ASPECTS OF SCET AND COLLIDER PHENOMENOLOGY

FACTORIZATION OF NON-GLOBAL LHC OBSERVABLES AND RESUMMATION OF SUPER-LEADING LOGARITHMS

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European Research Council

AdG EFT4jets

UK ANNUAL THEORY MEETING

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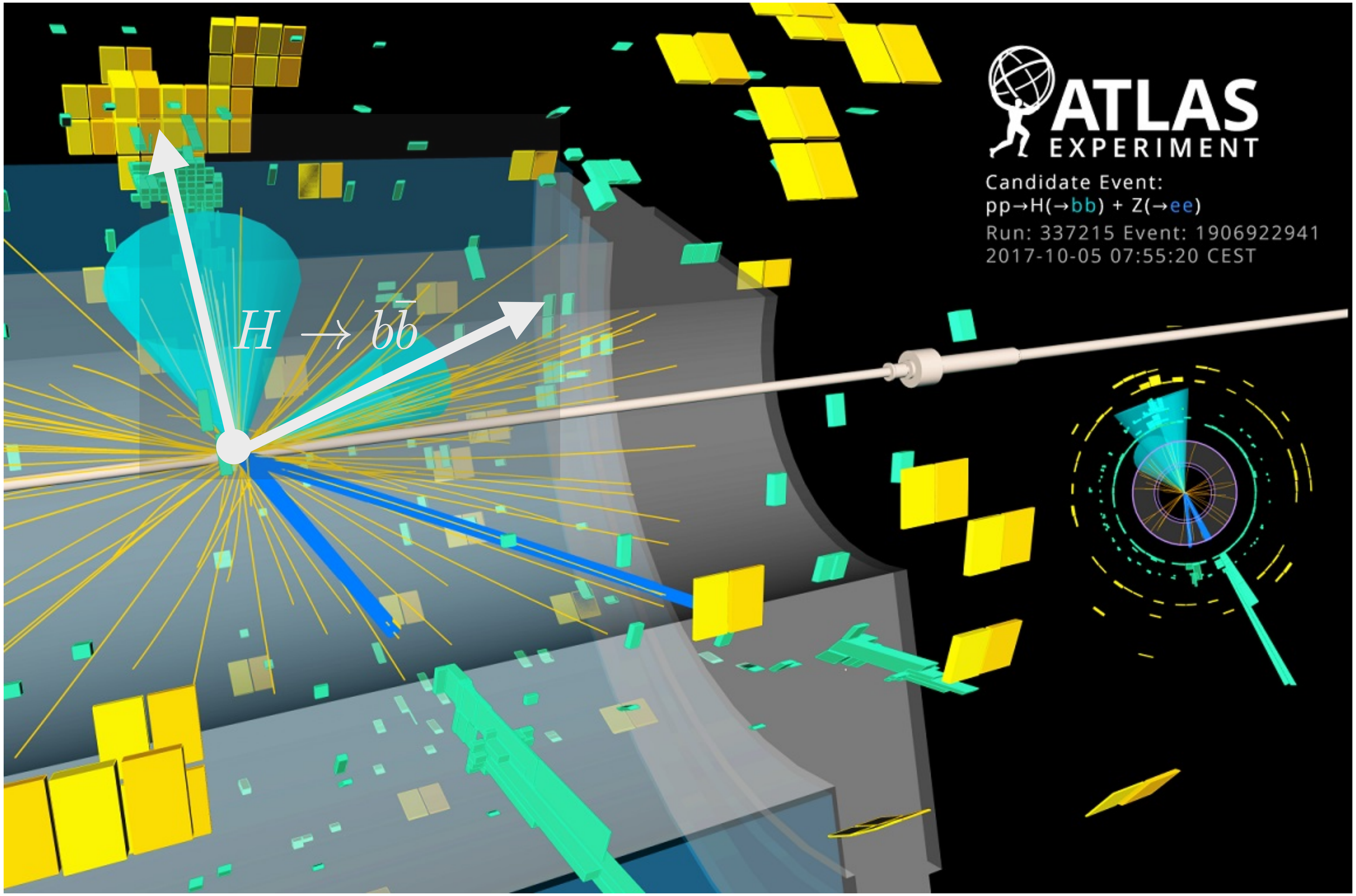
based on:

T. Becher, MN, D. Shao [2107.01212]; T. Becher, MN, D. Shao, M. Stillger [2307.06359]

P. Böer, MN, M. Stillger [2307.11089]; P. Böer, P. Hager, MN, M. Stillger, X. Xu [2311.18811]

and ongoing work

LARGE LOGARITHMS IN LHC JET PROCESSES



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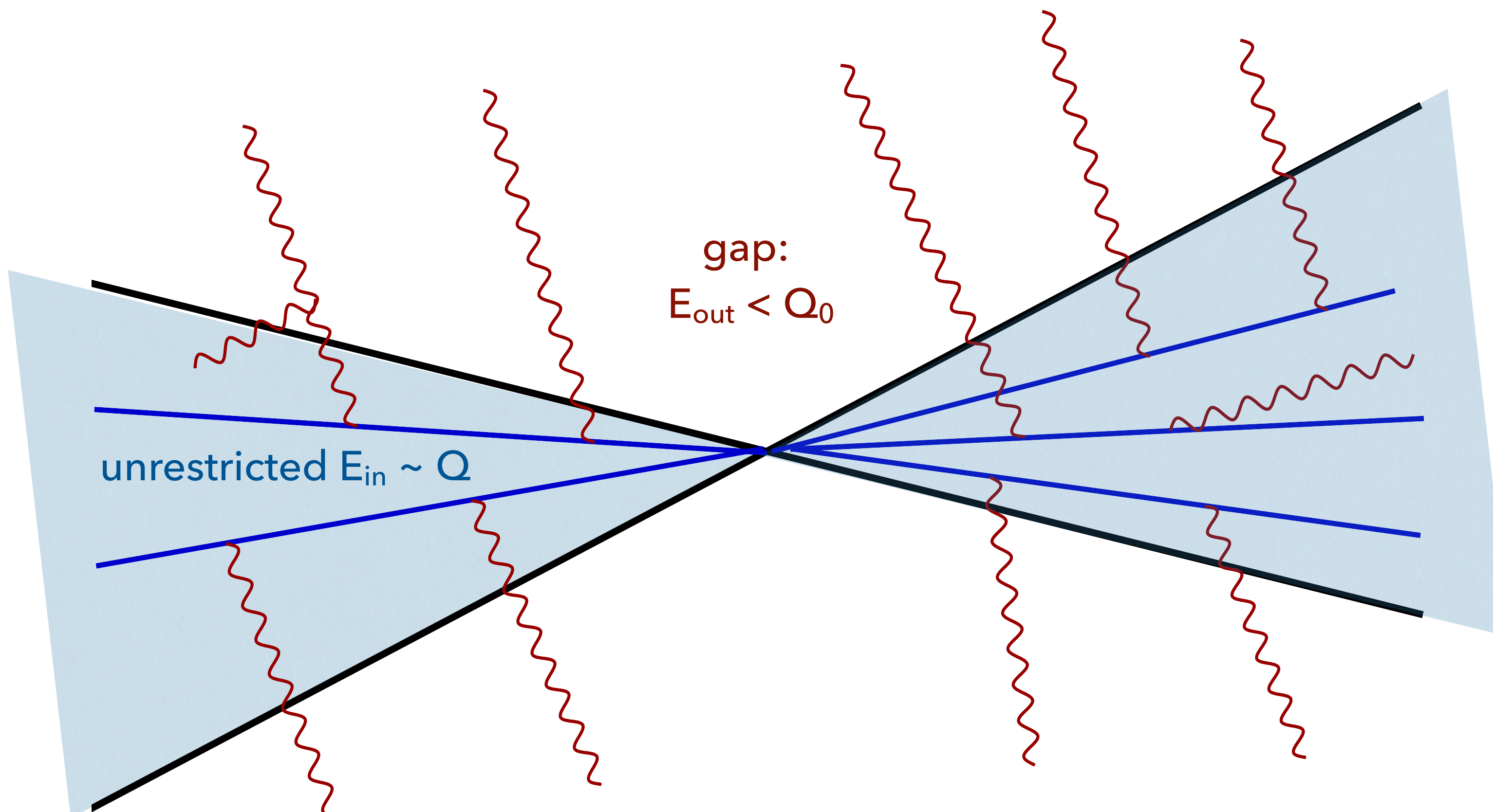
LARGE LOGARITHMS IN LHC JET PROCESSES



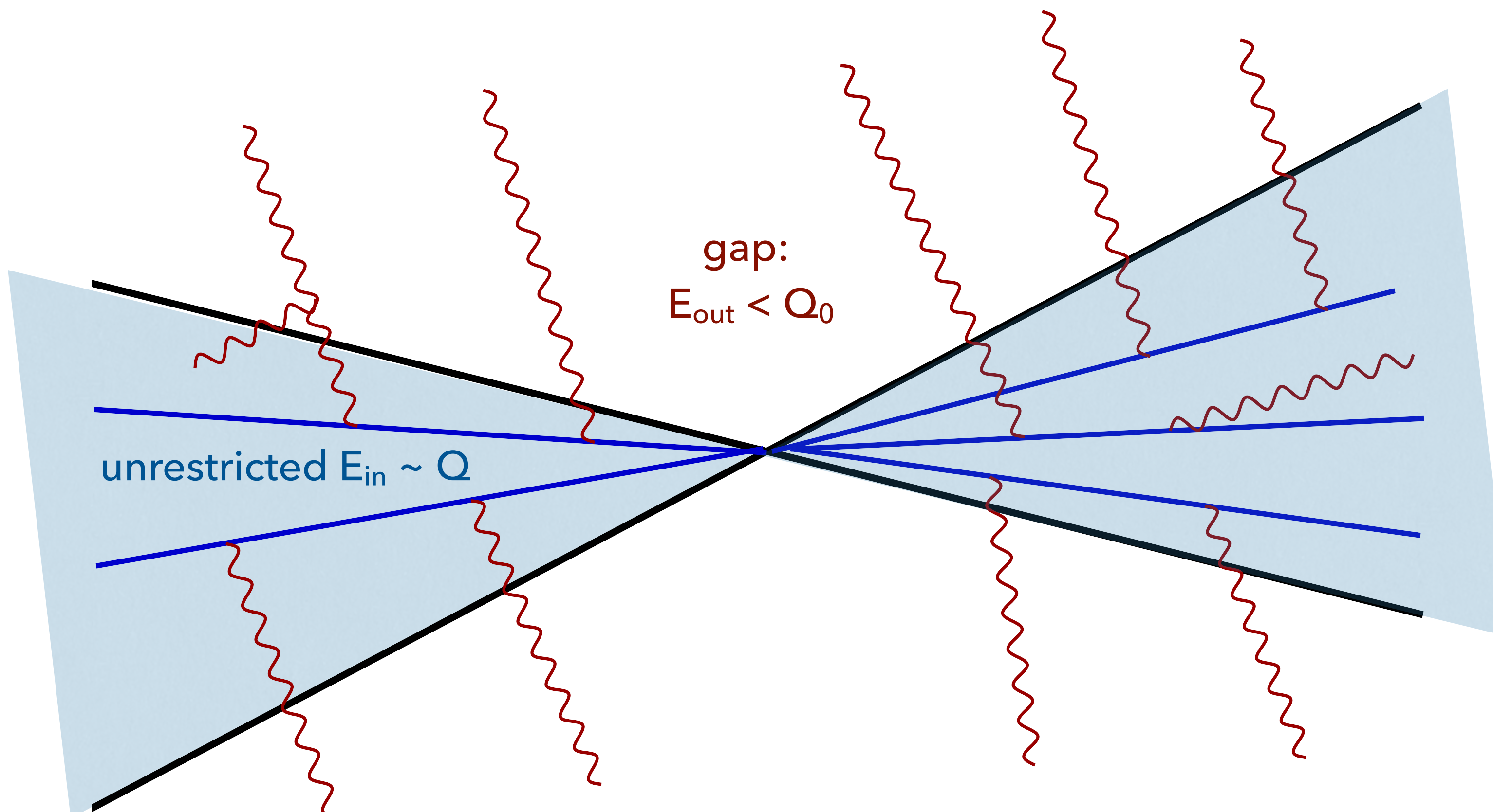
$$L = \ln(Q/Q_0) \gg 1$$

Perturbative expansion includes "super-leading" logarithms:

$$\sigma \sim \sigma_{\text{Born}} \times \left\{1 + \alpha_s L + \alpha_s^2 L^2 + \alpha_s^3 L^3 + \ldots\right\}$$

