## Imperial College London



# Event Generation on a Quantum Computer 

Simon Williams

Quantum Computing for High Energy Physics IPPP, Durham - 20th September 2023

## Imperial College London



- Event Generation - What's the problem?
- The Parton Shower framework
- The Quantum Walk
- Discretising QCD
G. Gustafson, S. Prestel, M. Spannowsky and SW, HHEP II (2022) 035
$\sim\left(P^{3} \sim\right.$


## Event Generation - What's the problem?

Typical hadron-hadron collisions are highly complex resulting in $\mathrm{O}(1000)$ particles

The theoretical description of collision
events is highly complex

## Monte Carlo Event

Generators have been the most
successful approach to simulating particle collisions

MC Event Generators exploit
factorisation theorems in QCD -

OHard Interaction

- Resonance Decays
- MECs, Matching \& Merging
- FSR


## ISR*

 - QEDWeak Showers
Hard Onium
OMultiparton Interactions
$\square$ Beam Remnants*
© Strings
$\mathbb{Q}$ Ministrings / Clusters
Colour Reconnections
String Interactions
Bose-Einstein \& Fermi-Dirac

- Primary Hadrons
- Secondary Hadrons
- Hadronic Reinteractions
(*: incoming lines are crossed)


## Monte Carlo Event Generation

## Monte Carlo Event Generation

Parton Density Functions


Phys. Rev.D 103, 034027

## Monte Carlo Event Generation



## Monte Carlo Event Generation

Parton Density Functions


Hard Process


Phys. Rev. D 103, 076020


## Monte Carlo Event Generation

Parton Density Functions


Hard Process


Phys. Rev.D 103, 034027

Parton Shower
Hadronisation


HEP || (2022) 035

## Monte Carlo Event Generation

Hard Process


Phys. Rev. D 103, 076020

Phys. Rev. D 106, 056002
Phys. Rev. Lett. 126, 06200 I

Parton Shower


## The Parton Shower



## Collinear mode:



Successive decay steps factorise into independent quasi-classical steps

## Soft mode:

Interference effects only allow for partial factorisation

Leading contributions to the decay rate in the collinear limit are included in the soft limit

In this limit, the decay from high energy to low energy proceeds as a colour-dipole cascade.

This interpretation allows for straightforward interference patterns and momentum conservation

## Parton shower - Collinear limit

## High-multiplicity final state:



The cross-section can be factorised in the collinear limit by defining splitting functions $P_{i j}(z)$

Using the splitting functions, we define the nonemission probability, known as the Sudakov:

$$
\Delta_{i}\left(z_{1}, z_{2}\right)=\exp \left[-\alpha_{s} \int_{z_{1}}^{z_{2}} \mathrm{~d} z^{\prime} P_{j i}\left(z^{\prime}\right)\right]
$$

It is now possible to build an MCMC algorithm for the collinear shower:
I) Determine whether an emission has occurred
2) Identify which emission has occurred
3) Update shower content

## Towards a quantum computing algorithm for helicity amplitudes and parton showers

Khadeejah Bepari•, ${ }^{1, *}$ Sarah Malik $\oplus{ }^{2, \dagger}$ Michael Spannowsky, ${ }^{1, \hbar}$ and Simon Williams $\oplus^{2, \S}$
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${ }^{2}$ High Energy Physics Group, Blackett Laboratory, Imperial College, Prince Consort Road, London SW7 2AZ, United Kingdom
(0) (Received 21 December 2020; accepted 18 March 2021; published 26 April 2021)

The interpretation of measurements of high-energy particle collisions relies heavily on the performance of full event generators, which include the calculation of the hard process and the subsequent parton shower step. With the continuous improvement of quantum devices, dedicated algorithms are needed to exploit the potential quantum that computers can provide. We propose general and extendable algorithms for quantum gate computers to facilitate calculations of helicity amplitudes and the parton shower process. The helicity amplitude calculation exploits the equivalence between spinors and qubits and the unique features of a quantum computer to compute the helicities of each particle involved simultaneously, thus fully utilizing the quantum nature of the computation. This advantage over classical computers is further exploited by the simultaneous computation of s - and t -channel amplitudes for a $2 \rightarrow 2$ process. The parton shower algorithm simulates collinear emission for a two-step, discrete parton shower. In contrast to classical implementations, the quantum algorithm constructs a wave function with a superposition of all shower histories for the whole parton shower process, thus removing the need to explicitly keep track of individual shower histories. Both algorithms utilize the quantum computers ability to remain in a quantum state throughout the computation and represent a first step towards a quantum computing algorithm describing the full collision event at the LHC.

