An introduction to Strong Field QED

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Outline

1. Introduction
   - QED in EM backgrounds
   - Laser experiments
   - Vacuum instability
2. Field assisted processes
   - Nonlinear Compton
   - Nonlinear Breit-Wheeler
3. Higher order processes
   - Higher multiplicity
   - Higher loop order
   - Summary

Introduction

Background field QED

Familiar action for Dirac field $\Psi(x)$ coupled to gauge potential $\mathrm{A}(x) = A_\mu(x)\mathrm{dx}^\mu$:

$$S[\Psi, A] = \int d^D x \left[\frac{1}{4}\mathrm{tr}F^2 + \bar{\Psi}(i\not{D} - m)\Psi\right] \qquad (D_\mu := \partial_\mu + ieA_\mu)\,.$$

- Split gauge field into **background** + quantised photons,

$$eA_\mu(x) = a_\mu(x) + eA^\gamma_\mu(x)\,.$$

[**Sauter**: Z . Phys. 69 (1931) 742, **Heisenberg, Euler**: Z. Phys. 98 11-12 (1936) 714, **Schwinger**: Phys. Rev. 82 (1951) 664 ...]

- Laser background: high occupation coherent state $\Longrightarrow$ High intensity.

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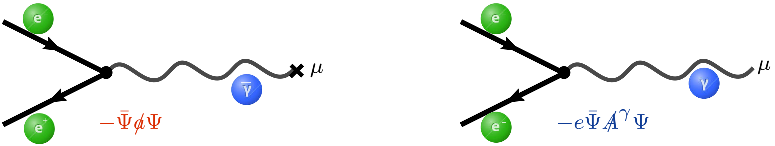
Aim: Quantise theory in the semi-classical background.

Two interactions between matter and photons:



Furry picture

- Laser interactions add coherently $\Longrightarrow$ background coupling $\alpha\rho_\gamma \gg 1$

  Non-perturbative QFT at low energies!

- In the Furry picture: background taken into account to all orders:

$$(i\partial\!\!\!/ - A\!\!\!/(x) - m)\Psi(x) = 0\,, \qquad \mathcal{S}(x', x) = \langle x' | i(\hat{p\!\!\!/} - A\!\!\!/ - m)^{-1} | x \rangle\,.$$

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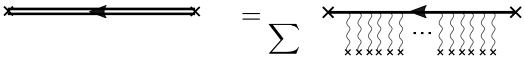

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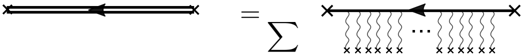
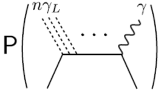


- Background enhances vertex coupling: $-ie\gamma^\mu \to -i\xi\gamma^\mu$ $\qquad (\xi \sim \sqrt{\alpha\rho_\gamma})$



$\sim \alpha\xi^{2n}$

- Note: still perturbative in α for quantised photons – **but see later**!

Nonlinearity parameters

Strong Field QED (**SFQED**) parameterised by two nonlinearity parameters:

- "Intensity parameter" – classical nonlinearity parameter, ξ or a_0:

$$\xi = \frac{eE\lambda_C}{\hbar\omega} \approx \frac{mc^2}{\hbar\omega}\frac{E}{E_{\rm cr}}$$

  Work done by background over 1 λ_C (units of photon energy $\hbar\omega$).

- Quantum nonlinearity parameter, χ:

$$\chi = \frac{eE\gamma\lambda_C}{mc^2} \approx \frac{E}{E_{\rm cr}}\Big|_{\rm Rest}$$

  Work done by background over 1 λ_C (units of electron rest mass energy).

Critical field strength: $E_{\rm cr} = \frac{m^2c^3}{e\hbar} \sim 1.3 \times 10^{18}$ V/m. [**Sauter, Heisenberg, Euler, Schwinger**]

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- $\xi \sim \mathcal{O}(1) \implies$ **high density** of interactions with background
- $\chi \sim \mathcal{O}(1) \implies$ **non-perturbative** quantum processes become likely

Plane waves – classical dynamics

Plane wave background – "good" approximation to field of intense laser.

- Plane wave characterised by null vector, n_μ, defining lightfront time:

$$x^+ = n \cdot x \,, \qquad n^2 = 0 \tag{1}$$

  Background gauge potential $a_\mu = a_\mu(x^+) = \delta_{\mu\perp} \int_{-\infty}^{x^+} d\varphi \, e\mathbf{E}_\perp(\varphi)$.

- Field invariants vanish: $\mathcal{F} = \frac{1}{4}\mathrm{tr}(f^2) = 0$ and $\mathcal{G} = \frac{1}{4}\mathrm{tr}(f \cdot \widetilde{f}) = 0$
  $\mathbf{E}^2 - \mathbf{B}^2 = 0$ $\qquad$ $\mathbf{E} \cdot \mathbf{B} = 0$

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- Classical dynamics – Lorentz force equation in lightfront time:

$$p_\mu(\varphi) = P_\mu - a_\mu(\varphi) + \frac{2P \cdot a(\varphi) - a^2(\varphi)}{2P^+} n_\mu \qquad p_\mu = m\dot{x}_\mu \,, \qquad p^2 = m^2 \,.$$

  $p^+ = p^+ = \mathrm{const}$ is conserved...