state-of-the-art

LARGE LOGARITHMS IN LHC JET PROCESSES



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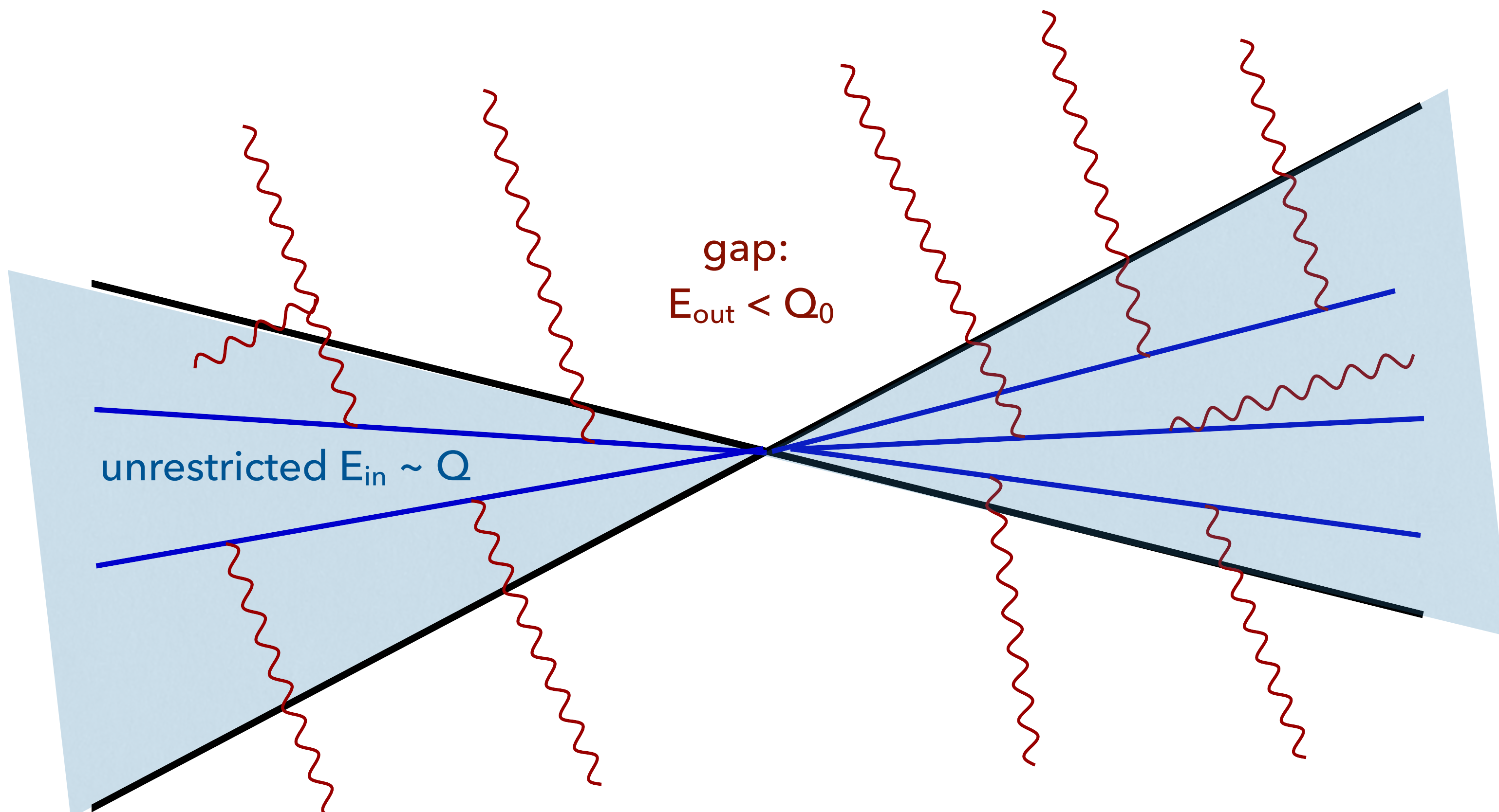
Perturbative expansion includes “super-leading” logarithms:

$$\sigma \sim \sigma_{\text{Born}} \times \{1 + \alpha_s L + \underset{\uparrow \text{ state-of-the-art}}{\alpha_s^2 L^2} + \alpha_s^3 L^3 + \underbrace{\alpha_s^4 \, L^5 + \alpha_s^5 \, L^7 + \dots}_{\text{formally larger than } O(1)}\}$$

J. R. Forshaw, A. Kyrieleis, M. H. Seymour (2006)

LARGE LOGARITHMS IN LHC JET PROCESSES



$$L = \ln(Q/Q_0) \gg 1$$

Really, a double logarithmic series starting at 3-loop order:

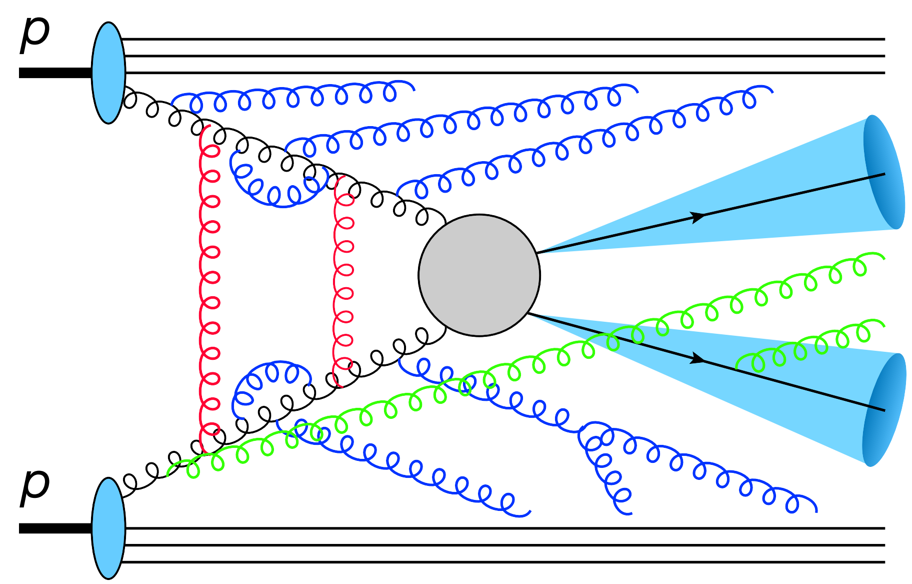
$$\sigma \sim \sigma_{\text{Born}} \times \left\{ 1 + \alpha_s L + \alpha_s^2 L^2 + (\alpha_s \pi^2) \underbrace{\left[\alpha_s^2 L^3 + \alpha_s^3 L^5 + \dots \right]}_{\text{formally larger than } O(1)} \right\}$$

$(\Im\mathrm{m}\, L)^2$

J. R. Forshaw, A. Kyrieleis, M. H. Seymour (2006)

COULOMB PHASES BREAK COLOR COHERENCE

Super-leading logarithms



- Breakdown of color coherence due to a subtle quantum effect: soft gluon exchange between initial-state partons

J. R. Forshaw, A. Kyrieleis, M. H. Seymour (2006)

- Soft anomalous dimension:

$$\boldsymbol{\Gamma}(\{\underline{p}\},\mu) = \sum_{(ij)} \frac{\boldsymbol{T}_i \cdot \boldsymbol{T}_j}{2}\, \gamma_{\text{cusp}}(\alpha_s)\, \ln \frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s) + \mathcal{O}(\alpha_s^3)$$

T. Becher, M. Neubert (2009)

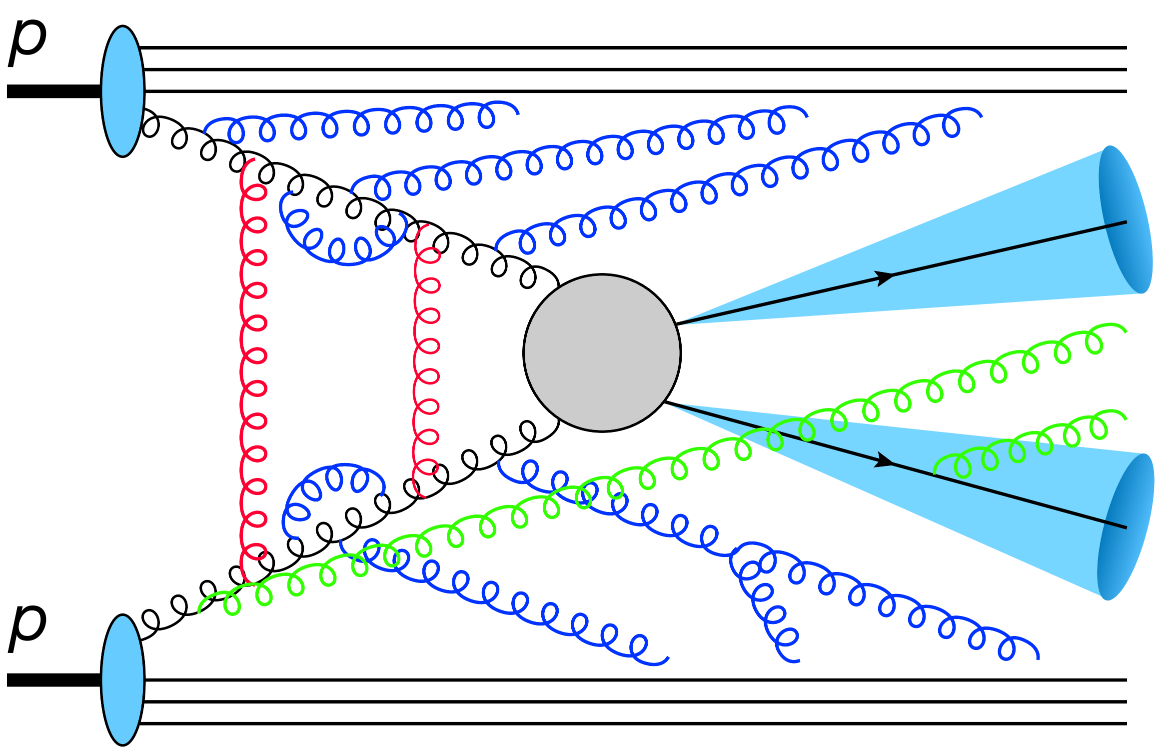
where $s_{ij} > 0$ if particles i and j are both in initial or final state

- Imaginary part (only at hadron colliders):

$$\boxed{\text{Im}\,\boldsymbol{\Gamma}(\{\underline{p}\},\mu) = +2\pi\, \gamma_{\text{cusp}}(\alpha_s)\, \boldsymbol{T}_1 \cdot \boldsymbol{T}_2} + (\dots)\, \boldsymbol{1}$$

irrelevant

THEORY OF JET PROCESSES AT LHC



red: Coulomb gluons

blue: gluons emitted along beams

green: soft gluons between jets

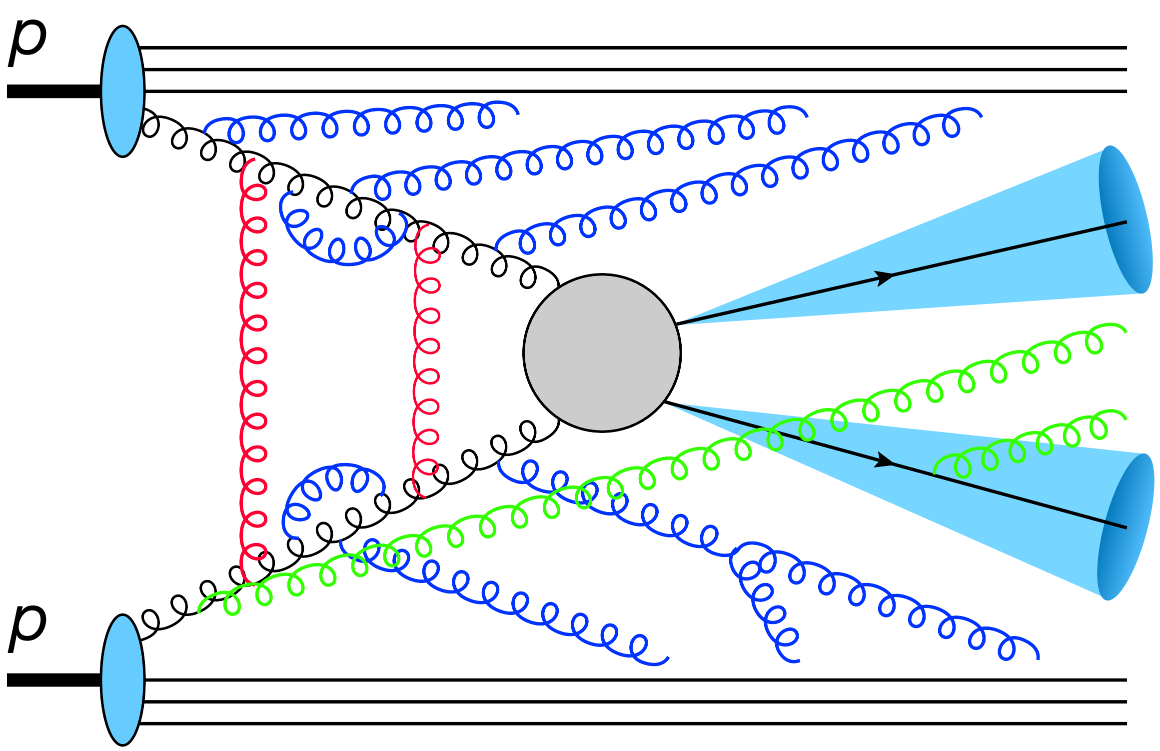
Loss of color coherence from initial-state Coulomb interactions

- Weird "super-leading logarithms"

$$d\sigma_{pp\to f}(s) = \sum_{a,b=q,\bar{q},g} \int dx_1 dx_2\, f_{a/p}(x_1,\mu)\, f_{b/p}(x_2,\mu)\, d\sigma_{ab\to f}(\hat{s}=x_1x_2s,\mu)$$

SLLs

THEORY OF JET PROCESSES AT LHC



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Loss of color coherence from initial-state Coulomb interactions

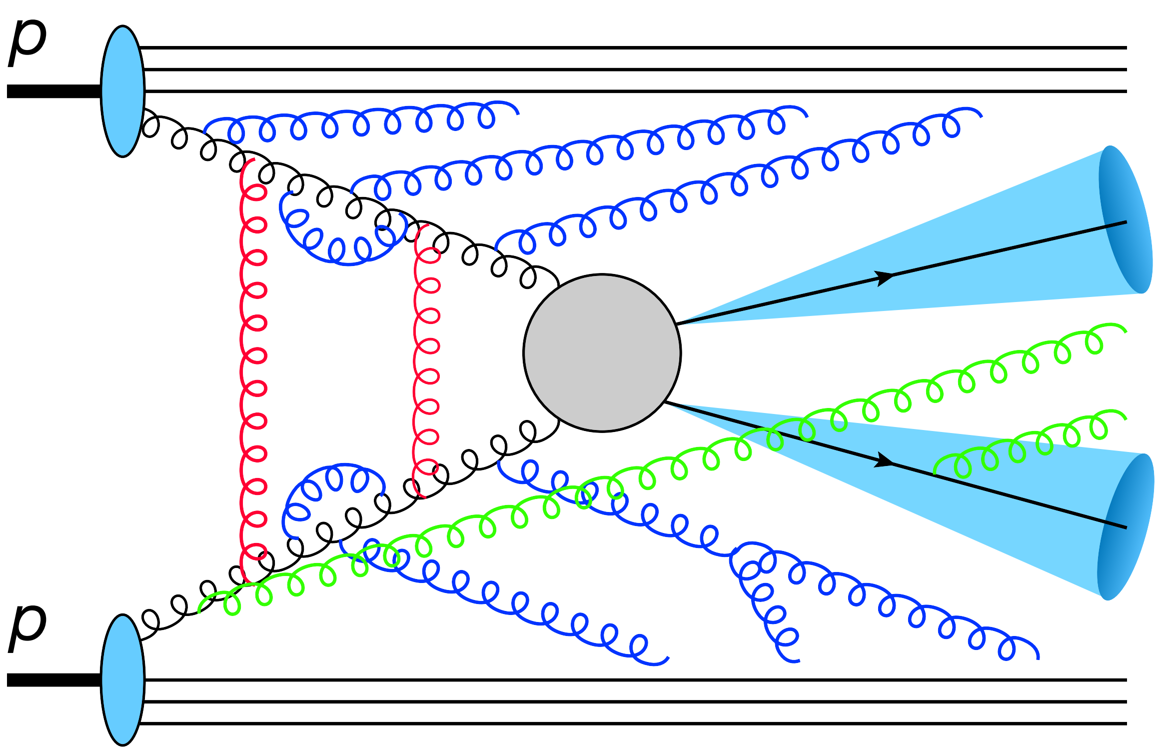
- Weird “super-leading logarithms”
- Breakdown of naive factorization

$$d\sigma_{pp\to f}(s) \boxed{\neq} \sum_{a,b=q,\bar{q},g} \int dx_1 dx_2\, f_{a/p}(x_1,\mu)\, f_{b/p}(x_2,\mu)\, \boxed{d\sigma_{ab\to f}(\hat{s}=x_1x_2s,\mu)}$$

with $\mu \approx \sqrt{\hat{s}} \equiv Q$

SLLs

THEORY OF JET PROCESSES AT LHC



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Loss of color coherence from initial-state Coulomb interactions