DOI: 10.1103/PhysRevD.103.076020

PHYSICAL REVIEW D 106, 056002 (2022)

## Quantum walk approach to simulating parton shower

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${ }^{2}$ Department of Physics and Astronomy, University College London, Gower Street, London WC1E 6BT, United Kingdom
${ }^{3}$ High Energy Physics Group, Blackett Laboratory, Imperial College,
Prince Consort Road, London SW7 2AZ, United Kingdom
(a) (Received 13 October 2021; accepted 16 August 2022; published 2 September 2022)

This paper presents a novel quantum walk approach to simulating parton showers on a quantum computer. We demonstrate that the quantum walk paradigm offers a natural and more efficient approach to simulating parton showers on quantum devices, with the emission probabilities implemented as the coin flip for the walker, and the particle emissions to either gluons or quark pairs corresponding to the movement of the walker in two dimensions. A quantum algorithm is proposed for a simplified, toy model of a 31-step, collinear parton shower, hence significantly increasing the number of steps of the parton shower that can be simulated compared to previous quantum algorithms. Furthermore, it scales efficiently: the number of possible shower steps increases exponentially with the number of qubits, and the circuit depth grows linearly with the number of steps. Reframing the parton shower in the context of a quantum walk therefore brings dramatic improvements, and is a step towards extending the current quantum algorithms to simulate more realistic parton showers
DOI: 10.1103/PhysRevD.106.056002

## The Quantum Walk

## The Quantum Walk




## The Quantum Walk



$$
\begin{gathered}
\text { Coin } \\
\text { Operation: } \\
C|0\rangle=\frac{1}{2}(|0\rangle+|1\rangle)
\end{gathered}
$$



## The Quantum Walk



$$
\left.\begin{array}{l}
\mathscr{H}_{P}=\{|i\rangle: i \in \mathbb{Z}\} \\
\mathscr{H}_{C}=\{|0\rangle,|1\rangle\}
\end{array}\right\} \mathscr{H}=\mathscr{H}_{C} \otimes \mathscr{H}_{P}
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Operation:

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Unitary
Transformation:
$U=S \cdot(C \otimes I)$

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## Quantum Walks with Memory



## Advantages:

- Arbitrary dynamics
- Classical dynamics in unitary evolution


## Disadvantages:

- Tight conditions on quantum advantage


## Qubit model:

Augment system further by adding an additional memory space

$$
\mathscr{H}=\mathscr{H}_{P} \otimes \mathscr{H}_{C} \otimes \mathscr{H}_{M}
$$

## Speedup via Quantum Walks

Szegedy Quantum Walks have been proven to achieve quadratic speed up for Markov Chain Monte Carlo

## The Dipole Shower

## Non-Emission Probability

The choice of the variables $\xi$ and $t$ is known as the phase space parameterisation

$$
\Delta\left(t_{n}, t\right)=\exp \left(-\int_{t}^{t_{n}} d t d \xi \frac{d \phi}{2 \pi} C \frac{\alpha_{s}}{2 \pi} \frac{2 s_{i k}(t, \xi)}{s_{i j}(t, \xi) s_{j k}(t, \xi)}\right)
$$

## Master Equation

$$
\mathcal{F}_{n}\left(\Phi_{n}, t_{n}, t_{c} ; O\right)=\Delta\left(t_{n}, t_{c}\right) O\left(\Phi_{n}\right)
$$

$$
+\int_{t_{c}}^{t_{n}} d t d \xi \frac{d \phi}{2 \pi} C \frac{\alpha_{s}}{2 \pi} \frac{2 s_{i k}(t, \xi)}{s_{i j}(t, \xi) s_{j k}(t, \xi)} \Delta\left(t_{n}, t\right) \mathcal{F}_{n}\left(\Phi_{n+1}, t, t_{c} ; O\right)
$$

## Inclusive Decay

## Probability

$d \mathcal{P}\left(q\left(p_{\mathrm{I}}\right) \bar{q}\left(p_{\mathrm{K}}\right) \rightarrow q\left(p_{i}\right) g\left(p_{j}\right) \bar{q}\left(p_{k}\right)\right) \simeq \frac{d s_{i j}}{s_{\mathrm{IK}}} \frac{d s_{j k}}{s_{\mathrm{IK}}} C \frac{\alpha_{s}}{2 \pi} \frac{2 s_{\mathrm{IK}}}{s_{i j} s_{j k}}$

