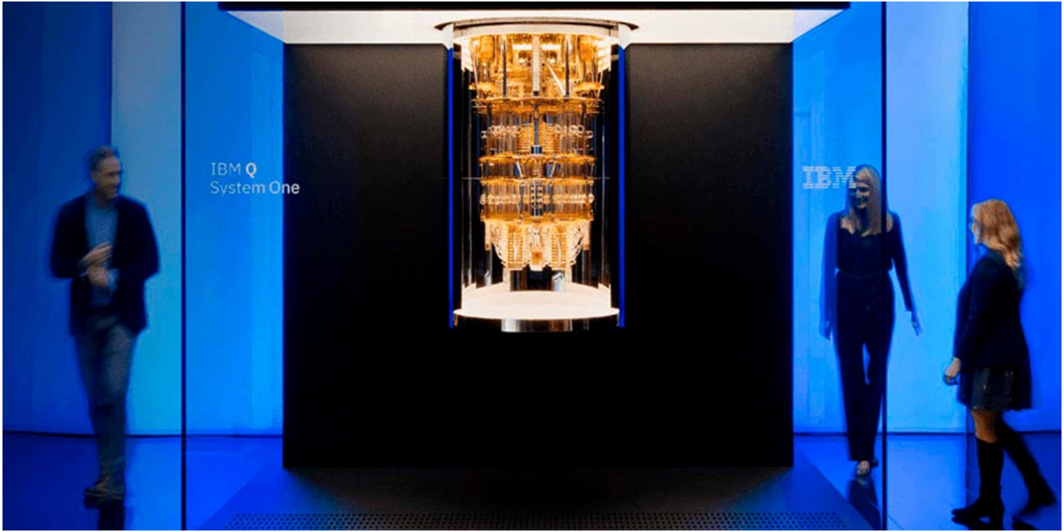
BY JEFF BEZOS, NASA AND THE CIA.
EACH ONE COSTS $10,000,000 AND OPERATES
AT 459° BELOW ZERO. AND NOBODY KNOWS
HOW IT ACTUALLY WORKS
THE INFINITY MACHINE
BY LEV GROSSMAN

Quantum computers



[IBM]

[Landscape with the worship of the Golden calf, Claude Lorrain, Staatliche Kunsthalle, Karlsruhe (Germany)]

Quantum Monte Carlo in HEP

- Is it possible?
- Is there a quantum advantage?
- Is it more resource efficient than CPU/GPU?

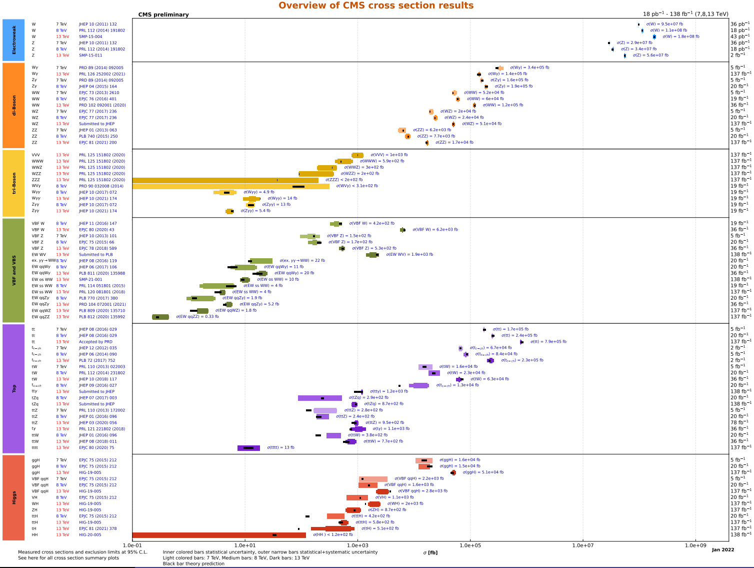
Reviews

- [Gray, Terashi; Gray:2022fou] (selected topics)
- [Delgado et al.; 2203.08805] (Snowmass)
- [Klco et al.; 2107.04769] (lattice)