- Weird "super-leading logarithms"
- Breakdown of naive factorization
- Phenomenological consequences?

erc AdG **EFT4jets**

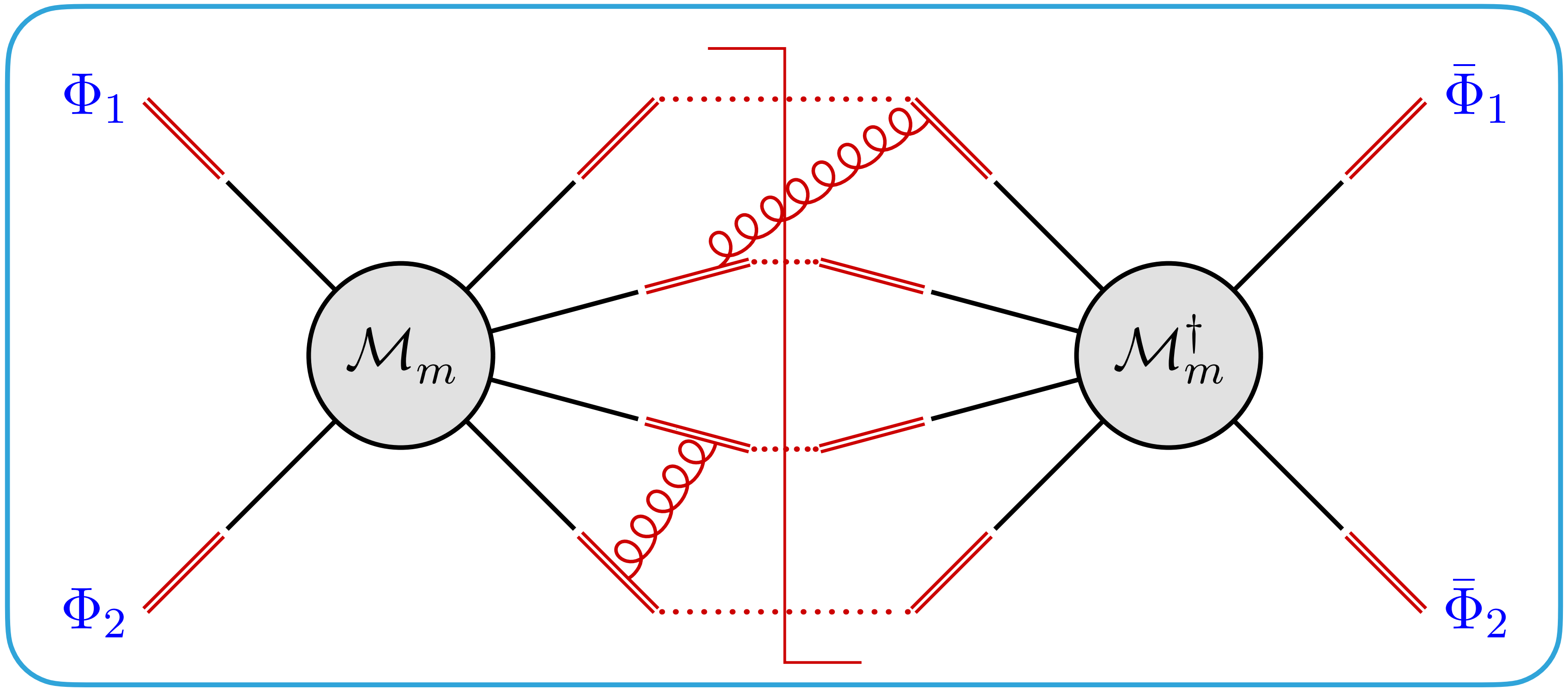
Need for a complete theory of quantum interference effects in jet processes!

THEORY OF NON-GLOBAL LHC OBSERVABLES

SCET factorization theorem

$$\sigma_{2\to M}(Q,Q_0)=\sum_{a,b=q,\bar{q},g}\int dx_1 dx_2 \sum_{m=2+M}^{\infty} \langle \underbrace{\boldsymbol{\mathcal{H}}_m^{ab}(\{\underline{n}\},Q,\mu)}_{\text{high scale}} \otimes \underbrace{\boldsymbol{\mathcal{W}}_m^{ab}(\{\underline{n}\},Q_0,x_1,x_2,\mu)}_{\text{low scale}} \rangle$$

T. Becher, M. Neubert, D. Shao (2021)
[see also: T. Becher, M. Neubert, L. Rothen, D. Shao (2015, 2016)]



THEORY OF NON-GLOBAL LHC OBSERVABLES

SCET factorization theorem

$$\sigma_{2\to M}(Q,Q_0)=\sum_{a,b=q,\bar q,g}\int dx_1 dx_2 \sum_{m=2+M}^{\infty}\Big\langle \boldsymbol{\mathcal{H}}_m^{ab}(\{\underline{n}\},Q,\mu)\otimes \boldsymbol{\mathcal{W}}_m^{ab}(\{\underline{n}\},Q_0,x_1,x_2,\mu)\Big\rangle$$

T. Becher, M. Neubert, D. Shao (2021)
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high scale

low scale

Rigorous operator definition:

$$\boldsymbol{\mathcal{H}}_m^{ab}(\{\underline{n}\},Q,\mu)=\frac{1}{2Q^2}\sum_{\text{spins}}\prod_{i=1}^{m}\int\frac{dE_i\,E_i^{d-3}}{(2\pi)^{d-2}}\,|\mathcal{M}_m^{ab}(\{\underline{p}\})\rangle\langle\mathcal{M}_m^{ab}(\{\underline{p}\})|\;(2\pi)^d\,\delta\Big(Q-\sum_{i=1}^{m}E_i\Big)\,\delta^{(d-1)}(\vec p_{\text{tot}})\,\Theta_{\text{in}}\big(\{\underline{p}\}\big)$$

density matrix involving hard-scattering amplitude in color space

$\Rightarrow$ new perspective to think about non-global observables

THEORY OF NON-GLOBAL LHC OBSERVABLES

SCET factorization theorem

$$\sigma_{2\to M}(Q,Q_0) = \sum_{a,b=q,\bar{q},g} \int dx_1 dx_2 \sum_{m=2+M}^{\infty} \big\langle \boldsymbol{\mathcal{H}}_m^{ab}(\{\underline{n}\},Q,\mu) \otimes \boldsymbol{\mathcal{W}}_m^{ab}(\{\underline{n}\},Q_0,x_1,x_2,\mu) \big\rangle$$

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high scale

low scale

Renormalization-group equation:

$$\mu \frac{d}{d\mu} \boldsymbol{\mathcal{H}}_l^{ab}(\{\underline{n}\},Q,\mu) = -\sum_{m\le l} \boldsymbol{\mathcal{H}}_m^{ab}(\{\underline{n}\},Q,\mu)\, \boldsymbol{\Gamma}_{ml}^H(\{\underline{n}\},Q,\mu)$$

operator in color space and in the infinite space of parton multiplicities

All-order summation of large logarithmic corrections, including the super-leading logarithms!

RESUMMATION OF SUPER-LEADING LOGARITHMS

Evaluate factorization theorem at low scale $\mu_s \sim Q_0$

- Low-energy matrix element:

$$\boldsymbol{\mathcal{W}}_m^{ab}(\{\underline{n}\}, Q_0, x_1, x_2, \mu_s) = f_{a/p}(x_1)\, f_{b/p}(x_2)\, \mathbf{1} + \mathcal{O}(\alpha_{\mathbf{s}})$$

- Hard-scattering functions:

$$\boldsymbol{\mathcal{H}}_m^{ab}(\{\underline{n}\}, Q, \mu_s) = \sum_{l \le m} \boldsymbol{\mathcal{H}}_l^{ab}(\{\underline{n}\}, Q, Q)\, \mathbf{P} \exp\left[\int_{\mu_s}^{Q} \frac{d\mu}{\mu}\, \mathbf{\Gamma}^H(\{\underline{n}\}, Q, \mu)\right]_{lm}$$

- Expanding the solution in a power series generates arbitrarily high parton multiplicities starting from the $2 \to M$ Born process

RESUMMATION OF SUPER-LEADING LOGARITHMS

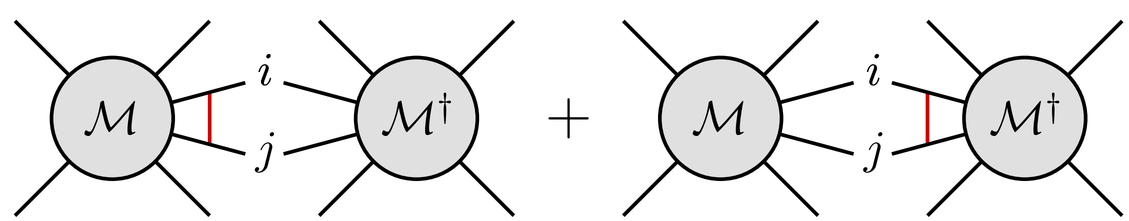
Evaluate factorization theorem at low scale $\mu_s \sim Q_0$

- Anomalous-dimension matrix:

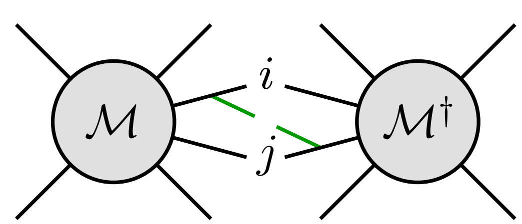
$$\boldsymbol{\Gamma}^H = \frac{\alpha_s}{4\pi} \begin{pmatrix} \boldsymbol{V}_{2+M} & \boldsymbol{R}_{2+M} & 0 & 0 & \dots \\ 0 & \boldsymbol{V}_{2+M+1} & \boldsymbol{R}_{2+M+1} & 0 & \dots \\ 0 & 0 & \boldsymbol{V}_{2+M+2} & \boldsymbol{R}_{2+M+2} & \dots \\ 0 & 0 & 0 & \boldsymbol{V}_{2+M+3} & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} + \mathcal{O}(\alpha_s^2)$$

- Action on hard functions:

$$\boldsymbol{\mathcal{H}}_m \, \boldsymbol{V}_m = \sum_{(ij)}$$



$$\boldsymbol{\mathcal{H}}_m \, \boldsymbol{R}_m = \sum_{(ij)}$$



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- Virtual and real contributions contain collinear singularities, which must be regularized and subtracted

$$\mathbf{\Gamma}^H(\xi_1,\xi_2) = \delta(1-\xi_1)\,\delta(1-\xi_2)\,\underbrace{\mathbf{\Gamma}^S}_{\text{soft / soft-collinear part}} + \underbrace{\mathbf{\Gamma}_1^C(\xi_1)\,\delta(1-\xi_2) + \delta(1-\xi_1)\,\mathbf{\Gamma}_2^C(\xi_2)}_{\text{collinear parts}}$$

RESUMMATION OF SUPER-LEADING LOGARITHMS

Detailed structure of the soft anomalous-dimension coefficients

$$\left.\begin{aligned} \boldsymbol{V}_m &= \overline{\boldsymbol{V}}_m + \boldsymbol{V}^G + \sum_{i=1,2} \boldsymbol{V}_i^c \ln\frac{\mu^2}{\hat{s}} \\ \boldsymbol{R}_m &= \overline{\boldsymbol{R}}_m + \sum_{i=1,2} \boldsymbol{R}_i^c \ln\frac{\mu^2}{\hat{s}} \end{aligned}\right\} \quad \boldsymbol{\Gamma} = \underbrace{\overline{\boldsymbol{\Gamma}}}_{\text{soft emission}} + \overbrace{\boldsymbol{V}^G}^{\text{Glauber phase}} + \underbrace{\boldsymbol{\Gamma}^c \ln\frac{\mu^2}{\hat{s}}}_{\text{soft \& collinear emission}}$$

where:

$$\boldsymbol{V}^G = -8i\pi \left(\boldsymbol{T}_{1,L}\cdot\boldsymbol{T}_{2,L} - \boldsymbol{T}_{1,R}\cdot\boldsymbol{T}_{2,R}\right) \qquad \text{Coulomb (Glauber) phase}$$

$$\left.\begin{aligned} \boldsymbol{V}_i^c &= 4C_i\,\boldsymbol{1} \\ \boldsymbol{R}_i^c &= -4\boldsymbol{T}_{i,L}\circ\boldsymbol{T}_{i,R}\,\delta(n_k - n_i) \end{aligned}\right\} \qquad \text{soft \& collinear terms}$$