Current interpretations of the veto
algorithm treat the phase space variables
$\xi$ and $t$ as continuous

## Collider events on a quantum computer

## Gösta Gustafson, ${ }^{a}$ Stefan Prestel, ${ }^{a}$ Michael Spannowsky ${ }^{b}$ and Simon Williams ${ }^{c}$

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AbStract: High-quality simulated data is crucial for particle physics discoveries. Therefore, parton shower algorithms are a major building block of the data synthesis in event generator programs. However, the core algorithms used to generate parton showers have barely changed since the 1980s. With quantum computers' rapid and continuous development, dedicated algorithms are required to exploit the potential that quantum computers provide to address problems in high-energy physics. This paper presents a novel approach to synthesising parton showers using the Discrete QCD method. The algorithm benefits from an elegant quantum walk implementation which can be embedded into the classical toolchain. We use the ibm_algiers device to sample parton shower configurations and generate data that we compare against measurements taken at the ALEPH, DELPHI and OPAL experiments. This is the first time a Noisy Intermediate-Scale Quantum (NISQ) device has been used to simulate realistic high-energy particle collision events.

## Discrete QCD - Abstracting the Parton Shower Method

I. Parameterise phase space in terms of gluon transverse momentum and rapidity:

$$
k_{\perp}^{2}=\frac{s_{i j} s_{j k}}{s_{\mathrm{IK}}} \quad \text { and } \quad y=\frac{1}{2} \ln \left(\frac{s_{i j}}{s_{j k}}\right)
$$

which leads to the inclusive probability:

$$
d \mathcal{P}\left(q\left(p_{\mathrm{I}}\right) \bar{q}\left(p_{\mathrm{K}}\right) \rightarrow q\left(p_{i}\right) g\left(p_{j}\right) \bar{q}\left(p_{k}\right)\right) \simeq=\frac{C \alpha_{s}}{\pi} d \kappa d y
$$

where $\kappa=\ln \left(\frac{k_{1}^{2}}{\Lambda^{2}}\right)$ and $\Lambda$ is an arbitrary mass scale
Due to the colour charge of emitted gluons, the rapidity span for subsequent dipole decays is increased. This is interpreted as


## Discrete QCD - Abstracting the Parton Shower Method

2. Neglect $g \rightarrow q \bar{q}$ splittings and examine transverse-momentum-dependent running coupling

$$
\alpha_{s}\left(k_{\perp}^{2}\right)=\frac{12 \pi}{33-2 n_{f}} \frac{1}{\ln \left(k_{\perp}^{2} / \Lambda_{\mathrm{QCD}}^{2}\right)}
$$

leads to the inclusive probability

$$
d \mathcal{P}\left(q\left(p_{\mathrm{r}}\right) \bar{q}\left(p_{\mathrm{K}}\right) \rightarrow q\left(p_{i}\right) g\left(p_{j}\right) \bar{q}\left(p_{k}\right)\right) \simeq=\frac{d \kappa}{\kappa} \frac{d y}{\delta y_{g}} \quad \text { with } \quad \delta y_{g}=\frac{11}{6}
$$

Interpreting the running coupling renormalisation group as a gainloss equation:

> Gluons within $\delta y_{g}$ act coherently as one effective gluon


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$$

leads to the inclusive probability

$$
d \mathcal{P}\left(q\left(p_{1}\right) \bar{q}\left(p_{K}\right) \rightarrow q\left(p_{i}\right) g\left(p_{j}\right) \bar{q}\left(p_{k}\right)\right) \simeq=\frac{d \kappa}{\kappa} \frac{d y}{\delta y_{g}} \quad \text { with } \quad \delta y_{g}=\frac{11}{6}
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Interpreting the running coupling renormalisation group as a gainloss equation:

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## Discrete QCD - Abstracting the Parton Shower Method

Folding out extends the baseline of the triangle to positive $y$ by $\frac{l}{2}$, where $l$ is the height at which to emit effective gluons


A consequence of folding is that the $\kappa$ axis is quantised into multiples of $2 \delta y_{g}$

Each rapidity slice can be treated independently of any other slice. The exclusive rate probability takes the simple form:

$$
\frac{d \kappa}{\kappa} \exp \left(-\int_{\kappa}^{\kappa_{\max }} \frac{d \bar{\kappa}}{\bar{\kappa}}\right)=\frac{d \kappa}{\kappa_{\max }}
$$