Selected references

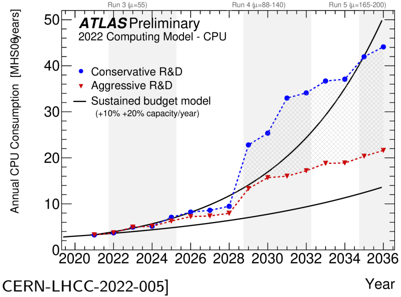
- Amplitude/loop integrals: [Ramirez-Uribe et al.; 2105.08703], [Bepari, Malik, Spannowsky, Williams; 2010.00046], [Chawdhry, MP; 2303.04818]
- Parton shower: [Bauer, de Jong, Nachman, Provasoli; 1904.03196], [Bepari, Malik, Spannowsky, Williams; 2010.00046], [Williams, Malik, Spannowsky, Bepari; 2109.13975], [Chigusa, Yamazaki; 2204.12500], [Gustafson, Prestel, Spannowsky, Williams; 2207.10694]
- Machine learning: [Filipek et al; 2105.04582], [Bravo-Prieto et al; 2110.06933], [Alvi, Bauer, Nachman; 2206.08391]
- Others: [Ciavarella; 2007.04447], [Perez-Salinas, Cruz-Martinez, Alhajri, Carrazza: 2011.13934], [Bauer, Freytsis, Nachman; 2102.05044], [Martenez de Lejarza, Cieri, Rodrigo; 2204.06496], [Agliardi, Grossi, MP, Prati; 2201.01547], [Martínez de Lejarza, Grossi, Cieri, Rodrigo; 2305.01686]

LHC legacy



Overview of CMS cross section results
CMS preliminary
18 pb^{-1} - 138 fb^{-1} (7,8,13 TeV)
Electroweak
di-Boson
tri-Boson
VBF and VBS
Top
Higgs
1.0e-01
1.0e+01
1.0e+03
1.0e+05
1.0e+07
1.0e+09
σ [fb]
Jan 2022
Measured cross sections and exclusion limits at 95% C.L.
See here for all cross section summary plots
Inner colored bars statistical uncertainty, outer narrow bars statistical+systematic uncertainty
Light colored bars: 7 TeV, Medium bars: 8 TeV, Dark bars: 13 TeV
Black bar theory prediction

Computing problem in high-energy physics



[ATLAS; CERN-LHCC-2022-005]

→ Event generation:
$\sim$ 15% of $\sim$ 3 billion cpuh.y^{-1}

→ More in: [Buckley; 1908.00167], [Valassi et al.; 2004.13687]

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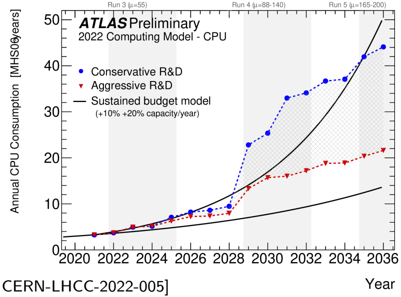


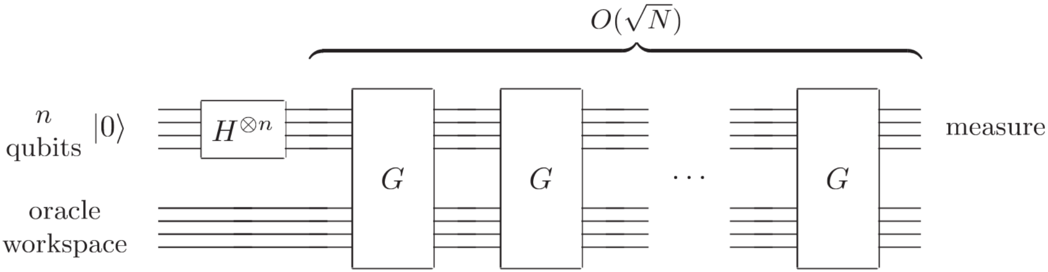
[ATLAS; CERN-LHCC-2022-005]