- *Canonical momentum* $\pi_\mu = p_\mu + a_\mu$:
  Three components π^+, $\pi^\perp$ conserved $\Longrightarrow$ integrable!

Plane waves – Volkov solutions

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- Plane wave characterised by null vector, n_μ, defining lightfront time:

$$x^+ = n\cdot x\,, \qquad n^2 = 0 \tag{1}$$

- Quantum dynamics – Volkov wavefunctions: [**Wolkow**: Z. Phys. 94 (1935) 250]

$$\Psi_{p,s}(x) = \Big(1 + \frac{\not n \not a(x^+)}{2p^+}\Big) u_s(p)\, \mathrm{e}^{-ip\cdot x - i\int_{-\infty}^{x^+} d\varphi\, \frac{2p\cdot a(\varphi) - a^2(\varphi)}{2p^+}}$$

(single particle wavefunctions)

- Quantum dynamics – Volkov propagator: $\mathcal{S}(x', x) = x'$  x

$$= i\int \frac{d^4p}{(2\pi)^4}\Big(1 + \frac{\not n \not a(x'^+)}{2p^+}\Big)\frac{\not p + m}{p^2 - m^2 + i\epsilon}\Big(1 + \frac{\not a(x^+)\not n}{2p^+}\Big)\, \mathrm{e}^{-ip\cdot(x'-x) - i\int_{x^+}^{x'^+} d\varphi\, \frac{2p\cdot a(\varphi) - a^2(\varphi)}{2p^+}}$$

- Early work with monochromatic plane waves misses non-trivial effects from beam profile: Finite duration / transverse structure etc.

High intensity laser facilities

Probes of SFQED – often scatter electrons off intense laser pulses (higher χ).

1. e^- beam sourced from ordinary accelerator: high precision measurements E144 at SLAC (46GeV); E320 at SLAC (13GeV); LUXE at DESY (16.5GeV).

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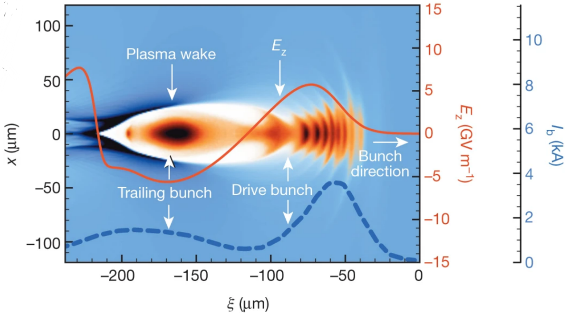
2. e^- beam from laser wakefield acceleration: facilities probing higher ξ CoRELS; ELI; Apollon; Vulcan; SEL; ZEUS; XCELS $\mathcal{O}(10s\,\text{MeV}) - \mathcal{O}(10s\,\text{GeV})$

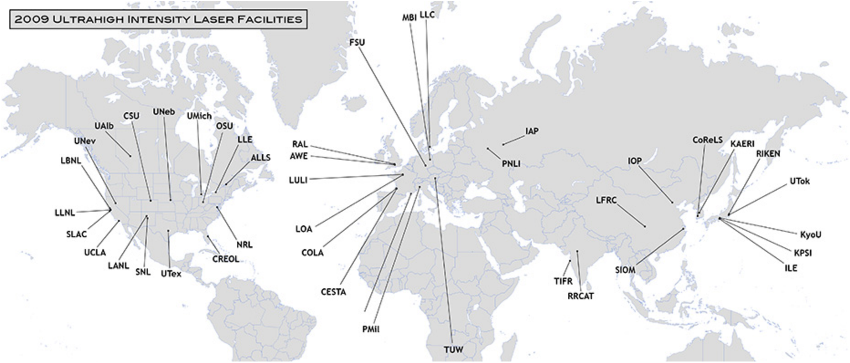


LWFA: Laser pulse initiates an electron plasma (ionisation) wave which produces "bubbles" with extremely high longitudinal electric fields ($c\tau < \lambda_p$). Electrons accelerated by "surfing" on the plasma wave.

[**Litos et al.:** Nature 515 (2014), 92]

Intense laser facilities – 2009

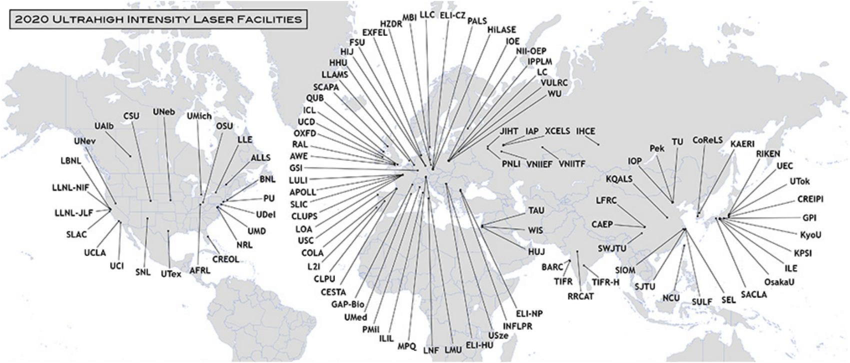
Significant growth in intense laser facilities (partly driven by CPA [Strickland, Mourou]).