RESUMMATION OF SUPER-LEADING LOGARITHMS

Detailed structure of the soft anomalous-dimension coefficients

Glauber phase

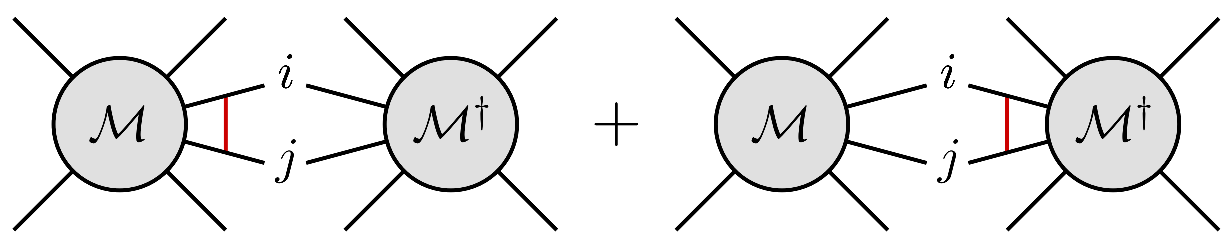
$$\left.\begin{aligned} \boldsymbol{V}_m &= \overline{\boldsymbol{V}}_m + \boldsymbol{V}^G + \sum_{i=1,2} \boldsymbol{V}_i^c \ln\frac{\mu^2}{\hat{s}} \\ \boldsymbol{R}_m &= \overline{\boldsymbol{R}}_m + \sum_{i=1,2} \boldsymbol{R}_i^c \ln\frac{\mu^2}{\hat{s}} \end{aligned}\right\} \quad \boldsymbol{\Gamma} = \boxed{\overline{\boldsymbol{\Gamma}}} + \boldsymbol{V}^G + \boldsymbol{\Gamma}^c \ln\frac{\mu^2}{\hat{s}}$$

soft emission

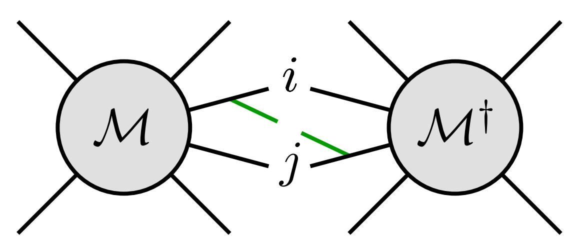
soft & collinear emission

where:

$$\mathcal{H}_m \, \overline{\boldsymbol{V}}_m = \sum_{(ij)}$$



$$\mathcal{H}_m \, \overline{\boldsymbol{R}}_m = \sum_{(ij)}$$



RESUMMATION OF SUPER-LEADING LOGARITHMS

Detailed structure of the soft anomalous-dimension coefficients

Glauber phase

$$\boldsymbol{V}_m = \overline{\boldsymbol{V}}_m + \boldsymbol{V}^G + \sum_{i=1,2} \boldsymbol{V}_i^c \ln\frac{\mu^2}{\hat{s}}$$

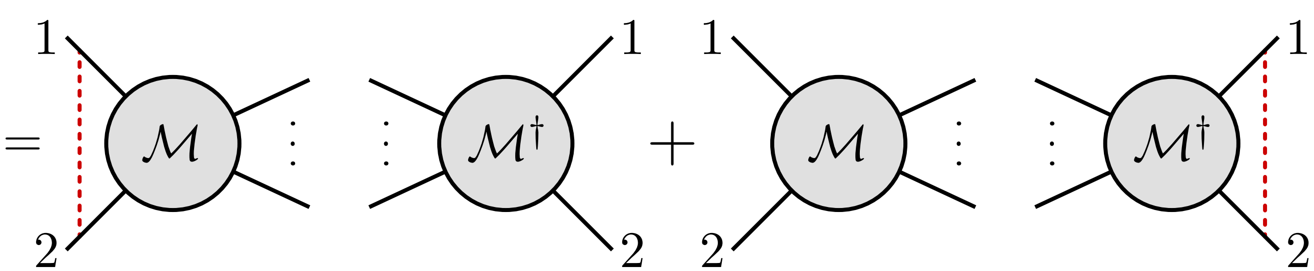
$$\boldsymbol{R}_m = \overline{\boldsymbol{R}}_m + \sum_{i=1,2} \boldsymbol{R}_i^c \ln\frac{\mu^2}{\hat{s}}$$

$$\boldsymbol{\Gamma} = \overline{\boldsymbol{\Gamma}} + \boxed{\boldsymbol{V}^G + \boldsymbol{\Gamma}^c} \ln\frac{\mu^2}{\hat{s}}$$

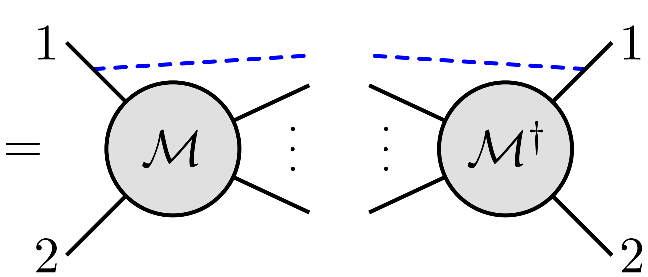
soft emission collinear emission

where:

$\boldsymbol{\mathcal{H}}_m\, \boldsymbol{V}^G =$



$\boldsymbol{\mathcal{H}}_m\, \boldsymbol{R}_1^c =$



RESUMMATION OF SUPER-LEADING LOGARITHMS

SLLs arise from the terms in $\mathbf{P}\exp\left[\int_{\mu_s}^{Q}\frac{d\mu}{\mu}\,\mathbf{\Gamma}^H(\{\underline{n}\},Q,\mu)\right]_{lm}$ **with the highest number of insertions of Γ^c**

- Three properties simplify the calculation:

  - color coherence in absence of Glauber phases (sum of soft emissions off collinear partons has same effect as soft emission of parent parton):
  $$\boldsymbol{\mathcal{H}}_m\,\mathbf{\Gamma}^c\,\overline{\mathbf{\Gamma}} = \boldsymbol{\mathcal{H}}_m\,\overline{\mathbf{\Gamma}}\,\mathbf{\Gamma}^c$$
  - collinear safety (singularities from real and virtual emissions cancel):
  $$\langle\boldsymbol{\mathcal{H}}_m\,\mathbf{\Gamma}^c\otimes\mathbf{1}\rangle = 0$$
  - cyclicity of the trace:
  $$\langle\boldsymbol{\mathcal{H}}_m\,\boldsymbol{V}^G\otimes\mathbf{1}\rangle = 0$$

RESUMMATION OF SUPER-LEADING LOGARITHMS

SLLs arise from the terms in $\mathbf{P}\exp\left[\int_{\mu_s}^{Q}\frac{d\mu}{\mu}\,\mathbf{\Gamma}^H(\{\underline{n}\},Q,\mu)\right]_{lm}$ **with the highest number of insertions of Γ^c**

- Under color trace, insertions of Γ_c are non-zero only if they come in conjunction with (at least) two Glauber phases and one $\overline{\Gamma}$
- Relevant color traces at $\mathcal{O}(\alpha_s^{n+3}L^{2n+3})$:

$$C_{rn} = \left\langle \boldsymbol{\mathcal{H}}_{2\to M}\,(\mathbf{\Gamma}^c)^r\,\boldsymbol{V}^G\,(\mathbf{\Gamma}^c)^{n-r}\,\boldsymbol{V}^G\,\overline{\mathbf{\Gamma}}\otimes\mathbf{1}\right\rangle$$

- Kinematic information contained in $(M+1)$ angular integrals from $\overline{\Gamma}$:

$$J_j = \int\frac{d\Omega(n_k)}{4\pi}\left(W_{1j}^k - W_{2j}^k\right)\Theta_{\text{veto}}(n_k)\,; \quad \text{with} \quad W_{ij}^k = \frac{n_i\cdot n_j}{n_i\cdot n_k\,n_j\cdot n_k}$$

RESUMMATION OF SUPER-LEADING LOGARITHMS

General result for $2 \to M$ hard processes

$$C_{rn} = -256\pi^2 \, (4N_c)^{n-r} \left[\sum_{j=3}^{M+2} J_j \sum_{i=1}^{4} c_i^{(r)} \left\langle \boldsymbol{\mathcal{H}}_{2\to M} \, \boldsymbol{O}_i^{(j)} \right\rangle - J_2 \sum_{i=1}^{6} d_i^{(r)} \left\langle \boldsymbol{\mathcal{H}}_{2\to M} \, \boldsymbol{S}_i \right\rangle \right]$$

T. Becher, M. Neubert, D. Shao, M. Stillger (2023)

Basis of 10 color structures:

$$\boldsymbol{O}_1^{(j)} = f_{abe} f_{cde} \, \boldsymbol{T}_2^a \, \{\boldsymbol{T}_1^b, \boldsymbol{T}_1^c\} \, \boldsymbol{T}_j^d - (1 \leftrightarrow 2)$$

$$\boldsymbol{O}_2^{(j)} = d_{ade} d_{bce} \, \boldsymbol{T}_2^a \, \{\boldsymbol{T}_1^b, \boldsymbol{T}_1^c\} \, \boldsymbol{T}_j^d - (1 \leftrightarrow 2)$$

$$\boldsymbol{O}_3^{(j)} = \boldsymbol{T}_2^a \, \{\boldsymbol{T}_1^a, \boldsymbol{T}_1^b\} \, \boldsymbol{T}_j^b - (1 \leftrightarrow 2)$$

$$\boldsymbol{O}_4^{(j)} = 2C_1 \, \boldsymbol{T}_2 \cdot \boldsymbol{T}_j - 2C_2 \, \boldsymbol{T}_1 \cdot \boldsymbol{T}_j$$

$$\boldsymbol{S}_1 = f_{abe} f_{cde} \, \{\boldsymbol{T}_1^b, \boldsymbol{T}_1^c\} \, \{\boldsymbol{T}_2^a, \boldsymbol{T}_2^d\}$$

$$\boldsymbol{S}_2 = d_{ade} d_{bce} \, \{\boldsymbol{T}_1^b, \boldsymbol{T}_1^c\} \, \{\boldsymbol{T}_2^a, \boldsymbol{T}_2^d\}$$

$$\boldsymbol{S}_3 = d_{ade} d_{bce} \left[\boldsymbol{T}_2^a \left(\boldsymbol{T}_1^b \boldsymbol{T}_1^c \boldsymbol{T}_1^d \right)_+ + (1 \leftrightarrow 2) \right]$$

$$\boldsymbol{S}_4 = \{\boldsymbol{T}_1^a, \boldsymbol{T}_1^b\} \, \{\boldsymbol{T}_2^a, \boldsymbol{T}_2^b\}$$

$$\boldsymbol{S}_5 = \boldsymbol{T}_1 \cdot \boldsymbol{T}_2$$

$$\boldsymbol{S}_6 = \boldsymbol{1}$$

RESUMMATION OF SUPER-LEADING LOGARITHMS

General result for $2 \to M$ hard processes

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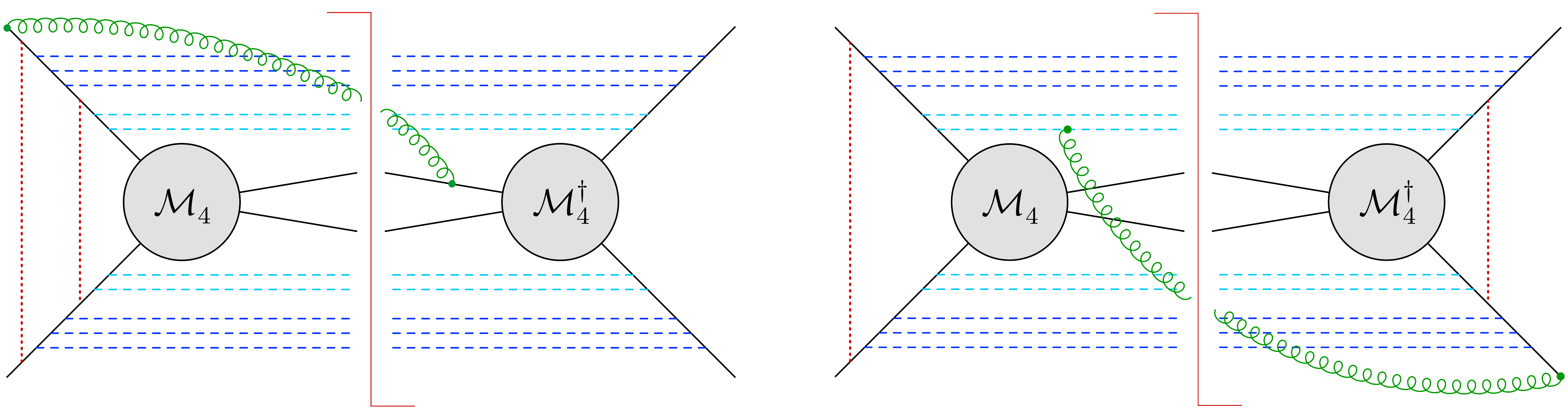
T. Becher, M. Neubert, D. Shao, M. Stillger (2023)

Basis of 10 color structures:



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T. Becher, M. Neubert, D. Shao, M. Stillger (2023)

Basis of 10 color structures:

$$c_1^{(r)} = 2^{r-1} \left[(3N_c+2)^r + (3N_c-2)^r \right]$$

$$c_2^{(r)} = 2^{r-2} N_c \left[\frac{(3N_c+2)^r}{N_c+2} + \frac{(3N_c-2)^r}{N_c-2} - \frac{(2N_c)^{r+1}}{N_c^2-4} \right]$$

$$c_3^{(r)} = 2^{r-1} \left[(3N_c+2)^r - (3N_c-2)^r \right]$$

$$c_4^{(r)} = 2^{r-1} \left[\frac{(3N_c+2)^r}{N_c+1} + \frac{(3N_c-2)^r}{N_c-1} - \frac{2N_c^{r+1}}{N_c^2-1} \right]$$

$$d_1^{(r)} = 2^{3r-1} \left[(N_c+1)^r + (N_c-1)^r \right] - 2^{r-1} \left[(3N_c+2)^r + (3N_c-2)^r \right]$$

$$d_2^{(r)} = 2^{3r-2} N_c \left[\frac{(N_c+1)^r}{N_c+2} + \frac{(N_c-1)^r}{N_c-2} \right] - 2^{r-2} N_c \left[\frac{(3N_c+2)^r}{N_c+2} + \frac{(3N_c-2)^r}{N_c-2} \right]$$

$$d_3^{(r)} = 2^{r-1} N_c \left[\frac{(3N_c+2)^r}{N_c+2} + \frac{(3N_c-2)^r}{N_c-2} - \frac{(2N_c)^{r+1}}{N_c^2-4} \right]$$

$$d_4^{(r)} = 2^{3r-1} \left[(N_c+1)^r - (N_c-1)^r \right] - 2^{r-1} \left[(3N_c+2)^r - (3N_c-2)^r \right]$$

$$\begin{aligned} d_5^{(r)} &= 2^r \, (C_1+C_2) \left[\frac{N_c+2}{N_c+1} (3N_c+2)^r - \frac{N_c-2}{N_c-1} (3N_c-2)^r - \frac{2N_c^{r+1}}{N_c^2-1} \right] \\ &\quad - \frac{2^{r-1} N_c}{3} \left[(N_c+4)\,(3N_c+2)^r + (N_c-4)\,(3N_c-2)^r - (2N_c)^{r+1} \right] \end{aligned}$$

$$\begin{aligned} d_6^{(r)} &= 2^{3r+1} C_1 C_2 \left[(N_c+1)^{r-1} + (N_c-1)^{r-1} \right] (1-\delta_{r0}) \\ &\quad - 2^{r+1} C_1 C_2 \left[\frac{(3N_c+2)^r}{N_c+1} + \frac{(3N_c-2)^r}{N_c-1} - \frac{2N_c^{r+1}}{N_c^2-1} \right] \end{aligned}$$