→ Event generation:
$\sim$ 15% of $\sim$ 3 billion cpuh.y^{-1}
→ More in: [Buckley; 1908.00167], [Valassi et al.; 2004.13687]

- One possible solution: GPU
  → Some references: [Borowka et al.; 1811.11720], [Carrazza et al.; 2002.12921, 2009.06635, 2106.10279], [Bothmann et al.; 2106.06507] + Talk1 + Talk2
- Can quantum integration be of any use in HEP?
  → Application mostly in finance: [Woerner and Egger; 1806.06893], [Stamatopoulos et al.; 1905.02666, 2111.12509], [Rebentrost, Gupt, Bromley; Phys.Rev.A 98 (2018) 022321]

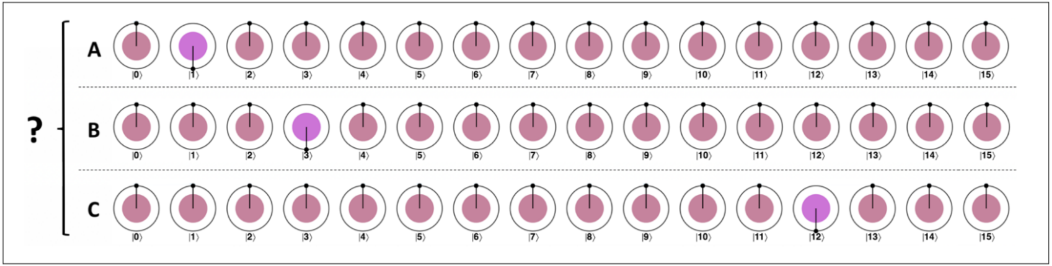
Grover algorithm/iteration

- Very general quantum algorithm
- Quadratic speed up
  → $\mathcal{O}(\sqrt{N})$ operations instead of $\mathcal{O}(N)$
- Most famous example: unstructured database search



[Nielsen, Chuang; Quantum Computation and Quantum Information]

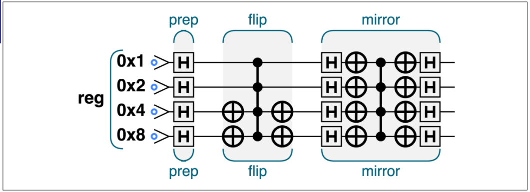
- Example (from [Johnston, Harrigan, Gimeno-Segovia; Programming Quantum Computers])

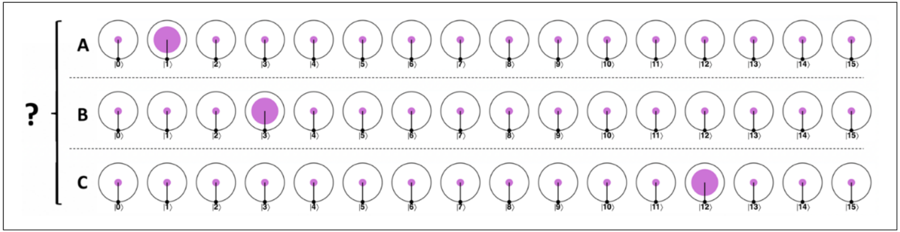


$\rightarrow$ What solution is contained in our quantum register?