[**ICUIL**: www.icuil.org]

Intense laser facilities – 2019

Significant growth in intense laser facilities (reaching multi-petawatt power).



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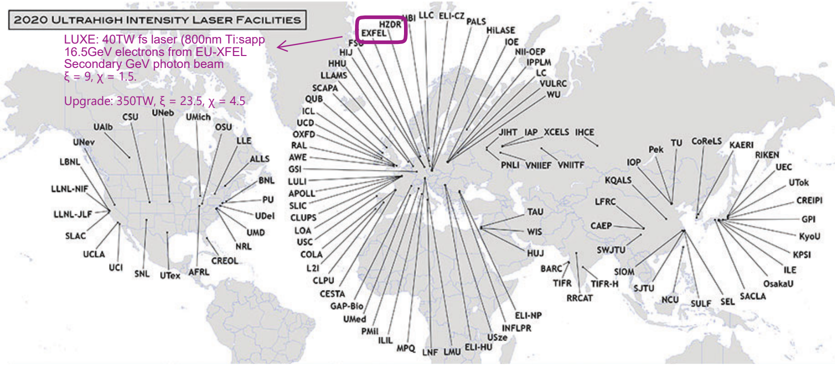

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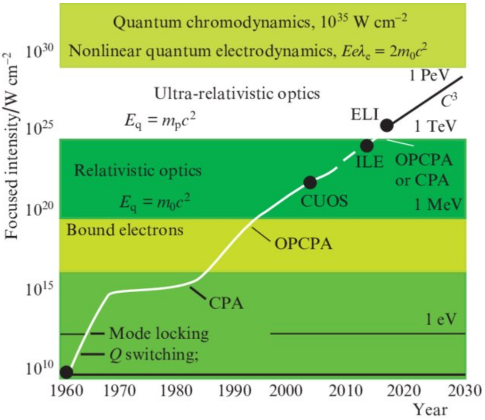

Evolution of laser intensity. Future experiment estimations included. [**Yakovlev:** Quant. Electron. 44 (2014), 393]

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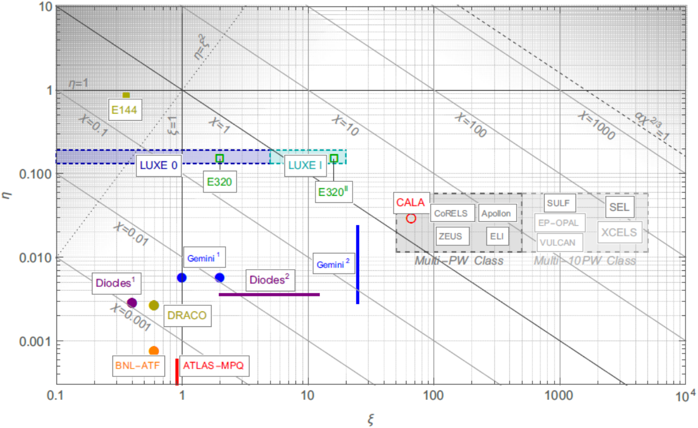
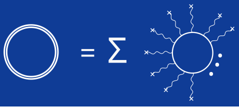


Laserscape for existing and upcoming laser-particle experiments.
For plane wave $\eta = \frac{\chi}{\xi}$ – CoM energy for photon-photon collision!

[**Fedotov et al.:** Phys. Rep. 1010 (2023), 1]

$ln\,\mathrm{Det}\,(i\not{D} - m) =$ 

Effective action

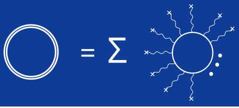
One loop effective action in arbitrary background:

$$e^{i\Gamma[a]} = \mathrm{Det}\left[i\not{\partial} - \not{a} - m\right] \tag{2}$$

- Vacuum bubbles contain field dependent physics:

$$_{\rm out}\langle\, 0 \,|\, 0 \,\rangle_{\rm in} = \int \mathscr{D}A \int \mathscr{D}\bar{\Psi} \int \mathscr{D}\Psi \, e^{\frac{i}{\hbar}S[A,\Psi]} \approx \mathrm{Det}\big(i\not{D} - m\big)\,.$$

Vacuum persistence probability $\left|_{\rm out}\langle 0|0\rangle_{\rm in}\right|^2 = e^{-2\Im\Gamma[a]} < 1 \implies$ vacuum decay!

ln Det $(i\not{D} - m) =$ 

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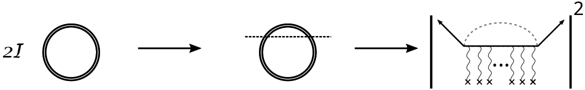
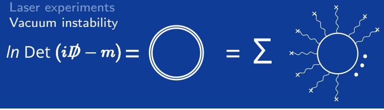
- Decay rate $\sim 2\Im\Gamma[a]$ related to pair creation by optical theorem: **[Sauter, Schwinger]**



This is the famous Schwinger mechanism.