RESUMMATION OF SUPER-LEADING LOGARITHMS

General result for $2 \to M$ hard processes

$$C_{rn} = -256\pi^2 \, (4N_c)^{n-r} \left[\sum_{j=3}^{M+2} J_j \sum_{i=1}^{4} c_i^{(r)} \, \langle \boldsymbol{\mathcal{H}}_{2\to M} \, \boldsymbol{O}_i^{(j)} \rangle - J_2 \sum_{i=1}^{6} d_i^{(r)} \, \langle \boldsymbol{\mathcal{H}}_{2\to M} \, \boldsymbol{S}_i \rangle \right]$$

T. Becher, M. Neubert, D. Shao, M. Stillger (2023)

- Series of SLLs, starting at 3-loop order:

$$\sigma_{\text{SLL}} = \sigma_{\text{Born}} \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{4\pi} \right)^{n+3} L^{2n+3} \, \frac{(-4)^n \, n!}{(2n+3)!} \sum_{r=0}^{n} \frac{(2r)!}{4^r \, (r!)^2} \, C_{rn}$$

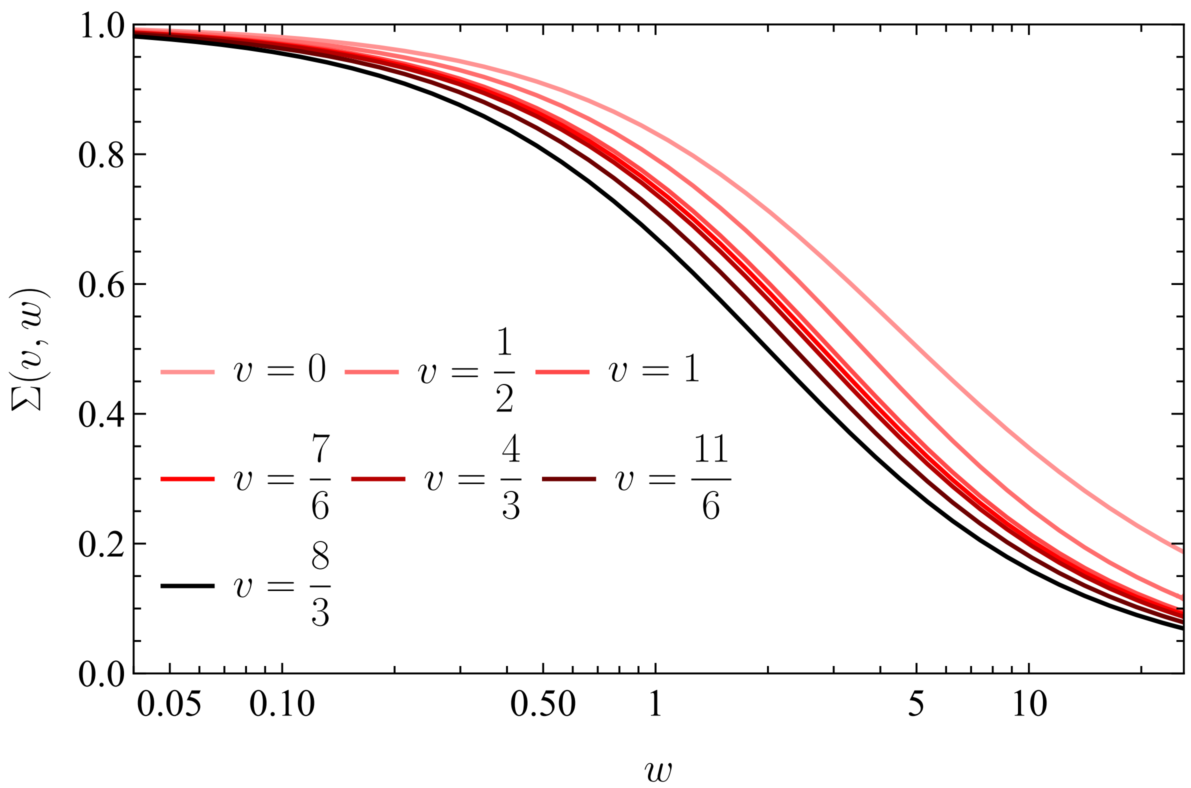
- Reproduces all that is known about SLLs (and much more…)

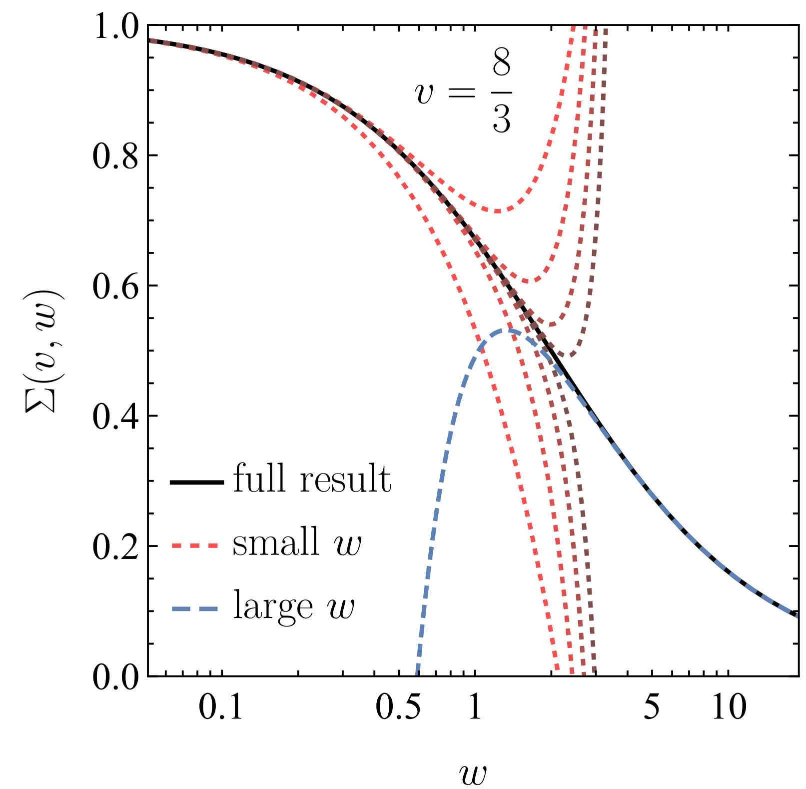
RESUMMATION OF SUPER-LEADING LOGARITHMS

Contribution to partonic cross sections

- Infinite series can be expressed in closed form in terms of a prefactor times Kampé de Fériet functions $\Sigma(v_i, w)$ with $w = \dfrac{N_c \alpha_s}{\pi} L^2$ and

$$v_0 = 0\,, \qquad v_1 = \frac{1}{2}\,, \qquad v_2 = 1\,, \qquad v_{3,4} = \frac{3N_c \pm 2}{2N_c}\,, \qquad v_{5,6} = \frac{2\,(N_c \pm 1)}{N_c}$$





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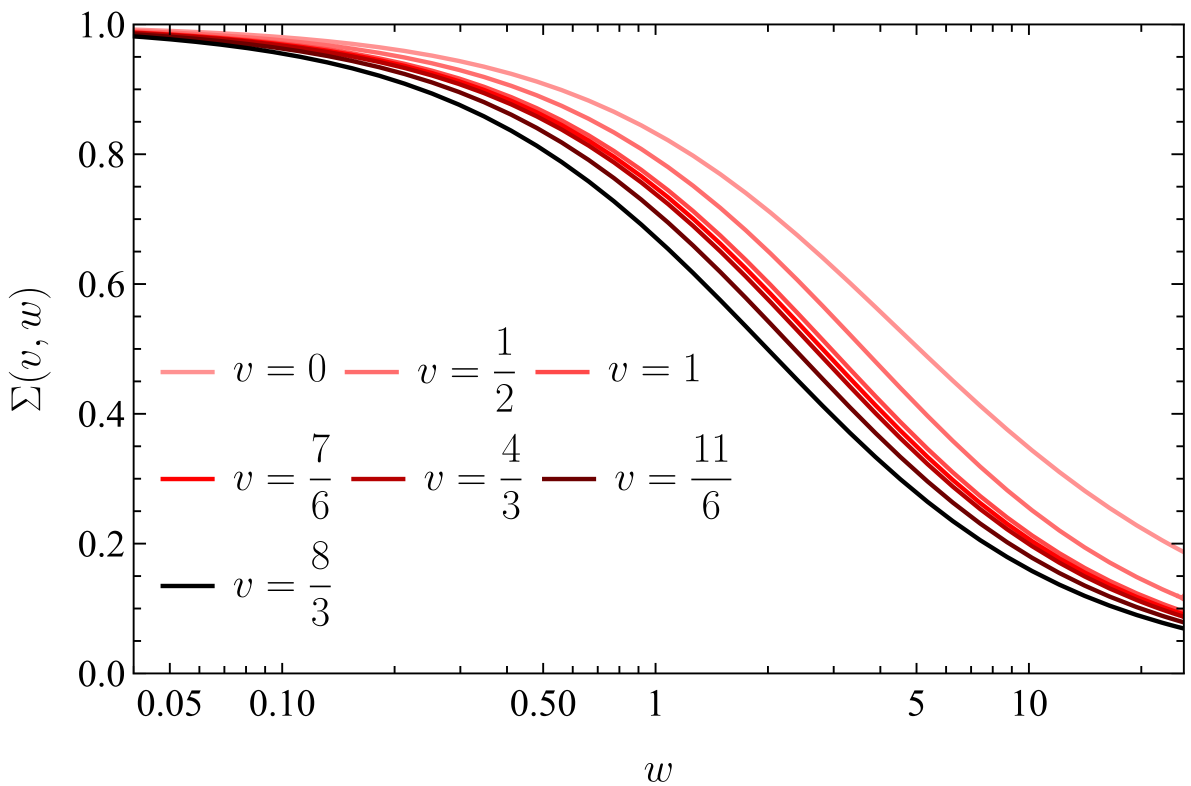

Asymptotic behavior for $w \gg 1$:

$$\Sigma_0(w) = \frac{3}{2w}\Big(\ln(4w) + \gamma_E - 2\Big) + \frac{3}{4w^2} + \mathcal{O}(w^{-3})$$

$$\Sigma(v,w) = \frac{3\arctan\left(\sqrt{v-1}\right)}{\sqrt{v-1}\,w} - \frac{3\sqrt{\pi}}{2\sqrt{v}\,w^{3/2}} + \mathcal{O}(w^{-2})$$

$\Rightarrow$ much slower fall-off than Sudakov form factors ~ e^{-cw}

PHENOMENOLOGICAL IMPACT (PARTON LEVEL)

Partonic channels contributing to $pp \to$ 2 jets (gap between jets)

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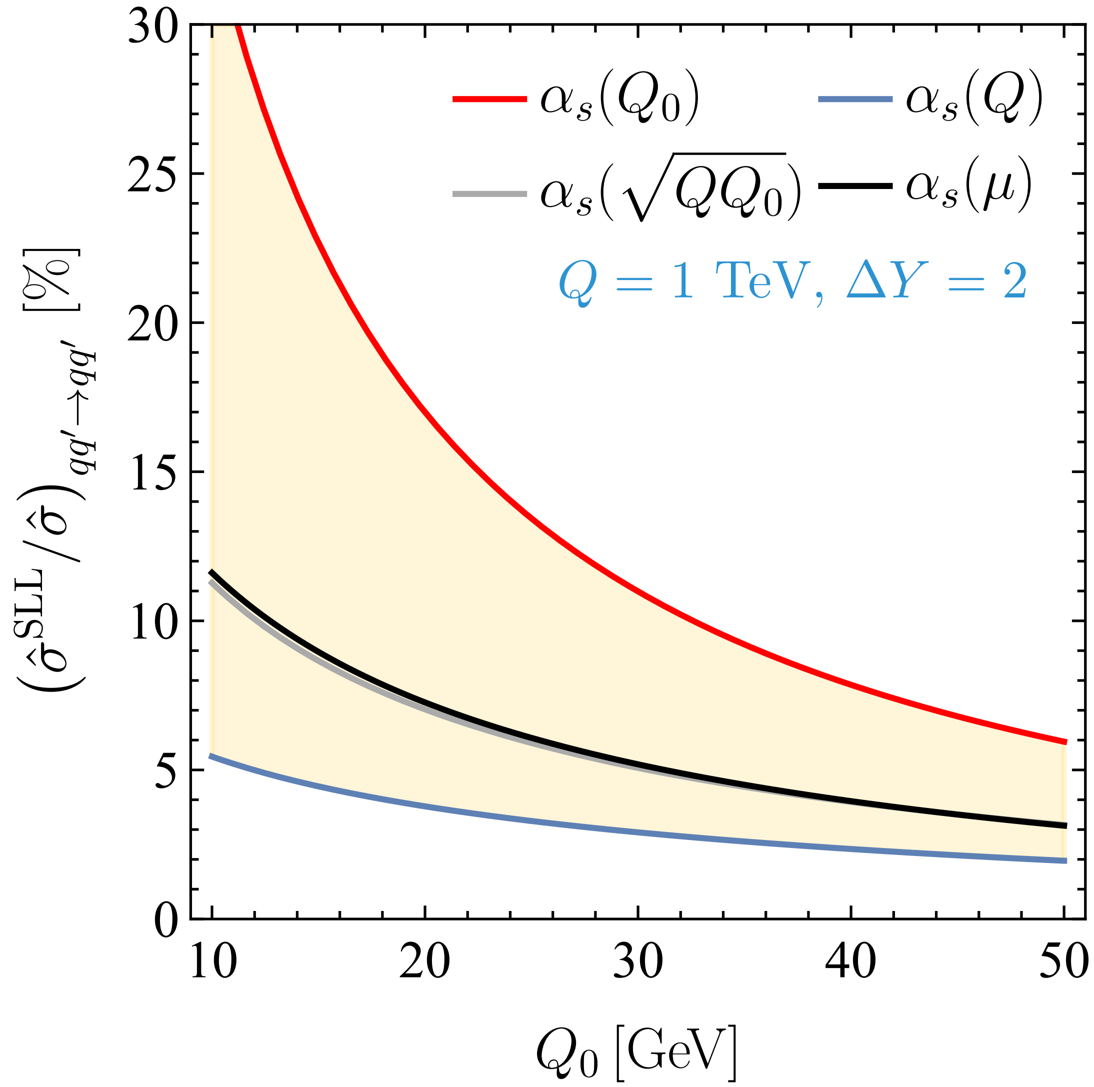