Grover algorithm/iteration

→ Applying a Grover iteration





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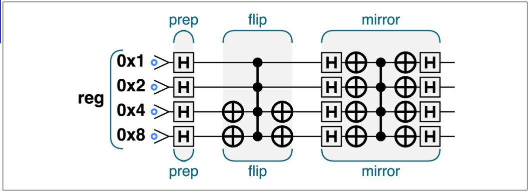
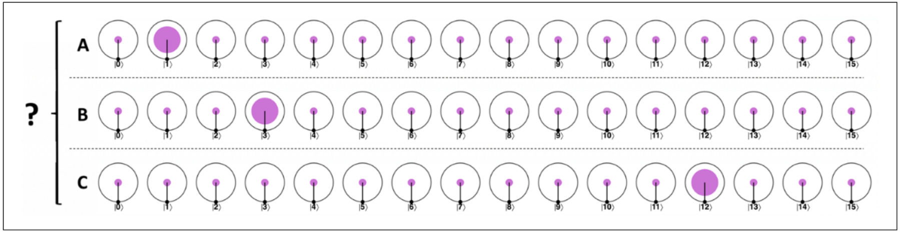
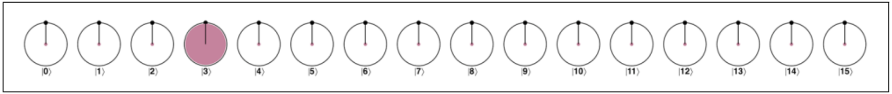


→ Applying a Grover iteration



→ Applying it twice



Quantum Amplitude Estimate (QAE)

[Brassard, Hoyer, Mosca, Tapp; Quantum Amplitude Amplification and Estimation]

$$\mathcal{A}|0\rangle = \sqrt{1-a}|\Psi_0\rangle + \sqrt{a}|\Psi_1\rangle$$

QAE estimates a with high probability such that the estimation error scales as $\mathcal{O}(1/M)$ [as opposed to $\mathcal{O}(1/\sqrt{M})$]

M: number of applications of $\mathcal{A}$

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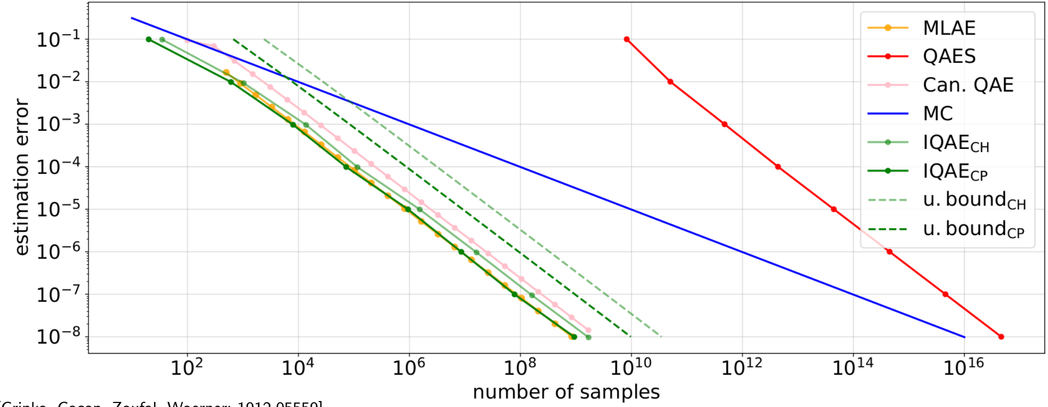
→ What the (orignal) algorithm provides:

- an estimate: $\tilde{a} = \sin^2(\tilde{\theta}_a)$
  with $\tilde{\theta}_a = y\pi/M$, $y \in \{0, ..., M-1\}$, and $M = 2^n$
- A success probability (that can be increased by repeating the algorithm)
- A bound: $|a - \tilde{a}| \leq \mathcal{O}\left(1/M\right)$

Quantum Amplitude Estimate (QAE)

→ Basis of quantum Monte Carlo integration and $\mathcal{O}(1/M)$ scaling
→ Various algorithms/implementations available



[Grinko, Gacon, Zoufal, Woerner; 1912.05559]

Resulting estimation error for $a = 1/2$ and 95% confidence level with respect to the required total number of oracle queries.