## Discrete QCD as a Quantum Walk



The baseline of the grove structure contains all kinematics information

For LEP data there are $\mathbf{2 4}$ unique grove structures for $\Lambda_{\mathrm{QCD}} \in[0.1,1] \mathrm{GeV}$
$\longrightarrow$


The Discrete-QCD dipole cascade can therefore be implemented as a simple

## Quantum Walk



Repeat for all slices in fold

## Discrete QCD as a Quantum Walk - Raw Grove Simulation



The algorithm has been run on the
IBM QASM 32-qubit simulator
The device simulates a fully fault
tolerant quantum computer without
a noise model

The 24 grove structures are generated for a $E_{C M}=91.2 \mathrm{GeV}$, corresponding to typical collisions at LEP.

We see exact agreement between the simulator and analytical rates

## Generating Scattering Events from Groves

Once the grove structure has been selected, event data can be synthesised in the following steps using the baseline:
I. Create the highest $\kappa$ effective gluons first (i.e. go from top to bottom in phase space)
2. For each effective gluon $j$ that has been emitted from a dipole $I K$, read off the values $s_{i j}$, $s_{j k}$ and $s_{I K}$ from the grove
3. Generate a uniformly distributed azimuthal decay angle $\phi$, and then employ momentum mapping (here we have used Phys. Rev.D 85, 014013 (2012), 1108.6172 ) to produce postbranching momenta

## Running on a Quantum Simulator

Transverse momentum of all particles


Rapidity of all particles


Running on a NISQ Quantum Device - Streamlined Circuit


Repeat for all slices in fold


## 15 qubits

116 gate operations
(I 02 multi-qubit, 14 single qubit)

## 10 qubits

21 gate operations
( 12 multi-qubit, 9 single qubit)

## Discrete QCD as a Quantum Walk - Raw Grove Simulation



The algorithm has been run on the

## IBM Falcon 5.IIr chip

The figure shows the uncorrected performance of the ibm_algiers device compared to a simulator

The 24 grove structures are generated for a $E_{C M}=91.2 \mathrm{GeV}$, corresponding to typical collisions at LEP.

Main source of error from CNOT errors from large amount of SWAPs

## Collider Events on a Quantum Computer



Differential 2-jet rate with Durham algorithm (91.2 GeV)


G Gustafson, S Prestel, M Spannowsky and SW, JHEP II (2022) 035

## IBMQ



## Summary

High Energy Physics is on the edge of a computational frontier, the High Luminosity Large Hadron Collider and FCC will provide unprecedented amounts of data

Quantum Computing offers an impressive and powerful tool to combat computational bottlenecks, both for theoretical and experimental purposes

The first realistic simulation of a high energy collision has been presented using a compact quantum walk implementation, allowing for the algorithm to be run on a NISQ device

Future Work: A dedicated research effort is required to fully evaluate the potential of quantum computing applications in HEP

## Imperial College London



# Backup slides 

Simon Williams

Quantum Computing for High Energy Physics IPPP, Durham - 20th September 2023

## Classical Random Walk

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## The Quantum Walk




## The Quantum Walk



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## The Quantum Walk



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H|0\rangle=\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)
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Unitary
Transformation:

$$
U=S \cdot(C \otimes I) \quad H=\frac{1}{\sqrt{2}}\left(\begin{array}{rr}
1 & 1 \\
1 & -1
\end{array}\right)
$$

Hadamard Coin:


## The Quantum Walk - Coin initialisation



## The Quantum Walk - Coin initialisation

Initialising the coin in the $-|1\rangle$ state

$$
H(-|1\rangle)=\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)
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Removing the asymmetry:

$$
|c\rangle=\frac{1}{\sqrt{2}}(|0\rangle-i|1\rangle)
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Left moving part $(|c\rangle=|0\rangle)$ propagates in real amplitudes. Right moving part $(|c\rangle=|1\rangle)$ propagates in imaginary amplitudes.


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## Collider Events on a Quantum Computer




## Collider Events on a Quantum Computer - Varying $\Lambda$




Varying values for the mass scale $\Lambda$. This leads to non-negligible uncertainties, however this is expected from a leading logarithm model.

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## Collider Events on a Quantum Computer



Differential 2-jet rate with Durham algorithm (91.2 GeV)


## Collider Events on a Quantum Computer - Changing tune



Observables dominated by non-perturbative dynamics show mild dependence on the mass scale $\Lambda$, but are highly sensitive to changes in the tune.

## Collider Events on a Quantum Computer



## Collider Events on a Quantum Computer