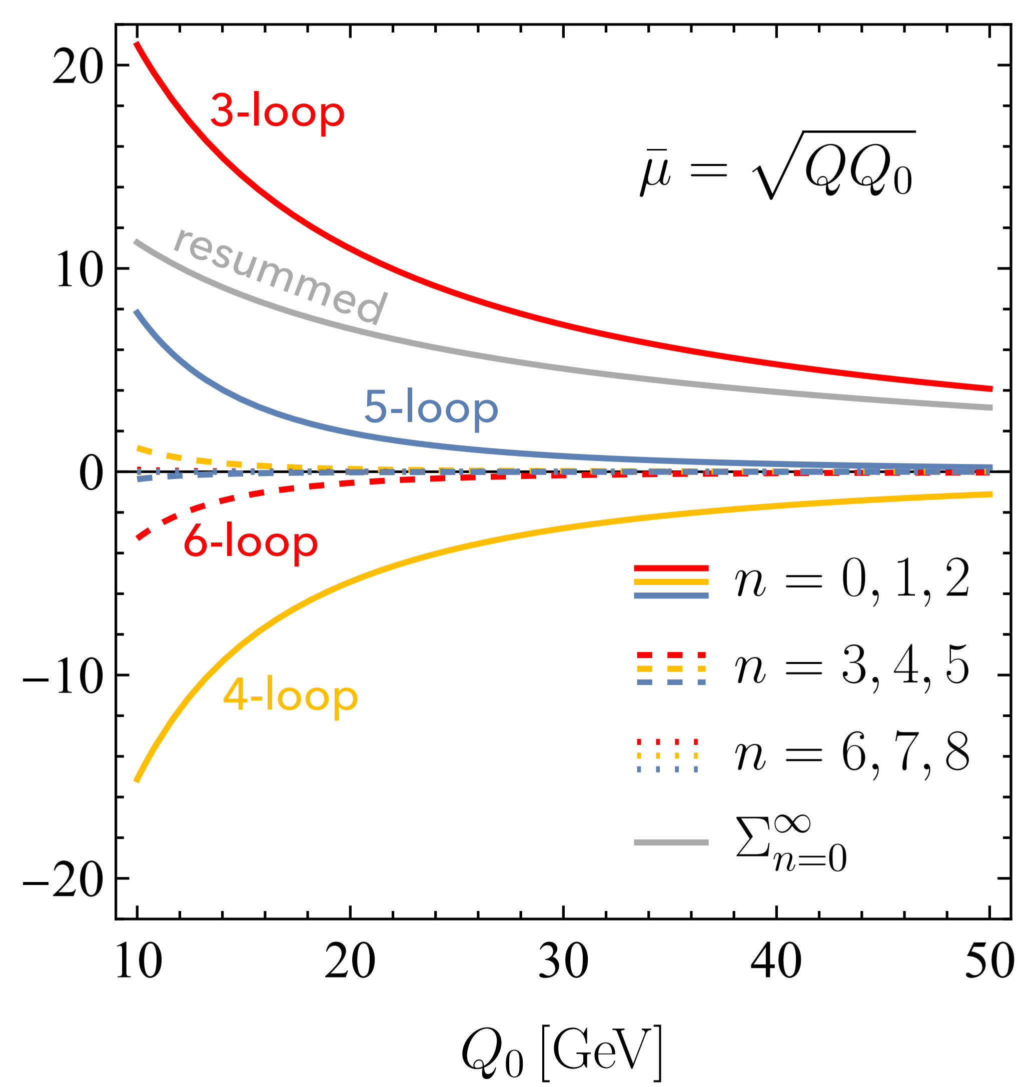

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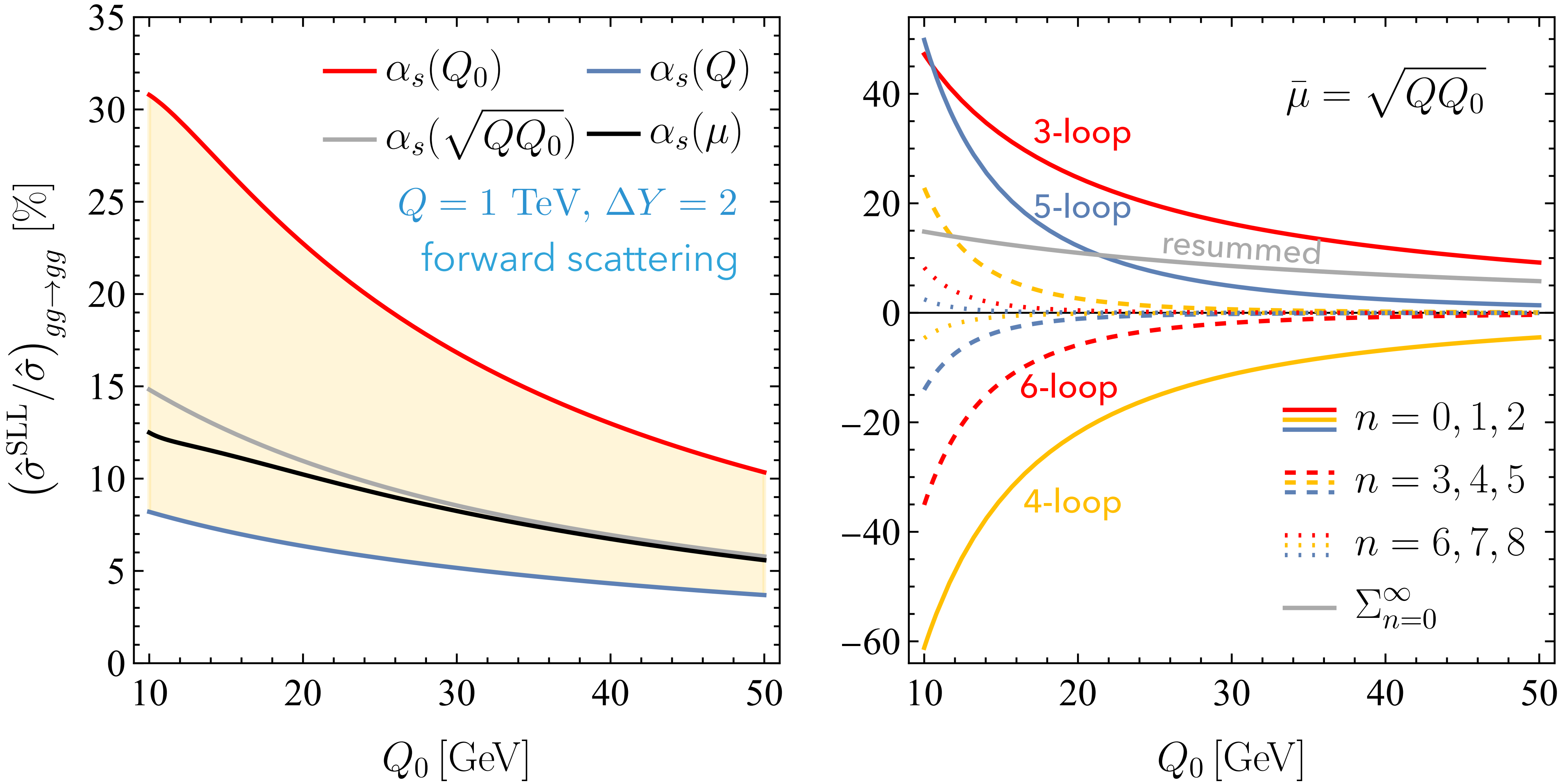

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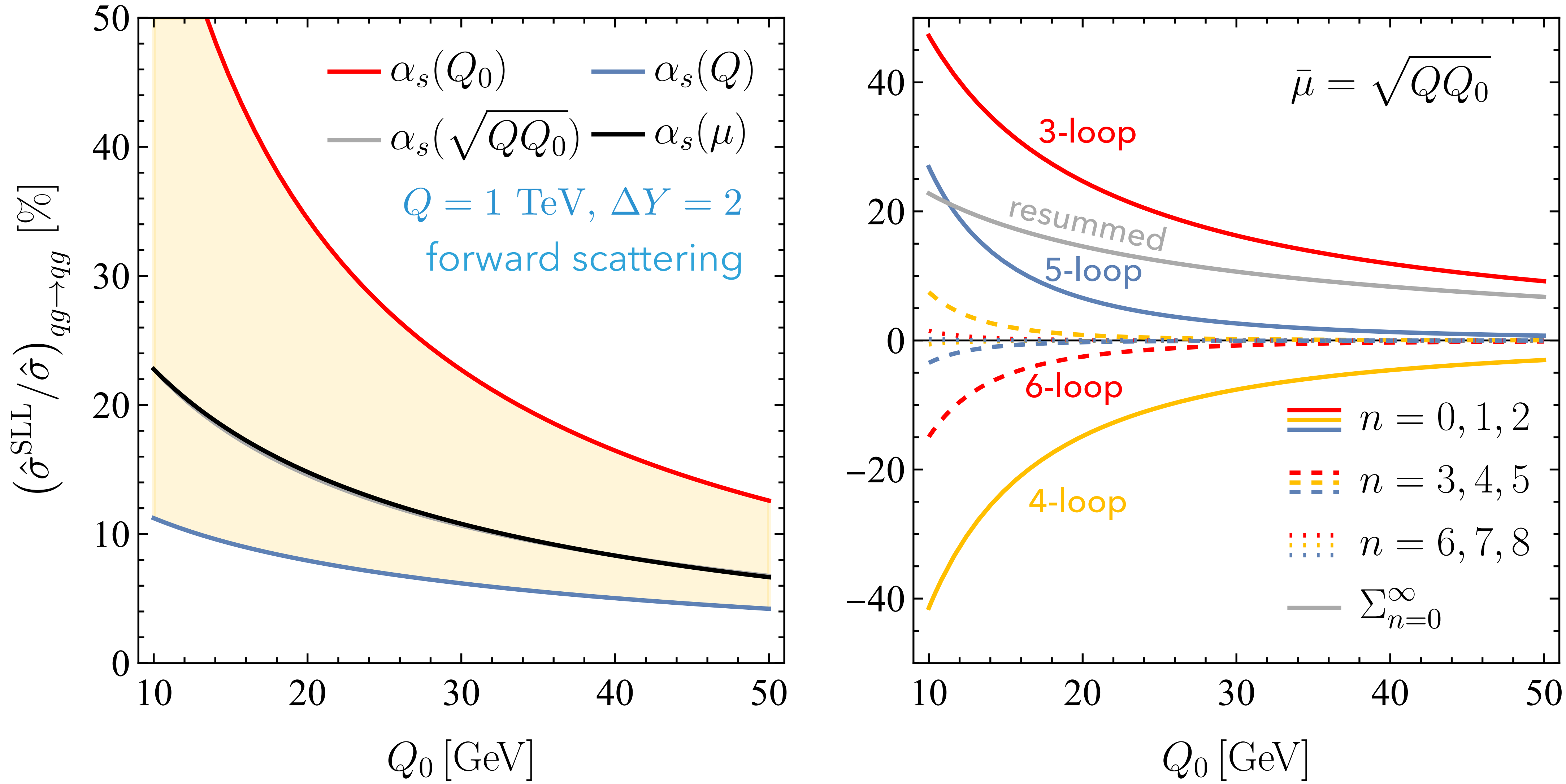

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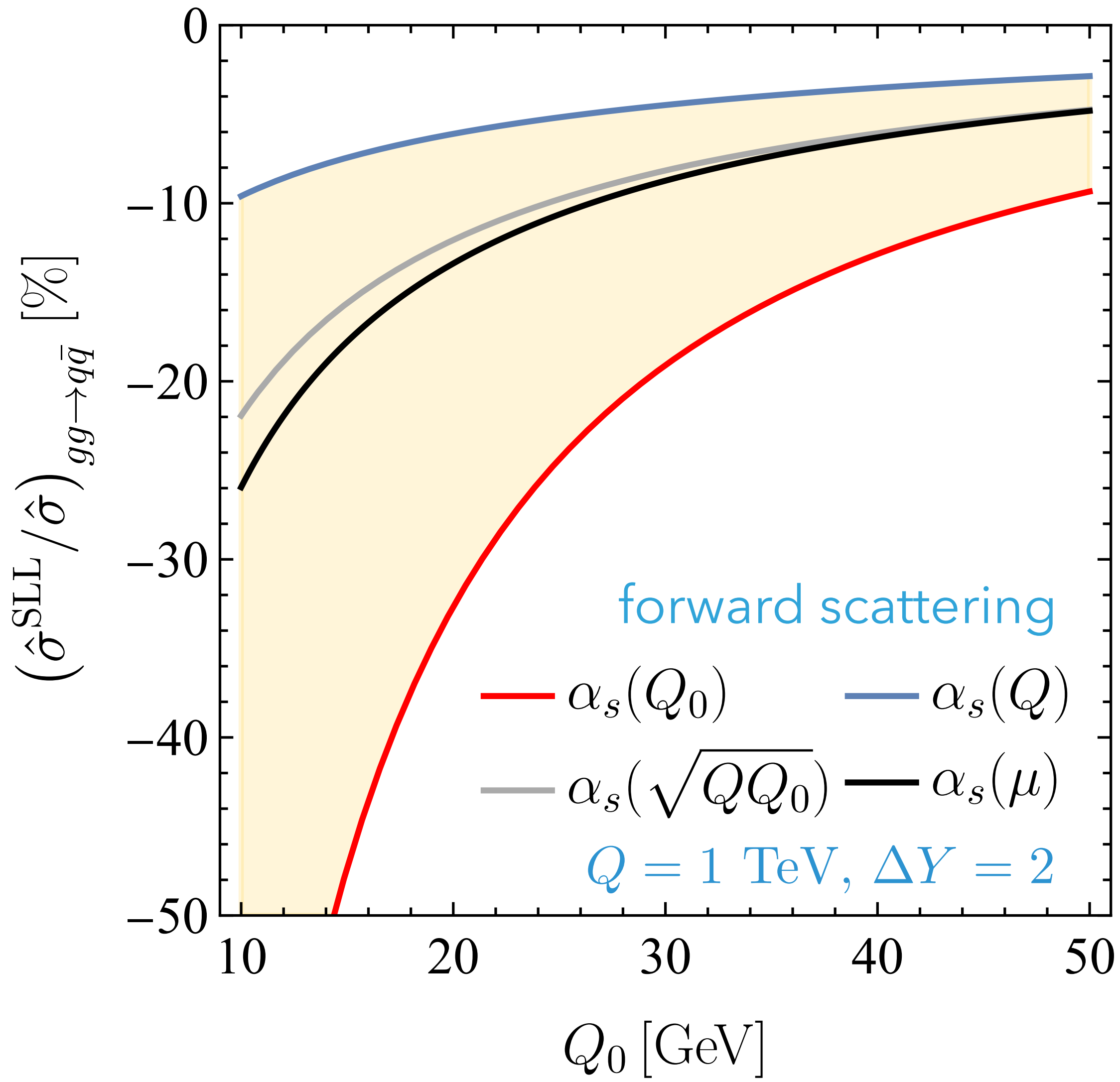