Euler-Heisenberg Lagrangian

One loop effective action in arbitrary background:

$$\mathrm{e}^{i\Gamma[a]} = \frac{1}{2}\mathrm{Det}\big[(\partial - a)^2 + m^2 + \frac{i}{2} f_{\mu\nu}\sigma^{\mu\nu}\big] \tag{2}$$

- First quantised representation: [**Strassler**: Nucl. Phys. B385 (1992), 45, **Schubert**: Phys. Rept. 355 (2001), 73]

$$\Gamma[a] = -\frac{1}{2}\int_0^\infty \frac{dT}{T}\mathrm{e}^{-im^2T}\oint_{\mathrm{PBC}}\mathscr{D}x(\tau)\oint_{ABC}\mathscr{D}\psi(\tau)\,\mathrm{e}^{\frac{i}{\hbar}S_{\mathrm{WL}}[x,\psi]}\,.$$

Worldline action $S_{\mathrm{WL}}[x,\psi] = -\int_0^T d\tau\Big[\frac{\dot{x}^2}{4} + \frac{i}{2}\psi\cdot\dot{\psi} + a(x(\tau))\cdot\dot{x}(\tau) + i\psi(\tau)\cdot f(x(\tau))\cdot\psi(\tau)\Big]$

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- Constant EM fields – solution for effective Lagrangian:

$$\mathcal{L}^{(1)}_{\mathrm{EH}} = -2\int_0^\infty \frac{dT}{T}(4\pi iT)^{-\frac{D}{2}}e^{-im^2T}\det{}^{-\frac{1}{2}}\left[\frac{\tanh(fT)}{fT}\right]$$

[**Heisenberg, Euler**]

- Electric field – poles at $eET = (2n+1)\frac{\pi}{2}$ produce imaginary part

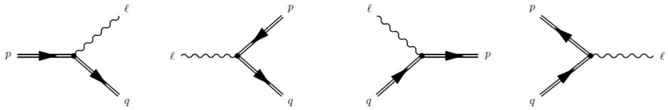
$$2\Im\mathcal{L}^{(1)}_{\mathrm{EH}} = \frac{(eE)^2}{(2\pi)^3}\sum_{n=1}^{\infty}\frac{1}{n^2}e^{-n\frac{\pi m^2}{eE}}\,.$$

[**Schwinger**, N.B. **Nikishov** Zh. Eksp. Teor. Fiz. 57 (1969) 1210]

Field assisted processes

Tree level scattering

First order processes (e.g. plane wave background) – one vertex:



Nonlinear Compton Nonlinear Breit-Wheeler Photon absorption Pair annihilation

Field induced: energy-momentum exchanged with background.

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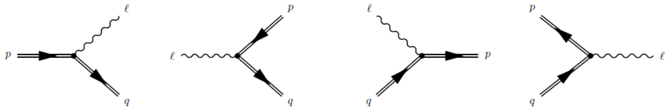

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Field induced: energy-momentum exchanged with background.

- S-matrix $S = -ie(2\pi)^3 \int du\, \delta^4(u\omega n - \sum_i p_i)\mathcal{M}(p_i, u)$
- For nonlinear Compton, $u = \frac{\ell \cdot p}{\omega q^+}$: [**E-144**: Phys. Rev. Lett. 76 (1996), 3116]
  Feynman Rules:

$$\mathcal{M} = \int d^4x\, \bar{\Psi}_q(x)\, \not{\epsilon}(\ell) e^{i\ell\cdot x}\, \Psi_p(x) \longrightarrow \int d\varphi\, \bar{u}(q(\varphi))\, \not{\epsilon}(\ell)\, u(p(\varphi))\, e^{\frac{i}{\hbar}\frac{\ell\cdot p(\varphi)}{\omega q^+}}\,.$$

  Frequency of outgoing photon: $\omega'(\ell) = \frac{\omega u p^+}{(p+\omega u n)\cdot\frac{\ell}{\ell^0}}$.

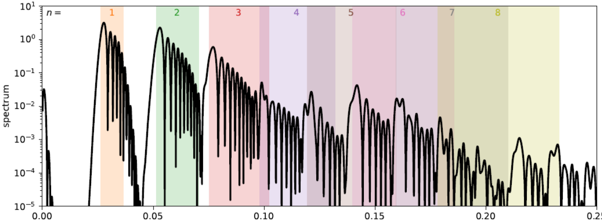
Nonlinear Compton scattering

Differential cross section for $e^-(p) \to e^-(q) + \gamma(\ell)$

$$\frac{d^3\sigma}{ds\, d^2r_\perp} = \frac{\alpha}{16\pi^2 m^2 \eta^2} \frac{s}{1-s} |\mathcal{M}|^2, \qquad s = \frac{q^+}{p^+}, \quad \mathbf{r}_\perp = \frac{\mathbf{q}_\perp}{ms}.$$

- For $\xi \gg 1$, scattering rate has distinct scalings: [**Nikishov, Ritus**: Sov. Phys. JETP 19 (1964) 529]

$$\sigma/T \sim \begin{cases} \alpha\chi & \text{for } \chi \ll 1 \\ \alpha\chi^{\frac{2}{3}} & \text{for } \chi \gg 1 \end{cases} \tag{3}$$



Differential spectrum, $\frac{d\sigma}{ds}$, as a function of lightfront momentum transfer, s, $(\mathbf{r}_\perp = (0.5, 0))$ with multiple Compton edges. [**Fedotov et al.**]