Quantum integration

Extension to

$$\mathcal{A}|0\rangle = \sum_i a_i |\Psi_i\rangle$$

$\rightarrow$ Definition of a piece-wise function with $f(x_i) = a_i$.

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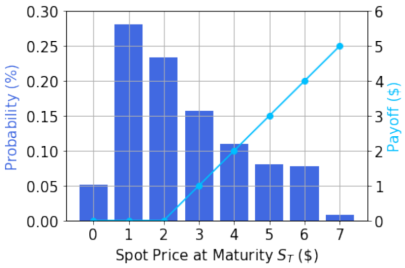
- So far used in finance for simple functions in 1D
  $\rightarrow$ Applicable to HEP? What are the limitations?



$$I = \int \mathrm{d}x f(x) g(x)$$

[Zoufal, Lucchi, Woerner; 1904.00043]

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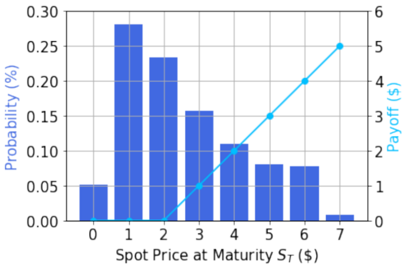

[Zoufal, Lucchi, Woerner; 1904.00043]

$$I = \int \mathrm{d}x f(x) g(x)$$

- In finance:
  - f: probability
  - g: payoff
- In HEP:
  - f: $|\mathcal{M}|^2$
  - g: $\Theta(\Phi - \Phi_c)$

- $\mathbf{e^+e^- \to q\bar{q}}$ (in QED)

$$\sigma \sim \int_{-1}^{1}\int_{0}^{2\pi} \mathrm{d}\cos\theta \mathrm{d}\phi \left(1+\cos^2\theta\right)$$

- $\mathbf{e^+e^- \to q\bar{q}'W}$

$$\sigma \sim \int_{M_W^2}^{s}\int_{0}^{s_1^{\mathrm{Max}}}\int_{-1}^{1}\int_{0}^{2\pi}\int_{0}^{2\pi} \mathrm{d}\Phi_3 \left|\mathcal{M}_{e^+e^-\to q\bar{q}'W}\right|^2$$

$$\sim \int_{M_W^2}^{s}\int_{0}^{s_1^{\mathrm{Max}}} \mathrm{d}\tilde{\Phi}_3 \left|\mathcal{M}'\right|^2$$

with $\mathcal{M}' = \mathcal{M}_{e^+e^-\to q\bar{q}'W}\,(\cos\theta_1 = 0,\ \phi_1 = \pi/2,\ \phi_2 = \pi/2)$.

⚠ We choose to output the variables of integration (maximal information)

→ Use Qiskit (IBM python software) subroutines and noiseless quantum simulation (perfect quantum computer)

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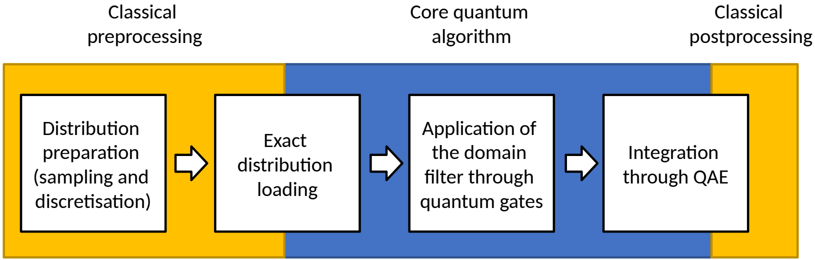
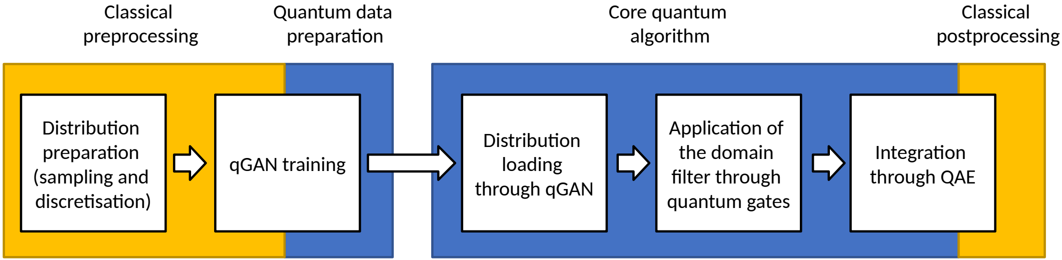
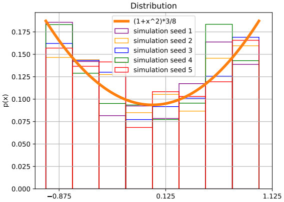
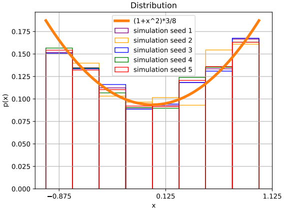