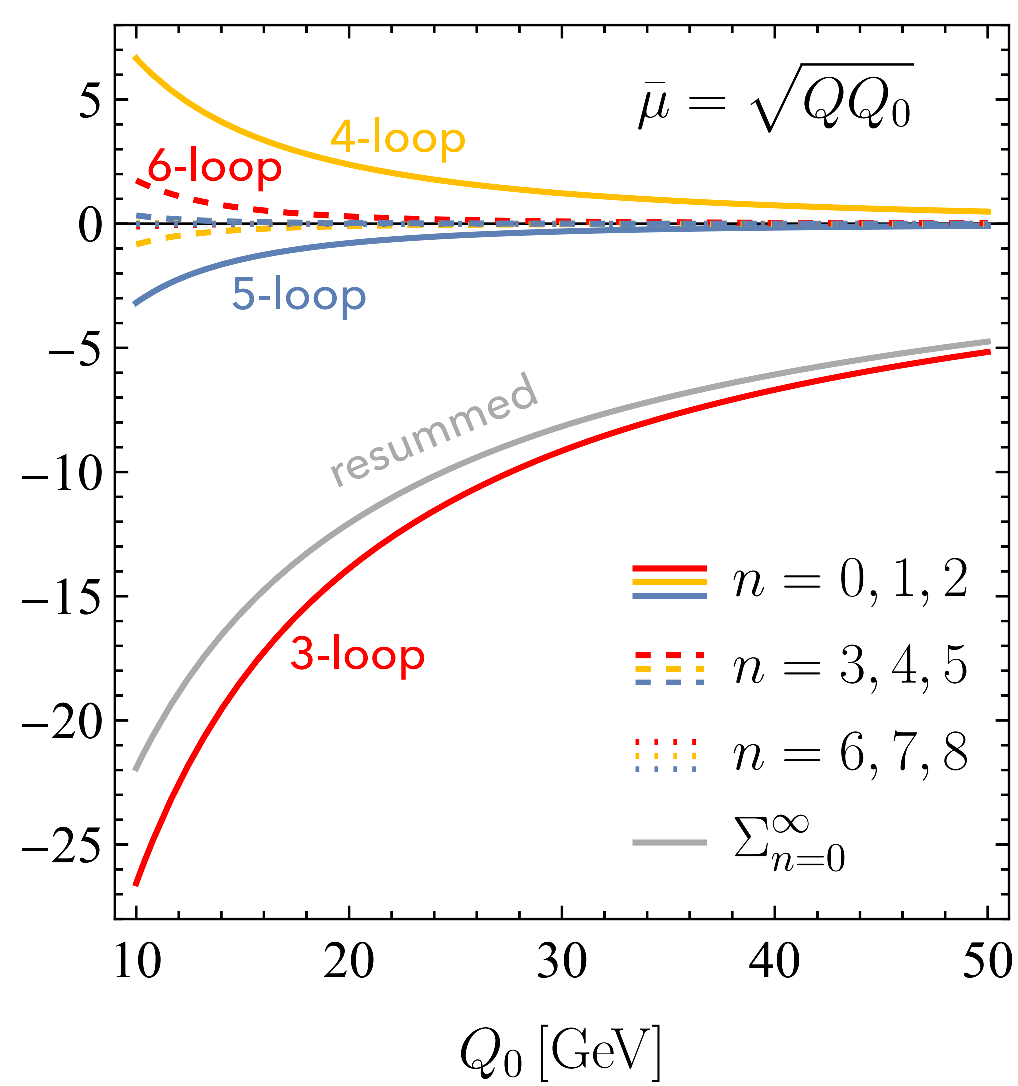

GLAUBER SERIES

Structure of the cross section

- We found:

$$\sigma \sim \sum_{n=0}^{\infty} \left[c_{0,n} \left(\frac{\alpha_s}{\pi} L \right)^n + c_{1,n} \left(\frac{\alpha_s}{\pi} L \right) \left(\frac{\alpha_s}{\pi} i\pi L \right)^2 \left(\frac{\alpha_s}{\pi} L^2 \right)^n + \dots \right]$$

- Introduce two $O(1)$ parameters:

$$w = \frac{N_c \alpha_s(\bar{\mu})}{\pi} L^2 \,, \qquad w_\pi = \frac{N_c \alpha_s(\bar{\mu})}{\pi} \pi^2$$

- Including multiple Glauber insertions:

$$\sigma^{\text{SLL+G}} \sim \frac{\alpha_s L}{\pi N_c} \sum_{\ell=1}^{\infty} \sum_{n=0}^{\infty} c_{\ell,n} \, w_\pi^\ell \, w^{n+\ell}$$

- Relevant color traces:

$$C^{\ell}_{\{\underline{r}\}} \equiv \langle \boldsymbol{\mathcal{H}}_{2\to M} \, (\boldsymbol{\Gamma}^c)^{r_1} \, \boldsymbol{V}^G \, (\boldsymbol{\Gamma}^c)^{r_2} \, \boldsymbol{V}^G \dots (\boldsymbol{\Gamma}^c)^{r_{2\ell-1}} \, \boldsymbol{V}^G \, (\boldsymbol{\Gamma}^c)^{r_{2\ell}} \, \boldsymbol{V}^G \, \overline{\boldsymbol{\Gamma}} \rangle$$

P. Böer, M. Neubert, M. Stillger (2023)

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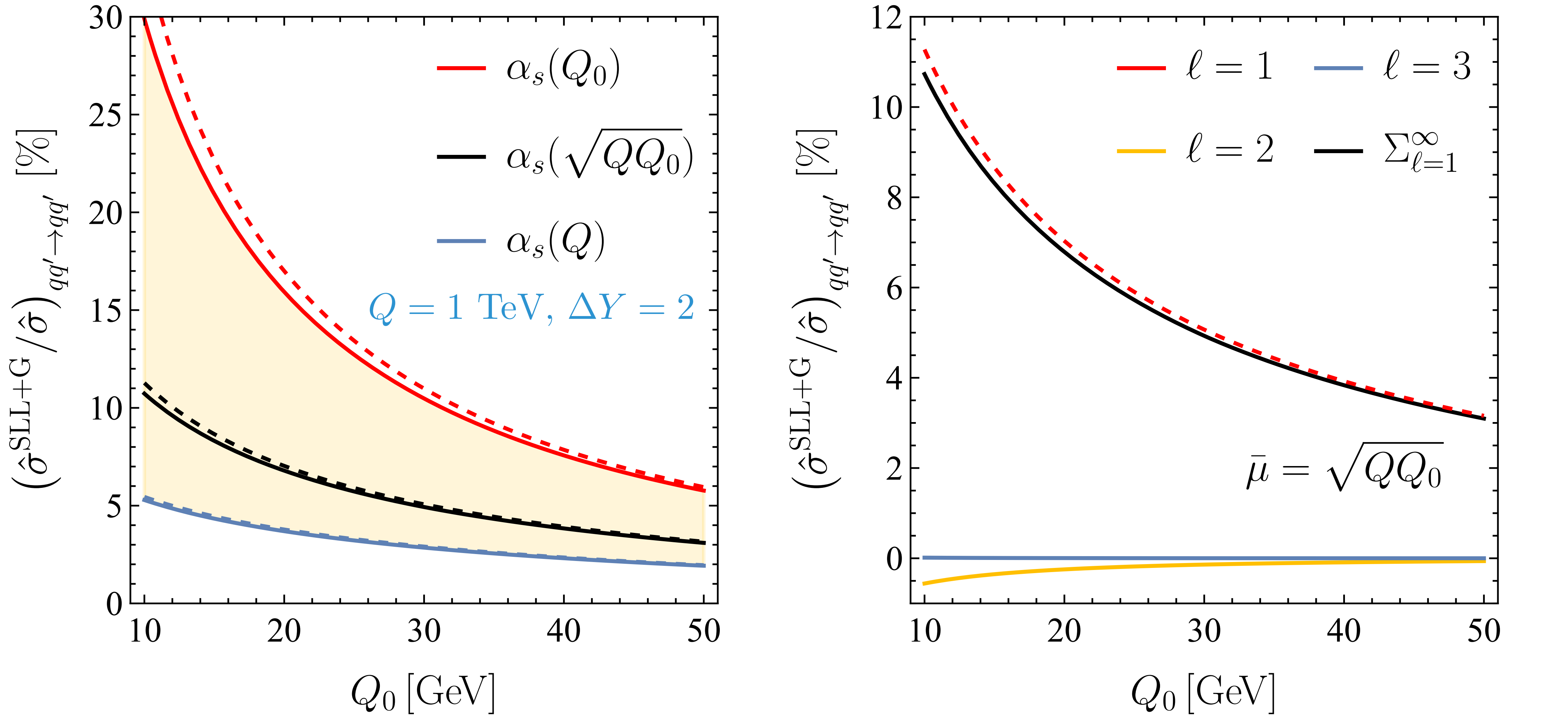
- These traces can be calculated for arbitrary exponents r_i in terms of **4** ($qq, \bar{q}\bar{q}, q\bar{q}$ scattering), **13** (gg scattering), and **11** ($qg, \bar{q}g$ scattering) **basis operators**, instead of 10 for $\ell = 1$ P. Böer, P. Hager, M. Neubert, M. Stillger, X. Xu (2023)

PHENOMENOLOGICAL IMPACT (PARTON LEVEL)

Impact on SLL resummation is small for quark-initiated processes …

P. Böer, M. Neubert, M. Stillger (2023)



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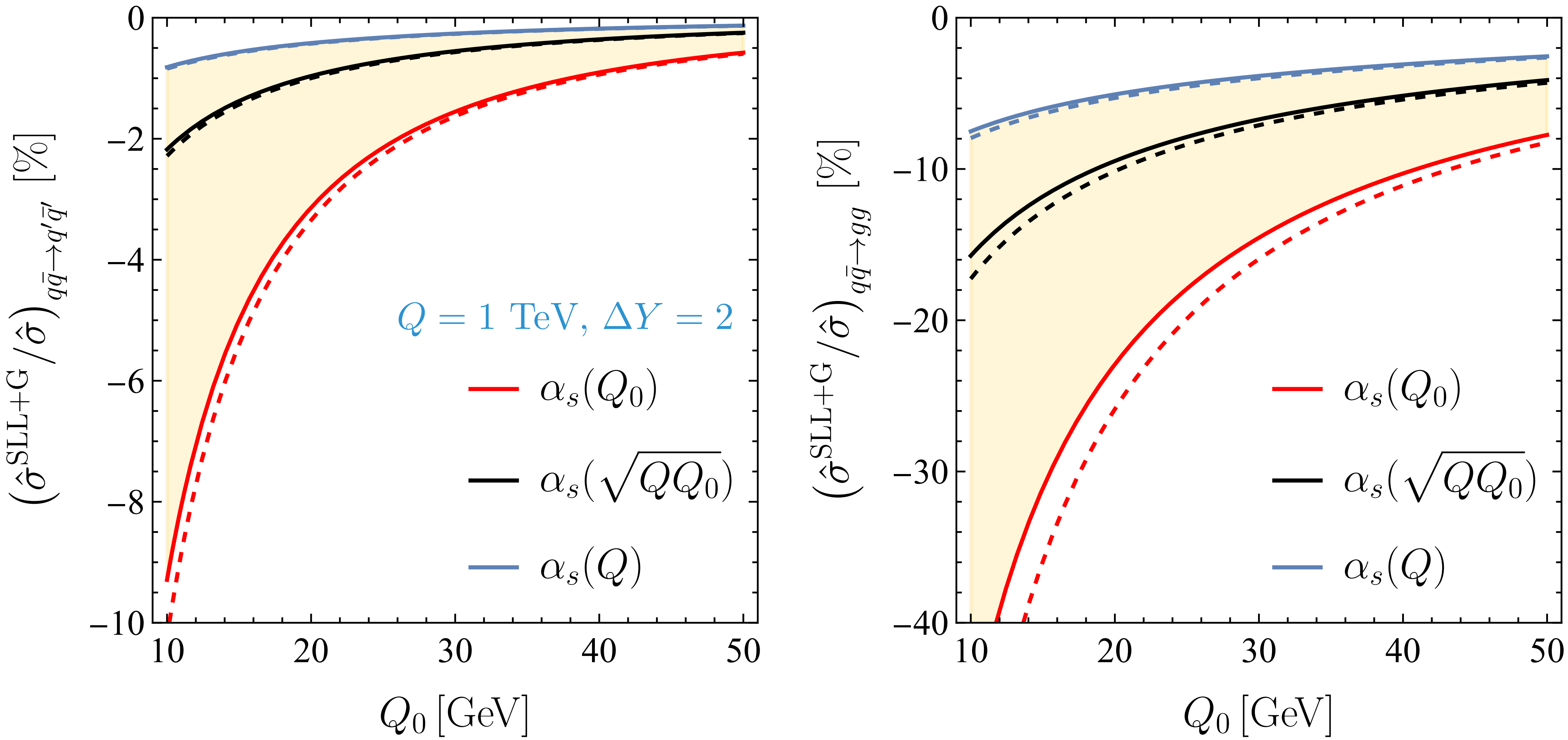