Nonlinear Breit-Wheeler

Field assisted pair production: $\gamma(\ell) \to e^{+}(p) + e^{-}(q)$ (from NLC by crossing)

$$\mathcal{M} = -\int d\varphi \, \bar{u}(q(\varphi)) \, \not{\epsilon}(\ell) \, \nu(-p(\varphi)) \, \mathrm{e}^{-i\frac{\ell\cdot(-p)(\varphi)}{\omega q^{+}}}, \qquad s = \frac{p^{+}}{\ell^{+}}$$

Threshhold energy: $\ell + u\omega n = p + q \Longrightarrow u > \frac{2}{\eta}$. [**E-144**: Phys. Rev. Lett. 79 (1997), 1626]

- Linear regime: $\xi \ll 1 \Longrightarrow$ 1 laser photon
- Multi-photon regime: $\xi \sim 1 \Longrightarrow n > 1$ laser photons
- Non-perturbative regime: $\xi \gg 1 \Longrightarrow$ many laser photons

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Production rate in intense plane wave: [**Nikishov, Ritus**: Sov. Phys. JETP 19 (1964) 529]

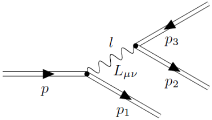
$$\sigma/T \sim \begin{cases} \alpha \mathrm{e}^{-\frac{8}{3\chi}} & \chi \ll 1 \\ \alpha \chi^{\frac{2}{3}} & \chi \gg 1 \end{cases} \tag{4}$$

Finite pulse effects can have significant impact on pair creation rate.

Higher order processes

Trident process

Nonlinear trident is the $1 \to 3$ process $e^-(p) \to e^-(p_1) + e^-(p_2) + e^+(p_3)$:



Internal photon conveniently taken in lightfront gauge:

$$D_{\mu\nu}(\ell) = \frac{-iL_{\mu\nu}(\ell)}{\ell^2 + i\epsilon}\,, \qquad L_{\mu\nu}(\ell) = g_{\mu\nu} - \frac{n_\mu \ell_\nu + \ell_\mu n_\nu}{\ell^+}\,.$$

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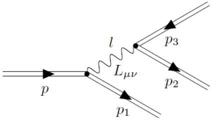

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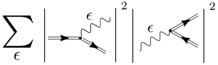
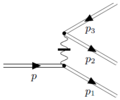
On-shell / Off-shell split $L_{\mu\nu}(\ell) = L_{\mu\nu}(\ell^\star) - \frac{\ell^2}{\ell^+} n_\mu n_\nu$:

- $L_{\mu\nu}(\ell^\star) \implies$ Causal "2-step process"



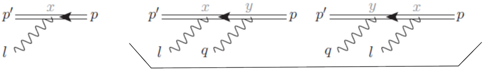
- $\frac{\ell^2}{\ell^+} n_\mu n_\nu$ Contact "1-step process"



[**Ilderton**: Phys. Rev. Lett. 106 (2011) 020404]

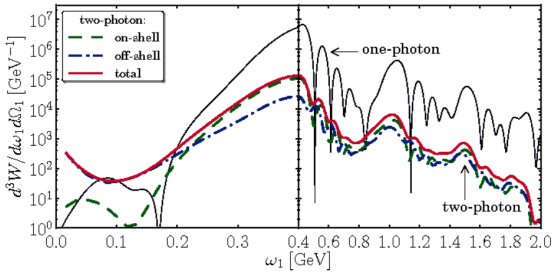
Double nonlinear Compton

Taken as a $1 \to 3$ process: $e^-(p) \to e^-(p_1) + \gamma(p_2) + \gamma(p_3)$



Lightfront integrals carried out numerically for plane wave background:

$\xi \sim 1$ [**Seipt, Kämpfer**: Phys. Rev. D 85 (2012) 101701], $\xi \gg 1$ [**Mackenroth, Di Piazza**: Phys. Rev. Lett. 110 (7) (2013) 070402]



Differential cross section for single & double NLC at fixed emitted photon frequency. [**Seipt, Kämpfer**]

Open worldlines

Higher multiplicity amplitudes involve multiple highly oscillatory integrals

- Locally Constant Field Approximation (LCFA): [**Ilderton, King, Seipt**: Phys. Rev. A 99 (4) (2019) 042121]
  Integrate rate in crossed constant fields (relativistic electron sees this!).
- Locally Monochromatic Field Approximation (LMA): [**Heinzl, King, Macleod**: Phys. Rev. A 102 (2020) 063110]
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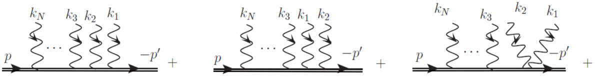
Worldline formalism extended: open lines $\Longrightarrow$ N-photon tree level amplitudes



$$\mathcal{D}_N \sim \int_0^\infty dT\,\mathrm{e}^{-im^2T}\int \mathcal{D}x(\tau)\,V(k_1,\varepsilon_1)\ldots V(k_N,\varepsilon_N)\,\mathrm{e}^{\frac{i}{\hbar}\int_0^T d\tau[-\frac{\dot{x}^2}{4}-a(x)\cdot\dot{x}(\tau)]} \quad (5)$$

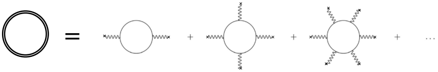
Photon vertex operator: $V(k,\varepsilon) = \int_0^T d\tau\,\varepsilon\cdot\dot{x}\,\mathrm{e}^{ik\cdot x(\tau)}$.

