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Event Generation on a Quantum Computer

Quantum Computing for High Energy Physics IPPP, Durham - 20th September 2023



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- The Quantum Walk
- Discretising QCD

G. Gustafson, S. Prestel, M. Spannowsky and SW, JHEP 11 (2022) 035



• Event Generation - What's the problem?

• The Parton Shower framework





Event Generation - What's the problem?

Typical hadron-hadron collisions are highly complex resulting in O(1000) particles

The theoretical description of collision events is **highly complex**

Monte Carlo Event

Generators have been the most successful approach to simulating particle collisions

MC Event Generators exploit factorisation theorems in QCD -

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Parton Density Functions





Phys. Rev. D 103, 034027

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JHEP 11 (2022) 035



Parton Density Functions



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Hard Process

Phys. Rev. D 103, 076020

Phys. Rev. D 106, 056002

Phys. Rev. Lett. 126, 062001

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Parton Shower

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The Parton Shower



Collinear mode:

$$k \stackrel{p}{-} \underbrace{ i }_{j} \qquad p_{i} = zP, \quad p_{j} = (1 - z)P$$

Successive decay steps factorise into independent quasi-classical steps

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Soft mode: $p_i \approx 0$

Interference effects only allow for partial factorisation

Leading contributions to the decay rate in the collinear limit are included in the soft limit

In this limit, the decay from high energy to low energy proceeds as a colour-dipole cascade.

This interpretation allows for straightforward interference patterns and momentum conservation





Parton shower - Collinear limit

High-multiplicity final state:

The cross-section can be factorised in the collinear limit by defining **splitting** functions $P_{ii}(z)$

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Using the splitting functions, we define the **non**emission probability, known as the Sudakov:

$$\Delta_i(z_1, z_2) = \exp\left[-\alpha_s \int_{z_1}^{z_2} \mathrm{d}z' P_{ji}(z')\right]$$

It is now possible to build an MCMC algorithm for the collinear shower:

1) **Determine** whether an emission has occurred 2) Identify which emission has occurred 3) Update shower content





Towards a quantum computing algorithm for helicity amplitudes and parton showers

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The interpretation of measurements of high-energy particle collisions relies heavily on the performance of full event generators, which include the calculation of the hard process and the subsequent parton shower step. With the continuous improvement of quantum devices, dedicated algorithms are needed to exploit the potential quantum that computers can provide. We propose general and extendable algorithms for quantum gate computers to facilitate calculations of helicity amplitudes and the parton shower process. The helicity amplitude calculation exploits the equivalence between spinors and qubits and the unique features of a quantum computer to compute the helicities of each particle involved simultaneously, thus fully utilizing the quantum nature of the computation. This advantage over classical computers is further exploited by the simultaneous computation of s- and t-channel amplitudes for a $2 \rightarrow 2$ process. The parton shower algorithm simulates collinear emission for a two-step, discrete parton shower. In contrast to classical implementations, the quantum algorithm constructs a wave function with a superposition of all shower histories for the whole parton shower process, thus removing the need to explicitly keep track of individual shower histories. Both algorithms utilize the quantum computers ability to remain in a quantum state throughout the computation and represent a first step towards a quantum computing algorithm describing the full collision event at the LHC.

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PHYSICAL REVIEW D 106, 056002 (2022)

Quantum walk approach to simulating parton showers

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This paper presents a novel quantum walk approach to simulating parton showers on a quantum computer. We demonstrate that the quantum walk paradigm offers a natural and more efficient approach to simulating parton showers on quantum devices, with the emission probabilities implemented as the coin flip for the walker, and the particle emissions to either gluons or quark pairs corresponding to the movement of the walker in two dimensions. A quantum algorithm is proposed for a simplified, toy model of a 31-step, collinear parton shower, hence significantly increasing the number of steps of the parton shower that can be simulated compared to previous quantum algorithms. Furthermore, it scales efficiently: the number of possible shower steps increases exponentially with the number of qubits, and the circuit depth grows linearly with the number of steps. Reframing the parton shower in the context of a quantum walk therefore brings dramatic improvements, and is a step towards extending the current quantum algorithms to simulate more realistic parton showers.