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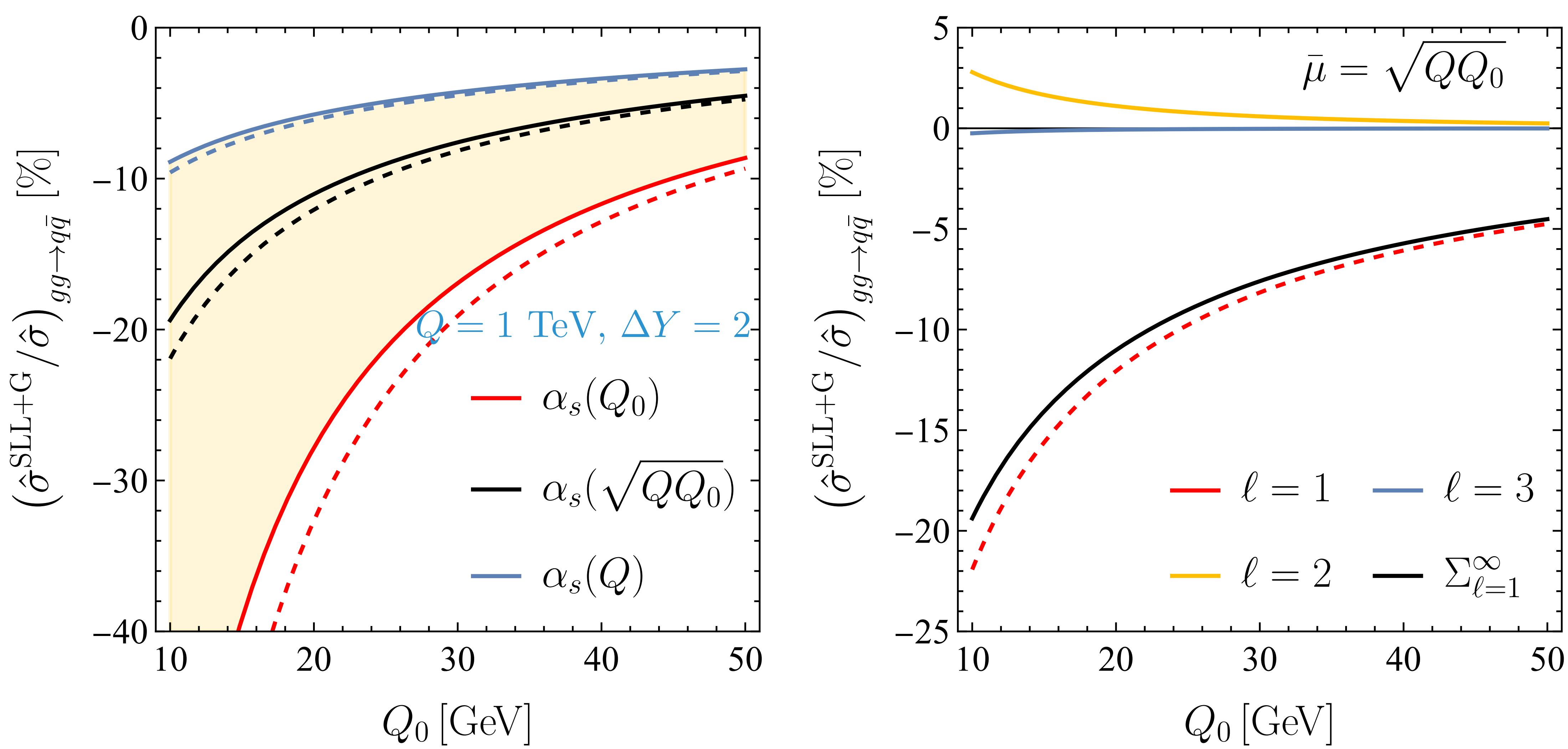

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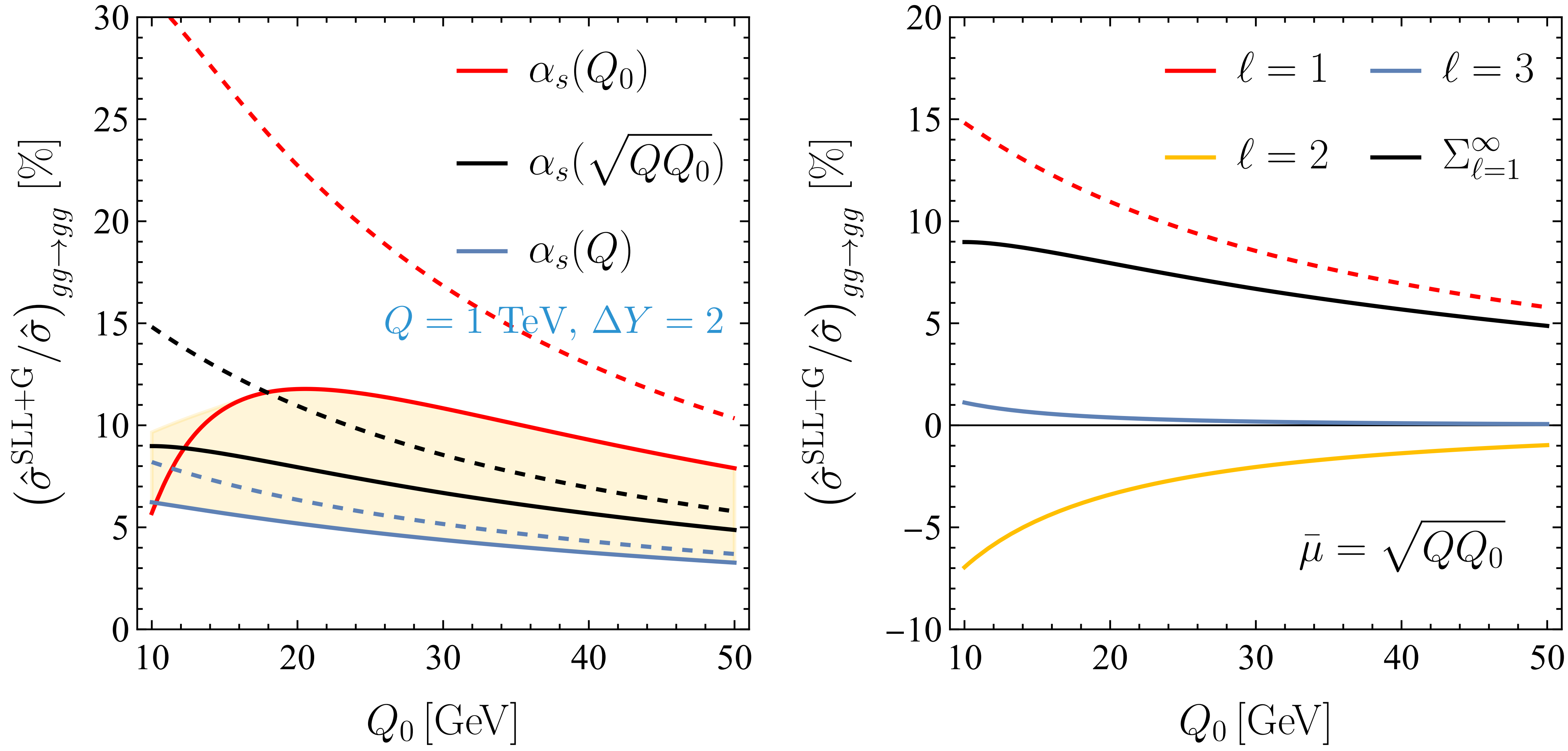

HIGHER-ORDER ASYMPTOTIC BEHAVIOR?

Standard Sudakov problems (with $\alpha_s L \sim 1$)

- RG-improved perturbation theory yields an expansion in exponent:

$$\begin{aligned}\sigma &\sim \sigma_0 \exp\left[-\frac{1}{\alpha_s(\mu_h)}\, g_0(x_s) + g_1(x_s) + \alpha_s(\mu_h)\, g_2(x_s) + \dots\right] \\ &= \sigma_0 \exp\left[-\frac{1}{\alpha_s(\mu_h)}\, g_0(x_s) + g_1(x_s)\right] \Big[1 + \alpha_s(\mu_h)\, g_2(x_s) + \dots\Big]\end{aligned}$$

where $x_s = \alpha_s(\mu_h)/\alpha_s(\mu_s)$

- Terms that are formally $\gg \mathcal{O}(1)$ are under control, whereas this is not the case for the perturbative expansion:

$$\sigma \sim \sigma_0 \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \sum_{n_3=0}^{\infty} \frac{1}{n_1!n_2!n_3!} \left[-\frac{g_0(x_s)}{\alpha_s(\mu_h)}\right]^{n_1} \left[g_1(x_s)\right]^{n_2} \left[\alpha_s(\mu_h)\, g_2(x_s)\right]^{n_3}$$

HIGHER-ORDER ASYMPTOTIC BEHAVIOR?

More complicated pattern for non-global observables (with $\alpha_s L \sim 1$)

- Resummation of SLLs $\sim (\alpha_s L)^3 (\alpha_s L^2)^n$ at fixed coupling yields:

$$\sigma \sim (\alpha_s L)^3\, \Sigma(v_i, w) \sim (\alpha_s L)^3\, \frac{\ln w}{w} \sim \alpha_s \ln \alpha_s \qquad (\text{for } i = 0)$$

  with $w = \dfrac{N_c \alpha_s}{\pi} L^2 \sim 1/\alpha_s$, which is not of exponential form

- Formally subleading-logs terms $\sim (\alpha_s L)^4 (\alpha_s L^2)^n$ sum of to:

$$\sigma \sim (\alpha_s L)^4\, G(w) \sim G(1/\alpha_s)$$

- **How do we know these terms are really subleading?**

HIGHER-ORDER ASYMPTOTIC BEHAVIOR?

Rewrite the evolution kernel for the SLLs

- Expand out all terms except the log-enhanced soft-collinear piece:

$$\mathbf{U}(\mu_h,\mu_s) = \mathbf{P}\exp\left(\int_{\mu_s}^{\mu_h}\frac{d\mu}{\mu}\,\mathbf{\Gamma}^H(\mu)\right)$$

$$\overset{\text{SLLs}}{=} \int_{\mu_2}^{\mu_h}\frac{d\mu_1}{\mu_1}\int_{\mu_3}^{\mu_h}\frac{d\mu_2}{\mu_2}\int_{\mu_s}^{\mu_h}\frac{d\mu_3}{\mu_3}\,\mathbf{U}_c(\mu_h,\mu_1)\,\mathbf{V}^G(\mu_1)\,\mathbf{U}_c(\mu_1,\mu_2)\,\mathbf{V}^G(\mu_2)\,\overline{\mathbf{\Gamma}}(\mu_3)$$

where:

$$\mathbf{U}_c(\mu_1,\mu_2) = \exp\left(\mathbf{\Gamma}^c\int_{\mu_2}^{\mu_1}\frac{d\mu}{\mu}\,\gamma_{\text{cusp}}(\mu)\,\ln\frac{\mu^2}{\mu_h^2}\right)$$

matrix in the space of basis operators

resums all double-logarithmic terms

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matrix in the space of basis operators

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For quark-initiated scattering:

$$\mathbf{\Gamma}^c = \begin{pmatrix} N_c & 0 & 0 & 0 & 0 \\ 0 & N_c & 0 & 0 & 0 \\ 0 & 0 & \frac{N_c}{2} & 0 & 0 \\ 0 & 0 & -N_c & N_c & 0 \\ 0 & 0 & -C_F N_c & 0 & 0 \end{pmatrix}$$

HIGHER-ORDER ASYMPTOTIC BEHAVIOR?

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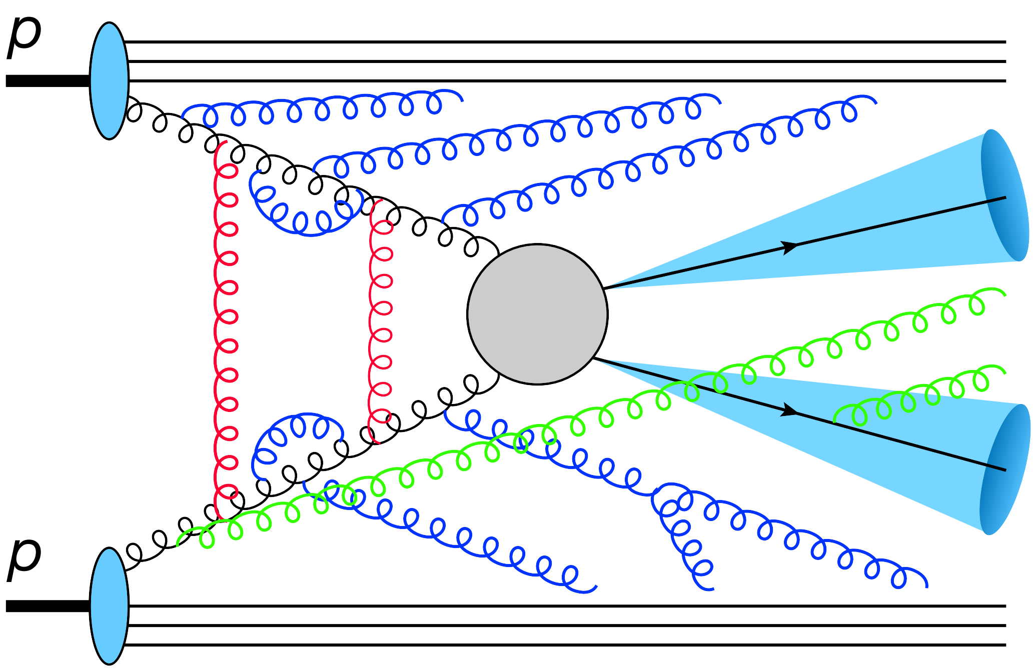
- Analogous relations hold for the Glauber series (more V^G factors) or other insertions of subleading parts of the anomalous dimension
- One scale integral for each insertion, suitable for numerical evaluation
- From asymptotic behavior of $\mathbf{U}_c(\mu_1,\mu_2)$ one can work out asymptotic behavior of the resummed series

EXPLORING UNCHARTERED TERRITORY

Important open questions

- Do the strong cancellations persist when subleading terms are included? How large is the remaining scale ambiguity?
- Can factorization violations be understood in a quantitative way? At what scale (Q_0 or $\Lambda_{\rm QCD}$) do they occur?
- What are the implications for LHC phenomenology?
- Results very relevant for future improvements of parton showers with quantum interference



Thank you!