[**Ahmadiniaz, JPE et al.**: JHEP 08 (2020) 08, 049 & JHEP 01 (2022) 050]

[**Ahmad et al.**: Nucl.Phys.B 919 (2017) 9-24, **Copinger, JPE et al.**: 2311.14638 [hep-th]]

Vacuum polarisation

Loop effects encode non-trivial quantum vacuum: quantum fluctuations.



Photon polarisation tensor, $\Pi^{\mu\nu}(a) \Longrightarrow$ response of probe photon: [**Toll**: Thesis (1952)]

$$\Pi^{\mu\nu}(\ell,\ell'|a) \sim \frac{\delta^2\Gamma[a+A^\gamma]}{\delta A^\gamma_\mu(\ell)\delta A^\gamma_\nu(\ell')}\Big|_{A^\gamma=0} \equiv$$

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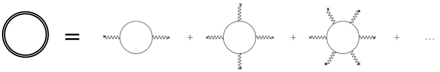

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  Linear polarisation $n_{1,2}(\ell) \sim 1 - 16\pi^2\alpha^2 c_{1,2} \frac{(\ell \cdot f)^2}{\ell^2 m^4} + \ldots$.
  [**Dinu et al.**: Phys. Rev. D 89 (12) (2014) 125003, **Macleod, JPE et al.**: New J.Phys. 25 (2023) 9, 093002]
- Helicity flip: genuine elastic scattering off background field.
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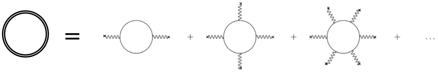

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Lab experiments: PVLAS close to QED sensitivity [**Della Valle et al.**: Phys. Rept. 871 1–74]
HIBEF [**Ahmadiniaz et al.**: Phys. Rev. D 104 (2021), L011902]
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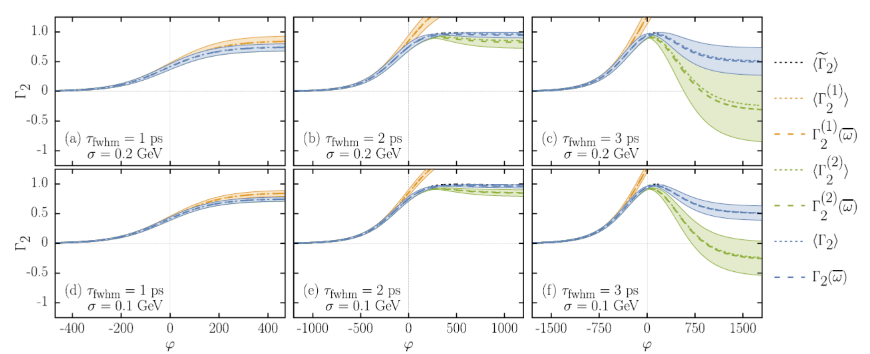

Figure: Stokes parameter evolution encoding polarisation asymmetry (multiple scattering expected if mean free path of the probe photon $\ll$ laser pulse length).

[**Macleod, JPE et al.**: New J.Phys. 25 (2023) 9, 093002]

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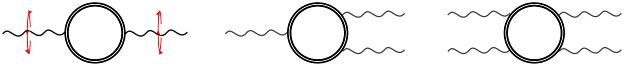
Light-by-light

External fields allow additional processes mediated by virtual loops:



- $1 \to 1$ scattering: elastic & inelastic [**E320** and **LUXE**: Eur. Phys. J. ST 230 (2021), 2445]
- Photon splitting in strong magnetic fields

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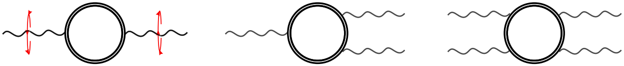

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Low energy limit of four-photon amplitude more accessible:

1. Lowest order contribution to vacuum polarisation!

   [**Ahmadiniaz et al.**: Phys. Rev. D 108 (2023) 7, 076005 and **Ahmadiniaz et al.**: Nucl.Phys.B 991 (2023) 116216]
2. Partially off-shell process detected at ATLAS

   [**Aaboud et al.**: Nature Phys. 13 (9) (2017) 852, **Aad, et al.**: Phys. Rev. Lett. 123 (5) (2019) 052001]
3. Contributes to g-2 at $\mathfrak{L} \geqslant 3$ loop order

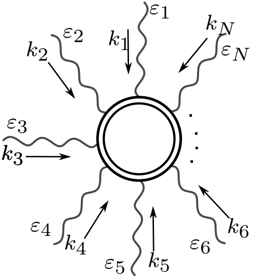
Closed worldlines

First quantised approach also well-suited to calculating loop corrections:

$$\Gamma[a] = -\frac{1}{2}\int_0^\infty \frac{dT}{T} \mathrm{e}^{-im^2T} \oint_{\mathrm{PBC}} \mathscr{D}x(\tau) \oint_{ABC} \mathscr{D}\psi(\tau)\, \mathrm{e}^{\frac{i}{\hbar} S_{\mathrm{WL}}[x,\psi]} + \text{Internal Photons}\,.$$