Loading of distribution - $1 + x^2$

- quantum Generative Adversarial Network (qGAN): default vs. optimised





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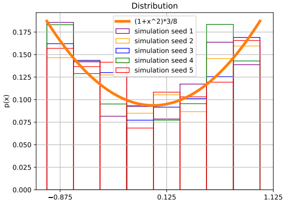
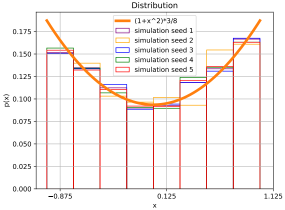
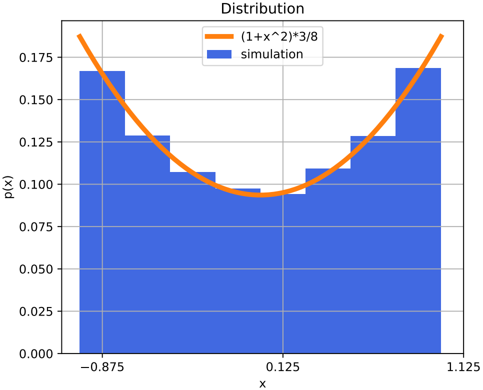




- Exact loading (more qubits needed)



Integration - $1 + x^2$

- Matching boundary of integration (3 qubits $\Rightarrow 2^3$ bins)

Domain	low stat.		high stat.		very high stat.		exact	
	σ	$\delta[\%]$	σ	$\delta[\%]$	σ	$\delta[\%]$	σ	$\delta[\%]$
$[-0.75; 0]$	0.345	-3.31	0.332	0.706	0.334	0.0331	0.334	-8.31×10^{-3}
$[-0.5; 0]$	0.215	-5.86	0.201	1.15	0.203	0.0986	0.203	-0.0161
$[-0.25; 0]$	0.112	-17.1	0.0939	1.87	0.0960	-0.284	0.0957	-0.0389

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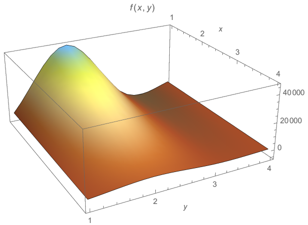
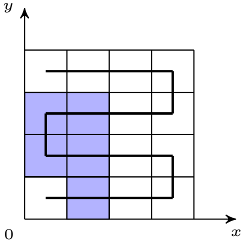
$[-0.25; 0]$	0.112	-17.1	0.0939	1.87	0.0960	-0.284	0.0957	-0.0389