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Coin Operation: $C|0\rangle = \frac{1}{2} \left(|0\rangle + |1\rangle \right)$

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Quantum Walks with Memory



Advantages:

- Arbitrary dynamics
- Classical dynamics in unitary evolution

Disadvantages:

- Tight conditions on quantum advantage

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Qubit model:

Augment system further by adding an additional memory space

 $\mathcal{H} = \mathcal{H}_P \otimes \mathcal{H}_C \otimes \mathcal{H}_M$

Speedup via Quantum Walks

Szegedy Quantum Walks have been proven to achieve quadratic speed up for Markov Chain **Monte Carlo**





The Dipole Shower

The choice of the variables ξ and t is known as the **phase space** parameterisation

> $\mathcal{F}_n(\Phi_n, t_n, t_c; O) = \Delta(t_n, t_c) O(\Phi_n)$ $+\int^{t_n} dt d\xi \frac{d\phi}{2\pi} C$

Inclusive Decay Probability

 $d\mathcal{P}\left(q(p_{\mathrm{I}})\bar{q}(p_{\mathrm{K}})\to q(p_{i})g(p_{j})\bar{q}(p_{k})\right)\simeq \frac{ds_{ij}}{s_{\mathrm{IK}}}\frac{ds_{jk}}{s_{\mathrm{IK}}}C\frac{\alpha_{s}}{2\pi}\frac{2s_{\mathrm{IK}}}{s_{ij}s_{jk}}$

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Non-Emission Probability

$$\Delta(t_n, t) = \exp\left(-\int_{t}^{t_n} dt d\xi \frac{d\phi}{2\pi} C \frac{\alpha_s}{2\pi} \frac{2s_{ik}(t, \xi)}{s_{ij}(t, \xi)s_{jk}(t, \xi)}\right)$$

Master Equation

$$\frac{\alpha_s}{2\pi} \frac{2s_{ik}(t,\xi)}{s_{ij}(t,\xi)s_{jk}(t,\xi)} \Delta(t_n,t) \mathcal{F}_n(\Phi_{n+1},t,t_c;O)$$

Current interpretations of the veto algorithm treat the phase space variables ξ and t as **continuous**







Collider events on a quantum computer

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ABSTRACT: High-quality simulated data is crucial for particle physics discoveries. Therefore, parton shower algorithms are a major building block of the data synthesis in event generator programs. However, the core algorithms used to generate parton showers have barely changed since the 1980s. With quantum computers' rapid and continuous development, dedicated algorithms are required to exploit the potential that quantum computers provide to address problems in high-energy physics. This paper presents a novel approach to synthesising parton showers using the Discrete QCD method. The algorithm benefits from an elegant quantum walk implementation which can be embedded into the classical toolchain. We use the ibm_algiers device to sample parton shower configurations and generate data that we compare against measurements taken at the ALEPH, DELPHI and OPAL experiments. This is the first time a Noisy Intermediate-Scale Quantum (NISQ) device has been used to simulate realistic high-energy particle collision events.

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• Parameterise phase space in terms of gluon transverse momentum and rapidity:

$$k_{\perp}^2 = \frac{s_{ij}s_{jk}}{s_{\rm IK}}$$
 and $y = \frac{1}{2}\ln x$

which leads to the inclusive probability:

$$d\mathcal{P}\left(q(p_{\mathrm{I}})\bar{q}(p_{\mathrm{K}}) \to q(p_{i})g(p_{j})\bar{q}(p_{k})\right) \simeq = \frac{Cd}{\pi}$$

where $\kappa = \ln \left(\frac{k_{\perp}^2}{\Lambda^2}\right)$ and Λ is an arbitrary mass scale

Due to the colour charge of emitted gluons, the rapidity span for subsequent dipole decays is increased. This is interpreted as "folding out"

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 $\frac{\chi_s}{-}d\kappa dy$





2. Neglect $g \rightarrow q\overline{q}$ splittings and examine transversemomentum-dependent running coupling

$$\alpha_s(k_{\perp}^2) = \frac{12\pi}{33 - 2n_f} \frac{1}{\ln(k_{\perp}^2 / \Lambda_{\rm QCD}^2)}$$

leads to the inclusive probability

$$d\mathcal{P}\left(q(p_{\mathrm{I}})\bar{q}(p_{\mathrm{K}}) \to q(p_{i})g(p_{j})\bar{q}(p_{k})\right) \simeq = \frac{d\kappa}{\kappa}\frac{dy}{\delta y_{g}} \quad \text{with}$$

Interpreting the running coupling renormalisation group as a gainloss equation:

Gluons within δy_g **act coherently** as one effective gluon

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leads to the inclusive probability

$$d\mathcal{P}\left(q(p_{\mathrm{I}})\bar{q}(p_{\mathrm{K}}) \to q(p_{i})g(p_{j})\bar{q}(p_{k})\right) \simeq = \frac{d\kappa}{\kappa}\frac{dy}{\delta y_{g}} \quad \text{with} \quad \delta y_{g}$$

Interpreting the running coupling renormalisation group as a gainloss equation:

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Folding out extends the baseline of the triangle to positive y by $\frac{l}{2}$, where l is the height at which to emit effective gluons

A consequence of folding is that the κ axis is quantised into multiples of $2\delta y_{g}$

Each rapidity slice can be treated independently of any other slice. The exclusive rate probability takes the simple form:

$$\frac{d\kappa}{\kappa} \exp\left(-\int_{\kappa}^{\kappa_{max}} \frac{d\bar{\kappa}}{\bar{\kappa}}\right) = \frac{d\kappa}{\kappa_{max}}$$

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Discrete QCD as a Quantum Walk



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The Discrete-QCD dipole cascade can therefore be implemented as a simple **Quantum Walk**







Discrete QCD as a Quantum Walk - Raw Grove Simulation



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The algorithm has been run on the **IBM QASM 32-qubit simulator**

The device simulates a fully fault tolerant quantum computer without a noise model

The 24 grove structures are generated for a $E_{CM} = 91.2$ GeV, corresponding to typical collisions at LEP.

We see exact agreement between the simulator and analytical rates





Generating Scattering Events from Groves

Once the grove structure has been selected, event data can be synthesised in the following steps using the baseline:

- and S_{IK} from the grove
- branching momenta

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1. Create the highest κ effective gluons first (i.e. go from top to bottom in phase space)

2. For each effective gluon j that has been emitted from a dipole IK, read off the values s_{ii} , s_{ik}

3. Generate a uniformly distributed azimuthal decay angle ϕ , and then employ momentum mapping (here we have used <u>Phys. Rev. D 85, 014013 (2012), 1108.6172</u>) to produce post-





Running on a Quantum Simulator



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Running on a NISQ Quantum Device - Streamlined Circuit



I5 qubits **I 6 gate operations** (102 multi-qubit, 14 single qubit)

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IO qubits **21** gate operations (12 multi-qubit, 9 single qubit)





Discrete QCD as a Quantum Walk - Raw Grove Simulation



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The algorithm has been run on the **IBM Falcon 5.11r chip**

The figure shows the uncorrected performance of the **ibm_algiers** device compared to a simulator

The 24 grove structures are generated for a $E_{CM} = 91.2$ GeV, corresponding to typical collisions at LEP.

Main source of error from CNOT errors from large amount of SWAPs





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Summary

High Energy Physics is on the edge of a computational frontier, the High Luminosity Large Hadron Collider and FCC will provide unprecedented amounts of data

Quantum Computing offers an impressive and powerful tool to combat computational bottlenecks, both for theoretical and experimental purposes

Future Work: A dedicated research effort is required to fully evaluate the **potential** of **quantum computing** applications in HEP

The first realistic simulation of a high energy collision has been presented using a compact quantum walk implementation, allowing for the algorithm to be run on a **NISQ device**











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Backup slides

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Classical Random Walk

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Classical Random Walk



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Classical Random Walk



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Unitary Transformation:

 $U = S \cdot (C \otimes I)$

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Initialising the coin in the $-|1\rangle$ state

$$H(-|1\rangle) = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle)$$

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$$H(-|1\rangle) = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle)$$

$$|c\rangle = \frac{1}{\sqrt{2}} (|0\rangle - i|1\rangle)$$

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Initialising the coin in the $-|1\rangle$ state

$$H(-|1\rangle) = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle)$$

Removing the asymmetry:

$$|c\rangle = \frac{1}{\sqrt{2}} (|0\rangle - i|1\rangle)$$

Left moving part $(|c\rangle = |0\rangle)$ propagates in real amplitudes. Right moving part $(|c\rangle = |1\rangle)$ propagates in **imaginary amplitudes**.

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Collider Events on a Quantum Computer

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Collider Events on a Quantum Computer - Varying Λ

Varying values for the mass scale Λ . This leads to non-negligible uncertainties, however this is expected from a leading logarithm model.

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Collider Events on a Quantum Computer - Changing tune

Observables dominated by non-perturbative dynamics show mild dependence on the mass scale Λ , but are highly sensitive to changes in the tune.

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