- 1-loop effective action calculated in constant field [**Schmidt, Schubert**: Phys. Lett. B 318 (1993), 438]
- Master Formulae for N-photon scattering [**Reuter et al.**:, Ann. Phys. (N.Y.) 259 (1997), 313]
- Generalised to plane wave background only recently

[**JPE, Schubert**: Phys. Lett. B 822 (2021), 136696 and **Schubert, Shaisultanov:** Phys. Lett. B 843 (2023), 137969]



Euler-Heisenberg

Resurgence in higher-loop calculations of Euler-Heisenberg Lagrangian:

$$\Gamma_{\mathrm{EH}}[a] = \int d^D x \, \mathcal{L}_{\mathrm{EH}}(f^2, \tilde{f}^2, \partial^2 f^2, \ldots) \, .$$

- For $D = 4$: **Two-loops** with **zero derivatives** [**Ritus**: Sov. Phys. JETP 42 (5) (1975) 774]
  Known to **one-loop** at **quadratic in derivatives**

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  [**Huet et al.**: Adv. High Energy Phys. 2017 (2017) 6214341, **Dunne, Schubert**: Phys. Lett. B526 (2002), 55]
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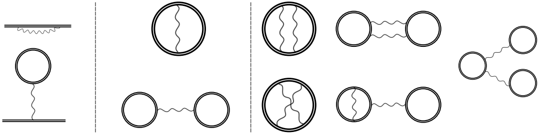
Recent discovery: 1PR contributions to $\mathcal{L}_{\mathrm{EH}}$ in constant fields...



[**Gies, Karbstein**: JHEP 03 (2016)]

[**Karbstein**: JHEP 10 (2017) 075]

[**JPE, Schubert**: Nucl. Phys. B 923 (2017) 339]

[**Ahmadiniaz, JPE et al.**: Nucl. Phys. B 924 (2017) 377]

[**Huet, JPE et al.**: Nucl. Phys. B935 (2018) 19]

- 1PR contributions found to be **dominant** in strong field limit!

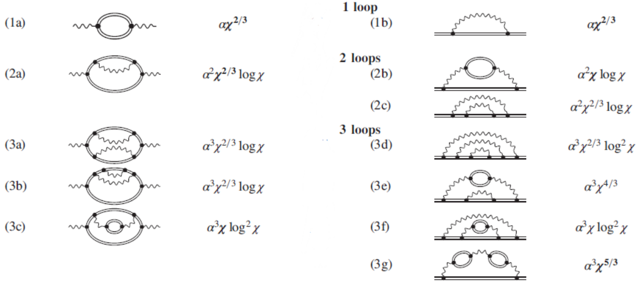
  [**Karbstein**: Phys. Rev. Lett. 122 (21) (2019) 211602]

Ritus-Narozhny Conjecture

Large ξ, χ limits of NLC & NLBW **rates** scale as $\alpha\chi^{\frac{2}{3}}$.

- General conjecture for crossed constant fields (LCFA):
  $\alpha \to \alpha\chi^{\frac{2}{3}} \implies$ Breakdown of strong field Furry expansion for large $\chi \gtrsim 1600$.

[**Ritus**: Ann. Phys. 69 (2) (1972) 555, **Narozhny**: Phys. Rev. D21 (4) (1980) 1176]



Asymptotic results in CCF (1969-1980). From [**Mironov et al.**: Phys. Rev. D 102 (2020), 053005]

- Already known not to hold in some processes away from CCF limit.

[**Podszus, Di Piazza**: Phys. Rev. D 99 (2019), 076004, **Ilderton**: Phys. Rev. D 100 (2019), 125018]

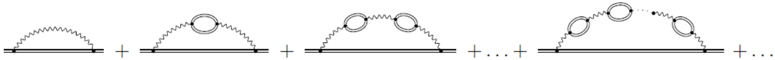
- Even in CCF – not seen for **inclusive** observables [**JPE**, **Ilderton**: Phys. Rev. D 103 (2021) 1, 01600]

Resummation

State of the art: all-orders resummation of vac. pol. "bubble diagrams"

[**Mironov et al.** Phys. Rev. D 102 (2020), 053005]]

- Mass operator for on-shell incoming particle (maximal saturation)



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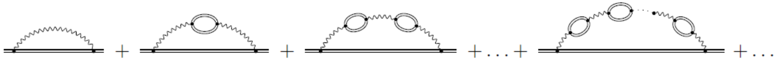

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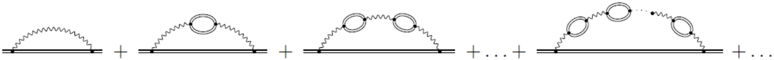

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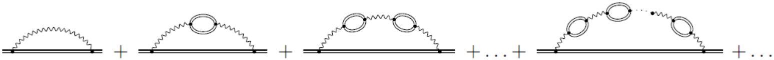

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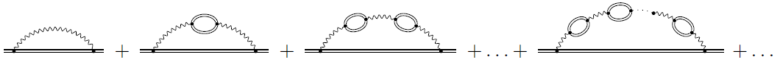

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- Resummed result in non-pertrubative limit: dominant contributions scale as $\chi^{-\frac{1}{3}}(\alpha\chi^{\frac{2}{3}})^{\frac{3}{2}}$ and $\chi^{-\frac{1}{3}}(\alpha\chi^{\frac{2}{3}})^{2}$

Recent work validating treatment of vertex provided at one-loop order.