- Non-matching boundary of integration

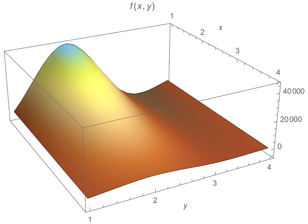
Qubits number	$[-0.7; 0.6]$				$[-0.625; 0.375]$			
	high stat.		exact		high stat.		exact	
	σ	$\delta[\%]$	σ	$\delta[\%]$	σ	$\delta[\%]$	σ	$\delta[\%]$
3	0.402	-28.0	0.406	-27.1	0.296	-28.1	0.299	-27.5
4	0.463	-17.0	0.468	-16.0	0.408	-1.07	0.412	5.96×10^{-3}
5	0.527	-5.46	0.532	-4.62	0.408	-1.07	0.412	5.96×10^{-3}
6	0.542	-2.76	0.547	-1.81	0.408	-1.07	0.412	5.96×10^{-3}

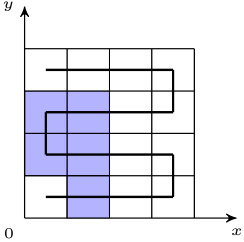
Integration - 2D





Integration - 2D





Qubits number	Grid dim.	$\mathcal{S}_1$		$\mathcal{S}_2$	
		σ	$\delta[\%]$	σ	$\delta[\%]$
4	4×4	0.55	0	0.70	-4.1
5	5×5	0.52	-4.92	0.53	-26.6
6	6×6	0.47	-14.1	0.79	9
6	7×7	0.62	-14.4	0.70	-3.0
6	8×8	0.55	0	0.78	7.6

$\mathcal{S}_1$: matching boundary of integration
$\mathcal{S}_2$: not matching boundary of integration

[Agliardi, Grossi, MP, Prati; 2201.01547]

Remarks

- For present application, too many qubits and too deep circuit for test on real hardware (**for free**)
  $\rightarrow$ 4 qubits for representation $\rightarrow$ 9 total qubits
  $\rightarrow$ 6 qubits for representation $\rightarrow$ 13 total qubits
- Best quantum computer on IBM quantum experience (we used Qiskit): 7 qubits (127 qubits for premium)
  $\rightarrow$ Simulators can go up to 5000 qubits

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Summary

- First application of quantum integration in HEP
- Theoretical quadratic speed-up
  $\rightarrow$ in practice, no gain because of generation of classical data
- Main challenge: error estimate
  $\rightarrow$ Interesting work in [Cruz-Martinez, Robbiati, Carrazza; 2308.05657]

Road map for quantum Monte Carlo in HEP

- Reliable error estimate
  $\rightarrow$ Taking into account binning effects / multi-dimension integrand

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- More natural definition of objects to be computed (matrix elements)
  $\rightarrow$ Example of colour algebra **[see Herschel's talk]**

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- Estimate of resources needed for actual computation on near-term quantum computers (noise, connections, ...)

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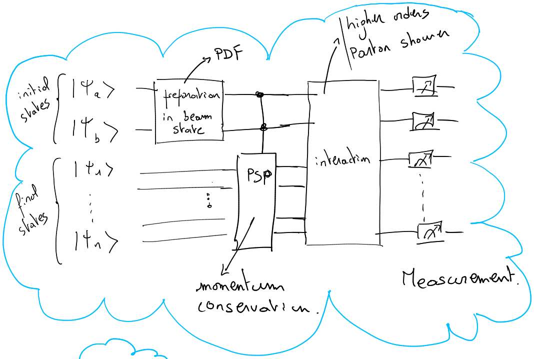
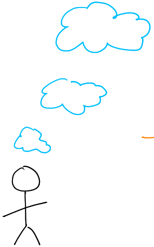
- Estimate of resources needed for actual computation on near-term quantum computers (noise, connections, ...)
- Can there be quantum advantage for event generation?


initial states
$|\psi_a\rangle$
$|\psi_b\rangle$
final states
$|\psi_1\rangle$
$|\psi_n\rangle$
PDF
preparation in beam state
PSP
interaction
higher orders
Parton shower
momentum conservation.
Measurement.



Thank you.

BACK-UP