[**Di Piazza, Lopez-Lopez**: Phys. Rev. D 102 (7) (2020) 076018]

Conclusion

Strong field QED – an open window onto **nonlinear** effects in QED!

Non-perturbative aspects of QED can be probed at relatively low energies by *enhancing* vacuum coupling.

Main points:

1. Low order processes – already part of experimental searches
2. Higher multiplicity processes – relevant for cascades
3. Loop effects – expose nonlinear nature of quantum vacuum
4. Vacuum birefringence – target of upcoming laser experiments

An invitation:

Worldline Formalism – especially useful for studying higher order processes.

[JPE, Schubert: arXiv:1912.10004 [hep-th]]

Not mentioned

Many other interesting aspects of this field:

- Details beyond plane waves: non-null fields, focussing etc
- Numerical simulations: Particle in Cell codes
- Semi-classical approaches (worldline instantons)
  Very useful for studying the effective action!

My key question:

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¡Thank you for your attention!

Additional Slides

CPA

Chirped Pulse Amplification – 2018 Nobel Prize

[**Strickland, Mourou**: Opt Commun. 56 (1985), 219.]

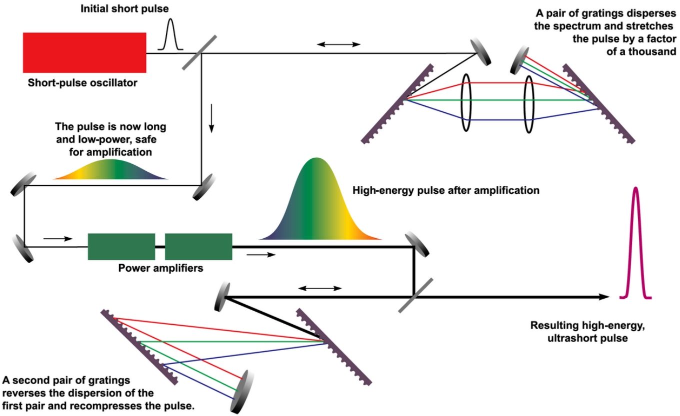

Figure: Schematic of CPA: ultra-short laser pulse amplified to PW level [Science and Technology Review: LLNL, Sept 1985]

Worldline Master Formulae

All multiplicity results obtained for tree level amplitudes:

- Master formulae obtained in constant fields: [**Ahmad et al.**: Nucl.Phys.B 919 (2017) 9-24]

$$D^{pp'}(F \mid k_1, \varepsilon_1; \cdots; k_N, \varepsilon_N) = (-ie)^N (2\pi)^D \delta\Big(p + p' + \sum_{i=1}^N k_i\Big) \int_0^\infty dT\, \mathrm{e}^{-m^2 T} \frac{1}{\det^{\frac{1}{2}}[\cos\mathcal{Z}]}$$
$$\times \int_0^T d\tau_1 \cdots \int_0^T d\tau_N\, \mathrm{e}^{\sum_{i,j=1}^N \left(k_i \underset{\smile}{\Delta}_{ij} k_j - 2i\varepsilon_i\, {}^\bullet\underset{\smile}{\Delta}_{ij} k_j - \varepsilon_i\, {}^\bullet\underset{\smile}{\Delta}{}^\bullet{}_{ij}\, \varepsilon_j\right)}\, \mathrm{e}^{-Tb\left(\frac{\tan\mathcal{Z}}{\mathcal{Z}}\right)b}\Big|_{\varepsilon_1\varepsilon_2\cdots\varepsilon_N}.$$

- LSZ-amputated formulae obtained for plane waves: [Copinger, **JPE et al.**: 2311.14638 [hep-th]]

$$\mathcal{A}_N^{p'p} = (-ie)^N (2\pi)^3 \delta_{\perp,-}(\tilde{p}' + K - p) \int_{-\infty}^{\infty} \mathrm{d}x^+ e^{i(K+p'-p)_+ x^+} \int_{-\infty}^{\infty} \prod_{i=1}^N \mathrm{d}\tau_i\, \delta\Big(\sum_{j=1}^N \frac{\tau_j}{N}\Big)$$
$$\times\, \mathrm{e}^{-i\int_{-\infty}^0 [2\tilde{p}'\cdot a(\tau) - a^2(\tau)]\mathrm{d}\tau - i\int_0^\infty [2p'\cdot\delta a(\tau) - \delta a^2(\tau)]\mathrm{d}\tau - 2i\sum_{i=1}^N [\int_{-\infty}^{\tau_i} k_i\cdot a(\tau)\mathrm{d}\tau - i\varepsilon_i\cdot a(\tau_i)]}$$
$$\times\, e^{i(\tilde{p}'+p)\cdot g - i\sum_{i,j=1}^N \left(\frac{|\tau_i - \tau_j|}{2} k_i\cdot k_j - i\,\mathrm{sgn}(\tau_i-\tau_j)\varepsilon_i\cdot k_j + \delta(\tau_i-\tau_j)\varepsilon_i\cdot\varepsilon_j\right)}\Big|_{\mathrm{lin.}\ \varepsilon}$$

Note: Momentum and lightfront integrals $\Longrightarrow$ Schwinger proper time integrals!